Parametrization of Political Representation

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Abstract

In this paper we introduce a way to map systems of political representation into mathematical models, using a framwork borrowed by racional choice theory. This shall allow us to compare, how various sets of parameters and initial conditions lead to different outcomes in respect to the distribution of power, efficiency of the representation of the beliefs, and other indexes. This approach should in particular open a way to model long term development and robustness of new forms of political participation and representation stemming from social networks on the internet, in which the dynamic of feedback loops and immediate effects might or might not lead to unstable situations.

Keywords. Computational Social Science, Simulation, Political Representation, Rational Choice, Delegation, Liquid Democracy

1 Introduction

"Code is Law" is the catch phrase of Lawrence Lessig famous bestseller on the future of democracy [4]. And from the beginning of the Internet revolution, there has been the discussion, wether our new forms of media and communication would lead to another revolution, too: a political one. Many of the media and platforms that rose over last decade show aspects of communal or even social systems - and hence might be called Social Media with good cause. Thus it does not come as a surprise that we start to see the development of the communication platforms that are genuinly ment to support and at the same time to experiment with new forms of political participation, like Proxy-Voting or Liquid Democracy, which had been hardly conceivable without the infrastructure of the Web [3]. Since these new forms of presenting, debating, and voting for policies have been occuring just recently, we can expect that many other varieties will appear, new concepts to translate the internet paradigm into social decision making. But how do these new forms of voting work? Are they really mapping the "volonté generale" into decisions, and if so, in a sustainable, stable, continuous way? And how to evaluate on system compared to another?

Let's take Lessig by the word. Our goal is to use mathematical formalization to define and measure perfomance indicators of political systems, to contribute to answering these questions.

Rational Choice. Over the course of the last six decades, there have been various apporaches to find an adequate mathematical formalization of politics, like rational choice theory, economic theory of voting, or rational politics, modelling political choice into a game theoretic framework (e.g. [5],[7] XXX). As a framework to model behavior, economic theory and rational choice have been criticized as leaning towards neo-liberalism, overemphasizing individual freedom and playing down the influence of social conditions like structural power with their construct of the rational actor. However, constructs like actors' preferences, their choosing the individually most prefered alternative, proof - as we shall see - reasonably useful, to find models for our task to compare systems of political decision making on a very simple, reductionist level.

Set Theory. Representation and voting can also be abstracted into set theory, as e.g. shown by the french mathematician and philosopher Alain Badiou [2]. Representative systems are based on the idea that people within a set of people that is defined e.g. by geographic region like a constituency, are to be viewed as sufficiently homogenous regarding their interests and needs to allow one element of the set to represent all others - thus them all to be counted as one. In opposition to this count as one, Badiou places the elements all to remain a presentation of one, i.e. each element to be counted for themselves.

From this first abstraction into sets, we can still go further on. And since the emerging participation platforms are basically databases, it makes even more sense to abstract form the actual political model to a more formalized view of objects and relations. We will abstract the political actors (voters, delegates) into mathematical objects and map their preferences and behavior into parameters that influence their interacting. Finally we can view the whole political system as a directed network graph, changing dynamically over time.

Forming a Model. To achieve our goal to parametrize and simulate different political systems, we have to set some restrictions. First: We will work with the items of our model as mathematical objects, such as vectors and matrices; we will not try to interpret, what in humanities would be the *meaning* of an object. That is, we will not discuss e.g. what a 'person' or 'people' are; we will just define their properties that are effective in our system. This is applies also when we define the task of a political system.

Second: We will reduce interdependencies to simple functions, such as linear combinations, so we will get linear, local approximations to the unknown correlations. Third: we will look at the system without external forcing, we will not feed the effects that result from the politics (like environmental change, economic success, poverty etc.), driven by the system, back into the system.

"The raison d'être of politics is freedom."[1] - We are aware of the fact that our aproach is rather positivistic, and without - at least at this time - taking into account the implicit ethical assumptions that underly our definitions of democratic representation in particular. These will have to be discussed seperately.

2 Model and Parametrization

2.1 Definitions

We will now formalize the objects and relations of the political system as described above.

The Political System

The *political system* Φ is the set of specific parameters and the value of its variables, over all points in time.

Actors and Delegates. Be $A = \{a_1, a_2, ..., a_N\}$ the set of people with suffragage, then all $a_i \in A$ are called *actors*. Depending on the political system, there is a subset of Actors $A^I \subseteq A$ who bear the $right of initiative$, that is they may issue an initiative. If $A^I = A$ the political system is called $grassroots democracy$. A second subset of actors $A^E \subseteq A$ is *elligable* - that is an actor $a_i \in A^E$ can get the vote of other actors *delegated* to decide on initiatives in their behalf. If $A^I \subset A^D \subset A$ and $A^I \neq A$ we speek of a *parliamentary system*. A political system with $A^E \subsetneq A$ is called *oligarchy*. The subset $A^D \subset A^E$of actors who actually get a *delegation* are called *delegates*. A Political System is called $representative$ if there is a partition $\bar{A} = \{A_1, \ldots, A_n\}$, $\cup A_i = A$ with the majority of actors in one subset $a_i \in A_i$ delegating to one actor of their subset who now represents the whole subset. The subsets A_i are called *constituencies*.

A political system is called *delegative* if every actor can delegate their vote to every other actor. An actor may get any number of delegations, but there are two different methods how to pass a delegation further. First: An actor delegates not only his own vote but also passes along the delegation he recieved from other, building chains of delegations. This is called *Liquid Democracy*. Second: An actor sets up a list of prefered delegates. If the most prefered delegate would pass her vote herself to another actor, her incoming delegations would fall to the respectively next preferd delegate.

If the delegation can be set arbitrarily by the actors according to their individual parameters (that we will define in the next paragraph as an actor's beliefs), we call the politcal system a *democracy*. If the delegation is set without relation to the actor's beliefs, it will be called *authoritarian*.

Topics and Beliefs. The task of a political System Φ, is to make decisions on *political topics*, $J = \{J_1, ..., J_H\}$, where a topic J_κ is a set of possible policies $J_k = \{j_{k,1}, ..., j_{k,G}\}$. For every Actor a_i all policies in each topic can be assigned a preference with $b(a_i, j_{k,l}) > 0$ so that $j_{k,M}$ is the most preferable policy for a_i if $b(a_i, j_{k,M}) = \max_\lambda b(a_i, j_{k,\lambda})$ etc.

$B = (b(a_\iota, j_{\kappa,\lambda}))_{\kappa,\lambda}$ is thus a matrix, showing a value for each policy in every topic for an actor, which we could call the actor's *beliefs*.

Likewise we define a weighting for the believes with $r(a_i, J_k) \in [0, 1]$ that will represent the *relevance* of topic J_k for the actor and with $r = (r(a_i, J_\kappa))_\kappa$ we get the *relevant beliefs* $B_r = Br^T$ for actor a_i, and we can measure the distance of two actors by measuring the distance in their relevant believes $\|a_i - a_j\| = \|B_r(a_i) - B_r(a_j)\|$ by using an appropriate metric $\|.\|$. The beliefs and relevances may change over time.

Ballots and Initiatives. By means of an *initative* $I_{k,l,i} = (J_k, j_{k,,l}, a_i)$, an actor $a_i \in A^I$ submits a specific policy $j_{k,l}$ for ballot to the topic J_k. Of course for each initiative, the policy $j_{l,k}$ proposed by actor a_i who issued $I_{k,l,i}$, should rank highest in a_i's relevant beliefs. So, if $A^D = A^I$, we could drop the concept of initiatives, because our actors would issue all of their top-ranking beliefs anyway.

If there are other initiatives, covering the same topic but offering a different policy, that are issued to vote at the same ballot, these called an *alternatives* $I_k = \{I_{k,\lambda,\iota} \ for \ all \ \lambda, \iota\}$.

The actors $a_j \in A^D$ will decide in a ballot, that in a result when completed. The ballot assigns the ruling policy to its topic $bal(J_k) = j_{k,l}$.

Some topics may require a quorum $quorum(J_k)$, i.e. a minimum percentage of supporters $a_i \in A^I$ that declare their support for a specific initiative issued in this topic, which otherwise would not qualify for the ballot. Likewise, each topic is also assignet to a set of rules, how to calculate the majority with $majority(J_k)$ as the percentage of votes that would be needed. This rule can be a percentage for a simple poll without an alternative. Usually it would be set to 50% for standard polls and 2/3 for polls that would change the political system. If alternatives are present, there could be run-off ballots to decide, if no majority is achieved in the first ballot. However it is also possible to define majority rules as scales of preferences. An actor could put her prefered initiative on top of an ordered list, and likewise sort those alternatives most unfavorable in a list of preferences what to turn down. alternatives of equal preference could be tied. We have a function $pref(a_i, I_k)$ that assinges an actor's rank of preference to an initiative's ballot, she would take part.

Table 1: List of Preferences: Given a set of Alternatives a, b, c, d, e, f, the Actor votes for a as top preference, b, c tied on second rank, d abstained, e as second worst, and f as worst Alternative to be turned down.

Preference	Alternative
1	a
2	b,c
-	d
-1	e
-2	f

Various methods exist to calculate the result of such single transferable vote (cf [6, 8]).

Secret vote and accountable vote. How the votes that lead to the outcome of a ballot are accounted for, has to be set initially in the system's parameters. The vote is said to be *secret* if only the sums are recorded and no link from an initiative to the actors voting for/against it is visible, and *accountable*, if the link for each actor to the initiative she voted for, is stored and visible. In the first case, initiatives can only succeed or be turned down. In the latter case, the individual voting behavior of all actors that interacted with the Initiative has to be recorded as a variable with the actor $vote(a_i, I_k) = \begin{cases} 1 & for \\ -1 & against \\ 0 & abstained \end{cases}$ for the simple ballot, or $vote(a_i, I_k) = pref(a_i, I_k)$ for the single transferable vote.

Publicity. We could define the system Φin a way such the relevant beliefs of the actors are generally visible from one to another. Alternatively, we could set the condition that the only visibility of beliefs would be through initiatives issued or supported.

2.2 The network perspective

We can draw a directed network graph with actors as nodes and edges that connect the actors. Also we define a set of weights $W = \{w_1, ..., w_\omega\}$ for nodes and edges. This network of actors could be called the *society*.

The nodes (i.e. actors) can weighted, e.g. by

1. n_i^1 the number of delegations an actor a_i has received,
2. n_i^2 the number of initiatives issued by one actor a_i had been successful,
3. n_i^3 based on the actor a_i 's activity (e.g. how many initiatives, how many ballots, etc.)

4. n_i^4 a function of topics' relevance. The shape of this function will impact the stability and vividness of the system. We could set $w^{\mathbf{3}} = mean(r(j))$ which would correspond to something like "general political interest" but it could as well be set to $w^{\mathbf{3}} = max(r(j))$ which would represent actors that have at least one prominent belief (like political activists and agitators) as the highest weighting.

5. n_i^5 psychological and social factors that would make actor a_i more or less attractive to get delegations (like social capital), which would not be built into the model and thus would have to be added as random values of specific distributions.

A first set of edges could be formed by the delegations, the edges getting a direction $a_i \to a_j$ if a_i delegates to a_j. A second set of edges would be the voting of a_i for an initiative of a_j (or voting against respectively). Both sets connect the actors can also be weighted:

1. e_{ij}^1 the number of delegations passed from actor a_i to a_j (if applicable),

2. e_{ij}^2 the distance which e.g. could simply be the Eukledian distance of the two actors' relevant beliefs $\|a_j - a_i\| = \sqrt{\frac{\sum_{\kappa,\lambda} \left(b(a_i, p_{\kappa,\lambda}) - b(a_j, p_{\kappa,\lambda})\right)^2 r(a_\iota, P_\kappa)}{\sum_{\iota,\kappa} r(a_\iota, P_\kappa)}}$.

e_{ij}^3 the distance of an actors beliefs with the initiatives of another actor $\|a_j - a_i\| = \sqrt{\frac{\sum_{\kappa, p_{\kappa,\lambda} \text{ was issued by } a_j} \left(b(a_i, p_{\kappa,\lambda}) - b(a_j, p_{\lambda,\mu})\right)^2 r(a_\iota, P_\kappa)}{\sum_{\iota,\kappa} r(a_\iota, P_\kappa)}}$

Participation. An actor can participate per active and passive vote. Passive vote means that an actor a_i would run for candidate in an election and/or would issue an initiative, and thus asks others to vote for him or his initiative or delegate their vote to him respecively. Active vote means that an actor a_j votes for/against an Initiative or delegates her vote to another actor. The distribution of the actors' propensity $p^{active}(a_i)$ for an actor a_i to participate actively or $p^{passive}(a_i)$ to participate passively could be depending merely on individual psychological factors (we would then represent as random noise), but we could also let it be a function of the degree, an actor's relevant beliefs have been matched by the policies that have been decided in the past. We could make it more likely for actors with high distance to the ruling policies of the political system to get active themselves, as well as the opposite case, where actors who were successfully represented with their relevant beliefs and thus get motivated to carry on -or any mixture of that. In any case the feedback of an actor's previous successes and their position in the society should play a role. Hence we could have $p^{active}(a_i) = f(w)$ with some weights $w_\iota$from above.

The expected turnout for an election would thus be $\frac{1}{N}\sum_{\iota} p^{active}(a_{\iota})$.

Delegations and elections. The act of delegation might be carried out only at specific points in time, called *elections*, between which the delegation remains fixed -this timespan is called *legislative period.* During a legislative period, the delegates decide on initiatives on behalf of the actors they represent (which might be those who directly delegated their vote to them or which had to delegate their vote because they would belong to the delegate's constituency).

In a democracy, the variable DEL should be influenced by the relations in the society, notably the actors' relevant beliefs and the policies, the delegates try to establish via the initiatives they issue, as well as their success in pushing these policies, hence the weights e_{ij} of the edges that connect the voting actor with the actors that candidate. And of course the delegation should also be influenced by the personal weights of the candidates' nodes n_j.

Legislation. Since issuing initiatives will consume time and therefore the delegates might not be able to issue all policies they might want to, the actors should start issuing the policies of their most relevant topic first. Thus it may occur, that an actor a_j who was delegated for his proximity to a_i by this actor's vote, disappoints a_i because a_j just did not issue the policies, that accounted for the percieved proximity.

3 Simulation

3.1 Initial Condidtions

Appart from setting up the political system's immediate parameters and rules, a simulation can start under various condidtions:

Complexity. How many topics J_k and how many policies $j_{k,l}$ for each topic exist? Are (some) preferences correlated, or are policies even mutually exclusive?

Volatility. How sticky are the topics? I.e. how often can one topic be up for vote? Or if there is no restriction: how is the mean and variance of topics coming up for revision? How often can delegations be reassigned, i.e. how long is the legislative period?

Variance and spacial structure.. How homogenous are the Actors regarding their beliefs? Are there minor differences in ranking Policies? Are there distinct groups of Actors with high correlation of preferences within the groups and large differences between, and how many of those groups are there?

Realistically, we can define a set $\hat{A}(a_i)$ of all actors that influence a_i. That could e.g. be the constituencies. In a society with instantanous commuication, we could get $\hat{A}(a_i) = A$ though.

Elasticity. How elastic do actors react on the system's changes, as represented in the weights? How do events past affect the weights? (What is the decay function?)

Sensitivity. What metric defines, how initiatives of one actor are ranked by another actor? Thus how sensitive is one's weighing regarding mean or extreme differences to another actor's belief system?

Arbitrariness. How likely do the actors change their beliefs at random (respectively triggered by external forcing that is not observed within the model)?

Autocorrelation and lagging. To what extent do previous states of the system influence the future state? And how does this influence decay over time?

3.2 Dynamics

Changes in Φ shall occur only at discrete points in time that form a system clock $t_n \in T = \{t_0, t_1, ...\}$. Most of the system's variables are dependent of values of past states of themselves in particular, and the whole system's state in general. We write all the systems values $v_1(a_\iota)$ to $v_\omega(a_\iota)$ that are dependent from the actors a_1 to a_N as well as the values $v_{\omega+1}$ to v_Ω that are not depending from actors, in one vector $\phi(n) = (v(a_1), v_1(a_2), ..., v_2(a_1), ...v_\omega(a_N), v_{\omega+1}, ..., v_\Omega)^T$. The transformation from one state to the next will be not only depending from the previous state, but from older states as well. When we set the time lag in the effect of some variables up to p periods, we have a $VAR(p)$ vector autoregressive model for a multivariate time serries $(\phi_\tau)_\tau = (\phi_{n-1}, \phi_{n-2}, ..., \phi_{n-p})$ which we write $\phi_n = \eta + \sum_{i=n-p}^{n-1} D(i)\phi(i)$.

3.3 Comparing Political Systems

Efficiency. A Political System should transport the Actor's beliefs into political decisions. We can define a measure of *efficiency* to describe how well a Political System would fulfill this task. Since we shall calculate such measures in our example 4, we will write out the formulae.

Let $P_{k,pref} = \left\{ p_{k,\lambda} | B(a_\iota, p_{k,l}) = \max_l \{B(a_\iota, p_{\kappa,\lambda})\} \right\}$ be the most prefered policies for topic J_k for all actors and $P_{k,dec} = \{p_{k,l} | j_{k,l} = BAL(J_k)\}$those policies, that have been successful in the ballot.

Accordingly we can now calculate the efficiency of the system by calculating the mean of beliefs of the policies that had been successful devided by the mean of all beliefs $e'_\Phi = \dfrac{\sum_{\iota,\kappa,p_{\kappa,\lambda} \in P_{\kappa,pref}} b(a_\iota, p_{\kappa,\lambda})}{\sum_{\iota,\kappa,p_{\kappa\lambda} \in P_{\kappa,dec}} b(a_\iota, p_{\kappa,\lambda})}$. We should of course bring the relevance into the measure, which we could do by setting $e''_{\Phi} = \frac{\sum\limits_{\iota,\kappa,p_{\kappa,\lambda}\in P_{\kappa,pref}} b(a_\iota, p_{\kappa,\lambda}) r(a_\iota, P_\kappa)}{\sum\limits_{\iota,\kappa,q\kappa\lambda\in P_{\kappa,dec}} b(a_\iota, q_{\kappa,\lambda}) r(a_\iota, P_\kappa)}$ to the ratio of the weighted means - which would become one for a perfectly efficient system. Furthermore, we could calculate the relevance-weighted mean difference $e'''_{\Phi} = \frac{\sum\limits_{\iota,\kappa,p_{\kappa,\lambda}\in P_{\kappa,pref},q_{\kappa,\mu}\in P_{\kappa,dec}} (b(a_\iota, p_{\kappa,\lambda}) - b(a_\iota, q_{\lambda,\mu}))\, r(a_\iota, P_\kappa)}{\sum\limits_{\iota,\kappa} r(a_\iota, P_\kappa)}$ between the prefered policies and the successful ones (which of course would be the smaller, the more efficient the system gets). If we would not have a metric scale of beliefs, we could as well use the difference in ranks. Accordingly we would set $e_\Phi(a_\iota)$ as the efficiency just caluculated for a single actor a_ι.

Fairness. Efficiency shows us, how well a Political System represents the average beliefs, while Fairness shows, how homogenous the representation is.

We could set $f'_\Phi = Var(e(a_{iota}))$ to the variance over the $e(a_{iota})$, or we take $f''_\Phi = \frac{\sum\limits_{\iota} \max\limits_{[\kappa,p_{\kappa,\lambda}\in P_{\kappa,pref},q_{\kappa,\mu}\in P_{\kappa,dec}]} (b(a_\iota, p_{\kappa,\lambda}) - b(a_\iota, q_{\lambda,\mu}))\, r(a_\iota, P_\kappa)}{\sum\limits_{\iota,\kappa} r(a_\iota, P_\kappa)}$ the mean maximal deviation of the actors' preferences to the outcome of the system. We could also look at the distribution and the histogram of the differences $b(a_\iota, p_{\kappa,\lambda}) - b(a_\iota, q_{\lambda,\mu})$.

Concentration. The concentration of delegative power, measured by one of the common metricses of concentration, like the Herfindahl-Index. If $v(a_i)$ is the number of votes an actor a_i has in a ballot, we define $H = \sum\limits_\iota \left(\frac{v(a_\iota)}{N}\right)^2$ which would give $H = \frac{1}{N^2}$ for the grassroots democracy, and $H = 1$ for a dictatorship.

Stability. The stability of a political system can be measured in various dimensions.

1. Continuous Efficiency, measured by the change of the system's efficiency during a period of time $\frac{\mathrm{d}e}{\mathrm{d}t}$, or some trend of the system's efficiency $e(t)$ over time (e.g. the slope of the linear trend)
2. Continuous Fairness, measured likewise.
3. Change in delegative concentration.

Vividness. The system should stay dynamic. If e.g. some delegates get dominant over an ongoing time span, the system might get to a halt.

We could measure the vividness if we look at the changing rates of participation $\frac{\mathrm{d}p}{\mathrm{d}t}$ for $p^{passive}_\Phi = \sum p^{passive}(a_\iota)$ and $p^{active}_\Phi = \sum p^{active}(a_\iota)$.

Table 2: Actors, topics with their relevance and beliefs for our example

Scalability. The properties of a political system should neither depend on the number of actors participating nor on changes in the number of actors given a sufficiently large number of actors througout the system's progress.

Network indexes. We could also look at the various measures that characterize a network, like centrality, degree, random walk, etc.

3.4 Optimization and Interpretation

All political systems with their related parameters span a multidimensional phase space. This should allow for optimization of the parameters to find the locus of "the best systems" for given initial or boundary conditions -at least numerically. An interesting question is, wether there are parameters or boundary conditions that would render the trajectories diverging in a chaotic way or keep them sufficiently linear.

We could not only optimize the full model but also find local optima under constraints or optimize partially e.g. by keeping all parameters except one fixed. Questions we would like to answer with partial optimiziation and constraints would be, if there are political systems that are optimal e.g. for a certain cardinality of the set of actors.

4 Example

Let's build two very simple versions of political systems with just seven actors $a_1, ..., a_7$, three topics $\{J_1, J_2, J_3\}$ and three policies for each topic, thus $\{j_{1,1}, j_{1,2}, \ldots, j_{3,3}\}$. We shall initialize the relevance $r(J_\kappa)$ for the topics, the actors' beliefs $B(j_{\kappa,\lambda})$ for all policies, and also the actors' motivation to run as candidate for an election $w^{\mathbf{1}}(a_\iota)$ with random numbers, equally distributed:

Actor a_ι	Topic J_κ	$r(a_\iota, J_\kappa)$	Policy $p_{\kappa,\lambda}$	$B(a_\iota, p_{\kappa,\lambda})$	Br	Rank of $p_{\kappa,\lambda}$ in J_κ
ι	κ		λ			
1	1	0.95	1	1.35	1.28	3
			2	2.47	2.33	2
			3	2.54	2.40	1
	2	0.78	1	0.19	0.15	3
			2	2.49	1.94	1
			3	1.75	1.36	2
	3	0.68	1	1.95	1.33	2

Actor a_ι	Topic J_κ	$r(a_\iota, J_\kappa)$	Policy $p_{\kappa,\lambda}$	$B(a_\iota, p_{\kappa,\lambda})$	Br	Rank of $p_{\kappa,\lambda}$in J_κ
			2	1.10	0.75	3
			3	2.67	1.82	1
2	1	0.86	1	0.88	0.76	2
			2	2.03	1.74	1
			3	0.67	0.57	3
	2	0.44	1	0.90	0.39	3
			2	1.48	0.64	2
			3	2.40	1.04	1
	3	0.28	1	1.46	0.41	3
			2	1.71	0.48	2
			3	2.20	0.62	1
3	1	0.52	1	0.20	0.10	3
			2	2.59	1.35	1
			3	1.50	0.78	2
	2	0.02	1	0.11	0.00	3
			2	2.08	0.04	2
			3	2.51	0.05	1
	3	0.37	1	1.75	0.65	1
			2	1.60	0.60	2
			3	0.16	0.06	3
4	1	0.20	1	1.22	0.25	3
			2	2.36	0.48	1
			3	1.87	0.38	2
	2	0.93	1	2.72	2.53	1
			2	2.22	2.06	3
			3	2.25	2.09	2
	3	0.20	1	1.49	0.29	2
			2	2.70	0.53	1
			3	0.03	0.01	3
5	1	0.52	1	0.67	0.35	1
			2	0.40	0.21	2
			3	0.04	0.02	3

Actor a_ι	Topic J_κ	$r(a_\iota, J_\kappa)$	Policy $p_{\kappa,\lambda}$	$B(a_\iota, p_{\kappa,\lambda})$	Br	Rank of $p_{\kappa,\lambda}$ in J_κ
	2	0.21	1	2.52	0.53	1
			2	1.59	0.34	3
			3	1.81	0.38	2
	3	0.52	1	2.17	1.14	1
			2	0.97	0.51	2
			3	0.75	0.39	3
6	1	0.50	1	0.57	0.29	3
			2	0.66	0.33	2
			3	1.98	1.00	1
	2	0.46	1	2.06	0.95	1
			2	1.35	0.62	3
			3	1.42	0.65	2
	3	0.74	1	1.79	1.33	1
			2	0.26	0.20	3
			3	0.87	0.65	2
7	1	0.37	1	2.67	0.98	1
			2	2.03	0.74	2
			3	1.81	0.66	3
	2	0.41	1	0.43	0.18	2
			2	0.05	0.02	3
			3	2.38	0.97	1
	3	0.31	1	2.39	0.73	1
			2	2.08	0.64	2
			3	1.29	0.39	3

We now compare two political systems.

Grassroot democracy. Let $\Phi_1$be a grassroot democracy where every actor has suffragage and bears the right of initiative but without delegation. So every actor issues their highest ranking policy as alternative for the topics (duplicate initiatives are avoided, since actors would just support an alternative, that would also cover their prefered policy). The relevant beliefs for the policies are shown in Table 4. Of course every actor shall then vote for his most prefered policy. The ballot should only require simple majority of votes. This leads to the following results (Tabel : 4):

Table 4: Results of the grassroot ballot

Table 5: Actors' motivations

Actor a_ι	$n(a_\iota)$	Rank
ι		
1	0.80	1
2	0.52	3
3	0.30	7
4	0.44	4
5	0.42	5
6	0.57	2
7	0.36	6

Topic	J_1			J_2			J_3		
Actor $a_\iota(\iota)$	$j_{1,1}$	$j_{1,2}$	$j_{1,3}$	$j_{2,1}$	$j_{2,2}$	$j_{2,3}$	$j_{3,1}$	$j_{3,2}$	$j_{3,3,}$
1			1		1				1
2		1				1			1
3		1				1	1		
4		1		1				1	
5	1			1			1		
6			1	1			1		
7	1					1	1		
Total	2	3	2	3	1	3	4	1	2

For J_2we have a tie of $j_{2,1}$with $j_{2,3}$, so there will be a run-off ballot. Since $j_{2,3}$ is still second best for a_1, we shall get $BAL(J_2) = j_{2,3}$ in the second ballot.

For topic $J_1$the policy $p_{1,2}$ is only prefered by actors a_2, a_3 and a_4, while for a_1, a_5, a_6 and a_7, it is only second best choice. We get the efficiency $e_{\Phi_1} = \frac{0.93}{1.10} = 0.85$. For the concentration, we get of course a grassroots' Herfindahl-Index $H = \frac{1}{49} \approx 2\%$

Representative democracy. In our second political system Φ_2, there is an election where we actors delegate their vote. Since hardly everyone will want to run a campaign, a weight $n(a_\iota)$ comes into the model that could represent motivation of actor a_ι to expose herself in an election:

We set the threthold to the first three ranks of motivation - thus a_1, a_2, a_6 would run for election, a_3, a_4, a_5 and a_7 abstain from trying and keep to only go voting. Every actor votes for the candidate closes to him regarding his relevant beliefs. We calculate the Euklidean distance $\|Br(a_i) - Br(a_j)\|^2$ as shown in Table 6.

Actors running as candidate should always vote for themself. To define the attraction $Q(a_i, a_j)$ of candidate a_j to voter a_i we set as the proximity (i.e. the inverse of the distence), weighted by the "psychological factor", thus

Table 6: Distance $\|Br(a_i) - Br(a_j)\|^2$ between the actors according to their relevant beliefs.

ιActora_ι	1	2	3	4	5	6	7
1		8.19	13.93	19.18	16.86	11.52	11.91
3	8.19		2.50	9.92	3.93	3.77	1.67
7	13.93	2.50		15.65	2.76	3.66	2.15
4	19.18	9.92	15.65		10.93	8.64	11.96
5	16.86	3.93	2.76	10.93		1.51	1.84
2	11.52	3.77	3.66	8.64	1.51		2.44
6	11.91	1.67	2.15	11.96	1.84	2.44	

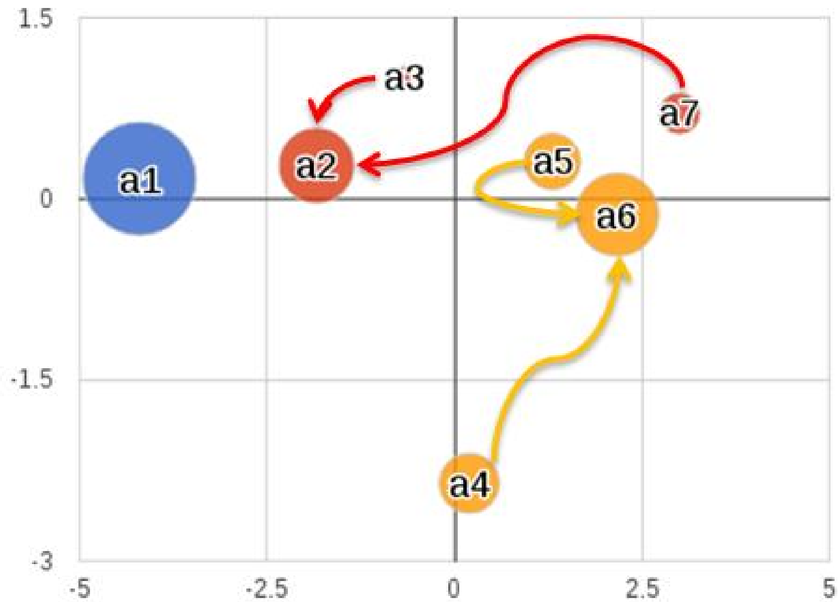
Table 7: $Q(a_i, a_j)$

ιActora_ι	1	2	3	4	5	6	7
1		9,79	5,76	4,18	4,76	6,96	6,74
2	6,40		20,98	5,29	13,35	13,92	31,51
3	2,18	12,16		1,94	11,02	8,31	14,18
4	2,31	4,46	2,83		4,05	5,13	3,70
5	2,49	10,67	15,18	3,83		27,83	22,72
6	4,94	15,10	15,54	6,59	37,79		23,36
7	3,03	21,64	16,79	3,01	19,54	14,80	

$Q(a_i, a_j) = \dfrac{n(a_j)}{\|Br(a_i) - Br(a_j)\|^2}$, so in this system, motivated actors are more attractive to the voters. By introducing a second weight ("social attractiveness") we could have made this relation asymetric, too.

The result is a set of delegations $a_1 \mapsto a_1, a_2 \mapsto a_2, a_3 \mapsto a_2, a_4 \mapsto a_6, a_5 \mapsto a_6, a_6 \mapsto a_6, a_7 \mapsto a_2$. as shown in Fig. 1.

Figure 1: Delegations



Actors are weighted by $n(a_\iota)$ and placed by multidimensional scaling on their Euklidean distances.

The delegates now have session in parliament where they issue their initiatives. For J_1, $a_1$and a_6 vote for $j_{1,3}$ while $a_2$votes for her prefered policy $j_{1,2}$. Since $a_1$and a_6 together have four votes, we get $BAL(J_1) = j_{1,3}$. For J_2we have a tie of $j_{2,1}$with $j_{2,3}$, so there will be a run-off ballot. Since $j_{2,3}$ is still second best for a_1, we shall get $BAL(J_2) = j_{2,3}$ in the second ballot, the third give us $BAL(J_3) = j_{3,3}$ with majority of the four votes of a_1 and a_2.

The efficiency for the second system is $e_{\Phi_1} = \dfrac{0.78}{1.10} = 0.71$, the concentration $H = \left(\frac{1}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{3}{7}\right)^2 = \frac{19}{49} \approx 39\%$.

5 Outlook

The outcome of this extremly simple model is not surprising. If we have just one iteration - no chance in re-electing the delegates if the voters are dissatisfied with their representation; we have no influencing of actors due to their social capital, no network effect, etc. So the next step in exploring this model democracies should be simulating more complex societies with their network effects and feedback loops over more iterations. Running many simulations with different initial conditions for the parameters will show, what system is stable and efficient, and what system tends to destabilize. It could also be uncovered, which distortions and constellations could crash a system.

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