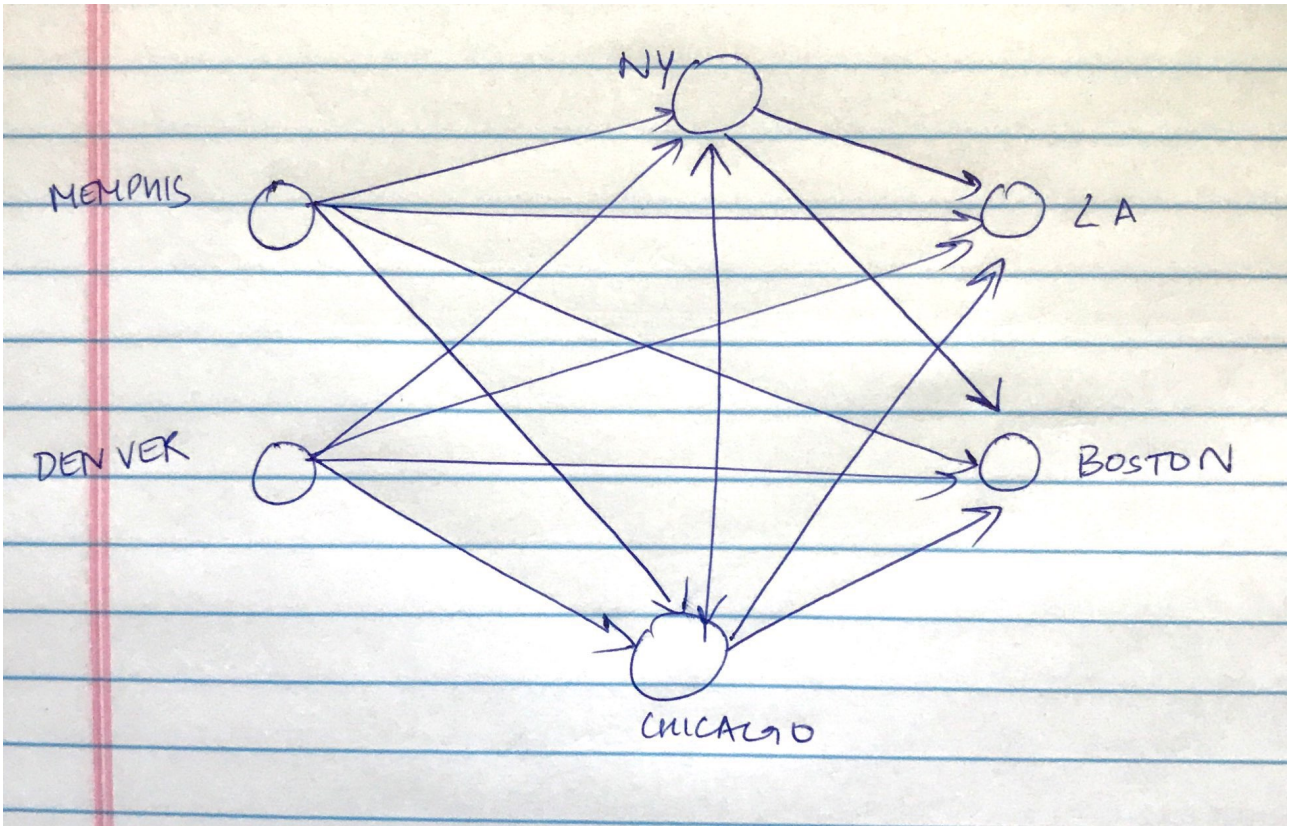
Networking. A company manufactures widgets at two factories, one in Memphis and one in Denver. The Memphis factory can produce up to 150 widgets per day, and the Denver factory can produce up to 200 widgets per day. The company are shipped by air to customers in Los Angeles and Boston. The customers in each city require 130 widgets per day. Because of the deregulation of airfares, the company believes that it might be cheaper to first fly some widgets to New York or Chicago and then fly them to their final destinations. The costs of flying a widget are shown below. Determine how to minimize the total cost of ship- ping the required widgets to the customers.

Widget shipping data							
Unit shipping costs (blanks mean those routes can't be used)							
		To					
		Memphis	Denver	NY	Chicago	LA	Boston
From	Memphis			$8	$13	$25	$28
	Denver			$15	$12	$26	$25
	NY				$6	$16	$17
	Chicago			$6		$14	$16
	LA						
	Boston						

Discussion.



This is an example of a networking problem where shipments are transferred from one location to another. Let us call each of these locations as nodes. Typically, in any networking problem there are source nodes, transmitter/intermediate nodes and destination nodes.

The function of any source node is to supply the shipments, hence the net flow (outflow-inflow) for a source node must be positive, and the amount that flows out of each source node must be within the capacity of each of the source node.

The intermediate nodes act as temporary stations for these shipments enroute from the source to the destination. Hence all shipments flowing into an intermediate node must eventually flow out, i.e. net flow of the intermediate node must be 0.

The destination nodes are the final destinations of the shipments, it accumulates the shipments from all other nodes. Hence the net flow (outflow-inflow) for a destination node must be negative and the net flow through the destination node must be able to satisfy the demands of the destination node.

These requirements are specified in the constraints. In the excel there are certain routes that are not allowed, to avoid allocating shipments through these routes we can create a new list of valid routes and force the solver to assign shipments to routes only present in this list. This helps to achieve faster computation and uses lesser storage space than placing very large costs for invalid routes in the cost matrix. Also, it is useful to keep in mind that there is not outflow once shipments reach a destination. i.e. shipments between destinations are not allowed.

Model.

Parameters:

S_S: *Capacity of source* s, *where* $s \in (Memphis,\ Denver)$

D : *Demand from destinations*

C_{ij} : *Cost of shipping one unit from source* i *to destination* j, *where* $i, j \in k$

Note: k is a list of all possible routes between i *to destination* j, *i.e.* $i \in (Memphis, Denver, Chicago, NY),\ j \in (Chicago, NY, LA, Boston)$

Decisions:

x_{ij} : *Number of widgets to be shipped from source* i *to destination* j, *where* $. i \in (Memphis, Denver, Chicago, NY),\ j \in (Chicago, NY, LA, Boston)$

Objective: *Minimize Cost*

$min \sum_{ij}\ x_{ij}\ *\ C_{ij}$

Constraints:

$\sum_j x_{sj} \leq S_s$ (1) Source capacity, $where\ s \in (Memphis,\ Denver)$

$\sum_i x_{ik} = \sum_j x_{kj}$ (2) Inflow = Outflow for any transmitter node k, k, ∈ (Denver, Chicago)

$\sum_i x_{ij} \geq$ D (3) Demand of destination j must be satisfied, , $j \in (LA, Boston)$

$x_{ij} \geq 0$ (4) Non-negative shipments

Optimal Solution. The following is the solution obtained from Excel Solver.

32(AP).xlsx

A minimum transportation cost of 6370 can be attained by distributing the widgets through the routes shown below.

From	To	cost	number of shipments
Memphis	NY	$8	150
Memphis	Chicago	$13	0
Memphis	LA	$25	0
Memphis	Boston	$28	0
Denver	NY	$15	0
Denver	Chicago	$12	0
Denver	LA	$26	0
Denver	Boston	$25	110
NY	Chicago	$6	0
NY	LA	$16	130
NY	Boston	$17	20
Chicago	NY	$6	0
Chicago	LA	$14	0
Chicago	Boston	$16	0

Widget shipping data							
Unit shipping costs (blanks mean those routes can't be used)							
		To					
		Memphis	Denver	NY	Chicago	LA	Boston
From	Memphis			$8	$13	$25	$28
	Denver			$15	$12	$26	$25
	NY				$6	$16	$17
	Chicago			$6		$14	$16
	LA						
	Boston						
From	To	cost	number of shipments				
Memphis	NY	$8	150				
Memphis	Chicago	$13	0				
Memphis	LA	$25	0				
Memphis	Boston	$28	0				
Denver	NY	$15	0				
Denver	Chicago	$12	0				
Denver	LA	$26	0				
Denver	Boston	$25	110				
NY	Chicago	$6	0				
NY	LA	$16	130				
NY	Boston	$17	20				
Chicago	NY	$6	0				
Chicago	LA	$14	0				
Chicago	Boston	$16	0				
Shipments from sources							
Memphis	150	<	150				
Denver	110	<	200				
Shipments through transmitting nodes							
TO		FROM					
NY	150	NY	150				
Chicago	0	Chicago	0				
Shioments to destination							
LA	130	>	130				
Boston	130	>	130				
OBJECTIVE	6370						