Integer Programming. At a machine tool plant, five jobs must be completed each day. The time it takes to do each job depends on the machine used to do the job. If a machine is used at all, a setup time is required. The relevant times (in minutes) are given below. Determine how to minimize the sum of the setup and machine operation times needed to complete all jobs.

Assigning jobs to machines							
Times to do jobs (large times mean machine can't do job) and setup times							
	Job 1	Job 2	Job 3	Job 4	Job 5		Setup time
Machine 1	42	70	93	1000	1000		30
Machine 2	1000	85	45	1000	1000		40
Machine 3	58	1000	1000	37	1000		50
Machine 4	58	1000	55	1000	38		60
Machine 5	1000	60	1000	54	1000		20

Discussion.

This is an example of an integer programming model to minimize time needed to set up and run the machines. We must decide the which machines should be chosen to do the jobs and also decide which machine should be assigned to which job. Note that a single machine can be used to do more than 1 job, this helps in reducing the setup time needed to set up a new machine. Since the time given in the inputs is the time needed to complete a job by a machine, we assume that a job can at most utilize only 1 machine. The constraints ensure that each of the 5 jobs are completed and if a machine is not chosen, then no jobs should be assigned to it.

Model.

Parameters:

S_j : *Set up time for machine j, where* $j \in$
$(Machine\ 1, Machine\ 2, Machine\ 3, Machine\ 4, Machine\ 5)$

t_{ji} : *Time taken by machine j to complete job i, where* $j \in$
$(Machine\ 1, Machine\ 2, Machine\ 3, Machine\ 4, Machine\ 5,),\ i \in (Job\ 1,\ Job\ 2,\ Job\ 3,$
$Job\ 4, Job\ 5)$

Decisions:

y_j: *Whether machine j should be chosen, where* $j \in$
$(Machine\ 1, Machine\ 2, Machine\ 3, Machine\ 4, Machine\ 5)$

x_{ji} : *Whether machine by machine j must be used to complete job i, where* $j \in (Machine\ 1, Machine\ 2, Machine\ 3, Machine\ 4, Machine\ 5,)$, $i \in (Job\ 1,\ Job\ 2,\ Job\ 3, Job\ 4, Job\ 5)$

Objective: *Minimize Time*

$min \sum_j\ y_j{*}S_J + \sum_{i,j}\ x_{ji} * t_{ji}$

Constraints:

$\sum_{i,j} x_{ji} \geq 5$ (1) All 5 jobs must be completed

$\sum_i x_{ji} \leq y_j * 5$ (2) A job can be done by only 1 machine at most and if a machine is not chosen, it must not be assigned to a job

$x_{ij} \in \{0,1\}$ (4) Binary decision

$y_j \in \{0,1\}$ (5) Binary decision

Notes:

1) Constraint (1) ensures that all 5 jobs must be completed. Please note that this constraint makes mathematical sense only because Constraint (2) ensures that a particular job can be done by at most one machine. Since all 5 jobs need to be completed, at least 1 machine will be assigned to 1 job.

Optimal Solution. The following is the solution obtained from Excel Solver.

A minimum time of 345 minutes can be attained by assigning jobs to machines as shown below.

	Whether machine must beused for the job						Whether machine must be chosen
	Job 1	Job 2	Job 3	Job 4	Job 5		
Machine 1	0	0	0	0	0		0
Machine 2	0	0	0	0	0		0
Machine 3	0	0	0	0	0		0
Machine 4	1	0	1	0	1		1
Machine 5	0	1	0	1	0		1
	1	1	1	1	1		
Objective	345						
Constraints	5	>=	5				