Integer Programming. Based on Bean et al. (1988). Simon's Mall has 9000 square feet of space to rent and wants to determine the types of stores that should occupy the mall. The minimum number and maximum number of each type of store (along with the square footage of each type) are shown below. The annual profit made by each type of store depends on how many stores of that type are in the mall. This dependence is given in the same file. For example, if two department stores are in the mall, each department store will earn $210,000 profit per year. Each store pays 5% of its annual profit as rent to Simon's. Determine how Simon can maximize its rental income from the mall.

Choosing stores for a mall																
Total profit for each possibility																
	Jewelry			Shoe			Department			Book				Clothing		
	1	2	3	1	2	3	1	2	3	0	1	2	3	1	2	3
	$90,000	$160,000	$210,000	$100,000	$180,000	$150,000	$270,000	$420,000	$600,000	$0	$160,000	$180,000	$210,000	$170,000	$260,000	$300,000
	J	S	D	B	C											
Square footage	500	600	1500	700	900											
% profit for rent	5%															

Discussion.

This is an example of an integer programming model to maximize income. We must decide how many stores of each type must occupy a mall. To make the mathematical model easier, we can create a parameter that specifies the space utilized by a specific number of stores of a particular type. This is derived from the space used by a single store of a particular type and the possible number of stores that we would like to decide from, as given in the question. Also ensure that once we decide to build n number of stores of a particular type, we cannot then again decide to build another m number of stores of that type. The question specifies that the numbers given are the minimum and maximum possible number of stores of that type that can be occupied in the mall. We must also ensure that the space occupied by the combination of store type and the number of stores we choose does not exceed the maximum space available.

Model.

Parameters:

S_s : *Space used by one store of type s, where* $s \in (J, S, D, B, C)$

S : *Maximum space available*

P_{is}: *Profit made by store type s in* i *numbers, where where* $(s, i) \in k$

k:

J	J	J	S	S	S	D	D	D	B	B	B	B	C	C	C
1	2	3	1	2	3	1	2	3	0	1	2	3	1	2	3

R: *Percentage of profit for rent*

Calculated variables:

$S_{is} = S_s * i$: *Space used by* i *stores of type s, where* $(s,i) \in K$, *and K is a set as listed below*

k:

J	*J*	*J*	*S*	*S*	*S*	*D*	*D*	*D*	*B*	*B*	*B*	*B*	*C*	*C*	*C*
1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*	*0*	*1*	*2*	*3*	*1*	*2*	*3*

Decisions:

x_{is}: *If store type s of* i *numbers must be chosen, where* $(s,i) \in k$

k:

J	*J*	*J*	*S*	*S*	*S*	*D*	*D*	*D*	*B*	*B*	*B*	*B*	*C*	*C*	*C*
1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*	*0*	*1*	*2*	*3*	*1*	*2*	*3*

Objective: *Maximize Rental Income*

$max \sum_{i,s} x_{is} * P_{is} *$ R

Constraints:

$\sum_i x_{is} \leq 1$ (1) For store type s, only one among the possible i's must be chosen

$\sum_{i,s} x_{is} * S_{is} \leq S$ (2) Allocation must not exceed space available

$x_{is} \epsilon \{0,1\}$ (3) Binary decision

Notes:

(1) Constraint 1 ensures that among all possible options for a store type, only one can be chosen. If we decide to build 2 Jewelry stores, then we cannot choose the option of 3 Jewelry stores along with it. For a Jewelry store, a maximum of 3 stores can only be built. The space utilized is linearly dependent on the number of stores of a particular type. Hence choosing 1 Jewelry store along with 2 Jewelry stores is equivalent to choosing 3 Jewelry stores. Thus we allow for only one of the possible specified combinations for a store type s.

Optimal Solution. The following is the solution obtained from Excel Solver.

46(AP).xlsx

A maximum rental income of 66000 $ can be attained by allocating stores in the mall as shown below.

Choosing stores for a mall																
Total profit for each possibility																
	Jewelry			Shoe			Department			Book				Clothing		
	1	2	3	1	2	3	1	2	3	0	1	2	3	1	2	3
	$90,000	$160,000	$210,000	$100,000	$180,000	$150,000	$270,000	$420,000	$600,000	$0	$160,000	$180,000	$210,000	$170,000	$260,000	$300,000
decision	0	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0
	J	S	D	B	C											
Square footage	500	600	1500	700	900											
Square footage effective	500	1000	1500	600	1200	1800	1500	3000	4500	0	700	1400	2100	900	1800	2700
% profit for rent	5%															
Space used																
objective	66000															
constraint	8800	<=	9000													
	jewellery	1	<=	1												
	shoe	1	<=	1												
	dept	1	<=	1												
	book	1	<=	1												