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Computational Logic Exercises
Module II – World models and logic systems

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Preamble: World Models

A **world model** is a triple $\mathscr{W}^{o} = \langle L, D, I \rangle$ where:

(intensional world model)

$L = \{a\}$	L is an assertional language, with descriptive assertions
$D = \{f\}$	D is a domain of interpretation, a set of facts
$I: L \rightarrow D$	I is an intepretation function that maps assertions to facts

(extensional world model)

$L = \langle E, \{C\}, \{P\} \rangle$	L is an assertional language with assertions about: $E = \{e\}$ is a set of (names of) entities, {C} is a set of concepts, where a concept is a name of a class, {P} and a set of (object/data) properties.
$D = \langle E, \{C\}, \{P\} \rangle$	D is a domain of interpretation, a set of facts about the entities E = {e}, their classes $C \subseteq E$ and properties $P \subseteq E \times \cdots \times E$
$I: L \rightarrow D$	I is an intepretation function that maps assertions to facts

Preamble: Logic Systems

A **logic system** is a triple $\mathcal{L} = <L, D, I, \models>$ or even $\mathcal{L} = < \mathcal{W}, \models>$ where:

$\mathcal{W} = < L, D, I>$	W is a world model
$\models \subseteq D \times L$	$\models$ is an entailment relation between subsets of the domain D (i.e. a specific model M) and assertions in the language L (i.e. a specific theory)

A **(logic) language L** can be formally defined by a set of **atomic formulas** and a **grammar**, i.e. a set of connectives and formation rules to combine atomic formulas into **complex formulas**.

For instance:
<wff> ::= <atomic>
<wff> ::= <wff> ⊔ <wff>
<wff> ::= <wff> ⊓ <wff>

World models (I)

Define an extensional world model W constituted by a language L, a domain D and an interpretation function I to formalize the following models M1, M2 and M3.

ANSWER: A possible W is as follows

L = <E = {i1, i2, i3}, C = {INS, R, G, B, SL}, R = ∅>

D = < E = {a, b, c}, C = {insect, red, green, blue, sixlegs}, R = ∅>

I(i1) = 1; I(i2) = b; I(i3) = c; I(INS) = insect = {a, b, c}; I(R) = red = {a}; I(G) = green = {b}; I(B) = blue = {c}; I(SL) = sixlegs = {a, b, c}.

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World models (II)

Define a language L’ ⊇ L and an interpretation function I’ for the same problem that includes at least a complex formula.

ANSWER:

L’ = L ∪ {INS ⊓ R}

I’ = (INS ⊓ R) = insect ∩ red = {a}

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Logic systems (I)

Define a logic system L = <W, ⊨ >, and in particular the entailment relation ⊨ starting from the world model W defined before.

ANSWER: we need to provide it for the models M1, M2 and M3:

M1 ⊨ INS(i1), M1 ⊨ R(i1), M1 ⊨ SL(i1)

M2 ⊨ INS(i2), M2 ⊨ G(i2), M2 ⊨ SL(i2)

M3 ⊨ INS(i3), M3 ⊨ B(i3), M3 ⊨ SL(i3)

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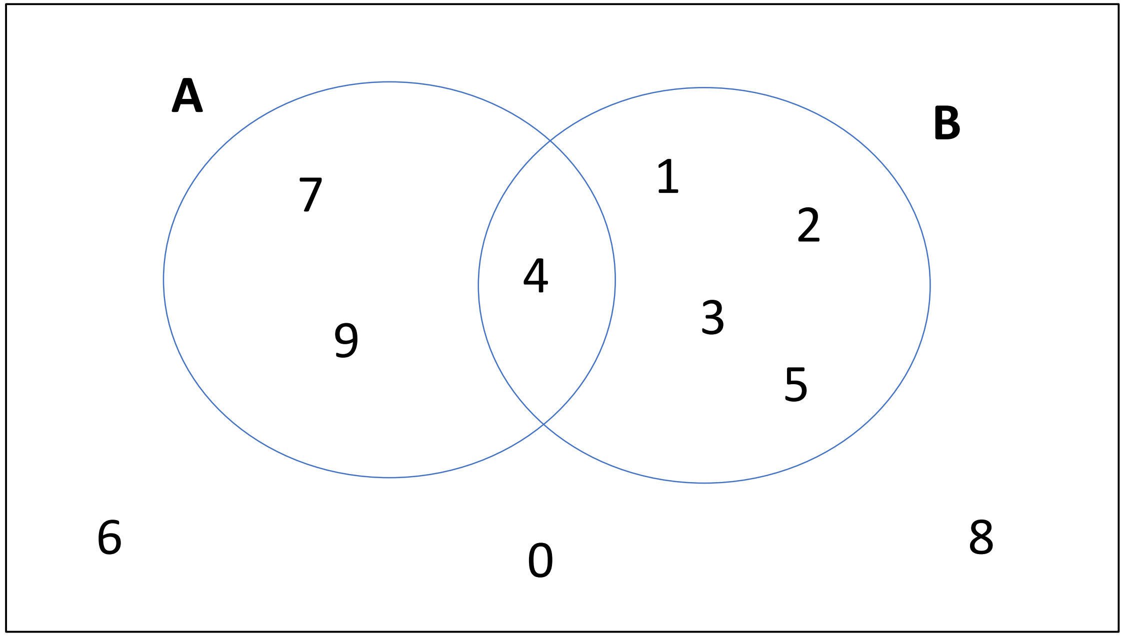
World Models (II)

Define an intentional world model W constituted by a language L, a domain D and an interpretation function I to formalize the following Venn Diagram V



ANSWER:

L = {A, B, A ⊓ B, A ⊔ B}

D = < E, {C}, {P} >

E = {0, 1, 2, 3, 4, 5, 6, 7, 8}

I(A) = {4, 7, 9} ∊ {C};

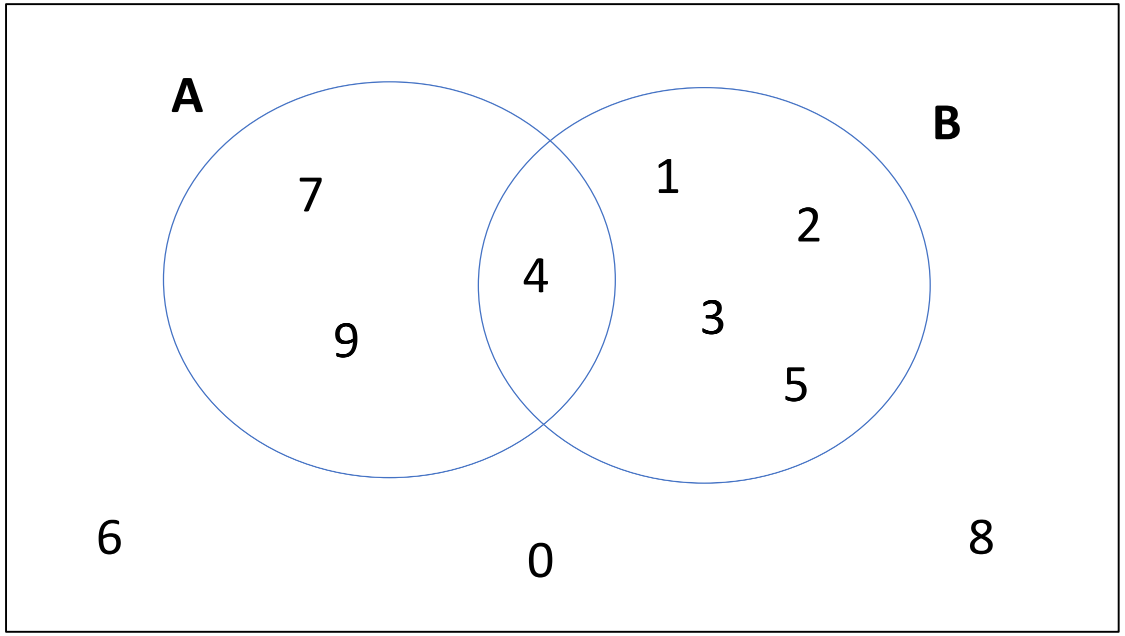
I(B) = {1, 2, 3, 4, 5} ∊ {C};

I(A ⊓ B) = I(A) ∩ I(B) = {4} ∊ {C};

I(A ⊔ B) = I(A) ∪ I(B) = {1, 2, 3, 4, 5, 7, 9} ∊ {C}

Logic systems (II)

Define a logic system L = <W, ⊨ >, and in particular the entailment relation ⊨ starting from the world model W defined before.



ANSWER:

$V \models A$

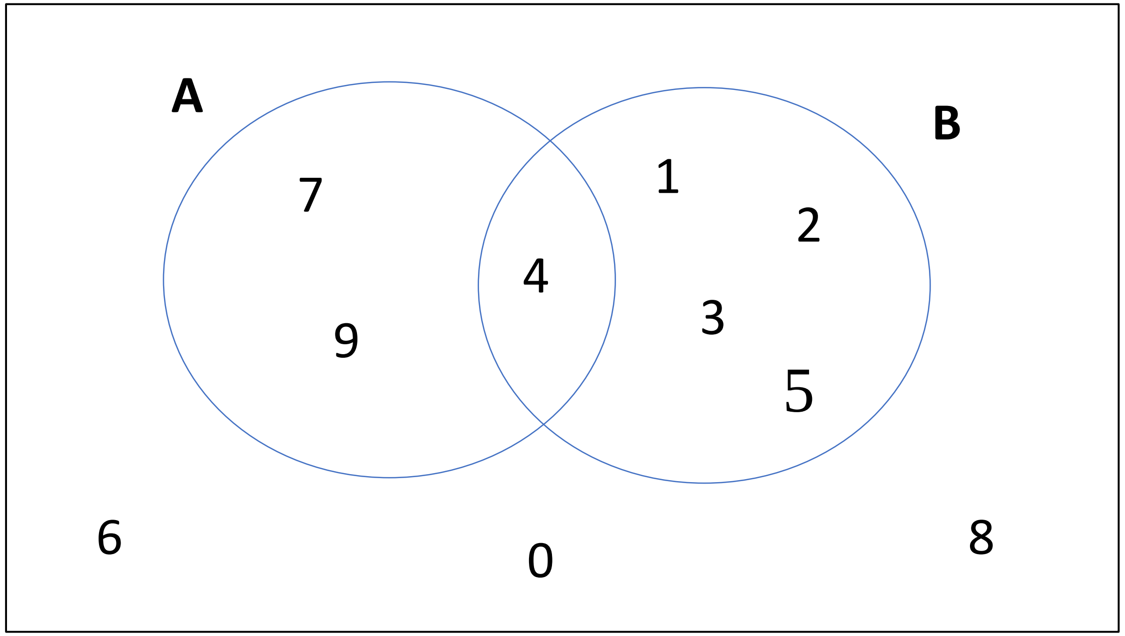
$V \models B$

$V \models A \sqcap B$

$V \models A \sqcup B$

Logic systems (III)

Define a theory T such that V ⊨ T.



ANSWER: In this case, we may take any T ⊆ L, for instance {A, B, A ⊔ B}

Preamble: Reasoning problems

Model checking. Given T and M, check whether M ⊨ T

Satisfiability. Given T, check whether there exists M such that M ⊨ T

Validity. Given T, check whether for all M, M ⊨ T

Unsatisfiability. Given T, check whether there is no M such that M ⊨ T

Logical consequence. Given T1, T2 and a set of reference models {M}, check whether

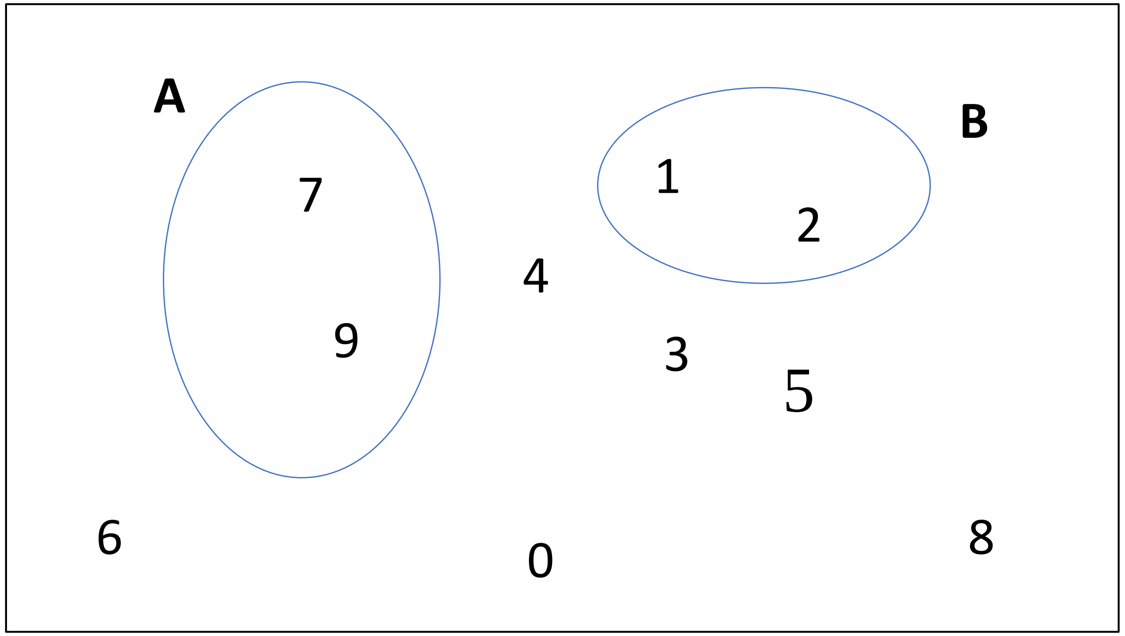
$$T1 \models_{\{M\}} T2$$

Logical equivalence. Given T1, T2 and a set of reference models {M}, check whether

$$T1 \models_{\{M\}} T2 \text{ and } T2 \models_{\{M\}} T1$$

Logic systems (IV): reasoning – model checking (a)

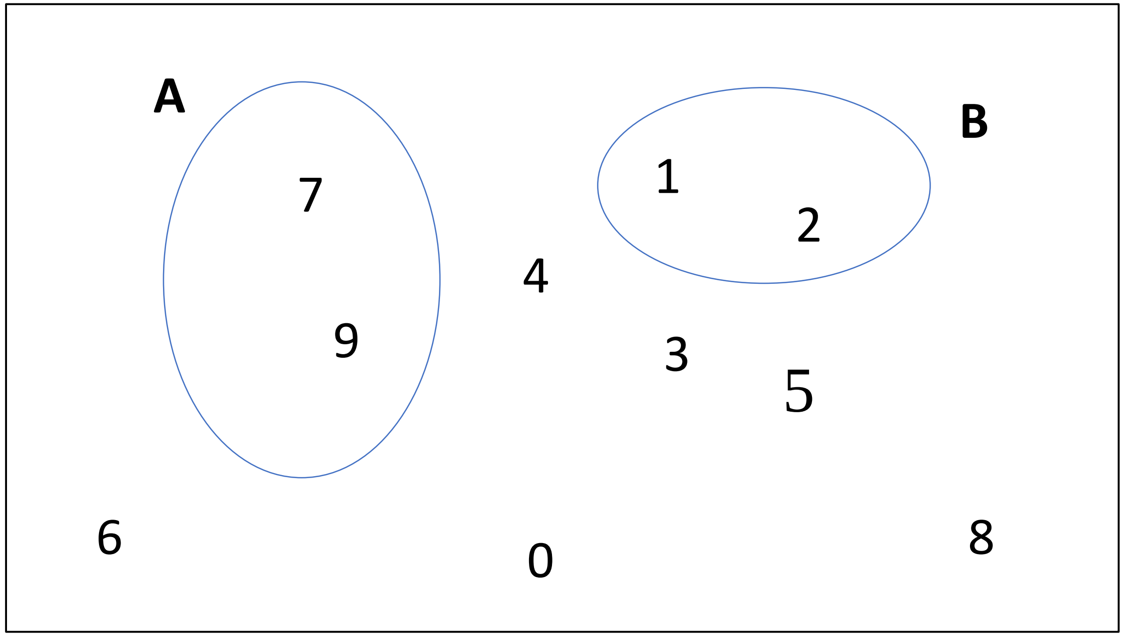
Given the theory T = {A, B, A ⊔ B}, and the model V below, check if V ⊨ T



ANSWER: yes

Logic systems (IV): reasoning – model checking (b)

Given the theory T = {A, B, A ⊓ B}, and the model V below, check if V ⊨ T



ANSWER: no

Logic systems (IV): reasoning – model checking (c)

Provide and example of Venn Diagram V' containing two sets A and B, and of a theory T' $\subseteq$ {A, B, A $\sqcap$ B, A $\sqcup$ B} such that V' $\not\models$ T'.

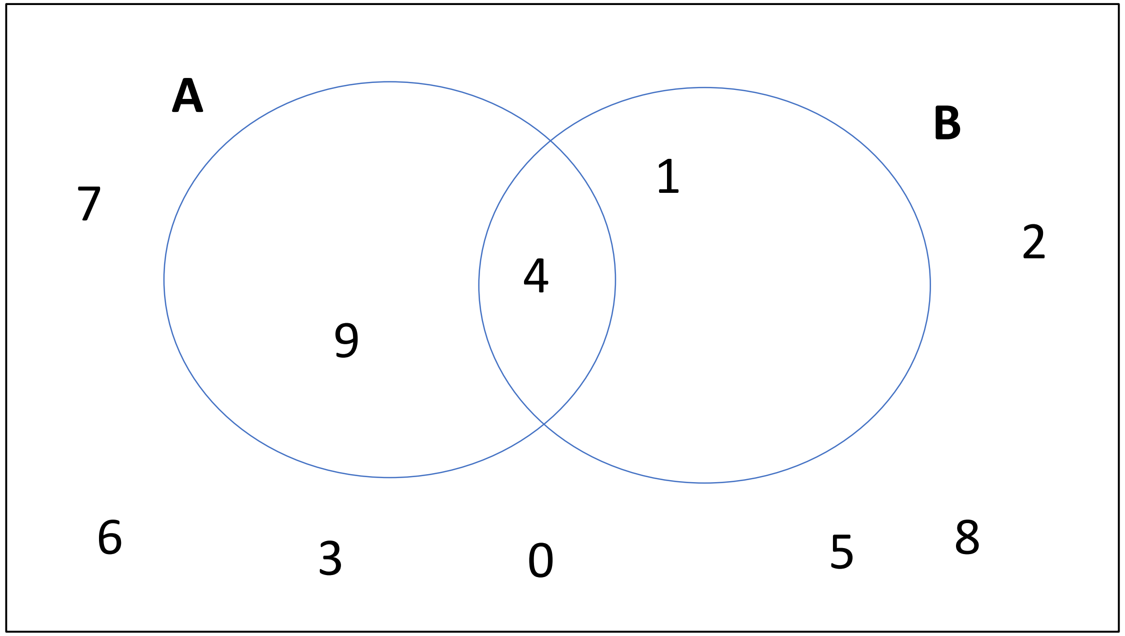
ANSWER: We may take the theory T' = {A $\sqcap$ B} and any V' in which A and B are disjoint.

In fact I(A $\sqcap$ B) = I(A) $\cap$ I(B) = $\emptyset$.

Logic systems (IV): reasoning – satisfiability

Given the theory T = {A, B, A ⊓ B}, check whether there exists V such that V ⊨ T

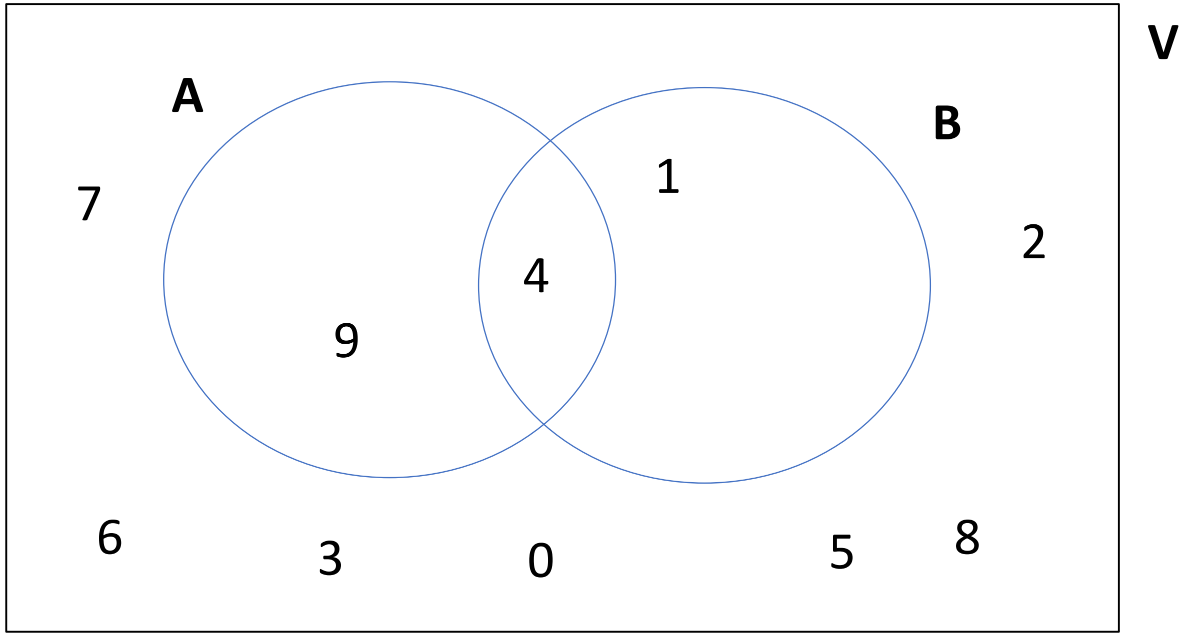
ANSWER



Logic systems (V)

Say which of the following statements are true given the formalization provided before.

a) $V \models D$
b) $V \models A$
c) $V \models \emptyset$
d) $I((A \sqcap B) \sqcap B) = I(A \sqcap B)$
e) $A \sqcap B$ is an atomic formula



ANSWER: b, d

Homework

Answer to the following questions

1. What are the main characteristics of a representation language?
2. What is the difference between extensional and intentional representations?
3. What is the difference between an atomic formula and a complex formula?
4. What is an interpretation function?
5. What is entailment and what are its properties?
6. What are the desired properties of logic languages?
7. When it is the case that a theory is correct and complete?
8. Can you describe the main reasoning problems?