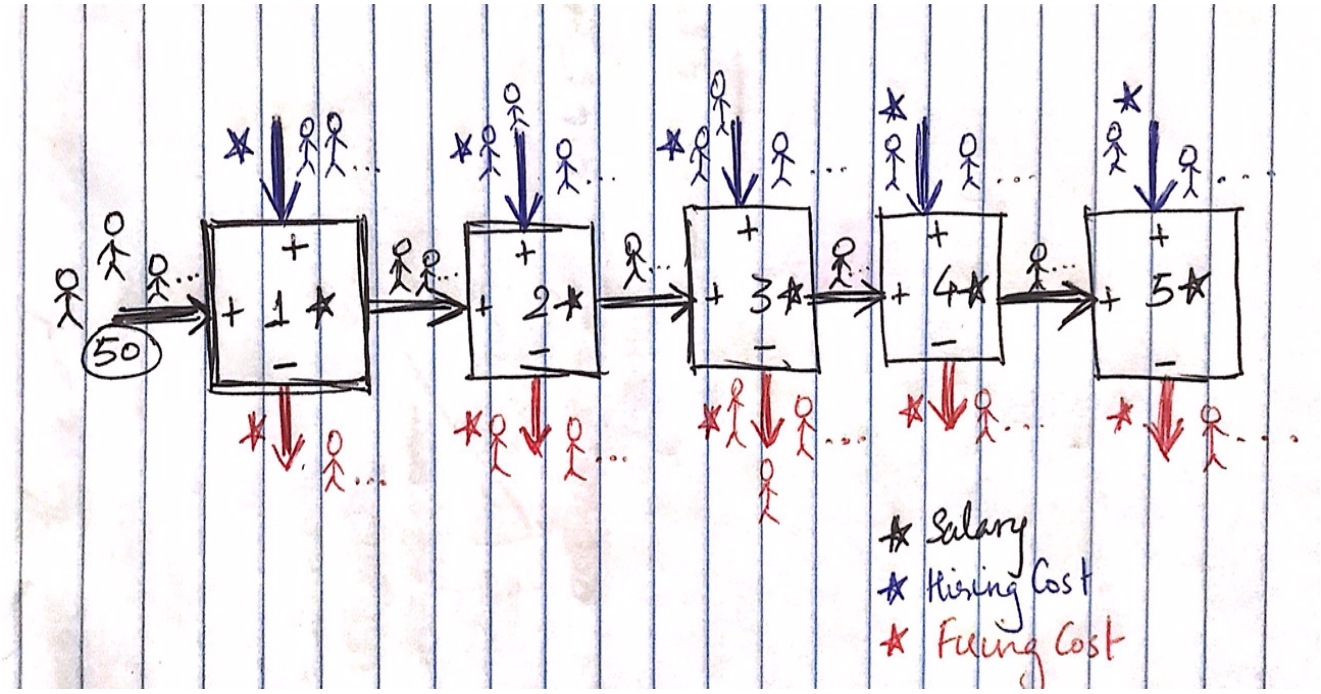
Aggregation. A bus company believes that it will need the following numbers of bus drivers during each of the next five years: 60 drivers in year 1; 70 drivers in year 2; 50 drivers in year 3; 65 drivers in year 4; and 75 drivers in year 5. At the beginning of each year, the bus company must decide how many drivers to hire or fire. It costs $4000 to hire a driver and $2000 to fire a driver. A driver's salary is $10,000 per year. At the beginning of year 1, the company has 50 drivers. A driver hired at the beginning of a year can be used to meet the current year's requirements and is paid full salary for the current year. Determine how to minimize the bus company's salary, hiring, and firing costs over the next five years.

Discussion.

Here we try to determine workforce levels in a multi period time horizon. Again, we need to use balancing equations to manage the flow of drivers throughout the years, i.e. At the beginning of each year, we have drivers from the previous year, some of whom we might decide to fire and/or hire new drivers for the current year. The effective number of workers after hiring and firing each year must satisfy each year's demand of drivers, which is the only major constraint in this problem. The objective is to minimize the total cost incurred due to paying salary to the drivers who are working in a particular year, the cost incurred due to hiring workers and the cost incurred due to firing workers.



Model.

Parameters:

D_i : Demand for drivers for year i , $where\ i \in (1,2,3,4,5)$

C_h: *Cost to hire a driver*
S : *Salary of a driver*
C_f: *Cost to fire a driver*
N_0*: Number of drivers available at the beginning of year 1*

Decisions:

x_i: *Number of drivers hired in year* i *,* $where\ i \in (1,2,3,4,5)$
y_i *: Number of drivers fired in year* i *,* $where\ i \in (1,2,3,4,5)$

Calculated Parameters:

N_i: *Number of employees working in year* i *,* $i \in (1,2,3,4,5)$
$N_i = N_{i-1} + x_i - y_i$

Objective: *Minimize Cost*
$min \sum_{i=1}^{5} N_i * S + x_i * C_h + y_i * C_f$

Constraints:

$N_i \geq D_i$	(1)Demand for drivers must be satisfied for each year
$x_i \geq 0$	(2) Number of drivers hired must be non-negative
$y_i \geq 0$	(3) Number of drivers fired must be non-negative

Optimal Solution. The following is the solution obtained from Excel Solver.


13(AP).xlsx

A minimum cost of $3,420,000 can be attained by hiring and firing workers over the 5 years as shown below.

Decision		Year 1	Year 2	Year 3	Year 4	Year 5
Number of drivers to hire		10	10	0	15	10
Number of drivers to fire		0	0	20	0	0

Inputs						
Cost to Hire	$4,000.00					
Cost to Fire	$2,000.00					
Salary	$10,000.00					
Drivers at the begining of year 1	50					
		Year 1	Year 2	Year 3	Year 4	Year 5
Demand for Drivers		60	70	50	65	75
Decision						
Number of drivers to hire		10	10	0	15	10
Number of drivers to fire		0	0	20	0	0
Number of drivers working		60	70	50	65	75
Cost due to Payroll		$600,000.00	$700,000.00	$500,000.00	$650,000.00	$750,000.00
Hiring Cost		$40,000.00	$40,000.00	$0.00	$60,000.00	$40,000.00
Firing Cost		$0.00	$0.00	$40,000.00	$0.00	$0.00
Total cost each year		$640,000.00	$740,000.00	$540,000.00	$710,000.00	$790,000.00
Objective						
Total cost	$3,420,000.00					