Pricing [Two Companies]: NLP-CH7-Q53

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(Two Companies) Two companies are producing widgets. It costs the first company q_1^2 dollars to produce widgets and the second company $(0.75 * q_2^2)$ dollars to produce widgets. If a total of q widgets are produced, consumers will pay 300 – q dollars for each widget. If the two companies want to collude in an attempt to maximize the sum of their profits, how many widgets should each company produce? (The model for this type of problem is called a *collusive duopoly model*.)

Discussion: -

Our objective in this problem is to maximize the total profit. Profit function is nonlinear because it includes a squared cost term. Generally, when we are using software there is a chance that solver can get struck at a local optimum and never find the global optimum. To overcome this, we must define our objective as convex or concave functions depending the problem we are solving. As it is a maximization problem, we must make sure our objective function is concave, and if there is any constraint make sure that it is linear. In similar way, we will define convex function for minimization problem.

Commented [YW1]: You can mention that this is the mechanism used by OPEC

Mathematical Model: -

Parameters (Inputs):

$i \in 1,2, \qquad (i: Index\ for\ companies)$

$q = q_1 + q_2 \quad (Total\ number\ of\ widgets\ produced)$
$P(q) : 300 - q \quad (Amount\ paid\ by\ consumer\ for\ each\ widget)$

Decision Variables:
$q_i : Widgets\ produced\ by\ company\ i$

Objective:

$$Maximize\ total\ profit = \left[\sum_{i=1}^{2} q_i * P(q)\right] - [\,q_1^2 + (0.75 * q_2^2)\,]$$

Constraints:

No Constraints

50[RA].xlsx

Excel Implementation: Please find the attached spreadsheet for solution.

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	Company 1	Company 2	Total		Inputs
Widgets produced	45	60	105		Decision variables
					Calculated Variables
Customers will pay	195				Constraints
Revenue	8775	11700	**20475**		Objective
Costs	2025	2700	**4725**		
		Profit	**15750**		