

Integer Programming. A medical supply company has customers in eight cities. It is trying to decide how many salespeople it needs to service these customers. Each salesperson needs to be located in one of the eight cities and needs to be assigned to a subset of the customers. For example, the company might base a salesperson in New York and have this person service customers in New York, Boston, and Philadelphia. Each salesperson receives an annual salary of \$50,000 and can work as many as 230 days per year. This includes days working with customers and days traveling to and from customers. Shown below is data on the annual travel costs (for example, \$15,900 for a salesperson based in New York traveling for customers in Orlando), the annual number of days of work required for the customers, and the annual number of days traveling to and from customers. Find an assignment that minimizes the total cost of salaries and traveling. The solution should indicate the number of salespeople to employ, where they should be based, and which cities they should serve. Assume that customers in a given city must be serviced by a single salesperson.

Assigning salespeople to locations								
Annual salary	\$50,000							
Max days worked	230							
Annual travel costs								
From\To	Boston	New Orleans	New York	Orlando	Philadelphia	Phoenix	Portland	Sacramento
Boston	\$1,800	\$19,340	\$1,920	\$15,940	\$3,080	\$33,890	\$38,360	\$39,280
New Orleans	\$19,340	\$1,800	\$19,360	\$9,810	\$18,100	\$21,530	\$32,730	\$30,040
New York	\$1,920	\$19,360	\$1,800	\$15,900	\$3,080	\$33,980	\$38,480	\$39,390
Orlando	\$15,940	\$9,810	\$15,900	\$1,800	\$14,700	\$29,490	\$39,740	\$37,720
Philadelphia	\$3,080	\$18,100	\$3,080	\$14,700	\$1,800	\$33,030	\$37,990	\$38,670
Phoenix	\$33,890	\$21,530	\$33,980	\$29,490	\$33,030	\$1,800	\$16,880	\$11,320
Portland	\$38,360	\$32,730	\$38,480	\$39,740	\$37,990	\$16,880	\$1,800	\$9,040
Sacramento	\$39,280	\$30,040	\$39,390	\$37,720	\$38,670	\$11,320	\$9,040	\$1,800
Days of work								
	120	80	160	70	130	65	75	65
Annual days traveling								
From\To	Boston	New Orleans	New York	Orlando	Philadelphia	Phoenix	Portland	Sacramento
Boston	4	24	12	21	13	33	36	37
New Orleans	24	4	24	17	23	25	33	31
New York	12	24	4	21	13	33	36	37
Orlando	21	17	21	4	21	30	37	36
Philadelphia	13	23	13	21	4	33	36	37
Phoenix	33	25	33	30	33	4	22	18
Portland	36	33	36	37	36	22	4	17
Sacramento	37	31	37	36	37	18	17	4

Discussion.

This is an example of an integer programming model to minimize total cost. We must decide number of salespersons that must be based at a source and also decide the salesperson at which source must serve which destination. Note that if a source has multiple salespersons, each of them must serve different destinations as a destination cannot be served by more than 1 sales person (as given in the question). But it is logical to have by 1 salesperson serve more than 1 destination as long as the maximum working days of that salesperson is not exceeded. We must ensure that the maximum working days of none of the sales persons are exceeded while assigning destinations. Also, if a source

has no sales person based from there, then logically no sales person will be assigned from that source to a destination.

Model.

Parameters:

D_j : Number of days of work needed by destination j , where $j \in (1,2,3)$

W : Maximum number of working days for a salesperson

T_{ij} : Number of working days needed to travel from source i to destination j , where $i, j \in$
(Boston, New Orleans, New York ... Sacramento)

C_{ij} : Travel cost from source i to destination j , where $i, j \in$
(Boston, New Orleans, New York ... Sacramento)

S : Annual Salary

Decisions:

n_i : Number of sales persons to be based in source i , where $i \in$
(Boston, New Orleans, New York ... Sacramento)

x_{ij} : Whether sales person in source i must serve destination j , where $i, j \in$
(Boston, New Orleans, New York ... Sacramento)

Objective: Minimize cost

$$\min \sum_i n_i * S + \sum_{i,j} x_{ij} * C_{ij}$$

Constraints:

$\sum_i x_{ij} \leq n_i$ (1) Sales person from a source should not be assigned if there are no sales persons based from the source

$\sum_i x_{ij} \leq 1$ (2) Only 1 sales person must serve in a destination

$\sum_j x_{ij} * [D_j + T_{ij}] \leq n_i * W$ (3) Total working days available

$n_i \in \text{Integer}$ (4) Non- negative number salespersons

$x_{ij} \in \{0,1\}$ (5) Binary decision

Notes:

- (1) Constraint 2 ensures that the destination is served by only 1 salesperson. This constraint is equivalent to $\sum_i x_{ij} = 1$, since the objective is to minimize the cost. We use the above form to make it computationally efficient.
- (2) Constraint 3 ensures that when salespersons based from a source are assignment to destinations, the assignment does not exceed the total working hours of each sales person. Note that a single sales person can be assigned to multiple destinations, but a destination can only be served by a single salesperson (Constraint 2) Also note that we only need to constrain the total hours. Because distinguishing which individual consultant was assigned to each city is not critical here for the model. Some follow up scheduling can make sure each person do not exceed max hours.

Optimal Solution. The following is the solution obtained from Excel Solver.



48(AP).xlsx

A minimum cost of \$ 281,050 can be attained by assigning salesmen to cities as shown below.

Number of sales person	Assignment								
	From\To	Boston	New Orleans	New York	Orlando	Philadelphia	Phoenix	Portland	sacramento
0	Boston	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	New Orleans	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00
2	New York	1.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00
0	Orlando	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	Philadelphia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	Phoenix	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
1	Portland	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
0	Sacramento	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	sales person serving	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		<==	<==	<==	<==	<==	<==	<==	<==
		1	1	1	1	1	1	1	1

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Portland	\$38,360	\$32,730	\$16,880	\$1,800	\$9,040
Sacramento	\$39,280	\$30,040	\$11,320	\$9,040	\$1,800
				total ppl	5
Days of work	120	80	65	75	65
Annual days traveling					
From\To	Boston	New Orleans	Phoenix	Portland	Sacramento
Boston	4	24	33	36	37
New Orleans	24	4	25	33	31
New York	12	24	33	36	37
Orlando	21	17	30	37	36
Philadelphia	13	23	33	36	37
Phoenix	33	25	4	22	18
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objective	\$281,050				