Networking. Sunco Oil produces oil at two wells. Well 1 can produce up to 150,000 barrels per day, and well 2 can produce up to 200,000 barrels per day. It is possible to ship oil directly from the wells to Sunco's customers in Los Angeles and New York. Alternatively, Sunco could transport oil to the ports of Mobile and Galveston and then ship it by tanker to New York or Los Angeles. Los Angeles requires 160,000 barrels per day, and New York requires 140,000 barrels per day. The costs of shipping 1000 barrels between various locations are shown in the file below, where a large number indicates shipments that are not allowed (The reasons for using a large number here will be explained in the solution). Determine how to minimize the transport costs in meeting the oil demands of Los Angeles and New York.

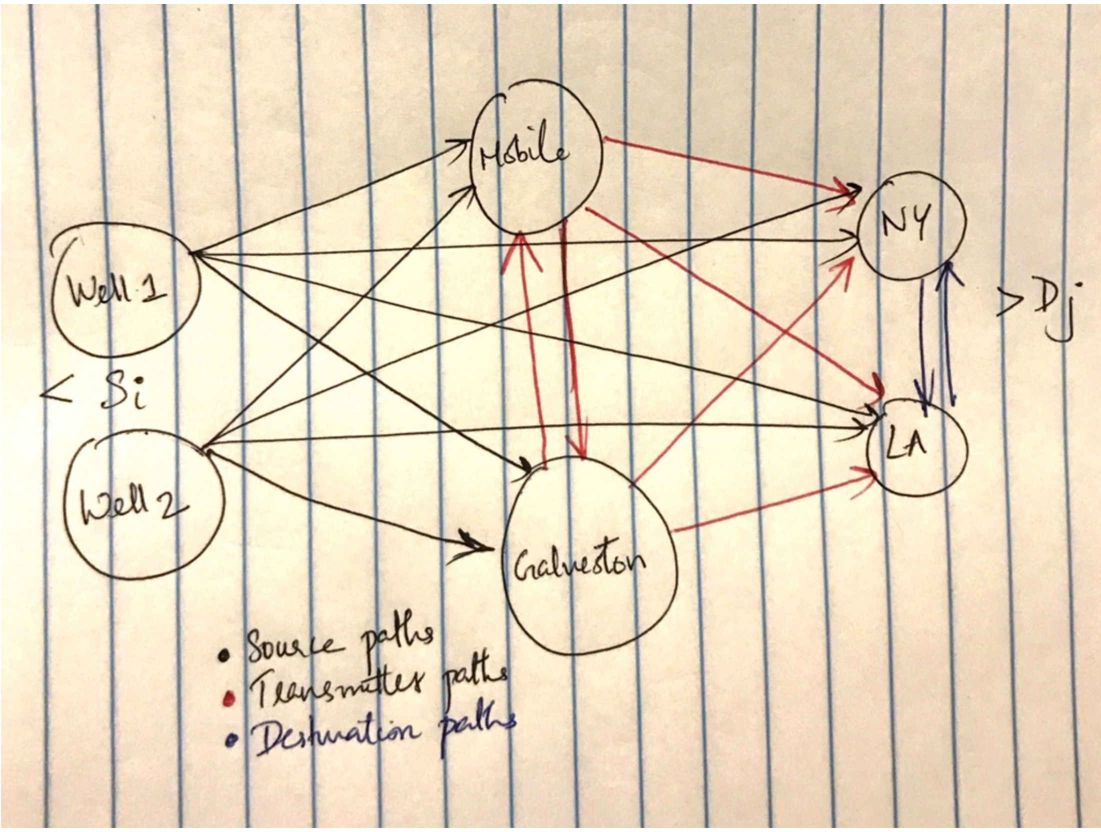
Discussion.

This is an example of a networking problem where shipments are transferred from one location to another. Let us call each of these locations as nodes. Typically, in any networking problem there are source nodes, transmitter/intermediate nodes and destination nodes. The function of any source node is to supply the shipments, hence the net flow (outflow-inflow) for a source node must be positive. the intermediate acts as temporary stations for these shipments enroute from the source to the destination. Hence all shipments flowing into an intermediate node must eventually flow out, i.e. net flow of the intermediate node must be 0. The destination nodes are the final destinations of the shipments, it accumulates the shipments from all other nodes. Hence the net flow (outflow-inflow) for a destination node must be negative and this net flow must satisfy the demands of the destination node. These requirements are specified in the constraints. The decision here is obviously how many units must be shipped through a particular route. The objective is straightforward and is to minimize the total distance that the students must be bussed.

In the excel there are certain routes that are not allowed, to avoid allocating shipments through these routes we can take either of the 2 approaches shown below.

Approach 1: Place very large numbers like 100000 in the cost matrix. Since this is a minimization problem, the routes having these large values will automatically be avoided by the solver.

Approach 1: We can force decisions to be taken on only those combinations of sources and destinations that are valid. For this, we create a list of valid combinations. This approach is more scalable and requires less storage.



Model.

Approach 1:

Parameters:

S_i : *Capacity of source* i, *where* $i \in (Well\ 1,\ Well\ 2)$

D_j : *Demand from destination* j , *where* $j \in (NY, LA)$

C_{ij} : *Cost of shipping one barrel* $from\ source\ s\ to\ destination\ k,\ where\ s, k, \in$ $(\ Well\ 1,\ Well\ 2,\ Mobile, Galveston, NY, LA)$.

Note: the cost cells in the excel having very large numbers (1000000) were previously blank indicating shipments in that path are not allowed

Decisions:

x_{sk} : *Number of barrels to be shipped from source* s *to destination* $k,\ where\ s, k, \in$ $(\ Well\ 1,\ Well\ 2,\ Mobile, Galveston, NY, LA)$.

Objective: *Minimize Cost*

$min \sum_{sk}\ x_{sk}\ *\ C_{sk}$

Constraints:

$\sum_k x_{ik} \leq S_i$ (1) Source capacity

$\sum_s x_{sk} = \sum_d x_{kd}$ (2) Inflow=Outflow for any transmitter node

Note: Here d denotes any destination node and k denotes any transmitter node. $k, \in$ (Mobile, alveston)

$\sum_s x_{sj} - \sum_k x_{jk} \geq D_j$ (4) Demand of destination must be satisfied

Note: s denotes all nodes except the final destinations LA, NY

$x_{sk} \geq 0$ (5) Non-negative shipments

Notes:

1) Constraint (1) ensures that the number of barrels shipped from the sources, Well 1 and Well 2 do not exceed its capacity
2) Constraint (2) ensures that all the barrels coming into an intermediate or transmitting node like Mobile, Galveston must flow out of that node
3) Constraint (4) ensures that demand of the destinations is satisfied. Also note that because shipment is allowed between the final destinations

Approach 2:

Model.

Parameters:

S_i : *Capacity of source* $i,\ where\ i \in (Well\ 1,\ \ Well\ 2)$

D_j : *Demand from destination* $j\ ,\ where\ j \in (NY, LA)$

C_{ij} : *Cost of shipping one barrel* $from\ source\ s\ to\ destination\ k,\ where\ s, k, \in (\ Well\ 1,\ \ Well\ 2,\ \ Mobile, Galveston, NY, LA)$.

Decisions:

x_{sk} : *Number of barrels to be shipped from source* $s\ to\ destination\ k,\ where\ s, k, \in L, list\ of\ all\ valid\ combination\ of\ sources\ and\ destinations$.

Objective: *Minimize Cost*

$$min \sum_{sk} \; x_{sk} \; * \; C_{sk}$$

Constraints:

$\sum_k x_{ik} \; \leq \; S_i$ (1) Source capacity

$\sum_s x_{sk} \; = \sum_d x_{kd}$ (2) Inflow=Outflow for any transmitter node

Note: Here d denotes any destination node and k denotes any transmitter node. $k, \in$ (Mobile, alveston)

$\sum_s x_{sj} \; - \sum_k x_{jk} \; \geq D_j$ (4) Demand of destination must be satisfied

Note: s denotes all nodes except the final destinations LA, NY

$x_{sk} \; \geq 0$ (5) Non-negative shipments

Optimal Solution. The following is the solution obtained from Excel Solver.

29(AP).xlsx

Approach 1:

A minimum transportation cost of 1124800 can be attained by distributing the oil barrels as shown below.

Amount of barrels to be transported							
		To					
		Well 1	Well 2	Mobile	Galveston	NY	LA
From	Well 1	0	0	0	0	100000	0
	Well 2	0	0	0	0	40000	160000
	Mobile	0	0	0	0	0	0
	Galveston	0	0	0	0	0	0
	NY	0	0	0	0	0	0
	LA	0	0	0	0	0	0
	total in destination	0	0	0	0	140000	160000

Sunco Oil data										
Unit shipping costs										
		To								
		Well 1	Well 2	Mobile	Galveston	NY	LA			
From	Well 1	$1,000,000.00	$1,000,000.00	$3.53	$3.42	$3.92	$3.71			
	Well 2	$1,000,000.00	$1,000,000.00	$3.65	$3.58	$3.84	$3.62			
	Mobile	$1,000,000.00	$1,000,000.00	$1,000,000.00	$1.20	$3.68	$3.74			
	Galveston	$1,000,000.00	$1,000,000.00	$1.50	$1,000,000.00	$3.57	$3.65			
	NY	$1,000,000.00	$1,000,000.00	$1,000,000.00	$1,000,000.00	$1,000,000.00	$4.12			
	LA	$1,000,000.00	$1,000,000.00	$1,000,000.00	$1,000,000.00	$3.98	$1,000,000.00			
Amount of barrels to be transported										
		To								
		Well 1	Well 2	Mobile	Galveston	NY	LA	total from source		constraint
From	Well 1	0	0	0	0	100000	0	100000	<=	150000
	Well 2	0	0	0	0	40000	160000	200000	<=	200000
	Mobile	0	0	0	0	0	0	0		
	Galveston	0	0	0	0	0	0	0		
	NY	0	0	0	0	0	0	0	<=	$140,000.00
	LA	0	0	0	0	0	0	0	<=	$160,000.00
	total in destination	0	0	0	0	140000	160000			
	constraint			0	0	>=	>=			
						140000	160000			
objective	1124800									

Approach 2:

The similar solution was obtained from the second approach as well as shown below

Amount of barrels to be transported					
	From	To	Cost	Decision	
	Well 1	Mobile	3.53	0	
	Well 1	Galveston	3.42	0	
	Well 1	NY	3.92	100000	
	Well 1	LA	3.71	0	
	Well 2	Mobile	3.65	0	
	Well 2	Galveston	3.58	0	
	Well 2	NY	3.84	60000	
	Well 2	LA	3.62	140000	
	Mobile	Galveston	1.2	0	
	Mobile	NY	3.68	0	
	Mobile	LA	3.74	0	
	Galveston	Mobile	1.5	0	
	Galveston	NY	3.57	0	
	Galveston	LA	3.65	0	
	LA	NY	3.98	0	
	NY	LA	4.12	0	