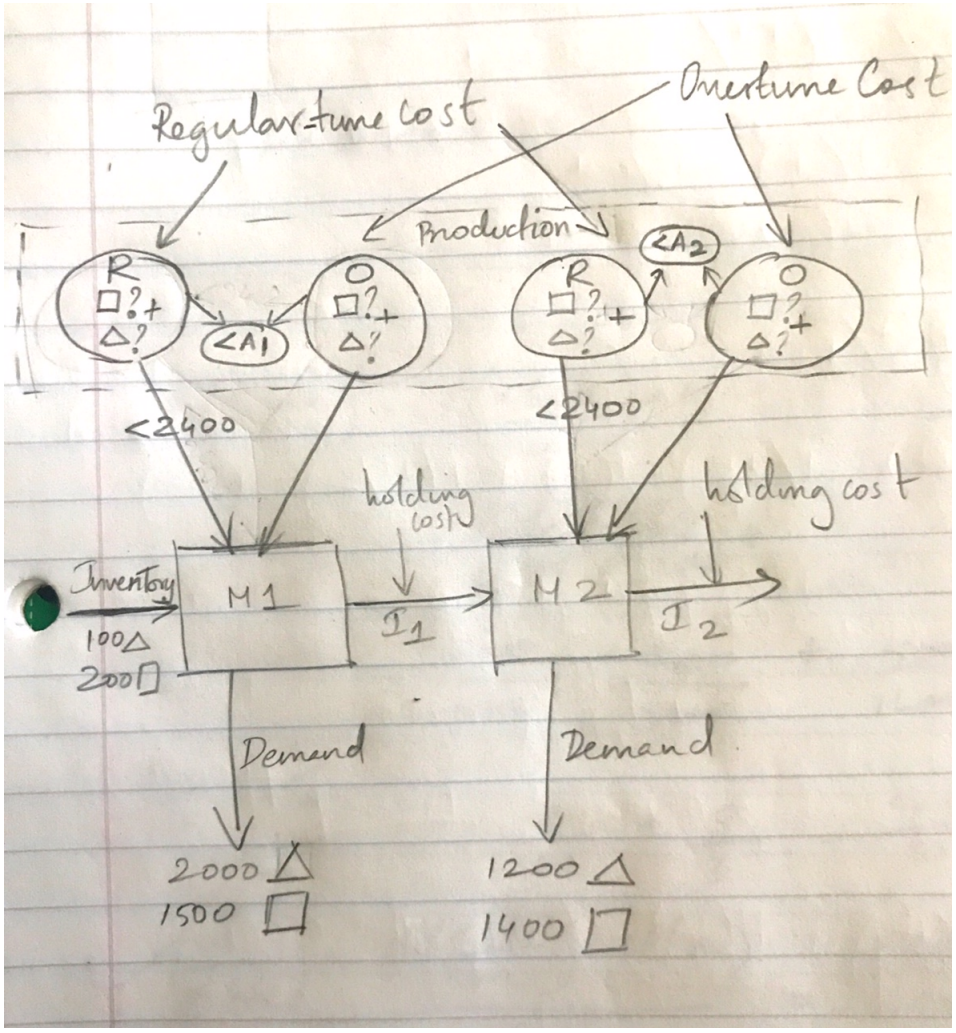
Inventory. A textile company produces shirts and pants. Each shirt requires three square yards of cloth, and each pair of pants requires two square yards of cloth. During the next two months the following demands for shirts and pants must be met (on time): month 1, 2000 shirts and 1500 pairs of pants; month 2, 1200 shirts and 1400 pairs of pants. During each month the following resources are available: month 1, 9000 square yards of cloth; month 2, 6000 square yards of cloth. In addition, cloth that is available during month 1 and not used can be used during month 2. During each month it costs $10 to produce a piece of clothing with regular-time labor and $16 with overtime labor. During each month a total of 2400 articles of clothing can be produced with regular time labor and an unlimited number of articles can be produced with overtime labor. At the end of each month, a holding cost of $1 per article of clothing is incurred. (There is no holding cost for cloth). Determine how to meet the demands of the next 2 months (on time) at a minimum cost. Assume that 100 shirts and 200 pairs of pants are already in inventory at the beginning of month 1.

Discussion.

As we have seen in the previous multi-period problems, we see that there is a flow of cloth articles from one month to the next. Hence, we have balancing equations for the same. The difference in this problem is that there are 2 kinds of cloth articles produced each month (shirt and pant) and there is a separate demand requirement for shirts and pants in each month, hence there must be balancing equations for each (shirt and pants) to control its flow over the months. A unique aspect here is that there is a cloth availability for each month, and the cloth left over after the production of shirts and pants in each month can flow into the next month and add on to the cloth available in the next month. Hence the net cloth available to produce articles each month is the sum of cloth available for that specific month and the cloth left over from the previous month. Along with cloth material that flows from one month to the next, shirts and pants that are left behind after satisfying the demand of that month also flows from one month to the next. Hence, we have abstract variables to account for the flow of cloth articles (shirt and pants) as well as the flow of cloth material. But it is important to note that the holding cost is only applied to the shirts and pants left behind in a month and not to the left-over cloth in a month.

Again, here we have two types for production methods, (1) the units can be produced by employees working in regular time and/or (2) the units can be produced by employees working in over time. Each of the above methods have a different cost associated with them. Since we have a limit for the number of cloth articles that can be produced by employees working in regular time in a month, to satisfy the demand requirements of a month, we might need to produce articles by making employees work over time. Hence, we must decide how many articles should be produced by employees working regular time and decide how many articles should be produced by employees working overtime for each month. Further, these articles can be shirts or pants. Hence our decision

boils down to how many shirts to be produced by regular time workers in a month, how many pants to be produced by regular time workers in a month, how many shirts to be produced by overtime workers in a month, how many pants to be produced by over time workers in a month. Since we have 2 months and 2 methods (regular time, over time) and 2 articles (shirts, pants), there are essentially 8 (2*2*2) decision variables, though we will be using a short hand mathematical notation to depict all 8 decision variables in the model. The objective is straight forward, we have to minimize the cost incurred through production cost and holding cost over the 2 months ensuring that demands for month is met.



Model.

Parameters:

T_i : *Amount of cloth required by article* i , *where* $i \in (shirt, pant)$

$D_{i\,m}$: *Demand for article* i *for month* m, *where* $i \in (shirt, pant), m \in (1,2)$

H : *Unit holding cost*

A_m : *Cloth availability for month* m, *where* $m \in (1, 2)$

C_l : *Unit cost of producing an article by labor type* l , *where* $l \in (R, O)$ *[Here we use R to denote regular time and O to denote overtime.]*
P: *Production capacity with regular time labour*
I_{Oi}: *Initial inventory of article* i, *where* $i \in (shirt, pant)$

Decisions:
p_{iml}: *Units of article* i *to to be produced in month* m *by labor type* l, *where* $i \in (shirt, pant), m \in (1,2), l \in (R, O)$

Calculated Parameters:
I_{mi}: *Inventory of article* i *at the end of month* m , *where* $m \in (1,2), i \in (shirt, pant)$
$I_{mi} = I_{m-1i} + \sum_l x_{iml}$ - D_{mi}
U_m : *Underutilized cloth from month* m, *where* $m \in (1,2)$
$U_m = A_m - \sum_i T_i * \sum_l x_{iml}$; $U_0 = 0$ (Underutilization of cloth occurs only after production has started in month 1)

Objective: *Minimize Cost*
$min \sum_m [\sum_{il} x_{iml} * C_l + \sum_i I_{mi} * H$

Constraints:

$I_{mi} \geq 0$	(1) Demand must be satisfied for each month for each article
$x_{iml} \geq 0$	(2) Units of products produced cannot be negative
$\sum_i x_{im\,R} \leq P$	(3) Production capacity for regular time for each month
$\sum_i T_i * \sum_l x_{iml} \leq A_m + U_{m-1}$	(4) Cloth availability for each month

Notes:

1) Constraint (4) ensures cloth articles produced in a month can utilize cloth material available for that month and cloth material left underutilized from previous month.
2) Constraint (3) ensures that the number of articles (shirts and pants) produced in regular time in each month is less that or equal to the monthly regular time production capacity

Optimal Solution. The following is the solution obtained from Excel Solver.

A minimum cost of $64000 can be attained by producing shirts and pants over the 2 months as shown below:

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		MONTH 1	MONTH 2
Regular time	Shirt	1900	1000
	Pant	500	1400
Overtime	Shirt	0	200
	Pant	800	0

Inputs				
	Textile	MONTH 0	MONTH 1	MONTH 2
Demand	Shirt		2000	1200
	Pant		1500	1400
Cloth Available		0	9000	6000
Cloth Utilized			8300	6400
Cloth left at end of month		0	700	300
Net cloth available at the beginning of month			9000	6700
Production cost	Regular time	Overtime		
	10	16		
Maximum articles produced in a month			2400	
Initial Inventory	Shirt	100		
	Pant	200		
Holding cost	1			
			MONTH 1	MONTH 2
Inventory	Shirt		0	0
	Pant		0	0
Total Inventory			0	0
Holding cost			0	0
Decision			MONTH 1	MONTH 2
	Regular time	Shirt	1900	1000

		Pant	500	1400
		Shirt	0	200
	Overtime	Pant	800	0
	Production cost		36800	27200
Objective	64000			