Non-Linear Programming. An oil company must determine how many barrels of oil to extract during each of the next two years. If the company extracts x1 million barrels during year 1, each barrel can be sold for 80 - x1 dollars. If the company extracts x2 million barrel during year 2, each barrel can be sold for 85 - x2 dollars. The cost of extracting x1 million barrels during year 1 is $2x_1^2$ million dollars, and the cost of extracting x2 million barrels during year 2 is $3x_2^2$ million dollars. A total of 20 million barrels of oil are available, and at most \$250 million can be spent on extraction. Determine how the company can maximize its profit (revenues less costs) for the next two years.

Discussion.

This is an example of a non-linear programming model to maximize profit. The model is non-linear because the objective function is an algebraic equation of degree greater than 1, making the boundary of the feasible region non-linear. The rest of the mathematical model remains similar to that of a linear model. The constraints ensure that the decision variable (barrels produced) is an integer and does not exceed maximum number of barrels that can be exceeded over the 2 years. In order to maximize the objective, we must try to extract as many barrels of oil as we can subject to the constraint that the cost price do not exceed the budget available. The decision is how barrels of oil needs to be extracted over the 2 years.

Model.
Parameters:
80 - x_1: Unit selling price of x_1 barrels extracted in year 1
85 - x_2: Unit selling price of x_2 barrels extracted in year 2
$2x_1^2$: Cost price of x_1 barrels
$3x_2^2$: Cost price of x_2 barrels
M: Maximum number of barrels that can be extracted, M = 20
B : Available budget

Decisions:
x_i : Barrels of oil to be extracted in year i, where i ∈ {1,2}

Objective: *Maximize profit*
$max \ [\ (80 - x_1) * x_1 + (85 - x_2) * x_2 - 2x_1^2 - 3x_2^2 \ ]$

Constraints:
$\sum_i x_i \le M$ (3) Maximum barrel capacity

$$2x_1^2 + 3x_2^2 \leq B \qquad\qquad\qquad\qquad (4) \text{ Budget available}$$

Optimal Solution. The following is the solution obtained from Excel Solver.

X
53(AP).xlsx

A maximum profit of $814 be attained by extracting barrels of oil over the 2 years as shown below..

	Year 1	Year 2		total number of barrels	
Barrels of oil to be extracted	8	6		14 <=	20
Unit SP	72	79		total cp	
CP	128	108		236 <=	250
SP	576	474			
Profit	448	366			
Total Profit	814				