**Ford Trucks**[1]

Ford manufactures two truck models, the F-150 and the F-250. Each truck made goes through the painting shop and the assembly shop. If the painting shop were completely devoted to painting F-150 trucks, 650 trucks could be painted per day, whereas if the painting shop were completely devoted to painting the F-250 model, 550 trucks could be painted per day. If the assembly shop were devoted to assembling F-150 engines, 1400 trucks could be assembled per day. Meanwhile, if the assembly shop were completely devoted to making F-250 engines, 1000 per day could be assembled. However, it is possible to paint both truck models in the painting shop and assemble both types in the assembly shop. Each F-150 contributes $2500 to profit while each F250 contributes $3000 to profit. Utilize Solver to maximize the company's profit.

## Discussion: -

The question tells us that there are two truck models and there are two steps involved in manufacturing them, painting and assembly. Because we know the unit profit for each Ford truck model, our objective is to maximize the profit; that is, we should decide on the numbers of each truck model that could be produced by Ford to maximize the profit. We have two models but only one painting shop and one assembly shop. Once a truck gets painted, it goes through the assembly shop. Per day production limits of painting shop and assembly shops are given, but the inputs given are the capacity of shops if they work on only one truck model-type. However, the question tells us that Ford can paint/assemble both types of trucks in these shops. As such, our real problem is to set a constraint which limits our decisions (number of trucks of each model) so that we will not overload the per day work which can be done by painting and assembly shops. This problem tests our mathematical skills. Do you remember "Work—Time" problems? Let's look at an example which might explain the concept more clearly. Suppose one painter can paint the entire house in seven hours, and the second painter could take 9 hours to paint the same house. How long would it take the two painters together to paint the house? If the first painter can do the entire job in seven hours and the second painter can do it in nine hours, then the first painter can do (1/7) of the job per hour, and the second painter can do (1/9) per hour. The question then becomes, how much then can they do per hour if they work together? Just do the summation (1/7) + (1/9). Mathematically, we can assume that "t" is the total time taken by two painters to complete the task, which means the above summation is nothing but equal to (1/t). (1/t) = (1/7) +(1/9) ---- This equation will help us find the time taken by the two painters to complete the job by working together. The important concept to understand about the above example is that the key to solving the problem was in converting how long each person took to complete the task into a rate/hour. You can use this concept while writing the per day limitations of painting and assembling.

[1] This exercise problem and related solutions were originally developed by Athira Praveen based on Practical Management Science 5th Edition. This current revision was revised by Nowed Patwary.

## Mathematical Model: -

*Parameters (Inputs):*

$i \in 1,2, (\,Index\ for\ Truck\ Model)$

$T_i$ : $Number\ of\ type\ i\ trucks\ that\ could\ be\ painted\ per\ day$

$A_i$ : $Number\ of\ type\ i\ trucks\ that\ could\ be\ assembled\ per\ day$

$P_i$ : $Profit\ expected\ from\ each\ type\ i\ truck$

*Decision Variables:*

$x_i$ : $Number\ of\ type\ i\ trucks\ manufactured$

*Objective:*

$$Minimize\ Total\ Profit = \sum_{i=1}^{2} (x_i * P_i)$$

*Constraints:*

$x_i \geq 0;$ (1) $Non\ Negative\ constraint$

$\sum_{i=1}^{2} (x_i / T_i) \leq 1$ (2) $Per\ day\ limitations\ for\ painting\ the\ trucks$

$\sum_{i=1}^{2} (x_i / A_i) \leq 1$ (3) $Per\ day\ limitations\ for\ assembling\ the\ trucks$

*Excel Implementation:*

7[RA].xlsx

Please find the attached spreadsheet for solution.

| | | | | | |
|---|---|---|---|---|---|
| | | | | | Inputs |
| | | | | | Decision variables |
| | Ford F-150 | Ford F-250 | | | Calculated Variables |
| Painting | 650 | 550 | | | Constraints |
| Assembling | 1400 | 1000 | | | Objective |
| Profit | $ 2,500 | $ 3,000 | | | |
| | | | | | |
| | | | | | |
| | Ford F-150 | Ford F-250 | | | |
| Trucks to be manufactured | 0 | 550 | | | |
| | | | | | |
| Maximize the Profit | $ 1,650,000 | | | | |
| | | | | | |
| Painting Constraint | 1 | <= | 1 | | |
| Assembling Constraint | 0.55 | <= | 1 | | |

As per the optimization model, solver suggests manufacturing 550 Ford F-250 trucks for the daily process, which would bring in a profit of $1.65 M. Even though the company could assemble 1000 F-250 trucks a day, there is a limiting constraint because of the number of trucks that could be painted per day. It is also not surprising that it recommends building zero (0) F-150 trucks; there is a higher per/unit profit on the F-250 model, and at the current painting levels for each model type, it is more profitable to make F-250 trucks. Recall previous Product Mix problems, such as the desk vs. chair problem, or the handbag vs purse model; in those problems, there was a requirement or constraint stating we needed to make a certain amount of each product (example, make 2 times more purses than handbags). Because that kind of constraint does not exist in this problem, Solver finds a solution that does not need both types of products to be made.

Extra question: If hypothetically, you could change the amount of F-150 trucks painted by the painting shop, at which point would it become more profitable to o