Random Variables

Saturday, October 1, 2022 7:57 PM

> Sometimes when an experiment is performed we are interested in some function of output rather than the actual outcome itself.

> A random variable is a rule / function which assigns a real number to each outcome of the experiment.

> A random variable is a real valued function whose domain is sample space of an experiment.

Ex: Tossing three coins

$$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$$

> Let X denote the number of heads obtained

$X(HHH) = 3$

$X(HHT) = X(HTH) = X(THH) = 2$

$X(HTT) = X(TTH) = X(THT) = 1$

$X(TTT) = 0$

Range of $X = \{0, 1, 2, 3\}$

Ex. Rolling 2 dies :-

> Let X denote the sum obtained.

Domain : 36 outcomes in S

Range : $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

* PROBABILITY DISTRIBUTIONS :-

> A probability distribution of a random variable is the description of the possible values of random variable along with their probabilities.

X	:	x_1	x_2	x_3	$\ldots\ldots$	x_n
$P(X=x)$	:	p_1	p_2	p_3	$\ldots\ldots$	p_n

$$\boxed{p_i = P(X = x_i)}$$

Ex: Tossing Three coins.

X : No. of Heads obtained

X	:	0	1	2	3
$P(X=x)$	:	$1/8$	$3/8$	$3/8$	$1/8$

Ex: Rolling 2 dies.

X : Sum Obtained

X :	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$:	$1/36$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

———————— X ———————— X ———————— X ————————

* MEAN (EXPECTATION) OF RANDOM VARIABLE :-

$$\boxed{\mu = \bar{X} = \sum_{1}^{n} x_i p_i}$$

$\mu = \bar{X} = E(X)$

Ex:

X :	0	1	2	3
$P(X=x)$:	$1/8$	$3/8$	$3/8$	$1/8$

$$\mu = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5$$

Ex:

X :	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$:	$1/36$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\mu = 7$

* VARIANCE AND STANDARD DEVIATION:-

$$Var(X) = \sigma^2 = \Sigma(x_i - \mu)^2 p(x_i)$$

$$Var(X) = E(X-\mu)^2$$

$$Var(X) = E(X^2) - E(X)^2$$

Ex:

X	:	0	1	2	3
$P(X=x)$	:	$1/8$	$3/8$	$3/8$	$1/8$

$$E(X) = \mu = \bar{X} = \frac{3}{2}$$

$$Var(X) = E(X^2) - E(X)^2$$

$$= \Sigma x_i^2 p(x_i) - 9/4$$

$$= 0 + 3/8 + \frac{12}{8} + \frac{9}{8} - \frac{9}{4}$$

$$= \frac{24-18}{8} = \frac{6}{8}$$

$$= \frac{3}{4} \text{ Ans.}$$

$$\therefore \sigma = \sqrt{Var(X)}$$

* TYPES OF RANDOM VARIABLES :-

1) DESCRETE RANDOM VARIABLE :-

> Finite number of descrete values.
> Countable
> Examples:
    > Number of heads obtained while tossing 3 coins
    > Sum obtained while rolling 2 dies
    > Number of products purchased

2) CONTINUOUS RANDOM VARIABLES :-

> All possible values in a given range
> Non countable
> Examples:
    > Time taken to address customer queries at a complain centre
    > Weight of students
    > Response time of a computer
    > Temperature Readings

* DESCRETE PROBABILITY DISTRIBUTIONS

* DESCRETE PROBABILITY DISTRIBUTIONS
* CONTINUOUS PROBABILITY DISTRIBUTIONS

* PROBABILITY DENSITY FUNCTIONS :-

1) PROBABILITY MASS FUNCTION :-

> PMF of a random variable X gives the probability that X is equal to some specific value.

$$P(X = x) \quad , \quad Ex \; P(X=1) , \; P(X = 2)$$

> PMF is used for descrete probability distributions.

2) CUMULATIVE DISTRIBUTION FUNCTION :-

* CDF of a random variable X gives the probability of X being less than a specific value.

$$P(X < x) \quad , \quad Ex. \; P(X < 3) , \; P(X < 8)$$

* CDF is mainly used for continuous probability distributions.

Ex:

X :	0	1	2	3
	[illegible]	3/8	3/8	1/8

Ex:

X :	0	1	2	3
$P(X=x)$:	$1/8$	$3/8$	$3/8$	$1/8$