Non Linear Programming. The costs of producing product A, product B, or products A and B bundled together are $50, $90, and $140, respectively. Show below are the sizes of the three market segments for these products and how much each of the segments is willing to pay for A alone, B alone, or the bundle. Under the assumptions that a market segment will buy the product combination that yields the maximum nonnegative surplus (value minus cost), and a segment will buy no product if no product has a nonnegative surplus, determine an optimal set of product prices. Should the company offer all products for sale?

Bundling products				
Segment	Size	A only	B only	Bundle
1	150	$100	$100	$195
2	100	$50	$96	$145
3	50	$80	$85	$195
	Cost	50	90	140

Discussion.

This is a non linear programming problem. From the information given, we need to decide which are the products we should release into the market and how much should we price these products. This depends on understanding which market segment will buy which product which will be one of the decision variables. Here we are assuming that a single segment can only by one product, which is one of our constraints. The price of the product is the second decision variable. Also note that the model is only optimizing the selling price for the product that is being bought by the segments i.e. only for the products that are pushed into the market. The product being bought depends on which product is viewed by the segment as most valuable based on its cost. That is, the product that a segment buys must have the highest value (surplus) compared to the other products. We must set a selling price for these products so that the profit is maximized, which is our objective. Another notable constraint is that a segment does not by any product is its valued (surplus) is less than zero.

Model.

Parameters:

C_i : *Cost of each product combination* i, *where* $i \in (A, B, A\&B)$

S_j : *Market size of each segment* j, *where* $j \in (1, 2, 3)$

W_{ij} : *Price each segment* j *is willing to pay for product, where* $i \in (A, B, A\&B)$, $j \in (1, 2, 3)$

Decisions:

p_i : *Prices for each product combination* i, *where* $i \in (A, B, A\&B)$

y_{ij}: Whether segment j buys product i, *where* $i \in (A, B, A\&B)$, $j \in (1, 2, 3)$

Calculated Parameters:

u_{ij}: *Surplus obtained for product combination* i *by segment* j, *where* $i \in (A, B, A\&B)$, $j \in (1, 2, 3)$

$u_{ij} = W_{ij} - p_i$ *for all* i *and* j

Objective: *Maximize Profit*

$max \sum_{ij} y_{ij} * S_j * (p_i - C_i)$

Commented [YW1]: Please re-run the solver as we have changed objective as well as the utility for the customers

Constraints:

$\sum_i y_{ij} \leq 1$	(1) One customer can buy max of 1 product
$\sum_i y_{ij} * u_{ij} \geq 0$ *for all* j	(2) Purchase product only if surplus is non negative
$u_{ij} \geq u_{kj} - M(1-y_{ij})$ *for all* i, k	(3) Purchase highest utility
$p_i \geq 0$	(4) Non negative selling price
$y_{ij} \in \{0,1\}$	(5) Binary decision

Commented [YW2]: Might be redundant because you are maximuizing

Commented [YW3]: Put a note saying that M is a large number that helps to enforce logical constraint

Optimal Solution. The following is the solution obtained from Excel Solver.

P08_23.xlsx

A maximum revenue of 30,000$ can be attained by scheduling choosing and pricing the products as shown below.

Commented [YW4]: Please update excel screen shot

Non liner programming [Based on Practical Management Science] Prepared by Athira Praveen

Bundling products

Segment	Size	A only	B only	Bundle
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	Cost	50	90	140

Price for each product

A only	B only	Bundle
100	0	0
<=	<=	<=
$100	$50	$80

Whether customer buys product

Segment	A only	B only	Bundle			
1	1.00	0.00	0.00	$1	<=	1
2	1.00	0.00	0.00	$1	<=	1
3	1.00	0.00	0.00	$1	<=	1

Value of product for customer

Segment	A only	B only	Bundle
1	$50	$50	$115
2	$0	$46	$65
3	$30	$35	$115

objective $30,000

revenue from segment

1	$15,000
2	$10,000
3	$5,000

Constraints to make utility positive

Segment	A only	B only	Bundle
1	$50	$0	$0
2	$0	$0	$0
3	$30	$0	$0
	>=	>=	>=
	0	0	0

M 10000

To pick highest utility

Segment	A only	B only	Bundle
1	$50	-$9,950	-$9,885
2	$0	-$9,954	-$9,935
3	$30	-$9,965	-$9,885