Data Structures

Divide and Conquer

Andres Mendez-Vazquez

November 19, 2016

Outline

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Gauss and the Beginning

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He devised a way to multiply two imaginary numbers as

$$(a+bi)(c+di) = ac + (ad+bc)i - bd \tag{1}$$

By realizing that

$$bc + ad = (a+b)(c+d) - ac - bd \tag{2}$$

Thus minimizing the number of multiplications from four to three.

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He developed a trick to decrease the speed up of the multiplication of two numbers in binary representation.

Simply Realizing

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We can represent numbers x, y as

- $x = x_L \circ x_R = 2^{n/2} x_L + x_R$
- $y = y_L \circ y_R = 2^{n/2} y_L + y_R$

Thus, the multiplication can be found by using

$$xy = \left(2^{n/2} x_L + x_R\right)\left(2^{n/2} y_L + y_R\right) = 2^n x_L y_L + 2^{n/2}(x_L y_R + x_R y_L) + x_R y_R \quad (3)$$

However

if we use the Gauss's trick, we only need $x_L y_L$, $x_R y_R$, $(x_L + x_R)(y_L + y_R)$ to calculate the multiplication:

- $x_L y_R + x_R y_L = (x_L + x_R)(y_L + y_R) - x_L y_L - x_R y_R$

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Now, You have this...

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xy can be calculated using $x_L x_L$, $x_L y_R$, $x_R y_L$ and $x_R y_R$.

An each of this multiplications

$x_L x_L$, $x_L y_R$, $x_R y_L$ and $x_R y_R$ can be calculated in a similar way

Recursion

This is know as a Recursive Procedure!!!

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Gauss' Algorithm.

The Recursive Method

```
function multiplyGauss(x,y)
    Input: Positive integers x and y, in binary
    Output: Their product

    n = max(size of x, size of y)

    if n = 1 then return xy

    xL , xR = leftmost ceil(n/2) , rightmost floor(n/2)
                                              bits of x
    yL , yR = leftmost ceil(n/2) , rightmost floor(n/2)
                                              bits of y

    P1 = multiplyGauss(xL,yL)
    P2 = multiplyGauss(xR,yR)
    P3 = multiply(xL + xR, yL + yR)

    return 2^n*P1 + 2^(n/2)*(P3 - P1 - P2) + P2
```

Outline

Complexities

Old Multiplication

$$T(n) = 4T\left(\frac{n}{2}\right) + \text{ Some Work} \tag{4}$$

New Multiplication with Gauss Trick

$$T(n) = 3T\left(n\right) + \text{ Some Work} \tag{5}$$

We will prove that

- For old style multiplications $O\left(n^2\right)$.
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What is $T(n)$?

In recursive theory

$T(n)$ is the amount of work that a procedure is doing.

Something Notable

We can actually use this function to represent the recursive work of any procedure!!!

In the case of the old multiplication

$$x_{LyL}\ x_{LyR}\ x_{RyL}\ x_{RyR} \tag{6}$$

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Four recursive multiplications

Clearly we are using half of the input!!!

Then

Each of this multiplications costs $n/2$

And as you go deeper in the recursion, we have the following sequence

$$n \longrightarrow \frac{n}{2} \longrightarrow \frac{n}{4} \longrightarrow \frac{n}{8} \longrightarrow \frac{n}{16} \longrightarrow \ldots \longrightarrow 1$$

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In the case of these algorithms

Some Work $=$ Summations that take constant time or $O(1)$.

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In a really un-clever way!!!

Or we can go and design something better

Thus, improving speedup!!!

The difference between

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- Or a crappy job...

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Recursion is the base of Divide and Conquer

This is the natural way we do many things

We always attack smaller versions first of the large one!!!

Stephen Cole Kleene

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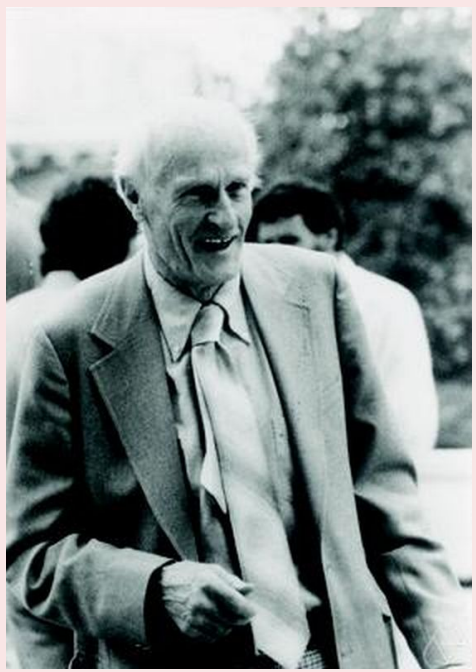

Kleene and Company

Some facts about him

- Stephen Cole Kleene (January 5, 1909 – January 25, 1994) was an American mathematician.
- One of the students of Alonzo Church!!!
  - Church is best known for the lambda calculus, Church–Turing thesis and proving the undecidability of the use of an algorithm to say Yes(Valid) or No(No Valid) to a first order logic statement on a FOL System (Proposed by David Hilbert).

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- This theory subsequently helped to provide the foundations of theoretical computer science.

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Basics

A clear application of Divide and Conquer

First, we explain the Merging

Merging

The key to Merge Sort is merging two sorted lists into one, such that if you have two lists:

$$X = (x_1 \leq x_2 \leq ... \leq x_m) \text{ and } Y = (y_1 \leq y_2 \leq ... \leq y_n)$$

into a resulting list $Z = (z_1 \leq z_2 \leq ... \leq z_{m+n})$

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Outline

Example: Merging

We have the following

X:

3	10	11	54

Y:

1	5	25	75

Result:

Example: Merging

We have the following

X:	3	10	11	54

Y:		5	25	75

Result:	1							

Example: Merging

We have the following

X:

	10	11	54

Y:

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Result:

1	3						

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Y:

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Result:

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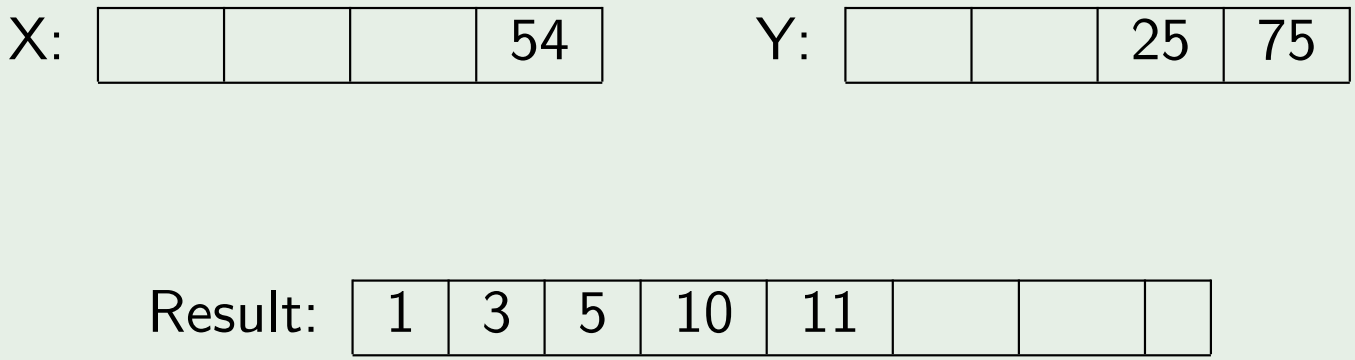

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X: | | | | |

Y: | | | | 75 |

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Outline

Divide and Conquer

Thus, basic step

Merging a two lists of one element each is the same as sorting them.

Strategy

Merge sort divides up an unsorted list until the above condition is met and then sorts the divided parts back together in pairs.

The Recursion

Specifically this can be done by recursively dividing the unsorted list in half, merge sorting the right side then the left side and then merging the right and left back together.

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Mergesort algorithm.

Pseudo-Code

MergeSort(List)

1. mid = len(alist)//2
2. lefthalf = alist[:mid]
3. righthalf = alist[mid:]
4. MergeSort(lefthalf)
5. MergeSort(righthalf)
6. Merging(List)

Mergesort algorithm.

The merge method

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    Input: Positive integers x and y, in binary
    Output: Their product

    n = max(size of x, size of y)

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    P1 = multiplyGauss(xL,yL)
    P2 = multiplyGauss(xR,yR)
    P3 = multiply(xL + xR, yL + yR)

    return 2^n*P1 + 2^(n/2)*(P3 - P1 - P2) + P2
```

Outline

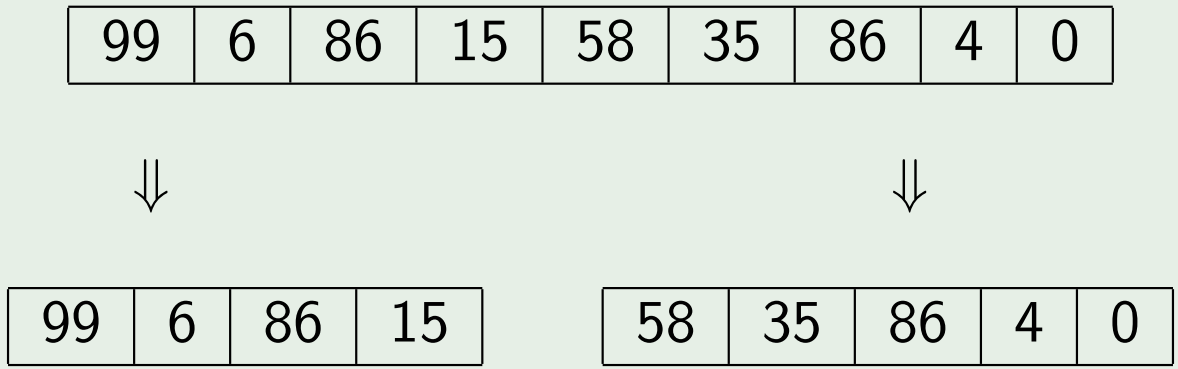
Example

The array

99	6	86	15	58	35	86	4	0

Example

First Split



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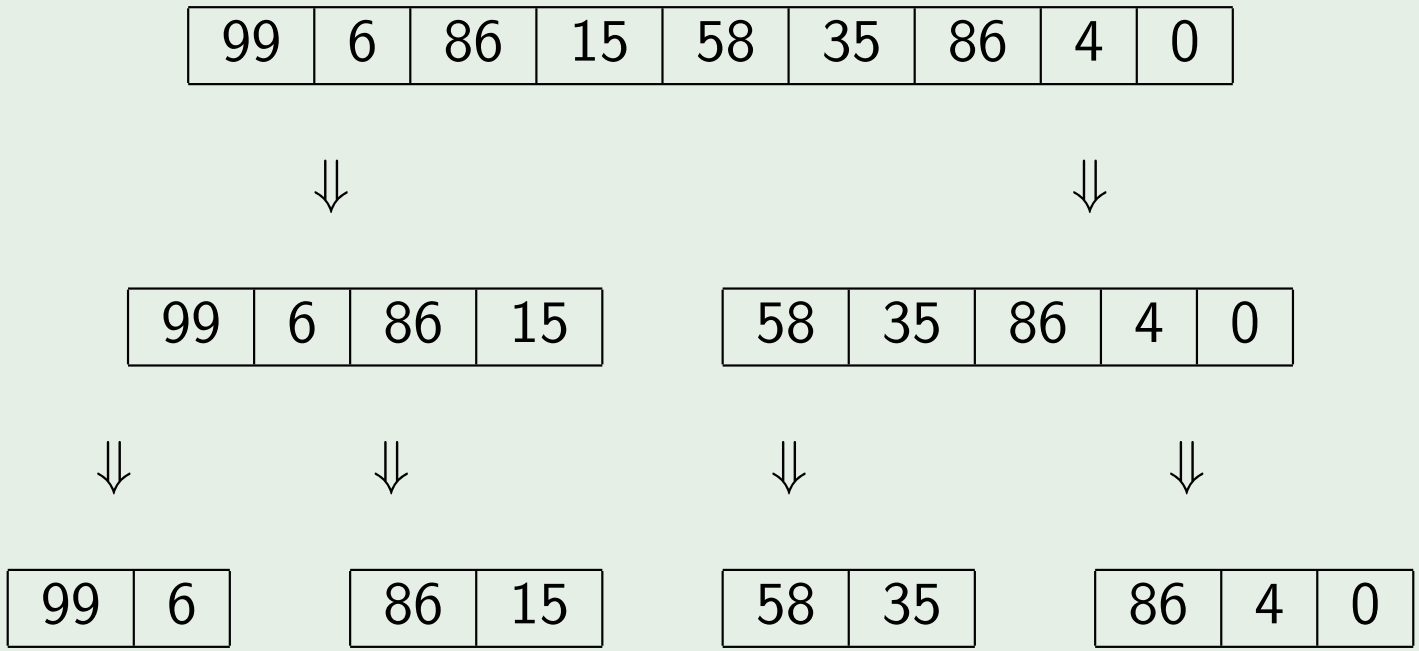
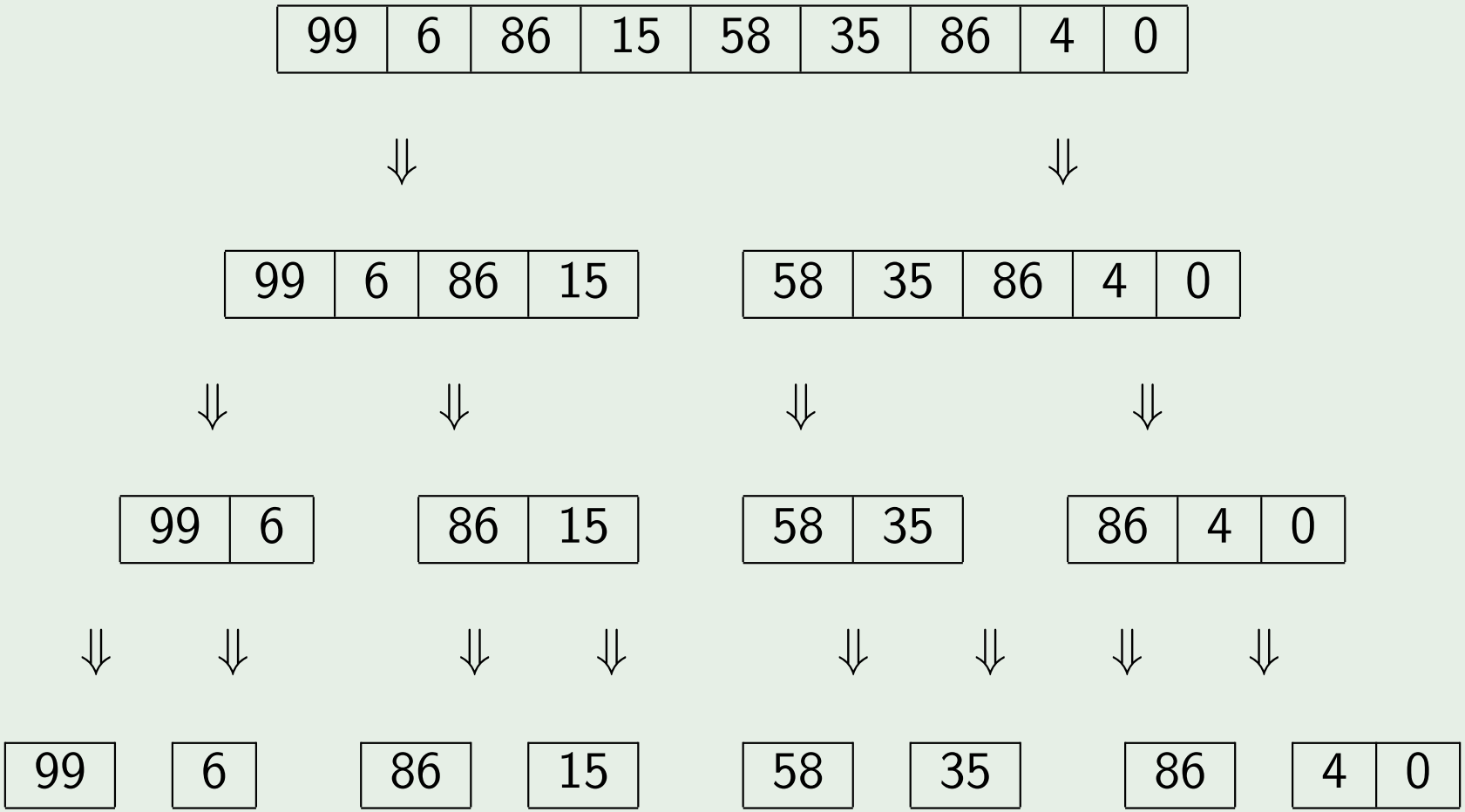
Second Split



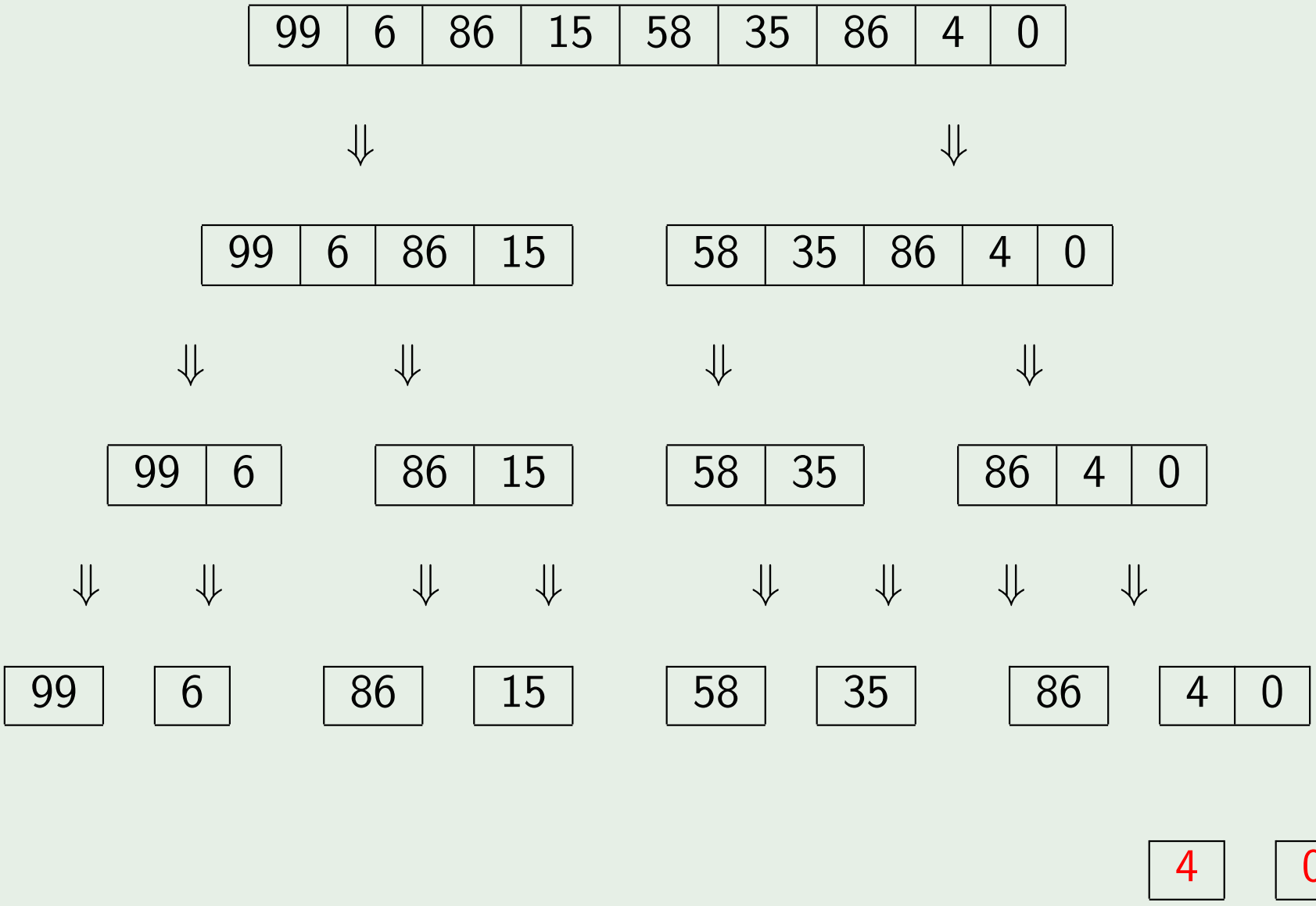
Example

Third Split



Example

Fourth Split



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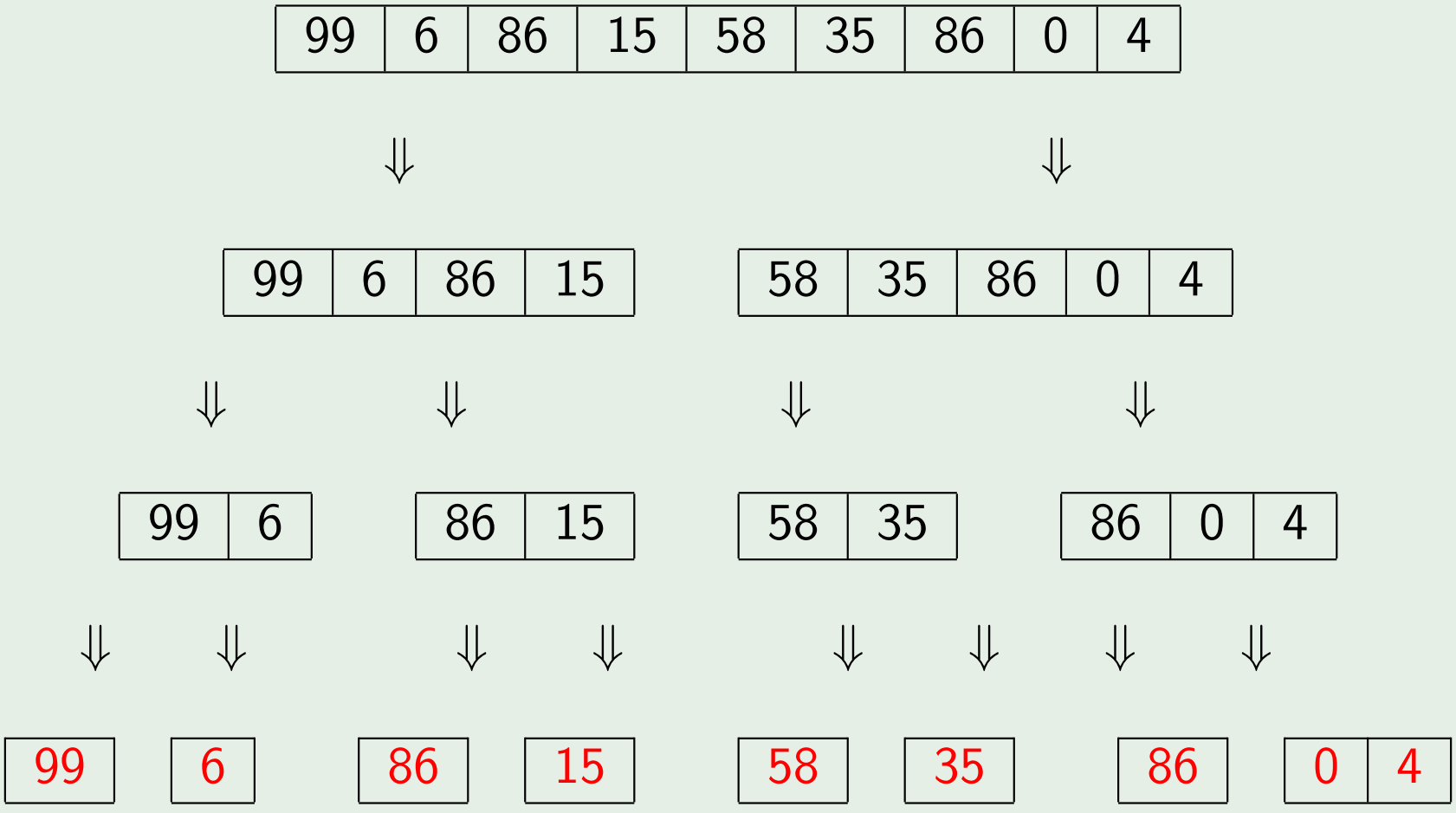

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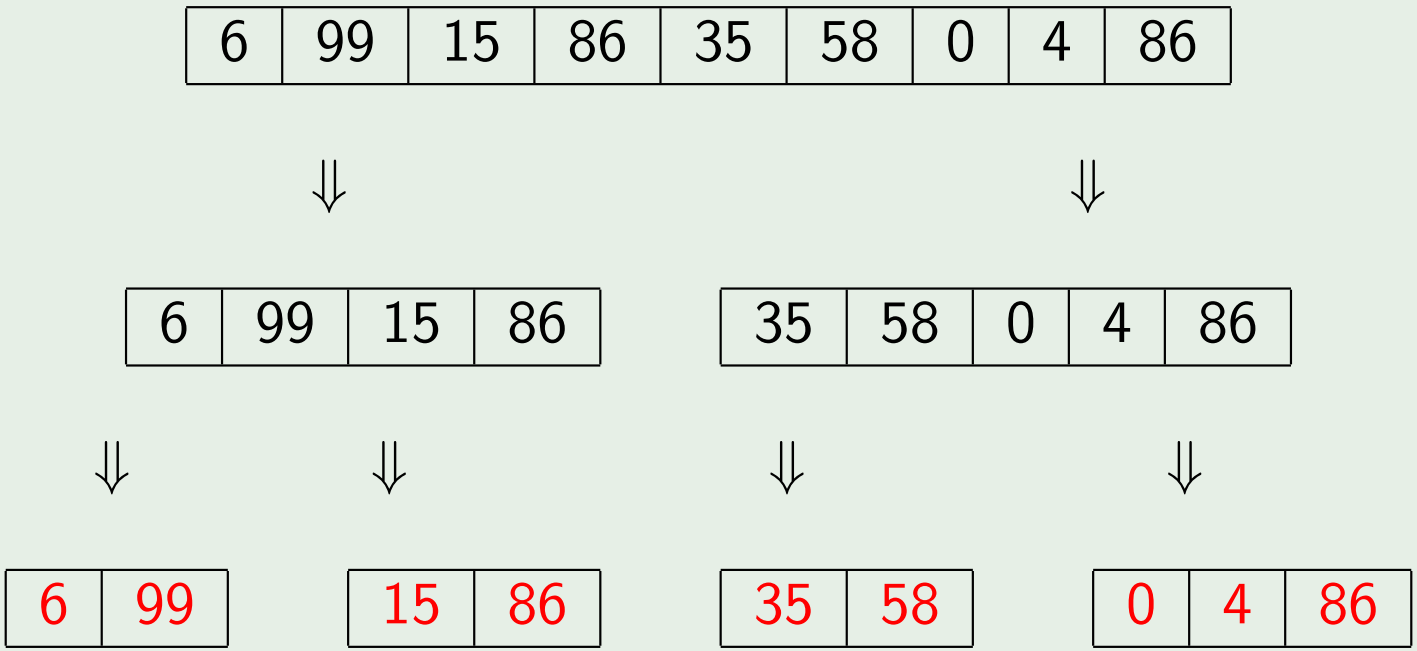

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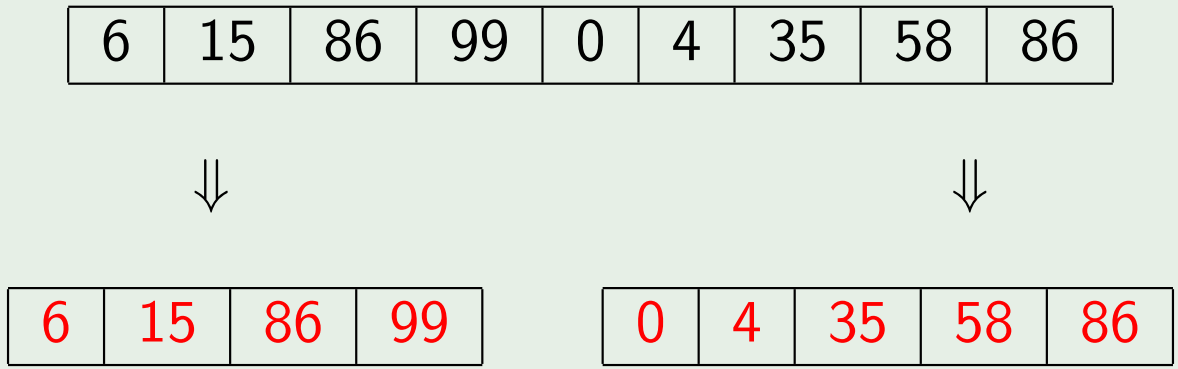

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Merging

0	4	6	15	35	58	86	86	99

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Complexity

We have the following

The Double Memory Merge Sort runs $O(N \log N)$ for all cases? (N being the number of numbers to be sorted) because of its Divide and Conquer approach.

Actually, we can prove the following recursion

$$T(N) = 2T\left(\frac{N}{2}\right) + N = O(N \log N)$$

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Who invented Quicksort?

Imagine this

The quicksort algorithm was developed in 1960 by Tony Hoare (He has a postgraduate certificate in Statistics) while in the Soviet Union, as a visiting student at Moscow State University.

Why?

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To do

He developed the algorithm in order to sort the words to be translated.

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Definitions

Divide

Partition (rearrange) the array $A[p, ..., r]$ into two (possibly empty) sub-arrays $A[p, ..., q-1]$ and $A[q+1, ..., r]$ such that each element of $A[p, ..., q-1]$ is less than or equal to $A[q]$, which is, in turn, less than or equal to each element of $A[q+1, ..., r]$. Compute the index q as part of this partitioning procedure.

Conquer

Sort the two sub-arrays $A[p, ..., q-1]$ and $A[q+1, ..., r]$ by recursive calls to quicksort.

Combine

Since the sub-arrays are sorted in place, no work is needed to combine them: the entire array $A[p, ..., r]$ is now sorted.

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Quicksort Algorithm

Quicksort Algorithm

Quicksort(A, p, r)

1. if $p < r$
2. $\quad q = Partition\,(A, p, r)$
3. $\quad$ **Quicksort**$(A, p, q-1)$
4. $\quad$ **Quicksort**$(A, q+1, r)$

Partition Algorithm

Quicksort Partition

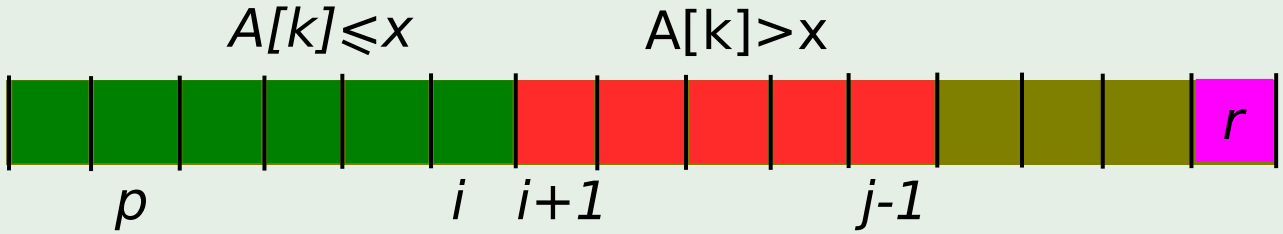
Partition(A, p, r)

1. $x = A[r]$
2. $i = p - 1$
3. for $j = p$ to $r - 1$
4. if $A[j] \leq x$
5. $i = i + 1$
6. exchange $A[i]$ with $A[j]$
7. exchange $A[i + 1]$ with $A[r]$
8. return $i + 1$

Quicksort: What is the Invariance?

Loop Invariance

1. **If** $p \leq k \leq$ **i, then** $A[k] \leq x$**.**
2. **If** $i+1 \leq k \leq j-1$**, then** $A[k] > x$**.**
3. **If** $k = r$ **, then** $A[k] = x$**.**
4. **UNKNOWN**



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Complexity

It is possible to prove that the expected complexity

$$O(N \log N)$$

However, the worst case is

$$O\left(N^2\right)$$

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Example

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99	6	86	15	58	35	86	4	10

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6	99	86	15	58	35	86	4	10

Example

Lets Sort the following sequence

6	4	86	15	58	35	86	99	10

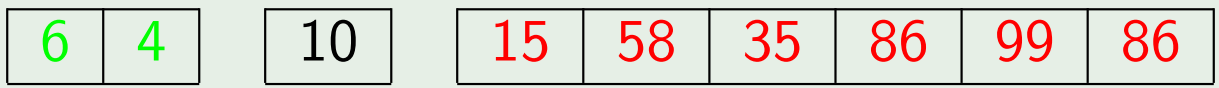
Example

Exchange pivot with the element at $i + 1$

6	4	10	15	58	35	86	99	86

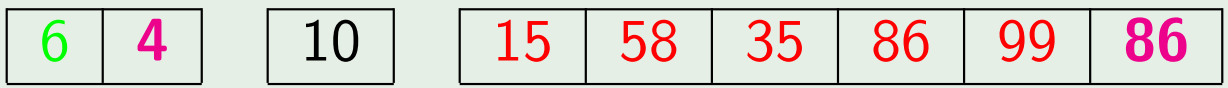
Example

Call the two recursions



Example

Thus, we have that



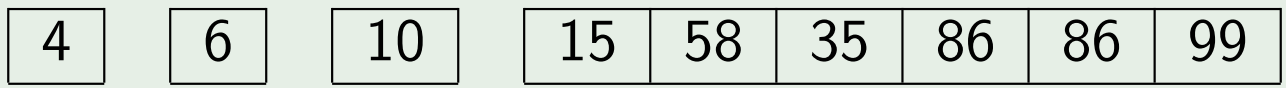
Example

Thus, we have that a simple exchange for the left part

4	6	10	15	58	35	86	99	**86**

Example

The right part after exchanging the pivot



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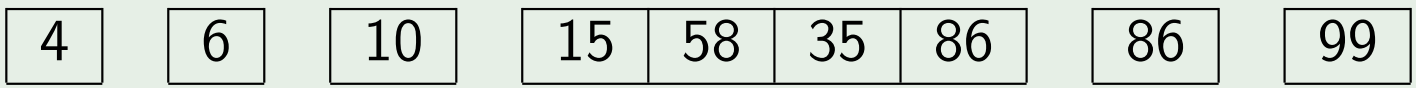
Split



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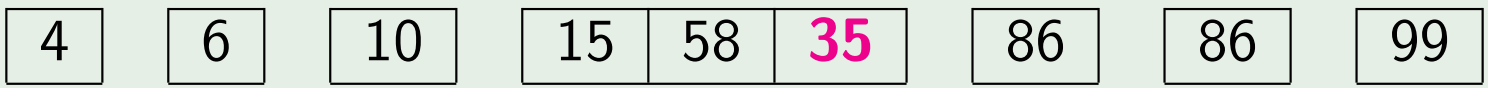
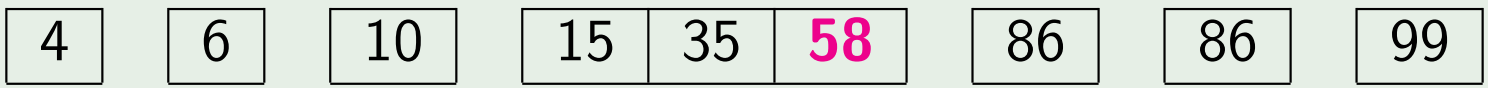


Example

Exchange



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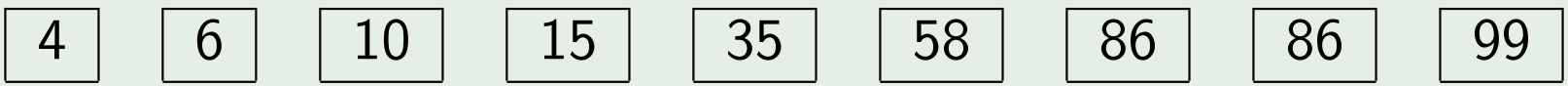

Example

Finally

4	6	10	15	35	58	86	86	99

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Choosing the Pivot

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For example, with input 8, 1, 4, 9, 6, 3, 5, 2, 7, 0

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