



线性代数

$Ax = \lambda x$
 A : $n \times n$ Matrix
 x : "Eigen Vector"
 λ : "Eigen Value"
 same thing $\rightarrow (A - \lambda I)x = 0$
 $\det(A - \lambda I) = 0$ ← "Characteristic Equation of A"
 $(A - \lambda I)$ is singular
 $(A - \lambda I)$ is not invertible
 $P(\lambda) = \det(A - \lambda I)$
 "Characteristic Polynomial"

$A = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix}$ $A - I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 25 \end{bmatrix}$
 $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$
 $A - \lambda I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{bmatrix}$
 $\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{vmatrix} = (4-\lambda)(26-\lambda) - (8)(6) = 104 - 30\lambda + \lambda^2 - 48 = \lambda^2 - 30\lambda + 56 = (\lambda - 28)(\lambda - 2) = 0$
 $\lambda_1 = 28$
 $\lambda_2 = 2$
 $N(A - \lambda_2 I) = \begin{bmatrix} 4-\lambda_2 & 8 & | & 0 \\ 6 & 26-\lambda_2 & | & 0 \end{bmatrix} = \begin{bmatrix} 2 & 8 & | & 0 \\ 6 & 26-2 & | & 0 \end{bmatrix} \xrightarrow{R1/2} \begin{bmatrix} 1 & 4 & | & 0 \\ 6 & 24 & | & 0 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 1 & 4 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$
 $x_1 + 4x_2 = 0 \rightarrow x_1 = -4x_2 \rightarrow \begin{bmatrix} -4x_2 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} -4 \\ 1 \end{bmatrix} x_2 \rightarrow x \begin{bmatrix} -4 \\ 1 \end{bmatrix}$
 Eigenspace \rightarrow Eigen Vector

标量



- 简单操作

$$c = a + b$$

$$c = a \cdot b$$

$$c = \sin a$$

- 长度

$$|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{otherwise} \end{cases}$$

$$|a + b| \leq |a| + |b|$$

$$|a \cdot b| = |a| \cdot |b|$$

向量



- 简单操作

$$c = a + b \quad \text{where } c_i = a_i + b_i$$

$$c = \alpha \cdot b \quad \text{where } c_i = \alpha b_i$$

$$c = \sin a \quad \text{where } c_i = \sin a_i$$

- 长度

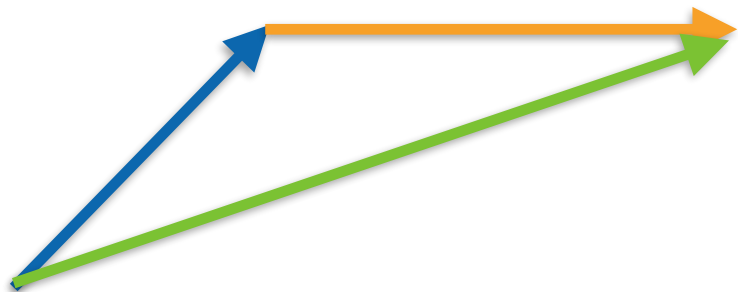
$$\|a\|_2 = \left[\sum_{i=1}^m a_i^2 \right]^{\frac{1}{2}}$$

$$\|a\| \geq 0 \text{ for all } a$$

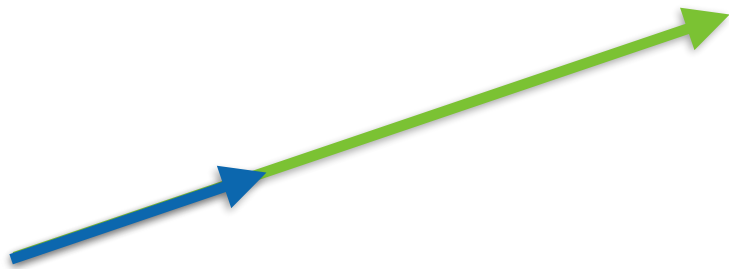
$$\|a + b\| \leq \|a\| + \|b\|$$

$$\|a \cdot b\| = |a| \cdot \|b\|$$

向量



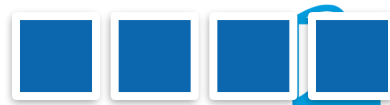
$$c = a + b$$



$$c = \alpha \cdot b$$

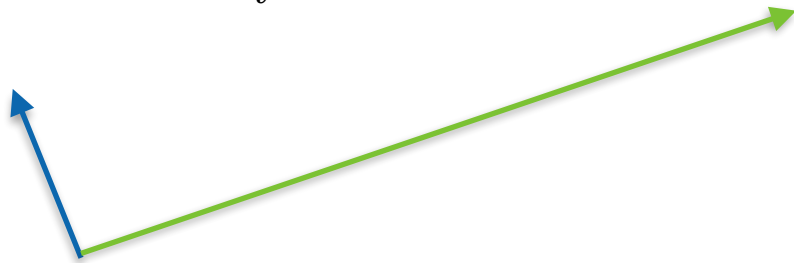
数学家的 '**parallel for all do**'

向量



- 点乘 $a^\top b = \sum_i a_i b_i$

- 正交 $a^\top b = \sum_i a_i b_i = 0$



矩阵



- 简单操作

$$C = A + B$$

where $C_{ij} = A_{ij} + B_{ij}$

$$C = \alpha \cdot B$$

where $C_{ij} = \alpha B_{ij}$

$$C = \sin A$$

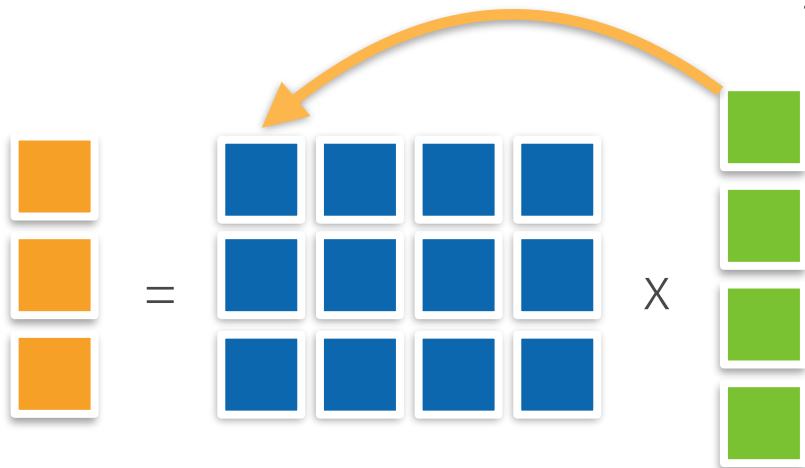
where $C_{ij} = \sin A_{ij}$

矩阵



- 乘法 (矩阵乘以向量)

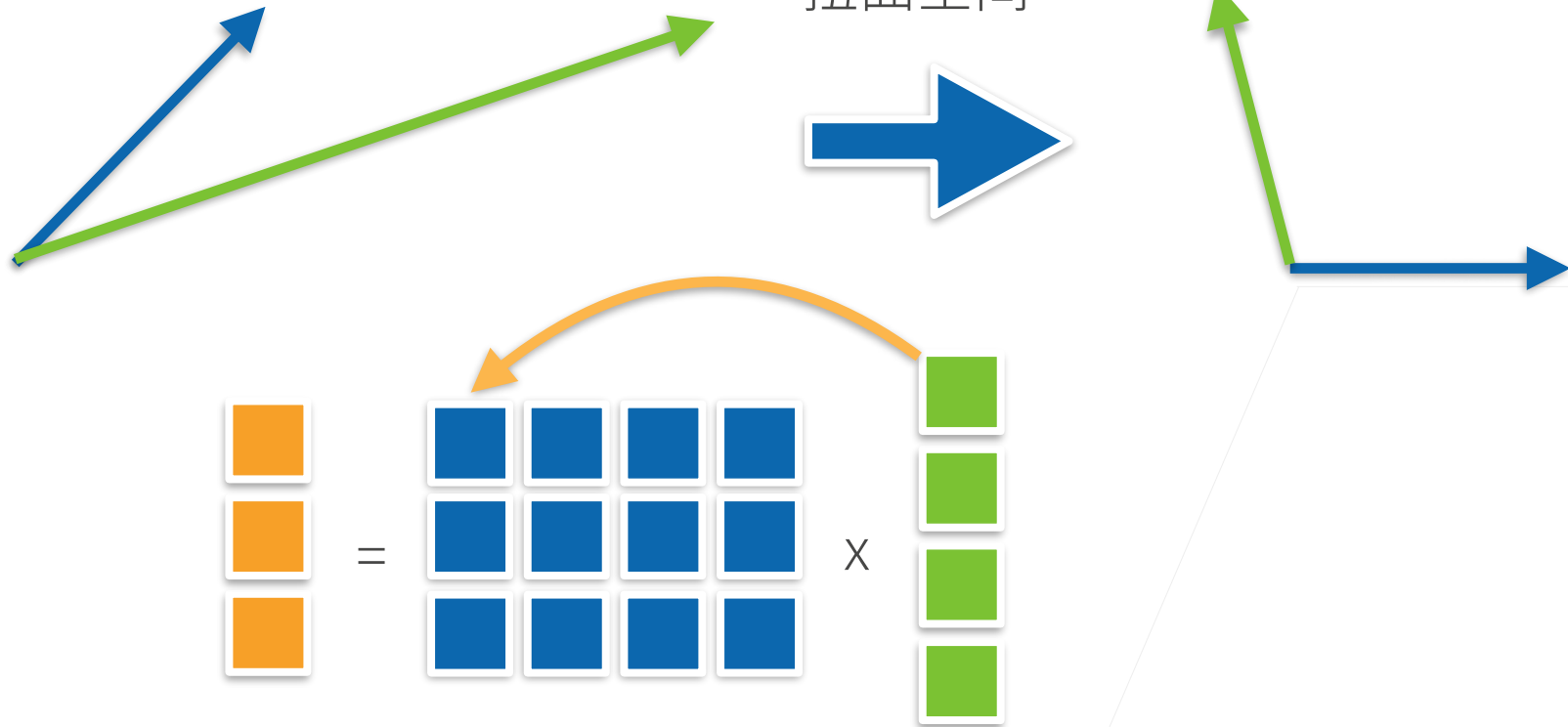
$$c = Ab \text{ where } c_i = \sum_j A_{ij} b_j$$



矩阵



扭曲空间

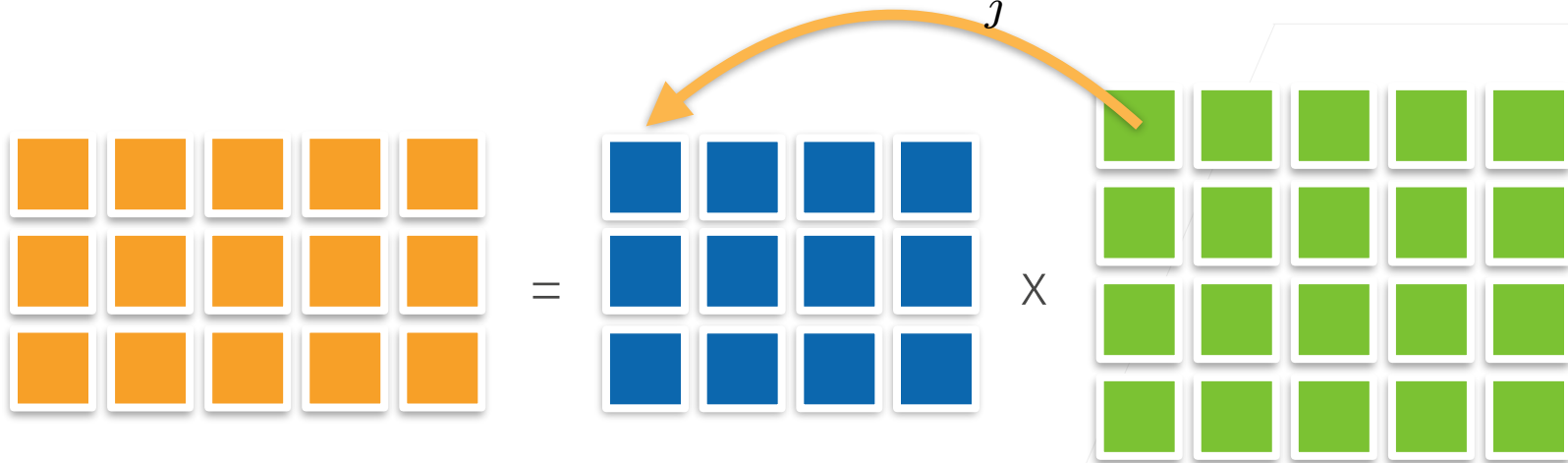


矩阵



- 乘法（矩阵乘以矩阵）

$$C = AB \text{ where } C_{ik} = \sum_j A_{ij} B_{jk}$$



矩阵



- 范数

$$c = A \cdot b \text{ hence } \|c\| \leq \|A\| \cdot \|b\|$$

- 取决于如何衡量 b 和 c 的长度
- 常见范数
 - 矩阵范数：最小的满足的上面公式的值
 - Frobenius 范数

$$\|A\|_{\text{Frob}} = \left[\sum_{ij} A_{ij}^2 \right]^{\frac{1}{2}}$$

特殊矩阵



- 对称和反对称



$$A_{ij} = A_{ji} \text{ and } A_{ij} = -A_{ji}$$



- 正定

$$\|x\|^2 = x^\top x \geq 0 \text{ generalizes to } x^\top A x \geq 0$$



特殊矩阵

- 正交矩阵

- 所以行都相互正交

- 所有行都有单位长度

U with $\sum_j U_{ij}U_{kj} = \delta_{ik}$

- 可以写成 $UU^T = \mathbf{1}$

- 置换矩阵

P where $P_{ij} = 1$ if and only if $j = \pi(i)$

- 置换矩阵是正交矩阵

矩阵



- 特征向量和特征值
 - 不被矩阵改变方向的向量

$$Ax = \lambda x$$

特征向量

- 对称矩阵总是可以找到特征向量

$$x^3 + x^2 + y^3 + z^3 + xyz - 6 = 0$$

$$\text{grad} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

$$\tan x \cdot \cot g x = 1$$

$$2x^2yy + y^2z = 2$$

$$Y_{i+1} = Y_i + b_i \cdot k_2$$

$$B = \begin{pmatrix} 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\sum_{i=0}^n (p_2(x_i) - y_i)^2$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$x - y + z = 1$$

$$x + \lambda y + z = \lambda$$

$$x + y + \lambda z = \lambda^2$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^3 + 1} + n}{\sqrt[n]{3n^2 + 2n - 1}}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$y = \sqrt[3]{x+1}; x = \tan t$$

$$x_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \beta \\ \beta \end{pmatrix}$$

$$\frac{\partial z}{\partial x} = 2; \frac{\partial z}{\partial y} = 0$$

$$\vec{n} = (F'_x, F'_y, F'_z)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}$$

$$|x| + |y| \neq 0; y \neq 0$$

$$\frac{\partial f}{\partial x} = 16 - x^2 + 16y^2 - 4z > 0$$

$$A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}, x=0, y=1, z=2$$

$$y' - \frac{\sqrt{y}}{x+2} = 0; y|_0$$

$$\cos p = \frac{(1,0)}{\sqrt{1}}$$

$$e^z - xyz = e; A[0; e; 1]$$

$$\frac{2x}{x^2 + 2y^2} = 2; z = \frac{1}{x} \text{ and}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\lambda_2 = i\sqrt{14}$$

$$f(x) = 2^{-x} + 1, \epsilon = 0.005$$

$$\alpha, \beta, \gamma \in \mathbb{C}$$

$$c = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$$

$$\delta(p_2) = \sqrt{0,16}$$

$$a^2 + b^2 = c^2$$

$$\lim_{n \rightarrow \infty} (1 + \frac{3}{n})^n$$

$$A = [1; 0; 3]$$

动手学深度学习 v2

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