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Computational Logic Exercises

Module V – LOD applications and LODE

Vincenzo Maltese

Well formed formulas

Which of the following complex formulas are syntactically correct in LOD? (TBOX formulas)

a) $A \equiv \exists R.C \sqcap \forall S.D$

b) $A \sqcap B \equiv C \sqcup D$

c) $A \equiv B \sqcap \neg C$

d) $A \sqsubseteq \neg C$

e) $A \sqsubseteq B \sqcap \exists R.C$

f) $A \sqsubseteq B \sqcap \exists R.(\forall S.D)$

g) $A \equiv B \sqcup \emptyset$

ANSWER:

a, c, d, e, f

Unfolding a Concept

Unfold ColouredGuitar:

$ElectricGuitar \equiv Guitar \sqcap \forall hasSoundAmplification.withInputJack$

$ColouredGuitar \equiv ElectricGuitar \sqcap \exists hasColour.String$

Answer:

$ColouredGuitar \equiv Guitar \sqcap \forall hasSoundAmplification.withInputJack \sqcap \exists hasColour.String$

Cyclic and acyclic TBOX

Is the following TBOX cyclic?

Woman ≡ Person ⊓ Female

Man ≡ Person ⊓ ¬Woman

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

Parent ≡ Father ⊔ Mother

ANSWER:

No, because by unfolding all concepts I never obtain the same concept on the left and on the right of the equivalences.

Cyclic and acyclic TBOX

Is the following TBOX cyclic?

Male ≡ ¬Female

Female ≡ ¬Male

ANSWER:
Yes, because by unfolding it I get Female ≡ ¬(¬Female) that is Female ≡ Female

Terminology

Is the following TBOX a terminology?

$\text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person}$

$\text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person}$

$\text{Parent} \equiv \text{Father} \sqcup \text{Mother}$

ANSWER:

Yes, because it is acyclic and there are only equivalences.

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Terminology

Is the following TBOX a terminology?

$Mother \equiv Woman \sqcap \exists hasChild.Person$

$Father \equiv Man \sqcap \exists hasChild.Person$

$Parent \sqsubseteq Father \sqcup Mother$

ANSWER:

No, because it contains a subsumption.

Creating a terminology by expansion

Given the previous TBOX, provided below, can I convert it to make it a terminology?

$\text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person}$

$\text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person}$

$\text{Parent} \sqsubseteq \text{Father} \sqcup \text{Mother}$

ANSWER:

Yes, for instance as follows:

$\text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person}$

$\text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person}$

$\text{StepMother} \equiv \text{Woman} \sqcap \exists \text{marriedWith.Father}$

$\text{StepFather} \equiv \text{Man} \sqcap \exists \text{marriedWith.Mother}$

$\text{Parent} \equiv \text{Father} \sqcup \text{Mother} \sqcup \text{StepFather} \sqcup \text{StepMother}$

Defining a terminology from natural language definitions

A lion is a large gregarious predatory feline of Africa and India having a shaggy mane in the male

$Lion \equiv Feline \sqcap Large \sqcap Gregarious \sqcap Predatory \sqcap \forall livesIn.(Africa \sqcup India) \sqcap \exists livesIn.(Africa \sqcup India)$

$MaleLion \equiv Lion \sqcap Male \sqcap \forall has.ShaggyMane \sqcap \exists has.ShaggyMane$

A penguin is a flightless bird of Antarctica having webbed feet

$Penguin \equiv Bird \sqcap \neg Fly \sqcap \forall livesIn.Antarctica \sqcap \exists livesIn.Antarctica \sqcap \forall has.WebbedFeet \sqcap \exists has.WebbedFeet$

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Defining a terminology from a schema

Thing > Event

Property	Expected Type
Properties from Event	
about	Thing
actor	Person
attendee	Organization or Person

ANSWER. By assuming the schema as complete (otherwise it is not a terminology) we have:

$$
\begin{aligned}
\text{Event} \equiv \text{Thing} \sqcap\ & \\
& \forall \text{about.Thing} \sqcap \exists \text{about.Thing} \sqcap \\
& \forall \text{actor.Person} \sqcap \exists \text{actor.Person} \sqcap \\
& \forall \text{attendee.(Person} \sqcup \text{Organization)} \sqcap \exists \text{attendee.(Person} \sqcup \text{Organization)}
\end{aligned}
$$

Logical consequences of a terminology (by unfolding)

Given the previous TBOX, replicated on the right, which of the following are logical consequences of the TBOX?

a) $\text{Event} \sqsubseteq \forall\text{about.Thing} \sqcap \exists\text{about.Thing}$

b) $\text{Event} \sqsubseteq \forall\text{about.Thing}$

c) $\text{Event} \sqsubseteq \exists\text{about.Thing}$

d) $\text{Event} \equiv \forall\text{about.Thing} \sqcap \exists\text{about.Thing}$

e) $\text{Event} \sqsubseteq \forall\text{attendee.Person} \sqcup \forall\text{attendee.Organization}$

f) $\text{Event} \sqsubseteq \forall\text{attendee.Person}$

g) $\text{Person} \sqsubseteq \neg\text{Organization}$

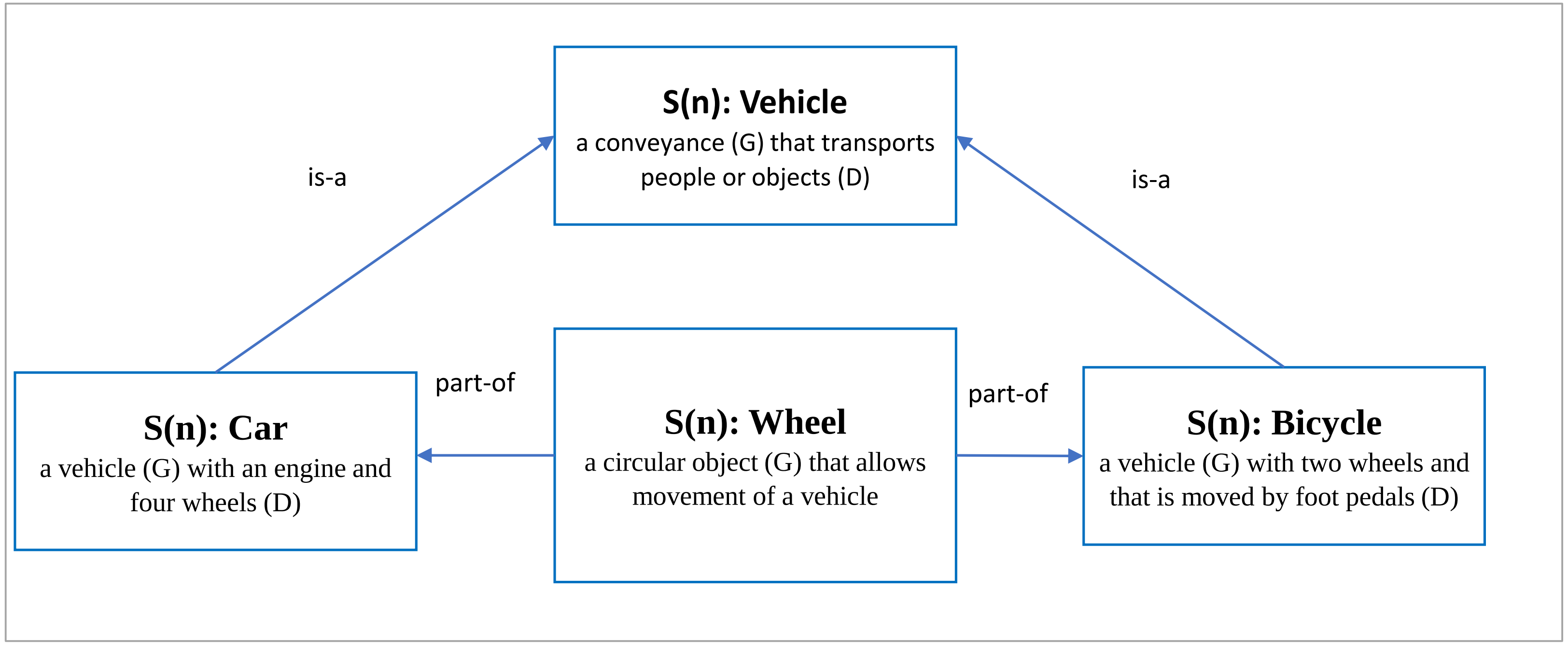
ANSWER.

a, b, c, e, f

TBOX

$\text{Event} \equiv \text{Thing}$
$\sqcap\ \forall\text{about.Thing} \sqcap \exists\text{about.Thing}$
$\sqcap\ \forall\text{actor.Person} \sqcap \exists\text{actor.Person} \sqcap$
$\forall\text{attendee.(Person} \sqcup \text{Organization)} \sqcap$
$\exists\text{attendee.(Person} \sqcup \text{Organization)}$

Formalizing a lexicon as a terminology (I)



Vehicle ≡ Conveyance ⊓ ∃transports.(Person ⊔ Object) ⊓ ∀transports.(Person ⊔ Object)

Car ≡ Vehicle ⊓ ¬Bicycle ⊓ ∃hasPart.Wheel ⊓ ∃hasPart.Engine

Bicycle ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃hasPart.FootPedal ⊓ ∃movedBy.FootPedal ⊓ ∀movedBy.FootPedal

Wheel ≡ Object ⊓ CircularShape ⊓ ∃moves.Vehicle ⊓ ∀moves.Vehicle

Reasoning in LOD

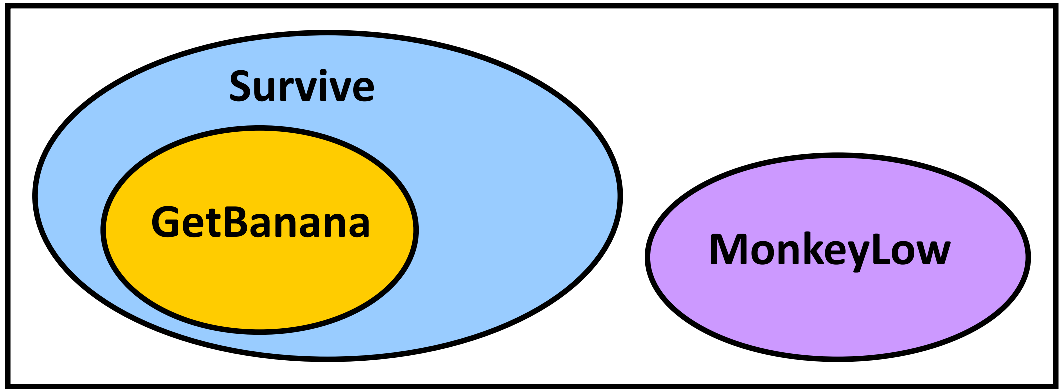
Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow $\sqsubseteq \neg$GetBanana
GetBanana $\sqsubseteq$ Survive

Is T satisfiasble?

ANSWER: Yes. It is enough to find one model for it, represented graphically with the Venn Diagram below.



Reasoning in LOD

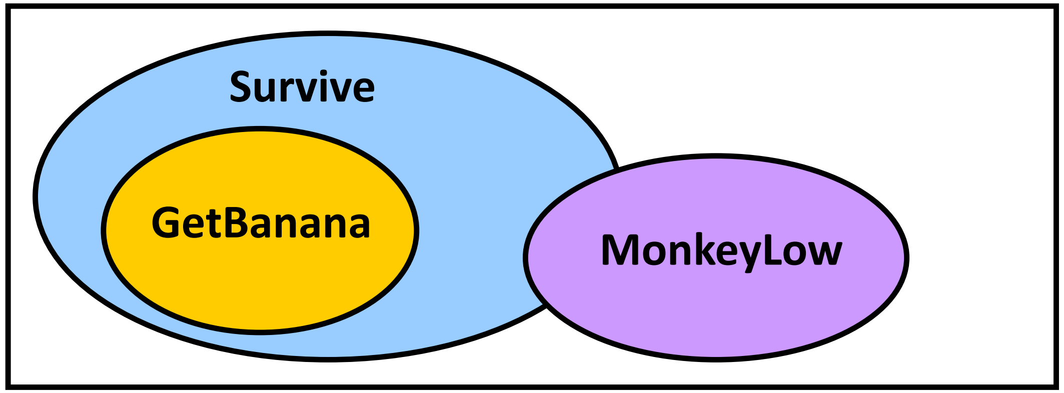
Suppose we model the Monkey-Banana problem as follows:

“If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives”.

Theory T:
MonkeyLow ⊑ ¬GetBanana
GetBanana ⊑ Survive

Is it possible for a monkey to survive even if it does not get the banana?

ANSWER: We can restate the problem as follow:
does T ⊨ ¬GetBanana ⊓ Survive at least in one model?
Yes. We can find <u>a model</u> in which both all the assertions in T and ¬GetBanana ⊓ Survive are not empty.



Reasoning in LOD

Suppose we describe students in a course as follows:

$$\begin{array}{ll} \text{Undergraduate} & \sqsubseteq \neg \text{Teach} \\ \text{Bachelor} & \equiv \text{Student} \sqcap \text{Undergraduate} \\ \text{Master} & \equiv \text{Student} \sqcap \neg \text{Undergraduate} \\ \text{PhD} & \equiv \text{Master} \sqcap \text{Research} \\ \text{Assistant} & \equiv \text{PhD} \sqcap \text{Teach} \end{array}$$

Are all assistants also undergraduates?

ANSWER: We can restate the problem as follow:
does $T \vDash \text{Assistant} \sqsubseteq \text{Undergraduate}$?

We need to prove that this is true in <u>all models</u> (via the method of *unfolding*)

$\text{Assistant} \equiv \text{PhD} \sqcap \text{Teach}$
$\equiv \text{Master} \sqcap \text{Research} \sqcap \text{Teach}$
$\equiv \text{Student} \sqcap \neg \text{Undergraduate} \sqcap \text{Research} \sqcap \text{Teach}$
Answer is No. Assistants are actually students who are not undergraduate.

Reasoning in LOD

Suppose we model the Monkey-Banana problem as follows:

“If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives”.

Theory T:

$MonkeyLow \equiv \neg GetBanana \sqcap \neg ClimbBox$

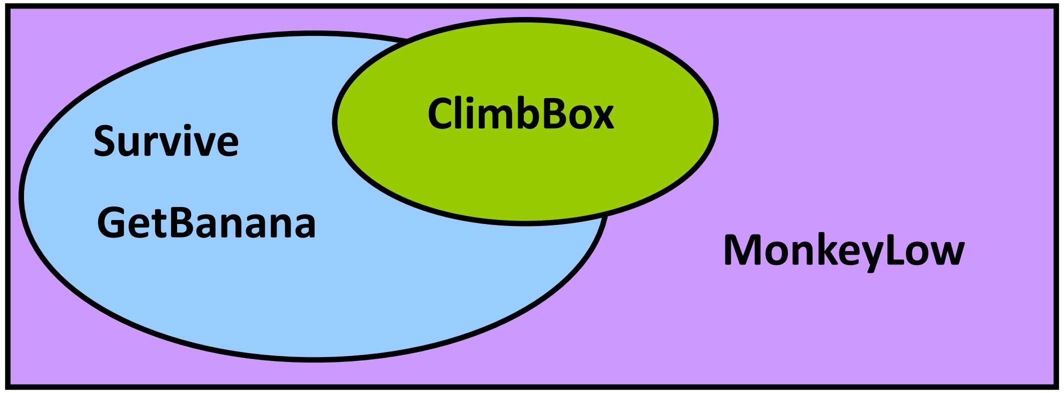
$GetBanana \equiv Survive$

Is it possible for a monkey to climb the box and not survive?

ANSWER: We can restate the problem as follow:

does $T \vDash ClimbBox \sqcap \neg Survive$ at least in one model?

Yes. We can find <u>a model</u> in which both all the assertions in T and $ClimbBox \sqcap \neg Survive$ are not empty.



Reasoning in LOD

Suppose we describe students in a course as follows:

$$
\begin{array}{ll}
\text{Undergraduate} & \sqsubseteq \neg\,\text{Teach} \\
\text{Bachelor} & \equiv \text{Student} \sqcap \text{Undergraduate} \\
\text{Master} & \equiv \text{Student} \sqcap \neg\,\text{Undergraduate} \\
\text{PhD} & \equiv \text{Master} \sqcap \text{Research} \\
\text{Assistant} & \equiv \text{PhD} \sqcap \text{Teach}
\end{array}
$$

Are bachelor and master disjoint?

ANSWER: We can restate the problem as follow:
does $T \models \text{Bachelor} \sqcap \text{Master} \sqsubseteq \bot$?

We need to prove that this is true in <u>all models</u> (via the method of *unfolding*)

Answer is obviously Yes because they contain two opposite constraints.

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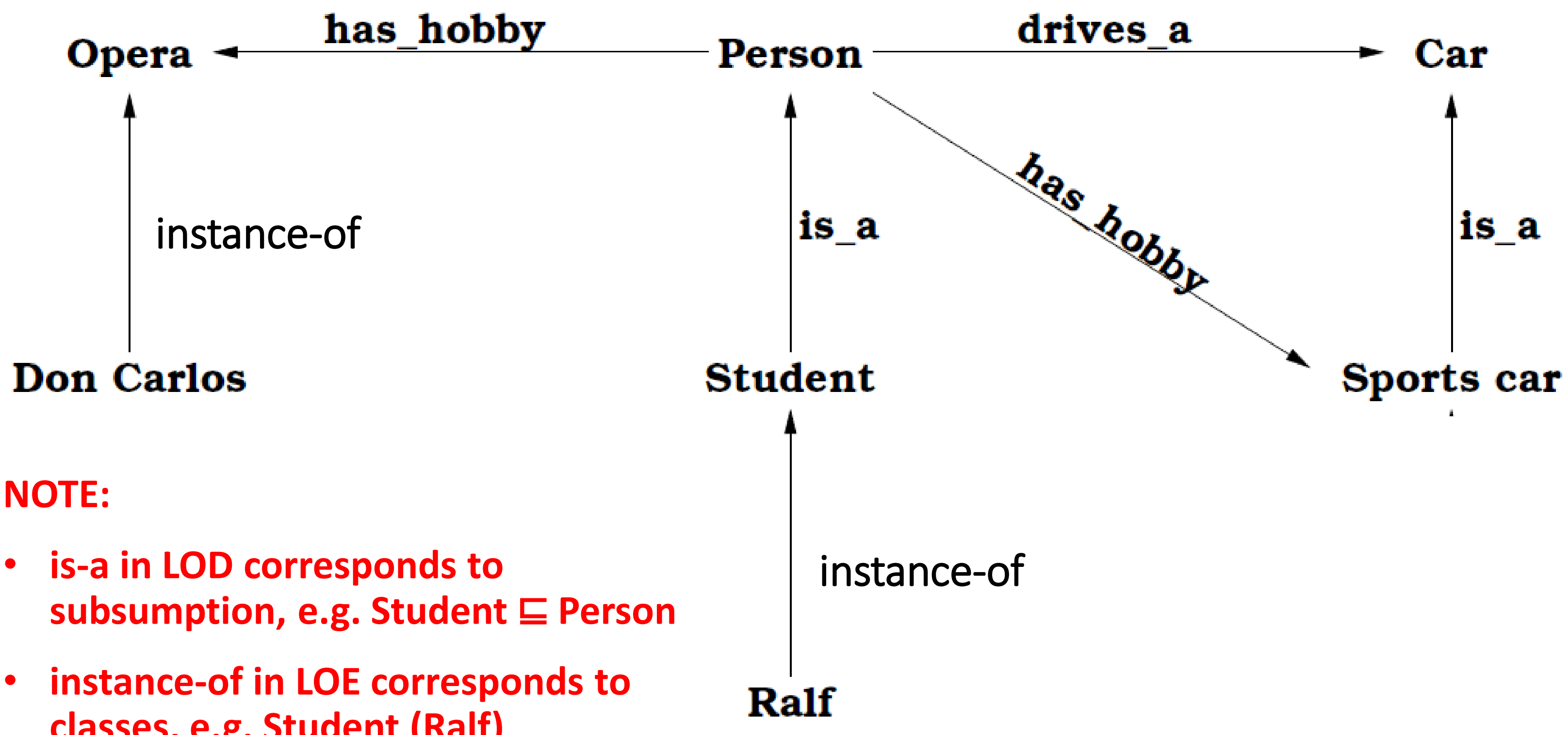
Define a LODE theory

Define a LODE theory for the following knowledge graph



NOTE:

- **is-a in LOD corresponds to subsumption, e.g. Student ⊑ Person**
- **instance-of in LOE corresponds to classes, e.g. Student (Ralf)**

ANSWER:

$$\begin{aligned} Person \sqsubseteq\ & \exists Drives.Car\ \sqcap \\ & \exists HasHobby.SportCar\ \sqcap \\ & \exists HasHobby.Opera \end{aligned}$$

$Student \sqsubseteq Person$

$SportCar \sqsubseteq Car$

$Student(Ralf)$

$Opera(DonCarlos)$

Define a LODE theory

Define a LODE theory for the following problem:

In a hospital patients, doctors and computers are equipped with proximity sensors able to detect whether doctors curated a patient or worked at their computer. The system detected that doctor Peter curated the patient Smith.

ANSWER:

Doctor $\sqsubseteq$ $\forall$cure.Patient $\sqcap$ $\forall$work.Computer

cure $\sqsubseteq$ detected

work $\sqsubseteq$ detected

Doctor (Peter) Patient (Smith) cure(Peter, Smith)

Expansion of a LODE concept

Given the following TBOX, compute the expansion of the ABox A = {StepMother(Mary)}

$Mother \equiv Woman \sqcap \exists hasChild.Person$

$Father \equiv Man \sqcap \exists hasChild.Person$

$StepMother \equiv Woman \sqcap \exists marriedWith.Father$

$StepFather \equiv Man \sqcap \exists marriedWith.Mother$

$Parent \equiv Father \sqcup Mother \sqcup StepFather \sqcup StepMother$

ANSWER:

StepMother(Mary), Woman(Mary), marriedWith(Mary, a1), Father(a1)

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Expansion of a LODE concept

Given the following TBOX, compute the expansion of the ABox A = {StepMother(Mary), marriedWith(Paul)}

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

StepMother ≡ Woman ⊓ ∀marriedWith.Father

StepFather ≡ Man ⊓ ∃marriedWith.Mother

Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

ANSWER:

StepMother(Mary), Woman(Mary), marriedWith(Mary, Paul), Father(Paul)

Instance checking in LODE

Given the following LODE theory T, does T |= Professor(John)?

Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor

Lecturer (John)

Teaches(John, Logics)

Course(Logics)

ANSWER:

The expansion of Lecturer (John) is {Teaches(John, Logics), Course(Logics), ¬Undergrad(John), Professor(John)}

Therefore the answer is **yes**.

Instance retrieval in LODE

Given the following LODE theory T, find all the instances of Lecturer.

Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor

Lecturer (John)

Teaches(John, Logics)

Course(Logics)

Teaches(Paul, Logics)

¬Undergrad(Paul)

Professor(Paul)

ANSWER:

{John, Paul}

In fact, John is in the ABox, while Paul satisfies all the constraints in the definition of Lecturer.

Concept realization in LODE

Given the following LODE theory T, find the most specific concept for Paul.

Lecturer $\equiv \forall$Teaches.Course $\sqcap$ $\neg$Undergrad $\sqcap$ Professor

Lecturer (John)

Teaches(John, Logics)

Course(Logics)

Teaches(Paul, Logics)

$\neg$Undergrad(Paul)

Professor(Paul)

ANSWER:

Given that Paul satisfies all the constraints in the definition of Lecturer, the answer is Lecturer.

Note that if we remove Professor(Paul), the answer becomes {$\neg$Undergrad}