



UNIVERSITY
OF TRENTO - Italy

Dipartimento di Ingegneria e Scienza dell'Informazione



World models

Intensional representation

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Extensional and intensional representation

Observation 6.1 (World model, extensional representation) World models, as defined so far are **extensional representations** of the world, namely they are defined as sets of assertions a and facts f , plus an interpretation function I_A which allows to define which assertions denote which facts in one or more reference models.

But what is a *fact*? How do we construct assertions about facts? The answer to this question requires defining an **intensional representation** of world models, namely the **representation mechanisms** which allow to construct assertions and facts starting from a finite set of primitive component elements.

Notation 6.1 (Extensional and intensional representation of a set) Let S be a set. Then by S_e we mean the **extensional representation** of S , i.e., as a set of elements (e.g., facts, assertions, but not only); by S_i we mean the **intensional representation** of S , where the elements of S_e are defined intensionally, starting from a set of primitive components. The notation is dropped when no confusion arises.

Domain – Intensional representation

Intuition (Domain, intensional representation) The intensional representation of a domain is composed of three components, as follows

- *entities*, associated with those elements of the representation which can be isolated and distinguished from the rest;
- *classes* (sets) of entities, characterized by the fact they have some common characteristics which is not shared by the entities of the other sets;
- *relations* among entities, which collect multiple entities sharing a common property.

Does it fit your intuition? How you would describe the world?

Domains and facts, intensional representation

Definition 6.1 (Domain, intensional representation) The intensional representation D_i of a domain D is defined as

$$D_i = \langle E, \{C\}, \{R\} \rangle$$

with

$$E = \{e\}, \quad C \subseteq E, \quad R \subseteq E \times \cdots \times E$$

where $E = \{e\}$ is a set of **entities**, $\{C\}$ is a set of **classes** of entities, $\{R\}$ is a set of n -ary **relations** R_n , for some n . E is called the **universe** of D_i or also the **universe of interpretation**.

Definition 6.2 (Fact, intensional representation) The intensional representation D_e of a fact f has one of the following four forms

$$e \in C, \quad \langle e_1, \dots, e_n \rangle \in R, \quad C \subseteq E, \quad R_n \subseteq C_1 \times \cdots \times C_n$$

with $e, e_i \in E$ and $C, C_i \subseteq E$.

Data and Knowledge domains

Definition. (Domain, data, knowledge, mixed) A **data domain** contains only facts of the form

$$e \in C$$
$$\langle e_1, \dots, e_n \rangle \in R.$$

A **knowledge domain** contains only facts of the form

$$C_1 \subseteq C_2,$$
$$R_n \subseteq C_1 \times \cdots \times C_n.$$

A **mixed domain** contains all types of facts.

Data and Knowledge domains

Example 6.2 (Data domain)

sofia \in Person,
sofia \in Woman,
 \langle paolo, rocky $\rangle \in$ HasDog
 \langle sofia, paolo $\rangle \in$ Near
paolo \in Man
 \langle paolo, sofia, stefania $\rangle \in$ Between

\langle rocky, paolo $\rangle \in$ DogOf
paolo \in Dog
rocky \in Dog
 \langle rocky, sofia $\rangle \in$ DogOf
paolo $\rangle \in$ FriendOf

Data and Knowledge domains

Example 6.2 (Knowledge domain)

Person \subseteq Entity

Dog \subseteq Entity

Animal \subseteq Entity

Near \subseteq Entity \times Entity

FatherOf \subseteq Person \times Person

HasDog \subseteq Person \times Dog

DogOf \subseteq Dog \times Person

FriendOf2 \subseteq person \times person \times person

FriendOf1 \subseteq Person \times Person

ChildOf \subseteq Person \times Person

where Entity stands for E.

Assertional Language

Definition (Assertional language, intensional representation) The **intensional representation** LiA of an assertional language LA is defined as

$$LiA = \langle E, \{C\}, \{P\} \rangle$$

where $E = \{e\}$ is a set of **(names of) entities**, $\{C\}$ is a set of **concepts**, where a concept is a **name of a class**, $\{P\}$ and a set of **properties**, where a property is a **name of a relation**.

Definition (Assertional language, extensional representation) The **extensional representation** LeA of an assertional is $LeA = \{a\}$ with a having one of the following four forms

$$C(e), \quad Pn(e1, \dots, en), \quad C, \quad Pn(C1, \dots, Cn)$$

Interpretation function

Definition (Interpretation function, intensional interpretation) The **Intensional representation** IA of an interpretation function $IA : LA \rightarrow D$ of an assertional language is defined as

$$IA = \langle I_e, I_C, I_P \rangle$$

with:

$$I_e : E \rightarrow E$$

$$I_C : \{C\} \rightarrow \{E\}$$

$$I_P : \{P_n\} \rightarrow \{E\} \times \cdots \times \{E\}$$

and such that:

$$IA(C(e)) = I_C(C)(I_e(e)) = e \in C$$

$$IA(P_n(e_1, \dots, e_n)) = I_P(P_n)(I_e(e_1), \dots, I_e(e_n)) = \langle e_1, \dots, e_n \rangle \in R_n$$

$$IA(C) = I_C(C) = C \subseteq E$$

$$IA(P_n(C_1, \dots, C_n)) = I_P(P_n)(I_C(C_1), \dots, I_C(C_n)) = R_n \subseteq C_1 \times \cdots \times C_n$$

World model, intensional representation

Definition 6.10 (World Model, intensional representation) Given a World Model

$$W = \langle LA, D, IA \rangle,$$

its intensional representation Wi is defined as

$$Wi = \langle LiA, Di, liA \rangle$$

World models, models and theories – The practice

1. Select the world model (crucial representation choice)

$$W_i = \langle LiA, Di, liA \rangle$$

2. Agree on LiA, liA (... and therefore D)

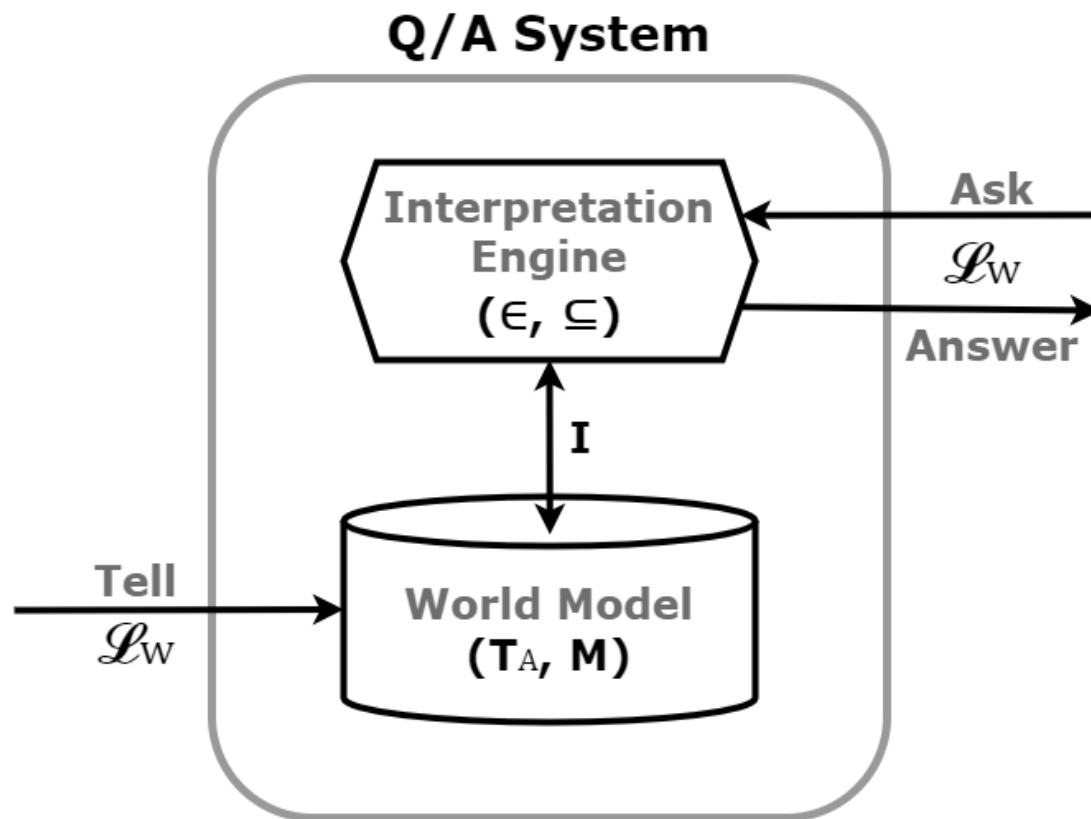
3. Construct $TA = \{a\} \subseteq LA$

4. The model $M = \{f\} \subseteq D$ is automatically defined

NOTE: Agreement is only on linguistic representation, based on a shared understanding of what language means

NOTE 2: agreement at different levels of formality depending on application

Using a world model



*Which
questions and
answers?*

*Reasoning
problems!*

Entailment

Definition (Interpretation and entailment) Let $W = \langle LA, D, IA \rangle$ be a world model. Let $T \subseteq LA$ be a theory and $M \in D$ a model of W . Let $a \in T$ be an assertion. Then, we write

$$M \models a \text{ to mean } IA(a) \in M$$
$$M \models T \text{ to mean } IA(a) \in M \text{ for all } a \in T$$

and say that M **entails** T , or also that M **entails** a .

Reasoning problems (with respect a world model)

Reasoning Problem (Model checking) Given T and M , check whether $M \models T$.

Reasoning Problem (Satisfiability) Given T , check whether there exists M such that $M \models T$.

Reasoning Problem (Validity) Given T , check whether for all M , $M \models T$.

Reasoning Problem 6.4 (Unsatisfiability) Given T , check whether there is no M such that $M \models T$.

Reasoning problems (with respect a world model) (cont)

Observation (Query answering in DBs) Query answering in DBs is a sophisticated form of model checking / satisfiability.

The contents of the DB are the reference world model, the query is the theory to be model checked, the answer is the set of instantiations which make the input theory correct.

This can be extended to knowledge graphs (both data and knowledge level, e.g. ER/UML like).



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