**(Two Products)** A company manufactures two products on two machines. The number of hours of machine time and labor depends on the machine and product as shown in the file P03_38.xlsx. The cost of producing a unit of each product depends on which machine produces it. These unit costs also appear in the same file. Each month, there are 200 hours available on each of the two machines, and there are 400 labor hours available total. This month at least 250 units of product 1 and at least 170 units of product 2 must be produced. Also, at least half of the product 1 requirement must be produced on machine 2, and at least half of the product 2 requirement must be produced on machine 1. Determine how the company can minimize the cost of meeting this month's requirements.

## Discussion: -

It is similar to the one for the smaller product mix model, but it is not the same. It might not be immediately obvious what the decision variables should be for this model. You might think that the company simply needs to decide how many products of each model to produce. However, because of the two machines, this is not enough information. The company must also decide how many of each model to produce on each machine. This is the type of reasoning you must go through to determine the appropriate decision variables for any LP model. You must choose the number of products of each model to produce on each machine. This choice determines the machine hours, labor hours, unit cost for product and machine combination. If you can put all your inputs in below format (Combination of product & machine), it will be easy to solve the problem. Let's us assume 'i' be the index for product and 'j' be the index for machines. Machine hours required to produce product 'i' using machine 'j' will be $M_{ij}$. $M_{12}$ indicates the machine hours required to produce product 1 using machine 2, which is 0.8.

| Hours of machine time required | Product 1 | Product 2 |
|---|---|---|
| Machine 1 | 0.70 | 0.75 |
| Machine 2 | 0.80 | 0.90 |
| | | |
| Hours of labor required | Product 1 | Product 2 |
| Machine 1 | 0.75 | 0.75 |
| Machine 2 | 1.20 | 1.00 |
| | | |
| Unit Cost | Product 1 | Product 2 |
| Machine 1 | 16.50 | 13.50 |
| Machine 2 | 18.00 | 14.50 |

Coming to Objective is to minimize the production cost. Production cost is nothing, but the product of Unit cost and products produced. Here, in this problem we have 2 types of products and 2 types of machine, so your total production cost is sum-product of "Unit cost to produce product i on machine j" and "Number of products i produced on machine j".

## Mathematical Model: -

*Parameters (Inputs):*

$i, j \in 1,2, (\ i\text{: } Index\ for\ type\ of\ products, j\text{: } Index\ for\ type\ of\ products)$
$M_{ij}$ : $Machine\ hours\ required\ to\ produce\ product\ 'i'\ on\ machine\ 'j'$

$L_{ij}$ : *Labor hours required to produce product 'i' on machine 'j'*
$C_{ij}$ : *Unit cost to produce product 'i' on machine 'j'*
$D_i$ : *Demand to produce product 'i'*
$M_j$ : *Machine hours available for machine 'j'*
$L$ : *Total Labor hours available*
$D_{ij}$ : *Demand to produce product 'i' on machine 'j'*

*Decision Variables:*
$x_{ij}$ : *Number of products 'i' produced on machine 'j'*

*Objective:*

$$Minimize\ Total\ Cost = \sum_{i=1}^{2}\sum_{j=1}^{2}(x_{ij} * C_{ij})$$

*Constraints:*

$x_{ij} \geq 0;$ (1) *Non Negative constraint*

$\sum_{i=1}^{2}(x_{ij} * M_{ij}) \leq M_j \quad for\ j\epsilon\{1,2\}$ (2) *Machine hours constraint*

$\sum_{i=1}^{2}\sum_{j=1}^{2}(x_{ij} * L_{ij}) \leq L$ (3) *Labor hours constraint*

$\sum_{j=1}^{2} x_{ij} \leq D_i \quad for\ i\epsilon\{1,2\}$ (4) *Production Demand constraint*

$x_{ij} \geq D_{ij};$ (5) *Demand to produce product 'i' on machine 'j'*

Constraints play a key role in solving this problem. As we have total labor hours in our problem as constraint, we can write a single mathematical equation which helps in calculating the sum-product of units produced and labors hours used. Coming to Machine hours & demand constraints, we can see that limitation for each individual machine or product, hence we need to have sperate constraints as given below.

$\sum_{i=1}^{2}(x_{i1} * M_{i1}) \leq M_1$

$\sum_{i=1}^{2}(x_{i2} * M_{i2}) \leq M_2$

$$\sum_{i=1}^{2}(x_{ij} * M_{ij}) \leq M_j \quad for\ j\epsilon\{1,2\}$$

$$\sum_{j=1}^{2} x_{1j} \le D_1$$
$$\sum_{j=1}^{2} x_{2j} \le D_2$$

$$\sum_{j=1}^{2} x_{ij} \le D_i \quad for\ i\epsilon\{1,2\}$$

*Excel Implementation:*

9[RA].xlsx

Please find the attached spreadsheet for solution.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Total Labor Hours available | 400 | | | | | | | | | |
| | | | | | | | | | | |
| | Hours Availability | | | | | | | | | |
| Inputs | Machine 1 | Machine 2 | | # of units Produced | Product 1 | Product 2 | | | | |
| Decision variables | 200 | 200 | | Machine 1 | 125 | 150 | **150** | >= | 85 | |
| Calculated Variables | | | | Machine 2 | 125 | 20 | **125** | >= | 125 | |
| Constraints | Demand | | | | **250** | **170** | | | | |
| Objective | Product 1 | Product 2 | | | >= | >= | | | | |
| | 250 | 170 | | | 250 | 170 | | | | |
| | | | | | | | | | | |
| Hours of machine time required | Product 1 | Product 2 | | Machine hours used | Product 1 | Product 2 | | | | |
| Machine 1 | 0.70 | 0.75 | | Machine 1 | 87.5 | 112.5 | **200** | <= | 200 | |
| Machine 2 | 0.80 | 0.90 | | Machine 2 | 100 | 18 | **118** | <= | 200 | |
| | | | | | | | | | | |
| Hours of labor required | Product 1 | Product 2 | | Labor hours used | Product 1 | Product 2 | | | | |
| Machine 1 | 0.75 | 0.75 | | Machine 1 | 93.75 | 112.5 | | | | |
| Machine 2 | 1.20 | 1.00 | | Machine 2 | 150 | 20 | **376.25** | <= | 400 | |
| | | | | | | | | | | |
| Unit Cost | Product 1 | Product 2 | | Production Cost | Product 1 | Product 2 | | | | |
| Machine 1 | 16.50 | 13.50 | | Machine 1 | 2062.5 | 2025 | | | | |
| Machine 2 | 18.00 | 14.50 | | Machine 2 | 2250 | 290 | 6627.5 | | | |