ASSIGNMENT - 1

1.What is probability? List down some laws of probability?

Probability is a measure that quantifies the likelihood of an event occurring. It is expressed as a number between 0 and 1, where 0 indicates impossibility (the event will not occur), and 1 indicates certainty (the event will occur). The probability of an event E is denoted by P(E).

Definition of Probability

The Probability can be defined as the ratio of the number of favorable outcomes to the total number of outcomes of any given event. The number of favorable outcomes can be expressed by “x” in an experiment and ‘n’ is expressed as the total number of outcomes.

Probability of any given event = Favorable Outcomes/Total Outcomes = x/n

By dividing the favorable number of possibilities by the entire number of possible outcomes, the probability of an occurrence can be estimated using the probability formula. Because the favorable number of outcomes can never exceed the entire number of outcomes, the chance of an event occurring can range between 0 and 1. Furthermore, the number of favorable outcomes cannot be negative.

The basic probability formula is:

$$P(E)= \frac{\{\text{Number of favorable outcomes}\}}{\{\{\text{Total number of possible outcomes}\}\}}$$

For example, if you roll a fair six-sided die, the probability of rolling a 4 is 1/6 , because there is one favorable outcome (rolling a 4) out of six possible outcomes (rolling a 1, 2, 3, 4, 5, or 6).

If we have a bag of balls. Some balls are red, and some are blue. Probability is like a way of chosing a particular ball without looking at the bag.

If there are 5 red balls and 5 blue balls, the probability of picking a red ball is like asking, "What's the chance of choosing a red one?" In this case, it's 5 out of 10, or y ou could say 1 out of 2 because half of the balls are red.

So, probability is "How likely is something to happen?" If something is certain, it has a probability of 1. If it can't happen at all, it has a probability of 0. If it's just as likely to happen as not, it's right in the middle at 0.5 (or 1 out of 2).

It's a way of choosing about chances and guessing what might happen when you do something.

Probability theory is a fundamental concept in mathematics and has applications in various fields such as statistics, physics, finance, and machine learning.

Terminology related to Probability

The terminology in probability listed below can help you comprehend probability ideas better.

- Experiment: An experiment is a trial or operation that is carried out to generate a specific result.
- Sample space: A sample space is the collection of all the probable outcomes of an experiment.. For example, a sample space of Tossing the coin is head and tail.
- Favorable Outcome: A favorable outcome is an event that has delivered the anticipated result or predicted event. When we roll two dice, for example, the possible/favorable outcomes of having the sum of the two dice as 4 are (1,3), (2,2), and (3,2). (3,1).
- Random Experiment: A random experiment is an experiment with a well-defined set of outcomes. When we toss a coin, for example, we know whether we will get heads or tails, but we don't know which one will appear.
- A random experiment's total number of results is referred to as an event.
- Equally Likely Events: Equally likely events are those that have the same possibility or probability of occurring. The outcome of one given event has no bearing to the outcome of the other. When we toss a coin, for example, we have an equal probability of getting a head or a tail.
- Exhaustive Events: The exhaustive event can be defined when the set of all possible outcomes of an experiment equals the sample space.

Types of Probability

Depending on the nature of the outcome or the method used to calculate the chance of an event occurring, several views or types of probabilities may exist. There are four major different types of probabilities:

- Classical Probability
- Axiomatic Probability
- Subjective Probability

- Empirical Probability

Let us understand each type of probability one by one,

Classical Probability

Classical probability which is also known as the theoretical probability which further states that if there are B equally likely outcomes in an experiment and event X has exactly A of them,

Then the probability of X is A/B, or P(X) = A/B.

It entails tossing a coin or rolling dice. It’s computed by making a list of all the possible outcomes of the activity and keeping track of what actually happens. When throwing a coin, for example, the possible outcomes are heads or tails. If you toss the coin ten times, you must keep track of which outcome occurred each time.

Explanation: This is like when you have a fair game or a spinner with equal sections. Classical probability is all about things being fair and equal.

Example:Imagine a spinner divided into four equal parts: red, blue, green, and yellow. The chance of landing on any color is the same, so it's classical probability. The probability of getting red is 1 out of 4 because there's one red section out of four sections.

Axiomatic Probability:

A series of rules or axioms by Kolmogorov are applied to all kinds of axiomatic probability. The probability of each event occurring or not occurring can be calculated using these axioms, which are written as,

- The smallest and greatest probabilities are 0 and one, respectively.
- The chance of a specific happening is equal to one.
- Only one of two mutually exclusive events can occur at the same time, according to the union of events.

- Example: Think of it as having rules for a game. If you have a six-sided die, axiomatic probability helps us figure out the chance of rolling any number because each side has an equal chance.

Subjective Probability

Subjective probability takes into account a person's personal belief on the probability of an event occurring. For example, a fan's opinion on the probability of a specific side winning a football match is based on their personal beliefs and feelings rather than a rigorous quantitative calculation.

This is like guessing based on what you feel or think might happen. It's your own personal idea about how likely something is.

Example: If you look at the sky and see dark clouds, you might feel like it's going to rain. Your subjective probability of rain is based on what you see and feel.

Empirical Probability

Through thinking experiments, the empirical probability or experimental perspective evaluates probability. If a weighted die is rolled and we don't know which side has the weight, we can gain an idea of the probability of each outcome by rolling the die a certain number of times and counting the proportion of times the die

This is like figuring out the chance of something by looking at past experiences or data. It's based on what actually happened before.

Example:If you flip a coin 100 times and it lands on heads 60 times, the empirical probability of getting heads is 60 out of 100 because that's what really happened in the past trials.

2.What is the difference between independent and dependent events?

1. Independent Events:

Explanation: Imagine events that don't care about each other. What happens in one event doesn't affect the other. They are like doing their own thing.

Example: Let's say we have a bag of colored balls. You pick a red ball and put it back before picking another one. The color of the first ball didn't change what color the second one could be. They're independent because one doesn't depend on the other.

Example: Flipping a Coin and Rolling a Die

Imagine you have a fair coin (with heads and tails) and a six-sided die. Now, let's say you want to know the outcomes of flipping the coin and rolling the die. In this case, these events are independent of each other.

- Flipping the Coin:

- Possible outcomes: Heads (H) or Tails (T)

- Probability of getting Heads: 1/2 (because there are two equally likely outcomes)

- Rolling the Die:

- Possible outcomes: 1, 2, 3, 4, 5, or 6

- Probability of rolling a 4: 1/6 (because there is one favorable outcome out of six possible outcomes)

Now, the key point is that whether we get Heads or Tails on the coin has no effect on what number you might roll on the die. The events are independent because the outcome of one event (flipping the coin) does not influence the outcome of the other event (rolling the die). Each event has its own separate probability, and what happens with the coin doesn't change the chances of getting any particular number on the die.

2. Dependent Events:

Example: If we pick a red ball and don't put it back before picking the second one. Now, the color of the first ball matters because there's one less marble in the bag. If you picked a red marble first, there's one less red marble for the second pick. The events are dependent because the outcome of the first event affects the second one.

In simple terms, independent events are like two separate stories where one doesn't affect the other. Dependent events are like stories where what happens in the first part can change what happens in the next part. So, when events are independent, they do their own thing, but when they are dependent, they are influencing each other

3. What is conditional probability? What is the formula to calculate conditional probability?

Conditional probability is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome.

Conditional probability of occurrence of an event A with respect on event B is the probability of a given B has already occurred.(A,B in sample space)

Conditional Probability Formula

P(B|A) = P(A and B) / P(A)

Or:

$P(B|A) = P(A \cap B) / P(A)$

Where

P = Probability

A = Event A

B = Event B

= n(A and B)/n(s)/ n(B)/n(s)

P(A\B)= n(A and B) /n(B)

5.According to hospital records, 30% of patients suffering from a disease will die. Find out the probability that 2 out of the 5 randomly selected patients survive.

This question falls under the binomial distribution because it satisfies the following conditions:

1. Fixed Number of Trials:The scenario involves a fixed number of trials, in this case, the selection of 5 patients.

2. Independence:The outcome of each trial (patient survival or death) is independent of the others. The survival or death of one patient does not affect the outcome for another patient.

3. Two Possible Outcomes: Each trial (patient) has two possible outcomes: survive or die.

4. Constant Probability of Success: The probability of success (a patient surviving) remains constant for each trial. In this case, it is stated that 30% of patients will die, which implies that 70% will survive. This probability remains the same for all patients.

Now, let's denote the probability of success (survival) as (p) and the probability of failure (death) as (q), where (p + q = 1 \). In this scenario, (p = 0.7) and

(q = 0.3).

The probability of getting exactly \(k\) successes in \(n\) trials in a binomial distribution is given by the binomial probability formula:

$[P(X = k)]= nC_x (p)^x*(q)^{n-x}$

where (binom{n}{k} \) represents the number of ways to choose (k) successes from \(n) trials (combinations).

In this case, we want to find the probability that 2 out of the 5 randomly selected patients survive, so (k = 2) and \(n = 5):

$P(X = 2) = 5\ C_2 * (o.7)^2 * (0.3)^{5-3}$

Calculating this expression will give you the probability of exactly 2 out of the 5 patients surviving.

$P(X = 2) = 5\ C_2 * (0.7)^2 * (0.3)^{5-2}$

$= 5 * 4*3 * 2*1/(5-2)! * 2! \quad * 0.7 * 0.7 * 0.3 * 0.3 * 0.3$

$= 10 \quad * \ 0.49 \ * \ 0.027$

$= 0.1323$

Probability of exactly 2 out of the5 patients surving = 0.1323

6. You pull two cards, one at a time, from a deck of cards, without replacement. What is the probability that the second card you pick has a different color, or different suit, than the first card?

When you draw two cards, one at a time, from a standard deck of 52 cards without replacement, the probability that the second card has a different color or a different suit than the first card can be calculated based on the principle of conditional probability.

1. First Card:

The probability of drawing any card as the first card is 1.

There are two possible colors for the first card: red or black.

2. Second Card:

Since the first card has been drawn without replacement, there are now 51 cards left in the deck.

The probability of drawing a card with a different color than the first card is (26/51).

There are 26 cards of the opposite color remaining in the deck (13 red cards if the first card was black, and 13 black cards if the first card was red).

Alternatively, the probability of drawing a card with a different suit than the first card is (39/51)

There are 39 cards with a different suit remaining in the deck (3 cards of the same rank and color as the first card have been removed).

Now, to find the overall probability of drawing a card with a different color or a different suit on the second draw, we can use the addition rule for mutually exclusive events:

[P({Different Color or Different Suit on Second Draw}) = P(Different Color) + P(Different Suit) - P(Both)]

=26/51 + 39/51 – 13/51

= 65/51-13/51

= 52/51

= 1.01

It's worth noting that the result is greater than 1 because the events of drawing a card with a different color and drawing a card with a different suit are not mutually exclusive (there are cards that satisfy both conditions). However, by subtracting the probability of both events occurring (drawing a card with both a different color and a different suit), we correct for double-counting, resulting in a valid probability.

5. How does Probability Mass Function and Probability Density Function differ?

Probability Density Function (PDF) and Probability Mass Function (PMF) are both mathematical concepts used in probability theory, but they are associated with different types of random variables.

Probability Mass Function (PMF):

Definition:The PMF is a function that gives the probability that a discrete random variable is exactly equal to a certain value.

Notation: Typically denoted by (P(X = x) \), where (X) is the random variable and

(x) is a specific value.

Domain:Applicable to discrete random variables, which take on distinct, separate values.

Example: When rolling a fair six-sided die, the PMF gives the probability of obtaining each possible outcome (1, 2, 3, 4, 5, or 6).

Summation: The PMF must sum to 1 over all possible values of the random variable.

Probability Density Function (PDF):

Definition: The PDF is a function that describes the likelihood of a continuous random variable falling within a particular range of values.

Notation: Typically denoted by (f(x)), where (x) is a specific value or a range of values.

Domain: Applicable to continuous random variables, which can take on any value within a certain interval.

-Example: When measuring the height of individuals, the PDF gives the likelihood of a person having a height within a specific range.

Integration: The area under the PDF curve over a given interval represents the probability of the random variable falling within that interval. The total area under the curve is 1.

Comparison:

1. Type of Random Variable:

- PMF is associated with discrete random variables.

- PDF is associated with continuous random variables.

2. Notation:

- PMF uses (P(X = x)) for probability mass.

- PDF uses (f(x)) for probability density.

3. Domain:

- PMF is defined for specific, separate values of the random variable.

- PDF is defined for intervals or continuous ranges of the random variable.

4. Summation vs. Integration:

- PMF values are summed over all possible values to equal 1.

- PDF values are integrated over the entire range to equal 1.

In summary, PMF is used for discrete random variables, providing the probability at specific points, while PDF is used for continuous random variables, providing probabilities over intervals or ranges of values.

8.You roll two dice. What's the probability of rolling at least one 4? What's the probability of rolling a 4 given a dice?

When we throw two dice total possible outcomes 36

To find the probability of getting atleast one 4

We can calculate the probability of not getting a 4 on 1 either die and subtract from 1.

Dice = {1,2,3,4,5,6}

Not getting 4 = 5/6

In 2 dice = 5/6 * 5/6

P(not getting 4) = 25/36

Probability of getting atleast one 4

$P(n^{1)}$ = 1 – p (n)

= 1 –(25/36)

= 36 – 26 / 36 = 11/36

9.What is the probability of getting a sum less than 8 when two dice are thrown?

The probability of 2 dice outcome is = 36

Getting sum less than 8 = [(1,1),(1,2),(1,3),(1,4)(1,5),(2,1),(2,2),(2,3) (2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)]

= 15/36

= 0.41

10. there are 12 horses in a race, numbered 1 to 12, what is the probability that horses bearing number 3, 5 or 8 will win the race?

To find the probability that a horse bearing number 3, 5, or 8 will win the race, you need to determine the number of favorable outcomes (winning horses) and divide it by the total number of possible outcomes (total number of horses).

Given that there are 12 horses in the race and you are interested in horses with numbers 3, 5, or 8, the favorable outcomes are 3, 5, and 8. Therefore, the probability (P) is given by:

P(horse with number 3, 5, or 8 wins) = Number of Favorable Outcomes\Total Number of outcomes]

P = 3/ 12 = 1/4

So, the probability that a horse bearing number 3, 5, or 8 will win the race is : 1/4