48. **Integer Programming.**

Based on Sonderman and Abrahamson (1985). In treating a brain tumor with radiation, physicians want the maximum amount of radiation possible to bombard the tissue containing the tumors. The constraint is, however, that there is a maximum amount of radiation that normal tissue can handle without suffering tissue damage. Physicians must therefore decide how to aim the radiation to maximize the radiation that hits the tumor tissue subject to the constraint of not damaging the normal tissue. As a simple example of this situation, suppose six types of radiation beams (beams differ in where they are aimed and their intensity) can be aimed at a tumor. The region containing the tumor has been divided into six regions: three regions contain tumors and three contain normal tis- sue. The amount of radiation delivered to each region by each type of beam is shown below. If each region of normal tissue can handle at most 60 units of radiation, which beams should be used to maximize the total amount of radiation received by the tumors?

Beam	Normal 1	Normal 2	Normal 3	Tumor 1	Tumor 2	Tumor 3
1	24	18	12	30	18	9
2	18	15	9	27	23	12
3	14	12	20	20	15	26
4	6	18	18	9	27	24
5	14	6	17	20	8	21
6	12	11	11	15	15	15

Discussion.

This is an example of an integer programming model. The basic objective if fairly simple, i.e. to maximize the radiation intensity. We must decide choosing which of the beams will help us achieve this without destroying the normal cells. In this case, we only need to decide which beams to select, i.e. a binary decision. The amount of intensity at each area cannot be controlled by us and is predefined in the question. We need to add constraints so that the intensity at normal areas due to the beams selected do not exceed its bearable value.

Model.

Parameters:

R_{ij} : *Radiation received in area* i *by beam* j*, where* $i \in (Normal\ 1, Normal\ 2, Normal\ 3, Tumour\ 1, Tumour\ 2, Tumour\ 3)$, $j \in (1,2,3,4,5,6)$

C *: Maximum beam intensity that can be tolerated by normal areas*

Decisions:

x_j: *Whether beam* j *must be used, where* $j \in (1,2,3,4,5,6)$

Objective: *Maximize beam intensity*

$max \sum_{i \in \{Tumor1,2,3\},j} \; x_j * r_{ij}$

Constraints:

$\sum_j r_{ij} \, * \, x_j \; \leq \; C \quad , where \; i \in (Normal\;1, Normal\;2, Normal\;3)$ (1) Maximum intensity for normal areas

$x_j \in \{0,1\}$ (2) Binary decision

Notes:

Optimal Solution. The following is the solution obtained from Excel Solver.

39(AP).xlsx

A maximum radiation intensity of 436.9 can be achieved by choosing the beams as shown below.

Beam	Normal 1	Normal 2	Normal 3	Tumor 1	Tumor 2	Tumor 3	Decision	
1	24	18	12	30	18	9	0	
2	18	15	9	27	23	12	1	
3	14	12	20	20	15	26	1	
4	6	18	18	9	27	24	1	
5	14	6	17	20	8	21	0	
6	12	11	11	15	15	15	1	
Total intensity in area	50	56	58	71	80	77		
	<=	<=	<=					
	60	60	60					
Objective	228							