Bernoulli Trials and Binomial Distribution

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* BERNOULLI TRIALS :-

> Trials of a random experiment are called Bernoulli trials if following conditions are satisfied :-

i) Finite number of trials

ii) Trials should be independent

iii) Each trial should have exactly two outcomes :-

> Success

> Failure

iv) The probability of success remains same in each trial.

* Dichotomous Experiments :-

> Coin Toss : {Head, Tails}

> Covid : {Infected, Not infected}

> Customer Churn : {Yes, NO}

> Email : {Spam, Not Spam}

> Loan default : {Yes, NO}

> Product : {Purchased, Not Purchased}

* Let $X = 1$ denote Success

$X = 0$ denote failure

$$P(X=1) = p$$

$$P(X=0) = q = (1-p)$$

* EXPECTATION OF BERNOULLI TRIALS:-

$$E(x) = \Sigma x_i p_i$$

$$= 0 * q + 1 * p$$

$$\boxed{E(x) = p}$$

* VARIANCE :-

$$VAR(x) = E(x^2) - E(x)^2$$

$$= 0^2 \times q + 1^2 \times p - p^2$$

$$= p - p^2$$

$$= p(1-p)$$

$$\boxed{VAR(x) = pq}$$

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* PERMATATIONS:-

> Number of arrangements

* $\{1,2,3\}$: 123, 132, 213, 231, 312, 321

* $\{A,B,C,D\}$: ABCD, ABDC, ACBD, ACDB, ADBC, ADCB — 6

B — 6

C — 6

B
C
D

24 arrangements

6
6

Permutation of n outcomes : $n!$

$$n! = n(n-1)(n-2)\dots 1$$

Ex: $3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Ex: $\{A, B, C, D\}$: 2 letter Arrangements

AB, AC, AD, BA, BC, BD

CA, CB, CD, DA, DB, DC   12

$\frac{4!}{2!} = 4 \times 3 = 12$

$\{A, B, C\}$: AB, AC, BA, BC, CA, CB   6 $\frac{3!}{1!} = 6$

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

* COMBINATIONS:-

> Number of ways of selecting / combining

> Order does not matter

Ex: {A, B, C} : AB, AC, BC

{A, B, C, D} : ABC, ABD, BCD, ACD

{A, B, C, D} : AB, AC, AD, BC, BD, CD

$$\boxed{{}^{n}C_{r} = \frac{n!}{(n-r)!\, r!}}$$

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* BINOMIAL DISTRIBUTION :-

> Probability distribution for n number of bernoulli trials.

> For n bernoulli trials let:

Probability of success = p

Probability of failure = q = $(1-p)$

> For n trials probability of getting x successes

$$\boxed{P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}}$$

Ex : Tossing coin two times.

Success : Getting a Head . $P(X=1) = p = \frac{1}{2}$

$S = \{HH, HT, TH, TT\}$

$S = \{HH, HT, TH, TT\}$

Probability of 1 success :- $\frac{1}{2}$ $\qquad$ ${}^2C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

Probability of 2 success :- $\frac{1}{4}$ $\qquad$ ${}^2C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^0 = \frac{1}{4}$

Ex: Tossing a coin 4 times.

$S = \{HHHH, HHHT, HHTH, HTHH, HHTT, HTHT, HTTH, HTTT,$
$TTTT, TTTH, TTHT, THTT, TTHH, THTH, THHT, THHH\}$

Probability of 3 success.

$$P(X=3) = \frac{4}{16} = \frac{1}{4}$$

$${}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = 4 \times \frac{1}{16} = \frac{1}{4}$$

* EXPECTATION OF BINOMIAL RANDOM VARIABLE:-

Let number of trials : n

Let probability of success : p

$$\boxed{E(X) = np}$$

* VARIANCE OF BINOMIAL RANDOM VARIABLE:-

$$\boxed{VAR(X) = npq}$$

Q.) Probability of a customer reaction to an add 0.1.

The add is pitched to 1000 customers. Find the probability:

> 50 purchase : $^{1000}C_{50} (0.1)^{50} (0.9)^{950}$

> 100 purchase : $^{1000}C_{100} (0.1)^{100} (0.9)^{900}$

- If an insurance company knows the probability of a claim being fraudulent (or not), binomial distribution can help determine the probability that there would be more than ‘k’ fraudulent claims in the next 1000 claims

- If an advertising company knows the probability of a customer buying a certain product, if he/she receives an advertisement on their Facebook account, binomial distribution can help them determine whether ‘k’ purchases would be made from a total of 10000 Facebook ads sent. Effectively, they can determine how many Facebook advertisements to send, if they want to achieve a particular sales target

- Suppose that a hardware store manager has 3 different suppliers. If he knows the probability of a newly purchased bolt being defective, based on the supplier it is bought from, binomial distribution can help him/her check whether there will be less than ‘k’ defective bolts in a box of 2000 bolts. This would help his decide which supplier to buy the bolts from.

1000   50 - 70   100   Sales target

~~100~~ = ✓   500   Qery

100   "   2000   0.4

$\boxed{10000 \quad 100 \quad 0.7}$