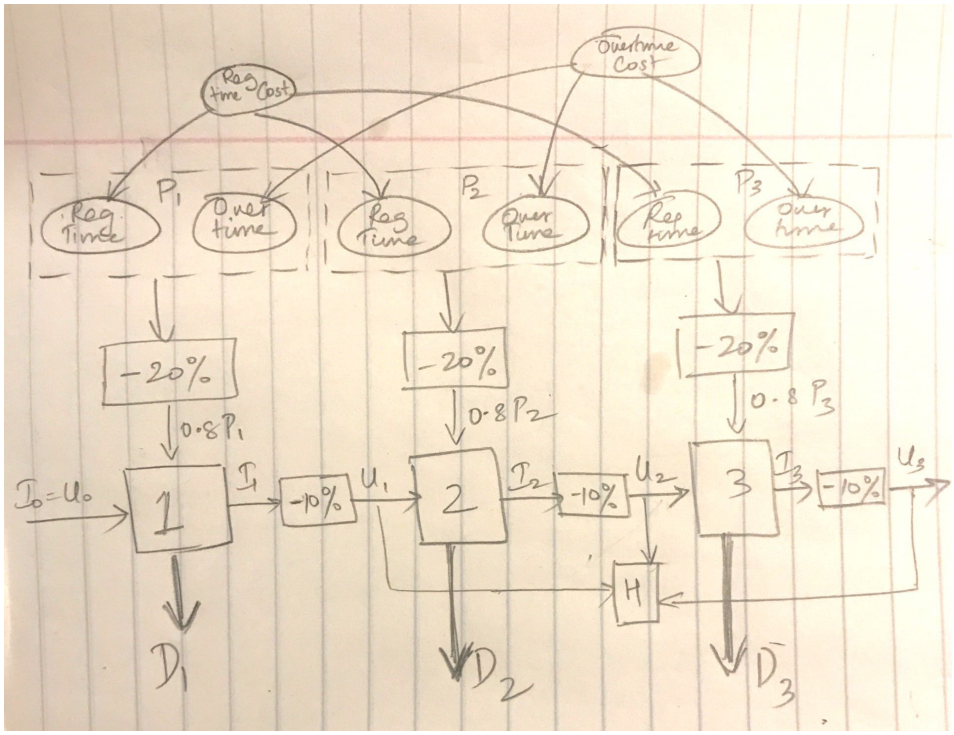
Inventory. A small appliance manufacturer must meet (on time) the following demands: quarter 1, 3000 units; quarter 2, 2000 unites; quarter 3, 4000 units. Each quarter, up to 2700 units can be produced with regular – time labor, at a cost of $40 per unit. During each quarter, an unlimited number of units can be produced with overtime labor, at a cost of $60 per unit. Of all units produced, 20% are unsuitable and cannot be used to meet demand. Also, at the end of each quarter, 10% of all units on hand spoil and cannot be used to meet any future demands. After each quarter's demand is satisfied and spoilage is accounted for, a cost of $15 per unit in ending inventory is incurred. Determine how to minimize the total cost of meeting the demands of the next 3 quarters. Assume that 1000 usable units are available at the beginning on quarter 1.

Discussion.

As we have seen in the previous multi-period problems, here there is a flow of appliances from one quarter to the next. Hence, we have balancing equations for the same. The difference in this problem is that not all appliances from one quarter after satisfying the demand of that quarter flows into the next quarter, A fraction of this inventory (10%) is spoiled, hence we have a usable inventory (remaining 90% of inventory) that flows in to the next quarter. Similarly, not all appliances produced in. a quarter are suitable to meet the demand of that quarter, a fraction (20%) of the produced units are unsuitable, hence only the remaining fraction (80%) of the produced appliances can be utilized in a quarter. The effective fraction of produced appliances and appliance from inventory of the previous quarter that flows into a quarter are illustrated in the figure below. Again, here we have two types for production methods, (1) the units can be produced by employees working in regular time and/or (2) the units can be produced by employees working in over time. Each of the above methods have a different cost associated with them. Since we have a limit for the number of appliances that can be produced by employees working in regular time in a quarter, to satisfy the demand requirements of a month, we might need to produce appliances by making employees work over time. Hence, we must decide how many appliances should be produced by employees working regular time and how many appliances should be produced by employees working overtime in each quarter. Since we have 3 months and 2 methods (regular time, over time), there are essentially 6 (3*2) decision variables, though we will be using a short hand mathematical notation to depict all 6 decision variables in the model. The objective is straight forward, we have to minimize the cost incurred through production cost and holding cost over the 3 quarters ensuring that demands for each quarter is met.



Model.

Parameters:

D_i : *Demand for quarter* i, *where* $i \in (1,2,3)$

H : *Unit holding cost*

C_l : *Unit cost of producing appliance by labor type* l , *where* $l \in (R, O)$ *[Here we use R to denote regular time and O to denote overtime.]*

R: *Production capacity with regular time labor*

α: *Fraction of appliances produced in a quarter that is suitable to meet the demands of the current quarter (0.8)*

β: *Fraction of appliances from the inventory of previous quarter that is not spoiled (0.9)*

U_O: *Initial usable inventory*

Decisions:

x_{il}: *Units of appliances that should be produced in quarter* i *by labor type* l, *where* $i \in (1,2,3)$, $l \in (R, O)$

Calculated Parameters:

I_i: *Inventory at the end of quarter* i, *where* $i \in (1,2,3)$

U_i *: Usable inventory at the end of quarter* i, *where* $i \in (1,2,3)$

$$I_i = U_{i-1} + [\sum_l x_{il} * \alpha] - D_i$$
$$U_i = I_i * \beta$$

Objective: *Minimize Cost*

$$min \sum_i [\sum_l [x_{il} * C_l] + U_i * H]$$

Constraints:

$I_i \geq 0$	(1) Demand must be satisfied for each quarter
$x_{il} \geq 0$	(2) Units of products produced cannot be negative
$x_{iR} \leq R$	(3) Production capacity for regular time in each quarter

Optimal Solution. The following is the solution obtained from Excel Solver.

A minimum cost of $445360 can be attained by appliances over the 3 quarters as shown below:

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Decision					
Appliances produced					
regular time			2500.00	2700.00	2700.00
over time			0.00	0.00	2120.00

Inputs							
		Q0	Q1	Q2	Q3		
Demand			3000.00	2000.00	4000.00		
Regular time production capacity	2700.00						
Production Cost							

Regular time	40.00						
Overtime	60.00						
% produced unsuitable	0.20						
% inventory spoiled	0.10						
Holding Cost	15.00						
% produced utilized	0.80						
% inventory utilized	0.90						
Decision							
Appliances produced							
regular time			2500.00	2700.00	2700.00	<=	2700.00
over time			0.00	0.00	2120.00		
total			2500.00	2700.00	4820.00		
Appliances utlilized			2000.00	2160.00	3856.00		
Inventory			0.00	160.00	0.00	>=	0.00
Usable Inventory		1000.00	0.00	144.00	0.00		
Total Production cost			100000.00	108000.00	235200.00		
Total holding cost			0.00	2160.00	0.00		
Total cost			100000.00	110160.00	235200.00		
Objective	445360.00						