iPhone Assembling Production[1]

One of Apple's production factories in Shenzen, China is assembling its' iPhone products: iPhone XR, iPhone XS, and iPhone XS Max. The devices are put together via two assembling processes: 1 and 2. Running process 1 for an hour costs $400 and assembles 300 units of XR, 100 units of XS and 100 units of XS Max. Running process 2 for an hour costs $100 and yields 100 units of XR and 100 units of XS. To meet customer demands in time for Black Friday in the US, at least 1000 units of XR, 500 units of XS and 300 units of XS Max must be produced daily. Determine the daily production plan that minimizes the cost of meeting Apple's daily demands.

Discussion.

This problem is straight-forward and we can mathematically try to solve for the number of hours to run process 1 and process 2 even without the solver. We see that process 2 does not assemble XS Max, but we have a minimum demand of 300. Since this needs to be produced solely by process 1, we need at least 3 hours of process 1 to meet the demand for XS Max.

At this point, because of process 1 we have 900 units of XR and 300 units of XS. XR is short of 100 units and XS is short of 200 units to meet the demand of 1000 units and 500 units, respectively.

Running either of the processes once will lead to satisfying demand of XR, hence let us first focus on satisfying demand of XS (which will consequentially lead to satisfying demand of XR). To produce 200 more units of XS, either of the processes needs to be run for 2 hours. Running process 1 for 2 hours costs 800$ and running process 2 for 2 hours costs 200$. Hence, we choose to run process 2 for 2 hours and satisfy demand of XS as well as product XR. Therefore, we need 3 hours of process 1 and 2 hours of process 2.

But this approach cannot be used for most problems as most business case scenarios are not fairly this simple and straight forward. Excel Solver can be used to solve many types of more complex situations. Let us look at how to do this.

In this problem, we first need to identify what we are trying to optimize per the problem statement. Our daily production plan must minimize cost of meeting demands. This cost is incurred while running the process. Since we have the hourly cost to run the two process, our production plan should be such that on a given day the combined cost of running the 2 processes to meet the demands of XR, XS and XS Max stays to a minimum.

We know that the objective is to minimize the total cost is which is:

---

[1] This exercise problem and related solutions were originally developed by Athira Praveen based on Practical Management Science 5th Edition. This current revision was developed by Nowed Patwary.

Total Cost = (unit cost for running process 1 * number of hours that process 1 is run) + (unit cost for running process 2 * number of hours that process 2 is run)

Here the unknowns are number of hours that process 1 is run and number of hours that process 2 is run. Hence, these will be our decision variables:

X_1: Number of hours process 1 is run
X_2: Number of hours process 2 is run

Next, we must ensure that the number of hours that process 1 and process 2 is run will assemble enough of the XR, XS, XS MAX to satisfy their respective demands. This will be part of our constraints in the model.

Column1	Process 1	Process 2
Cost/hour	$400.00	$100.00
XR Assembled/hour	300	100
XS Assembled/hour	100	100
XS MAX Assembled/hour	100	0

Model.

Parameters:
C_i : *Cost to run process* i *for 1 hour,* $i \in (1,2)$
A_{ij}: *Assemblage of product* j *from process* i, *where* $j \in (XR, XS, XS\ MAX)$
d_j : *Daily demand for product,* $j \in (XR, XS, XS\ MAX)$

Decisions:
x_i : *Number of hours to run process* i *in a given day,* $i \in (1,2)$

Objective: *Minimize total cost of running two processes*

$$min \sum_{i=1}^{2} C_i * x_i$$

Constraints:

$x_i \geq 0, i \in \{1,2\}$ (1) Process hours are non-negative
$\sum_{i=1}^{2} A_{ij} * x_i \geq \; d_j, j \in \{XR, XS, XS\ MAX\}$ (2) Demand must be satisfied for each product

Notes:

1. $\sum_{i=1}^{2} A_{ij}$ is a short hand for summing up similar terms indexed by i. Here the objective function can also be written as $C_1 * x_1 + C_2 * x_2$. Similarly, the constraint (2) can also be written out more elaborately as $A_{1j} * x_1 + A_{2j} * x_2 \geq d_j, where\ j \in \{XR, XS, XS\ MAX\}$.
2. The objective function minimizes the total cost incurred in running process 1 and 2 in a single day.
3. The constraint (2) ensures that the number of hours the processes are run to assemble for products XR, XS and XS MAX that satisfies their respective customer demand
4. Although we have two constraints above, in effect there are five constraints: two for (1) and three for (2). A more elaborate (but cumbersome) list of constraints is the following. $x_1 \geq 0, x_2 \geq 0, A_{1A} * x_1 + A_{2A} * x \geq d_A, A_{1B} * x_1 + A_{2B} * x_2 \geq d_B, A_{1C} * x_1 + A_{2C} * x_2 \geq d_C$.

Optimal Solution. The following is the solution obtained from Excel Solver.

Inputs			
	Process 1	**Process 2**	
Cost/hr	$400.00	$100.00	
Assembled XR/hr	300	100	
Assembled XS/hr	100	100	
Assembled XS MAX/hr	100	0	
Decision			
Number of hrs/day	3	2	
Objective	$1,400.00		
iPhone Assembled /day			**Constraints**
XR	1100	>=	1100
XS	500	>=	500
XS MAX	300	>=	300

The optimal solution is to run process 1 for 3 hours and run process 2 for 2 hours as discussed above and validated by the Excel solver.