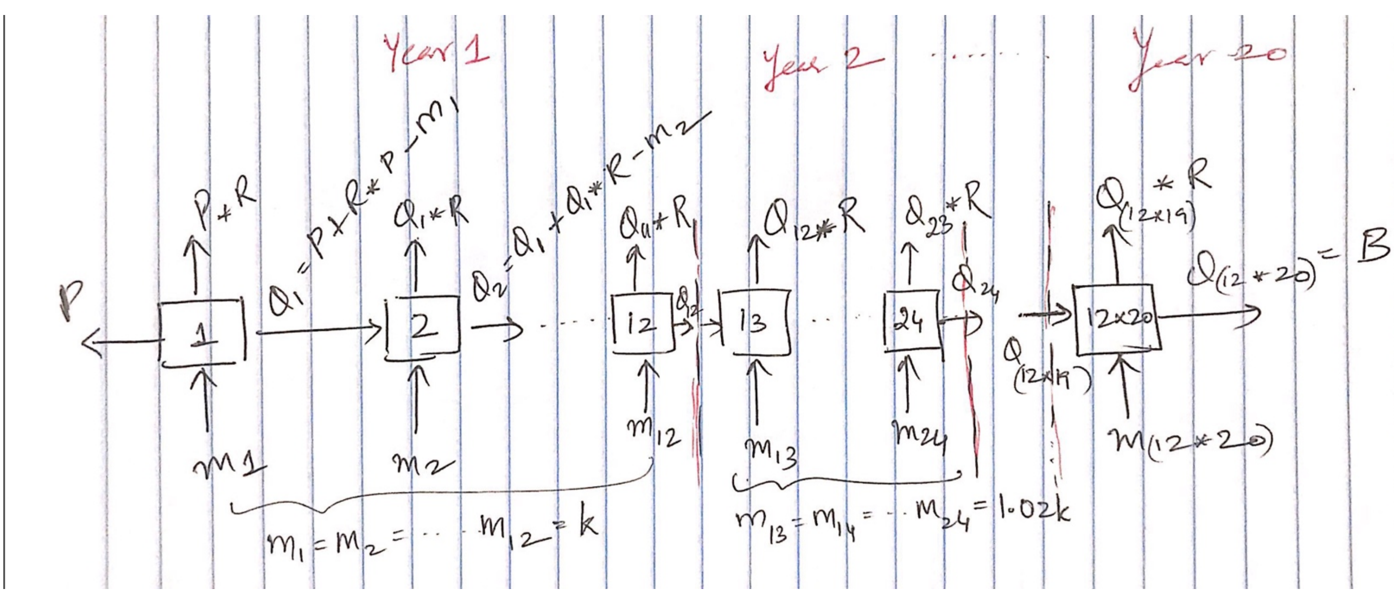
Financial. You want to take out a $450,000 loan on a 20-year mortgage with end-of month payments. The annual rate of interest is 3%. Twenty years from now, you will need to make a $50,000 ending balloon payment. Because you expect your income to increase, you want to structure the loan so at the beginning of each year, your monthly payments increase by 2%. a. Determine the amount of each year's monthly payment. You should use a lookup table to look up each year's monthly payment and to look up the year based on the month (e.g., month 13 is year 2, etc.).

Discussion.

This is an example of a financial modeling problem, where the customer wants to plan his monthly installments over the next 20 years to repay his mortgage of 450000$. Since 50,000$ is to be paid as balloon payment, i.e. payed in bulk at the end of the 20 years. The installments need to cover only the 400000$ plus the interest over the 20 years. The culture of balloon payments in mortgages helps to reduce the monthly installments that need to be paid. The balancing equation here is that of the ending balance after each monthly payment. Each monthly payment helps to cover a part of the principal and the interest incurred that month. All we have to ensure is that by the end of 20 years, we have paid off at least the principal – balloon payment amount, which acts as a constraint. Hence, there is no explicit other objective for this problem. We are also given that the monthly payments are fixed during a year and each year's monthly payment increases by 1.02. For example, for year 1, month 1: monthly payment is x, month 2: monthly payment is x……., month 12: monthly payment is x. For year 2, month 13: monthly payment is 1.02*x, month 14: monthly payment is 1.02*x……., month 24: monthly payment is 1.02*x. For year 3, month 25: monthly payment is 1.02^2*x, month 26: monthly payment is 1.02^2*x ……., month 36: monthly payment is 1.02^2*x. Hence, we need to decide only the monthly payment of year 1, as the monthly payments of subsequent years are directly dependent on this amount. In excel, we can model this using a look up table, with each month over the 20 years pointing to its appropriate installment.



Look up Table

Y_1	$m_1 = k$
	$m_2 = k$
	$m_3 = k$
	⋮
	$m_{12} = k$
Y_2	$m_{13} = 1.02k$
	$m_{14} = 1.02k$
	⋮
	$m_{24} = 1.02k$
Y_3	$m_{25} = 1.02^2 * k$
	$m_{26} = 1.02^2 * k$
	⋮
	$m_{36} = 1.02^2 * k$
Y_4	$m_{37} = 1.02^3 * k$
⋮	⋮
Y_{20}	$m_{20 \times 12} = 1.02^{20} * k$

Model.

Parameters:

L : *Actual loan amount*

B: *Balloon payment*

R: *monthly interest rate= 0.03/12*

P: *yearly increase in monthly payments* $= 1.02$

Decisions:

m_1: *Monthly payment for year 1*

Calculated Parameters:

Q_i: *Ending balance after month i, where i* $\in$ *(1,2,....20*12)*

$Q_i = Q_{i-1} - (m_i - Q_{i-1} * R)$

$Q_0 = L$

m_i: *Monthly payment for year i*

m_j= P * m_{j-1} where j $\in$ *(1,2,....20) : Each year the monthly payment increases P times. Please note monthly payments are fixed within a year.*

Objective: NONE*

Constraints:

$Q_{20*12} - B = 0$ (1) Amount remaining after balloon payment should be fully repaid after 20 years

$m_1 \geq 0$ (2) Non- negative monthly payment

Notes:

* This is a very special case where there is no objective defined. The reason here is that the problem is fully constrained: the loan must be paid off in 20 years and the balloon payment is already specified.

Constraint (2) should not be binding, as constraint (1) ensures that a positive payment is needed.

Optimal Solution. The following is the solution obtained from Excel Solver.

20 (AP).xlsx

The monthly payment for year 1 must be 1967.25$ to pay off the loan in 20 years.

Inputs			
Loan	450000		
Balloon payment	50000		
monthly interest rate	0.0025		
Decision			
monthly payment for year 1	1967.254571		
Month	Monthly pay	Ending balance	
1	1967.254571	449157.745	
2	1967.254571	448313.385	
3	1967.254571	447466.914	
240	2865.918438	50000	
	Ending balance after balloon paymenyt after	9.9851E-07	