

(Shoe) Each year, a shoe manufacturing company faces demands (which must be met on time) for pairs of shoes as shown in the file P03_40.xlsx. Employees work three consecutive quarters and then receive one quarter off. For example, a worker might work during quarters 3 and 4 of one year and quarter 1 of the next year. During a quarter in which an employee works, he or she can produce up to 600 pairs of shoes. Each worker is paid \$6000 per quarter. At the end of each quarter, a holding cost of \$10 per pair of shoes is incurred. Determine how to minimize the cost per year (labor plus holding) of meeting the demands for shoes. To simplify the model, assume that at the end of each year, the ending inventory is 0. (You can assume that a given worker gets the same quarter off during each year.)

Discussion: -

It is similar to the multi period production plan (commodity, CH3-Q28) and we need to calculate the month end inventory to calculate the holding cost. Also, we will define a constraint saying that month end inventory is greater than or equal to zero which helps us in meeting our demand. Our objective is to minimize the total cost (worker cost + Inventory holding cost). Our production is directly related to the number of workers working in that quarter. We must make sure that we meet our demand with limited workers and controlling the holding cost. The main concern in this problem is that we must make sure that our workers who start their work in quarter 1 should continue till quarter 3 and take off in quarter 4. In similar way workers who start their work in quarter 3 will continue their work in quarter 4, quarter 1 of next year and will take off in quarter 2. We must convert this info in to information table A_{ij} which illustrates the employee work duration from quarter i to j . This helps in finding the number of workers available quarter. Please check below mathematical model for detailed calculations.

Mathematical Model: -

Parameters (Inputs):

$i, j \in 1, 2, 3, 4$ (i : Index for quarters of year)

D_j : Demand to produce product in quarter ' j '

P : Pair of Shoes produced by one worker in a quarter

W : Wage paid to worker per quarter

H : Holding cost for each pair of shoes

I_0 : Starting inventory in quarter 1, $I_0 = 0$

A_{ij} : Information table which illustrates the employee work duration from quarter i to j

| Information Table | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 |
|-------------------|-----------|-----------|-----------|-----------|
| Quarter 1 | 1 | 1 | 1 | 0 |
| Quarter 2 | 0 | 1 | 1 | 1 |
| Quarter 3 | 1 | 0 | 1 | 1 |
| Quarter 4 | 1 | 1 | 0 | 1 |

Decision Variables:

x_i : Number of workers who start working in quarter ' i '

Calculated Variables:

$$N_j = \sum_{i=1}^4 (x_i * A_{ij}) \text{ for } j \in \{1,2,3,4\}$$

Number of workers available in quarter 'j'

$$P_j = N_j * P \text{ for } j \in \{1,2,3,4\}$$

Number of shoes produced in quarter 'j'

$$W_j = N_j * W \text{ for } j \in \{1,2,3,4\}$$

Worker cost in quarter 'j'

$$I_j = I_{j-1} + P_j - D_j \text{ for } j \in \{1,2,3,4\}$$

Ending inventory in quarter 'j'

$$H_j = I_j * H \text{ for } j \in \{1,2,3,4\}$$

Holding cost in quarter 'j'

Objective:

$$\text{Minimize Total Cost} = \sum_{j=1}^4 (W_j + H_j)$$

Constraints:

$$x_i \geq 0;$$

(1) Non Negative constraint

$$I_j \geq 0;$$

(2) Ending inventory constraint

As it is minimizing the cost problem, if we must make sure that we give non-negative constraint to the decision variable. Ending inventory $I_j = I_{j-1} + P_j - D_j$ should always be greater than or equal to zero which helps in meeting the demand for that month.

Excel Implementation:

11[RA].xlsx

Please find the attached spreadsheet for solution.

| | | | | | Inputs | | | | |
|----------------------------|-----------|-----------|-----------|-----------|-------------------------|---|-----------|------------|-----------|
| | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 | | # workers who start work in Quarter 'i' | | | |
| Demand | 4000 | 7000 | 8000 | 3000 | Decision variables | Quarter 1 | 7.22 | | |
| Shoes produced by 1 worker | 600 | | | | Calculated Variables | Quarter 2 | 5.00 | | |
| Worker Cost \$ | 6,000 | | | | Constraints | Quarter 3 | 0.00 | | |
| Holding Cost \$ | 10 | | | | Objective | Quarter 4 | 0.00 | Total Cost | \$230,000 |
| Information Table | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 | | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 |
| Quarter 1 | 1 | 1 | 1 | 0 | Total Workers available | 7 | 12 | 12 | 5 |
| Quarter 2 | 0 | 1 | 1 | 1 | # shoes produced | 4333 | 7333 | 7333 | 3000 |
| Quarter 3 | 1 | 0 | 1 | 1 | Worker Cost | \$ 43,333 | \$ 73,333 | \$ 73,333 | \$ 30,000 |
| Quarter 4 | 1 | 1 | 0 | 1 | Starting Inventory | 0 | | | |
| | | | | | Ending Inventory | 333 | 667 | 0 | 0 |
| | | | | | >= | >= | >= | >= | |
| | | | | | | 0 | 0 | 0 | 0 |
| | | | | | Holding Cost | \$ 3,333 | \$ 6,667 | \$ - | \$ - |