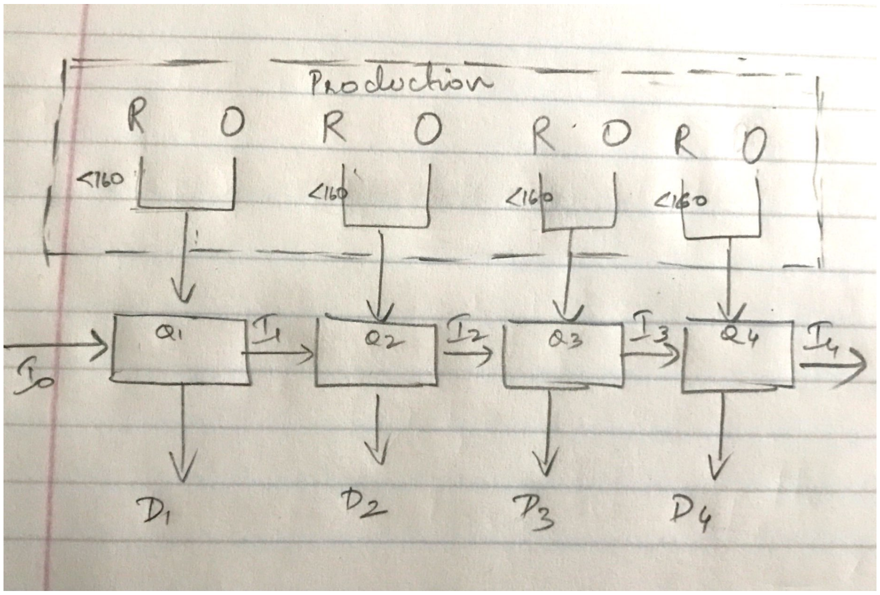
Inventory. A company that builds sailboats wants to determine how many sailboats to build during each of the next four quarters. The demand during each of the next four quarter is as follows: first quarter, 160 sailboats; second quarter, 240 sailboats; third quarter, 300 sailboats; fourth quarter, 100 sailboats. The company must meet demands on time. At the beginning of the first quarter, the company has an inventory of 40 sailboats. At the beginning of the first quarter, the company must decide how many sailboats to build during that quarter. For simplicity, assume that sailboats built during a quarter can be used to meet demand for that quarter. During each quarter, the company can build upto160 sailboats with regular time labor at a total cost of $1600 per sailboat. By having employees work overtime during a quarter, the company can build additional sailboats with overtime labor at a total cost of $1800 per sailboat. At the end of each quarter (after production has occurred and current quarter's demand has been satisfied), a holding cost of $80 per sailboat is incurred. Determine a production schedule to minimize the sum of production and inventory holding costs for the next four quarters.

Discussion.
The unique aspect of this problem lies in not only identifying the calculated or abstract parameters to draft the balancing equation, but also to identify the variables that control optimal solution. As we have seen in multi-period models, it is a common technique to use abstract variables that accommodate for materials that flow through time like inventory (as in this problem). These variables control the flow from one period to another, hence we must ensure that these materials (inventory/abstract variables) that comes into one period must either be consumed in that period or transferred to the next period. To depict this behavior, we use balancing equations for the abstract variables.

Here the units of sailboats produced per month determine the cost incurred in that quarter. But here we have two types for production methods, (1) the units can be produced by employees working in regular time and/or (2) the units can be produced by employees working in over time. Each of the above methods have a different cost associated with them. Since we have a limit for the number of sailboats that can be produced by employees working in regular time in a month, to satisfy the demand requirements of a month, we might need to produce sailboats by making employees work over time. Hence, we must decide how many sailboats should be produced by employees working regular time and decide how many sailboats should be produced by employees working overtime for each quarter.



Costs in a quarter is also incurred due to the holding cost of inventory. This is the cost of storing the sail boats produced that have not been needed to satisfy the demand of that quarter and flows into the next quarter and can be used to satisfy the demands of the next quarters. Since we have 4 quarters and 2 methods (regular time, over time) in each quarter, there are essentially 8 decision variables, though we will be using a short hand mathematical notation to depict all 8 decision variables in the model.
The objective is to minimize the cost incurred through production cost and holding over the 4 quarters ensuring that demands of each quarter is met.

Model.

Parameters:

D_i : *Demand for quarter* i *, where* $i \in (1,2,3,4)$
C_t : *Unit cost for production type t, where* $t \in (R, O)$ *[Here we use R to denote regular time and O to denote overtime.]*
H : *Unit holding cost*
RK : *Production capacity for regular time per quarter*
I_O: *Initial inventory*

Decisions:

x_{ti} : *Units of sailboats to produce through production type t for quarter i, where* $i \in (1, 2, 3, 4)$

Calculated Parameters:

I_i: *Inventory at the quarter* i *, where* $i \in (1,2,3,4)$

$I_i = I_{i-1} + \sum_{t=R,O} x_{ti} - D_i$

Objective: *Minimize Cost*

$max \sum_{i=1,2,3,4}[[\sum_{t=R,O} x_{ti} * C_t + I_i * H]$

Constraints:

$I_i \geq 0$	(1) Demand of each quarter must be satisfied
$x_{ti} \geq 0$	(2) Number of units produced cannot be negative
$x_{R\,i} \leq RK$	(3) Maximum production capacity for regular time each quarter

Notes:

1) Constraint (1) can also be stated as $I_{i-1} + \sum_{t=R,O} x_{ti} \geq D_i$, either notations means that demand for each quarter must be satisfied

Optimal Solution. The following is the solution obtained from Excel Solver.

8(AP).xlsx

UNITS TO PRODUCE	Q1	Q2	Q3	Q4
Regular time	160	160	160	100
Over time	0	40	140	0

Commented [YW1]: Put excel screen shot as a 'picture'?

Inputs					
Cost of unit production					

Inventory [Based on Practical Management Science] Prepared by Athira Praveen

Regular time	$1,600.00				
Over time	$1,800.00				
Maximum production / quarter in regular time	160				
Holding cost	$80.00				
Initial Inventory	40				
		Quarter 1	Quarter 2	Quarter 3	Quarter 4
Demand		160	240	300	100
Decision:					
Number of sailboats produced	Regular time	160	160	160	100
		<=	<=	<=	<=
		160	160	160	160
	Over time	0	40	140	0
Total production		160	200	300	100
Inventory		40	0	0	0
		>=	>=	>=	>=
		0	0	0	0
Regular time cost		$256,000.00	$256,000.00	$256,000.00	$160,000.00
Over time cost		$0.00	$72,000.00	$252,000.00	$0.00
Holding cost		$3,200.00	$0.00	$0.00	$0.00
Total Cost		$259,200.00	$328,000.00	$508,000.00	$160,000.00
Objective	$1,255,200.00				