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Computational Logic Exercises
Module IV – The Logic of Descriptions (LOD)

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Syntax of LOD

Which of the following symbols are used in LOD ?

$\sqcap \neg \top \vee \equiv \sqcup \sqsubseteq \rightarrow \leftrightarrow \bot \wedge \models$

ANSWER:

$\sqcap \neg \top \equiv \sqcup \sqsubseteq \bot \models$

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Syntax of LOD

Which of the following is not a wff in LOD?

1. $\neg$ MonkeyLow $\sqcup$ BananaHigh
2. $\neg \neg$ MonkeyLow $\sqcap$ BananaHigh $\sqsubseteq \neg$ GetBanana
3. MonkeyLow $\neg \sqcap$ BananaHigh
4. MonkeyLow $\vee \neg$ GetBanana

ANSWER:

2, 3, 4

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Formalization of simple sentences in LOD

The set of games which are not legal	$Game \sqcap \neg Legal$
Lakes are locations	$Lake \sqsubseteq Location$
Lakes are locations made of water	$Lake \sqsubseteq Location \sqcap \exists Madeof.Water$
Persons can be distinguished into male and female	$Person \sqsubseteq Male \sqcup Female$
Male and Female are disjoint	$Male \sqsubseteq \neg\ Female$
Persons have a birthplace	$Person \sqsubseteq \exists hasBirthPlace.\top$
The set of documents about “programming in Java” are a subset of the documents about “programming languages” and “computer science”	$JavaProgramming \sqsubseteq ProgrammingLanguage \sqcap ComputerScience$

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Formalization of a problem in a LOD theory

Unicorns are mythical horses having a horn.

$\text{Unicorn} \sqsubseteq \text{mythical} \sqcap \text{horse} \sqcap \exists \text{has.Horn}$

There are two kinds of students: master students and PhD students. All PhD students' task is research.

$\text{MasterStudent} \sqsubseteq \text{Student}$

$\text{PhDStudent} \sqsubseteq \text{Student} \sqcap \exists \text{hasTask.Research}$

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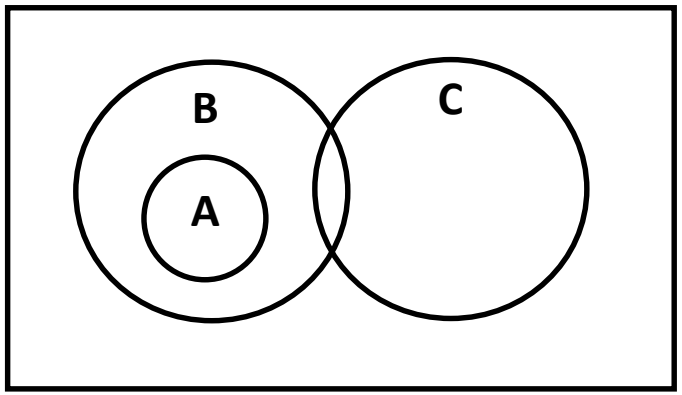
Venn Diagrams and LOD

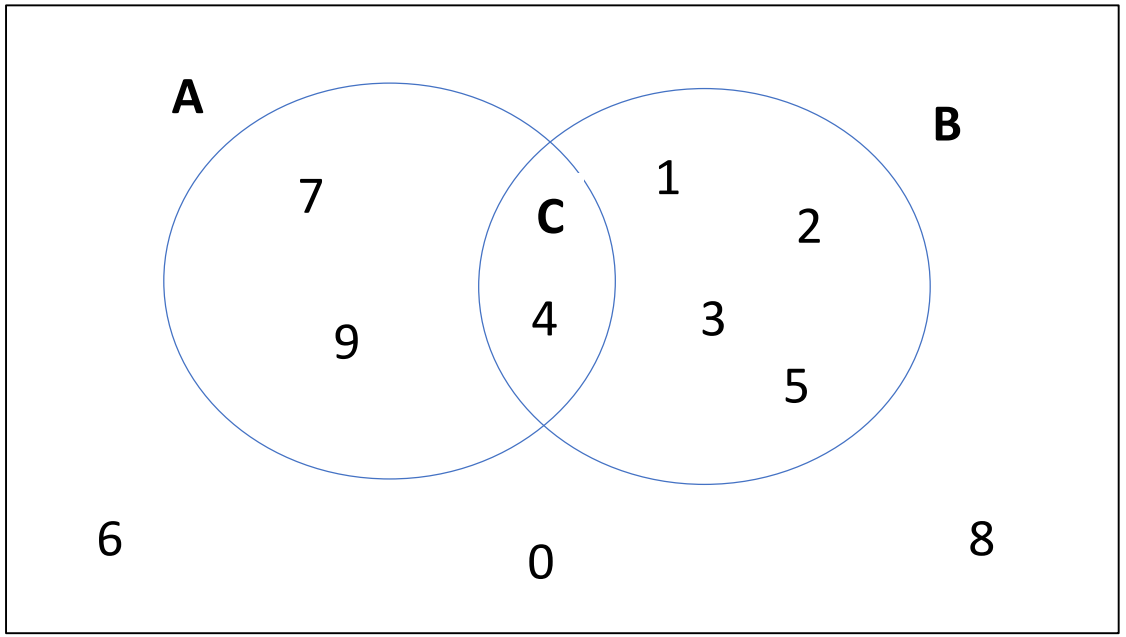
Provide the Venn diagram for $A \sqsubseteq B \sqcap \neg C$

ANSWER:



Define a LOD domain and theory

Define a plausible LOD domain D and a theory T for the Venn diagram



ANSWER:

D = <E, {C}, {P}>

E = {0, …9} C = {A, B, C} P = ∅

T = { C ≡ A ⊓ B }

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Define a LOD domain and theory

Define a plausible LOD domain D and a theory T for the database table

Employee			
Name	**Role**	**Nationality**	**Supervises**
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-

ANSWER:

D = <E, {C}, {P} >;

E = {Fausto, Rui, Bisu, Italian, Chinese, Indian}

C = {Employee, Professor, Student, Nationality} P = {hasNationality, hasSupervisor}

T = { Professor ⊑ Employee; Student ⊑ Employee;
Employee ⊑ ∃hasNationality.Nationality ⊓ ∃hasSupervisor.Employee }

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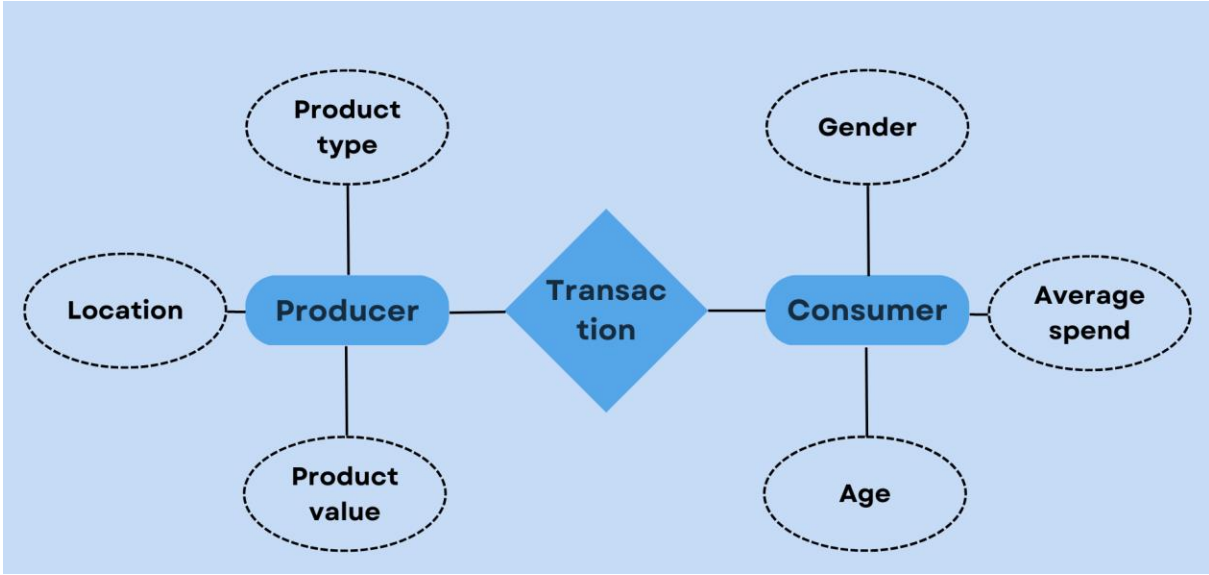
Define a LOD domain and theory

Define a plausible theory T for the ER diagram



ANSWER:

Producer ⊑ Entity ⊓ (∃ProductType.⊤ ⊔ ∃Location.⊤ ⊔ ∃ProductValue.⊤ ⊔ ∃Transaction.Consumer)

Consumer ⊑ Entity ⊓ (∃Gender.⊤ ⊔ ∃Age.⊤ ⊔ ∃AverageSpend.⊤)

Define a LOD domain and theory

Define a plausible LOD domain D for the knowledge graph

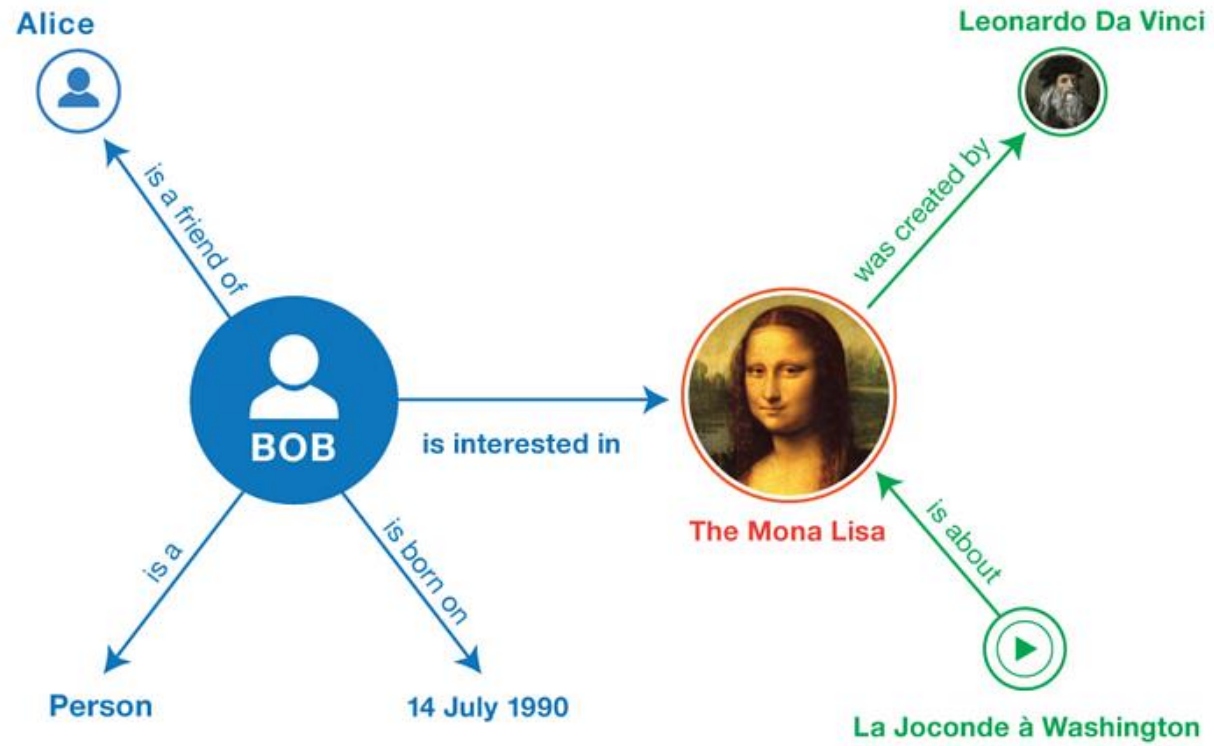
ANSWER (part I):

D = <E, {C}, {P}>

E = {Alice, Bob, The Mona Lisa, Leonardo Da Vinci, La Joconde à Washington, 14 July 1990}

C = {Entity, Person, Picture, File, Date}

P = {isFriendOf, interestedIn, isAbout, wasCreatedBy, isBornOn}



Define a LOD domain and theory

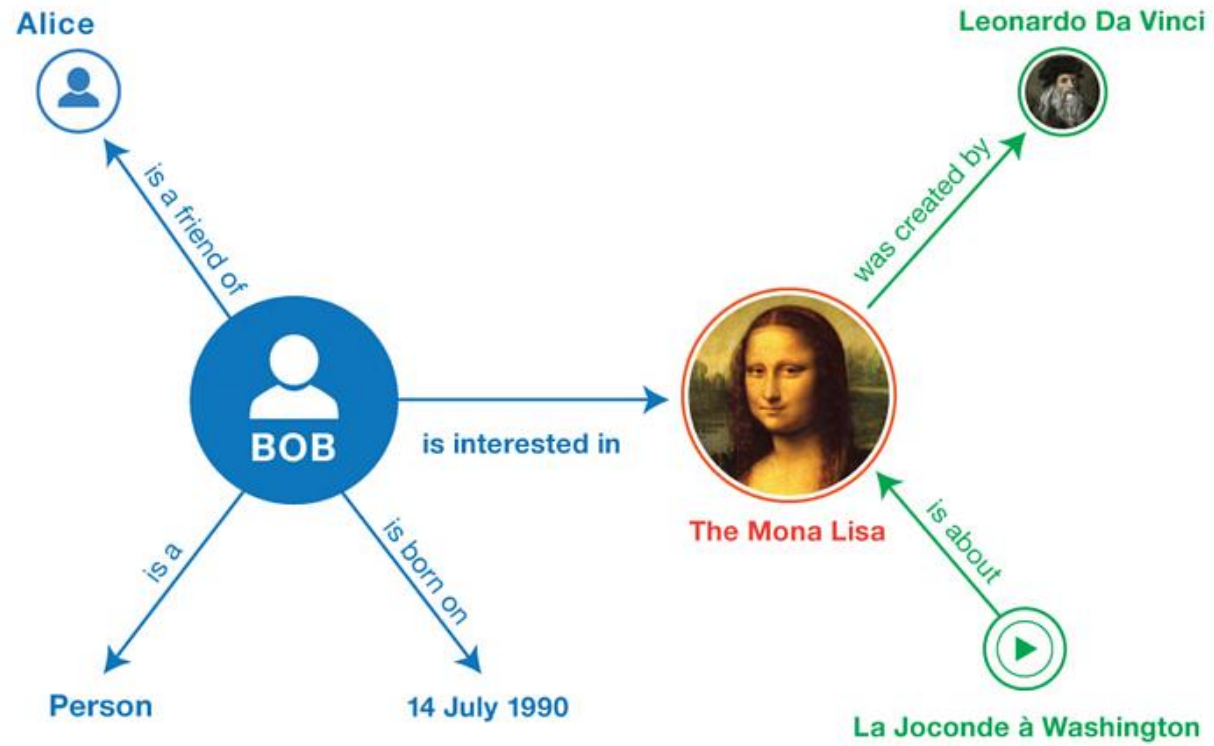
Define a plausible LOD theory T for the knowledge graph

ANSWER (part II):
The theory T can be as follows:

Person ⊑ Entity ⊓ (∃isFriendOf.Person ⊔ ∃interestedIn.Picture ⊔ ∃isBornOn.Date)

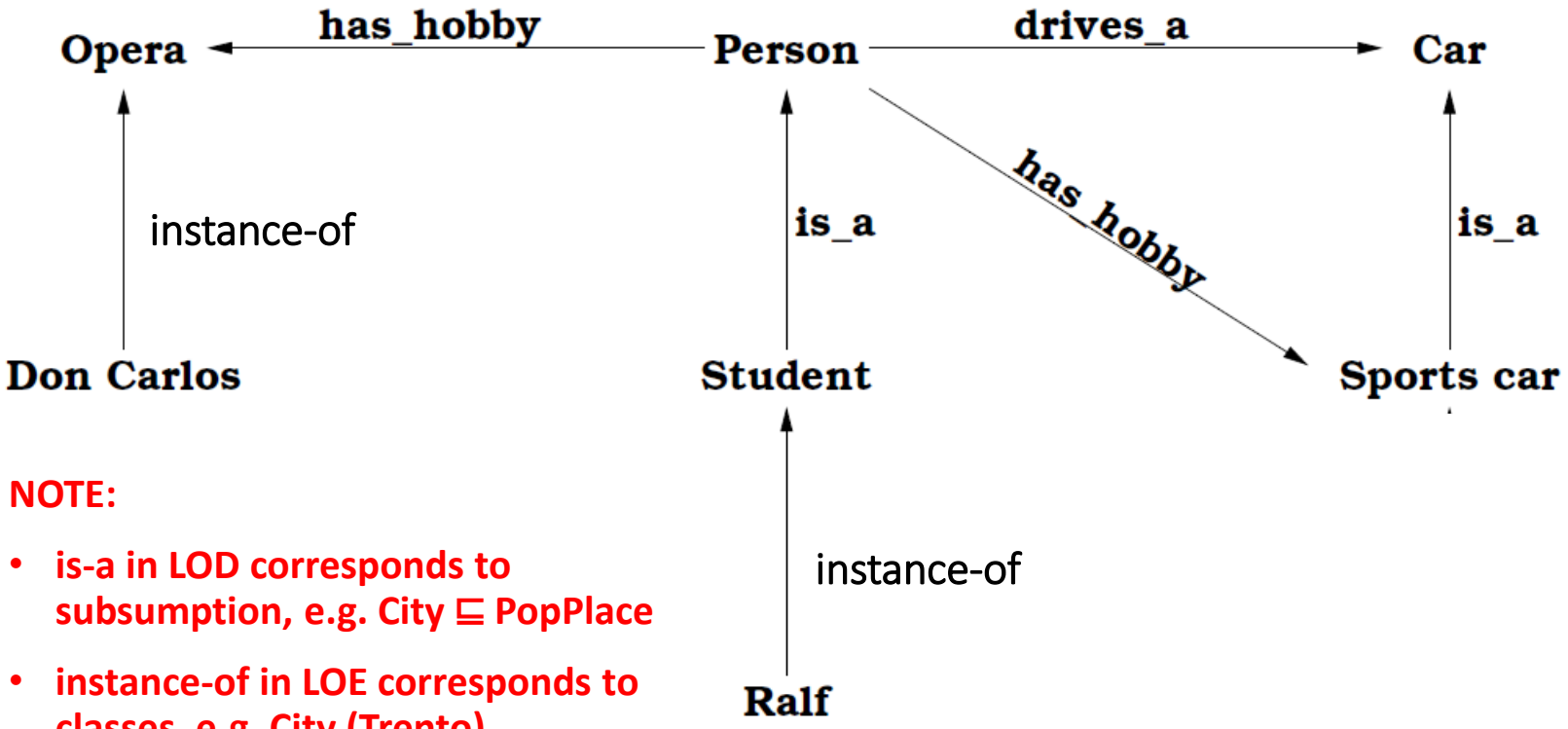
Picture ⊑ Entity ⊓ ∃wasCreatedBy.Person

File ⊑ Entity ⊓ ∃isAbout.Picture



Define a LOD + LOE theory

Define a LOD + LOE theory for the following knowledge graph



NOTE:

- **is-a in LOD corresponds to subsumption, e.g. City ⊑ PopPlace**
- **instance-of in LOE corresponds to classes, e.g. City (Trento)**

ANSWER:

$\text{Person} \sqsubseteq \exists \text{Drives.Car} \sqcup \exists \text{HasHobby.SportCar} \sqcup \exists \text{HasHobby.Opera}$

$\text{Student} \sqsubseteq \text{Person}$

$\text{SportCar} \sqsubseteq \text{Car}$

Student(Ralf)

Opera(DonCarlos)

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Define a LOD + LOE theory

Define a LOD + LOE theory for the following problem:

In a hospital patients, doctors and computers are equipped with proximity sensors able to detect whether doctors curated a patient or worked at their computer. The system detected that doctor Peter curated the patient Smith.

ANSWER:

$\text{Doctor} \sqsubseteq \forall \text{cure.Patient} \sqcup \forall \text{work.Computer}$

$\text{cure} \sqsubseteq \text{detected}$

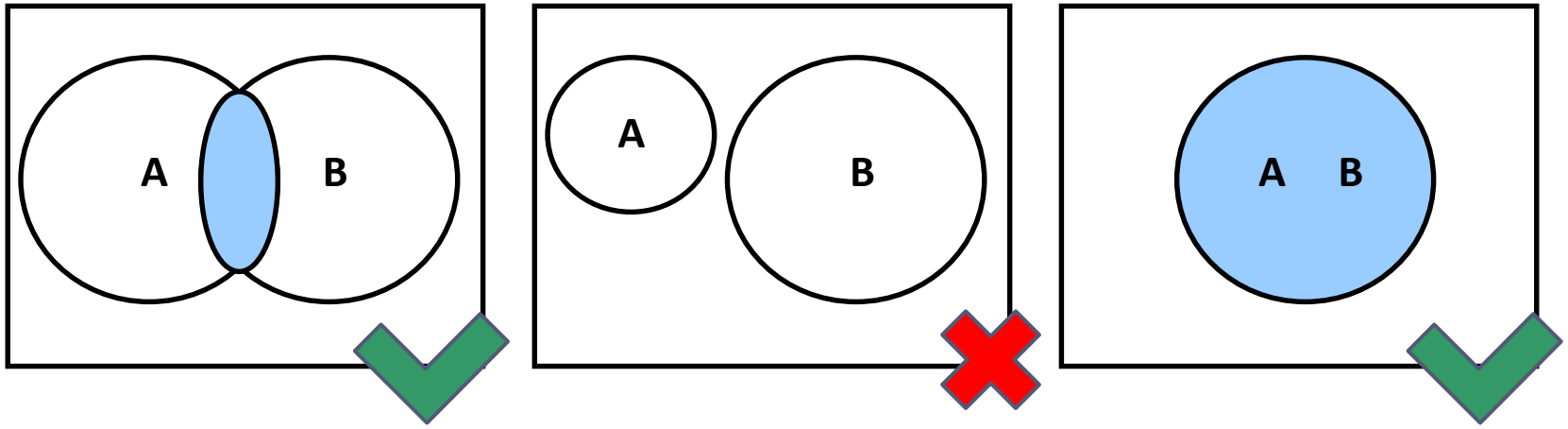
$\text{work} \sqsubseteq \text{detected}$

Doctor (Peter) Patient (Smith) cure(Peter, Smith)

Reasoning in LOD

Suppose you have that M ⊨ A and M ⊨ B. Does M ⊨ A ⊓ B?

ANSWER:

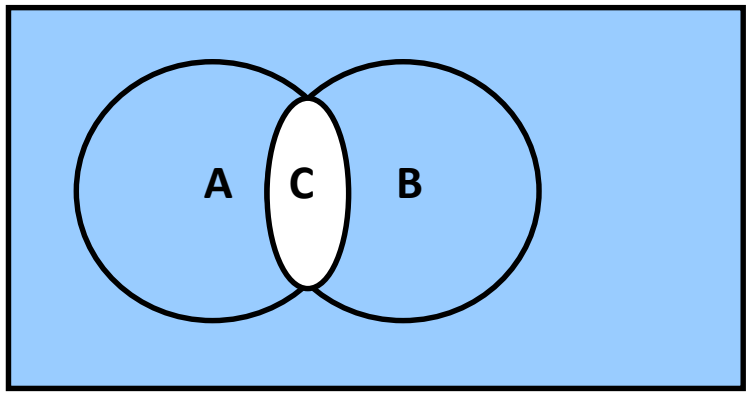


By using Venn Diagrams, we can easily observe that the fact that A and B are not empty does not imply that A ⊓ B is also not empty. Think to the case in which their extensions are disjoint.

Reasoning in LOD

Is the theory T= {C⊑A, C⊑B, ¬(A⊓B)} satisfiable?

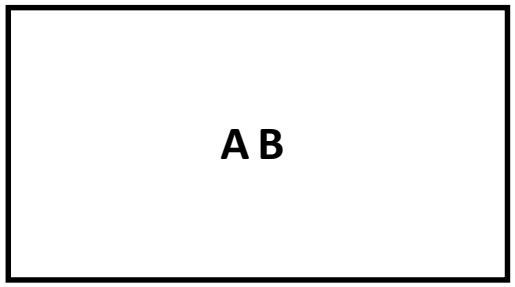
ANSWER: Yes. A case is described below with a Venn Diagram.



Reasoning in LOD

Given the theory T={A⊑B, B⊑A}, does T ⊨ ¬(A⊓B)?

ANSWER: No. In fact, we can find a counterexample in which I(A) = I(B) but the I(¬(A⊓B)) is empty.



Reasoning in LOD

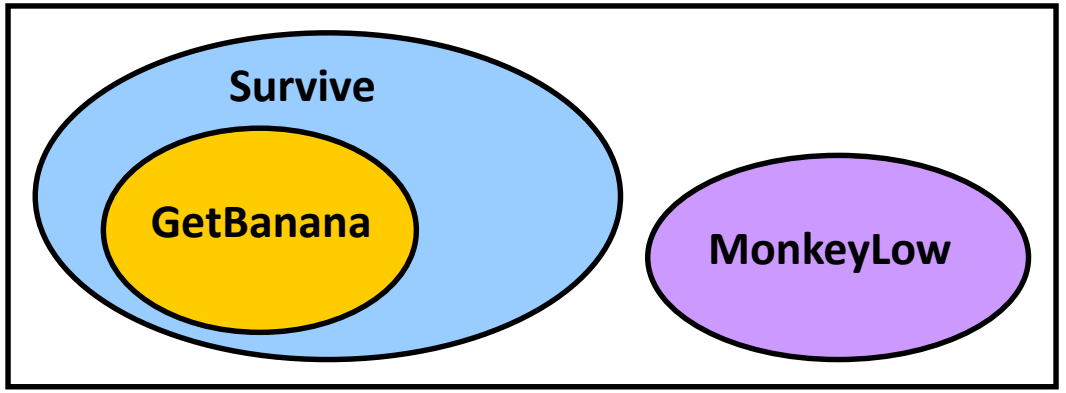
Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow $\sqsubseteq \neg$GetBanana
GetBanana $\sqsubseteq$ Survive

Is T satisfiasble?

ANSWER: Yes. It is enough to find one model for it, represented graphically with the Venn Diagram below.



Reasoning in LOD

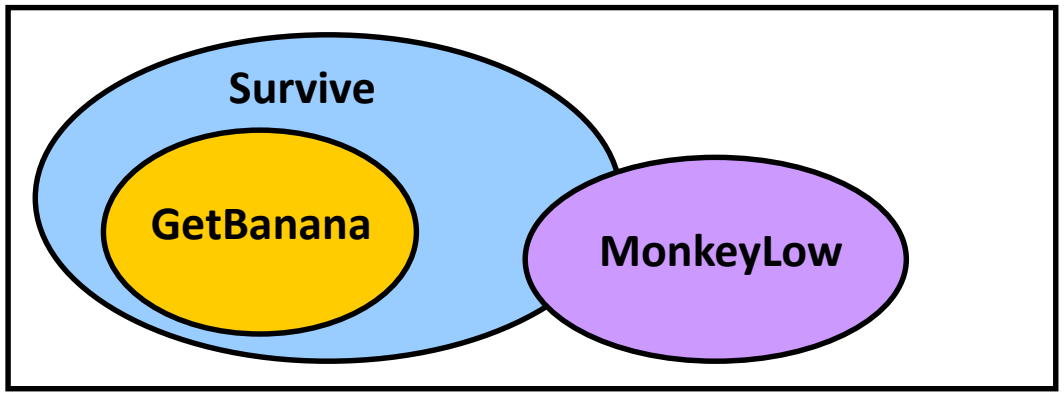
Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow $\sqsubseteq \neg$GetBanana
GetBanana $\sqsubseteq$ Survive

Is it possible for a monkey to survive even if it does not get the banana?

ANSWER: We can restate the problem as follow:
does T $\models \neg$GetBanana $\sqcap$ Survive at least in one model?
Yes. We can find a model in which both all the assertions in T and $\neg$GetBanana $\sqcap$ Survive are not empty.



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Reasoning in LOD

Suppose we describe students in a course as follows:

Undergraduate	$\sqsubseteq \neg$ Teach
Bachelor	$\equiv$ Student $\sqcap$ Undergraduate
Master	$\equiv$ Student $\sqcap \neg$ Undergraduate
PhD	$\equiv$ Master $\sqcap$ Research
Assistant	$\equiv$ PhD $\sqcap$ Teach

Are all assistants also undergraduates?

ANSWER: We can restate the problem as follow:
does $T \models$ Assistant $\sqsubseteq$ Undergraduate ?

We need to prove that this is true in <u>all models</u> (via the method of *unfolding*)

Assistant $\equiv$ PhD $\sqcap$ Teach
$\equiv$ Master $\sqcap$ Research $\sqcap$ Teach
$\equiv$ Student $\sqcap \neg$ Undergraduate $\sqcap$ Research $\sqcap$ Teach
Answer is No. Assistants are actually students who are not undergraduate.

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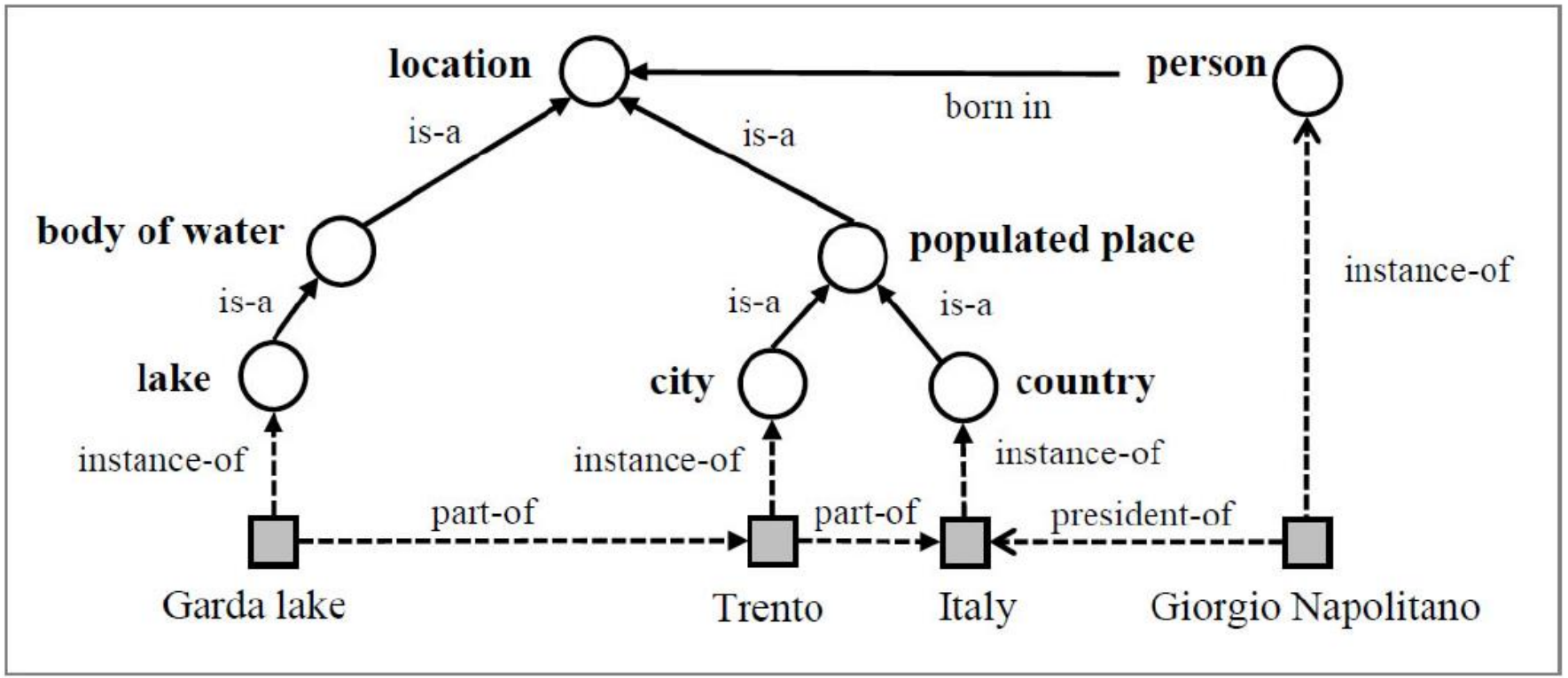
Homework

Answer to the following questions

1. What is the purpose of the Logic of Descriptions?
2. What are the key elements of the Logic of Descriptions?
3. What is the form of facts in a domain of the Logic of Descriptions?
4. Do we have negative facts in the Logic of Descriptions?
5. What is the form of assertions in a language of the Logic of Descriptions?
6. What is the form of a theory in the Logic of Descriptions?
7. What is the form of an interpretation function in the Logic of Descriptions?
8. What is entailment in the Logic of Descriptions?
9. What are the reasoning problems in the Logic of Descriptions?

Homework

Define a LOD + LOE theory for the following knowledge graph



Homework - Reasoning in LOD

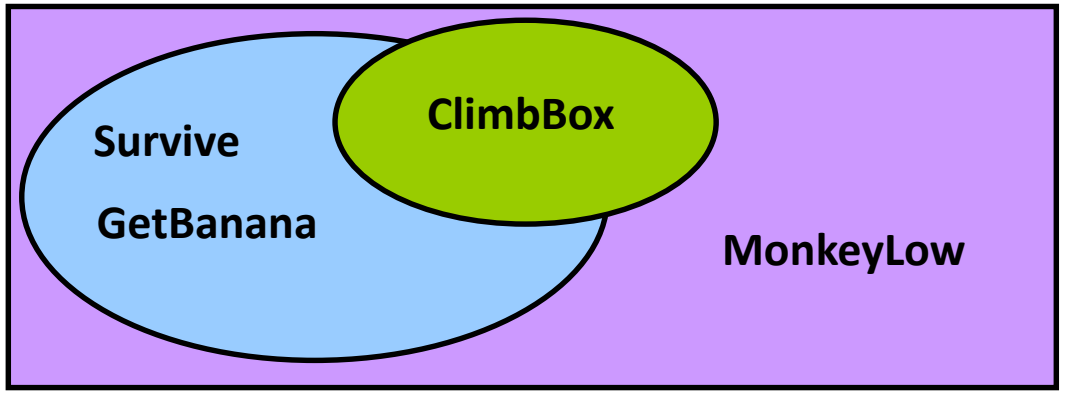
Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow ≡ ¬GetBanana ⊓ ¬ClimbBox
GetBanana ≡ Survive

Is it possible for a monkey to climb the box and not survive?

ANSWER: We can restate the problem as follow:
does T ⊨ ClimbBox ⊓ ¬Survive at least in one model?
Yes. We can find a model in which both all the assertions in T and ClimbBox ⊓ ¬Survive are not empty.



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Homework - Reasoning in LOD

Suppose we describe students in a course as follows:

Undergraduate	$\sqsubseteq \neg$ Teach
Bachelor	$\equiv$ Student $\sqcap$ Undergraduate
Master	$\equiv$ Student $\sqcap \neg$ Undergraduate
PhD	$\equiv$ Master $\sqcap$ Research
Assistant	$\equiv$ PhD $\sqcap$ Teach

Are bachelor and master disjoint?

ANSWER: We can restate the problem as follow:
does $T \models$ Bachelor $\sqcap$ Master $\sqsubseteq \bot$?

We need to prove that this is true in <u>all models</u> (via the method of expansion)

Bachelor $\sqcap$ Master
$\equiv$ (Student $\sqcap$ Undergraduate) $\sqcap$ (Student $\sqcap \neg$ Undergraduate)
$\equiv$ Student $\sqcap$ (Undergraduate $\sqcap \neg$ Undergraduate) $\equiv \bot$
Answer is therefore Yes.