(Bus 99) Bus 99 serves towns 1 through 10. We assume that town k is $|k-j|$ miles from town j. The numbers of people in the towns who want to take the bus each hour are listed in the file P08_29.xlsx. Bus 99 will make two stops and anyone who wants to take the bus will walk to the closest bus stop. If the goal is to minimize the total distance people walk, where should the bus stop?

Town	Customers
1	43
2	84
3	71
4	57
5	39
6	13
7	90
8	35
9	54
10	44


P08_29.xlsx

Discussion: -

Our problem is to identify two towns to build bus stops with minimum total distance that people must travel to bus stop. So, our decision variable should be whether to build bus stop in town or not. To calculate the total distance, we must assign each town to the bus stop. We must create a calculated variable and create Distance matrix D_{ij}.

Distance	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	0	1	2	3	4	5	6	7	8	9
2	1	0	1	2	3	4	5	6	7	8
3	2	1	0	1	2	3	4	5	6	7
4	3	2	1	0	1	2	3	4	5	6
5	4	3	2	1	0	1	2	3	4	5
6	5	4	3	2	1	0	1	2	3	4
7	6	5	4	3	2	1	0	1	2	3
8	7	6	5	4	3	2	1	0	1	2
9	8	7	6	5	4	3	2	1	0	1
10	9	8	7	6	5	4	3	2	1	0

Mathematical Model: -

Parameters (Inputs):

$i, j \in 1,2,3,..n$ $\quad$ (i: *Index for towns*)
V_i : *Number of customers from town* i

Decision Variables:

z_j : *Decision to pick town* j *as bus stop*
e_{ij} : *Whether town* $'i'$ *is assigned to bus stop* $'j'$

Calculated Variables:

D_{ij} : $Distance\ from\ town\ i\ to\ j$

Objective:

$$Minimize\ total\ \ distance = \sum_{j=1}^{9}\sum_{i=1}^{9}(V_i * e_{ij} * D_{ij})$$

Constraints:

$z_j\ , e_{ij}\ \ \epsilon\{0,1\}$ (1) $Binary\ constraint$

$\sum_{j=1}^{9} z_j \leq 2\ ;$ (2) $Number\ of\ bus\ stops\ constraint$

$\sum_{j=1}^{9} e_{ij} \geq 1\ ;\quad for\ i\ \epsilon\{1,2,3,.n\}$ (3) $Atleast\ one\ bus\ stop\ is\ assigned\ to\ each\ town\ constraint$

$\sum_{i=1}^{9} e_{ij} \leq n * z_j\ ;\quad for\ j\ \epsilon\{1,2,3,.n\}$ (4) $Assigning\ constraint\ for\ bus\ stop$

Constraint 2 will make sure that our optimal solution will consider only two towns as bus stop. Constraint 3 will make sure that at least one bus stop is assigned to each town. Constraint 4 will bound the number of towns assigned to each bus stop.

Excel Implementation: Please find the attached spreadsheet for solution.

59[RA].xlsx

Routing [Bus 99]: NLP-CH8-Q29

Prepared By: Ramesh Alla

Bus problem data

Town	Customers
1	43
2	84
3	71
4	57
5	39
6	13
7	90
8	35
9	54
10	44

Distance	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
1	0	1	2	3	4	5	6	7	8	9
2	1	0	1	2	3	4	5	6	7	8
3	2	1	0	1	2	3	4	5	6	7
4	3	2	1	0	1	2	3	4	5	6
5	4	3	2	1	0	1	2	3	4	5
6	5	4	3	2	1	0	1	2	3	4
7	6	5	4	3	2	1	0	1	2	3
8	7	6	5	4	3	2	1	0	1	2
9	8	7	6	5	4	3	2	1	0	1
10	9	8	7	6	5	4	3	2	1	0

Bus Stop (Yes/No)	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***			
	0	0	1	0	0	0	0	1	0	0	2	<=	2

Town->Bus Stop	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***			
1	0	0	1	0	0	0	0	0	0	0	1	>=	1
2	0	0	1	0	0	0	0	0	0	0	1	>=	1
3	0	0	1	0	0	0	0	0	0	0	1	>=	1
4	0	0	1	0	0	0	0	0	0	0	1	>=	1
5	0	0	1	0	0	0	0	0	0	0	1	>=	1
6	0	0	0	0	0	0	0	1	0	0	1	>=	1
7	0	0	0	0	0	0	0	1	0	0	1	>=	1
8	0	0	0	0	0	0	0	1	0	0	1	>=	1
9	0	0	0	0	0	0	0	1	0	0	1	>=	1
10	0	0	0	0	0	0	0	1	0	0	1	>=	1
	0	0	5	0	0	0	0	5	0	0			
	<=	<=	<=	<=	<=	<=	<=	<=	<=	<=			
	0	0	10	0	0	0	0	10	0	0			

1	86
2	84
3	0
4	57
5	78
6	26
7	90
8	0
9	54
10	88
	563

Inputs
Decision variables
Calculated Variables
Constraints
Objective