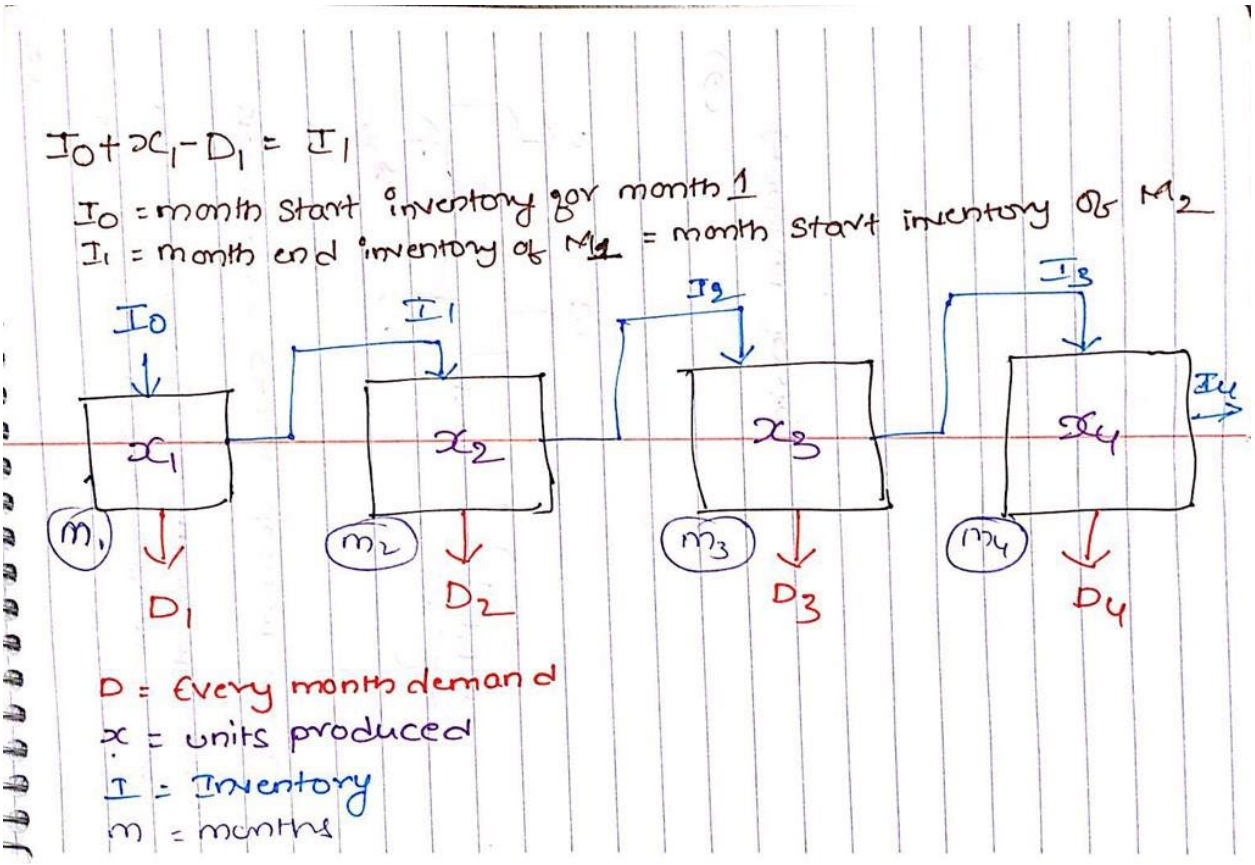
(Automobile) During the next two months, ASN automobile manufacturer must meet (on time) the following demands for trucks and cars: month 1, 400 trucks and 800 cars; month 2, 300 trucks and 300 cars. During each month at most 1000 vehicles can be produced. Each truck uses two tons of steel, and each car uses one ton of steel. During month 1, steel costs $700 per ton; during month 2, steel is projected to cost $800 per ton. At most 2500 tons of steel can be purchased each month. (Steel can be used only during the month in which it is purchased.) At the beginning of month 1, 100 trucks and 200 cars are in the inventory. At the end of each month, a holding cost of $200 per vehicle is assessed. Each car gets 35 miles per gallon (mpg), and each truck gets 15 mpg. During each month, the vehicles produced by the ASN automobiles must average at least 23 mpg. Determine how to meet the demand and mileage requirements at minimum total cost.

Discussion: -

It is similar to the earlier solved aggregate planning problems, only difference is we have two types of vehicles which needs to be produced in two different months. If you can create two indexes, one for vehicle type and one for months and place the inputs in matrix form it is easy to solve this problem.

You can refer to Multiperiod planning problems for understanding the end inventory concept. We will be using similar concept in solving this problem. In this problem, instead of one product ASN automobiles in producing two types of products (trucks & cars).



Our objective is to minimize the cost, in order to calculate the same we need the number of the vehicles produced in each month. So this will be our decision variable.

Mathematical Model: -

Parameters (Inputs):

$i \in 1,2 \quad (i: Index\ for\ type\ of\ vehicles; trucks, cars)$
$j \in 1,2 \quad (i: Index\ for\ months)$
$D_{ij} : Demand\ for\ vehicle\ i\ in\ month\ j$
$P : Maximum\ production\ in\ each\ month; 1000\ units$
$S : Maximum\ steel\ that\ can\ be\ \ purchased\ in\ a\ month; 2500\ tons$
$C_j : Cost\ to\ purchase\ one\ ton\ steel\ in\ month\ j$
$M : Avarage\ mpg\ expected\ in\ each\ month; 23\ mpg$
$S_i : Steel\ used\ in\ tons\ to\ produce\ each\ unit\ of\ vehicle\ i$
$M_i : mpg\ from\ each\ vehicle\ i$
$H : Holding\ cost\ \ for\ each\ vehicle\ ; \$200$
$I_{ij} : Ending\ Inventory\ of\ vehicle\ i\ in\ month\ j$
$I_{10} : Starting\ inventory\ of\ trucks; 100$
$I_{20} : Starting\ inventory\ of\ cars; 200$

Decision Variables:
$x_{ij} : Vehicle\ i\ produced\ in\ month\ j$

Calculated Variables:

$I_{ij} = I_{i(j-1)} + x_{ij} - D_{ij}\ \ for\ i\epsilon\{1,2\}\ \&\ j\epsilon\{1,2\}$ — $Ending\ inventory\ of\ vehicle\ i\ in\ month\ j$

$H_j = \left(\sum_{i=1}^{2} I_{ij}\right) * H\ \ for\ j\epsilon\{1,2\}$ — $Holding\ cost\ in\ month\ j$

$Q_j = \left(\sum_{i=1}^{2} x_{ij} * S_i\right) * C_j\ \ for\ j\epsilon\{1,2\}$ — $Production\ cost\ in\ month\ j$

Objective:

$$Minimize\ total\ cost = \sum_{j=1}^{2} (H_j + Q_j)$$

Constraints:

$x_{ij} \geq 0;$ — (1) $Non\ Negative\ constraint$
$I_{ij} \geq 0;$ — (2) $Ending\ Inventory\ constraint$

$$\sum_{i=1}^{2}(x_{ij} * M_i) \geq (\sum_{i=1}^{2} x_{ij}) * M \; ; \; for \; j\epsilon\{1,2\} \qquad (3)\; Average\; mpg\; constraint$$

$$\sum_{i=1}^{2} x_{ij} \leq P \; ; \; for \; j\epsilon\{1,2\} \qquad (4)\; Production\; constraint$$

$$\sum_{i=1}^{2}(x_{ij} * S_i) \leq S \; ; \; for \; j\epsilon\{1,2\} \qquad (5)\; Steel\; purchase\; constraint$$

As we need to meet the demand our solution should make sure that ending inventory is always greater than or equal to zero. During each month, the vehicles produced by the ASN automobiles must average at least 23 mpg, so we have written a Avg mpg constraint. On top of this ASN automobiles have limits to produce the vehicles and purchase raw material (steel) for their production. These constraints are listed above.

Excel Implementation: Please find the attached spreadsheet for solution.

19[RA].xlsx

ASN Automobile Manufacturer

	Month 1	Month 2
Production Cost	$980,000	$560,000
Holding Cost	$20,000	$0
Total Cost	$1,560,000	

Demand	**Month 1**	**Month 2**
Trucks	400	300
Cars	800	300
Production Constraint per month	1000	1000
Steel purchase Constraint	2500	2500
Steel Cost per ton	$700	$800
Average mpg	23	23

	Trucks	**Cars**
Steel used per vehicle in tons	2	1
Miles per gallon (mpg)	15	35
Inventory at the beginning on Month 1	100	200
Holding Cost per vehicle	$200	$200

		Month 1	Month 2
	Trucks	400	200
	Cars	600	300

		Month 1	Month 2
Starting Inventory	Trucks	100	100
	Cars	200	0

		Month 1	Month 2
Ending Inventory	Trucks	100	0
	Cars	0	0
		>=	>=
		0	0

Inputs
Decision variables
Calculated Variables
Constraints
Objective

	Month 1	Month 2
	27000	13500
	>=	>=
Avg mpg constraint	23000	11500

	Month 1	Month 2
	1000	500
Production	<=	<=
Constraint	1000	1000

	Month 1	Month 2
	1400	700
Steel purchase	<=	<=
Constraint	2500	2500