Chemical. A chemical company manufactures 3 chemicals: A, B and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs $400 and yields 300 units of A, 100 units of B, and 100 units of C. Running process 2 for an hour costs $100 and yields 100 units of A, 100 units of B. To meet customer demands, at least 1000 units of A, 500 units of B, and 300 units of C must be produced daily. Determine the daily production plan that minimizes the cost of meeting the company's daily demands.

Discussion.

We can see that this problem is straightforward and that we can intuitively try solve for the number of hours to run process 1 and process 2 even without the solver. We see that Process 2 does not yield product C, but we have a minimum demand of 300 for product C. Since this needs to be produced solely by process 1, we need at least 3 hours of process 1 to meet the demand of product C.

At this point, because of process 1 we have 900 units of A and 300 units of B. Product A is short of 100 units and Product B is short of 200 units to meet the demand of product A (1000 units) and Product B (500 units) respectively.

Running either of the processes once will lead to satisfying demand of product A, hence let us first focus on satisfying demand of product B (which will consequentially lead to satisfying demand of Product A). To produce 200 more units of product B, either of the processes needs to be run for 2 hours. Running process 1 for 2 hours costs 800$ and running process 2 for 2 hours costs 200$. Hence, we choose to run process 2 for 2 hours and satisfy demand of product B as well as product A. This we need 3 hours of process 1 and 2 hours of process 2.

But this approach cannot be used for most problems as most business case scenarios are not fairly this simple and straight forward. Excel Solver can be used to solve many types of more complex situations. Let us take a look at how to do this.

In this problem, first, we need to identify what we are trying to optimize as per the given problem statement. Our daily production plan must minimize cost of meeting demands. This cost is incurred while running the process. Since we have the hourly cost to run the two process, our production plan should be such that on a given day the combined cost of running the 2 processes to meet the demands of A, B and C stays to a minimum. We know that the objective is to minimize the total cost is which is = (unit cost for running process 1 * number of hours that process 1 is run) + (unit cost for running process 2 * number of hours that process 2 is run). Here the unknowns are number of hours that process 1 is run and number of hours that process 2 is run. Hence, these will be our decision variables. Next, we must ensure that the number of hours that process 1 and process

2 is run is capable pf producing enough of the chemicals A, B, and C to satisfy their respective demands. This will be part of our constraints in the model.

Column1	Process 1	Process 2
Cost/hour	$400.00	$100.00
Yield A/hour	300	100
Yield B/hour	100	100
Yield C/hour	100	0

Note that throughout the discussion, we only care to yield the least possible units of products that can satisfy demand as our objective is to minimize cost. In other words, our objective tends to pull the yield down while our constraints try to push the yield above a certain threshold.

Model.

Parameters:

C_i : *Cost to run process i for 1 hour, i* $\in (1,2)$

Y_{ij}: $Yield\ of\ product\ j\ from\ process\ i, where\ j \in (A, B, C)$

d_j : *Daily demand for product, j* $\in (A, B, C)$

Commented [YW1]: Use capital letter for d_j (i.e., D_j)

Decisions:

n_i : *Number of hours to run process i in a given day, i* $\in (1,2)$

Commented [YW2]: As a general rule, try to use x, y, z as decisions, and other letters as parameters. This is just a convention in most mathematical programming community

Objective: *Minimize total cost of running two processes*

$$min \sum_{i=1}^{2} C_i * n_i$$

Constraints:

$n_i \geq 0, i \in \{1,2\}$ (1) Process hours are non-negative

$\sum_{i=1}^{2} Y_{ij} * n_i \geq \quad d_j , j \in \{A, B, C\}$ (2) Demand must be satisfied for each product

Notes:

1. $\sum_{i=1}^{2}$ is a short hand for summing up similar terms indexed by i. Here the objective function can also be written as $C_1 * n_1 + C_2 * n_2$. Similarly, the constraint (2) can also be written out more elaborately as $Y_{1j} * n_1 + Y_{2j} * n_2 \geq d_j, where\ j \in \{A, B, C\}$.

2. The objective function minimizes the total cost incurred in running process 1 and 2 in a given day
3. The constraint (2) ensures that the number of hours the processes are run produces a yield for products A, B and C that satisfies their respective customer demand
4. Although we have two constraints above, in effect there are five constraints: two for (1) and three for (2). A more elaborate (but cumbersome) list of constraints is the following. $n_1 \geq 0, n_2 \geq 0, Y_{1A} * n_1 + Y_{2A} * n_2 \geq d_A, Y_{1B} * n_1 + Y_{2B} * n_2 \geq d_B, Y_{1C} * n_1 + Y_{2C} * n_2 \geq d_C$.

Optimal Solution. The following is the solution obtained from Excel Solver.

<excel needs to be inserted>

The optimal solution is to run process 1 for 3 hours and run process 2 for 2 hours as discussed above and validated by the Excel solver.

Commented [YW3]: For the excel screen shot, it might be better looking to insert as a 'picture'.

Inputs			
	Process 1	Process 2	
Cost/hr	$400.00	$100.00	
Yield A/hr	300	100	
Yield B/hr	100	100	
Yield C/hr	100	0	
Decision			
Number of hrs/day	3	2	
Objective	1400		
Yield produced/day			
A	1100	>=	1000
B	500	>=	500
C	300	>=	300