Probability

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* Experiment
* Outcome
* Sample Space :
* Event :

Ex: Rolling a dice

Outcomes : 1, 2, 3 ... 6

S : {1, 2, 3, 4, 5, 6}

Events : Getting an even number {2, 4, 6}

: Getting a number > 4 {5, 6}

: Getting a 4 {4}

PROBABILITY: Probability of occurence of an event is the ratio of number of favourable outcomes to that event w.r.t total number of outcomes in the sample space.

$$\boxed{P(E) = n(E) / n(S)}$$

Q.1 Rolling a dice

i) Getting an even number $P(E) = 3/6$

ii) Getting a prime number $P(P) = 3/6$

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

$P = \{2, 3, 5\}$

Q.2 Tossing three coins

(i) Getting two heads

(ii) Getting at least one head

(iii) Getting at least two tails

(iv) Getting at most two heads

Ans. $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

(i) $3/8$

(ii) $7/8$

(iii) $1/2$

(iv) $7/8$

Q.3 Rolling two dice:

(i) Sum > 8 — $10/36$ — $5/18$

(ii) Getting an even sum — $18/36$ — $1/2$

(iii) Getting a prime sum — $15/36$ — $5/12$

$S =$

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

* TYPES OF EVENTS:-

1) Impossible event : $P(E) = 0$

2) Equally likely events / outcomes : Same Prob.

3) Mutually exclusive events :

> A set of events $E_1, E_2 \ldots E_n$ is said to be mutually exclusive if they can not occur at the same time.

> Events are pairwise disjoint

$$\boxed{E_i \cap E_j = \phi}$$

Ex: Rolling a die

> $E_1 : \{2, 4, 6\}$, $E_2 : \{1, 3, 5\}$ ✓

> $E_1 : \{2, 4, 6\}$, $E_2 : \{2, 3, 5\}$ ✗

4> Mutually Exhaustive Events:

* A set of Events $E_1, E_2 \ldots E_n$ is said to be mutually exhaustive if they do not occur at the same time but one must occur.

i> Mutually Exclusive
ii> One must occur

i> $E_i \cap E_j = \phi$

ii> $E_1 \cup E_2 \ldots \cup E_n = S$

Ex: $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$

Ex: $S = \{1, 2, 3, 4, 5, 6\}$

	Exc.	Exh.
i> $E_1 = \{1, 2, 3\}$, $E_2 = \{4, 5, 6\}$	✓	✓
ii> $E_1 = \{1, 2\}$, $E_2 = \{4, 5\}$, $E_3 = \{3\}$	✓	✗
(iii) $E_1 = \{1, 2, 3\}$, $E_2 = \{4, 6\}$, $E_3 = \{2, 5\}$	✗	✗
(iv) $E_1 = \{1, 2, 3\}$, $E_2 = \{4, 5\}$	✓	✗
(v) $E_1 = \{1, 2\}$, $E_2 = \{3, 4\}$, $E_3 = \{5\}$, $E_4 = \{6\}$	✓	✓

X X

* CONDITIONAL PROBABILITY :-

> Conditional probability of occurrence of an event A w.r.t an event B is the probability of occurrence of A given B has already occurred.

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

S

B { }

A

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$\boxed{P(A/B) = \frac{n(A \cap B)}{n(B)}}$$

Q.1 Deck of Cards

A = Drawing a card of hearts $\quad P(A) = \frac{1}{4}$

B = Drawing a red card $\quad P(B) = \frac{1}{2}$

$$P(A/B) = \frac{13}{26} = \frac{1}{2}$$

Q.2 Rolling a Dice $\quad C = \{2, 4$

A : Getting a prime number $\quad \{2, 3, 5\}$

B : Getting an odd number $\quad \{1, 3, 5\}$

$P(A/B) = 2/3$

$P(B/A) = 2/3$

Q.3 Tossing three coins

A : Two tails occur

B : At least one Head occurs

$P(A/B) = 3/7$

Q.4 A family has 2 children. (Consider the order $GB \neq BG$)

> Both are boys given at least one is boy.

Sol. $S = \{BB, BG, GG, GB\}$

$A = \{BB\}$, $B = \{BB, BG, GB\}$

$P(A/B) = \frac{1}{3}$

Q.5 Two dies are thrown:-

Prob. that 4 occurs given the sum is 6.

Ans. A = Sum is 6 $\{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

B = 4 occurs

$$P(B/A) = \frac{2}{5}$$

* FORMULAS :-

> $P(A/S) = P(A)$

$\frac{n(A \cap S)}{}$

$\Rightarrow P(A|S) = P(A)$

$$\Rightarrow P(A/S) = \frac{P(A\cap S)}{P(S)} = \frac{\frac{n(A\cap S)}{n(S)}}{\frac{n(S)}{n(S)}}$$

$$= \frac{n(A)}{n(S)} = P(A)$$

(ii) $P(S|A) = 1$

$$\Rightarrow P(S|A) = \frac{n(S\cap A)}{n(A)} = \frac{n(A)}{n(A)} = 1$$

(iii) $P((A\cup B)|F) = P(A|F) + P(B|F) - P((A\cap B)|F)$

$$= \frac{n[(A\cup B)\cap F]}{n(F)}$$

$$= \frac{n[(A+B-A\cap B)\cap F]}{n(F)}$$

$$= \frac{n[(A\cap F)+(B\cap F)-(A\cap B)\cap F]}{n(F)}$$

$$= \frac{n(A\cap F)}{n(F)} + \frac{n(B\cap F)}{n(F)} - \frac{n[(A\cap B)\cap F]}{n(F)}$$

$$= P(A|F) + P(B|F) - P((A\cap B)|F)$$

(iv) $P(A'|B) = 1 - P(A|B)$

$$= n(A'\cap B)$$

$$\overline{\quad n(B) \quad}$$

$$= \frac{n((S-A) \cap B)}{n(B)}$$

$$= \frac{n(S \cap B - A \cap B)}{n(B)}$$

$$= \frac{n(B)}{n(B)} - \frac{n(A \cap B)}{n(B)}$$

$$= 1 - P(A|B)$$

—————— X —————— X ——————

*** MULTIPLICATION THEOREM:-**

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B) \cdot P(A|B) \rightarrow (1)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \cdot P(B|A) \rightarrow (2)$$

$$\boxed{\begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) \\ &= P(B) \cdot P(A|B) \end{aligned}}$$

Q.1 Deck of cards, drawing two cards in succession. (Without replacement)

(i) KQ $\quad \frac{4}{52} \times \frac{4}{51} \quad (P(K) \cdot P(Q|K))$

(i) K Q $\frac{4}{52} \times \frac{4}{51}$

(ii) K K $\frac{4}{52} \times \frac{3}{51}$

(iii) K Q K $\frac{4}{52} \times \frac{4}{51} \times \frac{3}{50}$

$$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|AB)}$$

Q. 2 Deck of cards. (With replacement)

(i) KKK $\frac{4}{52} * \frac{4}{52} * \frac{4}{52}$

(ii) K Q J $\frac{4}{52} * \frac{4}{52} * \frac{4}{52}$

(iii) H D H $\frac{13}{52} * \frac{13}{52} * \frac{13}{52}$

* In case A and B are independent :-

$$\boxed{\begin{aligned} P(A|B) &= P(A) \\ P(B|A) &= P(B) \end{aligned}}$$

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* PARTITION OF A Sample Space :-

> A set of events $E_1, E_2 \ldots E_n$ is said to be a partition of sample space if

$E_1, E_2 \ldots E_n$:

i) Are pairwise disjoint

ii) Exhaustive

iii) Non zero probability

$$\boxed{\begin{array}{ll} \text{i)}\ E_i \cap E_j = \phi & , i,j \in 1,2..n \\ \text{ii)}\ E_1 \cup E_2 \ldots \cup E_n = S & \\ \text{iii)}\ P(E_i) \neq 0 & , i \in 1,2,\ldots n \end{array}}$$

* THEOREM OF TOTAL PROBABILITY :-

> Given $E_1, E_2 \ldots E_n$ are partition of sample space S

> Let $A \in S$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \ldots P(A \cap E_n)$$

$$\boxed{P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + \ldots\ldots P(E_n)P(A|E_n)}$$

Q.1 Probability of strike $= 0.65$

> Probability of Job completion on time given there no strike $= 0.80$

> Probability of Job completion on time if there is a strike $= 0.32$

> Find Probability of Job completion on time.

Sol. A = Job completion on time

E_1 = Strike happens

E_2 = no strike

E_1 = Strike happens

E_2 = NO Strike

$P(E_1) = 0.65$, $P(E_2) = 0.35$

$P(A|E_2) = 0.80$, $P(A|E_1) = 0.32$

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$$

$$= 0.65 * 0.32 + 0.35 * 0.80$$

$$= 0.488$$

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* BAYES THEOREM :-

> Reverse Probability

B_1	B_2
6 W	3 W
4 B	7 B

A = White Ball

B = Black Ball

∴ Given a sample space S

∴ Let $E_1, E_2 \ldots E_n$ Partition of sample space

∴ Let $A \in S$

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i)\,P(A|E_i)}{P(A)}$$ (By multiplication Theorem)

$$\boxed{P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{1}^{n} P(E_j)\,P(A|E_j)}}$$ (By Total Probability theorem)

Spam Assassin works by having users train the system. It looks for patterns in the words in emails marked as spam by the user. For example, it may have learned that the word "free" appears in 30% of the mails marked as spam, i.e., P(Free | Spam) = 0.30. Assuming 1% of non-spam mail includes the word "free" and 50% of all mails received by the user are spam, find the probability that a mail is spam if the word "free" appears in it.

$P(\text{free} / \text{sham}) = 0.3$

$P(\text{sham}) = 0.5$

$P(\text{not sham}) = 0.5$

$P(\text{free} \mid \text{non sham}) = 0.01$

$$P(\text{Sham} / \text{free}) = \frac{P(\text{sham})\, P(\text{free} / \text{shm})}{P(\text{sham})\, P(\text{free} / \text{sham}) + P(NS)\, P(b/NS)}$$

$$= \frac{0.5 * 0.3}{0.5 * 0.3 + 0.5 * 0.01}$$

$$= 0.967$$

Suppose that the reliability of a HIV test is specified as follows:
Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV?

E : HIV +

E' : HIV -

A : Diagnosed +ve

$P(E) = 0.001$

$P(E') = 0.999$

$P(E/A) = ?$

$$P(E|A) = \frac{P(E)\, P(A|E)}{P(E)\, P(A|E) + P(E')\, P(A|E')}$$

$$= \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$P(E/A) = ?$

$P(A|E) = 0.9$

$P(A/E') = 0.01$

$$= \frac{}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$= 0.08$

Sim

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six.
Find the probability that it is actually a six.

$P(S_1) = 1/6$

$P(S_2) = 5/6$

Truth $= 3/4$

Not truth $= 1/4$

E = Man reports six occurs

$$P(S_1|E) = ? \quad \frac{P(S_1)\,P(E|S_1)}{P(S_1)P(E|S_1) + P(S_2)P(E|S_2)}$$

$$= \frac{\frac{1}{6} * 3/4}{\frac{1}{6} * \frac{3}{4} + \frac{5}{6} * \frac{1}{4}} = \frac{3}{8}$$