Normal Distribution

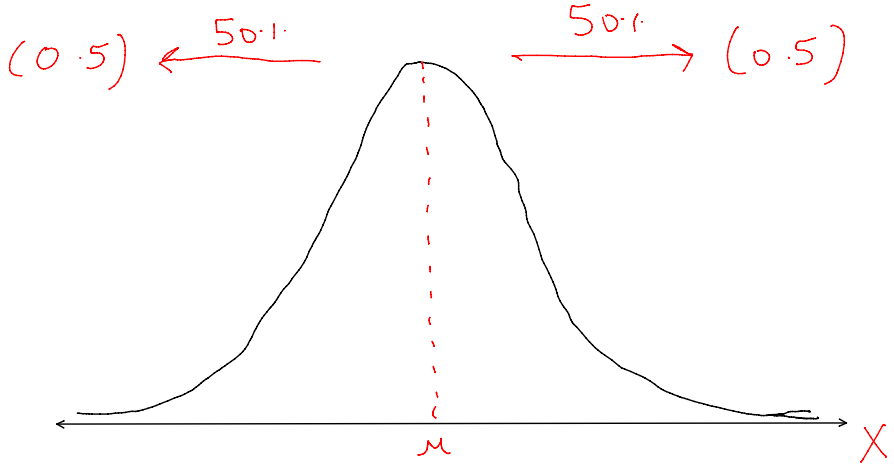
30 December 2023 09:22 AM

* It is a continuous probability distribution
* X is said to be normally distributed if the probability density function is given by

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

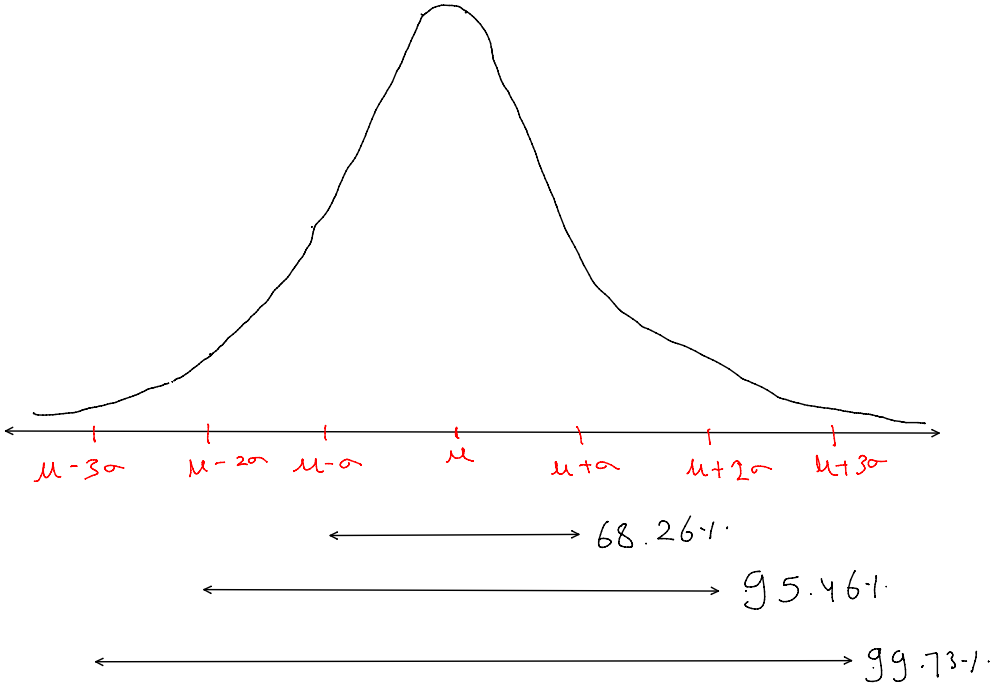
$$N(X) \cong N(\mu, \sigma^2)$$

* BELL SHAPED CURVE
* SYMMETRIC ABOUT MEAN



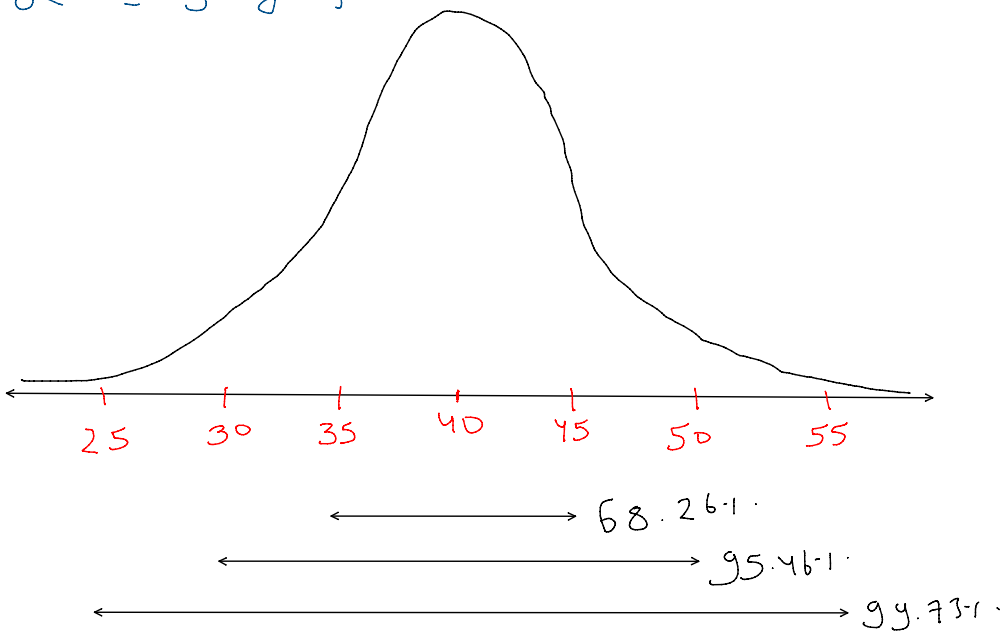
* TOTAL AREA UNDER THE CURVE = 1
  - AREA UNDER THE LEFT HALF = 0.5
  - AREA UNDER THE RIGHT HALF = 0.5

* 68.26% of data is between $(\mu-\sigma)$ to $(\mu+\sigma)$
* 95.46% of data is between $(\mu-2\sigma)$ to $(\mu+2\sigma)$
* 99.73% of data is between $(\mu-3\sigma)$ to $(\mu+3\sigma)$



Ex: Ages of employes in an organization

- μ = 40 years
- σ = 5 years



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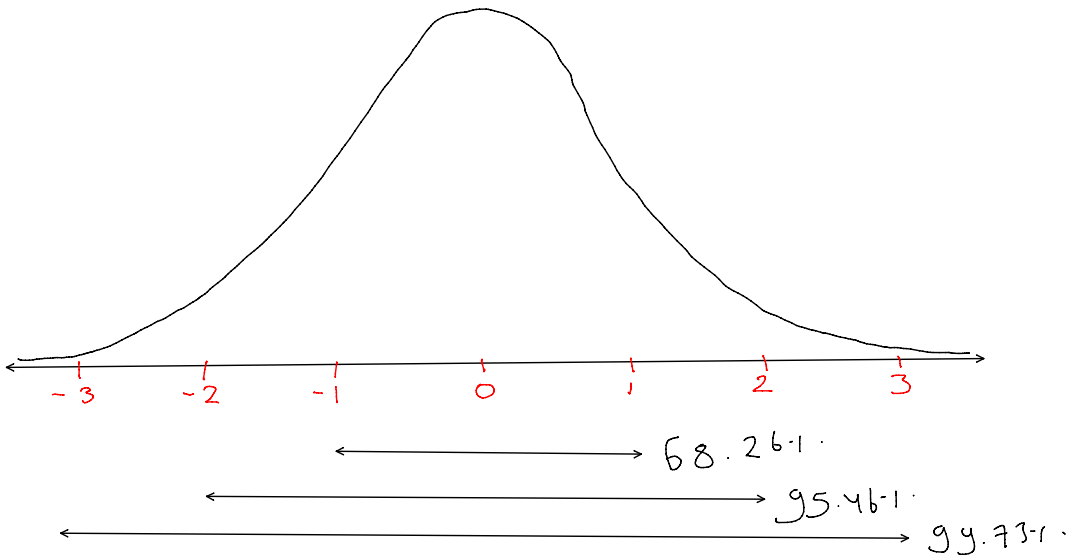
* STANDARD NORMAL DISTRIBUTION:

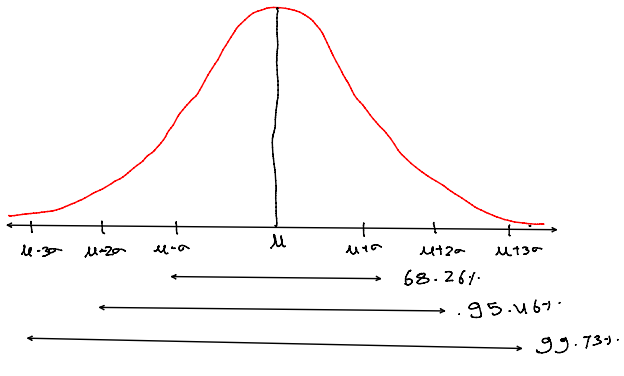
* It is the distribution of z scores of a normal random variable X: (N(0,1))

$$Z = \frac{X - \mu}{\sigma}$$

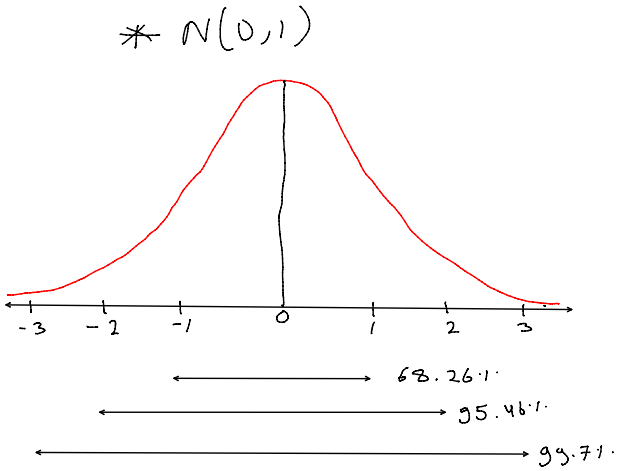
- THE DISTRIBUTION WILL BE NORMAL
- THE MEAN IS 0
- THE VARIANCE (AND STANDARD DEVIATION) IS 1
- $N(0,1)$



* $N(\mu, \sigma^2)$



* $N(0,1)$



Normal Distribution Recap

- Normal Distribution is defined by Mean and Variance: N(μ, σ^2)
- Bell Shaped Curve
- Symmetric About mean
- Total area under the curve is 1 (0.5 left of mean, 0.5 right of mean)

- 68 % of data is between (μ - σ) to (u + σ)
- 95 % of data is between (μ - 2* σ) to (μ+ 2* σ)
- 99 % of data is between (μ - 3* σ) to (μ + 3 * σ)

Standard Normal Distribution Recap

- N(0,1)
- Bell Shaped Curve
- Symmetric About mean
- Total area under the curve is 1 (0.5 left of mean, 0.5 right of mean)

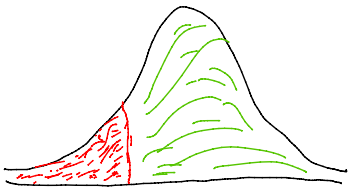
- 68 % of data is between -1 to +1
- 95 % of data is between -2 to +2
- 99 % of data is between -3 to +3

* PRACTICE :-

> $P(Z \leq 2.5) = 0.9937$

> $P(Z \leq -1.57) = 0.0582$

> $P(Z > 2) = 1 - P(Z \leq 2)$
$= 1 - 0.9772$
$= 0.0227$

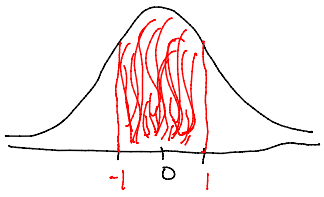


$P(Z \leq a)$ √√

$P(Z > a) = 1 - P(Z \leq a)$

> $P(Z > -0.68) = 1 - P(Z \leq -0.68)$
$= 1 - 0.2482$
$= 0.7518$

> $P(-1 < Z < 1) = P(Z \leq 1) - P(Z \leq -1)$
$= 0.8413 - 0.1586$
$= 0.6826$



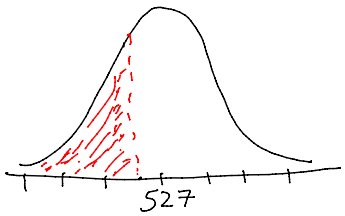
Most Graduate Schools of business require applicants to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are approximately normally distributed with a mean of 527 and a standard deviation of 112.

Q.1 What is the probability of a candidate scoring less than 500?

Sol. Let X be a random variable denoting scores

$P(X \leq 500) = ?$

$$Z = \frac{500 - 527}{112} = -0.241$$



$P(X \leq 500) = P(Z \leq -0.241)$
$= 0.405$

Q.2 What is the probability of a candidate scoring less than 450?

Sol. $P(X \leq 450) = ?$

$$Z = \frac{450 - 527}{112} = -0.69$$

$$P(X \leq 450) = P(Z \leq -0.69)$$
$$= 0.245 \text{ Ans.}$$

Q.3 What is the probability of a candidate scoring above 650 ?

Sol. $P(X > 650) = ?$

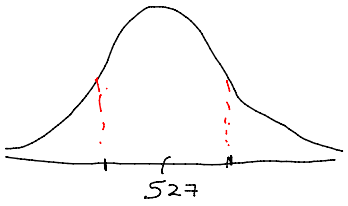
$$Z = \frac{650 - 527}{112} = 1.10$$

$$P(X > 650) = P(Z > 1.10)$$
$$= 1 - P(Z \leq 1.10)$$
$$= 1 - 0.8643$$
$$= 0.135$$

Q.4 What is the probability of a candidate scoring between 400 and 600?

Sol. $P(400 < X < 600) = ?$

$$Z_1 = \frac{400 - 527}{112} = -1.13$$



$$Z_2 = \frac{600 - 527}{112} = 0.65$$

$$P(400 < X < 600) = P(-1.13 < Z < 0.65)$$
$$= P(Z \leq 0.65) - P(Z \leq -1.13)$$
$$= 0.742 - 0.129$$
$$= 0.613$$

Q.5 Candidates falling in the bottom 30% will be considered as failing the test. How much a candidate has to score to pass?

Sol. $P(X \leq x) = 0.3$, $x = ?$

$P(X \leq x) = P(Z \leq z) = 0.3$

$P(Z \leq -0.52) = 0.3$

$z = -0.52$

$$-0.52 = \frac{x - 527}{112}$$

$$x = -0.52 \times 112 + 527$$

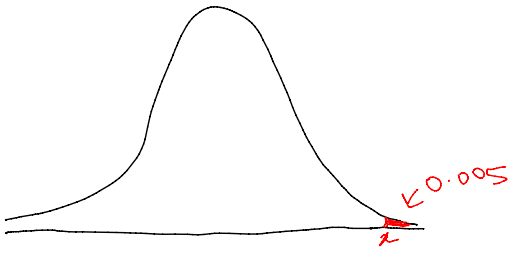
$$= -58.8 + 527$$

$$= 468.76 \simeq 469$$

* A Candidate has to score above 469 to pass

Q.6 A premier B School only accepts top 0.5 % of candidates. How Much a candidate has to score to be in that B School?

Sol. $P(X > x) = 0.005$, $x = ?$



$P(Z > z) = 0.005$

$P(Z \leq z) = 1 - 0.005$

$= 0.995$

$z = 2.58$

$$2.58 = \frac{x - 527}{112}$$

$$x = 2.58 \times 112 + 527$$

$$= 288.96 + 527$$

$$= 815.96 \simeq 816$$

* A Candidate has to score above 816 to be in that B School.