Assignment. You have put four valuable paintings up for sale. Four customers are bidding for the paintings. Customer 1 is willing to buy two paintings, but each other customer is willing to purchase at most one painting. The prices that each customer is willing to pay are shown below. Determine how to maximize the total revenue you receive from the sale of the paintings.

Painting data				
Bids (where a blank means a customer doesn't bid on a painting)				
	Painting			
Customer	1	2	3	4
1			$132,019	$86,711
2	$96,977	$100,820		
3	$108,283		$146,622	
4	$141,521	$131,075		$141,823

Discussion.

This is an example of an assignment problem where paintings are assigned to buyers. The objective is to maximize the revenue from the buyers. The paintings have to be assigned considering factors like once a painting is assigned to a buyer, it cannot be assigned to any other buyer. Customer 1 is willing to buy 2 paintings whereas the other customers are willing to buy a maximum of 1 painting each. The decision variable is straightforward as it is to decide which painting must be sold to which customer.

We must also consider that few customers are not willing to pay for certain paintings, to account for this we can take either of the 2 approaches below:

Approach 1: We can assign these costs to a negative value in the cost matrix so that Excel solver does not choose those assignments as it is trying to maximize the revenue.

Approach 2: We can make a list of valid biddings that is containing only valud combinations of customers and paintings and force the model to make decisions from this list

Model.

Approach 1:

Parameters:

R_{ij} : Revenue received from customer $i\ for\ painting\ j, where\ i, j \in (1, 2, 3, 4)$

Decisions:

x_{ij} : $Whether\ painting\ paining\ j\ is\ sold\ to\ customer\ i,\ where\ .\ i, j \in (1, 2, 3, 4)$

Objective: *Maximize revenue*

$max \sum_{ij} x_{ij} * R_{ij}$

Constraints:

$\sum_i x_{ij} \leq 1$ (1) A painting can be sold to maximum 1 customer

$\sum_j x_{1j} \leq 2$ (2) Customer 1 can buy maximum of 2 paintings

$\sum_j x_{ij} \leq 1$ (3) Demand of destination j must be satisfied, $i \in (2,3,4)$. Customers are willing to buy at most 1 painting

$x_{ij} \in (0,1)$ (4) Binary variable for assignments

Approach 2:

Parameters:

R_{ij} : Revenue received from customer $i \; for \; painting \; j, where \; i,j \in (1,2,3,4)$

Decisions:

x_{ij} : $Whether \; painting \; paining j \; is \; sold \; to \; customer \; i, \; where \; . \; i,j \in K, list \; opf \; valiud \; biddings$

Objective: *Maximize revenue*

$max \sum_{ij} x_{ij} * R_{ij}$

Constraints:

$\sum_i x_{ij} \leq 1$ (1) A painting can be sold to maximum 1 customer

$\sum_j x_{1j} \leq 2$ (2) Customer 1 can buy maximum of 2 paintings

$\sum_j x_{ij} \leq 1$ (3) Demand of destination j must be satisfied, $i \in (2,3,4)$. Customers are willing to buy at most 1 painting

$x_{ij} \in (0,1)$ (4) Binary variable for assignments

Optimal Solution. The following is the solution obtained from Excel Solver.

Approach 1:

A maximum revenue of 4,82,945 $ attained by selling the paintings to the customers as shown below.

	Painting			
Customer	1	2	3	4
1	0.00	0.00	1.00	0.00
2	0.00	1.00	0.00	0.00
3	1.00	0.00	0.00	0.00
4	0.00	0.00	0.00	1.00

Painting data								
Bids (where a blank means a customer doesn't bid on a painting)								
	Painting							
Customer	1	2	3	4				
1			$132,019	$86,711				
2	$96,977	$100,820						
3	$108,283		$146,622					
4	$141,521	$131,075		$141,823				
	Painting							
Customer	1	2	3	4				
1	-$1	-$1	$132,019	$86,711				
2	$96,977	$100,820	-$1	-$1				
3	$108,283	-$1	$146,622	-$1				
4	$141,521	$131,075	-$1	$141,823				
DECISION								
	Painting							
Customer	1	2	3	4	Number of paintings bought by a customer			
1	0.00	0.00	1.00	0.00	1	<=	2	
2	0.00	1.00	0.00	0.00	1	<=	1	
3	1.00	0.00	0.00	0.00	1	<=	1	
4	0.00	0.00	0.00	1.00	1	<=	1	
Number of customers that can buy a painting	1.00	1	1	1				
	<=	<=	<=	<=				
	1	1	1	1				
OBJECTIVE	482945							

Approach 2:

Similar solution is obtained by Approach 2.

Painting data

Bids (where a blank means a customer doesn't bid on a painting)

	Painting			
Customer	1	2	3	4
1			$132,019	$86,711
2	$96,977	$100,820		
3	$108,283		$146,622	
4	$141,521	$131,075		$141,823

Customer	Painting	Price	Decision
1	3	$132,019	1
1	4	$86,711	0
2	1	$96,977	0
2	2	$100,820	1
3	1	$108,283	1
3	3	$146,622	0
4	1	$141,521	0
4	2	$131,075	0
4	4	$141,823	1

Painting	Number of customers that can buy a painting		
1	1	<=	1
2	1	<=	1
3	1	<=	1
4	1	<=	1

DECISION

Customer	Number of paintings bought by a customer		
1	1.00	<=	2
2	1	<=	1
3	1	<=	1
4	1	<=	1

objective 482945