(Cook County) Cook County needs to build two hospitals. There are nine cities where the hospitals can be built. The number of hospital visits made annually by the inhabitants of each city and the *x* and *y* coordinates of each city are listed in the file P08_24.xlsx. To minimize the total distance that patients must travel to hospitals, where should the hospitals be located? Solve the problem when people can travel in straight lines ("as the crow flies") between cities. Then solve it when people must travel along a horizontal/vertical grid of roads. (*Hint*: Use lookup functions to generate the distances between each pair of cities.)

City	x	y	Visits
1	0	0	3000
2	10	3	4000
3	12	15	5000
4	14	13	6000
5	16	9	4000
6	18	6	3000
7	8	12	2000
8	6	10	4000
9	4	8	1200

P08_24.xlsx

Discussion: -

Our problem is to identify two cities to build hospitals with minimum total distance that patients must travel to hospitals. So, our decision variable should be whether to build hospital in city or not. To calculate the total distance, we must assign each city to the hospital. We have multiple methods (Euclidean distance) to calculate the distance between two cities. We must create a calculated variable and create Distance matrix D_{ij}.

Distance between Cities	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
1	0.00	10.44	19.21	19.10	18.36	18.97	14.42	11.66	8.94
2	10.44	0.00	12.17	10.77	8.49	8.54	9.22	8.06	7.81
3	19.21	12.17	0.00	2.83	7.21	10.82	5.00	7.81	10.63
4	19.10	10.77	2.83	0.00	4.47	8.06	6.08	8.54	11.18
5	18.36	8.49	7.21	4.47	0.00	3.61	8.54	10.05	12.04
6	18.97	8.54	10.82	8.06	3.61	0.00	11.66	12.65	14.14
7	14.42	9.22	5.00	6.08	8.54	11.66	0.00	2.83	5.66
8	11.66	8.06	7.81	8.54	10.05	12.65	2.83	0.00	2.83
9	8.94	7.81	10.63	11.18	12.04	14.14	5.66	2.83	0.00

Mathematical Model: -

Parameters (Inputs):

$i, j \in 1,2,3,..n$ (i: *Index for cities*)

V_i : $Number\ of\ visits\ from\ city\ i$
$\{x_i, y_i\}$: $Coordinates\ of\ city\ i$

Decision Variables:

z_j : $Decision\ to\ pick\ city\ i\ to\ build\ hospital$
e_{ij} : $Whether\ city\ 'i' is\ assigned\ to\ hospital\ 'j'$

Calculated Variables:
D_{ij} : $Distance\ from\ city\ i\ to\ j$

Objective:

$$Minimize\ total\ \ distance = \sum_{j=1}^{9}\sum_{i=1}^{9}(V_i * e_{ij} * D_{ij})$$

Constraints:

$z_j\ , e_{ij}\ \ \epsilon\{0,1\}$ (1) $Binary\ constraint$

$\sum_{j=1}^{9} z_j \leq 2\ ;$ (2) $Number\ of\ hospitals\ constraint$

$\sum_{j=1}^{9} e_{ij} \geq 1\ ;\quad for\ i\ \epsilon\{1,2,3,.n\}$ (3) $Atleast\ one\ Hospital\ is\ assigned\ to\ each\ city\ constraint$

$\sum_{i=1}^{9} e_{ij} \leq n * z_j\ ;\quad for\ j\ \epsilon\{1,2,3,.n\}$ (4) $Assigning\ constraint\ for\ Hospital$

Constraint 2 will make sure that our optimal solution will consider only two cities for building hospital. Constraint 3 will make sure that at least one hospital is assigned to each city. Constraint 4 will bound the number of cities assigned to each hospital.

Excel Implementation: Please find the attached spreadsheet for solution.

57[RA].xlsx

Facility Location [Cook County]: NLP-CH8-Q24

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Cook County Hospitals

City	x	y	Visits
1	0	0	3000
2	10	3	4000
3	12	15	5000
4	14	13	6000
5	16	9	4000
6	18	6	3000
7	8	12	2000
8	6	10	4000
9	4	8	1200

Distance between Cities	1	2	3	4	5	6	7	8	9
1	0.00	10.44	19.21	19.10	18.36	18.97	14.42	11.66	8.94
2	10.44	0.00	12.17	10.77	8.49	8.54	9.22	8.06	7.81
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6	18.97	8.54	10.82	8.06	3.61	0.00	11.66	12.65	14.14
7	14.42	9.22	5.00	6.08	8.54	11.66	0.00	2.83	5.66
8	11.66	8.06	7.81	8.54	10.05	12.65	2.83	0.00	2.83
9	8.94	7.81	10.63	11.18	12.04	14.14	5.66	2.83	0.00

Hospital (Yes/No)	1	2	3	4	5	6	7	8	9			
	0	0	0	1	0	0	0	1	0	2	<=	2

City->Hospital	1	2	3	4	5	6	7	8	9			
1	0	0	0	0	0	0	0	1	0	1	>=	1
2	0	0	0	0	0	0	0	1	0	1	>=	1
3	0	0	0	1	0	0	0	0	0	1	>=	1
4	0	0	0	1	0	0	0	0	0	1	>=	1
5	0	0	0	1	0	0	0	0	0	1	>=	1
6	0	0	0	1	0	0	0	0	0	1	>=	1
7	0	0	0	0	0	0	0	1	0	1	>=	1
8	0	0	0	0	0	0	0	1	0	1	>=	1
9	0	0	0	0	0	0	0	1	0	1	>=	1
	0	0	0	4	0	0	0	5	0			
	<=	<=	<=	<=	<=	<=	<=	<=	<=			
	0	0	0	9	0	0	0	9	0			

1	34986
2	32249
3	14142
4	0
5	17889
6	24187
7	5657
8	0
9	3394
	132503

Inputs
Decision variables
Calculated Variables
Constraints
Objective