(Fitzsimmons and Allen) Based on Fitzsimmons and Allen (1983). The State of Texas frequently audits companies doing business in Texas. Because these companies often have headquarters located outside the state, auditors must be sent to out-of-state locations. Each year, auditors must make 500 trips to cities in the Northeast, 400 trips to cities in the Midwest, 300 trips to cities in the West, and 400 trips to cities in the South. Texas is considering basing auditors in Chicago, New York, Atlanta, and Los Angeles. The annual cost of basing auditors in any city is $100,000. The cost of sending an auditor from any of these cities to a given region of the country is given in the file P06_71.xlsx. Determine how to minimize the annual cost of conducting out-of-state audits.

From\To	Northeast	Midwest	West	South
NY	$1,100	$1,400	$1,900	$1,400
Chicago	$1,200	$1,000	$1,500	$1,200
LA	$1,900	$1,700	$1,100	$1,400
Atlanta	$1,300	$1,300	$1,500	$1,050

P06_71.xlsx

Discussion: -

This is like Easting house problem. We must decide on the city, whether to use it as a base for auditor or not and decide on the regions covered by auditor based of a particular city.

Mathematical Model: -

Parameters (Inputs):

$i \in 1,2,3,4$ (i: Index for cities {NY, Chicago, LA, Atlanta})
$j \in 1,2,3,4$ (i: Index for regions {northeast, midwest, west, south})
F :   Annual cost for basing auditors , $100,000
V_{ij} :   Cost for sending an auditor based out of city i to region j
D_j :   Minimum trips audiotrs must take in each region j

Decision Variables:
x_i : Decision on whether auditors are based out of city i
y_{ij} : Decision on whether auditor based out of city i travelled to region j

Objective:

$$Minimize\ total\ \ cost = (\sum_{i=1}^{4} x_i) * F + \sum_{j=1}^{4}\sum_{i=1}^{4}(y_{ij} * V_{ij} * D_j)$$

Constraints:

$$\sum_{i=1}^{4} y_{ij} \geq 1 \ ; For\ j \in \{1,2,3,4\} \qquad (1)\ Region\ constraint$$

x_i & y_{ij} $\in\{0,1\}$ (2) *Binary Constraint*

$$\sum_{j=1}^{4} y_{ij} \leq A * x_i \quad ; For\ i \in \{1,2,3,4\} \quad \text{(3) } City\ constraint\ where\ A = 4\ (Number\ of\ regions)$$

Constraint 1 will make sure that company is providing services to all the regions.
In constraint 3, if we don't multiply the decision variable with constant on R.H.S of the constraint, as it is a minimization problem, our optimal solution will make sure that there is no Annual fixed cost by consider all the binary decisions as Zero (0).

Excel Implementation: Please find the attached spreadsheet for solution.

45[RA].xlsx

Texas auditor data										Inputs	
From\To	Northeast	Midwest	West	South		Annual Cost		Annual Cost for basing Auditors	$200,000	Decision variables	
NY	$1,100	$1,400	$1,900	$1,400		$100,000		Travelling Cost	$1,810,000	Calculated Variables	
Chicago	$1,200	$1,000	$1,500	$1,200				Total Cost	$2,010,000.00	Constraints	
LA	$1,900	$1,700	$1,100	$1,400						Objective	
Atlanta	$1,300	$1,300	$1,500	$1,050							
Num of Trips	500	400	300	400							
From\To	Northeast	Midwest	West	South					Based in City		
NY	0	0	0	0	0 <=		0			0	
Chicago	1	1	0	1	3 <=		4			1	
LA	0	0	1	0	1 <=		4			1	
Atlanta	0	0	0	0	0 <=		0			0	
	1	1	1	1							
	>=	>=	>=	>=							
	1	1	1	1							
Cost	1200	1000	1100	1200							