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Computational Logic

Module II – Set theory and knowledge graphs

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Set theory in a nutshell: defining sets

We can define sets in two ways

Listing: The set is described by listing all its elements (for instance, $A = \{a, e, i, o, u\}$).

Abstraction: The set is described through a property of its elements (for instance, $A = \{x | x$ is a vowel of the Latin alphabet$\}$).

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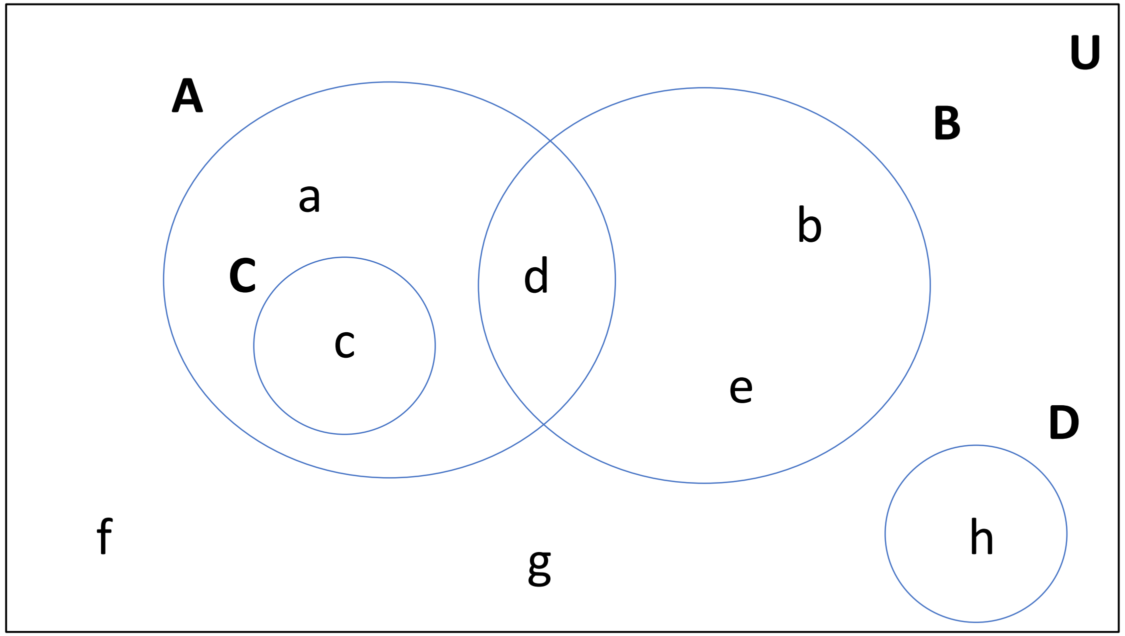
Set theory in a nutshell: basic notions

- **Empty Set.** $\emptyset$ is the set containing no elements.
- **Membership**. $\boldsymbol{a} \in \boldsymbol{A}$, element $\boldsymbol{a}$ belongs to the set $\boldsymbol{A}$.
- **Non-membership**. $\boldsymbol{a} \notin \boldsymbol{A}$, element $\boldsymbol{a}$ doesn’t belong to the set $\boldsymbol{A}$.
- **Equality**. $\boldsymbol{A} = \boldsymbol{B}$, if and only if $\boldsymbol{A}$ and $\boldsymbol{B}$ contain the same elements.
- **Inequality**. $\boldsymbol{A} \neq \boldsymbol{B}$, if and only if it is not true that $\boldsymbol{A} = \boldsymbol{B}$.
- **Subset**. $\boldsymbol{A} \subseteq \boldsymbol{B}$, if and only if all elements in $\boldsymbol{A}$ also belong to $\boldsymbol{B}$.
- **Proper Subset**. $\boldsymbol{A} \subset \boldsymbol{B}$, if and only if $\boldsymbol{A} \subseteq \boldsymbol{B}$ and $\boldsymbol{A} \neq \boldsymbol{B}$.
- **Universal Set**. The universal set is the set of all elements or members of all related sets and is denoted by the letter U.

Set theory in a nutshell: Venn Diagrams

Sets are typically represented with **Venn Diagrams**

EXAMPLE:



$\boldsymbol{a} \in \boldsymbol{A}$,

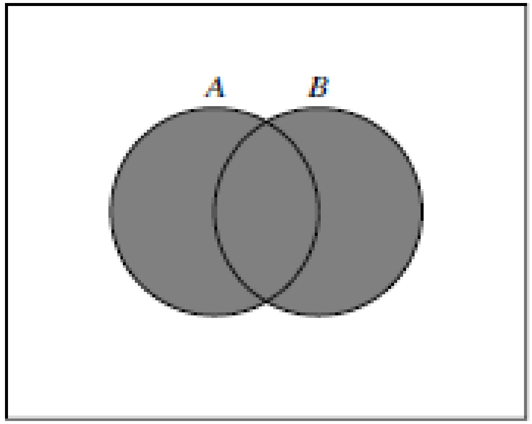
$\boldsymbol{a} \notin$ B

$\boldsymbol{A} \neq \boldsymbol{B}$,

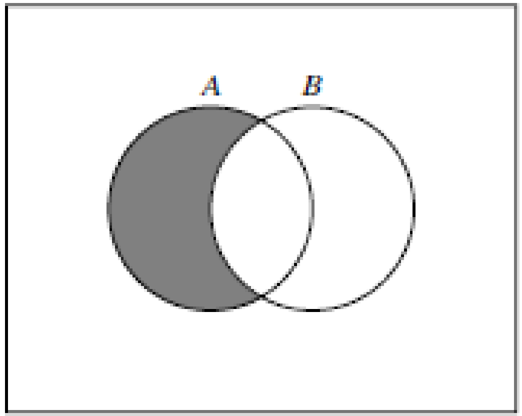
C $\subseteq$ A,

C $\subset$ A

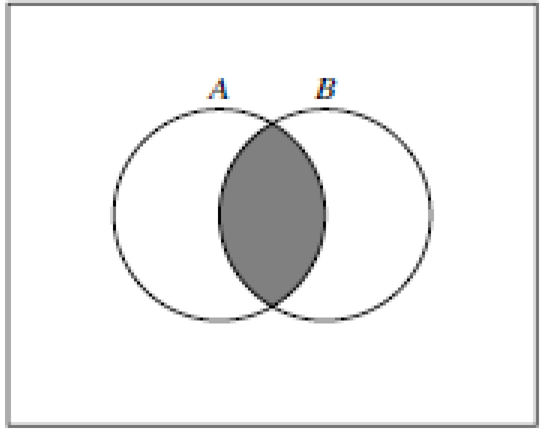
Set theory in a nutshell: operations



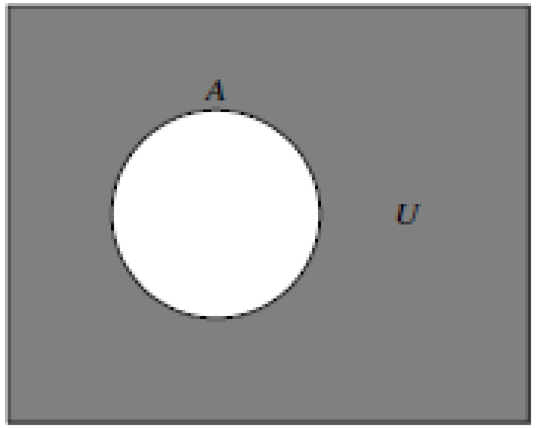
Union. Given two sets $\boldsymbol{A}$ and $\boldsymbol{B}$, the union of $\boldsymbol{A}$ and $\boldsymbol{B}$ is the set containing the elements belonging to $\boldsymbol{A}$ or to $\boldsymbol{B}$ or to both, and is denoted with $\boldsymbol{A} \cup \boldsymbol{B}$.



Difference. Given two sets $\boldsymbol{A}$ and $\boldsymbol{B}$, the difference of $\boldsymbol{A}$ and $\boldsymbol{B}$ is the set containing all the elements which are members of $\boldsymbol{A}$, but not members of $\boldsymbol{B}$, and is denoted with $\boldsymbol{A} \setminus \boldsymbol{B}$.



Intersection. Given two sets $\boldsymbol{A}$ and $\boldsymbol{B}$, the intersection of $\boldsymbol{A}$ and $\boldsymbol{B}$ is the set containing the elements that belong both to $\boldsymbol{A}$ and $\boldsymbol{B}$, and is denoted with $\boldsymbol{A} \cap \boldsymbol{B}$.



Complement. Given a universal set $\boldsymbol{U}$ and a set $\boldsymbol{A}$, the complement of $\boldsymbol{A}$ in $\boldsymbol{U}$ is the set containing all the elements in $\boldsymbol{U}$ that do not belong to $\boldsymbol{A}$, and is denoted with $\boldsymbol{U} \setminus \boldsymbol{A}$.

Set theory in a nutshell: cartesian product and relations

Cartesian product. Given two sets A and B, the Cartesian product of A and B is the set of ordered couples (a, b) where $a \in A$ and $b \in B$, formally:

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Relation. A relation R from the set A to the set B is a subset of the Cartesian product of A and B, formally:

$$R \subseteq A \times B$$

If $(x, y) \in R$, then we will write xRy and we say 'x is R-related to y'.

Set theory in a nutshell: porperties of relations

Let R be a binary relation. R is:

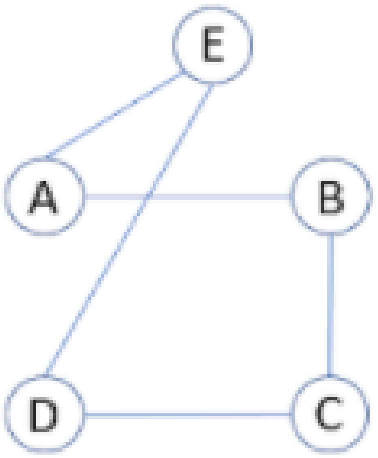
- **reflexive** iff aRa for all $a \in A$
- **symmetric** iff aRb implies bRa for all $a, b \in A$
- **transitive** iff aRb and bRc imply aRc for all $a, b, c \in A$
- **anti-symmetric** iff aRb and bRa imply $a = b$ for all $a, b \in A$

EXAMPLES

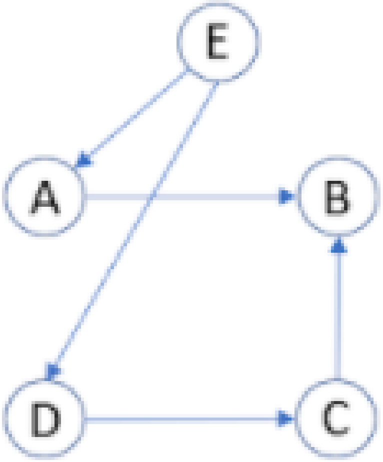
- reflexive: equalTo
- symmetric: friendOf, roommateOf, siblingsOf
- transitive: ancestorOf, kindOf, partOf
- anti-symmetric: isDivisibleBy, subsetOf

Graph theory in a nutshell: defining graphs (I)

A **graph** $\boldsymbol{G}$ is an ordered pair $\boldsymbol{G} =< \boldsymbol{V}, \boldsymbol{E} >$, where $\boldsymbol{V}$ is the set of vertices (or nodes) and $\boldsymbol{E}$ is the set of edges (or links). Edges are pairs of vertices.



A **directed graph** is a graph where edges are ordered pairs of distinct vertices $(\boldsymbol{x}, \boldsymbol{y})$. $\boldsymbol{x}$ and $\boldsymbol{y}$ are called the end points, where $\boldsymbol{x}$ is the tail and $\boldsymbol{y}$ is the head.



https://www.mygreatlearning.com/blog/representing-graphs-in-data-structures/

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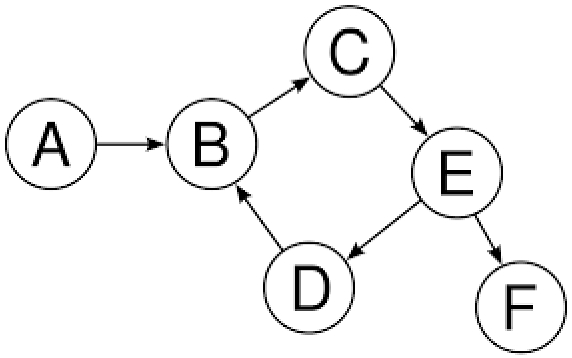
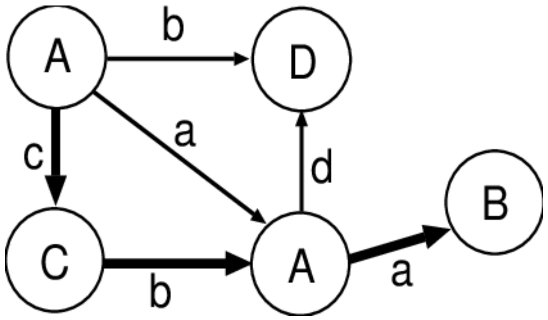
Graph theory in a nutshell: defining graphs (II)

A **cycle** is a path in which only the first and last vertices are equal. A **cyclic graph** is a graph which contains a cycle.



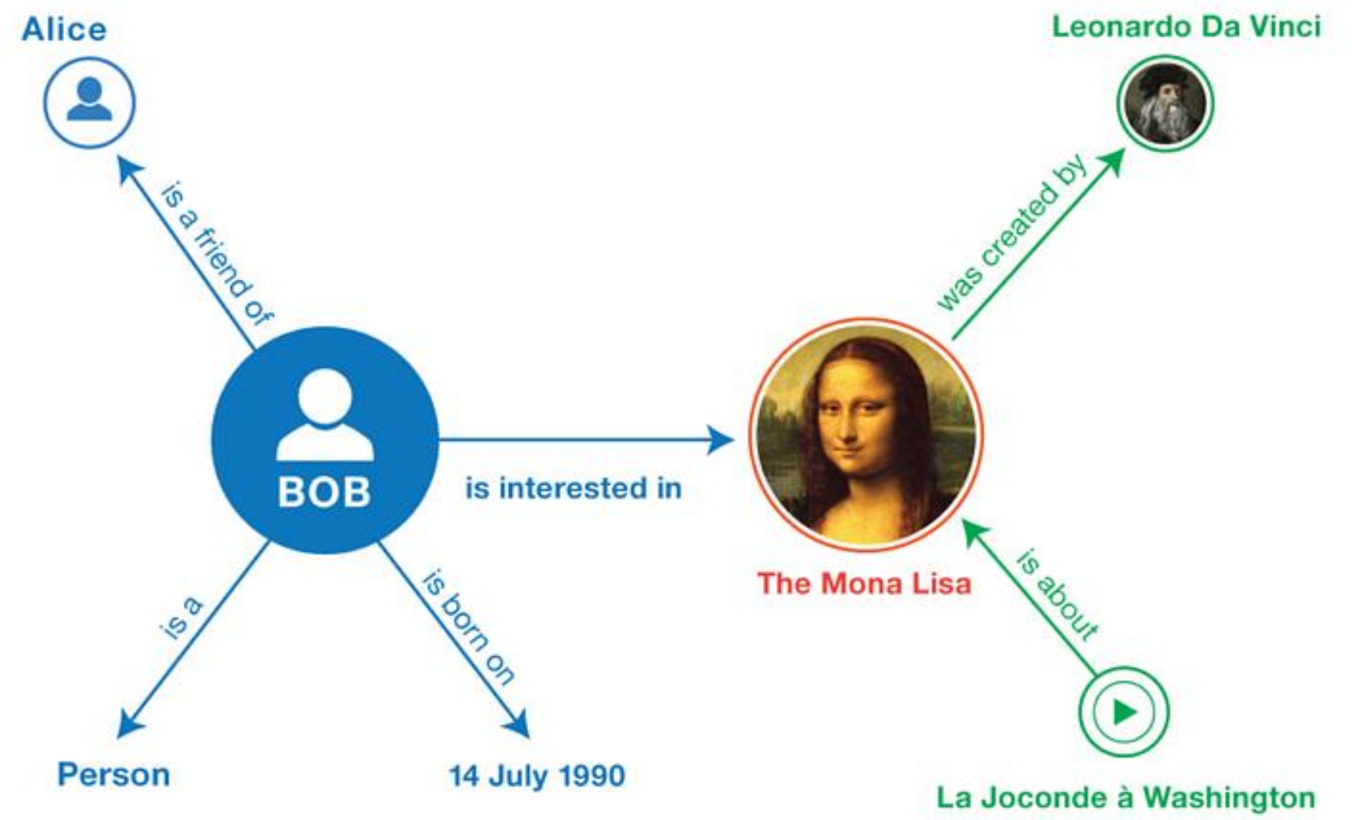
A **directed acyclic graph** (DAG) is a directed graph that does not contain any cycles.

A **labeled graph** is a graph where each vertex and edge is assigned a label.



Knowledge graphs

A **knowledge graph** is a labelled graph representing real world knowledge.



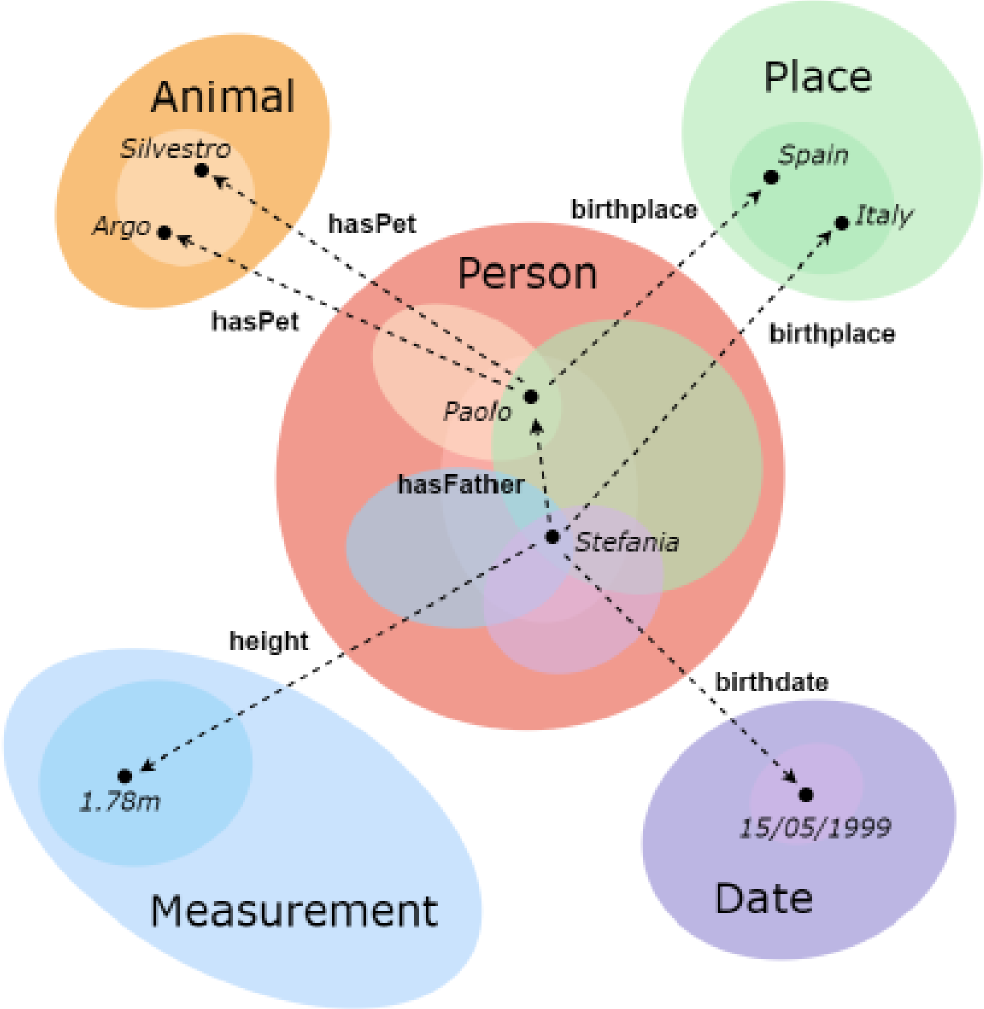
Vertexes belong to two disjoint sets:

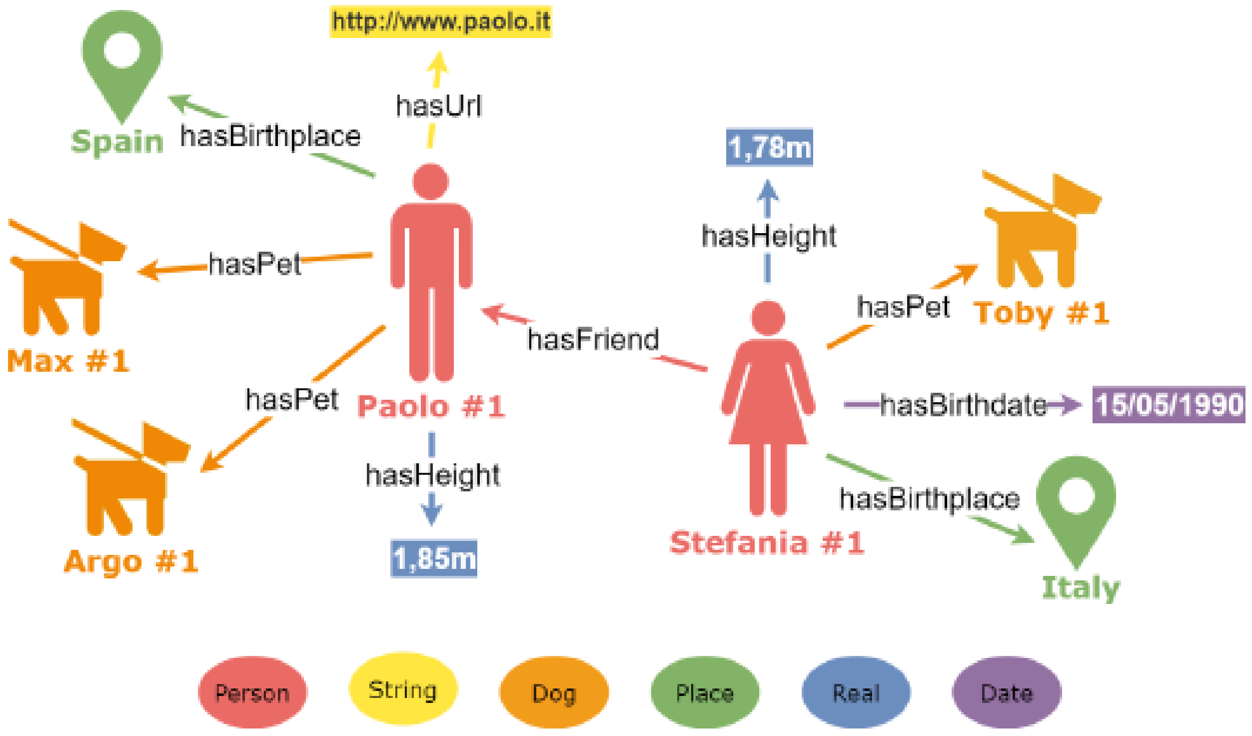
- **Entity types**: the set of (named) entities in the world, such as a person, location, artifact…
- **Data types**: the set of values of the data properties of the entities, such as dates, numbers, texts…

Edges are relations of two types:

- **Object properties**: relations between two entities, such as createdBy, friendOf
- **Data properties**: relations between and entity and a values, such as bornOn

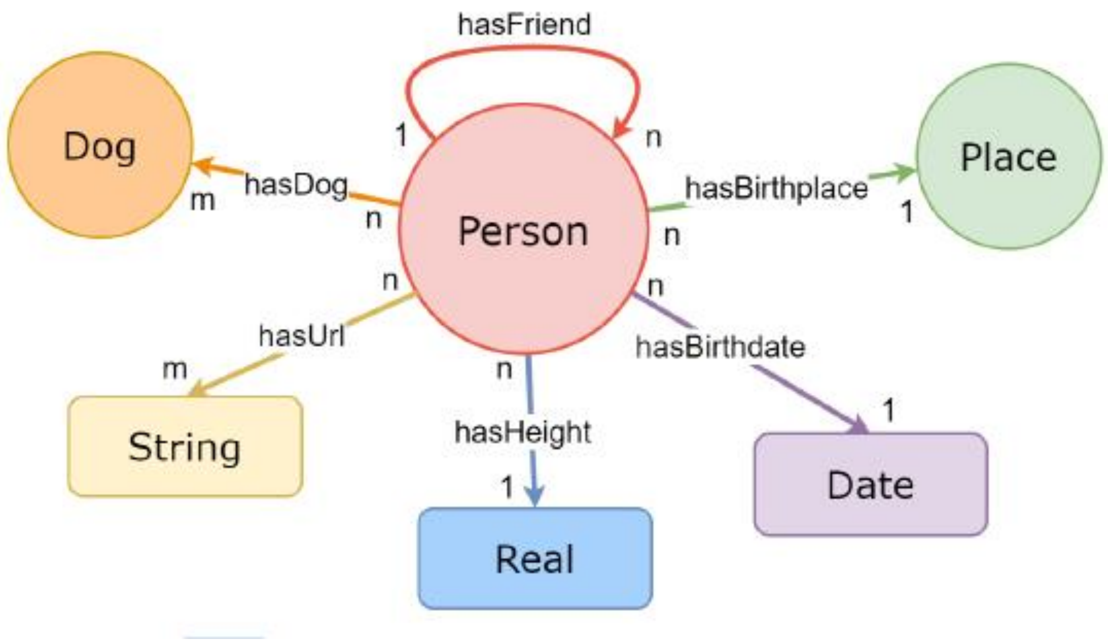
Examples of a knowledge graph and corresponding sets





Etype graphs

An **Etype Graph**, or **schema**, is a knowledge graph that focuses only on Entity types, Data types, Object properties and Data properties. It provides constraints on knowledge graphs with corresponding instances.



NOTICE: an ER diagram is a schema