

Simulation Exercise_Statistical Inference Project_Part 1

Overview:

The Exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The purpose of this data analysis is to investigate the exponential distribution and compare it to the Central Limit Theorem. For this study, `lambda` will be set = 0.2 for all simulations. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. We will investigate the distribution of averages of 40 exponentials by doing over thousand simulations.

Simulations:

```
set.seed(394651)
lambda <- 0.2
#n is the number of exponentials
n <- 40
number_of_simulations <- 1000
```

Create a matrix with 1000 simulations(1000 rows) each with 40(columns) samples drawn from the exponential distribution.

```
newdata <- matrix(rexp(n * number_of_simulations, rate = lambda), number_of_simulations)
```

Calculating the mean

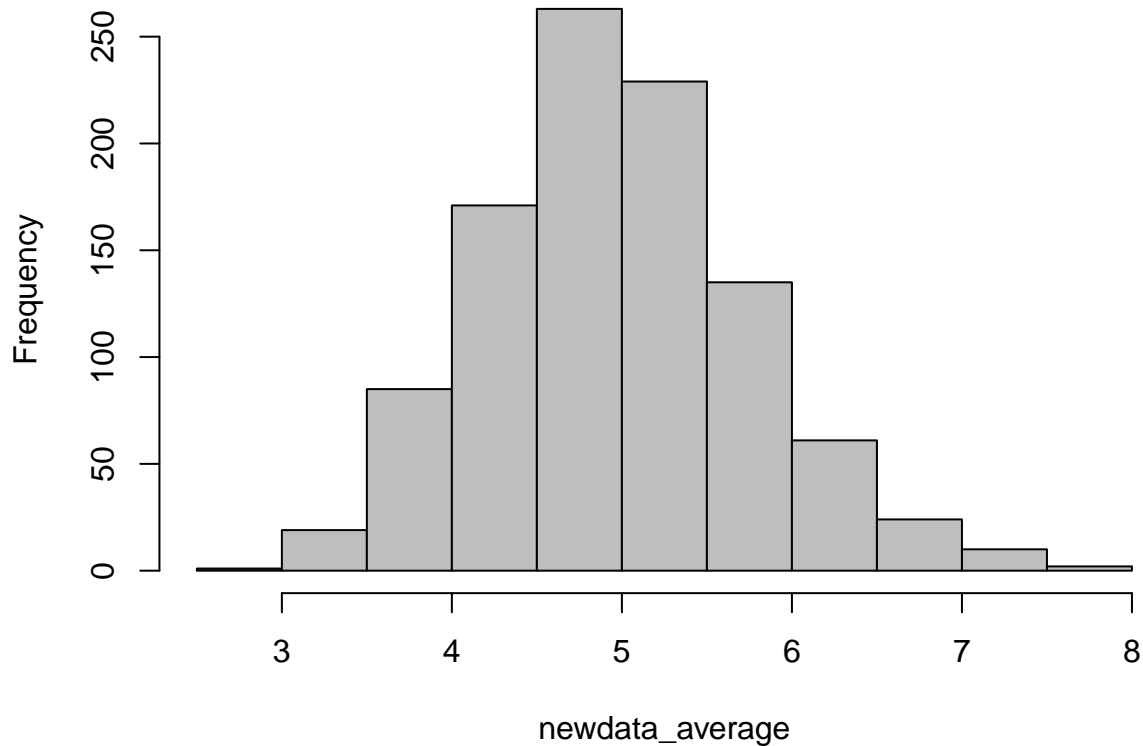
```
newdata_average <- rowMeans(newdata)
```

Sample Mean versus Theoretical Mean:

Show the sample mean and compare it to the theoretical mean of the distribution.

```
SampleMean <- mean(newdata_average)
hist(newdata_average,col="gray", main="Histogram of Sample Mean",adj = 0.5)
```

Histogram of Sample Mean



```
paste('Sample mean is', round(SampleMean,3))  
  
## [1] "Sample mean is 4.97"  
TheoreticalMean <- 1/lambda  
paste('The Theoretical Mean is', round(TheoreticalMean,3))  
  
## [1] "The Theoretical Mean is 5"
```

Based on the values of the Sample Mean and Theoretical Mean, we can say that both means are very close.

Sample Variance versus Theoretical Variance:

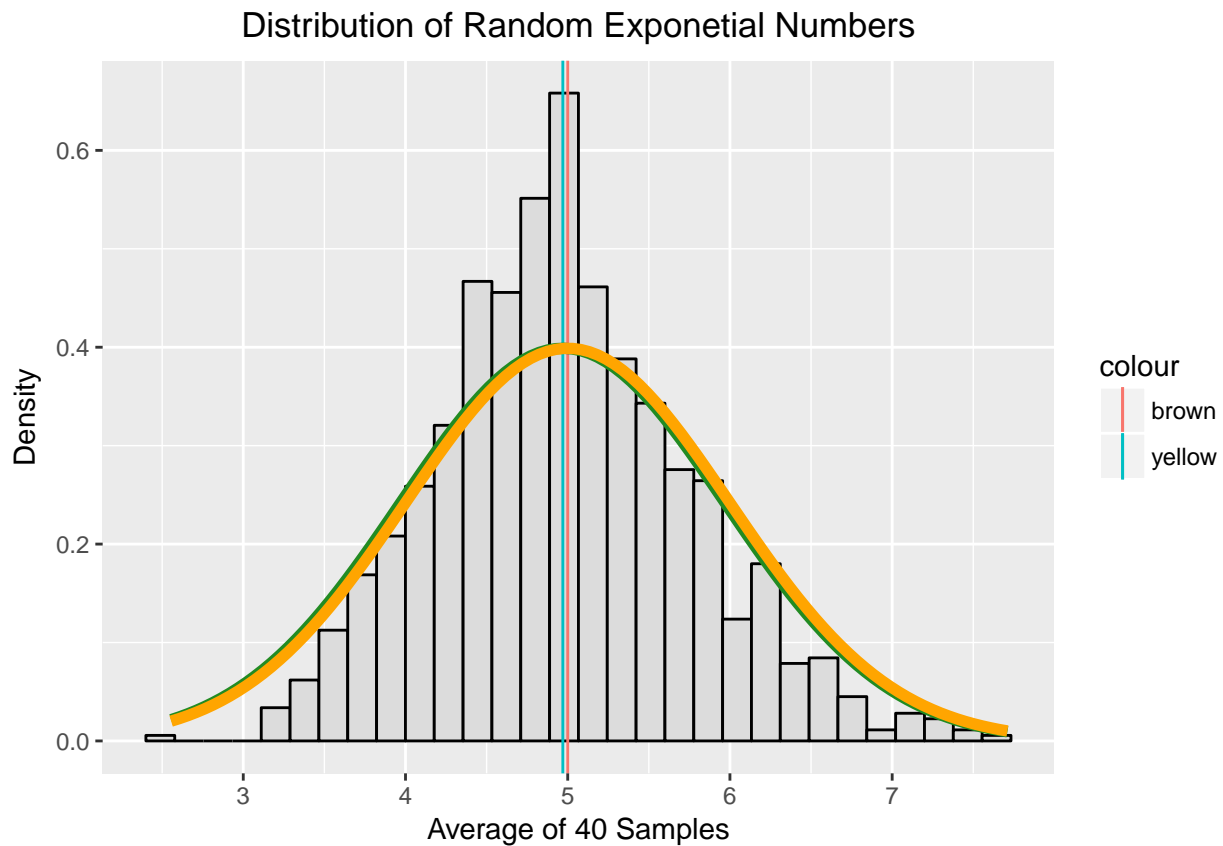
```
SampleVariance <- var(newdata_average)  
TheoreticalVariance <- (1 / lambda)^2 / (n)  
  
## [1] "The Sample Variance is 0.61"  
## [1] "The Theoretical Variance is 0.625"
```

Based on the values of the above, we can say that the Sample Variance and the Theoretical Variance are very close.

Distribution

```
library(ggplot2)
newdata_average_df <- data.frame(newdata_average)
ggplot(newdata_average_df, aes(x =newdata_average)) +
  geom_histogram(aes(y=..density..), colour="black",fill="gainsboro") +
  labs(title = "Distribution of Random Exponential Numbers", x = "Average of 40 Samples", y = "Density")
  theme(plot.title = element_text(hjust = 0.5))+
  geom_vline(aes(xintercept = SampleMean, colour = "yellow")) +
  geom_vline(aes(xintercept = TheoreticalMean, colour = "brown"))+
  stat_function(fun = dnorm, args = list(mean = SampleMean), colour = "forestgreen", size = 2.0)+
  stat_function(fun = dnorm, args = list(mean = TheoreticalMean), colour = "orange", size = 2.0)
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



Conclusion

1. From the plot above, we observe both distributions almost overlap each other.
2. The distribution of sample means follows a normal distribution
3. also see that the curve resembles a bell curve or normal distribution.

Hence based on the findings above we can state that the distribution does indeed demonstrate Central Limit Theorem.