

# Welcome!

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An overview of the course

Daniel Anderson

Week 1

# Agenda

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- Getting on the same page
- Syllabus
- A little bit of R



# whoami

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- Research Assistant  
Professor: Behavioral  
Research and Teaching
- Dad (two daughters: 8  
and 6)
- Pronouns: he/him/his
- Primary areas of interest:  
    ♥♥R♥♥,  
    computational research,  
    achievement gaps,  
    systemic inequities, and  
    variance between  
    educational institutions





# whoisyou?

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- Introduce yourself
- Why are you here? I know you probs took HLM 1, but give me a little more than that.
- What pronouns would you like us to use for you for this class?
- What is one fun thing you have done not related to academic work recently?
- Do you have a furry friend you want to introduce?

# A few class policies

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- Be kind
- Be understanding and have patience, with others and yourself
- Help others whenever possible

Truly the most important part of this class. Important not just in terms of decency, but also in your learning, and most importantly, for equity.

# A bit more specific

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Normally I would have information here about welcoming kids into class.

Because we're virtual, that part is both easier and harder.

If you need to not attend class, or a portion of class, for any reason, that is fine.

Ideally you would let me know ahead of time. But we're in the middle of a pandemic and life is cray. Please try to contact me beforehand. If this isn't possible, please check in with me after.

# Last intro thing

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- I'm here for you
- We won't have specific office hours, but know I'm always willing to meet
- This course, like all in the sequence, can be difficult. Don't suffer in silence. Don't do this alone.

# Syllabus

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# Course Website(s)

website

repo



## EDLD 629

*Multilevel modeling II*

Welcome to the second course on multilevel modeling taught through the University of Oregon. This course will be taught through R, a free and open-source statistical computing environment. We will cover the theory and practice of fitting various types of multilevel models, including how to specify the variance-covariance matrices of multilevel models, how predictions are made from multilevel models, and how to interpret the results. Both frequentist and Bayesian approaches to estimation will be covered. See the [schedule](#) for a complete listing of topics.

# Materials

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- Nearly everything will be distributed through the website.  
Go there now! Get the slides!
- If you feel comfortable, it would be best if you could clone the repo , then just pull each week for the most recent changes.
- We'll use Canvas for grading and announcements. I'll post a few things there too, but not much.
- The slides will always be available through the website, and you can click the button in the footer to download them as a PDF.

# A method by many names

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All of these refer to the **same thing!**

- Hierarchical linear modeling
- Multilevel modeling
- Mixed effects modeling

And then there's a bunch of similar variants!

- Hierarchical linear regression
- Mixed effects regression

etc.

# The vocabulary I'll use

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This class is technically called

**Hierarchical linear modeling II.**

I don't like that term



# Multilevel modeling

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Instead, I'll use the term multilevel modeling

- Keeps it distinct from hierarchical regression (not the same thing at all)
- Provides a useful heuristic
  - Like many heuristics, can also be a bit limiting

# Course Learning objectives

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- Fit and interpret multilevel models in R using both frequentist and Bayesian approaches
- Fit and interpret multilevel logistic regression models
- Visualize the fitted model predictions
- Understand the assumptions of multilevel models and be able to simulate data assuming the data were generated from a multilevel model

# Course Learning objectives

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- Understand various variance–covariance specification of the random effects and how this relates to overfitting
- Understand and be able to translate equations between the Raudenbush & Bryk notation and the Gelman & Hill notation
- Be able to model growth flexibly within a multilevel framework, including discontinuous and non–linear trends

# A bit of a caveat

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The previous two slides outline a lot of objectives.

My goal is to get you exposure to all of these concepts and help you dive deeper if you so choose

Not all learning objectives will be covered equally – this is an advanced class, you get to decide where you'd like to focus

# Weekly learning objectives

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Provide you a frame for what you should be working to learn for that specific week.

## This week's objectives

- Understand the requirements of the course
- Understand the requirements of the final project
- Be ready to go with R and understand the basics of fitting a multilevel model

# Textbooks

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I will only require reading from textbooks that you can access for free.

# Mixed models with R

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## Mixed Models with R

### Getting started with random effects

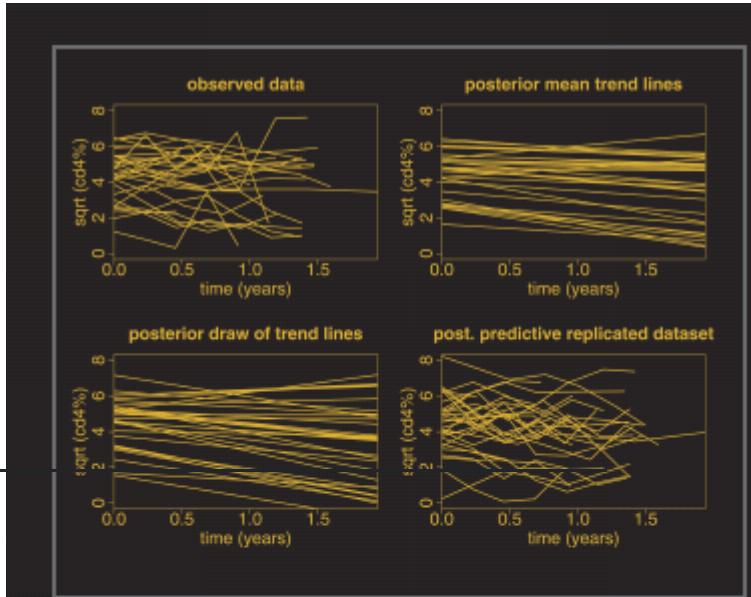
Michael Clark

[m-clark.github.io](https://m-clark.github.io)

# Gelman and hill

Not freely available but I will

provide select readings.



## Data Analysis Using Regression and Multilevel/Hierarchical Models

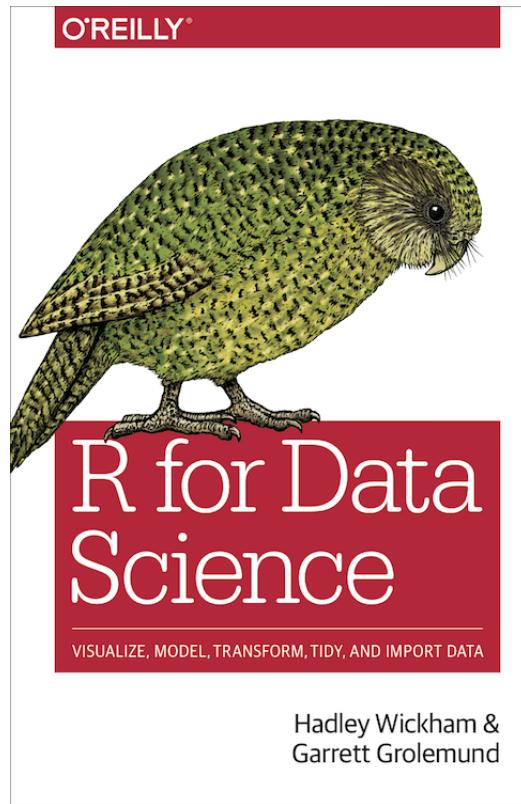
ANDREW GELMAN  
JENNIFER HILL

# Other books (also free)

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Bryan

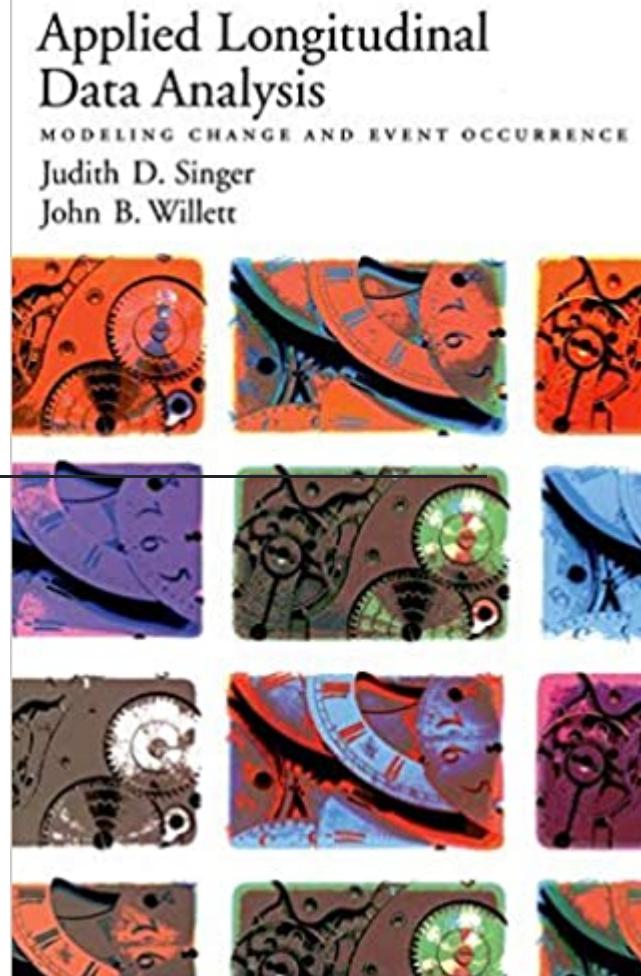


Wickham & Grolemund

# One more

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Not free, but recommended



# Assignments

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100 points possible

# Participation (10%)

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Each week, I'll ask you to follow along for specific parts of the slides.

You might also have notes you take on specific topics.

Please turn in your script and/or notes to canvas each week (1 point each).

Note – this is expected whether you attend "live" or watch the recording later.

# Homework

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I am expecting we will have time to devote to the homework assignments in class.

You are welcome to work in groups.

Please use RMarkdown.

15 points each (45 points total; 45%)

<b>Lab</b>	<b>Date Assigned</b>	<b>Date Due</b>	<b>Topic</b>
1	Fri, April 09	Fri, April 23	Basic multilevel modeling with R
2	Fri, April 23	Fri, May 07	Growth models and variance–covariance matrices
3	Fri, May 07	Fri, May 21	Bayesian estimation & multilevel logistic regression models

# Final Project

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45 points total

Three parts

- Proposal/outline (5 points): Due 4/30/21
- Script (20 points): Due 6/9/21
- Writeup (20 points): Due 6/9/21

# Proposal

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## Four components

- Data source identified, which must be shareable with me
- Research Question(s) identified (no more than three), which must be addressable through a multilevel model
- A description of any data processing that must occur before you can fit the given model
- Your current status with the project (e.g., challenges you are facing, what steps still need to occur, feasibility of finishing, etc.)

# Script

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See the assignments page for full details

- Reproducibility: 2 points
- Exploratory and descriptive analyses: 3 points
- Analysis: 10 points
  - Model should map directly to the research question and be properly specified.
  - Judgments are clear and justified from the evidence.
- Plots: 5 points
  - One plot **from your model** that communicates the coefficients, model predictions, or other features.

# Writeup

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See the assignments page for full details

**Must not** exceed five pages, double-spaced, w/standard margins and font size.

- Introduction: 2 points
- Research Question(s): 2 points
- Method: 5 points
- Results: 5 points
- Discussion: 3 points
- General style: 3 points

# Grading

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# Points

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100 points total

- Participation (10 points total; one point per class)
- 3 Homework assignments at 15 points each (45 points)
- Final project proposal/outline (5 points)
- Final project script (20 points)
- Final project writeup (20 points)

# Grading

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Lower percent	Lower point range	Grade	Upper point range	Upper percent
0.97	(97 pts)	A+		
0.93	(93 pts)	A	(97 pts)	0.97
0.90	(90 pts)	A-	(93 pts)	0.93
0.87	(87 pts)	B+	(90 pts)	0.90
0.83	(83 pts)	B	(87 pts)	0.87
0.80	(80 pts)	B-	(83 pts)	0.83
0.77	(77 pts)	C+	(80 pts)	0.80
0.73	(73 pts)	C	(77 pts)	0.77
0.70	(70 pts)	C-	(73 pts)	0.73
		F	(69 pts < )	0.70

# A note on deadlines

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Life be cray – don't worry about any deadlines except the final project, and make sure you get all assignments in by the final.

Do worry about the final project deadline – it can't be moved. I'd suggest starting on it ASAP. You can turn things like your proposal in anytime.



# A few final notes

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- This is the first time I've taught this course
- I don't have a great feel for how long things will take
- Please be patient with me if we end up needing to make some changes to the course schedule
- I do plan on this being highly applied, with you running lots of models and playing with data immediately

# My approach

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The first basically half of the quarter will be on

- Moving to R
- Deepening your current understanding This means that for many of you, there probably won't be a lot of completely "new" content at first (e.g., Bayesian approaches)

That will come, but I want to focus on depth of understanding first

# Differentiation

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- I know most of you, but not all
- Please understand that there is a wide range of comfort levels in this class both with R and multilevel model
  - Occasionally it may feel slow, or way too fast
  - Please be patient in either case, but **do** speak up if you are feeling lost – formative feedback is much more difficult through zoom and I may not know I'm losing you if you don't speak up.

# Final note

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This is a long class – we'll try to take at least one 5-minute break each time

q u e s t i o n s

My

assumptions

about you

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# I assume you

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- Have a strong foundational knowledge of multiple regression
  - Example: You can interpret two-way interactions without twisting your brain too much
- Understand how multilevel modeling extends the basic multiple regression model
- Can correctly interpret two- and three-level models
- Have at least a basic understanding of how multilevel modeling can be used to estimate change over time

# Pop quiz

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In groups, discuss each of the following, then we'll discuss as a class:

- When is multilevel modeling preferred. Is it always better than multiple regression?
- What does it mean for something to be **nested**. Can multilevel modeling be used if the data are not nested?
- Describe the differences between multiple regression and a model with
  - Random intercepts
  - Random slopes
  - Random intercepts and slopes
- Why does multilevel modeling allow you to investigate changes over time? What specifically makes it different?

07:00

# Pop Quiz #2

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Describe the model below in plain words.

$$\text{math}_{ijk} = \pi_{0jk} + \pi_{1jk}(\text{week}) + e_{ijk}$$

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}(\text{FRL}) + r_{0jk}$$

$$\pi_{1jk} = \beta_{10k} + \beta_{11k}(\text{FRL}) + r_{1jk}$$

$$\beta_{00k} = \gamma_{000} + \gamma_{001}(\text{Size}) + u_{00k}$$

$$\beta_{01k} = \gamma_{010} + \gamma_{011}(\text{Size}) + u_{01k}$$

$$\beta_{10k} = \gamma_{100} + \gamma_{101}(\text{Size}) + u_{10k}$$

$$\beta_{11k} = \gamma_{110}$$

where  $i$ ,  $j$ , and  $k$  index time points, students, and schools;  
 $\text{FRL}$  is free/reduced lunch eligibility, and  $\text{Size}$  is the school size

05:00

# Fixed effects

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The model on the previous slide estimates:

- the relation between time and students' math scores in weekly units,  $\gamma_{1000}$ 
  - Each week students gained, on average, X math points
- the relation between FRL eligibility and students' initial math scores,  $\gamma_{010}$ , and their weekly rate of growth,  $\gamma_{110}$  (cross-level interaction)
- the relation between school size and students' initial math achievement,  $\gamma_{001}$ , rate of growth,  $\gamma_{101}$ , and the school-size relation with student-level FRL eligibility on initial achievement,  $\gamma_{011}$  (cross-level interaction)

# Random effects

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It also estimates:

- Between-student variability in initial math scores,  $r_{0jk}$  and rate of weekly change,  $r_{1jk}$ , as well as their covariance (not shown)
- Between-school variability in initial math scores,  $u_{00k}$ , rate of weekly change,  $u_{10k}$ , and the relation between student FRL eligibility and initial achievement,  $u_{10k}$ , as well as all covariances (not shown)

# Back to assumptions

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Related to R: I assume you

- Have a basic understanding of the R package ecosystem (how to find, install, load, and learn about them)
- Can read "flat" (i.e., rectangular) datasets into R
- Know and use RStudio Projects & the `{here}` package
  - See [Jenny Bryan's blog post](#) for why.

- Can perform basic data wrangling and transformations in R, using the tidyverse
  - Leverage appropriate functions for introductory data science tasks (pipeline)
  - "clean up" the dataset using scripts and reproducible workflows
- Use R Markdown to create at least basic reproducible and dynamic documents



# Let's play with R

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- Open RStudio
- Install any of the packages below you don't already have installed
- General recommendation when installing packages –  
Type 1 at prompt to update old packages – select "no" if asked to compile from source

```
install.package("tidyverse")
install.packages("lme4")
install.packages("equatiomatic")
remotes::install_github("easystats/easystats")

# Or for the latest and greatest features for equatiomatic
remotes::install_github("datalorax/equatiomatic")
```

Load **equatiomatic** and you should have access to the **hsb** dataset

```
library(equatiomatic)
head(hsb)
```

```
##      sch.id    math  size sector meanses minority female     ses
## 1     1224  5.876  842        0 -0.428         0       1 -1.528
## 2     1224 19.708  842        0 -0.428         0       1 -0.588
## 3     1224 20.349  842        0 -0.428         0       0 -0.528
## 4     1224  8.781  842        0 -0.428         0       0 -0.668
## 5     1224 17.898  842        0 -0.428         0       0 -0.158
## 6     1224  4.583  842        0 -0.428         0       0  0.022
```

# HSB

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- I'm assuming you're all familiar with it. Perhaps painfully so.
- We're just doing some basic R stuff here
- Let's load the **tidyverse** and compute:
  - mean math scores by school
  - standard error of the mean, which is  $\frac{\sigma}{\sqrt{n}}$

```
library(tidyverse)
sch_means <- hsb %>%
  group_by(sch.id) %>%
  summarize(sch_mean = mean(math, na.rm = TRUE),
            sch_mean_se = sd(math, na.rm = TRUE)/sqrt(n()))
sch_means

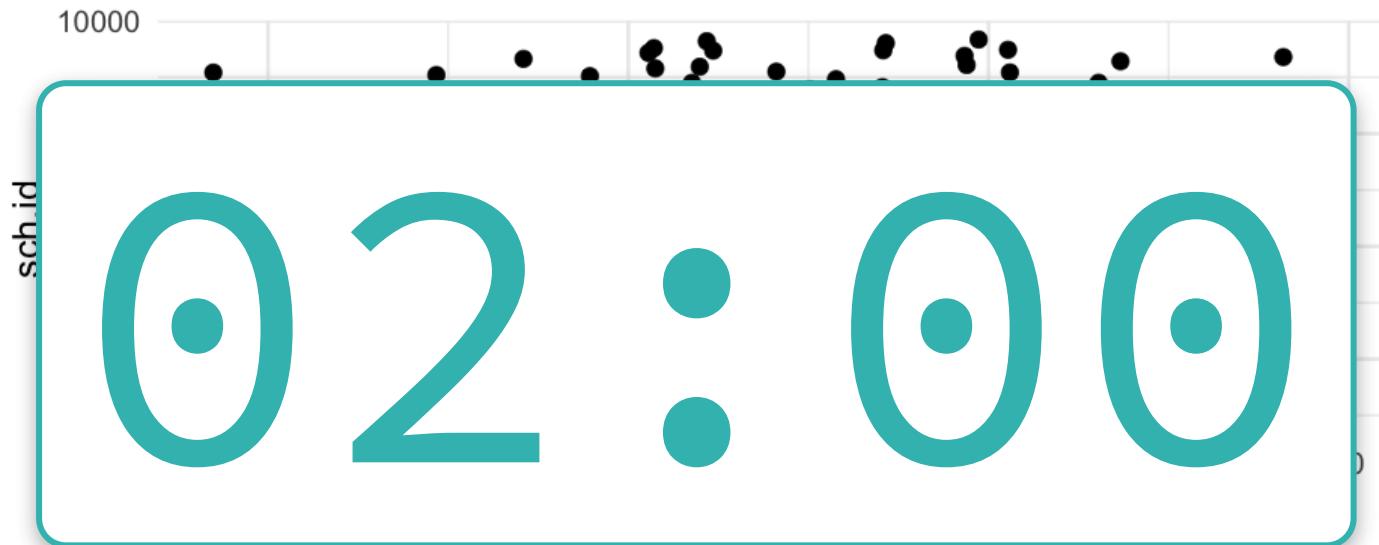
## # A tibble: 160 x 3
##       sch.id   sch_mean   sch_mean_se
##       <int>     <dbl>        <dbl>
## 1     1224    9.715447    1.107521
## 2     1288   13.5108     1.404369
## 3     1296    7.635958    0.7723605
## 4     1308   16.2555     1.367186
## 5     1317   13.17769    0.7884564
## 6     1358   11.20623    1.072893
## 7     1374    9.728464    1.579465
## 8     1433   19.71914    0.6551441
## 9     1436   18.11161    0.6856883
## 10    1461   16.84264    1.209306
## # ... with 150 more rows
```

# Plot the means by school

---

Not particularly helpful

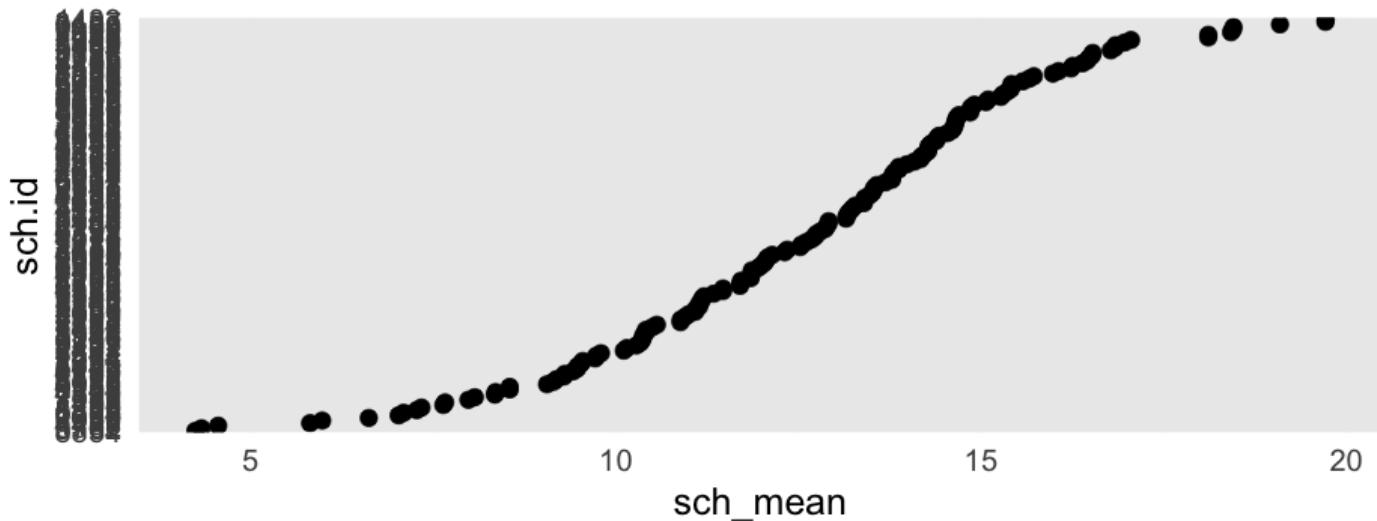
```
ggplot(sch_means, aes(sch_mean, sch.id)) +  
  geom_point()
```



# Order schools

---

```
sch_means %>%
  mutate(sch.id = factor(sch.id),
        sch.id = reorder(sch.id, sch_mean)) %>%
ggplot(aes(sch_mean, sch.id)) +
  geom_point()
```



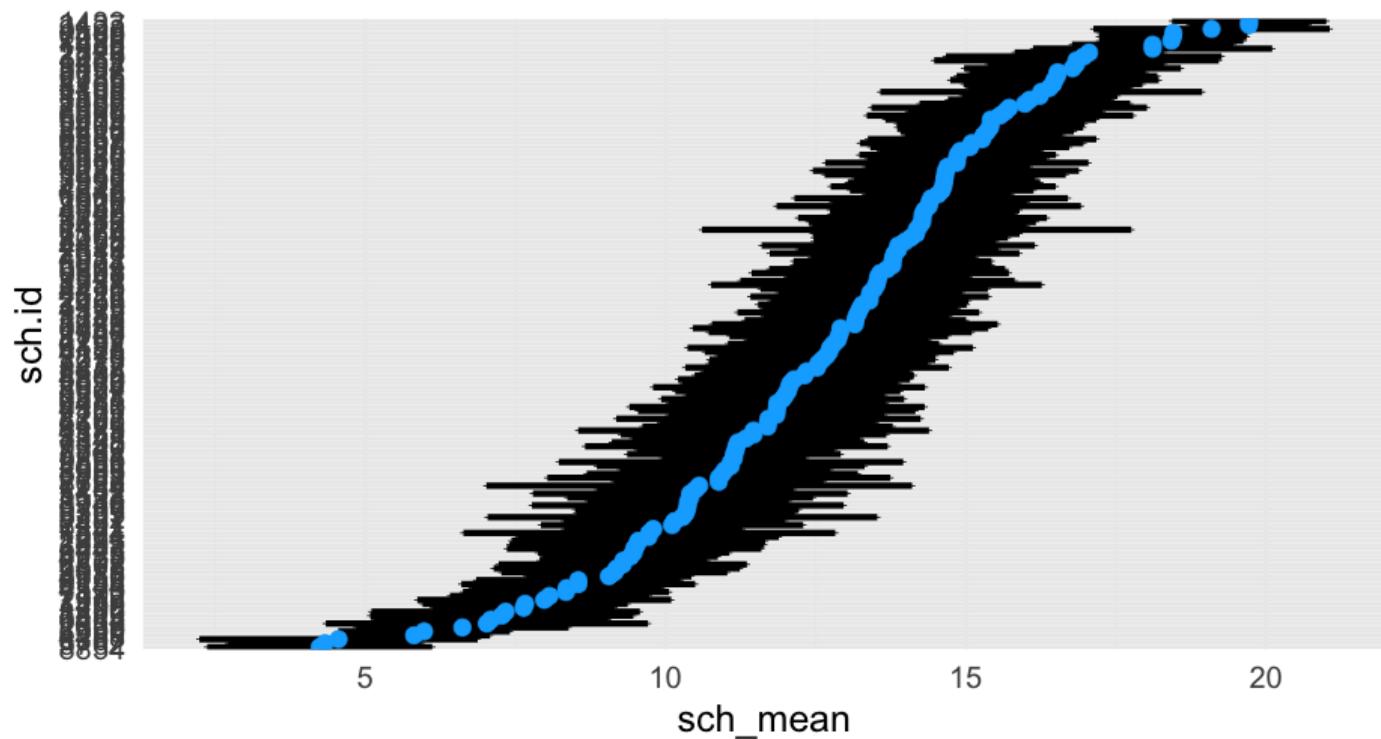
# Add SE of mean

---

```
sch_means %>%
  mutate(sch.id = factor(sch.id),
        sch.id = reorder(sch.id, sch_mean)) %>%
ggplot(aes(sch_mean, sch.id)) +
  geom_errorbarh(
    aes(xmin = sch_mean - 1.96*sch_mean_se,
        xmax = sch_mean + 1.96*sch_mean_se),
    ) +
  geom_point(color = "#0aadff")
```

# Add SE of mean

---



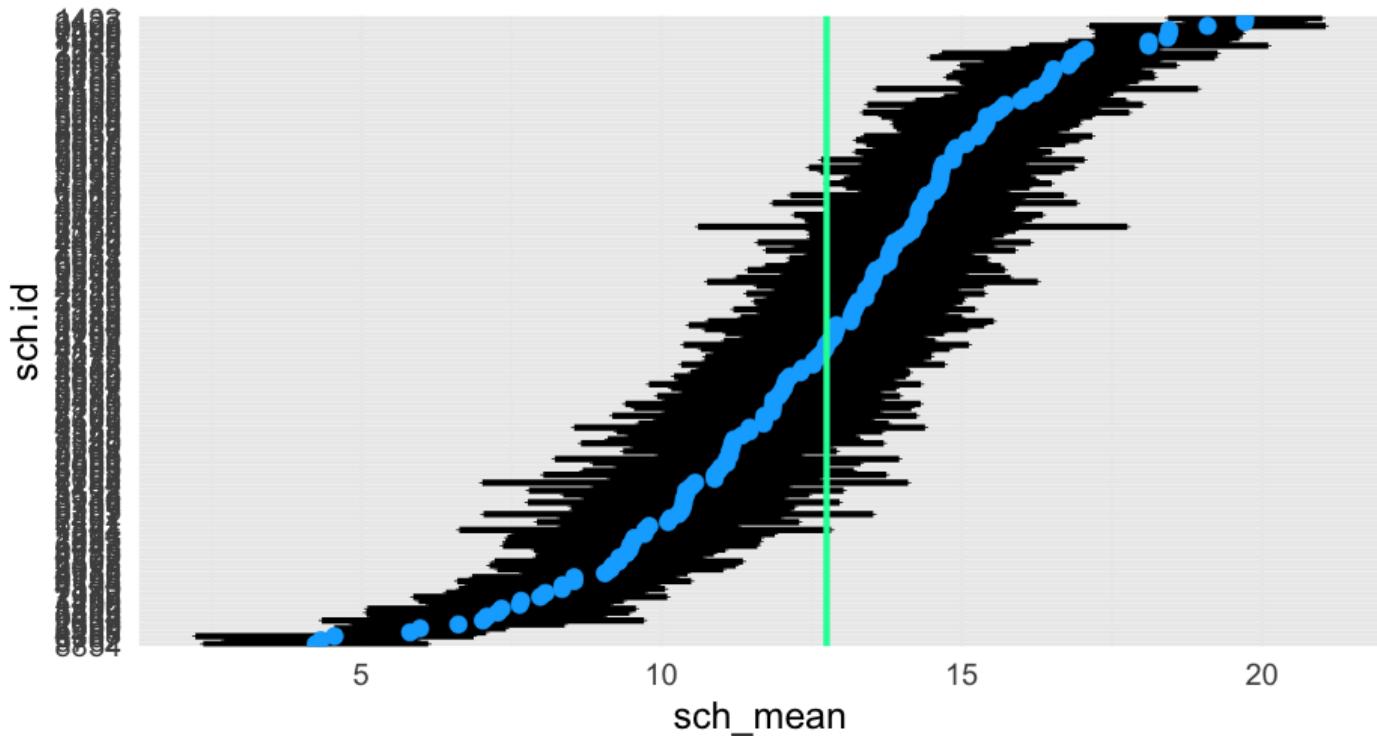
# Add sample mean

---

```
sch_means %>%
  mutate(sch.id = factor(sch.id),
        sch.id = reorder(sch.id, sch_mean)) %>%
ggplot(aes(sch_mean, sch.id)) +
  geom_errorbarh(
    aes(xmin = sch_mean - 1.96*sch_mean_se,
        xmax = sch_mean + 1.96*sch_mean_se),
    height = 0.3
  ) +
  geom_point(color = "#0aadff") +
  geom_vline(xintercept = mean(hsb$math, na.rm = TRUE),
             color = "#0affa5",
             size = 2)
```

# Add sample mean

---



# Why did we do this?

---

We could make the prior plot prettier, but that's not really why we were doing this.

Do you think there's between-school variation in math scores?

Let's model it!

# Fit a basic model

---

```
# install.packages("lme4")
library(lme4)
```

Can we fit a model equivalent to what we just did descriptively?

What would that look like? (I don't mean code, I mean statistically/conceptually)

# Unconditional model

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An unconditional model just estimates a mean score for each school.

If we have no other variables in our model, the prediction for each student would equal the school mean

Using the Gelman and Hill notation, the unconditional model we want would look like this:

$$\begin{aligned}\text{math}_i &\sim N(\alpha_{j[i]}, \sigma^2) \\ \alpha_j &\sim N(\mu_{\alpha_j}, \sigma_{\alpha_j}^2), \text{ for sch.id } j = 1, \dots, J\end{aligned}$$

which we'll get into more later.

# Estimate it!

---

When we use **lme4** to estimate this model, it looks like this:

```
m0 <- lmer(math ~ 1 + (1|sch.id), hsb)
```

We learn more about this model with **summary(m0)**.

If you're used to the HLM software, this will seem *blazingly* fast.

```
summary(m0)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: math ~ 1 + (1 | sch.id)
##   Data: hsb
##
## REML criterion at convergence: 47116.8
##
## Scaled residuals:
##    Min     1Q Median     3Q    Max
## -3.0631 -0.7539  0.0267  0.7606  2.7426
##
## Random effects:
##   Groups   Name        Variance Std.Dev.
##   sch.id   (Intercept) 8.614    2.935
##   Residual           39.148    6.257
## Number of obs: 7185, groups: sch.id, 160
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) 12.6370    0.2444  51.71
```

# In equation form

---

$$\widehat{\text{math}}_i \sim N(12.64_{\alpha_{j[i]}}, 6.26)$$
$$\alpha_j \sim N(0, 2.93), \text{ for sch.id } j = 1, \dots, J$$

Part of why I like the above notation more is that it makes our assumptions explicit

What's your guess – how will our model–estimated school means compare to our descriptive stats?

# Pull estimated means

---

We won't worry about the standard errors for now.

```
estimated_means <- coef(m0)$sch.id  
head(estimated_means)
```

```
##          (Intercept)  
## 1224      9.973039  
## 1288     13.376384  
## 1296      8.068508  
## 1308     15.585491  
## 1317     13.130920  
## 1358     11.394462
```

# Join them

---

```
estimated_means <- estimated_means %>%
  mutate(sch.id = as.integer(rownames(.))) %>%
  rename(intercept = `Intercept`)

left_join(sch_means, estimated_means)

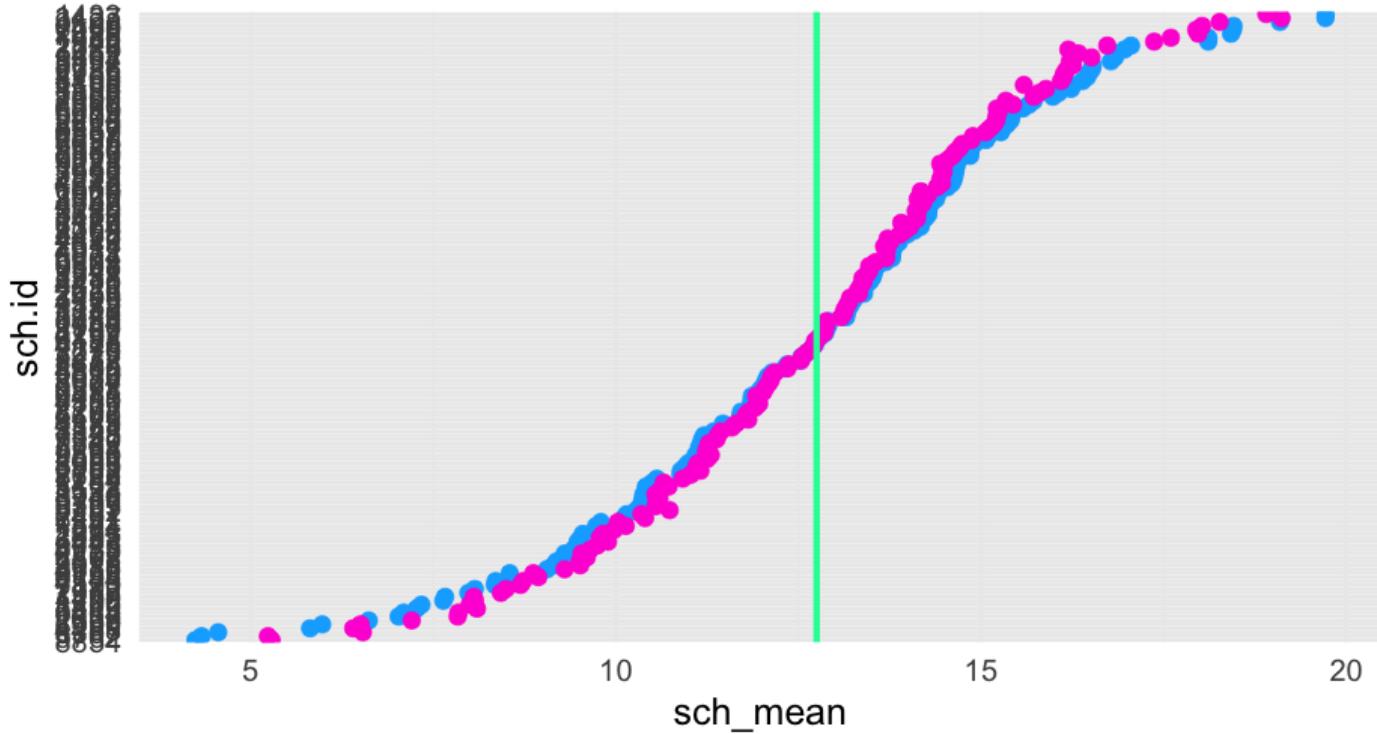
## Joining, by = "sch.id"

## # A tibble: 160 x 4
##   sch.id    sch_mean sch_mean_se intercept
##   <int>      <dbl>       <dbl>      <dbl>
## 1 1224     9.715447   1.107521   9.973039
## 2 1288    13.5108     1.404369  13.37638
## 3 1296     7.635958   0.7723605  8.068508
## 4 1308    16.2555     1.367186  15.58549
## 5 1317    13.17769   0.7884564 13.13092
## 6 1358    11.20623   1.072893  11.39446
## 7 1374     9.728464   1.579465  10.13462
## 8 1433    19.71914   0.6551441 18.90522
## 9 1436    18.11161   0.6856883 17.59908
## 10 1461    16.84264   1.209306  16.33355
## # ... with 150 more rows
```

# Plot it

---

```
left_join(sch_means, estimated_means) %>%
  mutate(sch.id = factor(sch.id),
        sch.id = reorder(sch.id, sch_mean)) %>%
  ggplot(aes(sch_mean, sch.id)) +
  geom_point(color = "#0aadff") +
  geom_point(aes(x = intercept),
             color = "#ff0ad6") +
  geom_vline(xintercept = mean(hsb$math, na.rm = TRUE),
             color = "#0affa5",
             size = 2)
```



Points are being pulled toward the grand mean

# ICC

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- We can estimate the ICC with the **performance** package
- Part of **easystats**

```
library(performance)
icc(m0)
```

```
## # Intraclass Correlation Coefficient
##
##      Adjusted ICC: 0.180
##      Conditional ICC: 0.180
```

So approximately 18% of the variability in math scores lies between schools.

# Next time

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- Data structuring – moving data to a longer format
- Random intercepts, random slopes, multiple levels
- Homework 1 is assigned – we'll hopefully have 30–45 minutes to work on it in class.