Computational Logic Exercises
Module V – LOD applications and LODE

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Which of the following complex formulas are syntactically correct in LOD? (TBOX formulas)

a) $A \equiv \exists R.C \sqcap \forall S.D$

b) $A \sqcap B \equiv C \sqcup D$

c) $A \equiv B \sqcap \neg C$

d) $A \sqsubseteq \neg C$

e) $A \sqsubseteq B \sqcap \exists R.C$

f) $A \sqsubseteq B \sqcap \exists R.(\forall S.D)$

g) $A \equiv B \sqcup \emptyset$

ANSWER:

a, c, d, e, f

Unfold ColouredGuitar:

ElectricGuitar ≡ Guitar ⊓ ∀hasSoundAmplification.withInputJack

ColouredGuitar ≡ ElectricGuitar ⊓ ∃hasColour.String

Answer:

ColouredGuitar ≡ Guitar ⊓ ∀hasSoundAmplification.withInputJack ⊓ ∃hasColour.String

Is the following TBOX cyclic?

Woman ≡ Person ⊓ Female

Man ≡ Person ⊓ ¬Woman

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

Parent ≡ Father ⊔ Mother

ANSWER:
No, because by unfolding all concepts I never obtain the same concept on the left and on the right of the equivalences.

Is the following TBOX cyclic?

Male ≡ ¬Female

Female ≡ ¬Male

ANSWER:
Yes, because by unfolding it I get Female ≡ ¬(¬Female) that is Female ≡ Female

Is the following TBOX a terminology?

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

Parent ≡ Father ⊔ Mother

ANSWER:
Yes, because it is acyclic and there are only equivalences.

Is the following TBOX a terminology?

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

Parent ⊑ Father ⊔ Mother

ANSWER:
No, because it contains a subsumption.

Given the previous TBOX, provided below, can I convert it to make it a terminology?

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

Parent ⊑ Father ⊔ Mother

ANSWER:
Yes, for instance as follows:

Mother ≡ Woman ⊓ ∃hasChild.Person
Father ≡ Man ⊓ ∃hasChild.Person
StepMother ≡ Woman ⊓ ∃marriedWith.Father
StepFather ≡ Man ⊓ ∃marriedWith.Mother
Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

A lion is a large gregarious predatory feline of Africa and India having a shaggy mane in the male

Lion ≡ Feline ⊓ Large ⊓ Gregarious ⊓ Predatory ⊓ ∀livesIn.(Africa ⊔ India) ⊓ ∃livesIn.(Africa ⊔ India)

MaleLion ≡ Lion ⊓ Male ⊓ ∀has.ShaggyMane ⊓ ∃has.ShaggyMane

A penguin is a flightless bird of Antarctica having webbed feet

Penguin ≡ Bird ⊓ ¬Fly ⊓ ∀livesIn.Antarctica ⊓ ∃livesIn.Antarctica ⊓ ∀has.WebbedFeet ⊓ ∃has.WebbedFeet

Defining a terminology from a schema

Property	Expected Type
Properties from Event	
about	Thing
actor	Person
attendee	Organization or Person

ANSWER. By assuming the schema as complete (otherwise it is not a terminology) we have:

Event ≡ Thing ⊓

$\forall$about.Thing ⊓ $\exists$about.Thing ⊓

$\forall$actor.Person ⊓ $\exists$actor.Person ⊓

$\forall$attendee.(Person ⊔ Organization) ⊓ $\exists$attendee.(Person ⊔ Organization)

Given the previous TBOX, replicated on the right, which of the following are logical consequences of the TBOX?

a)   Event ⊑ ∀about.Thing ⊓ ∃about.Thing

b)   Event ⊑ ∀about.Thing

c)   Event ⊑ ∃about.Thing

d)   Event ≡ ∀about.Thing ⊓ ∃about.Thing

e)   Event ⊑ ∀attendee.Person ⊔ ∀attendee.Organization

f)   Event ⊑ ∀attendee.Person

g)   Person ⊑ ¬Organization

ANSWER.

a, b, c, f

TBOX

Event ≡ Thing
⊓ ∀about.Thing ⊓ ∃about.Thing
⊓ ∀actor.Person ⊓ ∃actor.Person ⊓
∀attendee.(Person ⊔ Organization) ⊓
∃attendee.(Person ⊔ Organization)
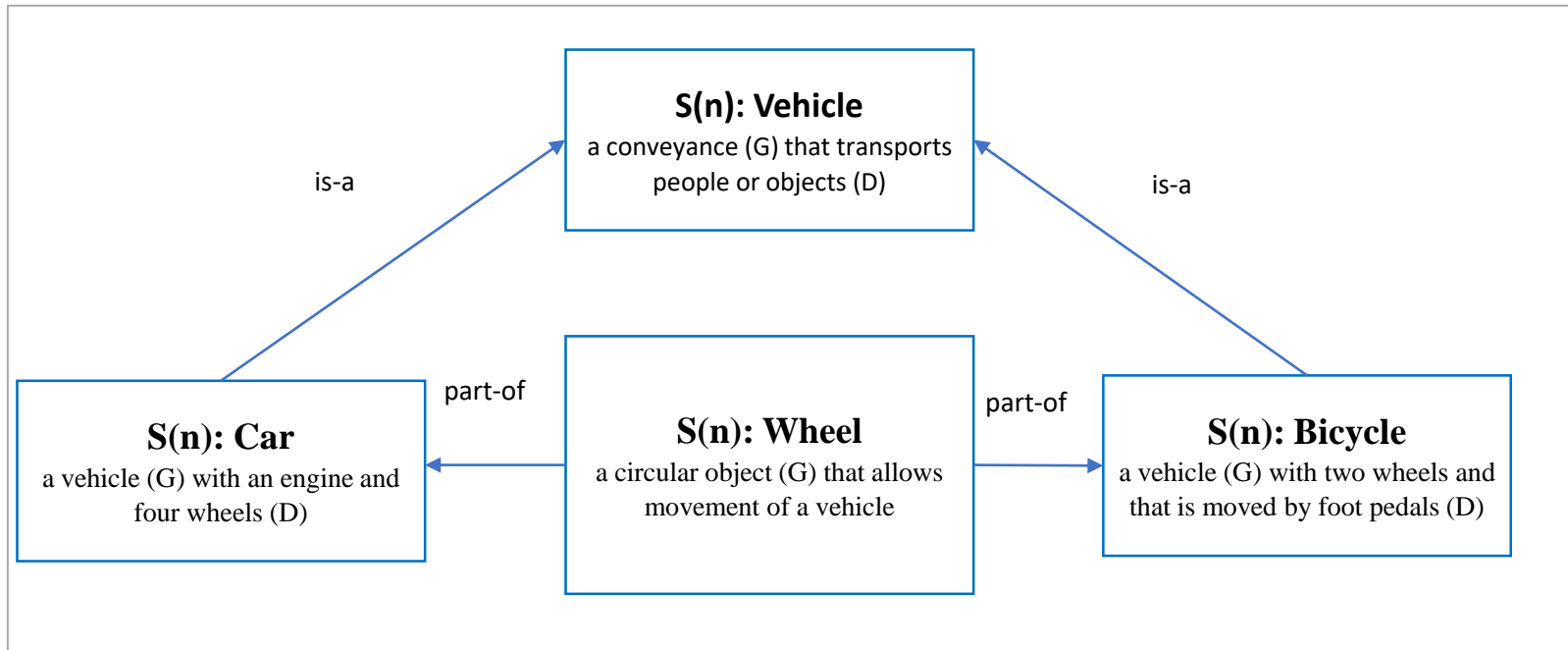
Vehicle ≡ Conveyance ⊓ ∃transports.(Person ⊔ Object) ⊓ ∀transports.(Person ⊔ Object)

Car ≡ Vehicle ⊓ ¬Bicycle ⊓ ∃hasPart.Wheel ⊓ ∃hasPart.Engine

Bicycle ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃hasPart.FootPedal ⊓ ∃movedBy.FootPedal ⊓ ∀movedBy.FootPedal

Wheel ≡ Object ⊓ CircularShape ⊓ ∃moves.Vehicle ⊓ ∀moves.Vehicle

Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow ⊑ ¬GetBanana
GetBanana ⊑ Survive

Is T satisfiasble?

ANSWER: Yes. It is enough to find one model for it, represented graphically with the Venn Diagram below.

Suppose we model the Monkey-Banana problem as follows:

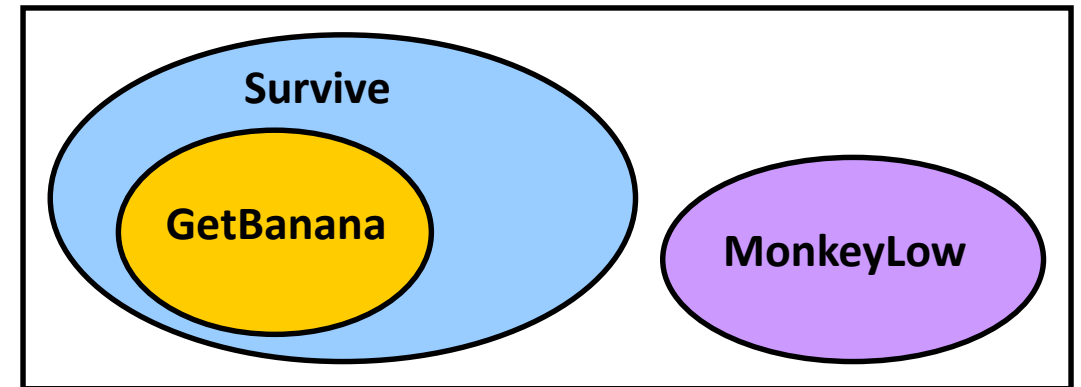"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow ⊑ ¬GetBanana
GetBanana ⊑ Survive

Is it possible for a monkey to survive even if it does not get the banana?

ANSWER: We can restate the problem as follow:
does T ⊨ ¬GetBanana ⊓ Survive at least in one model?
Yes. We can find a model in which both all the assertions in T and ¬GetBanana ⊓ Survive are not empty.
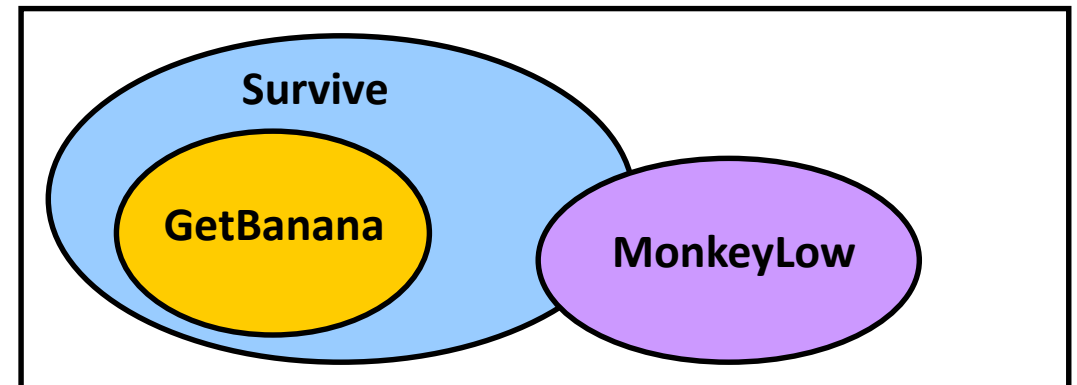
Suppose we describe students in a course as follows:

Undergraduate $\sqsubseteq \neg$ Teach
Bachelor $\equiv$ Student $\sqcap$ Undergraduate
Master $\equiv$ Student $\sqcap \neg$ Undergraduate
PhD $\equiv$ Master $\sqcap$ Research
Assistant $\equiv$ PhD $\sqcap$ Teach

Are all assistants also undergraduates?

ANSWER: We can restate the problem as follow:
does T $\vDash$ Assistant $\sqsubseteq$ Undergraduate ?

We need to prove that this is true in <u>all models</u> (via the method of *unfolding*)

Assistant $\equiv$ PhD $\sqcap$ Teach
$\equiv$ Master $\sqcap$ Research $\sqcap$ Teach
$\equiv$ Student $\sqcap \neg$ Undergraduate $\sqcap$ Research $\sqcap$ Teach
Answer is No. Assistants are actually students who are not undergraduate.

Suppose we model the Monkey-Banana problem as follows:

"If the monkey is low in position then it cannot get the banana. If the monkey gets the banana it survives".

Theory T:
MonkeyLow ≡ ¬GetBanana ⊓ ¬ClimbBox
GetBanana ≡ Survive

Is it possible for a monkey to climb the box and not survive?

ANSWER: We can restate the problem as follow:
does T ⊨ ClimbBox ⊓ ¬Survive at least in one model?
Yes. We can find a model in which both all the assertions in T and ClimbBox ⊓ ¬Survive are not empty.
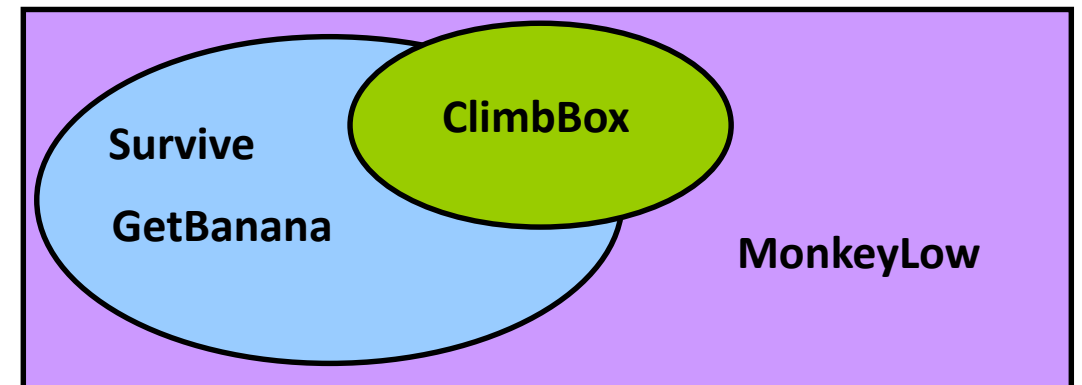
Suppose we describe students in a course as follows:

Undergraduate	$\sqsubseteq \neg$ Teach
Bachelor	$\equiv$ Student $\sqcap$ Undergraduate
Master	$\equiv$ Student $\sqcap \neg$ Undergraduate
PhD	$\equiv$ Master $\sqcap$ Research
Assistant	$\equiv$ PhD $\sqcap$ Teach

Are bachelor and master disjoint?

ANSWER: We can restate the problem as follow:
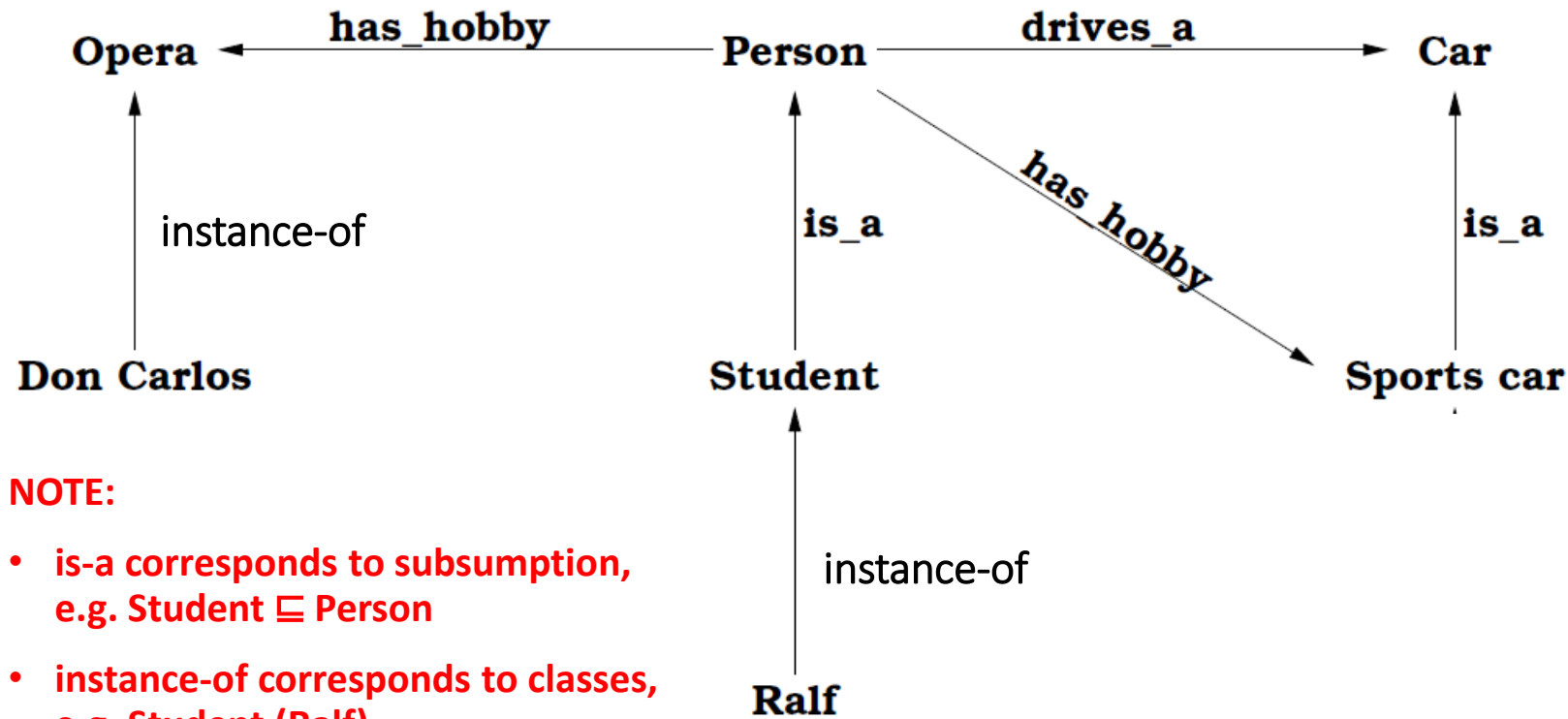does $T \vDash$ Bachelor $\sqcap$ Master $\sqsubseteq \perp$?

We need to prove that this is true in <u>all models</u> (via the method of *unfolding*)

Answer is obviously Yes because they contain two opposite constraints.

Define a LODE theory for the following knowledge graph

ANSWER:

Person ⊑ ∃Drives.Car ⊓
∃HasHobby.SportCar ⊓
∃HasHobby.Opera

Student ⊑ Person

SportCar ⊑ Car

Student(Ralf)

Opera(DonCarlos)

NOTE:

- **is-a corresponds to subsumption, e.g. Student ⊑ Person**

- **instance-of corresponds to classes, e.g. Student (Ralf)**

Define a LODE theory for the following problem:

In a hospital patients, doctors and computers are equipped with proximity sensors able to detect whether doctors curated a patient or worked at their computer. The system detected that doctor Peter curated the patient Smith.

ANSWER:

Doctor $\sqsubseteq$ $\forall$cure.Patient $\sqcap$ $\forall$work.Computer

cure $\sqsubseteq$ detected

work $\sqsubseteq$ detected

Doctor (Peter)   Patient (Smith)   cure(Peter, Smith)

Given the following TBOX, compute the expansion of the ABox A = {StepMother(Mary)}

Mother ≡ Woman ⊓ ∃hasChild.Person

Father ≡ Man ⊓ ∃hasChild.Person

StepMother ≡ Woman ⊓ ∃marriedWith.Father

StepFather ≡ Man ⊓ ∃marriedWith.Mother

Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

ANSWER:

StepMother(Mary), Woman(Mary), marriedWith(Mary, a1), Father(a1)

Given the following TBOX, compute the expansion of the ABox A = {StepMother(Mary), marriedWith(Paul)}

Mother ≡ Woman ⊓ ∃hasChild.Person
Father ≡ Man ⊓ ∃hasChild.Person
StepMother ≡ Woman ⊓ ∀marriedWith.Father
StepFather ≡ Man ⊓ ∃marriedWith.Mother
Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

ANSWER:
StepMother(Mary), Woman(Mary), marriedWith(Mary, Paul), Father(Paul)

Given the following LODE theory T, does T |= Professor(John)?

Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor

Lecturer (John)

Teaches(John, Logics)

Course(Logics)

ANSWER:

The expansion of Lecturer (John) is {Teaches(John, Logics), Course(Logics), ¬Undergrad(John), Professor(John)}

Therefore the answer is **yes**.

Given the following LODE theory T, find all the instances of Lecturer.
Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor
Lecturer (John)
Teaches(John, Logics)
Course(Logics)
Teaches(Paul, Logics)
¬Undergrad(Paul)
Professor(Paul)

ANSWER:
{John, Paul}
In fact, John is in the ABox, while Paul satisfies all the constraints in the definition of Lecturer.

Given the following LODE theory T, find the most specific concept for Paul.

Lecturer $\equiv$ $\forall$Teaches.Course $\sqcap$ ¬Undergrad $\sqcap$ Professor

Lecturer (John)

Teaches(John, Logics)

Course(Logics)

Teaches(Paul, Logics)

¬Undergrad(Paul)

Professor(Paul)

ANSWER:

Given that Paul satisfies all the constraints in the definition of Lecturer, the answer is Lecturer.

Note that if we remove Professor(Paul), the answer becomes {¬Undergrad}