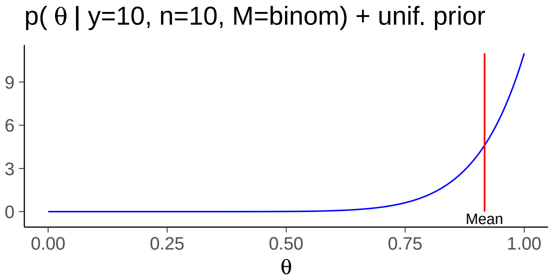
Benefits of integration and prior

Example: $n = 10, y = 10$ - uniform vs Beta(2,2) prior



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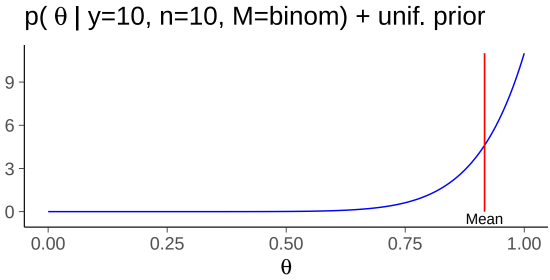
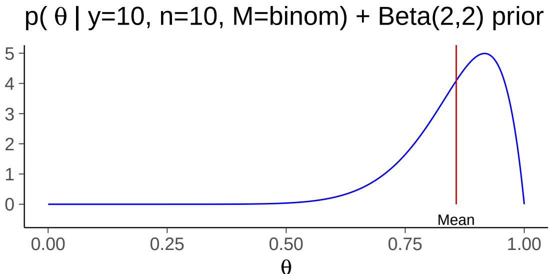
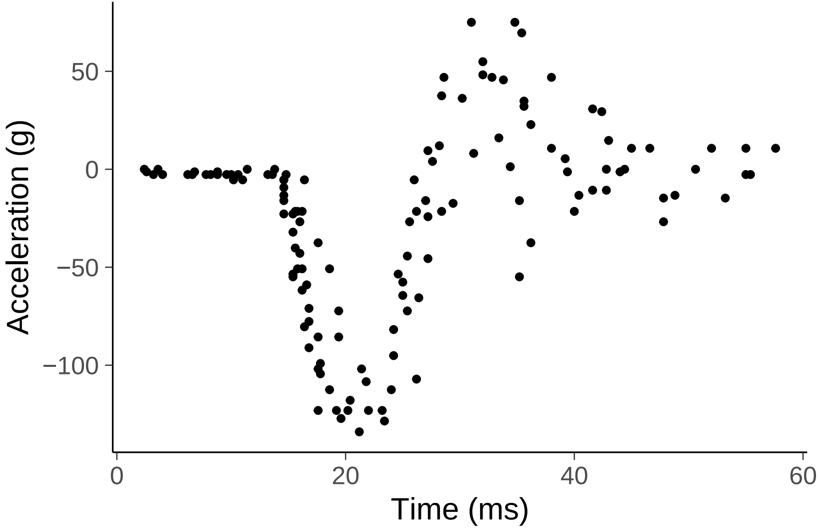




Benefits of integration: Gaussian process example

Simulated data of *g* forces in a motorcycle accident



Benefits of integration: Gaussian process example

- Gaussian process is a hierarchical normal model with multivariate normal population prior, where the off-diagonal covariances are determined by similarity measure (in the this example distance)

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\begin{aligned}
y &\sim \text{normal}(f(x), \sigma) \\
f &\sim GP(0, K_1) \\
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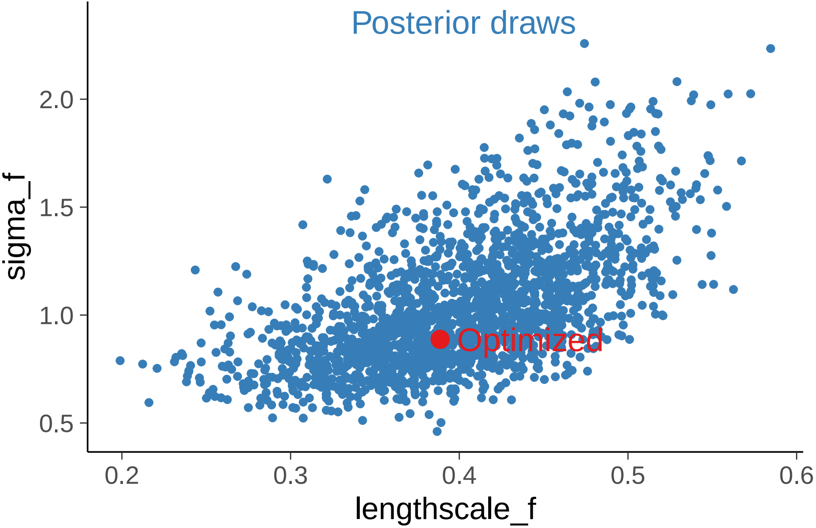
- K_1 is a covariance matrix, defined by a covariance function, that has parameters lengthscale l and magnitude σ_f
- Latent values f can be integrated out analytically, and the remaining marginal posterior is 3 dimensional

Benefits of integration: Gaussian process example

Plenty of data: the mode of th posterior can be representative



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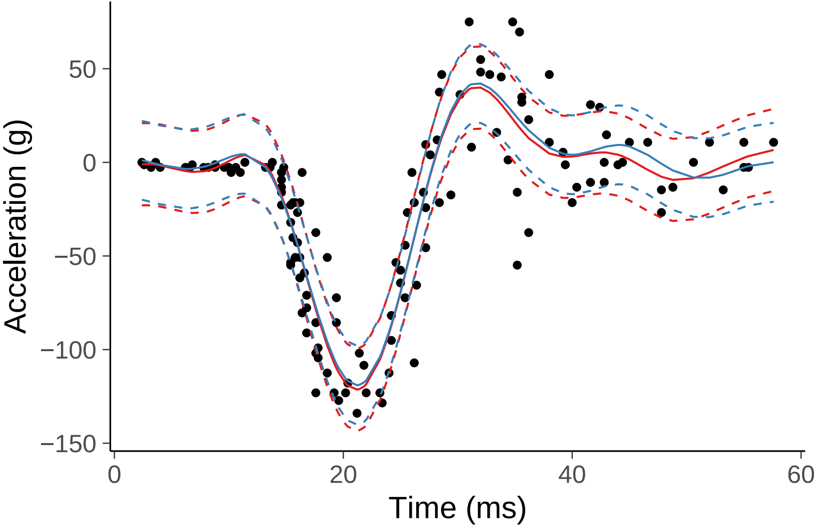
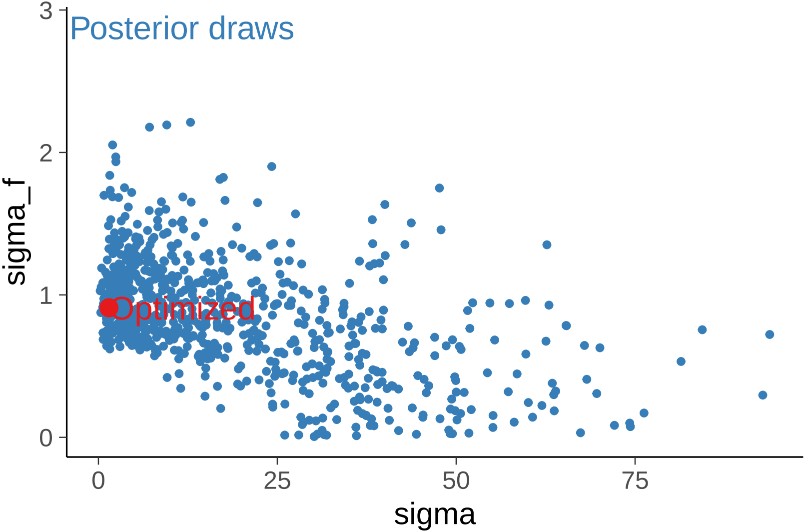


Benefits of integration: Gaussian process example

Small data: the mode of the posterior is not representative



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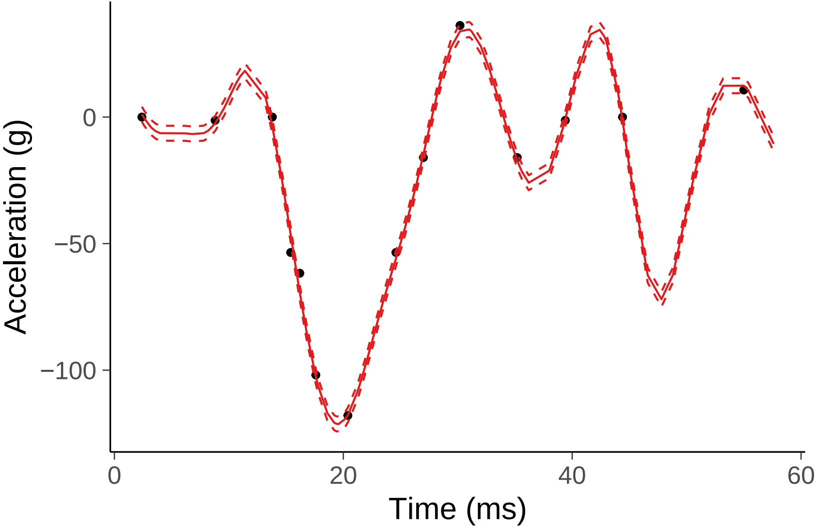

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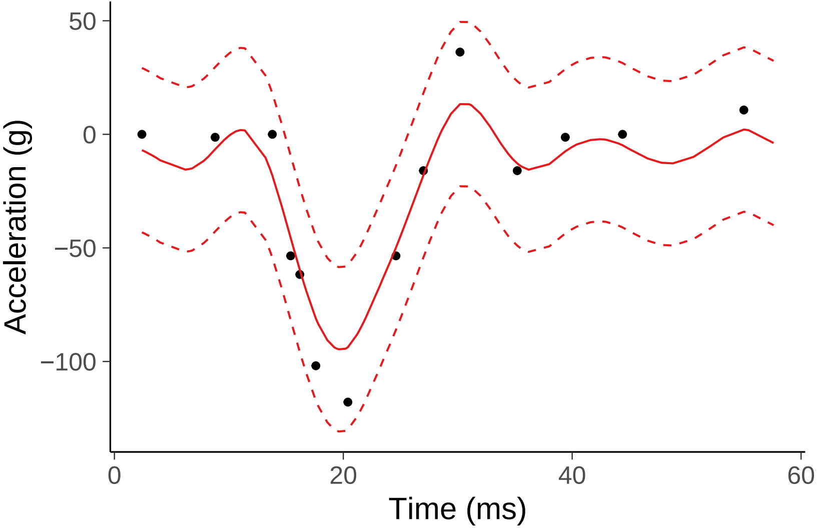

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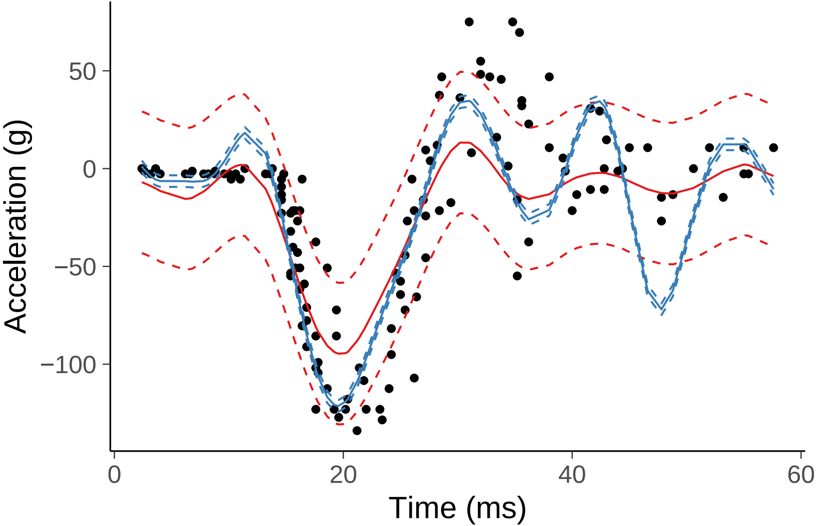

Benefits of integration: Gaussian process example

- More complex model, with time dependent residual variance $\exp(g(x))$

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Benefits of integration: Gaussian process example

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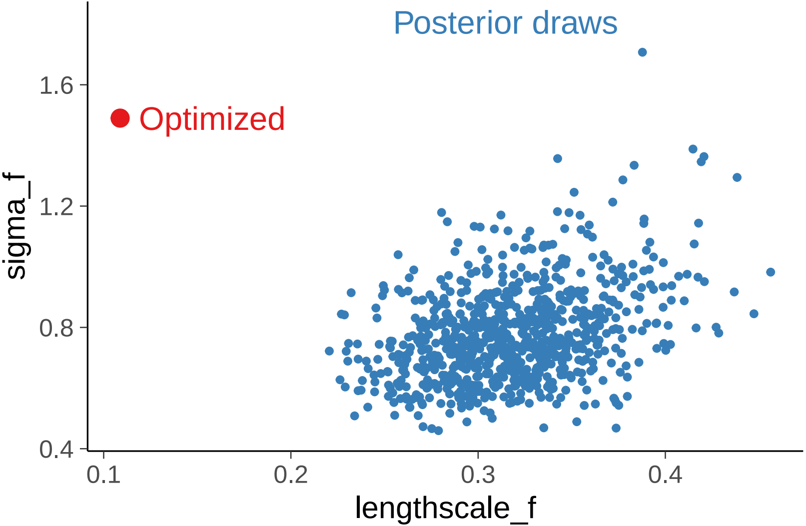
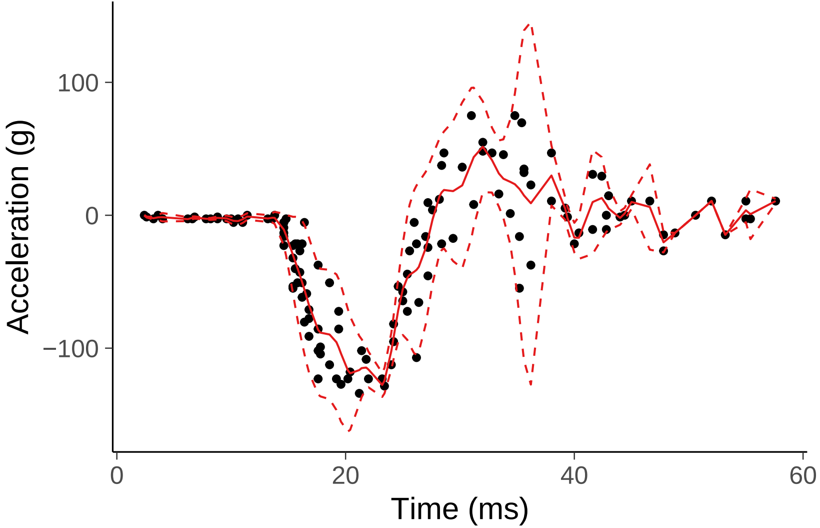


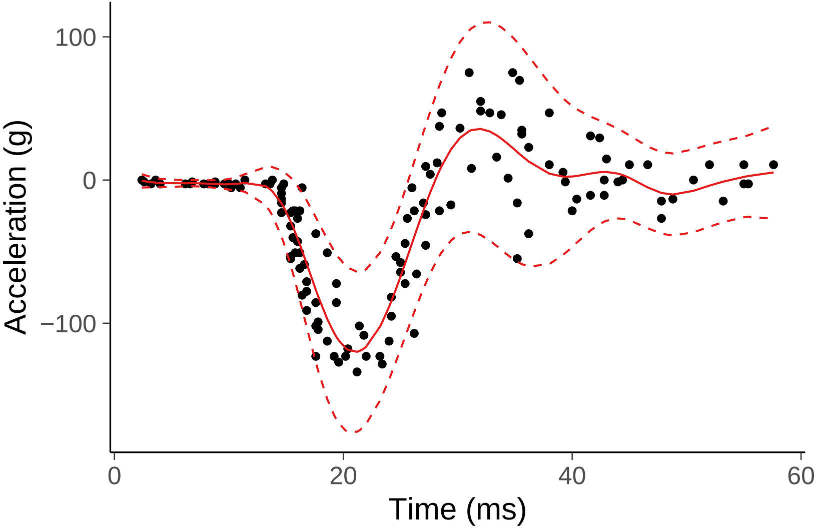
Benefits of integration: Gaussian process example

Optimization overfits



Benefits of integration: Gaussian process example

Integration works well



Benefits of better priors: logistic regression

$$y \sim \text{Bernoulli}\left(\text{logit}^{-1}(\alpha + \beta X)\right)$$

where α is a scalar intercept,
and β is a vector of coefficients

Model selection is needed to avoid overfitting?

logistic regression: 30 **completely irrelevant** variables, 100 observations

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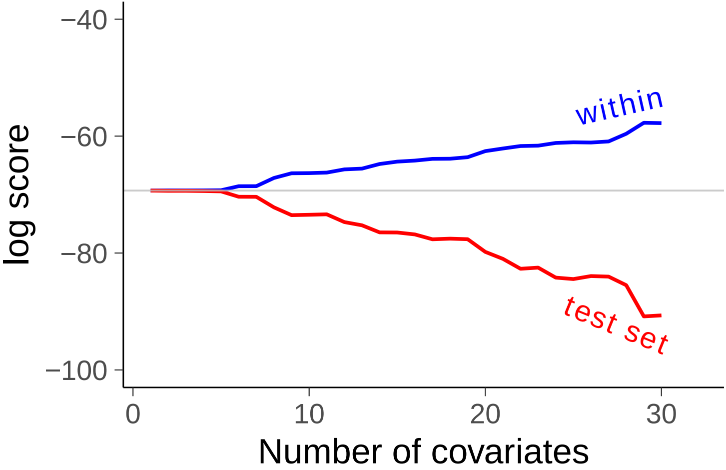
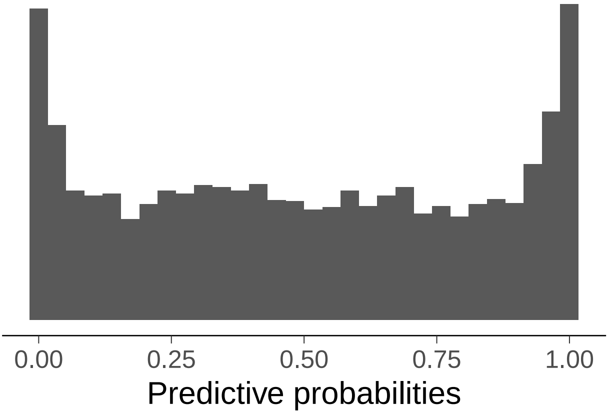


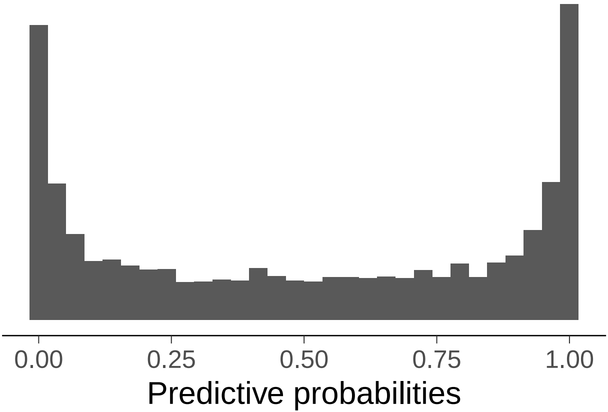
Prior on parameters vs predictions

N(0,3) prior on each coefficient
1 variable



Prior on parameters vs predictions

N(0,3) prior on each coefficient
2 variables



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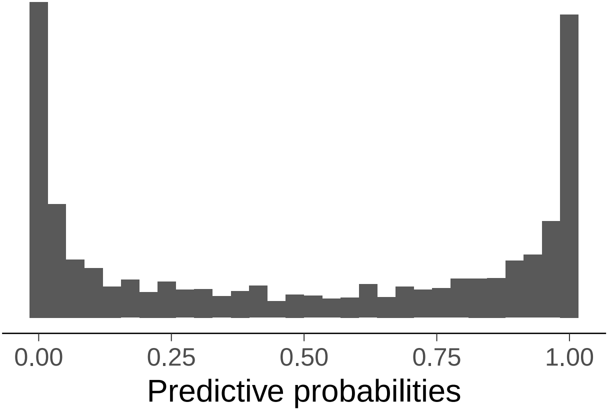

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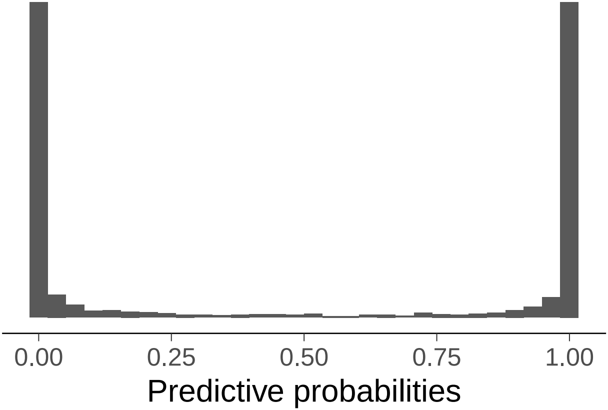

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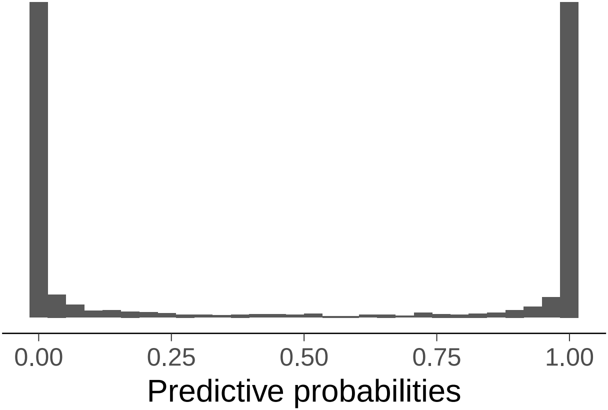
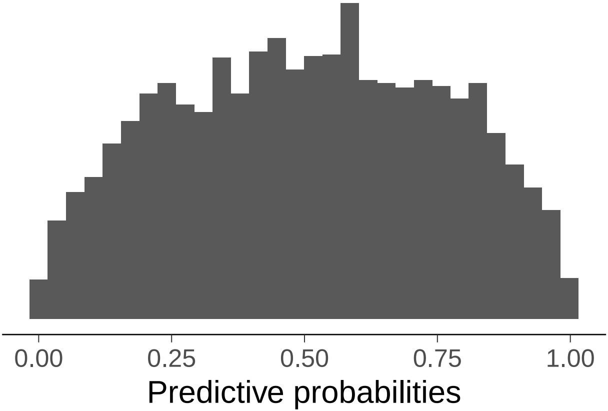


A weak prior on parameters can be a strong prior on predictions

Better priors

$N(0, \frac{1}{\sqrt{p}})$ prior on each coefficient
1 variable



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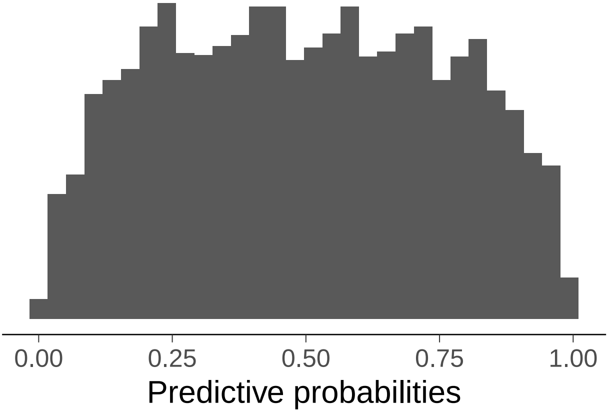

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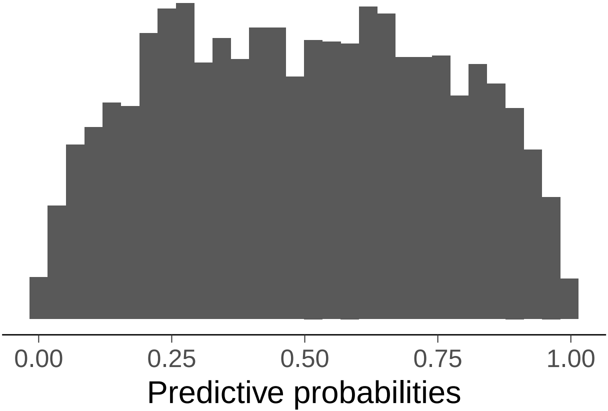

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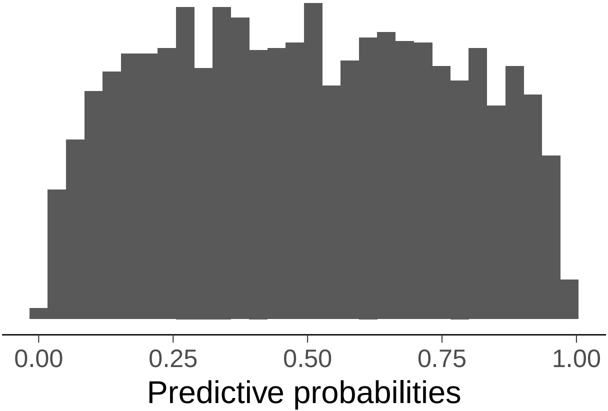

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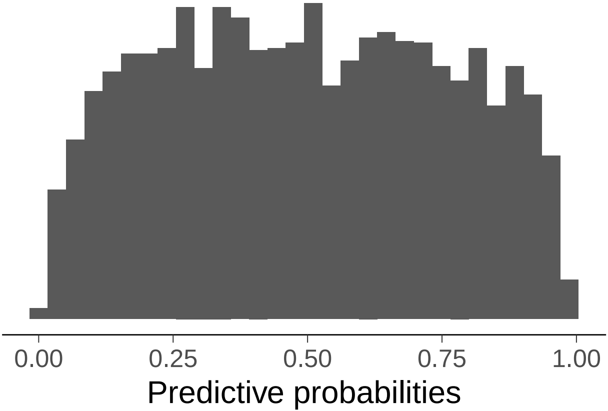
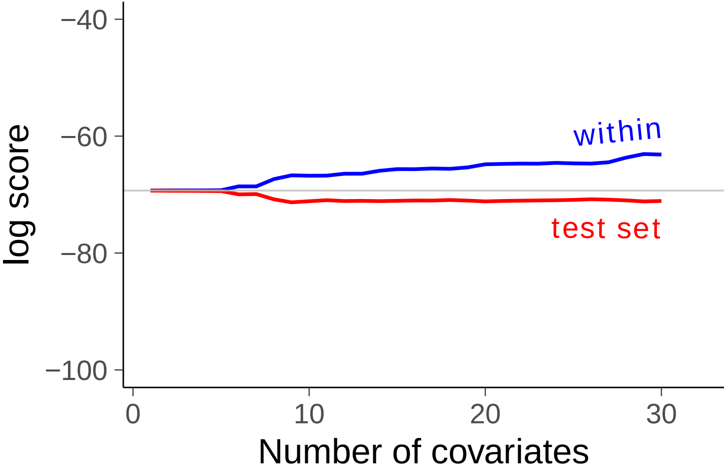


Prior on predictions (almost) fixed when the model gets bigger

Better priors, no overfitting

logistic regression: 30 **completely irrelevant** variables, 100 observations



Many weak effects, wide prior on parameters

logistic regression: 30 **weakly relevant** variables,
100 observations, wide prior

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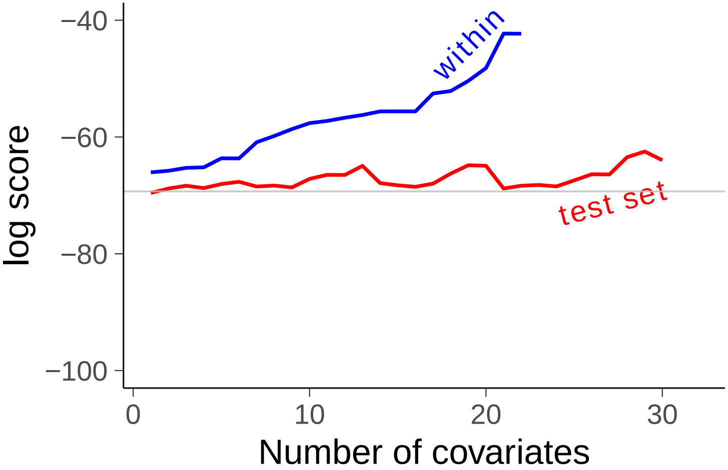

Many weak effects, better prior

logistic regression: 30 **weakly relevant** variables,
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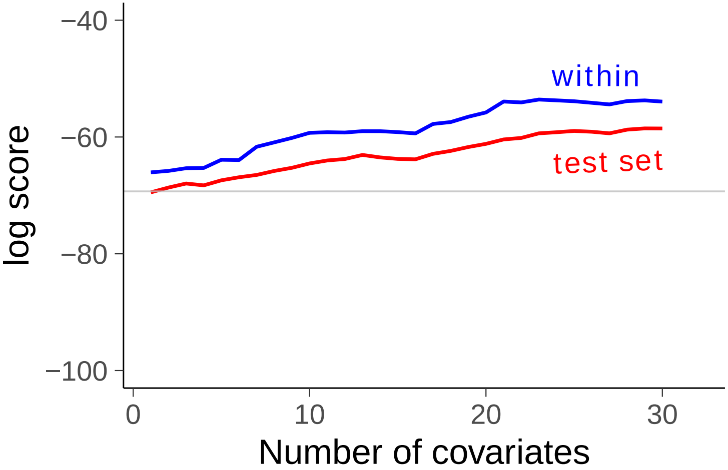

Correlating variables, wide prior on parameters

logistic regression: 30 **correlating relevant** variables,
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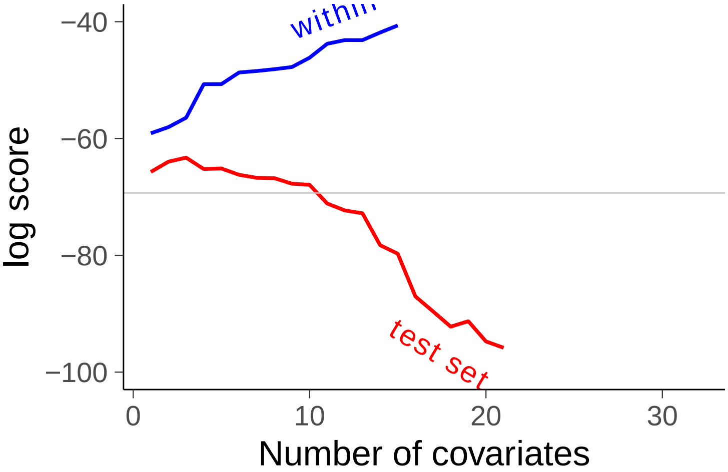

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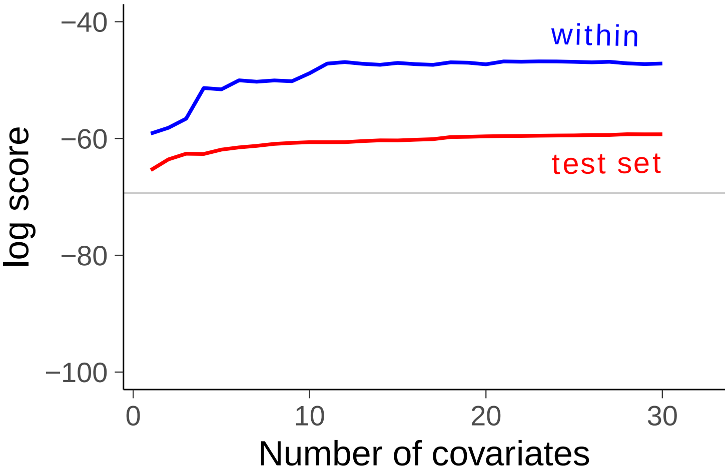

Benefits of integration and prior

- Integration helps to avoid overfitting
- Integration is not able to counter bad priors