

Bill Carey¹

Bill Carey is planning to run in the annual ASU Pi Day 5k Run. To get stronger and fitter to be able to run in the event, Bill has decided to start taking protein shakes to gain lean muscle mass. Bill is trying to decide between two protein powders to help him increase muscle mass. For the first protein powder (chocolate flavor), one scoop weighs 37 grams, contains 120 calories and 5 grams of protein. For the second protein powder (vanilla flavor), one scoop contains 65 grams, 160 calories and 10 grams of protein. However, Bill has decided he wants no more than 450 calories and 25 grams of protein for his protein shakes during the day. He does, however, need at least 120 grams of powder per day to help him gain muscle.

Having sampled the two protein powders, he also assigned a “taste rating” to each gram of powder, where 0 is the lowest, and 100 is the highest. He rates the chocolate-flavored protein powder a 95, and he rates the vanilla powder 85 (he likes chocolate more than vanilla). Because he found a good deal, he wants to buy both and use them together for his diet plan.

Find the optimum plan that stays within Bill’s constraints and maximizes the total taste rating of his protein shakes.

Discussion:

In this problem, Bill has two options to choose between for his daily protein powder intake (chocolate flavor or vanilla flavor). Each serving size (one scoop) has very specific parameters (weights, calories, protein, taste index), which will be the inputs. As per Bill’s plan, he wants to take 120 grams of protein powder in a day. Chocolate and vanilla flavors contain 37 and 65 grams respectively. Therefore, the decision variable will be the number of scoops to calculate the total intake of protein powder. This will help to calculate the taste index as well. If we have the number of scoops Bill gets in a day, we can calculate the total taste index. As such, the decision variable should be the number of scoops, and the objective is to maximize the taste index value by deciding the number of scoops of each item.

¹ This exercise problem and related solutions were originally developed by Athira Praveen based on Practical Management Science 5th Edition. This current revision was revised by Nowed Patwary.

Mathematical Model:

Parameters (Inputs):

$i \in 1, 2$, (Index for protein powders: chocolate flavor, vanilla flavor)

W_i : Weights of protein powder i

C_i : Calories in protein powder i

P_i : Protein in protein powder i

T_i : Taste index to each gram of protein powder i

W : Per day requirement of weight

C : Per day limitations of Calories

P : Per day limitations of protein

Decision Variables:

x_i : Number of servings of protein powder i

Objective:

$$\text{Maximize Total Taste Index} = \sum_{i=1}^2 (x_i * W_i * T_i)$$

Constraints:

$$x_i \geq 0 \quad (1) \text{Non Negative constraint}$$

$$\sum_{i=1}^2 (x_i * W_i) \geq W \quad (2) \text{Per day limitations of weight constraint}$$

$$\sum_{i=1}^2 (x_i * C_i) \leq C \quad (3) \text{Per day limitations of Calories constraint}$$

$$\sum_{i=1}^2 (x_i * P_i) \leq P \quad (4) \text{Per day limitations of Protein constraint}$$

Excel Implementation:

Protein Powder flavors	Chocolate	Vanilla							Inputs
Weight (grams) of each scoop	37	65							Decision variables
Calories in each scoop	120	160							Calculated Variables
Protein in each scoop	5	10							Constraints
Taste index to each gram	95	85							Objective
	Chocolate	Vanilla							
# of scoops	1	2							
Total Weight (grams)	37	130	167	>=		120			Constraints
Total calories	120	320	440	<=		450			
Total Protein content	5	20	25	<=		25			
Taste Index	3515	11050							
Total Taste Index	14565								

As per the optimization model Bill can take 1.25 scoops of Chocolate flavor and 1.875 scoops of Vanilla flavor. As the objective is to maximize the total taste index, Solver suggests more Vanilla flavor over Chocolate. This may seem surprising, but a basic calculation will show that Vanilla has a higher gram/taste index ratio than Chocolate, and so, it would maximize the Taste Index to have more grams of Vanilla protein powder.

Realistically though, it makes more sense to use whole scoops instead of partial scoops, so we should have an integer constraint. If we put integer constraint for our decision variables, solution would be as given in screenshot 2. As per the optimization model Bill can take 1 scoop of Chocolate flavor and 2 scoops of Vanilla flavor. Without the Calories and Protein constraint, Solver would maximize for taste and disregard the Chocolate flavor powder all together; however, because of those constraints, more than 2 scoops of Vanilla protein powder would not meet the needs.

Protein Powder flavors	Chocolate	Vanilla							Inputs
Weight (grams) of each scoop	37	65							Decision variables
Calories in each scoop	120	160							Calculated Variables
Protein in each scoop	5	10							Constraints
Taste index to each gram	95	85							Objective
	Chocolate	Vanilla							
# of scoops	1.25	1.875							
Total Weight (grams)	46.25	121.875	168.125	>=		120			Constraints
Total calories	150	300	450	<=		450			
Total Protein content	6.25	18.75	25	<=		25			
Taste Index	4393.75	10359.38							
Total Taste Index	14753.13								