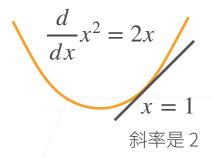
## 标量导数



У	a	$\chi^n$	$\exp(x)$	log(x)	$\sin(x)$
$\frac{dy}{dx}$	0	$nx^{n-1}$	$\exp(x)$	$\frac{1}{x}$	$\cos(x)$
	a = a	不是 x 的	<b>勺函数</b>		

## 导数是切线的斜率



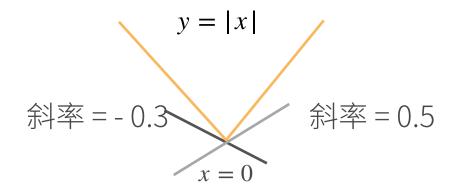
$$y \qquad u + v \qquad uv \qquad y = f(u), u = g(x)$$

$$\frac{dy}{dx} \qquad \frac{du}{dx} + \frac{dv}{dx} \qquad \frac{du}{dx}v + \frac{dv}{dx}u \qquad \frac{dy}{du}\frac{du}{dx}$$

### 亚导数



• 将导数拓展到不可微的函数



$$\frac{\partial |x|}{\partial x} = \begin{cases} 1 & \text{if } x > 0\\ -1 & \text{if } x < 0\\ a & \text{if } x = 0, \quad a \in [-1, 1] \end{cases}$$

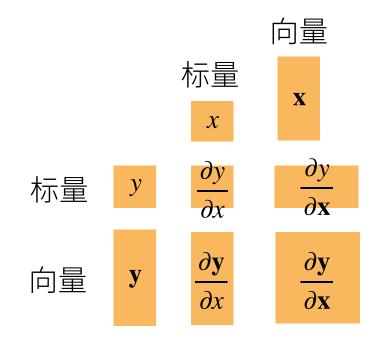
#### 另一个例子

$$\frac{\partial}{\partial x} \max(x,0) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \\ a & \text{if } x = 0, \quad a \in [0,1] \end{cases}$$

# 梯度



• 将导数拓展到向量



$$\partial y/\partial \mathbf{x}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \frac{\partial y}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \dots, \frac{\partial y}{\partial x_n} \end{bmatrix}$$



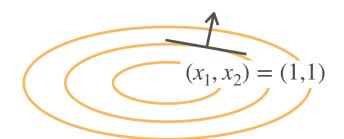


$$\frac{\partial y}{\partial x}$$
  $\frac{\partial y}{\partial x}$ 

$$\frac{o}{\hat{c}}$$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$$

$$\frac{\partial}{\partial \mathbf{x}} x_1^2 + 2x_2^2 = [2x_1, 4x_2]$$
 方向 (2, 4) 跟等高线正交



# 样例



<i>y</i> 		аи	sum(x)	$\ \mathbf{x}\ ^2$	a is not a	function of x
$\frac{\partial y}{\partial \mathbf{x}}$	$0^{T}$	$a\frac{\partial u}{\partial \mathbf{x}}$	$1^T$	$2\mathbf{x}^T$	<b>0</b> and <b>1</b> a	ire vectors
$\partial \mathbf{x}$		$\partial \mathbf{x}$	1	<b>2X</b>		
У	<u>u</u> -	+ <i>v</i>	uv		$\langle \mathbf{u}, \mathbf{v} \rangle$	

 $\frac{\partial y}{\partial \mathbf{x}} \mid \frac{\partial u}{\partial \mathbf{x}} + \frac{\partial v}{\partial \mathbf{x}} \quad \frac{\partial u}{\partial \mathbf{x}} v + \frac{\partial v}{\partial \mathbf{x}} u \quad \mathbf{u}^T \frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \mathbf{v}^T \frac{\partial \mathbf{u}}{\partial \mathbf{x}}$ 

$$\partial \mathbf{y}/\partial x$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \qquad \frac{\partial \mathbf{y}}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \vdots \\ \frac{\partial y_m}{\partial x} \end{bmatrix}$$

 $\partial y/\partial x$  是行向量, $\partial y/\partial x$  是列向量这个被称之为分子布局符号,反过来的版本叫分母布局符号

$$\partial y/\partial x$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial \mathbf{x}} \\ \frac{\partial y_2}{\partial \mathbf{x}} \\ \vdots \\ \frac{\partial y_m}{\partial \mathbf{x}} \end{bmatrix} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1}, \frac{\partial y_1}{\partial x_2}, \dots, \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1}, \frac{\partial y_2}{\partial x_2}, \dots, \frac{\partial y_2}{\partial x_n} \\ \vdots \\ \frac{\partial y_m}{\partial x_1}, \frac{\partial y_m}{\partial x_2}, \dots, \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$



r

 $\frac{\partial y}{\partial x}$ 

 $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ 

X

y

 $\frac{\partial y}{\partial z}$ 

 $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ 

# 样例



<b>y</b>	a	X	Ax	$\mathbf{x}^T \mathbf{A}$
$\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$	0	I	A	$\mathbf{A}^T$

$$\mathbf{x} \in \mathbb{R}^n, \quad \mathbf{y} \in \mathbb{R}^m, \quad \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \in \mathbb{R}^{m \times n}$$

a, a and A are not functions of x

0 and I are matrices

$$\begin{array}{c|cccc}
\mathbf{y} & a\mathbf{u} & \mathbf{A}\mathbf{u} & \mathbf{u} + \mathbf{v} \\
\hline
\frac{\partial \mathbf{y}}{\partial \mathbf{x}} & a\frac{\partial \mathbf{u}}{\partial \mathbf{x}} & \mathbf{A}\frac{\partial \mathbf{u}}{\partial \mathbf{x}} & \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{x}}
\end{array}$$

# 拓展到矩阵



		标量	向量	矩阵
		<i>x</i> (1,)	<b>x</b> (n,1)	$\mathbf{X}$ $(n,k)$
标量	y (1,)	$\frac{\partial y}{\partial x}$ (1,)	$\frac{\partial y}{\partial \mathbf{x}}$ (1,n)	$\frac{\partial y}{\partial \mathbf{X}}$ $(k, n)$
向量	<b>y</b> (m,1)	$\frac{\partial \mathbf{y}}{\partial x}$ (m,1)	$\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ $(m, n)$	$\frac{\partial \mathbf{y}}{\partial \mathbf{X}}$ $(m, k, n)$
矩阵	$\mathbf{Y}$ $(m,l)$		$\frac{\partial \mathbf{Y}}{\partial \mathbf{x}}$ $(m, l, n)$	



# 链式法则和自动求导

