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UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Final Project Report

Bayesian Hierarchical Model Analysis of NBA Player Statistics in 2009-2010

STAT 430 Applied Bayesian Analysis

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I. Background Introduction

Nowadays, basketball is an increasingly popular sport around the world. The National Basketball Association is considered to be the pinnacle of professional basketball leagues. In NBA, each player belongs to a certain team and teams are divided into six divisions, called Atlantic, Central, Northwest, Pacific, Southeast, Southwest respectively. Owing to the existence of trading system, the managers of each team would like to know how is a player's performance so they can assess his trade value and further strengthen the team by trading players with the other teams. The coaches also want to evaluate the technical and tactical skills of players and teams from statistical analysis. Evaluating the performance of teams and players is therefore an important task for each team. The purpose of this paper is to help determine whether division and team would affect players' performance.

II. Data Preparation

The data is obtained from 'NBAPlayerStatistics0910' dataset of 'SportsAnalytics' library in R[1]. The original dataset contains records of 25 variables for 441 different NBA players. We planned to use conference and teams as two stages at the beginning. But since conference has only two levels (East and West), placing a distribution on parameters with only two levels might not be proper. Therefore, we choose to manually divide teams into six divisions according to NBA division record in 2009-2010[2]. Finally, we will use Player, Team, Division (newly created) as three stages in our project. The team category is the dataset while division category is collected from external source. We also regenerate the response variable Points Per Game(PPG), which is calculated from dividing the value of the toal points (TotalPoints) by the number of games played(GamesPlayed) in the dataset. Considering that each team has a maximum number of thirteen players in a game but the number of players is varying from teams (for example, Boston Celtics(BOS) has 15 players, while team Denver Nuggets(DEN) only has 13 players), we randomly select 13 individual players from each team. Performance regarding the players are made using the posterior distributions of the division and team parameters.

III. Model Construction

Bayesian Hierarchical Model is used in examination of whether the Points Per Game (PPG) is significantly different between teams in different divisions. Two different Bayesian Hierarchical Models are performed in this study.

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[1] Retrevied from: https://www.rdocumentation.org/packages/SportsAnalytics/versions/0.2/topics/NBAPlayerStatistics0910 HYPERLINK "https://www.rdocumentation.org/packages/SportsAnalytics/versions/0.2/topics/NBAPlayerStatistics0910"
Based on NBA official website: http://www.nba.com/standings/2009/team_record_comparison/conferenceNew_Std_Div.html

The first one is normal hierarchical model where Y_{ijk} is the PPG of each player calculated from the dataset. We place a normal distribution to Y_{ijk} with μ_{ij} as the mean parameter. and σ_A^2 as variance parameter. μ_{ij} can be deemed as the average PPG of all players in team j of division i. It follows a normal distribution with the μ_i and σ_B^2 as parameters. Using the same notation, μ_i is the average PPG of all players of division i. μ_i follows a normal distribution with the μ_0 and σ_A^2 as parameters. The prior distributions of variance parameters σ_A^2, σ_B^2, σ_C^2 follow a very thin inverse gamma distribution, and the prior distribution of population mean μ_0 follows flat uniform distribution. The Directed Acyclic Graph of the normal model one is demonstrated in Figure 1.

$$Y_{ijk}|\mu_{ij},\sigma_C^2 \sim N\,(\mu_{ij},\sigma_C^2)$$
$$\mu_{ij}|\mu_i,\sigma_B^2 \sim N\,(\mu_i,\sigma_B^2)$$
$$\mu_i|\mu_0,\sigma_A^2 \sim N\,(\mu_0,\sigma_A^2)$$

Prior:

$$\mu_0 \sim 1\ du$$
$$\sigma_A^2 \sim IG\,(0.001,0.001)$$
$$\sigma_B^2 \sim IG\,(0.001,0.001)$$
$$\sigma_C^2 \sim IG\,(0.001,0.001)$$

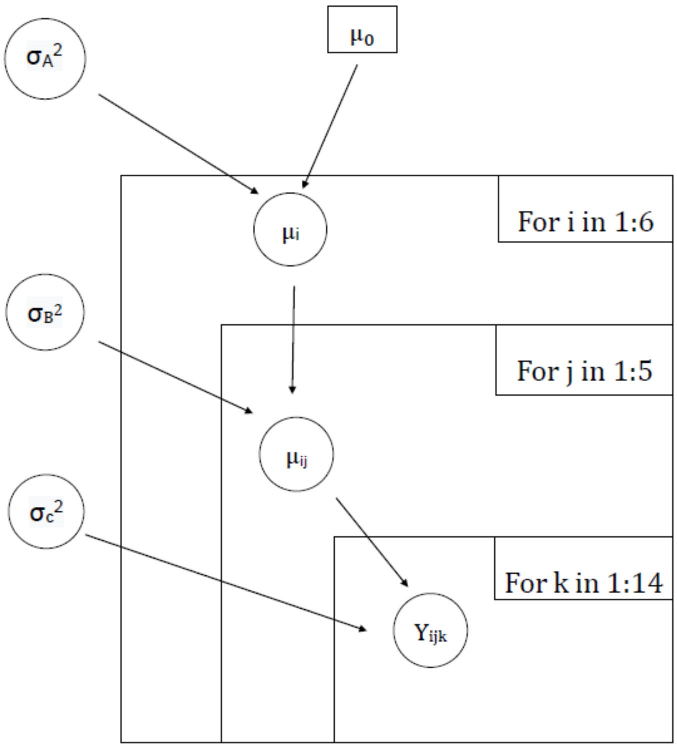

Figure 1. Directed acyclic graph for Normal Bayesian Hierarchical Model

The second one is exponential hierarchical model where the PPG of each player follows exponential distribution with parameter λ_{ij}, which follows gamma distribution with scale parameter 1 and shape parameter φ. The prior distributions of φ follows uniform distribution with range from 1 to 5. The DAG of exponential model is shown in Figure 2.

$$Y_{ijk}|\lambda_{ij} \sim Exp\ (\lambda_{ij})$$
$$\lambda_{ij}|\varphi \sim Gamma\ (\varphi, 1)$$

Prior:

$$\varphi \sim Uniform\ (1,5)$$

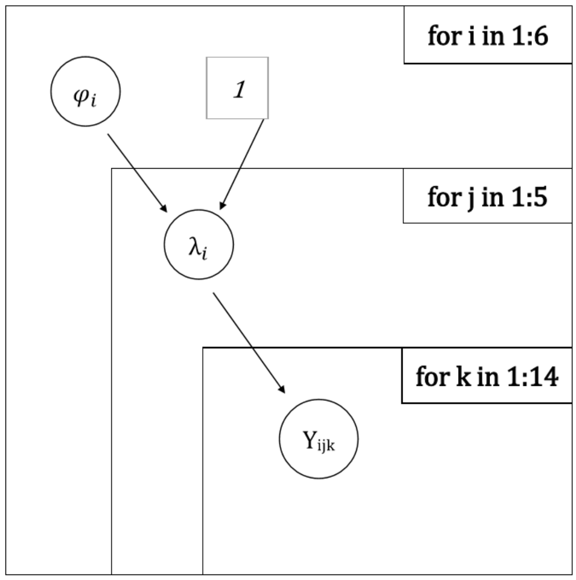

Figure 2. Directed acyclic graph for Gamma Bayesian Hierarchical Model

IV. Experiment and Result

Executions of both models are performed in OpenBUGS. Each model uses three chains in the simulation and the initialization of parameters in both models are shown in Figure 3.

```
# Initialization for Normal model
list(mu0=10,tausq_a=1,tausq_b=1,tausq_c=1)
list(mu0=9,tausq_a=1,tausq_b=2,tausq_c=3)
list(mu0=9,tausq_a=3,tausq_b=2,tausq_c=1)
# Initialization for Exponential model
list(psi=1)
list(psi=3)
list(psi=5)
```

Figure 3. Initialization of parameters in OpenBUGS

In Normal Bayesian Hierarchical Model, we monitor the convergence using Brooks Gelman Rubin diagnostic (BGR). Figure 4 and Figure 5 are BGR plot with 50,000 iterations for four parameters (μ_{ij},σ_A^2,$\sigma_B^2$and σ_C^2). After 10,000 iterations, the values of all parameters stabilize. Thus, this initial 10,000 iterations should be discarded as burn-in.

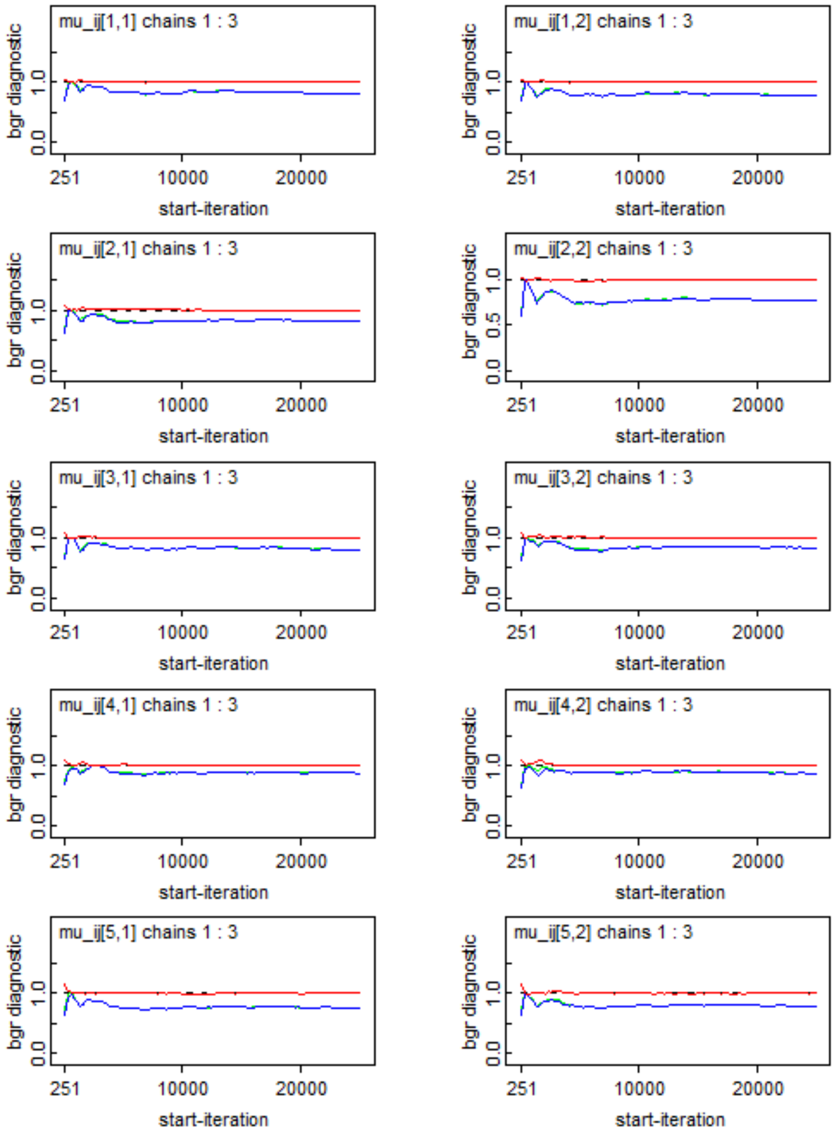

Figure 4. Partial BGR in OpenBUGS for μ_{ij}
in Normal Bayesian Hierarchical Model

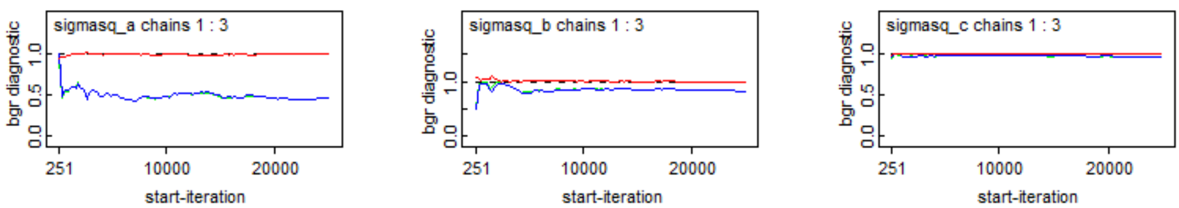

Figure 5. Output of Brooks Gelman Rubin Diagnostic in OpenBUGS for σ_A^2,σ_B^2 and σ_C^2
in Normal Bayesian Hierarchical Model

History plots give us preliminary run for four parameters (μ_{ij},σ_A^2,$\sigma_B^2$and σ_C^2). To check if the parameters are stablizied after 10,000 iterations, we examine the plots for each parameter from 10,000 iteration to 50,000 iteration after burn-in phase. We can see that within 10,000 iterations, all three chains for each parameter came together and started drawing from the same range of values. See the following Figure 6, Figure 7, Figure 8, and Figure 9 for μ_{ij},σ_A^2,$\sigma_B^2$and σ_C^2) respectively.

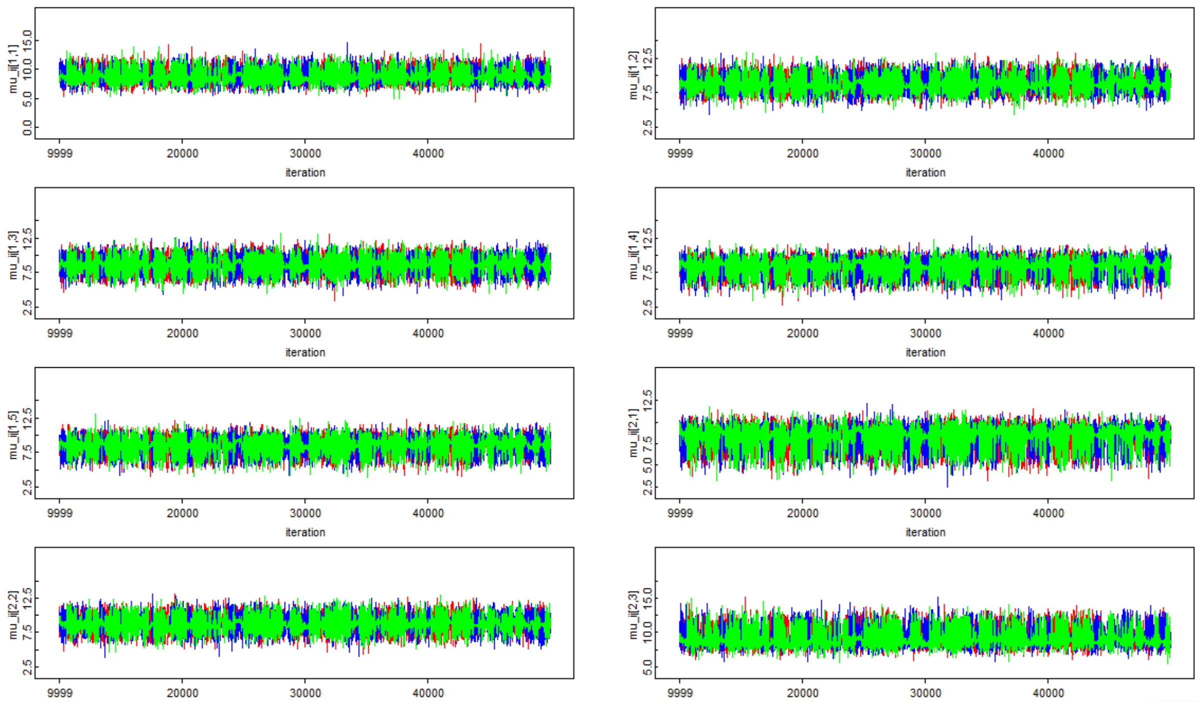

Figure 6. Sample History Diagnostic in OpenBUGS for μ_{ij}
in Normal Bayesian Hierarchical Model

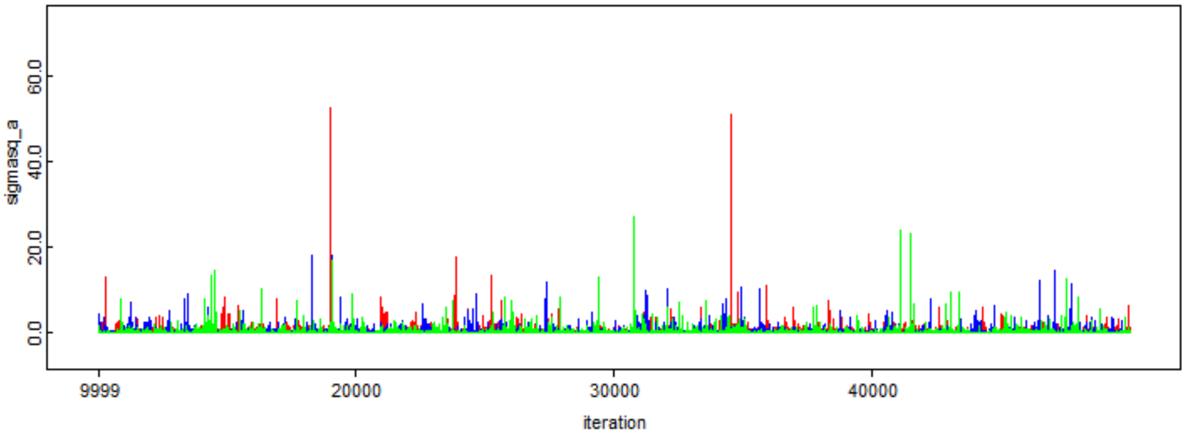

Figure 7. History Diagnostic in OpenBUGS for σ_A^2
in Normal Bayesian Hierarchical Model

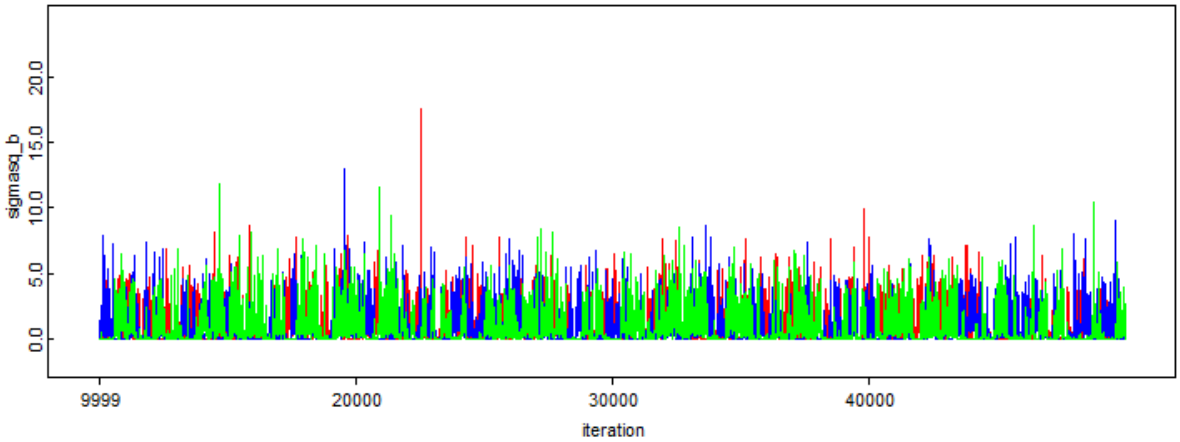

Figure 8. Output of History Diagnostic in OpenBUGS for σ_B^2
in Normal Bayesian Hierarchical Model

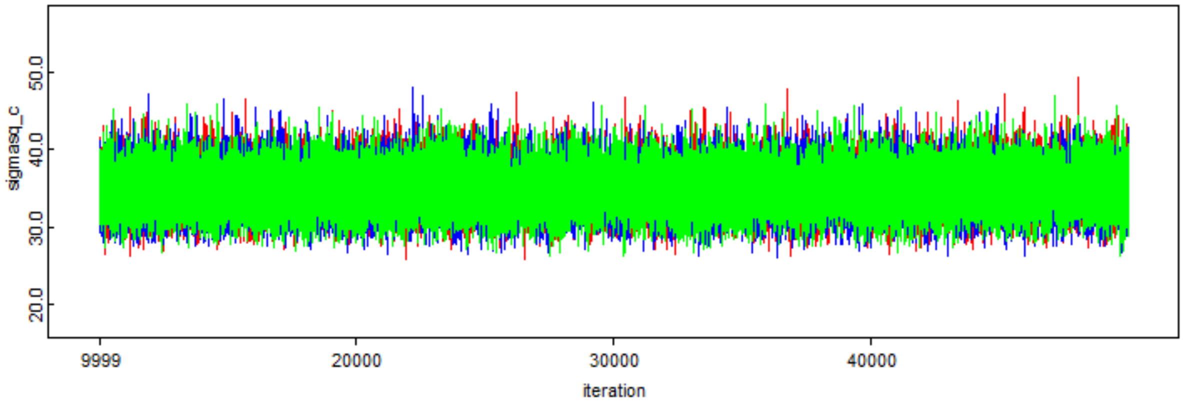

Figure 9. History Diagnostic in OpenBUGS for σ_C^2
in Normal Bayesian Hierarchical Model

From the summary of estimators of the posterior distributions after performing 50,000 iterations we also excluded first 10,000 iterations. The posterior μ_{ij} are within 7.9 and 9.3 and standard deviations are around 0.8. The MCMC error are very small and the posterior 95% credible intervals are given in Table 1 and Table 2. The differences between μ_i and standard deviation for μ_i for all i are even smaller.

Based on the simulation result, there is not any significant difference between the average Points Per Game of players in each team. We do not see any difference between the average Points Per Game of players in different divisions as well.

	mean	sd	MC_error	val2.5pc	median	val97.5pc
mu_ij[1,1]	8.881	0.7957	0.01008	7.419	8.792	10.74
mu_ij[1,2]	8.764	0.7695	0.008594	7.23	8.719	10.49
mu_ij[1,3]	8.588	0.7639	0.008167	6.902	8.612	10.16
mu_ij[1,4]	8.357	0.81	0.01085	6.434	8.472	9.755
mu_ij[1,5]	8.467	0.7793	0.009248	6.659	8.54	9.917
mu_ij[2,1]	8.296	0.8728	0.0137	6.171	8.448	9.73
mu_ij[2,2]	8.743	0.7622	0.007972	7.169	8.719	10.38
mu_ij[2,3]	9.223	0.9068	0.01487	7.873	9.03	11.45
mu_ij[2,4]	8.955	0.7935	0.009702	7.533	8.86	10.81
mu_ij[2,5]	8.846	0.7738	0.008373	7.341	8.789	10.59
mu_ij[3,1]	8.944	0.7851	0.00944	7.525	8.845	10.79
mu_ij[3,2]	8.325	0.8561	0.01273	6.243	8.473	9.735
mu_ij[3,3]	8.696	0.7612	0.007765	7.059	8.697	10.29
mu_ij[3,4]	8.888	0.7733	0.008685	7.42	8.814	10.66
mu_ij[3,5]	9.136	0.8644	0.01289	7.785	8.969	11.25
mu_ij[4,1]	7.91	1.065	0.02057	5.279	8.205	9.35
mu_ij[4,2]	9.637	1.233	0.02594	8.069	9.276	12.63
mu_ij[4,3]	8.936	0.804	0.01041	7.507	8.828	10.86
mu_ij[4,4]	8.379	0.8056	0.0105	6.461	8.49	9.778
mu_ij[4,5]	8.487	0.7756	0.008817	6.694	8.553	9.945
mu_ij[5,1]	8.612	0.7577	0.007294	7.018	8.61	10.23
mu_ij[5,2]	8.397	0.7825	0.008989	6.568	8.482	9.84
mu_ij[5,3]	8.163	0.8701	0.01335	6.021	8.337	9.507
mu_ij[5,4]	8.798	0.7811	0.008822	7.352	8.722	10.62
mu_ij[5,5]	8.327	0.7996	0.01017	6.435	8.438	9.723
mu_ij[6,1]	8.76	0.7691	0.008093	7.259	8.713	10.49
mu_ij[6,2]	8.689	0.7589	0.007573	7.141	8.666	10.34
mu_ij[6,3]	8.624	0.7558	0.007325	7.011	8.628	10.21
mu_ij[6,4]	8.725	0.7595	0.007789	7.222	8.687	10.42
mu_ij[6,5]	8.035	0.9629	0.01706	5.653	8.266	9.412

Table 1. Summary Results of Normal Model Estimates of Points Per Game on Team Level

	mean	sd	MC_error	val2.5pc	median	val97.5pc
mu_i[1]	8.635	0.415	0.008736	7.788	8.639	9.429
mu_i[2]	8.716	0.4226	0.008673	7.928	8.699	9.611
mu_i[3]	8.708	0.4134	0.008433	7.921	8.695	9.566
mu_i[4]	8.656	0.4074	0.00839	7.836	8.655	9.451
mu_i[5]	8.58	0.4156	0.00809	7.679	8.595	9.337
mu_i[6]	8.619	0.4111	0.00817	7.758	8.625	9.395

Table 2. Summary Results of Normal Model Estimates of Points Per Game on Division Level

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
sigmasq_a	0.1318	0.4794	0.005124	7.242E-4	0.02266	0.932	10000	120003
sigmasq_b	0.7632	0.9708	0.02071	0.001339	0.3712	3.341	10000	120003
sigmasq_c	34.7	2.597	0.01773	29.95	34.58	40.14	10000	120003

Table 3. Summary Results of Normal Model Estimates of σ_A^2, σ_B^2 and σ_C^2

In Exponential Bayesian Hierarchical Model, because it seems like after 2,000 iterations all $\mu_i$and μ_{ij} converge, the initial 2,000 iterations should be discarded as burn-in (Figure 10, 11).

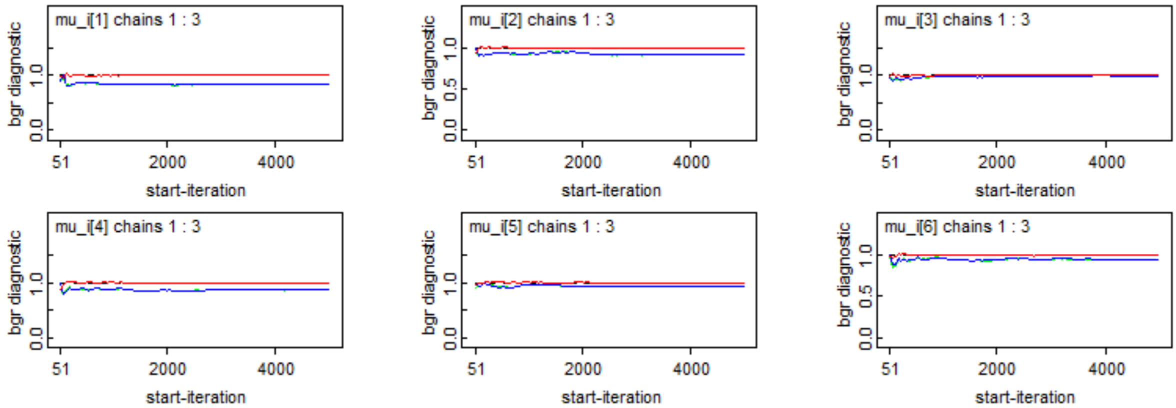

Figure 10. Partial Output of Brooks Gelman Rubin Diagnostic in OpenBUGS for μ_i in Exponential Bayesian Hierarchical Model

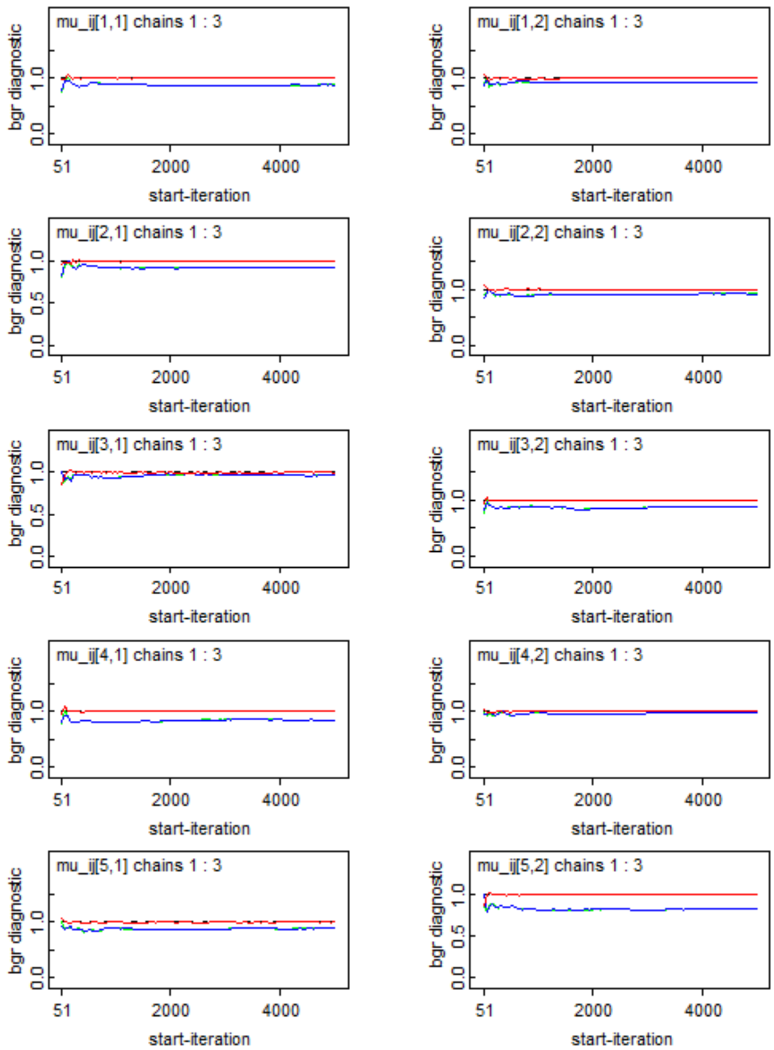

Figure 11. Partial Output of Brooks Gelman Rubin Diagnostic in OpenBUGS for μ_{ij} in Exponential Bayesian Hierarchical Model

Looking at History plot of sampler, History plots give us preliminary run for μ_i and μ_{ij}. To check if the parameters are stablizied after 2,000 iterations, we examine the plots for each parameter from 2,000 iteration to 10,000 iteration after burn-in phase. We can see that within 8,000 iterations, all three chains for each parameter came together and started drawing from the same range of values. See the following Figure 12 and Figure 13 for μ_i and μ_{ij} respectively.

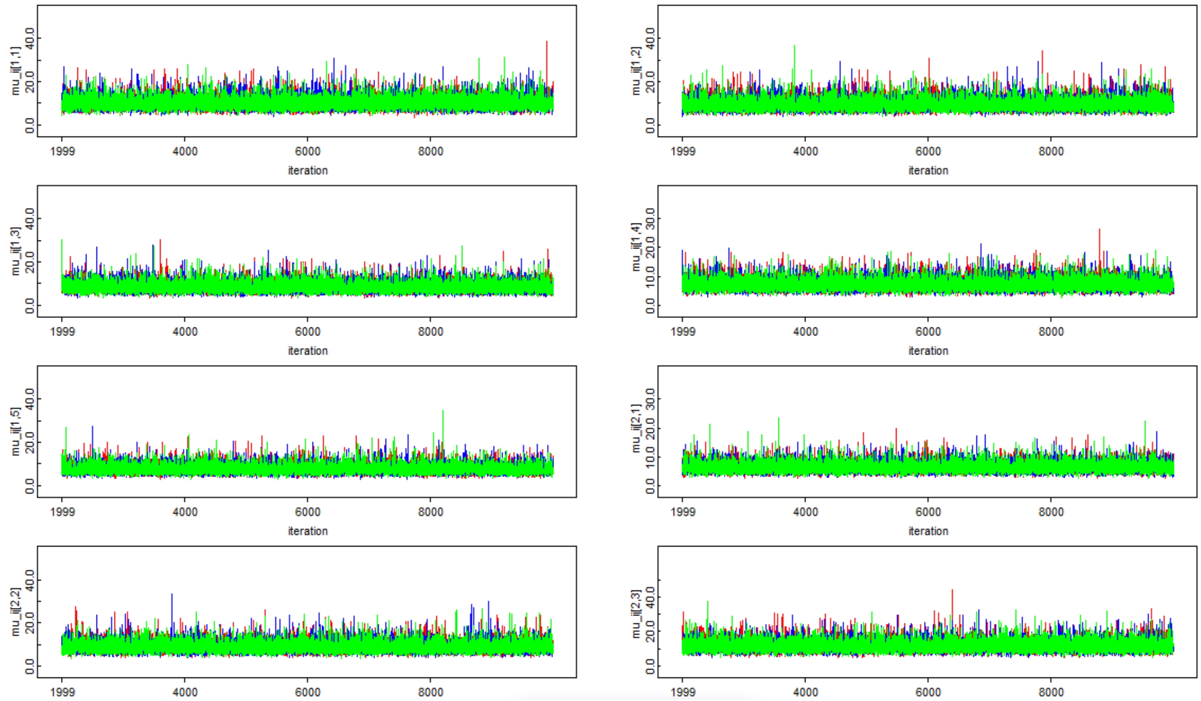

Figure 12. Output of History Diagnostic in OpenBUGS for μ_i in Exponential Bayesian Hierarchical Model

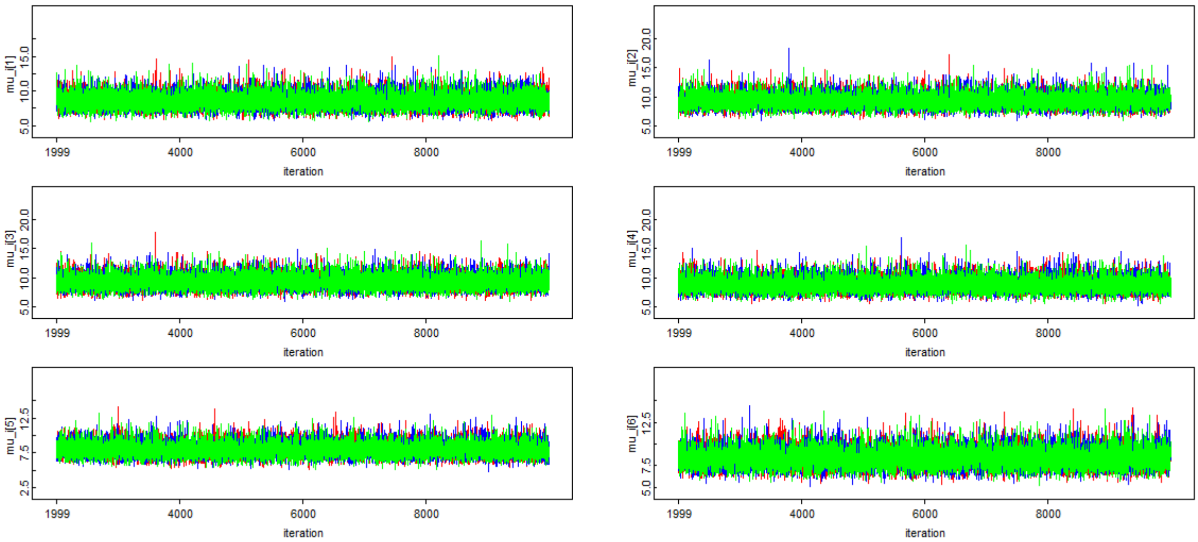

Figure 13. Output of History Diagnostic in OpenBUGS for μ_{ij} in Exponential Bayesian Hierarchical Model

The posterior μ_i varies a lot with the minimum of 5.343 and the maximum of 14.22. Most of the standard deviations are around 2 but with few exceptions that have standard deviations of one and four (Table 3, 4). The MCMC errors are very small and also the posterior 95% credible intervals are given in Table 3 and Table 4. Based on the simulation result, although there is not any significant difference between the average Points Per Game of players on division level, we do see noticeable differences between the average Points Per Game of players in different teams.

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
mu_ij[1,1]	10.06	2.877	0.01841	5.896	9.573	16.99	2000	24003
mu_ij[1,2]	9.424	2.748	0.0163	5.51	8.961	16.05	2000	24003
mu_ij[1,3]	8.429	2.457	0.01688	4.953	7.997	14.29	2000	24003
mu_ij[1,4]	7.14	2.044	0.01329	4.176	6.792	12.14	2000	24003
mu_ij[1,5]	7.74	2.25	0.01455	4.537	7.368	13.13	2000	24003
mu_ij[2,1]	6.384	1.819	0.01169	3.741	6.073	10.74	2000	24003
mu_ij[2,2]	8.897	2.567	0.01669	5.214	8.467	15.08	2000	24003
mu_ij[2,3]	11.6	3.326	0.02006	6.823	11.04	19.64	2000	24003
mu_ij[2,4]	10.1	2.904	0.01921	5.931	9.605	17.16	2000	24003
mu_ij[2,5]	9.468	2.721	0.01795	5.591	9.031	16.02	2000	24003
mu_ij[3,1]	10.08	2.907	0.01895	5.888	9.59	17.04	2000	24003
mu_ij[3,2]	6.613	1.899	0.01223	3.868	6.291	11.19	2000	24003
mu_ij[3,3]	8.68	2.499	0.01571	5.09	8.245	14.79	2000	24003
mu_ij[3,4]	9.778	2.829	0.01691	5.735	9.289	16.65	2000	24003
mu_ij[3,5]	11.16	3.245	0.0202	6.491	10.62	19.09	2000	24003
mu_ij[4,1]	4.51	1.293	0.008594	2.64	4.291	7.63	2000	24003
mu_ij[4,2]	14.22	4.118	0.02625	8.279	13.53	24.04	2000	24003
mu_ij[4,3]	10.33	2.991	0.01846	6.02	9.827	17.43	2000	24003
mu_ij[4,4]	7.16	2.07	0.01305	4.155	6.819	12.17	2000	24003
mu_ij[4,5]	7.768	2.227	0.01432	4.542	7.405	13.19	2000	24003
mu_ij[5,1]	8.815	2.534	0.01626	5.158	8.391	15.0	2000	24003
mu_ij[5,2]	7.605	2.195	0.01525	4.432	7.229	12.91	2000	24003
mu_ij[5,3]	6.3	1.808	0.01144	3.709	5.994	10.69	2000	24003
mu_ij[5,4]	9.891	2.861	0.01934	5.765	9.423	16.72	2000	24003
mu_ij[5,5]	7.241	2.088	0.01388	4.234	6.881	12.25	2000	24003
mu_ij[6,1]	9.457	2.749	0.01795	5.502	8.981	16.05	2000	24003
mu_ij[6,2]	9.03	2.589	0.01794	5.314	8.588	15.27	2000	24003
mu_ij[6,3]	8.697	2.506	0.01688	5.109	8.259	14.82	2000	24003
mu_ij[6,4]	9.293	2.711	0.01777	5.489	8.817	15.87	2000	24003
mu_ij[6,5]	5.343	1.535	0.009854	3.122	5.086	9.094	2000	24003

Table 4. Summary Results of Exponential Model Estimates of Points Per Game of players on Team Level

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
mu_i[1]	8.558	1.112	0.007177	6.678	8.456	11.03	2000	24003
mu_i[2]	9.289	1.207	0.007725	7.262	9.185	11.97	2000	24003
mu_i[3]	9.261	1.223	0.007526	7.194	9.145	11.96	2000	24003
mu_i[4]	8.797	1.209	0.00773	6.778	8.683	11.5	2000	24003
mu_i[5]	7.97	1.038	0.007189	6.193	7.877	10.27	2000	24003
mu_i[6]	8.364	1.108	0.007352	6.499	8.26	10.81	2000	24003

Table 5. Summary Results of Exponential Model Estimates of Points Per Game of players on Division Level

V. Model Comparison

Based on Table 6 and Table 7, the deviance information criterion (DIC) value of Normal Bayesian Hierarchical Model is 2498 while DIC value of Exponential Bayesian Hierarchical Model is 2500. We should choose the model with minimal DIC. That indicates that these two Bayesian models are pretty similar and Normal Bayesian Hierarchical Model is slightly better compared with Exponential Bayesian Hierarchical Model. Also, from Table 3, mean and variance estimators of μ_{ij} are not stable, so we conclude that Normal Bayesian Hierarchical Model is a better Bayesian model choice for analyzing NBA Player Statistics data.

	Dbar	Dhat	DIC	pD
y	2489.0	2481.0	2498.0	8.625
total	2489.0	2481.0	2498.0	8.625

Table 6. DIC result of Normal Bayesian Hierarchical Model

	Dbar	Dhat	DIC	pD
y	2472.0	2444.0	2500.0	28.0
total	2472.0	2444.0	2500.0	28.0

Table 7. DIC result of Exponential Bayesian Hierarchical Model

VI. Further Exploration on Frequentist Inference

For normal Bayesian hierarchical model, we perform frequentist method on it. The maximum likelihood estimators (MLE) and $(1-\alpha/2)$ confidence intervals are listed below:

$$\mu_i \approx X_i$$
$$\mu_{ij} \approx X_{ij}$$
$$\sigma^2_i \approx S^2_i$$
$$\sigma^2_{ij} \approx S^2_{ij}$$

Confidence interval of μ_{ij}: $$(\widehat{\mu_{ij}} - t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}, \widehat{\mu_{ij}} + t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}})$$

Confidence interval of σ^2_{ij}: $$(\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}})$$

Division	Team	mean	variance	n	CI_mu_low	CI_mu_high	CI_var_low	CI_var_high
Atlantic	BOS	8.94	28.63	13	6.03	11.85	7.51	21.41
Atlantic	NJN	8.47	27.85	13	5.60	11.34	7.31	20.83
Atlantic	NYK	9.87	31.92	13	6.80	12.94	8.38	23.87
Atlantic	PHI	8.53	22.75	13	5.94	11.12	5.97	17.01
Atlantic	TOR	8.61	40.82	13	5.14	12.08	10.71	30.53
Central	CHI	7.09	23.22	13	4.47	9.71	6.09	17.36
Central	CLE	9.46	54.28	13	5.46	13.46	14.25	40.59
Central	DET	9.29	26.5	13	6.49	12.09	6.96	19.82
Central	IND	9.62	29.58	13	6.66	12.58	7.76	22.12
Central	MIL	8.44	31.25	13	5.40	11.48	8.20	23.37
Northwest	DEN	9.67	62.52	13	5.37	13.97	16.41	46.75
Northwest	MIN	8.41	23.56	13	5.77	11.05	6.18	17.62
Northwest	OKL	7.58	60.52	13	3.35	11.81	15.88	45.26
Northwest	POR	7.6	22.63	13	5.01	10.19	5.94	16.92
Northwest	UTA	8.92	36.32	13	5.64	12.20	9.53	27.16
Pacific	GSW	11.03	46.69	13	7.32	14.74	12.25	34.91
Pacific	LAC	8.45	34	13	5.28	11.62	8.92	25.43
Pacific	LAL	9.02	56.09	13	4.95	13.09	14.72	41.94
Pacific	PHO	7.56	29.02	13	4.63	10.49	7.62	21.70
Pacific	SAC	7.35	36.28	13	4.08	10.62	9.52	27.13
Southeast	ATL	7.67	51.84	13	3.76	11.58	13.61	38.77
Southeast	CHA	6.97	29.07	13	4.04	9.90	7.63	21.74
Southeast	MIA	8.37	47.35	13	4.63	12.11	12.43	35.41
Southeast	ORL	9.56	21.09	13	7.06	12.06	5.54	15.77
Southeast	WAS	8.83	29.73	13	5.87	11.79	7.80	22.23
Southwest	DAL	9.61	43.38	13	6.03	13.19	11.39	32.44
Southwest	HOU	9.28	42.4	13	5.74	12.82	11.13	31.71
Southwest	MEM	8.56	54.65	13	4.54	12.58	14.34	40.87
Southwest	NOR	8.87	39.59	13	5.45	12.29	10.39	29.61
Southwest	SAN	7.73	29.15	13	4.80	10.66	7.65	21.80

Table 8. Frequentist approach results of μ_{ij} in each division and team

Compared with the results of normal Bayesian hierarchical model in Table 1, frequentist method presents much greater estimators than these of Bayesian method. And all the confidence intervals of frequentist method are wider than these of normal Bayesian hierarchical model. Large variance shows that the scoring ability of players in each team are varying greatly.

Methods	Atlantic	Central	Northwest	Pacific	Southeast	Southwest
Frequentist	8.88	8.78	8.43	8.68	8.28	8.81
Bayesian	8.635	8.716	8.708	8.656	8.58	8.619

Table 9. Comparison between Frequentist and Bayesian approaches for μ_i

Both the frequentist and Bayesian methods results for μi are approximately 8.5 and the pairwise differences are very small. That indicates that results of Bayesian model seem good and the differences of Players' Points Per Game (PPG) between different divisions are small. Players in each division have close average levels.

Methods	σ_A^2 (Between Division Standard Deviation)	σ_B^2 (Between Team Standard Deviation)	σ_C^2(Between Players Standard Deviation)
Frequentist	0.2472	0.9319	37.09
Bayesian	0.1318	0.7632	34.7

Table 10. Comparison between Frequentist and Bayesian approaches for σ^2

Based on Table 5, we can clearly see that normal Bayesian hierarchical model has slightly greater standard deviation between different teams and inside each team than frequentist model, but the differences are very small. But for between different divisions, result of frequentist method is almost twice as much as that of Bayesian method. That indicates that normal Bayesian hierarchical model shows representative results compared to frequentist model. Additionally, between different divisions and between teams, the results in the set are close to the mean and each other. But in each team, Players' Points Per Game (PPG) in each set are far from the mean and each other. So the differences of players in each team are obvious.

Appendix

```
# Normal Model
model
{
  for(i in 1:6){
          for(j in 1:5){
                    for(k in 1:13){
                              y[k,j,i] ~ dnorm(mu_ij[j],tausq_c)
                    }
               mu_ij[j] ~ dnorm(mu_i[i],tausq_b)
          }
           mu_i[i] ~ dnorm(mu0,tausq_a)
  }
  mu0 ~ dflat()
  tausq_a ~ dgamma(0.001,0.001)
  tausq_b ~ dgamma(0.001,0.001)
  tausq_c ~ dgamma(0.001,0.001)

  sigmasq_a <- 1/tausq_a
  sigmasq_b <- 1/tausq_b
  sigmasq_c <- 1/tausq_c
```

```
}

list(mu0=10,tausq_a=1,tausq_b=1,tausq_c=1)
list(mu0=9,tausq_a=1,tausq_b=2,tausq_c=3)
list(mu0=9,tausq_a=3,tausq_b=2,tausq_c=1)


# Exponential Model
model
{
	for(i in 1:6){
		for(j in 1:5){
			for(k in 1:13){
			y[k,j,i] ~ dexp(lambda[i,j])
			}
			lambda[i,j] ~ dgamma(psi,1)
			mu_ij[i,j] <- 1/lambda[i,j]
		}
		mu_i[i] <- mean(mu_ij[i,])
	}
	psi~dunif(1,5)
}

# Initialization for Gamma model
list(psi=1)
list(psi=3)
list(psi=5)

data
list(y=structure(
	.Data =
c(16.300000,6.111111,5.647059,6.296296,4.391304,14.347826,7.000000,2.444444,10.141026,18.25
3521,10.125000,13.703704,1.500000,2.400000,4.015873,6.867925,9.805970,16.875000,4.461538,7.
777778,7.057971,11.980769,12.464789,18.804878,2.240000,5.304348,11.714286,4.720000,15.2769
23,3.714286,8.589286,7.373134,15.074074,2.156250,17.722222,6.955882,20.246914,8.166667,6.57
5758,13.105263,4.691176,8.134146,1.000000,8.739726,8.041096,17.085366,13.750000,5.684211,4.
683333,3.375000,8.580645,14.031250,4.954545,17.200000,7.106061,23.971429,10.279412,8.59740
3,3.392857,11.353659,6.231707,1.950000,3.880952,1.727273,11.283784,0.500000,7.980000,17.571
4286,8.963415,10.878378,1.000000,3.938462,4.3461538,8.768293,9.933333,10.734375,5.460317,2.
055556,6.339286,2.000000,8.5061728,7.406250,29.710526,18.712121,4.852459,12.0000000,7.3456
79,3.950000,4.837209,8.565789,8.800000,4.025000,3.270833,9.9682540,5.057971,13.790323,18.06
5217,9.262500,6.7500000,13.489796,16.643836,3.045455,11.935897,5.521739,9.940299,10.297872
,3.0625000,24.145161,8.482759,7.595745,11.679012,10.2105263,3.980769,4.261905,14.583333,7.3
21429,9.439024,6.464789,15.869565,0.9047619,11.000000,2.750000,10.370370,2.113636,0.821428
6,15.463415,6.205479,11.944444,10.390244,15.370370,8.829268,2.058824,5.894737,28.159420,1.0
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```

```
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7,7.500000,5.825397,14.888889,7.970149,19.5609756,8.878378,1.200000,1.500000,4.365854,5.191
489,5.314286,9.117647,20.565217,16.207317,4.468750,8.810345,2.943662,11.9875000,14.608696,
19.587500,1.000000,1.722222,2.320000,2.523810,17.463415,20.753086,3.058824,12.447368,3.250
000,0.400000,0.7142857,10.365854,18.688889,7.108696,5.181818,7.240000,12.645161,14.479452,
19.024691,3.808824,7.780488,4.392405,7.030769,17.8846154,16.493333,2.106383,2.500000,12.35
8974,1.666667,12.271605,3.884615,6.341772,5.792208),
        .Dim=c(13,5,6)
        )
)
```