**Blending.** Capsule Drugs manufactures two drugs. The drugs are produced by blending two chemicals. By weight, drug 1 must contain at least 75% chemical 1, and drug 2 must contain at least 45% chemical 1. Drug 1 sells for $10 per ounce, and drug 2 sells for $7 per ounce. Chemicals 1 and 2 can be produced by one of two production processes. Running process 1 for an hour requires 8 ounces of raw material and 3 hours of skilled labor, and it yields 4 ounces of each chemical. Running process 2 for an hour requires 6 ounces of raw material and 2 hours of skilled labor, and it yields 5 ounces of chemical 1 and 2 ounces of chemical 2. A total of 2000 hours of skilled labor and 6000 ounces of raw material are available. Determine how to maximize Capsule's sales revenues.
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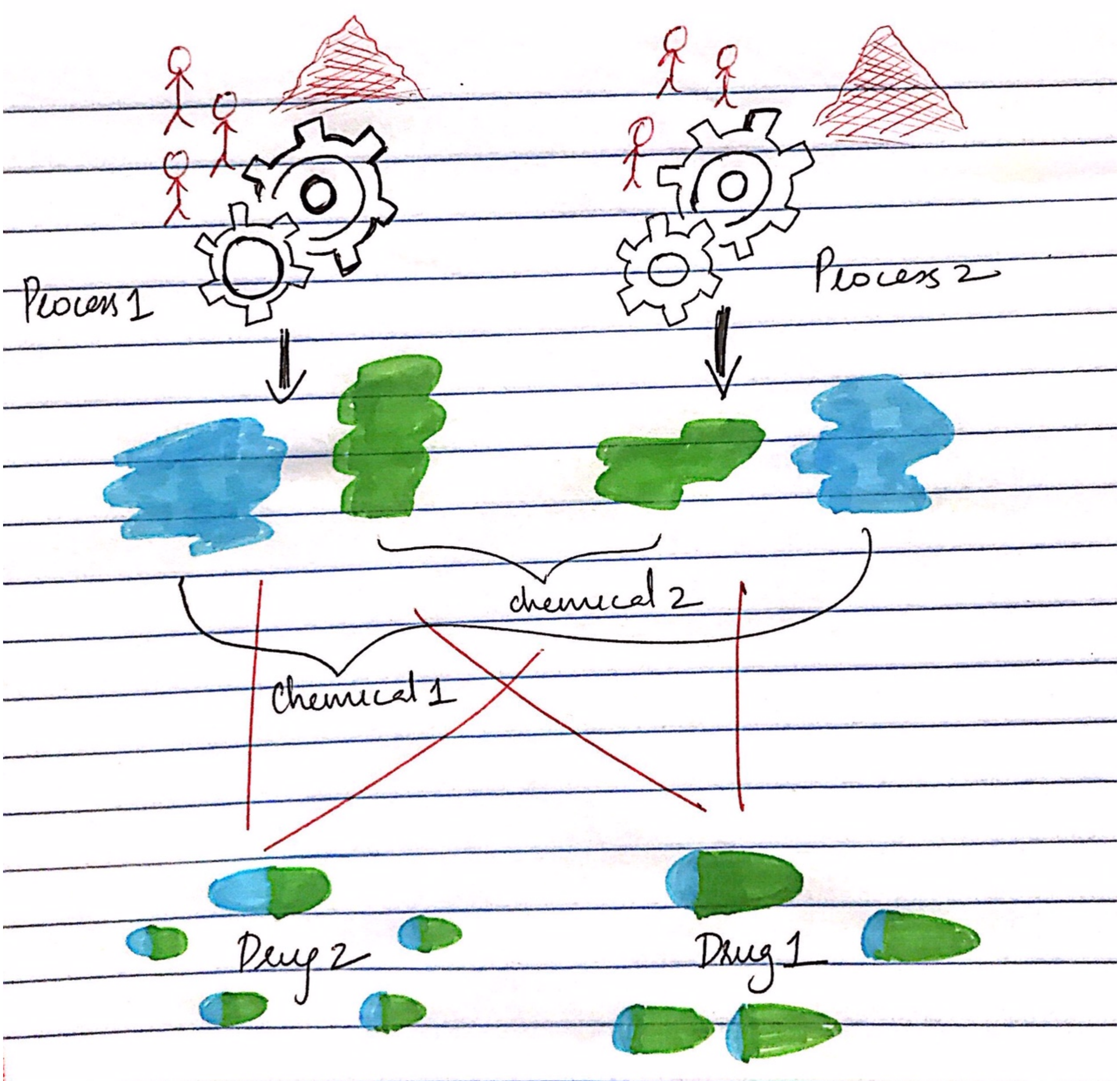
**Discussion.**

This is an example of a blending problem where a particular product is a mix of two or more materials. Here, there are 2 types of drug, each made from a mixture of 2 chemicals. We have the requirement that at least a certain fraction of each drug of must be composed of a specific chemical. Instead of directly giving us the availability of each of the chemical, here we are given details of how the chemicals can be produced, i.e. by running processes 1 and 2. Since we have a limit of the labor and raw materials available to run these processes, we can determine what is the maximum amount of each type of chemical that can be produced given these constraints. Since the question does not specify any cost associated with running these processes, we can simply aim to maximize the production of these chemicals, which in turn would maximize the amount of drugs that can be produced from these chemicals, thereby maximizing the revenue from selling these drugs, which is our main objective. Hence our decisions will include the number of hours to run process 1 and 2 to produce the chemicals so that the above can be achieved. We must also decide amount of a chemical type that must be contained in each drug so that the required proportions mentioned in the question are satisfied. Once a chemical is allocated to both the drugs, we must ensure that the total amount of this type of chemical present in both the drugs does not exceed the initial availability of this chemical produced by processes 1 and 2.




**Model.**

Parameters:

$R_i$ : *Raw material required for running process* $i$ *for 1 hour, where* $i \in (1,2)$

$L_i$: *Labor required for running process* $i$ *for* 1 *hour, where* $i \in (1,2)$

$A_{ci}$ : *Amount of chemical* $c$ *produced by process* $i$ *when run for* 1 *hour, where* $i \in (1,2)$, $c \in (1,2)$

$P_y$ : *Selling price of drug* $y$*, where* $y \in (1,2)$

*L: Labor availability*

*R: Raw material availability*

$M_{cy}$ : *minimum fraction of chemical* $c$ *to be present in drug* $y$*, where* $y \in (1,2), c \in (1,2)$

Decisions:

$x_i$: *Number of hours to run process i , where* $i \in (1,2)$

$z_{cy}$ : *Amount of chemical c to be used in drug y, where* $y \in (1,2), , c \in (1,2)$

Calculated Parameters:

$P_c$: *Amount of chemical c produced , where* $c \in (1,2)$

$P_c = \sum_i A_{ci} * x_i$

$N_y$ : *Ounces of drug y, where* $y \in (1,2)$

$N_y = \sum_c z_{cy}$

Objective: *Maximize Revenue*

$max \sum_{y \in (1,2)} N_y * P_y$

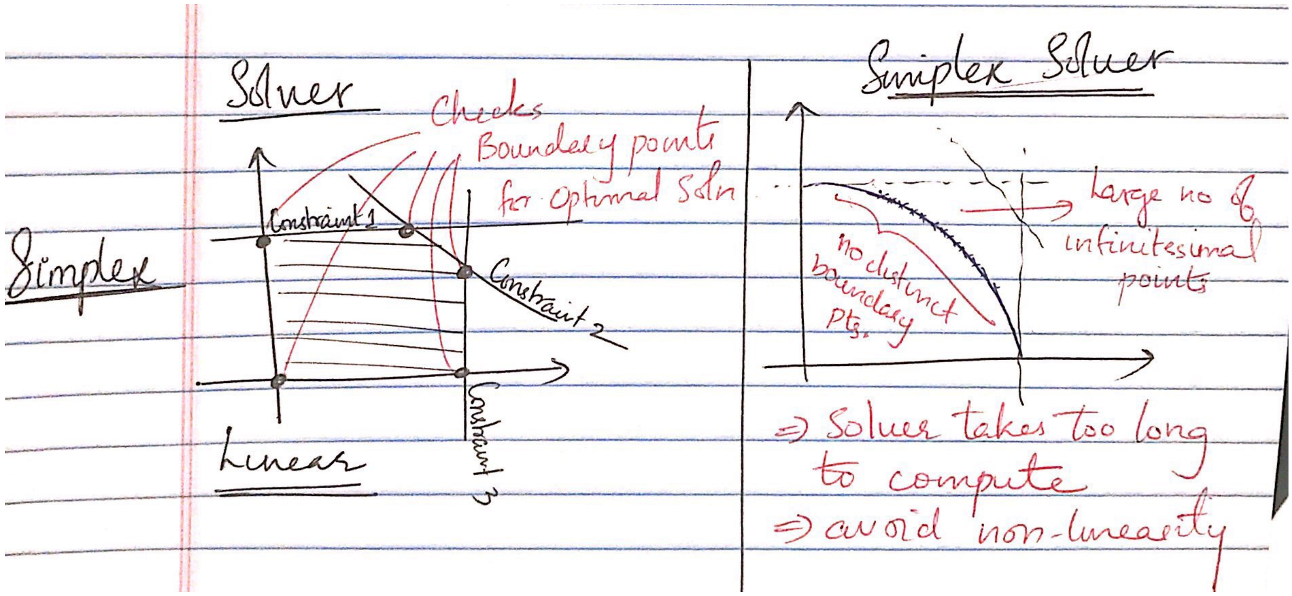
Constraints:

| | |
|---|---|
| $\sum_i x_i * R_i \leq R$ | (1) Raw material availability |
| $\sum_i x_i * L_i \leq L$ | (2) Labor availability |
| $z_{cy} \geq M_{cy} * N_y$ | (3) Chemical proportions required in each drug |
| $\sum_y z_{cy} \leq P_c$ | (4) Availability of Chemical $c$ |
| $x_i, z_{cy} \geq 0$ | (5) Non- negative hours and ounces to be allocated |

Notes:

1) Constraint (1) and (2) ensures that the number of hours process 1 and 2 is run takes into account the available raw material and labor required to run the processes
2) Constraint (4) ensures that the total amount of a particular chemical distributed among both the drugs do not exceed the availability of the chemical, which is produced by running the processes 1 and 2.
3) Constraint (3) ensures that units of each drug type has the required percentage of chemicals mentioned. It is important to denote this equation in a linear format, i.e., a decision variable cannot be present in a denominator of the fraction. It is natural to write this constraint as $\frac{z_{cy}}{N_y} \geq M_{cy}$ ,which is a non-linear representation, since a decision variable is present in the denominator. To convert this to the linear format, simply move this denominator to the right-hand side of the equation to get constraint (3). Below the implications of a linear and non-linear representations when using simplex solver is illustrated. It is important to note

that the simplex solver computes the optimal solution by checking only the boundary points of the feasible region.




**Optimal Solution.** The following is the solution obtained from Excel Solver.

22 (AP).xlsx

A maximum revenue of $75000 can be attained by allocating each of the checmicals to each of the drugs as follows, and running processes 1 and 2 as shown below.

| **Decision** | | |
|---|---|---|
| materials to be allocated to each candy | Chemical 1 | Chemical 2 |
| Drug 1 | 1250 | 416.666667 |
| Drug 2 | 3750 | 4583.33333 |
| Total amt of material | 5000 | 5000 |
| | Process 1 | Process 2 |
| Number of hrs to run | 0 | 1000 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Inputs | | | | | | |
| | Chemical 1 | Chemical 2 | | | Process 1 | Process 2 |
| Available onces of material | 5000 | 5000 | | Labor | 3 | 2 |
| | constraint | constraint | | Raw material | 8 | 6 |
| Selling Price | | | | Chemical 1 | 4 | 5 |
| Drug 1 | 10 | | | Chemical 2 | 4 | 5 |
| Drug 2 | 7 | | | | | |
| | | | | | | |
| Minimum fraction of material needed | Chemical 1 | Chemical 2 | | | | |
| Drug 1 | 0.75 | 0 | | | | |
| Drug 2 | 0.45 | 0 | | | | |
| | | | | | | |
| | | | | | | |
| Availibility | | | | | | |
| Labor | 2000 | | 0 | >constraint | | |
| Raw material | 6000 | | 0 | >constraint | | |
| | | | | | | |
| | | | | | | |
| **Decision** | | | | | | |
| materials to be allocated to each candy | Chemical 1 | Chemical 2 | total amt of drug type | Minimum proportion of chemical 1 | Minimum proportion of chemical 2 | |
| Drug 1 | 1250 | 416.666667 | 1666.666667 | 1250 | 0 | |
| Drug 2 | 3750 | 4583.33333 | 8333.333333 | 3750 | 0 | |
| Total amt of material | 5000 | 5000 | | constraint | constraint | |
| | Process 1 | Process 2 | | | | |
| Number of hrs to run | 0 | 1000 | | | | |
| | | | | | | |
| | | | | | | |
| **Objective** | 75000 | | | | | |
| | | | | | | |
| Raw material used | 6000 | < | 6000 | | | |
| Labour used | 2000 | < | 2000 | | | |