Networking. Allied Freight supplies goods to three customers. The company has two warehouses. The warehouse availabilities, the customer requirements, and the unit shipping costs from warehouses to customers are shown below. There is a penalty for each unsatisfied customer unit of demand, and these penalties are also listed in the same file. Determine how to minimize the sum of penalty and shipping costs.

Allied Freight data							
Unit shipping costs							
		To					
		Customer 1	Customer 2	Customer 3		Available units	
From	Warehouse 1	$35	$15	$25		Warehouse 1	400
	Warehouse 2	$30	$10	$40		Warehouse 2	200
Requirements							
		Customer 1	Customer 2	Customer 3			
		150	250	300			
Penalty cost per unit of requirement not satisfied							
		Customer 1	Customer 2	Customer 3			
		$125	$100	$90			

Discussion.

In this problem the determination of decision variable is straightforward as we need to understand how much goods from each warehouse should be assigned to each customer. The peculiarity of the problem is that, here the available goods is lesser than the demand, meaning there would definitely be dissatisfied customers because of whom we would incur a penalty for each unit of demand that is not catered to. Hence for each customer, apart from the shipping cost from each warehouse, we also need to account for the penalty cost caused by the units of demand that have not been satisfied. This adds on to our total cost incurred in the objective which we would like to minimize.

It is important to note that each customer receives goods from both warehouses and it is the demand for that customer which is left unsatisfied after receiving goods from both the warehouses that we need to account for.

Also note that we cannot give the usual constraint of goods being assigned to each customer to be greater than or equal to their demand. This is because this constraint is impossible to satisfy with the given amount of available goods at the ware houses. If we add this constraint, solver will not be able to find an optimal solution.

We also give the obvious constraint (apart from the non-negative constraint) that the number of goods from each warehouse that is being assigned to each customer, does not exceed the available number of goods in each warehouse.

Model.

Parameters:

C_{ij} : Unit Shipping Cost from ware house i to customer j, *where* $i \in \{1,2\}$, $j \in \{1,2,3\}$

R_j: Requirements of each customer j, where $j \in \{1,2,3\}$

A_i : Units available in each warehouse i, where $i \in \{1,2$

P_j: Penalty cost for each dissatisfied customer per unit demand where $j \in \{1,2,3\}$

Decisions:

x_{ij} : Amount of units from warehouse i that must be distributed to customer j, *where* $i \in \{1,2\}$, $j \in \{1,2,3\}$

Objective: *Minimize Cost*

$\min [\sum_{ij} x_{ij} * C_{ij} + \sum_j (R_j - \sum_i x_{ij}) P_j]$

Constraints:

$\sum_j x_{ij} \leq A_i$ (1) Warehouse capacity

$\sum_i x_{ij} \leq R_j$ (2) Avoid demand overflow

$x_{ij} \geq 0$ (3) Non-negative shipments

Notes:

1) Constraint (2) It is important to give this constraint, which ensures the number of dissatisfied units to be non-negative. This is to avoid all goods from being assigned to one customer and we cannot have negative number of dissatisfied demands (that is, when assignment is more than demand, the penalty cost has no relevance in this case, but will result in a negative term in the objective which will be used to minimize overall cost. To avoid this situation, we add this constraint)

Optimal Solution. The following is the solution obtained from Excel Solver.

A minimum transportation cost of 22000 can be attained by distributing the goods as shown below.

Allied Freight data							
Unit shipping costs							
		To					
		Customer 1	Customer 2	Customer 3		Available units	
From	Warehouse 1	$35	$15	$25		Warehouse 1	400
	Warehouse 2	$30	$10	$40		Warehouse 2	200
Requirements							
		Customer 1	Customer 2	Customer 3			
		150	250	300			
Penalty cost per unit of requirement not satisfied							
		Customer 1	Customer 2	Customer 3			
		$125	$100	$90			
No of goods from warehouse j assigned to customer		Customer 1	Customer 2	Customer 3			
	Warehouse 1	0	200	200	400	<=	400
	Warehouse 2	150	50	0	200	<=	200
		150	250	200			
	Dissatisfied demand	0	0	100			
	Penality Cost	9000					
	Shipping Cost from						
	Warehouse 1	8000					
	Warehouse 2	5000					
	Total Cost	22000					