Nama : Faranisa Ramadhani
NIM : 2011511009
Kelas : 01

Tugas Aljabar Linier

* Kalkulasi Eigen dan Vektor Eigen

2. $A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$

$$\lambda I - A = \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \lambda+5 & -2 \\ -2 & \lambda+2 \end{pmatrix}$$

$$\det(\lambda I - A) = \det \begin{pmatrix} \lambda+5 & -2 \\ -2 & \lambda+2 \end{pmatrix}$$

$$= \lambda^2 + 7\lambda + 6$$
$$= (\lambda+1)(\lambda+6)$$

$\lambda = -1 \qquad \lambda = -6$

* vektor eigen

$\lambda = -1$

$$\begin{pmatrix} 4 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4V_1 - 2V_2 = 0$$
$$-2V_1 - 4V_2 = 0$$
$$2V_1 = V_2$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \vec{v}$$

$\lambda = -6$

$$\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-V_1 - 2V_2 = 0$$
$$-2V_1 - 4V_2 = 0$$
$$V_1 = -2V_2$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \vec{v}$$

6. $A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$

$$(\lambda I - A) = \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda-1 & 0 & 2 \\ 0 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{pmatrix}$$

$$\det(\lambda I - A) = \left(\begin{array}{ccc|cc} \lambda-1 & 0 & 2 & \lambda-1 & 0 \\ 0 & \lambda-2 & -1 & 0 & \lambda-2 \\ -1 & 0 & \lambda-3 & -1 & 0 \end{array}\right)$$

$$= ((\lambda-1)\cdot(\lambda-2)\cdot(\lambda-3)) - (-2(\lambda-2))$$
$$= (\lambda-2)((\lambda-2)\cdot(\lambda-3)+2)$$
$$= (\lambda-2)(\lambda^2 - 4\lambda + 5)$$

$$D = b^2 - 4ac$$
$$= 16 - 4 \cdot 1 \cdot 5$$
$$= 16 - 20 = -4$$
$$\boxed{D < 0}$$
↳ akar imajiner

nilai eigen $\lambda = 2$ //

* vektor eigen

$\lambda = 2$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-V_1 - V_3 = 0$

$V_1 = -V_3$

misal :

$V_2 = t$

$V_3 = s$

$V_1 = -s$

$V_2 = t$

$V_3 = s$

$$\vec{v} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -s \\ t \\ s \end{pmatrix} = \begin{pmatrix} -s \\ 0 \\ s \end{pmatrix} + \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix}$$

$$= s\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\Rightarrow$ vektor reigen //

Diagonalisasi

1. $A = \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}$

$$(\lambda I - A) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{pmatrix}$$

$\det |A| = (\lambda - 3)(\lambda + 1) - 0$

nilai eigen $\lambda = 3$; $\lambda - 1$

* vektor eigen

$\lambda = 3$

$$\begin{pmatrix} 0 & 0 \\ -8 & 4 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$-8V_1 = -4V_2$

$8V_1 = 4V_2$

$2V_1 = V_2$

$$\bar{v} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\lambda = -1$

$$\begin{pmatrix} -4 & 0 \\ -8 & 0 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-4V_1 + 0V_2 = 0$

$V_1 = 0$

misal $V_2 = t$

$$\vec{v} = \begin{pmatrix} 0 \\ t \end{pmatrix} = t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$D = P^{-1} A P$$

$$= \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix} //.$$

2.

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$$

$$(\lambda I - A) = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda-3 & 1 & 2 \\ -2 & \lambda-0 & 2 \\ -2 & 1 & \lambda+1 \end{pmatrix}$$

$$\det(\lambda I - A) = \left(\begin{array}{ccc|cc} \lambda-3 & 1 & 2 & \lambda-3 & 1 \\ -2 & \lambda+0 & 2 & -2 & \lambda \\ -2 & 1 & \lambda+1 & -2 & 1 \end{array}\right)$$

$$= \left(\left((\lambda-3)(\lambda)(\lambda+1)\right) - 4 - 4\right) - \left((-4\lambda) + (2\lambda-6) + (-2\lambda-2)\right)$$

$$= (\lambda-3)(\lambda)(\lambda+1) + 4\lambda$$
$$= \lambda^3 + \lambda^2 - 3\lambda^2 - 3\lambda + 4\lambda$$
$$= \lambda^3 - 2\lambda^2 + \lambda$$
$$\lambda(\lambda-1)(\lambda-1)$$

$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 1$

* vektor eigen

$\lambda_1 = 0$

$$\begin{pmatrix} -3 & 1 & 2 \\ -2 & 0 & 2 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -3 & 1 & 2 & 0 \\ -2 & 0 & 2 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right) \begin{array}{l} b_1/-3 \\ b_2 - b_3 \\ 3b_3 - 2b_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1/3 & -2/3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & -3 & 0 \end{array}\right) \begin{array}{l} \\ b_2/-1 \\ b_3/-3 \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & -1/3 & -2/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1/3 & 1 & 0 \end{array}\right) \quad \begin{array}{l} v_1 - 1/3\, v_2 - 2/3\, v_3 = 0 \\ v_1 - 1/3\, v_3 - 2/3\, v_2 = 0 \\ v_1 - v_2 = 0 \end{array}$$

$v_1 = v_2$

$v_2 - v_3 = 0$

$v_2 = v_3$

$$\bar{v}_1 = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

untuk $\lambda = 1$

$$\begin{pmatrix} -2 & 1 & 2 \\ -2 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

jadi, $P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$

$$P^{-1} = \begin{pmatrix} -2 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

$$D = P^{-1} A P$$

$$= \begin{pmatrix} -2 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ tidak dapat didiagonalkan

- Bentuk kuadrat.

1. $x^2 + xy - y^2 = 1$

$$[x \quad y]\begin{bmatrix} 1 & 1/2 \\ 1/2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 1/2 \\ 1/2 & -1 \end{bmatrix} \qquad A - \lambda I = \begin{pmatrix} 1 & 1/2 \\ 1/2 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & 1/2 \\ 1/2 & -1-\lambda \end{pmatrix}$$

$$= \left((1-\lambda)(-1-\lambda)\right) - \left(1/2 \cdot 1/2\right)$$

$$= \lambda^2 - 1 - \frac{1}{4}$$

$$\lambda^2 = \frac{5}{4}$$

$$\lambda = \sqrt{\frac{5}{4}}$$

$$\lambda = \pm \frac{\sqrt{5}}{4}$$

* vektor reigen

$\lambda = \frac{\sqrt{5}}{4} = \frac{2,2}{4} = 0,55$

$$\begin{pmatrix} 1-0{,}55 & 1/2 \\ 1/2 & -1-0{,}55 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$0,45\ V_1 + 1/2\ V_2 = 0$

$1/2\ V_1 - 1,55\ V_2 = 0$

$1/2\ V_1 = 1,55\ V_2$

$V_1 = 3,1\ V_2$

$\vec{V} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 3.1 \\ 1 \end{pmatrix}$

$\lambda = -\frac{\sqrt{5}}{4} = -0,55$

$$\begin{pmatrix} 1{,}55 & 1/2 \\ 1/2 & -0{,}45 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$1,55\ V_1 + 1/2\ V_2 = 0$

$1/2\ V_1 - 0,45\ V_2 = 0$

$1/2\ V_1 = 0,45\ V_2$

$V_1 = 0,9\ V_2$

$\vec{V} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0,9 \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} 3,1 & 0,9 \\ 1 & 1 \end{pmatrix}$