Handbag Production[1]

A Louis Vuitton factory produces handbags and purses. Each handbag uses 4kgs of leather, and each purse uses 3kgs of leather. A handbag contributes $400 to profit, and a purse contributes $250. Marketing restrictions require that the number of purses produced be at least twice the number of handbags produced. There are 2000kg of leather available. Setup a model to maximize the company's profit.

Discussion

The objective is to design a production plan in such a way that Louis Vuitton maximizes the profit from production of the two items, handbags and purses. The parameters given in this problem are the amount of leather required to produce each item and the profit that Louis Vuitton would get from each item sold. In simple terms, profit is the difference between the selling price and cost; however, because we are not given selling price and cost of production, we can use profit of each item as the input.

Total Profit is the product of 'units produced' and 'unit profit'. Our objective is to increase the total profit, for which we need to come up with an expression that is dependent on how many items to produce. Since we do not know how many items of each to produce, these will be the decision variables.

Now that we have clear inputs and decision variables, let's look at other details highlighted in the problem. It was given that the company has only 2000kg of leather available for production, which is a constraint for our production plan. Assume if we do not have this constraint, we cannot find the final solution as there is no boundary to the decision variables (items produced) and Solver will keep on computing to maximize the profit. It will throw an error stating that "The Objective cell values do not converge".

In addition to the above constraint, there is a marketing restriction that stipulates the number of purses produced be at least twice the number of handbags. If we do not add this constraint to our mathematical model, Solver will calculate an incorrect production plan depending on the inputs we have provided. For every 1 kg of leather the company gets $100 in profit for handbags, whereas it only gets $83.33 on purses; hence if the problem did not describe that there is a marketing constraint, Solver would have suggested to only produce handbags as it gives a higher profit on each kg of leather.

[1] This exercise problem and related solutions were originally developed by Ramesh Alla based on Practical Management Science 5th Edition. This current revision was revised by Nowed Patwary.

Mathematical Model

Parameters (Inputs):

$i \in 1,2$ ($Index\ for\ items;\ 1 = handbag\ , 2 = Purse$)
L_i : $kgs\ of\ Leather\ used\ to\ produce\ item\ i$
P_i : $Profit\ from\ each\ item\ i$
K : $Total\ kg\ of\ Leather\ available\ for\ production$

Decision Variables:

x_i : $Number\ of\ items\ produced\ in\ each\ item - type\ i$

Objective:

$$Max\ Total\ Profit = \sum_{i=1}^{2}(x_i * P_i)$$

Constraints:

$x_i \geq 0;$ (1) $Non\ Negative\ constraint$

$\sum_{i=1}^{2}(x_i * K_i) \leq K$ (2) $Leather\ availability\ constraint$

$x_1 \geq 2 * x_2$ (3) $Marketing\ restriction\ constraint$

If we do not use the non-negative constraint, Solver gives us no solution to maximize the profit because it does not know how much leather total to use for producing the items. Because the company only has 2000kgs of leather which can be used for production we have to make sure that sum-product of items produced, and material required to produce each item is less than or equal to total units of raw material (leather) available. $\sum_{i=1}^{2}$ is a short hand for summing up similar terms indexed by i.

Excel Implementation

Handbag Problem	Purses	Handbags			
kgs of leather required to produce each	3	4			
Profit for each item sold	$ 250	$ 400			
kgs of leather available	2,000				
Marketing restrictions : Purses produced be atleast twice the number of Handbags					
	Purses	Handbags			
# of items produced	400	200			
	>=				
Marketing Constraints	400	200	Total Leather used		Constraint
kgs of leather used to produce	1,200	800	2,000	<=	2,000
Profit from each type of item based on amount sold	$ 100,000	$ 80,000			
Total Profit	$ 180,000				

As per the optimization model Louis Vuitton can produce 400 handbags and 200 purses to get the maximum profit.

Solver's sensitivity report performs two types of sensitivity analyses:

(1) On the co-efficient of the objective
(2) On the right sides of the constraints

The chart below gives you the sensitivity analysis. In the variable cells table, the allowable increase and allowable decrease indicate how much the coefficient of item profit for Purses in the objective, currently 250, could change before the optimal product mix would change. If the coefficient of Purses stays within the allowable range, the set of values in the decision variable cells does not change at all. However, outside of these limits, the optimal mix between Purses and handbags might change.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	# of items produced Purses	400	0	250	50	450
C13	# of items produced Handbags	200	0	400	1E+30	66.66666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B13	# of items produced Purses	400	-20	0	666.6666667	1000
D17	kgs of leather used to produce Total Leather used	2000	90	2000	1E+30	2000

Table 2 (Constraints) in above picture indicates how much these right-side constants can change before the optimal solution changes. R.H side of each constraints should be numeric constant (not formulae). Shadow price indicates the change in the objective when a right-side constant is changed.