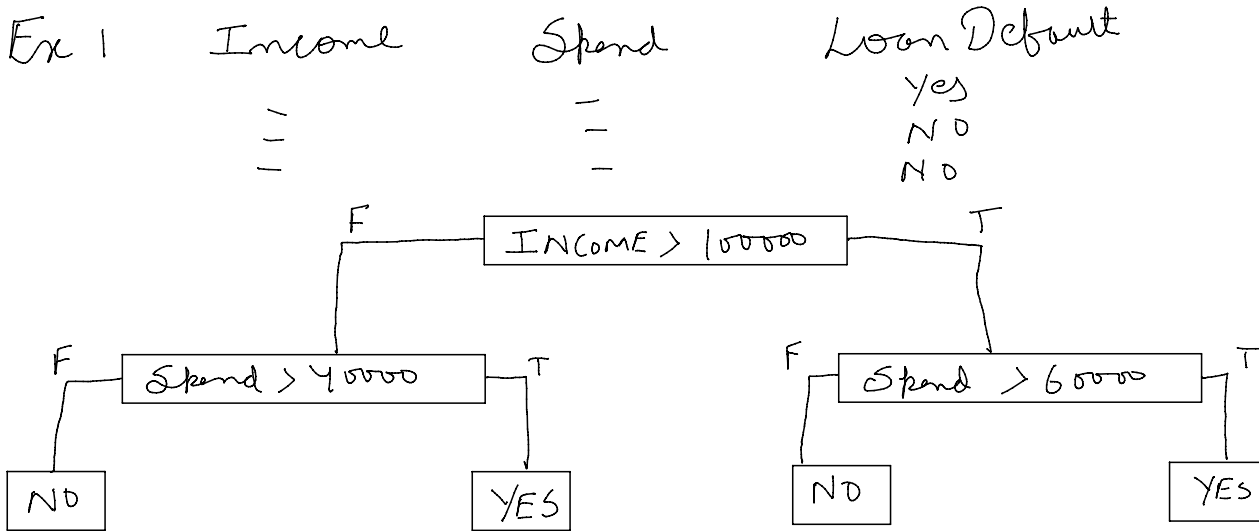


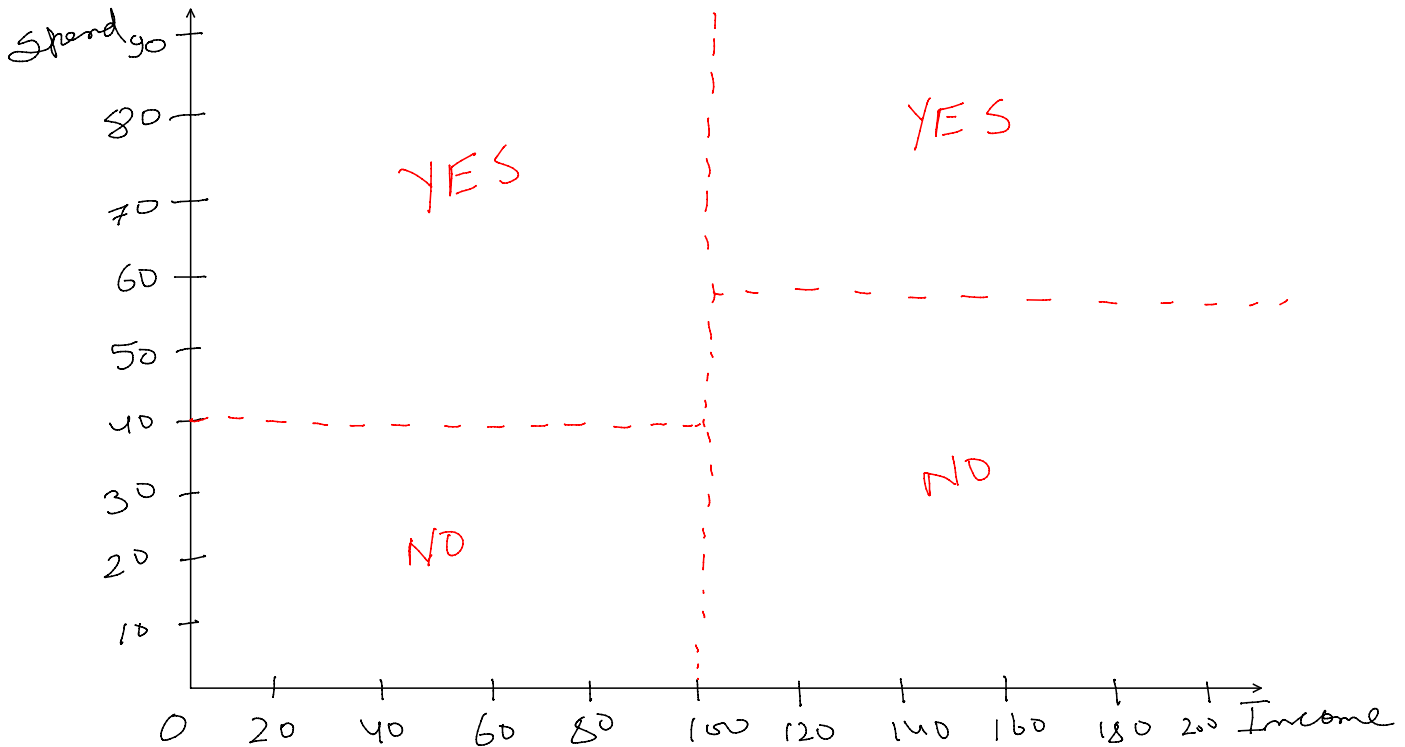
# Decision Trees

2 January 2024 08:13 AM

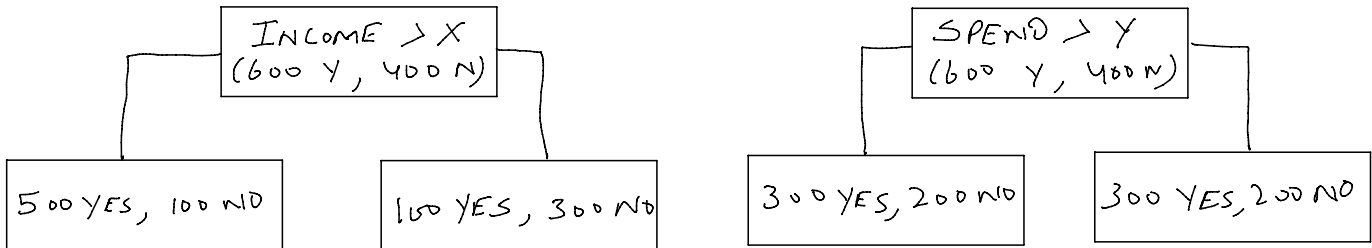
- Decision Trees recursively partition the dataset into multiple segments.
- Each internal node corresponds to a test or a condition.
- Each branch is a result of the test.
- Each leaf node assigns a category.



- i) Income: 95000, Spend: 50000 YES
- ii) Income: 120000, Spend: 50000 NO
- iii) Income: 100000, Spend: 45000 YES



- The algorithm works by recursively partitioning the dataset into multiple segments.
- At each step the algorithm selects the most predictive feature to split the data.
- The most predictive feature is the one which gives maximum separation between the classes.



- At each step it is important to determine the best test condition, which feature and value are most significant.
  - Criteria for classification : Entropy or Gini Index
  - Criteria for Regression : MSE (RSS)

## Decision Tree algorithm for classification

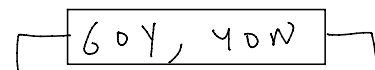
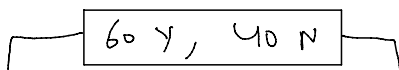
- Choose attributes and values from dataset.
- Calculate Entropy before and after split.
- Choose an attribute and value (Test Condition) which give us maximum reduction in entropy.
  - ▶ Purer Split
  - ▶ Maximum Separation
- Perform the split based on that test condition.
- Repeat the steps recursively.

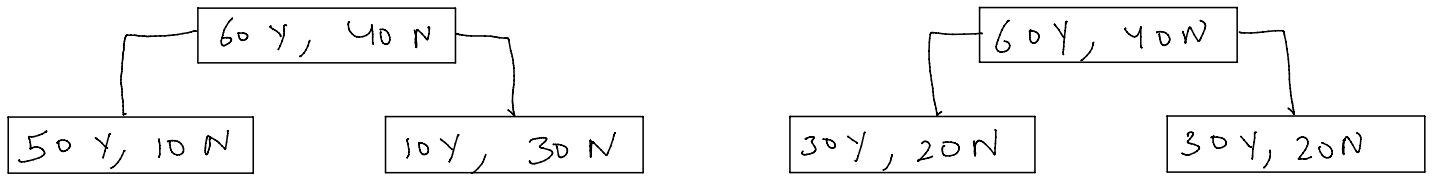
## Entropy

- Entropy is measure of purity / impurity of a node
- Lower the entropy, purity is more
- Higher the entropy, purity is less

$$ENTROPY = - \sum p_i \log_2 (p_i)$$

Ex: 100, 60 YES, 40 NO





$$\text{Entropy} = - (p_Y \cdot \log_2(p_Y) + p_N \cdot \log_2(p_N))$$

$$\text{Entropy}_B = - \left( \frac{6}{10} \cdot \log_2\left(\frac{6}{10}\right) + \frac{4}{10} \cdot \log_2\left(\frac{4}{10}\right) \right)$$

$$= 0.97$$

$$\text{Entropy}_{\text{left}} = 0.65$$

$$\text{Entropy}_{\text{right}} = 0.81$$

$$\text{Entropy}_A = \frac{60 \times 0.65 + 40 \times 0.81}{100}$$

$$= 0.71$$

$$\text{Entropy}_B - \text{Entropy}_A = 0.97 - 0.71$$

$$= 0.26$$

$$\text{Entropy}_{\text{left}} = 0.97$$

$$\text{Entropy}_{\text{right}} = 0.97$$

$$\text{Entropy}_A = \frac{50 \times 0.97 + 50 \times 0.97}{100}$$

$$= 0.97$$

$$\text{Entropy}_B - \text{Entropy}_A = 0.97 - 0.97$$

$$= 0$$

$$\text{INFORMATION GAIN} = E_B - E_A$$

\* GINI INDEX :-

$$GI = 1 - \sum p_i^2$$

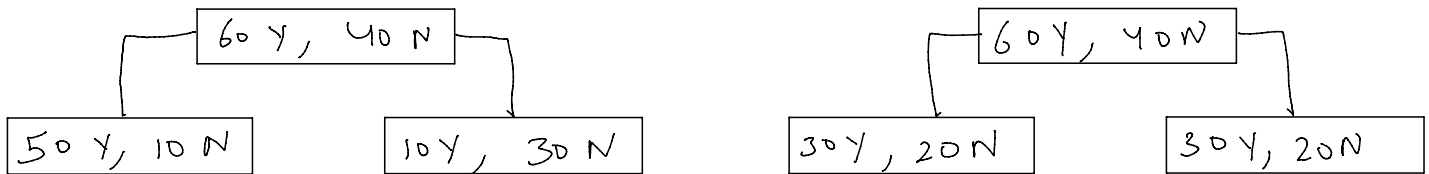
$$GI = \sum p_i(1 - p_i)$$

$$GI = p_1(1 - p_1) + p_2(1 - p_2)$$

$$= p_1 - p_1^2 + p_2 - p_2^2$$

$$\begin{aligned}
 &= P_1 - P_1^2 + P_2 - P_2^2 \\
 &= P_1 + P_2 - P_1^2 - P_2^2 \\
 &= 1 - (P_1^2 + P_2^2) \\
 &= 1 - \sum p_i^2
 \end{aligned}$$

Ex: 100, 60 YES, 40 NO



$$GI = 1 - (P_Y^2 + P_N^2)$$

$$\begin{aligned}
 GI_B &= 1 - \left( \left( \frac{60}{100} \right)^2 + \left( \frac{40}{100} \right)^2 \right) \\
 &= 0.48
 \end{aligned}$$

$$GI_{\text{left}} = 0.27$$

$$GI_{\text{right}} = 0.37$$

$$\begin{aligned}
 GI_A &= \frac{0.27 \times 60 + 0.37 \times 40}{100} \\
 &= 0.31
 \end{aligned}$$

$$\begin{aligned}
 GI_B - GI_A &= 0.48 - 0.31 \\
 &= 0.17
 \end{aligned}$$

$$GI_{\text{left}} = 0.48$$

$$GI_{\text{right}} = 0.48$$

$$\begin{aligned}
 GI_A &= \frac{0.48 \times 50 + 0.48 \times 50}{100} \\
 &= 0.48
 \end{aligned}$$

$$\begin{aligned}
 GI_A - GI_B &= 0.48 - 0.48 \\
 &= 0
 \end{aligned}$$

~~~~~ X ~~~~~ X ~~~~~ X ~~~~~

### Decision Tree algorithm for Regression

- Choose attributes and values from dataset.
- Calculate MSE/RSS before and after split.

- Choose an attribute and value (Test Condition) which gives us maximum reduction in MSE.
- Perform the split based on that test condition.
- Repeat the steps recursively.