DEA. The Salem Board of Education wants to evaluate the efficiency of the town's four elementary schools. The three outputs of the schools are:

- Output 1 = average reading score
- Output 2 = average mathematics score
- Output 3 = average self-esteem score

The three inputs to the school are:

- Input 1 = average educational level of mothers (defined by highest grade completed: 12= high school graduate; 16= college graduate etc.)
- Input 2 = number of parent visits to school (per child)
- Input 3 = teacher-to-student ratio

The relevant information for the four schools is given in the file. Determine which (if any) schools are inefficient.

Discussion.

This is an example of a data envelopment analysis. This method can be used to determine whether an organization is operating efficiently. Here the idea is that we present each organization that we compare in the best light, i.e. the inputs and outputs are valued such that it can maximize the efficiency of the organization. Then, we compare the efficiency of each of the hospitals to each other. Hence to determine whether the school is efficient, the model must determine a value (or price per unit) for each output and a value (or cost per unit) for each input. Then the efficiency of the hospital is value of hospital's outputs divided by the value of the hospital's inputs. We must ensure that the efficiency of a hospital cannot be more than 100%. By scaling the value of inputs to 1, we provide an upper boundary for the value of outputs. i.e. the value of outputs must always be lesser than the value of inputs. We can scale the inputs to any value, scaling it to 1 simply helps to visualize the efficiency of the school as equal to the value of its outputs. Thus, we put each school in the best light and compare their efficiencies.

Model.

Parameters:

I_{ij} : *units of input i used by school j , where* $i \in (1,2,3)$, $j \in (1,2,3,4)$

O_{kj} : *units of output k from school j, where* $k \in (1,2,3)$, $j \in (1,2,3,4)$

Decisions:

x_i: *unit value of input i , where* $i \in (1, 2, 3)$

y_k : *unit value of output k , where* $k \in (1, 2, 3)$

Objective: *Maximize Output Value*

$max \sum_{k\in(1,2,3)} O_{kj} * y_k$

Note that the objective is set to a particular school j, and this has to be run individually for each school j

Constraints:

$\sum_k y_k * O_{kj} \leq \sum_i x_i * I_{ij}$ (1) Value of output cannot be more than value of input

$\sum_i x_i * I_{ij} \leq 1$ (2) Scale inputs of a school to 1

$x_i,\ y_k \geq 0$ (3) Non-negative values to inputs and outputs

Notes:

1) Constraint (1) and (2) ensures that the number of hours process 1 and 2 is run takes into account the available raw material and labor required to run the processes
2) Constraint (2) scales the value across inputs to 1, it can be scaled to any value of preference. It is scaled to 1 here so that the efficiency of the school to be equal to the value of the outputs

Optimal Solution. The following is the solution obtained from Excel Solver.

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None of the schools are inefficient since each of the hospitals produces the maximum output value possible.

Salem Board of Education									
School	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3		Output value	Input value
1	13	4	0.05	9	7	6		1	1
2	14	5	0.05	10	8	7		1	1
3	11	6	0.06	11	7	8		1	1
4	15	8	0.08	9	9	9		1	1
								>constraint	
Decision									
Value	0.066667	0	0	0.111111	0	0			
	> None of the schools are inefficient since none of the output values are less than 1								