(Three Investments) Consider three investments. You are given the following means, standard deviations, and correlations for the annual return on these three investments. The means are 0.12, 0.15, and 0.20. The standard deviations are 0.20, 0.30, and 0.40. The correlation between stocks 1 and 2 is 0.65, between stocks 1 and 3 is 0.75, and between stocks 2 and 3 is 0.41. You have $10,000 to invest and can invest no more than half of your money in any single stock. Determine the minimum-variance portfolio that yields a mean annual return of at least 0.14.

Discussion: -

A company has $10,000 to invest and mean, standard deviation and correlations for the annual return on these three investments were given. Create a correlation matrix using the inputs given. Our objective is to minimize the variance and make sure the optimal solution has the expected mean annual return. Let the decision variable be the fraction of the investment amount invested. Create a calculated field for actual return mean value and make sure that it is at least 0.14. Create necessary calculated fields like weighted standard deviation which will help in making our objective function (here variance) simple.

Mathematical Model: -

Parameters (Inputs):

$i, j \in 1,2,3 \qquad (i: Index\ for\ investment\ options\ )$

$M_i = Mean\ annual\ return\ on\ invetment\ option\ i\ (0.12, 0.15, 0.20)$

$S_i = Standard\ deviations\ on\ invetment\ option\ i\ (0.20, 0.30, 0.40)$

$C_{ij} = Correlation\ matrix$

Correlation	Inv 1	Inv 2	Inv 3
Inv 1	1	0.65	0.75
Inv 2	0.65	1	0.41
Inv 3	0.75	0.41	1

$M\ : Expected\ mean\ annual\ return\ (0.14)$
$T\ : Total\ investment\ value\ \ (\$\ 10{,}000)$

Decision Variables:
$x_i : Fraction\ invested\ in\ investment\ \ i$

Calculated Variables:

$$A = \left[\sum_{i=1}^{3} x_i * M_i\right] ; Actual\ Return\ mean\ value$$

$$W_i = S_i * x_i \quad ; for\ i \epsilon \{\ 1,2,3\}\ Weighted\ Standard\ Deviation$$

$$E_j = \sum_{i=1}^{3} (W_i * C_{ij})\ ; for\ j \epsilon \{\ 1,2,3\}$$

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Objective:

$$Minimize\ variance = \sum_{i,j=1}^{3} (W_i * E_j)$$

Constraints:

$$\sum_{i=1}^{3} x_i = 1 \qquad (1)\ Sum\ of\ fraction\ of\ investments\ should\ be\ 1$$

$$A \geq M \qquad (2)\ Actual\ return\ should\ be\ greater\ than\ Required\ return$$

$$x_i \leq 0.5 \qquad (3)\ Fraction\ of\ investment\ should\ not\ be\ more\ than\ 50\%$$

Excel Implementation: Please find the attached spreadsheet for solution.

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	Mean	S.D					Inputs
Inv 1	0.12	0.2					Decision variables
Inv 2	0.15	0.3					Calculated Variables
Inv 3	0.2	0.4					Constraints
							Objective
Correlation	Inv 1	Inv 2	Inv 3		Decision		
Inv 1	1	0.65	0.75		0.50	<=	0.50
Inv 2	0.65	1	0.41		0.40	<=	0.50
Inv 3	0.75	0.41	1		0.10	<=	0.50
					1.00		
					=		
Actual Return	0.14	>=	0.14		1		
					Weighted S.D		
				Inv 1	0.10		
				Inv 2	0.12		
				Inv 3	0.04		
	Inv 1	Inv 2	Inv 3				
	0.21	0.20	0.16				
Variance	0.05						