(Swimming Coach) Based on Machol (1970). A swimming coach is putting together a relay team for the 400-meter relay. Each swimmer must swim 100 meters of breaststroke, backstroke, butterfly, or freestyle, and each swimmer can swim only one race. The coach believes that each swimmer can attain the times given in below table. To minimize the team's total time for the race, which swimmers should swim which strokes?

Times	Free	Breast	Butterfly	Back
Hall	54	54	51	53
Spitz	51	57	52	52
Montgomery	50	53	54	56
Jastremski	56	54	55	53

Discussion: -

Our objective in the problem is to minimize the total time. Time taken by swimmer to finish 100 meters in each swimming style is given as out input. Our model should decide which swimmer needs to be picked for each swimming style.

Mathematical Model: -

Parameters (Inputs):

$i \in 1,2,3,4$ (i: Index for style of swimming)
$j \in 1,2,3..8$ (j: Index for swimmers)
A_{ij} :   Time taken by swimmer j to finish 100 mtrs in swimming style i

Decision Variables:
x_{ij} : Decision whether to pick swimmer j for swimming style i

Objective:

$$Minimize\ total\ time = \sum_{j=1}^{4}\sum_{i=1}^{4}(x_{ij} * A_{ij})$$

Constraints:

$x_{ij} \in \{0,1\}$ (1) Binary constraint

$\sum_{i=1}^{4} x_{ij} = 1 ;$ for $j \in \{1,2,3,4\}$ (2) Number of races for each swimmer constraint

$\sum_{j=1}^{4} x_{ij} = 1 ;$ for $i \in \{1,2,3,4\}$ (3) Should cover all the swimming styles

Constraint 2 will make sure that a swimmer can go for only one type of race. Constraint 3 makes sure that swimmers were assigned to all the styles to complete the relay.

Excel Implementation: Please find the attached spreadsheet for solution.

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						Inputs	
Times	Free	Breast	Butterfly	Back		Decision variables	
Hall	54	54	51	53		Calculated Variables	
Spitz	51	57	52	52		Constraints	
Montgomery	50	53	54	56		Objective	
Jastremski	56	54	55	53			

Times	Free	Breast	Butterfly	Back			
Hall	0	0	1	0	1 =	1
Spitz	0	0	0	1	1 =	1
Montgomery	1	0	0	0	1 =	1
Jastremski	0	1	0	0	1 =	1
	1	1	1	1		
	=	=	=	=		
	1	1	1	1		

Total Time	207