1. **Non Linear Programming.** Nine jobs need to be completed within eight weeks. The number of weeks required to complete each job is given below. For example, job 2 requires five weeks. Each job requires 40 hours of labor per week. Each week, 160 hours of regular time labor are available. Up to 40 hours of overtime labor can be purchased each week at a cost of $18 per hour. Additional overtime hours cost $22 per hour. Determine how to minimize the overtime cost incurred in completing the jobs within eight weeks. A penalty of $500 is incurred for each day a job is late. Determine how to minimize the sum of overtime and due date penalties

Completing jobs		
Job	Weeks	Due date
1	3	6
2	5	5
3	4	4
4	4	5
5	2	4
6	3	4
7	2	4
8	5	5
9	4	7

Discussion.

This is a nonlinear programming model. The objective is to minimize the cost incurred due to over time labor and additional over time labor used. We should decide how many regular, over time, additional overtime hours need to be used each week. These labor hours can be distributed among the jobs being performed in that week so that each job gets more than 40 hours of labor hours. Note that we need not be interested in how the labor hours are distributed among the jobs of that week. We simply need to understand how many each type of labor hours is used in each week to calculate the total cost. We should also decide which week each job should start as it determined when the job will end depending on which there will be a penalty cost if the job ends after the due date of that job. The constraints must ensure that more than the available number of each type of labor hours is not used each week.

Model.

Parameters:

R_j : *Number of weeks required for job* j, *where* $j \in (1,2,3,4,5,6,7,8,9)$

D_j: *Due week for job* j, *where* $j \in (1,2,3,4,5,6,7,8,9)$

P: Penalty cost per late day
C_O *: Cost to purchase one hour of over time*
C_a *: Cost to purchase one hour of additional overtime*
H: Maximum overtime hours that can be purchased/week
L: Labor hours needed for each job/week (40hours)
T: Regular time hours available /week (160hours)

Decisions:

x_j: *Week to start job* j, *where* $j \in (1,2,3,4,5,6,7,8,9)$
w_i: *Regular time labor hours for week* i, *where* $i \in (1,2,3,4,5,6,7,8)$
o_i: *Overtime time labor hours purchased for week* i, *where* $i \in (1,2,3,4,5,6,7,8)$
a_i: *Additional over time labor hours for job week* i, *where* $i \in (1,2,3,4,5,6,7,8)$

Calculated Parameters:

y_j : *Week that start job* j *ends, where* $j \in (1,2,3,4,5,6,7,8,9)$
$y_i = x_j + R_j - 1$
P_j *: Effective Penalty for job* j, *where* $j \in (1,2,3,4,5,6,7,8,9)$
$P_j = P * \max(y_j - D_{ij}, 0)$
z_{ij} *: if job* j *is done in week* i *where* $j \in (1,2,3,4,5,6,7,8,9),\ i \in (1,2,3,4,5,6,7,8)$

Objective: *Minimize cost*

$min \sum_i (o_i * C_O + a_i * C_a) + \sum_j P_j$

Constraints:

$T + o_i + a_i \geq L * \sum_j z_{ij}$ (1) Labor hours required/week
$o_i \leq H$ (3) Over time availability per week
$o_i, a_i \geq 0$ (4) Non- negative number of hours
$x_j \in$ Integer, $x_j \geq 1$ (5) Week number is an integer

Notes:

1) $\sum_j z_{ij}$ in constraint 1 gives the number of jobs being performed in week i

Optimal Solution. The following is the solution obtained from Excel Solver.


56(AP).xlsx

A minimum cost of 16699.57$ can be attained by scheduling the jobs as shown below.

Completing jobs						
Job	Weeks	Due	Hours neede	Started	Ended	Penalty
1	3	6	40	1	4	0
2	5	5	40	0	5	0
3	4	4	40	0	4	0
4	4	5	40	0	4	0
5	2	4	40	1	3	0
6	3	4	40	0	3	0
7	2	4	40	1	3	0
8	5	5	40	0	5	0
9	4	7	40	1	5	0
						0
Hours neded by each job in a week		40		Cost		
Regular time hours available/week			160			
Max overyime hours purchasable/week			40	18		
Penalty cost/late day			500			
Additional overtime cost				22		
Decision						
Week	Regular hou	Overtime purchase	Additional C	Cost Overtime purchas	Cost Additional Overtime/w	Overall cost
1	160	40	160	720	3520	4240
2	160	40	160	720	3520	4240
3	160	40	160	720	3520	4240
4	160	25.10710517	94.89289	451.9278931	2087.643686	2539.571579
5	160	40	0	720	0	720
6	40	40	0	720	0	720
7	0	0	0	0	0	0
8	0	0	0	0	0	0
						16699.57158
Objective	16699.57					