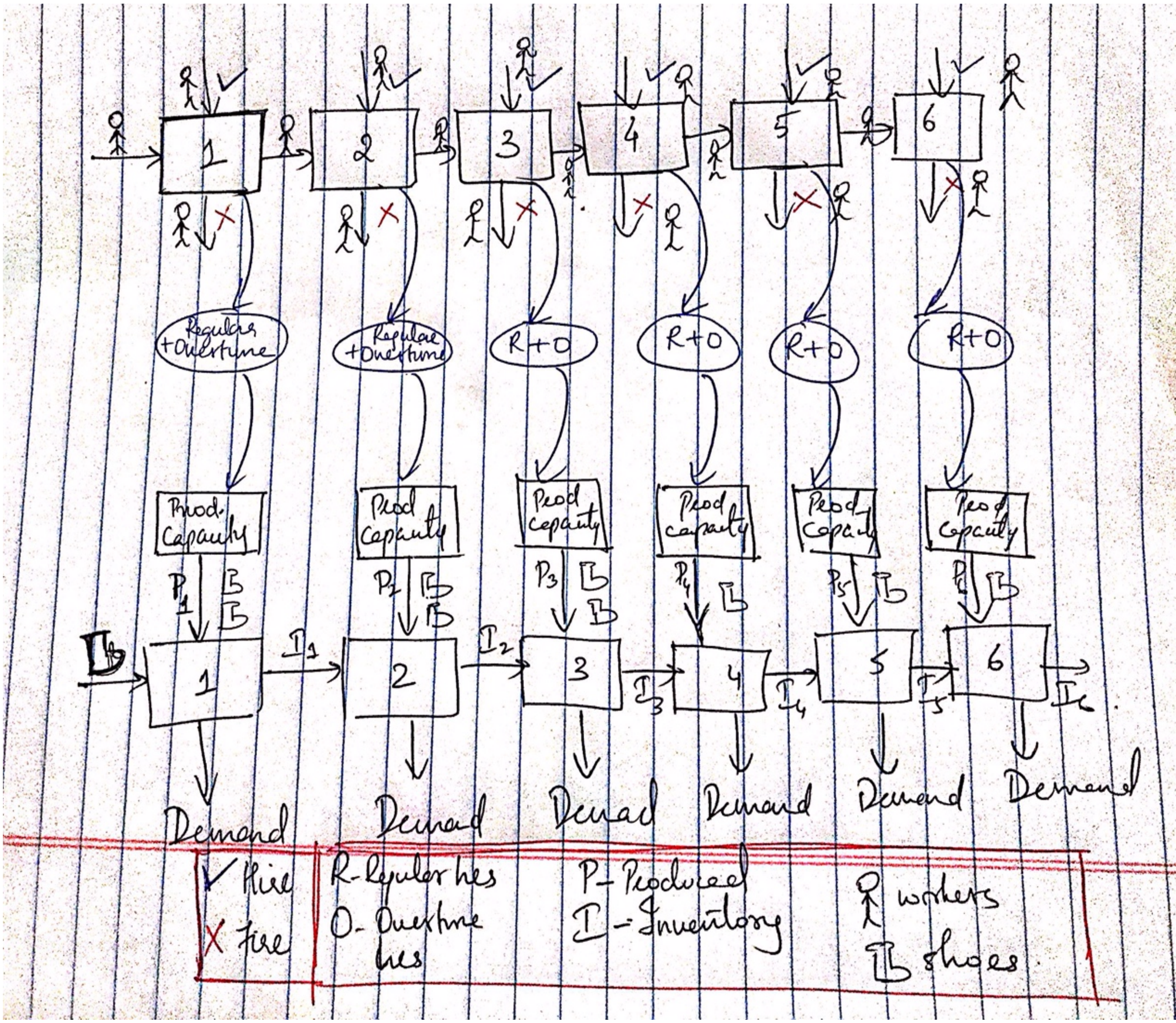
Aggregation. Shoemakers of America forecasts the following demand for the next six months: 5000 pairs in month 1; 6000 pairs in month 2; 7000 pairs in month 3; 9000 pairs in month 4; 6000 pairs in month 5; 5000 pairs in month 6. It takes a shoemaker 20 minutes to produce a pair of shoes. Each shoemaker works 150 hours per month plus up to 40 hours per month of overtime. A shoemaker is paid a regular salary of $2000 per month plus $20 per hour for overtime. At the beginning of each month, Shoemakers can either hire or fire workers. It costs the company $1000 to hire a worker and $1200 to fire a worker. The monthly holding cost per pair of shoes is 5% of the cost of producing a pair of shoes with regular-time labor. The raw materials in a pair of shoes cost $10. At the beginning of month 1, Shoemakers has 15 workers and 500 pairs of shoes in inventory. Determine how to minimize the cost of meeting (on time) the demands of the next six months

Discussion.

In this problem, we have to determine the workforce levels as well as production levels over a multiperiod time horizon. Hence, we need balancing equations for both the workforce as well as production units. The balancing equation for the workforce, ensures the flow of workers through the years after hiring and firing at the beginning of each year as we have seen before. The balancing equation for production levels ensure that the newly produced units in a month and the units from inventory of the previous month can be used to satisfy the demands of that particular month, and any left-over units again flows into the next month.
Again, here we have two types for production methods, (1) the units can be produced by employees working in regular time and/or (2) the units can be produced by employees working in over time. Each of the above methods have a different cost associated with them. And each worker has a fixed number of regular hours he can serve and a maximum limit for the overtime hours he can serve. This gives us a maximum available hour to produce shoes for a month depending on the number of workers working in that month. This poses a constraint on the number of shoes that can be produced in a month. Hence to understand how many workers need to work in a month and how many units need to be produced satisfy demand and to minimize overall production cost, we must decide numbers of workers to hire and fire in each month, number of overtime hours used in each month and the number of units to be produced in each month. Hence, we have 4 * 6 (number of months) = 24 decision variables. The workers and the shoes produced are related through the number of hours available to produce the shoes and the number of hours needed to produce a shoe. The number of shoes to be produced each month along with the inventory from previous month must satisfy the demands of that month, and the number of shoes produced each month must be within the capacity of the workers available in that month. The objective is straight forward, i.e. to minimize the overall cost due to firing, hiring and paying the workers each month, overtime salary for the workers, cost incurred due to raw material cost to produce a shoe and the holding cost of shoes in inventory.



Model.

Parameters:

D_i *: Demand for each month i , where ,i ϵ{1,2,3,4,5,6}*

H_r *: Max Regular Working Hours*

H_o *: Max Overtime Working Hours*

l *: labor hours needed to make a pair of shoe= 20/60 hrs.*

S : Fixed Monthly Salary

S_o*: Overtime Salary per hour worked overtime*

C_h*: Cost to hire a worker*

C_f*: Cost to fire a worker*

C_p: *Raw material cost for a pair of shoe*
W_o: *Workers available at the beginning of month 1*
I_0: *Shoes available at the beginning on month 1*
k: *Holding cost*
[k=5% of regular making charge of one shoe = 0.05(((20/60) *(2000/150))+10)]*

Decisions:

N_{hi}: *Number of workers hired in month i , where ,i ϵ{1,2,3,4,5,6}*
N_{fi}: *Number of workers fired in month i, where ,i ϵ{1,2,3,4,5,6}*
N_i: *Number of shoes produced each month i, where ,i ϵ{1,2,3,4,5,6}*
N_{oi}: *Number of overtime labor hours used each month i, where ,i ϵ{1,2,3,4,5,6}*

Calculated Parameters:

I_i: *Inventory of shoes at the end of each month i , where ,i ϵ{1,2,3,4,5,6}*
$I_i = I_{i-1} + N_i - D_i$
W_i: *Workers available to work in each month i , where ,i ϵ{1,2,3,4,5,6}*
$W_i = W_i + N_{hi} - N_{fi}$

Objective: *Minimize Cost*

$$min \sum_{i=1}^{6} (N_i * C_p + N_{hi} * C_h + N_{fi} * C_f + W_i * S + N_{oi} * S_o + I_i * k)$$

Constraints:

$I_i \geq 0$, $W_i \geq 0$	(1)$Non-negative\ inventory$ and *workforce*
$N_i * l \leq W_i * H_r + N_{oi}$	(2)$Maximum\ production\ capacity$
$N_{oi} \leq W_i * H_o$	(3)$Maximum\ overtime\ hours\ available$

Notes:

1) Constraint (2) ensures that number of shoes produced in a month is within the capacity that can be produced by the number of workers working in that month
2) Constraint (3) ensures that the overtime hours used in a month is within the maximum overtime hours available due to the workers working in that particular month
3) Each term in objective explained below:

OBJECTIVE:

$$\min \sum_{i=1}^{6} \overbrace{N_i * C_p}^{\text{Raw material cost}} + \underbrace{N_{hi} * C_h}_{\text{Hiring Cost}} + \overbrace{N_{fi} * C_f}^{\text{Firing Cost}} + \underbrace{W_i * S}_{\text{Regular Salary}} + \overbrace{N_{oi} * S_o}^{\text{Overtime Salary}} + \underbrace{T_i * k}_{\text{holding Cost}}$$

Optimal Solution. The following is the solution obtained from Excel Solver.

15(AP).xlsx

A minimum cost of $551027.2000 can be attained by producing shirts and pants over the 2 months as shown below:

Decision		Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Number of workers to hire each month		0	0	0	0	0	0
Number of employees to fire each month		0	0	1	0	1	2
No of shoes produced each month		6818.18182	6818.18182	6363.63636	6500	6000	5000
No of over time hours used each month		0	0	0	45	30	0

Inputs							
For a shoe maker:							
Regular hours	150						
Over time hours max	40						
Time to produce a pair of shoes(hrs)	0.33						
Regular salary	2000						
Over time salary/ hr	20						
Cost to hire a worker	1000						
Cost to fire a worker	1200						
Holding cost of a shoe	0.72	5% of cost to produce a shoe in regular time					
Raw material cost of a shoe	10						
		Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Demand for shoes		5000	6000	7000	9000	6000	5000
Shoes available at the beginning of month 1	500						
Workers available at the beginning of month 1	15						
Decision		Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Number of workers to hire each month		0	0	0	0	0	0
Number of employees to fire each month		0	0	1	0	1	2
No of shoes produced each month		6818.18182	6818.18182	6363.63636	6500	6000	5000
No of over time hours used each month		0	0	0	45	30	0
Number of workers working each month		15	15	14	14	13	11
Number of shoes in inventory		2318.18182	3136.36364	2500	0	0	0
Regular working hours available		2250	2250	2100	2100	1950	1650
Overtime hours available		600	600	560	560	520	440
Actual total number of hours available each month		2250	2250	2100	2145	1980	1650
Max production possible in each month		6818.18182	6818.18182	6363.63636	6500	6000	5000
Costs							
Regular Salary		30000	30000	28000	28000	26000	22000
Overtime salary		0	0	0	900	600	0
Hiring Cost		0	0	0	0	0	0
Firing Cost		0	0	1200	0	1200	2400
Material Cost		68181.8182	68181.8182	63636.3636	65000	60000	50000
Holding Cost		1669.09091	2258.18182	1800	0	0	0
Total cost each month		99850.9091	100440	94636.3636	93900	87800	74400
Objective							
Total Cost	551027.273						