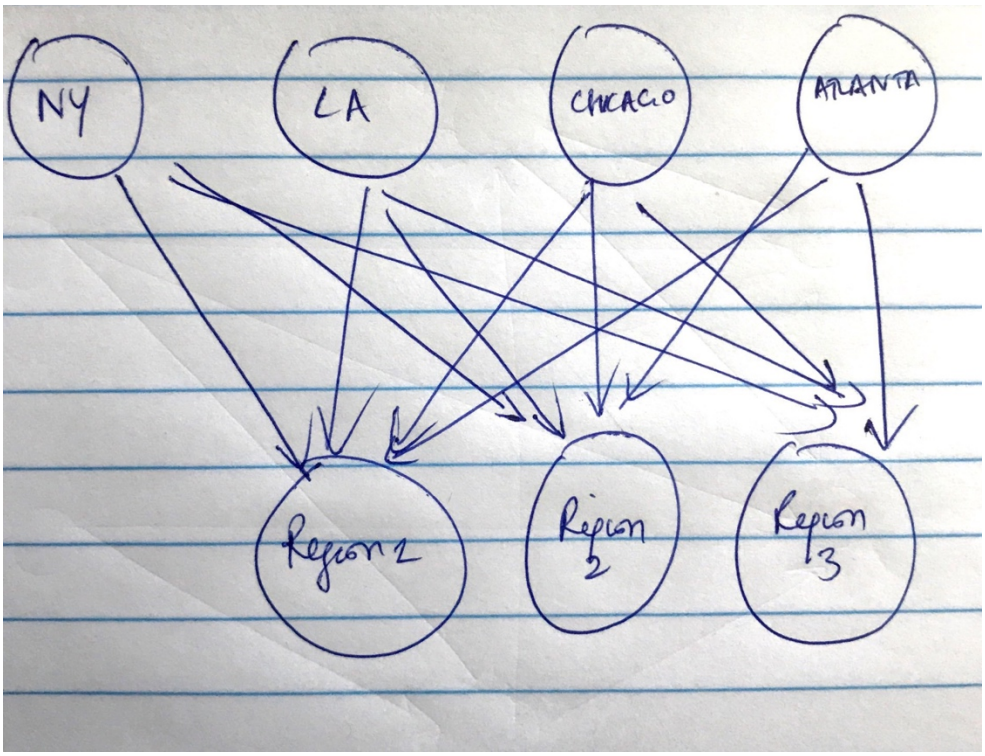
Integer Programming. A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 15,000 units per week. The weekly fixed cost of keeping each warehouse open is $60,000 for New York, $50,000 for Los Angeles, $40,000 for Chicago, and $35,000 for Atlanta. Region 1 of the country requires 8000 units per week, region 2 requires 9000 units per week, and region 3 requires 7000 units per week. The costs (including production and shipping costs) of sending one unit from a warehouse to a region are shown below. The company wants to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions: ■ If the New York warehouse is opened, then the Los Angeles warehouse must be opened. ■ At most two warehouses can be opened. ■ Either the Atlanta or the Los Angeles warehouse must be opened.

Unit production and shipping costs				
		To		
		Region 1	Region 2	Region 3
From	NY	$26	$41	$39
	LA	$59	$27	$27
	Chicago	$28	$32	$43
	Atlanta	$28	$40	$38

Discussion.

This is an example of an integer programming model to minimize total cost. We must decide the warehouses that needs to be opened and the amount that needs to be shipped from each warehouse. In line with the usual integer programming models, we must ensure that nothing is being shipped from warehouses that are not opened, for this purpose instead of using a large number, here we choose the upper limit of the shipments from a particular warehouse to be the maximum capacity of the warehouse. We must also ensure that the demand of each region is being satisfied. The additional constraints we add are due to the conditions explicitly mentioned in the question.



Model.

Parameters:

D_j : *Demand of region j, where* $j \in (1,2,3)$

F_i: *Fixed cost to open warehouse* i *,* $where\ i \in (LA, NY, Chicago,\ Atlanta)$

C_{ij} :*Cost to ship one unit from warehouse* i *to Region j,* $where\ i \in (LA, NY, Chicago, Atlanta),\ j \in (1,2,3)$

S: Weekly capacity of warehouse

Decisions:

x_i: *Whether warehouse* i *should be opened,* $where\ i \in (LA, NY, Chicago,\ Atlanta)$

n_{ij} : *Unit to ship from warehouse* i *to region* j*,* $where\ i \in (LA, NY, Chicago,\ Atlanta),\ j \in (1,2,3)$

Objective: *Maximize Revenue*

$min \sum_{i,j}\ x_i * F_i + n_{ij} * C_{ij}$

Constraints:

$\sum_i n_{ij} \geq D_j$ (1) Satisfy demand of a region

$\sum_j n_{ij} \leq S * x_i$	(2) No shipments allowed from closed warehouses
$x_{LA} \geq x_{NY}$	(3) If NY is open, LA must be open
$\sum_i x_i \leq 2$	(4) At most 2 warehouses can be open
$\sum_i x_i \geq 1, where\ i \in (LA, Atlanta)$	(5) Either LA or Atlanta must always be open
$n_{ij} \geq 0$	(6) Non- negative number of shipments
$x_i \in \{0,1\}$	(7) Binary decision

Notes:

(1) Constraint 2 ensures that the shipment cannot exceed logical capacity constraint. When an investment is not chosen, x_i=0 then n_{ij} is forced to be 0. If an investment is chosen, x_i=1 then n_i can be any value less than S, that is the weekly capacity of warehouse i.

Optimal Solution. The following is the solution obtained from Excel Solver.

A minimum cost of 15000$ can be attained by selecting the routes below with number of shipments as shown below.

Warehouse data									
Unit production and shipping costs					Decision1: if warehouse is chosen				
		To				Fixed cost of warehouse			
		Region 1	Region 2	Region 3				Sum of atlanta and la	
From	NY	$26	$41	$39	0.00	$60,000		1.00	
	LA	$59	$27	$27	0.00	$50,000			
	Chicago	$28	$32	$43	0.00	$40,000			
	Atlanta	$28	$40	$38	1.00	$35,000			
					Total # warehouses chosen				
Demand		$8,000	$9,000	$7,000	1.00	<=	2.00		
Weekly capacity	15000								
Decision 2									
Amount to ship									
		To			Total from warehouse				
		Region 1	Region 2	Region 3					
From	NY	8000	0	1000	9000	<=	0		
	LA	0	9000	6000	15000	<=	0		
	Chicago	0	0	0	0	<=	0		
	Atlanta	0	0	0	0	<=	15000		
	TOTAL	8000	9000	7000					
Fixed cost	35000								
Variable cost	652000				IF NY IS OPEN LA MUST BE OPEN				
Cost	687000				Either la or atlanta must be open				
Large number	15000								