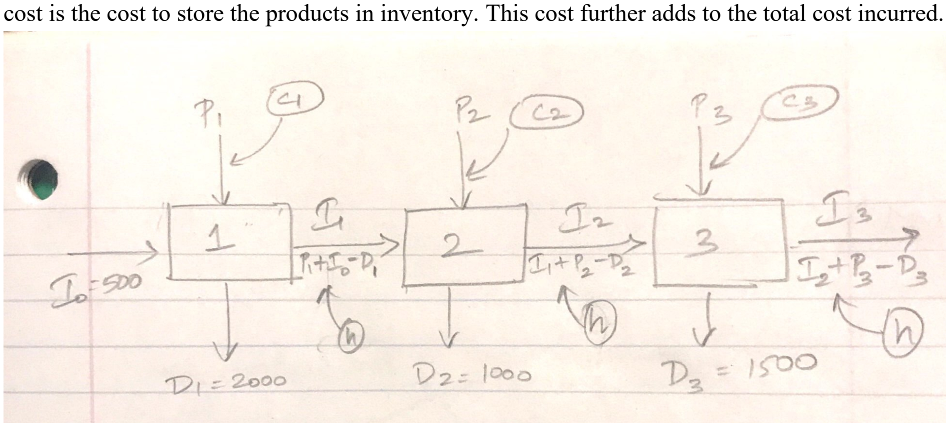
Inventory. A company faces the following demands during the next three weeks: week 1, 2000 units; week 2, 1000 units; week 3, 1500 units. The unit production cost for each week are as follows: week 1, \$130; week 2, \$140, week 3, \$150. A holding cost of \$20 per unit is assessed against each week's ending inventory. At the beginning of week 1, the company has 500 units on hand. In reality, not all goods produced during a month can be used to meet the current month's demand. To model this fact, assume that only half of the goods produced during a week can be used to meet the current week's demand. Determine how to minimize the cost of meeting the demand for the next three weeks.

Discussion.

The unique aspect of this problem lies in identifying the calculated or abstract parameters that are not explicitly mentioned in the input and drafting a balancing equation for them. What this means is that, calculated parameters are those parameters that flow through the timeline of the problem, keeps updating itself by the balancing equation and impacts the objective and the decision variables. In this case, we see that all the products produced that are not used to meet the demand of week 1 is stored as inventory and is utilized to meet the demand of week 2 along with the new products produced in week 2. Note that there can be an initial inventory as in this case before starting production too. It only means that to satisfy demands of week 1, we can utilize products produced in week 1 as well as the initial inventory (i.e. Inventory from week 0, I_0) . The holding cost is the cost to store the products in inventory. This cost further adds to the total cost incurred.



Model.

Parameters:

D_i : *Demand for week* i *, where* $i \in (1,2,3)$

C_i: *Unit production cost for week* i *, where* $i \in (1,2,3)$

H : *Unit holding cost*

α : *Fraction of the goods produced during a week that can be used to meet the current week's demand (here* $\alpha = 50\%$)
I_0: *Initial inventory*

Decisions:
p_i : *Units of products to be produced in week* i, *where* $i \in (1, 2, 3)$

Commented [YW1]: It is customary to use x, y, z etc to denote decisions

Calculated Parameters:
I_i: *Inventory at the end of week* i , *where* $i \in (1,2,3)$
$I_i = I_{i-1} + P_i - D_i$

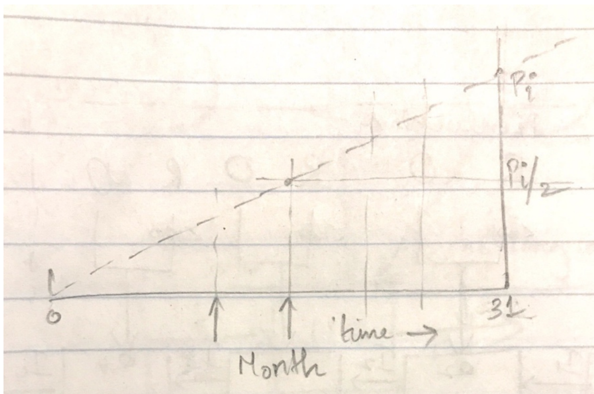
Objective: *Minimize Cost*
$max \sum_{i=1,2,3} P_i * c_i + I_i * h$

Constraints:

$P_i \geq 0$	(1) Units of products produced cannot be negative
$I_{i-1} + \alpha * p_i \geq D_i$	(2) Demand must be satisfied for each week

Notes:

1) The problem states that only half of the units produced in a week can be used to meet the demand of that week. Hence constraint 2 ensures that units from inventory of the previous week and half of the units produced in the current week can meet the demand of the current week.
2) In theory, we can use $I_i \geq 0$, which means that demand of each week is satisfied, i.e. $I_{i-1} + P_i \geq D_i$, sum of inventory flown in from previous week and current week's production must satisfy the current week's demand. But in reality, not all the products produced in a month can be used to satisfy the demand of that month since production occurs throughout the course of the month, and at a particular moment in time in the month, not all units that should be produced will be available. To account for this, the second equation considers only a fraction (half in this case) of the units produced in a month to be available to satisfy the demand of that month. See figure below for further illustration.



Optimal Solution. The following is the solution obtained from Excel Solver.

The optimal solution is to produce 3000 units in week 1, 1000 units in week 2 and no units in week 3.

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	WEEK 0	WEEK 1	WEEK 2	WEEK 3
Demand		2000	1000	1500
Inventory	500	1500	1500	0
Units Produced		3000	1000	0
Unit cost		130	140	150
Holding Cost/week		20	20	20
	Units available	2000	2000	1500
		>=	>=	>=
		2000	1000	1500
objective	590000			