Networking. The town of Busville has three school districts. The numbers of students and teachers in each district are shown below. The Supreme Court requires the schools in Busville to be balanced between students and teachers. Thus, each school must have exactly 300 people, and each school must have the same number of students. The distances between districts are shown below. Determine how to minimize the total distance that people must be bussed while still satisfying the Supreme Court's requirements. Assume that a person who remains in his or her own district does not need to be bussed.

Numbers of people in districts

	students	Teachers
District 1	210	120
District 2	210	30
District 3	180	150

Busing distances (miles)

		To			
		District 1	District 2		District 3
From	District 1	0	3		5
	District 2	3	0		4
	District 3	5	4		0

Discussion.

This is an example of an assignment problem where students of different color in a school district are assigned to other school districts to meet the diversity requirements of all the school districts. The decision is hence how many students of each color from a school district must be assigned to each of the school district. The constraints must ensure that the number of students of each color assigned to other school districts from a particular school district does not exceed the total number of students of that color present in that particular school district. The assignment must also ensure that number of black students in all school districts are equal and that the total number of students in each school district must be exactly 300. The objective is straightforward and is to minimize the distance that these students must be bussed from their school district to their assigned school district.

Model.

Parameters:

A_{ij} : $Number\ of\ students\ of\ people\ of\ type\ i\ in\ district\ j,\ \ where\ i \in (students, teachers),$
$j \in (1,2,3)$
R_k : $Number\ of\ people\ required\ in\ district\ k,\ where\ k \in (1,2,3)\ [R_k = 300]$
D_{jk} : $Distance\ between\ district\ j\ and\ district\ k,\ where\ j,k \in (1,2,3)$

Decisions:

x_{ijk}: $Number\ of\ people\ of\ type\ i\ belonging\ to\ district\ j\ that\ must\ be\ assigned\ to\ district\ k$
$,\ where\ i \in (students\ , teachers),\ \ j,k \in (1,2,3)$

Objective: $Minimize\ distance$

$$min \sum_{i,j,k\in(1,2,3)} x_{ijk} * D_{jk}$$

Constraints:

|	
$\sum_{i,j} x_{ijk} = R_k$	(1) Total people needed in each school district
$\sum_j x_{blackjk} = \sum_{j,k} x_{blackjk} / 3$	(2) Equal number of students needed in all schools
$\sum_k x_{ijk} \geq A_{ij}$	(3) Availability of people of type i from source district j
$x_{ijk} \geq 0$	(4) Non- negative number of people allocated

Notes:

1) Constraint (2) is same as ensuring number of students in each of the k districts is equal. Since there are 3 school districts, number of students in each school must be one-third the total number of students over all the k districts.
2) Constraint (3) ensures that the total number of people of each type (students / teachers) assigned to each of the district k from the district j is equal to and no more than the number of people of that type available in a district j
3) Note on constraint (3), Since we are minimizing the objective, using greater than or equal to sign instead of equal to sign the equation helps to run the solver faster

Optimal Solution. The following is the solution obtained from Excel Solver.

X
25(AP).xlsx
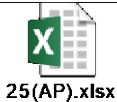
A minimum distance of 350 units is acquired by allocating students as shown below.

Number of students of district allocated to school

ORIGIN\DEST	1	2	3	Total students in district
1	100	20	0	120
2	0	30	0	30
3	0	50	100	150
Total in school	100	100	100	300
constraint	100	100	100	

Number of teachers of district allocated to school

ORIGIN\DEST	1	2	3	Total teachers in district
1	200	0	10	210
2	0	200	10	210
3	0	0	180	180
Total in school	200	200	200	

Number of students of district allocated to school

ORIGIN\DEST	1	2	3	Total students in district		
1	100	20	0	120	<	120
2	0	30	0	30	<	30
3	0	50	100	150	<	150
Total in school	100	100	100	300		
constraint	100	100	100			

Number of teachers of district allocated to school

ORIGIN\DEST	1	2	3	Total teachers in district		
1	200	0	10	210	<	210
2	0	200	10	210	<	210
3	0	0	180	180	<	180
Total in school	200	200	200			
Total people	300	300	300	>		300
Objective	350					