(Furniture) A furniture company manufactures desks and chairs. Each desk uses four units of wood, and each chair uses three units of wood. A desk contributes $400 to profit, and a chair contributes $250. Marketing restrictions require that the number of chairs produced be at least twice the number of desks produced. There are 2000 units of wood available. Use Solver to maximize the company's profit.

Discussion: -

Our Objective is to design a production plan in such a way that company gets more profit. Company is producing two items here (chairs & desks). The input values given in this problem are units of wood required to produce each item and profit that company would get from each unit. In simple terms profit is the difference between the selling price and cost. We are not going in detailed about the selling price and cost as we have already got unit profit as one of our inputs.

Total Profit is the product of 'Number of Units Produced' and 'Unit Profit'. Our objective is to increase the Total Profit which we need to come up with an expression for the total profit, which depends on how many units to produce. Yet, we do not know how many units to produce. To resolve this issue, we define a variable the represents how many units to produce and call this a decision variable. Then we can 'pretend' we know how much to produce and hence will be able to express total profit.
Software tool (Excel Solver) will compute and find us the production plan which will maximize the profit.

We are having clear Inputs and decision variables, let's look at other details highlighted in the problem. It was given that company is having only 2000 units of wood available for production, which is a constraint for our production plan. Assume if we don't have this constraint, we can't give final solution as there is no boundary to the decision variables (units produced) and software will keep on computing to maximize the profit. It will throw an error stating that "The Objective cell values do not converge". In addition to above constraint, there is a marketing restriction saying that the number of chairs produced be at least twice the number of desks. If we miss the constraint to add in our mathematical model, solver still calculates and gives us the incorrect production plan depending on the inputs we have provided. For every 1 unit of raw material company gets $100 profit on desk, where as it gets only $83.33 on chairs, hence if the problem didn't describe that there is a marketing constraint, we would have suggested to go with desks only as it is giving higher profit on each unit of raw material.

Mathematical Model: -

Parameters (Inputs):

$i \in 1,2\ (\ Index\ for\ items;\ 1 = Chairs, 2 = Desks)$
$U_i : Units\ of\ wood\ used\ to\ produce\ item\ i$
$P_i : Unit\ profit\ from\ item\ i$
$U\ : Total\ units\ of\ wood\ available\ for\ production$

Decision Variables:

$x_i : Number\ of\ units\ produced\ in\ each\ item\ i$

Objective:

$$Max\ Total\ Profit = \sum_{i=1}^{2}(x_i * P_i)$$

Constraints:

$x_i \geq 0;$ (1) *Non Negative constraint*

$\sum_{i=1}^{2}(x_i * U_i) \leq U$ (2) *Wood availability constraint*

$x_1 \geq x_2$ (3) *Marketing restriction constraint*

If we don't give the non-negative constraint also solver gives us the correct solution as we are maximizing the profit. As company is having only 2000 units which can be used for production we have to make sure that sum-product of units produced, and material required to produce each unit is less than or equal to total units of raw material available. $\sum_{i=1}^{2}$ is a short hand for summing up similar terms indexed by i.

Excel Implementation:

2[RA].xlsx

Please find the attached spreadsheet for solution.

									Inputs
	Chairs	Desks							Decision variables
Units of wood used to produce	3	4							Calculated Variables
Unit profit	$ 250	$ 400							Constraints
									Objective
Units of wood available	2,000								
Marketing restrictions : Chairs produced be atleast twice the number of desks									
	Chairs	Desks							
# of Units produced	400	200							
	>=								
Marketing restrictions	400	400							
Units of wood used to produce	1,200	800	2,000	<=	2,000				
Profit from each item	$ 100,000	$ 80,000							
Total Profit	$ 180,000								

As per the optimization model company can produce 400 units of chairs and 200 units of desks to get the maximum profit.

Solver's sensitivity report performs two types of sensitivity analysis:

(1) On the co-efficient of the objective
(2) On the right sides of the constraints

Below picture gives you the sensitivity analysis. In the variable cells table, the allowable increase and allowable decrease indicate how much the coefficient of unit profit for Chairs in the objective, currently 250, could change before the optimal product mix would change. If the coefficient of Chairs stays within the allowable range, the set of values in the decision variable cells does not change at all. However, outside of these limits, the optimal mix between Chairs and Desks might change.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [3(RA).xlsx]CH3-Q26
Report Created: 1/12/2019 4:09:56 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	# of Units produced Chairs	400	0	250	50	450
C13	# of Units produced Desks	200	0	400	1E+30	66.66666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B16	Marketing restrictions >=	400	-20	0	666.6666667	1000
D17	Units of wood used to produce	2000	90	2000	1E+30	2000

Table 2 (Constraints) in above picture indicates how much these right-side constants can change before the optimal solution changes. R.H side of each constraints should be numeric constant (not formulae). Shadow price indicates the change in the objective when a right side constant changes by one unit.