线性代数

"Eigen Vector"
Vector

$$Ax = \lambda x \xrightarrow{\text{same thing}} (A-\lambda I)x = 0$$

n x n Matrix

Scalar
"Eigen Value"

$\det(A-\lambda I) = 0$ ← "Characteristic equation of A"

$(A-\lambda I)$ is singular
$(A-\lambda I)$ is not invertable

$P(\lambda) = \det(A-\lambda I)$
"Characteristic Polynomial"

$$A = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} \qquad A - I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4-1 & 8 \\ 6 & 26-1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 25 \end{bmatrix}$$

$$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{vmatrix} = (4-\lambda)(26-\lambda) - (8)(6) = 104 - 30\lambda + \lambda^2 - 48 = \lambda^2 - 30\lambda + 56 = (\lambda-28)(\lambda-2) = 0$$

$$\lambda_1 = 28$$
$$\lambda_2 = 2$$

$$N(A-\lambda_2 I) = \left[\begin{array}{cc|c} 4-\lambda_2 & 8 & 0 \\ 6 & 26-\lambda_2 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 4-2 & 8 & 0 \\ 6 & 26-2 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 2 & 8 & 0 \\ 6 & 24 & 0 \end{array}\right] \xrightarrow[R2/6]{R1/2} \left[\begin{array}{cc|c} 1 & 4 & 0 \\ 1 & 4 & 0 \end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{cc|c} 1 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 + 4x_2 = 0 \longrightarrow \begin{matrix} x_1 = -4x_2 \\ x_2 = x_2 \end{matrix} \longrightarrow \begin{bmatrix} -4x_2 \\ x_2 \end{bmatrix} \longrightarrow \underbrace{\begin{bmatrix} -4x \\ x \end{bmatrix}}_{\text{Eigenspace}} \longrightarrow x \underbrace{\begin{bmatrix} -4 \\ 1 \end{bmatrix}}_{\text{Eigen Vector}}$$

标量

- 简单操作

$$c = a + b$$

$$c = a \cdot b$$

$$c = \sin a$$

- 长度

$$|a| = \begin{cases} a & \text{if} a > 0 \\ -a & \text{otherwise} \end{cases}$$

$$|a + b| \leq |a| + |b|$$

$$|a \cdot b| = |a| \cdot |b|$$

向量

- 简单操作

$$c = a + b \quad \text{where } c_i = a_i + b_i$$

$$c = \alpha \cdot b \quad \text{where } c_i = \alpha b_i$$

$$c = \sin a \quad \text{where } c_i = \sin a_i$$

- 长度

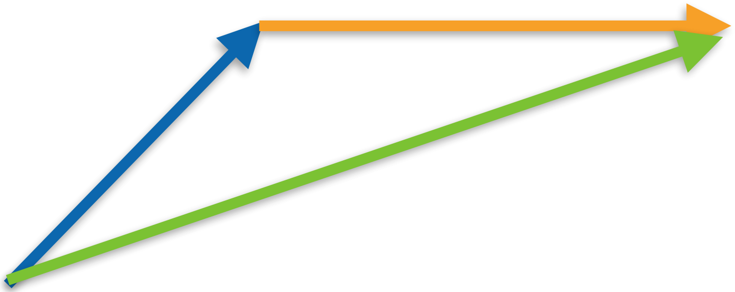
$$\|a\|_2 = \left[\sum_{i=1}^{m} a_i^2\right]^{\frac{1}{2}}$$

$$\|a\| \geq 0 \text{ for all } a$$

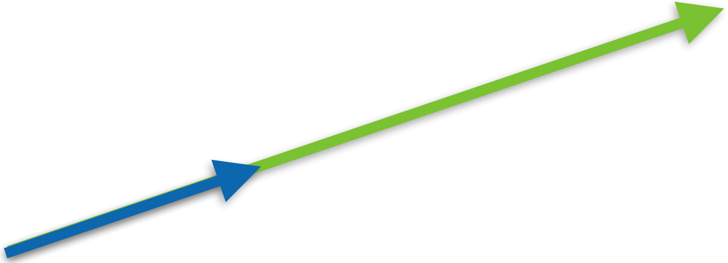
$$\|a + b\| \leq \|a\| + \|b\|$$

$$\|a \cdot b\| = |a| \cdot \|b\|$$

向量



$$c = a + b$$



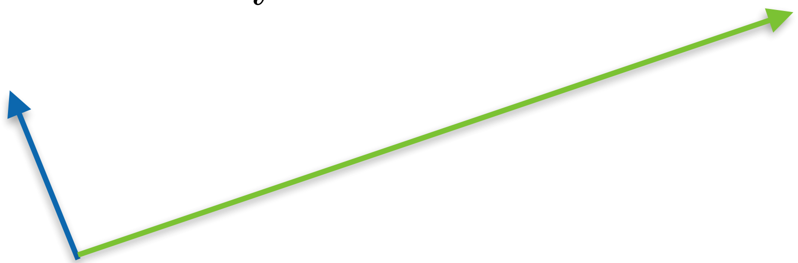
$$c = \alpha \cdot b$$

数学家的‘**parallel for all do**’

向量

- 点乘 $a^\top b = \sum_i a_i b_i$

- 正交 $a^\top b = \sum_i a_i b_i = 0$



矩阵

- 简单操作

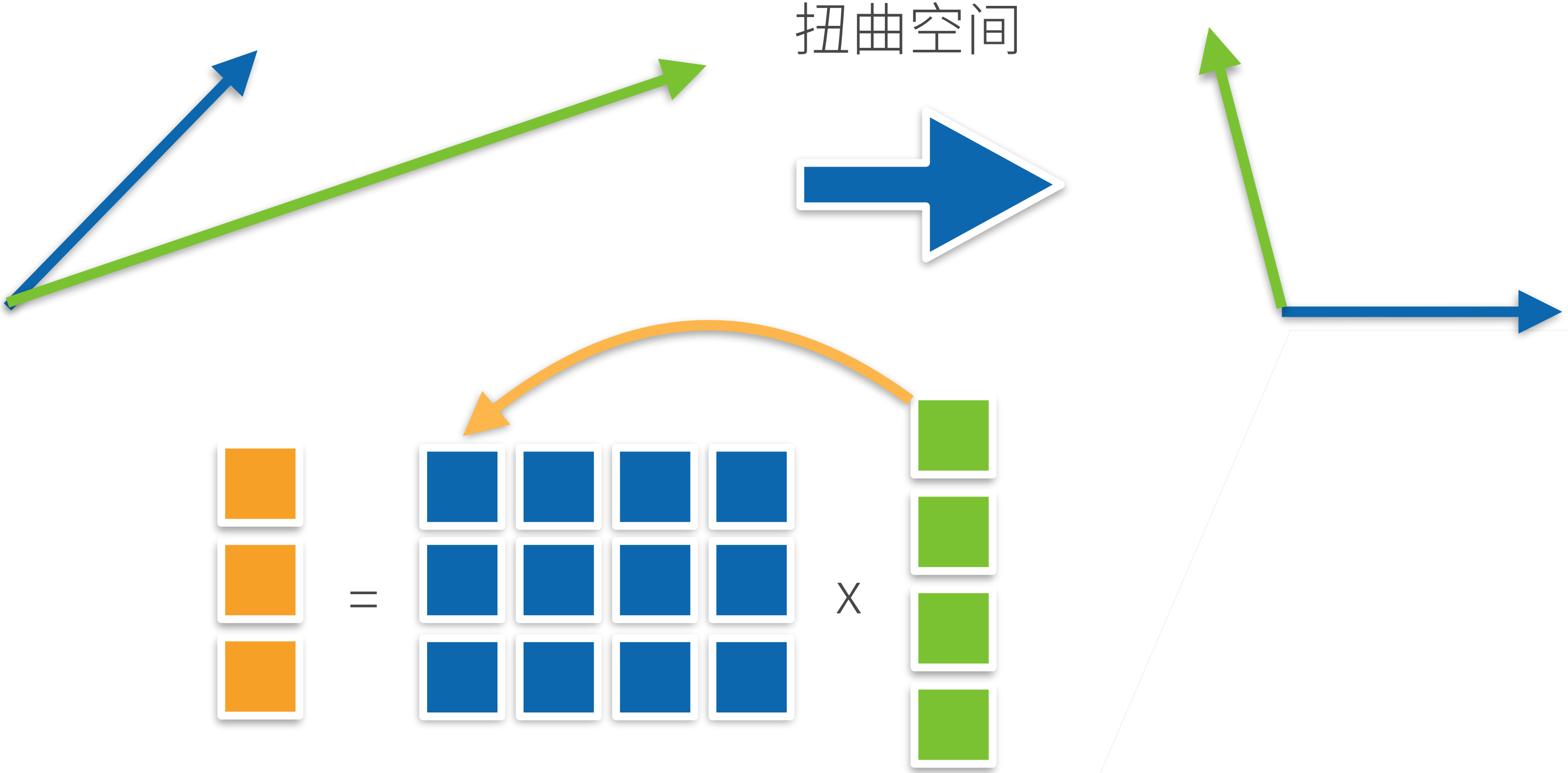
$$
\begin{aligned}
C &= A + B && \text{where } C_{ij} = A_{ij} + B_{ij} \\
C &= \alpha \cdot B && \text{where } C_{ij} = \alpha B_{ij} \\
C &= \sin A && \text{where } C_{ij} = \sin A_{ij}
\end{aligned}
$$

矩阵

- 乘法 （矩阵乘以向量）

$$c = Ab \text{ where } c_i = \sum_j A_{ij} b_j$$

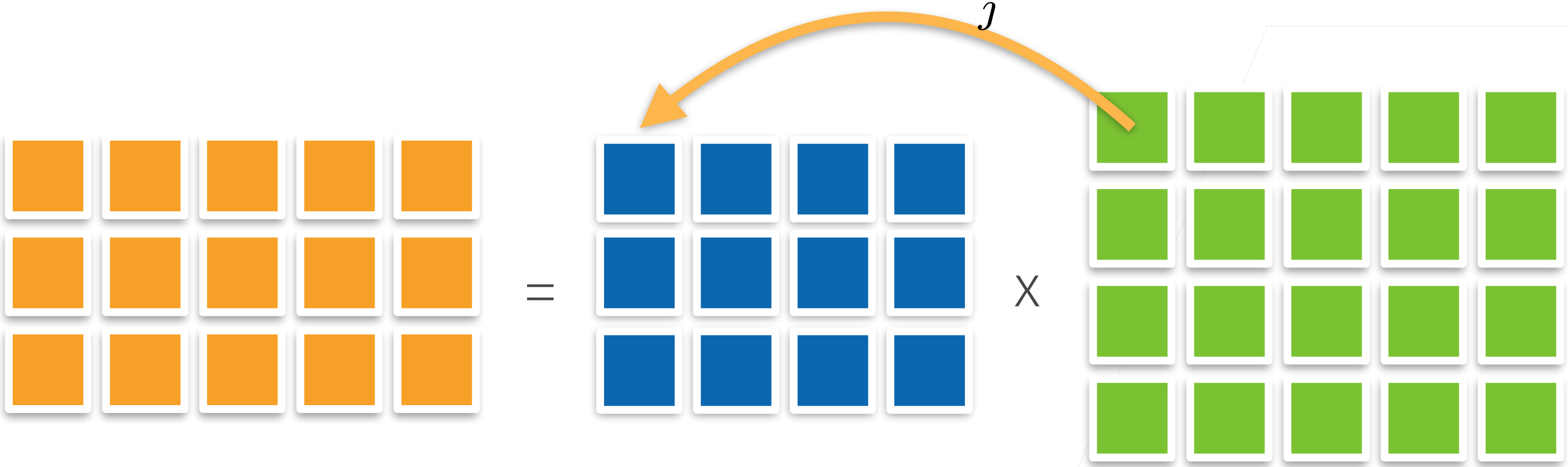
矩阵



矩阵

- 乘法（矩阵乘以矩阵）

$$C = AB \text{ where } C_{ik} = \sum_j A_{ij} B_{jk}$$



矩阵

- 范数

$$c = A \cdot b \text{ hence } \|c\| \leq \|A\| \cdot \|b\|$$

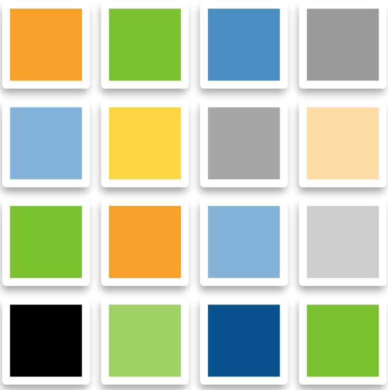
  - 取决于如何衡量 b 和 c 的长度
- 常见范数
  - 矩阵范数：最小的满足的上面公式的值
  - Frobenius 范数

$$\|A\|_{\text{Frob}} = \left[\sum_{ij} A_{ij}^2\right]^{\frac{1}{2}}$$

特殊矩阵

- 对称和反对称

$$A_{ij} = A_{ji} \text{ and } A_{ij} = -A_{ji}$$



- 正定

$$\|x\|^2 = x^\top x \geq 0 \text{ generalizes to } x^\top A x \geq 0$$

特殊矩阵

- 正交矩阵
  - 所以行都相互正交
  - 所有行都有单位长度 $U \text{ with } \sum_j U_{ij}U_{kj} = \delta_{ik}$
  - 可以写成 $UU^\top = \mathbf{1}$
- 置换矩阵

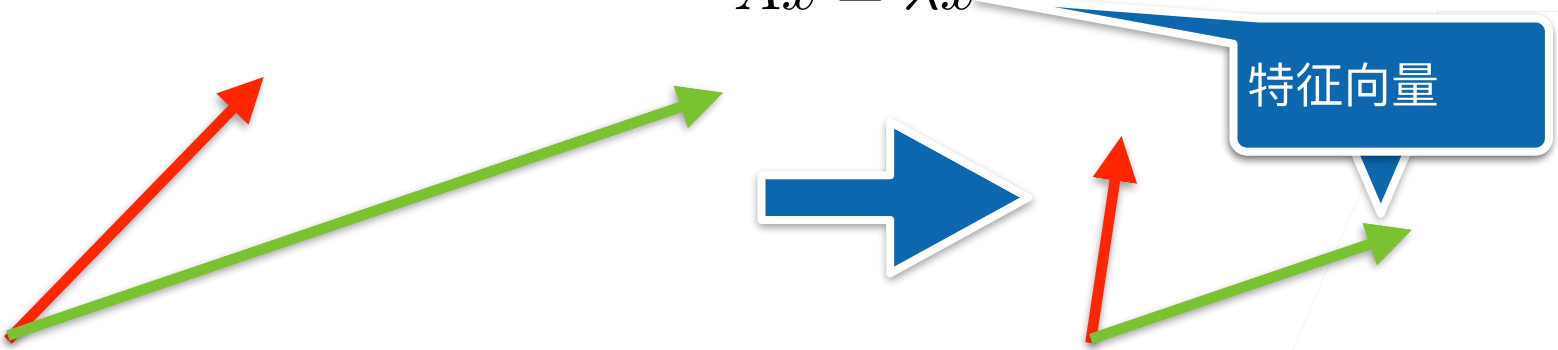
$$P \text{ where } P_{ij} = 1 \text{ if and only if } j = \pi(i)$$

  - 置换矩阵是正交矩阵

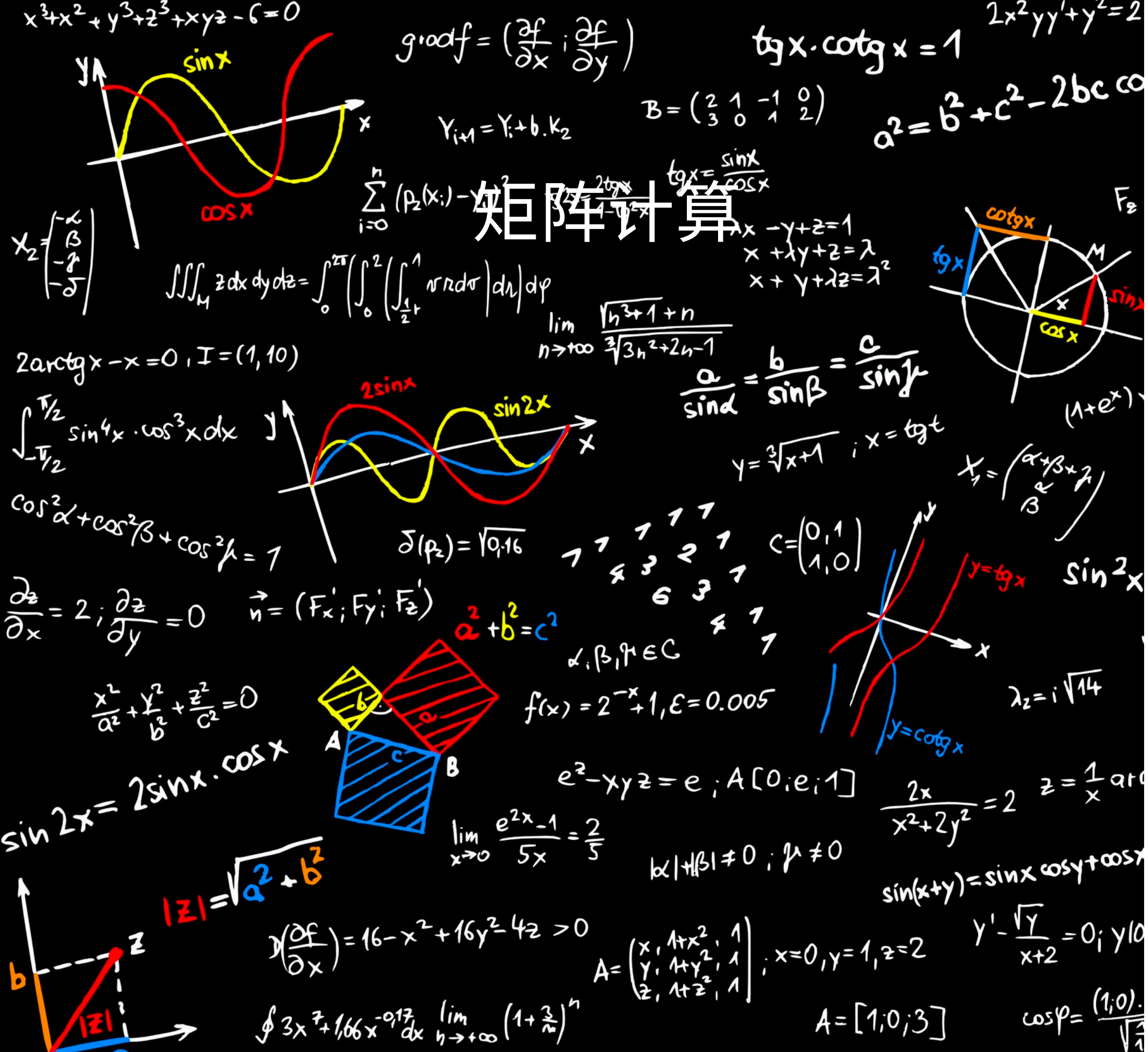
矩阵

- 特征向量和特征值
  - 不被矩阵改变方向的向量

$$Ax = \lambda x$$



  - 对称矩阵总是可以找到特征向量



矩阵计算

动手学深度学习 v2

李沐 · AWS