Practical File

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<tr><td>2.</td><td>The XYZ Company produces two types of tape recorders: A reel-to-reel model and a cassette model on two assembly lines. The company must process each tape recorder on each assembly line and it has found that the following time is required:
<table>
<tr><td>Assembly line</td><td>Reel-to-reel</td><td>Cassette</td></tr>
<tr><td>1</td><td>6 hours</td><td>2 hours</td></tr>
<tr><td>2</td><td>3 hours</td><td>2 hours</td></tr>
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The production manager says that line 1 will be available 40 hours per week and line 2 only 30 hours. After these hours of operation each line must be checked for repairs. The company realizes a profit of Rs 300 on each reel-to-reel recorder and Rs 120 on each cassette recorder. Formulate and solve this problem as an integer LP problem to determine the number of recorders of each type to be produced each week in order to maximize profit.</td></tr>
<tr><td>3.</td><td>Solve Q2 using Gomory’s cutting plane method.</td></tr>
<tr><td>4.</td><td>ABC manufacturing company faces the following problem: Should they make or buy each of their several products? The company policies specify that they will either make or buy the whole lot of each product in a complete lot. The company has four products to make or buy with six machines involved in making these products if they are made in the shop. The time per unit (in hours) required are as follows:</td></tr>
</table>

Product	Machine					
	A	B	C	D	E	F
1	0.04	0.02	0.02	0	0.03	0.06
2	0	0.01	0.05	0.15	0.09	0.06
3	0.02	0.06	0	0.06	0.02	0.02
4	0.06	0.04	0.15	0	0	0.05

Forty hours are available of each machine. One hundred ten units of each product are needed. The costs of making the products are listed below:

Products:	1	2	3	4
Cost/unit (in Rs.):	2.25	2.22	4.50	1.90

The costs to buy the products are:

Products:	1	2	3	4
Cost/unit (in Rs.):	3.10	2.60	4.75	2.25

Formulate this problem as a zero-one integer programming problem.

5. Consider the following production data:

Product	**Profit Per Unit (Rs.)**	**Direct Labor Requirement (in Hours)**
1	8	15
2	10	14
3	7	17

Fixed Cost	**Direct Labor Requirement**
10,000	Up to 20,000 hours
20,000	20,000-40,000 hours
30,000	40,000-70,000 hours

Formulate an integer programming problem to determine the production schedule so as to maximize the total net profit.

6.	Solve the following problem for profit maximization:

Model	Quantity	Plant A	Plant B	Plant C	Capacity	Selling Price	Plant
Standard	450	8	7.95	8.10	800	14.95	A
Deluxe	1050	8.50	8.60	8.45	600	18.85	B
New Deluxe	600	9.25	9.20	9.30	700	21.95	C

7.	Find the minimum cost for covering all zones:

	i	ii	iii	iv	v	vi
A	73	91	87	82	78	80
B	81	85	69	76	74	85
C	75	72	83	84	78	91
D	93	96	86	91	83	82
E	90	91	79	89	89	76

8.	A manufacturer has distribution centers at Agra, Allahabad and Kolkata. These centers have availability of 40, 20 and 40 units of his product respectively. His retail outlets at A, B, C, D, E require 25, 10, 20, 30 and 15 units of the products respectively. The transport cost (in rupees) per unit between each center outlet is given below: -

Distribution Centre	Retail Outlets				
	A	**B**	**C**	**D**	**E**
Agra	55	30	40	50	40
Allahabad	35	30	100	45	60
Kolkata	40	60	95	35	30

Determine the optimal distribution so as to minimize the cost of transportation.

9.

Find minimum production cost.

	Jan	Feb	Mar	Apr	May
Production cost	24	27	32	50	34
Demand	200	250	150	80	120
Capacity	250	225	250	200	225

Inventory carrying cost: 5 per unit per month

10.

Find minimum production cost.

Month	**Max Production**	**Demand**	**Production Cost**	**Inventory**
Jan	120	100	60	15
Feb	120	130	60	15
March	150	160	55	20
April	150	160	55	20
May	150	140	50	20
June	150	140	50	20

11.

ABC Company wishes to develop a monthly production schedule for the next three months depending upon the sales commitments, the company can keep the production constant, allowing fluctuations in inventory or inventories can be maintained at constant level, with fluctuating production. Fluctuating production necessitates, working overtime, the cost of which is estimated to be double the normal production cost of 12 Rupee per unit. Fluctuating inventories result in inventory carrying of 2 Rupee per unit per month. If the company fails to fulfill its sales commitment it incurs a shortage cost of 4 Rupee per unit per month. The production capacities for the next three month are shown in table: -**Production Capacity**

Month	Regular	Overtime	Sales
1	50	30	60
2	50	0	120
3	60	50	40

Determine optimal production schedule.

12.

Consider the game with the following payoff table:

	Player B	
Player A	B1	B2
A1	2	6
A2	-2	λ

i) Show that the game is strictly determinable, whatever λ may be.

ii) Determine the value of the game.

13.

Determine which of the following two-person zero sum games are strictly determinable and fair. Give the optimum strategies for each player in the case of strictly determinable.

a)

	Player B	
Player A	B1	B2
A1	1	2
A2	4	-3

b)

	Player B	
Player A	B1	B2
A1	-5	2
A2	-7	-4

14.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below:
Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	1	7	3	4
A2	5	6	4	5
A3	7	2	0	3

15.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below:
Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

	Player B				
Player A	B1	B2	B3	B4	B5
A1	-2	0	0	5	3
A2	3	2	1	2	2
A3	-4	-3	0	-2	6
A4	5	3	-4	2	6

16. Solve the following game by using maximin(minimax) principle whose payoff matrix are given below:
Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	3	-5	0	6
A2	-4	-2	1	2
A3	5	4	2	3

17. Solve the following game by using maximin(minimax) principle whose payoff matrix are given below:
Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

	Player B		
Player A	B1	B2	B3
A1	-2	15	-2
A2	-5	-6	-4
A3	-5	20	-8

18. Solve the following game by using maximin(minimax) principle whose payoff matrix are given below:
Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	-5	3	1	10
A2	5	5	4	6
A3	4	-2	0	-5

19.	Two competitive manufacturers are producing a new toy under license from a patent holder. In order to meet the demand, they have the option of running the plant for 8, 16 or 24 hours a day. As the length of production increases so does the cost. One of the manufacturers, say A, has set up the matrix given below. He uses the matrix to estimate the percentage of the market that he could capture and maintain the different production schedules:

	Manufacturer B		
Manufacturer A	C1: 8 hrs.	C2:16 hrs.	C3:24 hrs.
S1:8 hrs.	60%	56%	34%
S2:16 hrs.	63%	60%	55%
S3:24 hrs.	83%	72%	60%

20.	Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	2	1	0	-2
A2	1	0	3	2

21.	Solve graphically, the rectangular game, whose payoff matrix is:

	Player B	
Player A	B1	B2
A1	1	-3
A2	3	5
A3	-1	6
A4	4	1
A5	2	2
A6	-5	0

22. Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	6	5	2	3
A2	1	2	6	3

23. Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	3	1	0	-2
A2	1	0	3	2

24. Two computer manufacturers A and B are attempting to sell computer systems to two banks 1 and 2. Company A has 4 salesmen; company B only has 3 salesmen available. The computer companies must decide upon how many salesmen to assign to sell computer to each bank. Thus, company A can assign 4 salesmen to bank 1 and none to bank 2 or three to bank 1 and one to bank 2, etc.

Each bank will buy one computer system. The probability that a bank will buy from a particular computer company is directly related to the number of salesmen calling from that company, relative to the total salesmen calling. Thus, if company A assigns three salesmen to bank 1 and company B assigns two salesmen, the odds would be three out of five that bank 1 would purchase company A's computer system. (If none calls from either company the odds are one-half for buying either computer.)

Let the payoff be the expected number of computer systems that company A sells. (2 minus this payoff is the expected number company B sells).

What strategy would company A use in allocating its salesmen? What strategy should company B use? What is the value of the game to company A? What is the meaning of the value of the game in this problem?

25.	The firms A and B have for years been selling's a competitive product which forms a part of both firms' total sales. The marketing executive of firm A raised the question, "What should be the firm's strategies in terms of advertising product in question?" The market research team of firm A developed the following data for varying degrees of advertising: I) No advertising, medium advertising, and large advertising for both firms will result in equal market shares. II) Firm A with no advertising: 40% of market with medium advertising by firm B and 28% of the market with large advertising by firm B III) Firm A using medium advertising: 70% of the market with no advertising by firm B and 45% of the market with large advertising by firm B IV) Firm A using large advertising: 75% of the market with no advertising by firm B and 47.5% of the market with medium advertising by firm B a) Based upon their foregoing information, answer the marketing executive's questions. b) What advertising policy should firm A pursue when consideration is given to the above factors: selling price Rs. 4 per unit: variable cost of product Rs. 2.5 per unit; annual volume of 30,000 units for firm A; cost of annual medium advertising Rs. 5,000 and cost of annual large advertising Rs. 15,000? What contribution before other fixed costs is available to the firm?
26.	Solve the given payoff matrix. Transfer the zero sum two-person game into equivalent linear programming problem. Solve using Simplex Method.

	B1	**B2**	**B3**
A1	**5**	**3**	**7**
A2	**7**	**9**	**1**
A3	**10**	**6**	**2**

27. For the following payoff matrix, transform the zero sum two-person game into an equivalent linear programming problem and solve it by simplex method.

	Player B		
Player A	B1	B2	B3
A1	9	1	4
A2	0	6	3
A3	5	2	8

28. For the following payoff matrix, transform the zero sum two-person game into an equivalent linear programming problem and solve it by simplex method.

	Company A		
Company B	A1	A2	A3
B1	2	-2	3
B2	-3	5	-1

29. A soft drink company calculated the market share of two of its products against its major competitor, which has three products. The company found out the impact of additional advertisement in any one of its products against the other.

	Company B		
Company A	B1	B2	B3
A1	6	7	15
A2	20	12	10

What is the best strategy for the company as well as the competitor? What is the payoff obtained by the company and the competitor in the long run? Use the graphical method to obtain the solution.

30.	Two Firms A and B make color and black & white television sets. Firm A can make either 150 color sets in a week or an equal number of black and white sets, and make a profit of Rs 400 per color set and Rs 300 per black & white set. Firm B can, on the other hand, make either 300 color sets, or 150 color and 150 black & white sets, or 300 black & white sets per week. It also has the same profit margin on the two sets as A. Each week there is a market of 150 color sets and 300 black & white sets. The manufacturers would share market depending upon the proportion in which they manufacture a particular type of set. Write the payoff matrix of A per week. Obtain, graphically, A's and B's optimal strategies and the value of the game.
31.	In a town there are only two discount stores ABC and XYZ. Both stores run annual pre-Diwali sales. Sales are advertised through local newspapers with the aid of an advertising firm. ABC stores constructed following payoff in units of Rs 1,00,000. Find the optimal strategies for both stores and the value of the game:

	Store XYZ		
Store ABC	B1	B2	B3
A1	1	-2	1
A2	-1	3	2
A3	-1	-2	3

32.	Assume that the two firms are competing for market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share for Firm A and the decrease in market share for Firm B. Determine the optimal strategies for each firm.

	Firm B		
Firm A	No Promotion	Moderate Promotion	Much Promotion
No Promotion	5	0	-10
Moderate Promotion	10	6	2
Much Promotion	20	15	10

Formulate a suitable linear programming model of the game, with respect to minimizing player B's losses and derive the optimal strategy for B.

33.	Firm X is fighting for its life against the determination of firm Y to drive it out of the industry. Firm X has the choice of increasing the price, leaving it unchanged, or lowering it. Firm Y has the same three options. Firm X's gross sales in the event of each of the pairs of choices are shown below:

Firm Y's Pricing Strategies	Firm Y's Pricing Strategies		
	Increase Price	Do not change	Reduce Price
Increase Price	90	80	110
Do not change	110	100	90
Reduce Price	120	70	80

Assuming firm X as the maximizing one, formulate and solve the problem as a linear programming problem.

Ques 1.

The owner of a readymade garment store sells two types of shirts Zee shirts and button-down shirts. He makes a profit of \$3 on zee shirts and \$12 on button down shirts. He has two tailor A & tailor B at his disposal to stitch the shirts. Tailor A can devote 7 hours and tailor B can devote 15 hours at most per day. Both these shirts need to be stitched by both tailors. Tailor A and tailor B take 2 & 5 hours on zee shirts and 4 & 3 hours on button shirts respectively. How many of shirts of both types should be stitched to maximize daily profit. Formulate and solve this as LP problem and if the solution is not integer value, derive the optimal integer problem.

Solution:

Let the zee shirts be denoted by variable = x_1

Let the button-down shirts be denoted by variable = x_2

So, the objective function will be to maximize profit, which can be written as: $3x_1 + 12x_2$

Time constraint for Tailor A: $2x_1 + 4x_2 \leq 7$

Time constraint for Tailor B: $5x_1 + 3x_2 \leq 15$

So, the formulated problem is:

$$Maximize\ Z = 3x_1 + 12x_2$$

$$Subject\ to\ constraints: 2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0\ and\ integers$$

Now solving this as an LPP in Tora, we get the following input table:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name	x1	x2		
Maximize	3.00	12.00		
Constr 1	2.00	4.00	<=	7.00
Constr 2	5.00	3.00	<=	15.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

First Iteration:

Iteration 1	x1	x2			
Basic	x1	x2	sx3	sx4	Solution
z (max)	-3.00	-12.00	0.00	0.00	0.00
sx3	2.00	4.00	1.00	0.00	7.00
sx4	5.00	3.00	0.00	1.00	15.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Second Iteration (Optimal):

Iteration 2	x1	x2			
Basic	x1	x2	sx3	sx4	Solution
z (max)	3.00	0.00	3.00	0.00	21.00
x2	0.50	1.00	0.25	0.00	1.75
sx4	3.50	0.00	-0.75	1.00	9.75
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Here, we get $x_1 = 0$ and $x_2 = 1.75$. This gives maximum Z = 21.

Since x_2 is non-integer, we proceed with integer programming.

Input Table:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name	x1	x2		
Maximize	3.00	12.00		
Constr 1	2.00	4.00	<=	7.00
Constr 2	5.00	3.00	<=	15.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		
Integer (y/n)?	y	y		

Output:

Variable	x1	x2
Var. Name	x1	x2
Value	0	1.75
Integer(y/n)?	y	y

Here as well, we get the same initial solution using simplex. Now we branch x_2 since it is non integer.

N10	
z= 21.00	
x2=1.75	
<N20, N21>	
N20	N21
x2<=1	x2>=2
z= 16.50	
x1=1.50	infeasible
<N30, N31>	
N30	N31
x1<=1	x1>=2
z= 15.00	z= 15.00
integer	z <= LBound
Best LBound	

Here, in the first iteration we add two constraints for x_2, $x_2 \leq 1$ and $x_2 \geq 2$. The latter is infeasible so we proceed with $x_2 \leq 1$. With this we get $x_1 = 1.50$, which is again non integer so in the next iteration we add two more constraints, $x_1 \leq 1$ and $x_1 \geq 2$.

The best solution is then found at $x_1 = 1$ and $x_2 = 1$, Z = 15 (represented by N30 in the above picture).

Variable	x1	x2
Var. Name	x1	x2
Value	1	1
Integer(y/n)?	y	y

We see that there is a difference of $6 when we move from a non-integer to an integer solution.

Final Solution:

For LPP: Maximized profit = 21, if 0 zee (x_1) and 1.25 button down (x_2) shirts are made.

For IPP: Maximized profit = 15, if 1 zee (x_1) and 1 button down (x_2) shirts are made.

Ques 2.

The XYZ Company produces two types of tape recorders: A reel-to-reel model and a cassette model on two assembly lines. The company must process each tape recorder on each assembly line and it has found that the following time is required:

Assembly line	Reel-to-reel	Cassette
1	6 hours	2 hours
2	3 hours	2 hours

The production manager says that line 1 will be available 40 hours per week and line 2 only 30 hours. After these hours of operation each line must be checked for repairs. The company realizes a profit of Rs 300 on each reel-to-reel recorder and Rs 120 on each cassette recorder. Formulate and solve this problem as an integer LP problem to determine the number of recorders of each type to be produced each week in order to maximize profit.

Solution:

Let the reel-to-reel recorders be denoted by variable = x_1

Let the cassette recorders be denoted by variable = x_2

So, the objective function will be to maximize profit, which can be written as: $300x_1 + 120x_2$

Time constraint for line 1: $6x_1 + 2x_2 \leq 40$

Time constraint for line 2: $3x_1 + 2x_2 \leq 30$

So, the formulated problem is:

$$Maximize\ Z = 300x_1 + 120x_2$$

$$Subject\ to\ constraints: 6x_1 + 2x_2 \leq 40$$

$$3x_1 + 2x_2 \leq 30$$

$$x_1, x_2 \geq 0\ and\ integers$$

Now solving this problem in Tora, we get the following input table:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name	x1	x2		
Maximize	300.00	120.00		
Constr 1	6.00	2.00	<=	40.00
Constr 2	3.00	2.00	<=	30.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		
Integer (y/n)?	y	y		

Output:

Click BRANCHING variable (N10)

Variable	x1	x2
Var. Name	x1	x2
Value	3.33	10
Integer(y/n)?	y	y

(MAX) B&B SEARCH TREE (Click any GREEN node)

N10
z= 2200.00
x?

Here, the initial solution that we get is Z = 2200 at $x_1 = 3.33$ *and* $x_2 = 10$. Since x_1 is non integer, we proceed with integer programming.

N10	
z= 2200.00	
x1=3.33	
<N20, N21>	
N20	N21
x1<=3	x1>=4
z= 2160.00	z= 2160.00
x?	integer
	Best LBound

We select x_1 since it was non integer (3.33) and add two constraints in the first iteration to make two new subproblems, $x_1 \leq 3$ *and* $x_1 \geq 4$. We get the best integer solution at $x_1 \geq 4$, which maximizes Z at 2160.

Variable	x1	x2
Var. Name	x1	x2
Value	4	8
Integer(y/n)?	y	y

Final Solution: Maximized profit = 2160, if 4 reel-to-reel (x_1) and 8 cassette (x_2) recorders are made.

Ques 3.

Solve Q2 using Gomory's cutting plane method.

Solution:

Let the reel-to-reel recorders be denoted by variable = x_1

Let the cassette recorders be denoted by variable = x_2

So, the objective function will be to maximize profit, which can be written as: $300x_1 + 120x_2$

Time constraint for line 1: $6x_1 + 2x_2 \leq 40$

Time constraint for line 2: $3x_1 + 2x_2 \leq 30$

So, the formulated problem is:

$$Maximize\ Z = 300x_1 + 120x_2$$

$$Subject\ to\ constraints: 6x_1 + 2x_2 \leq 40$$

$$3x_1 + 2x_2 \leq 30$$

$$x_1, x_2 \geq 0\ and\ integers$$

Now solving this problem in Tora, we get the following input table:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name	x1	x2		
Maximize	300.00	120.00		
Constr 1	6.00	2.00	<=	40.00
Constr 2	3.00	2.00	<=	30.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Output:

Solving this we get, the following iterations:

Iteration 1:

Iteration 1	x1	x2			
Basic	x1	x2	sx3	sx4	Solution
z (max)	-300.00	-120.00	0.00	0.00	0.00
sx3	6.00	2.00	1.00	0.00	40.00
sx4	3.00	2.00	0.00	1.00	30.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Iteration 2:

Iteration 2	x1	x2			
Basic	x1	x2	sx3	sx4	Solution
z (max)	0.00	-20.00	50.00	0.00	2000.00
x1	1.00	0.33	0.17	0.00	6.67
sx4	0.00	1.00	-0.50	1.00	10.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Iteration 3 (Optimal):

Iteration 3	x1	x2			
Basic	x1	x2	sx3	sx4	Solution
z (max)	0.00	0.00	40.00	20.00	2200.00
x1	1.00	0.00	0.33	-0.33	3.33
x2	0.00	1.00	-0.50	1.00	10.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Now we see that the x_1 is non-integer. So we add another constraint known as Gomory's cut in the original table to solve further.

Gomory's cut is found to be:

$$-\frac{1}{3}S_1 + \frac{1}{3}S_2 + S_3 = -\frac{1}{3}$$

This is added in the original table and solved.

Input in Tora:

	x1	x2	x3	x4	x5	Enter <, >, or =	R.H.S.
Var. Name	x1	x2	s1	s2	s3		
Maximize	300.00	120.00	0.00	0.00	0.00		
Constr 1	6.00	2.00	1.00	0.00	0.00	=	40.00
Constr 2	3.00	2.00	0.00	1.00	0.00	=	30.00
Constr 3	0.00	0.00	-0.33	0.33	1.00	=	-0.33
Lower Bound	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n		

Output:

Iteration 1 and 2:

Phase 1 (Iter 1	x1	x2	s1	s2	s3				
Basic	x1	x2	x3	x4	x5	Rx6	Rx7	Rx8	Solution
z (min)	9.00	4.00	1.33	0.67	-1.00	0.00	0.00	0.00	70.33
Rx6	6.00	2.00	1.00	0.00	0.00	1.00	0.00	0.00	40.00
Rx7	3.00	2.00	0.00	1.00	0.00	0.00	1.00	0.00	30.00
Rx8	0.00	0.00	0.33	-0.33	-1.00	0.00	0.00	1.00	0.33
Lower Bound	0.00	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n	n				
Phase 1 (Iter 2	x1	x2	s1	s2	s3				
Basic	x1	x2	x3	x4	x5	Rx6	Rx7	Rx8	Solution
z (min)	0.00	1.00	-0.17	0.67	-1.00	-1.50	0.00	0.00	10.33
x1	1.00	0.33	0.17	0.00	0.00	0.17	0.00	0.00	6.67
Rx7	0.00	1.00	-0.50	1.00	0.00	-0.50	1.00	0.00	10.00
Rx8	0.00	0.00	0.33	-0.33	-1.00	0.00	0.00	1.00	0.33
Lower Bound	0.00	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n	n				

Iteration 3 and 4:

Phase 1 (Iter 3	x1	x2	s1	s2	s3				
Basic	x1	x2	x3	x4	x5	Rx6	Rx7	Rx8	Solution
z (min)	0.00	0.00	0.33	-0.33	-1.00	-1.00	-1.00	0.00	0.33
x1	1.00	0.00	0.33	-0.33	0.00	0.33	-0.33	0.00	3.33
x2	0.00	1.00	-0.50	1.00	0.00	-0.50	1.00	0.00	10.00
Rx8	0.00	0.00	0.33	-0.33	-1.00	0.00	0.00	1.00	0.33
Lower Bound	0.00	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n	n				
Phase 1 (Iter 4	x1	x2	s1	s2	s3				
Basic	x1	x2	x3	x4	x5	Rx6	Rx7	Rx8	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	0.00
x1	1.00	0.00	0.00	0.00	1.01	0.33	-0.33	-1.01	3.00
x2	0.00	1.00	0.00	0.50	-1.52	-0.50	1.00	1.52	10.50
x3	0.00	0.00	1.00	-1.00	-3.03	0.00	0.00	3.03	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n	n				

Iteration 5 (Optimal):

Phase 2 (Iter 5	x1	x2	s1	s2	s3				
Basic	x1	x2	x3	x4	x5	Rx6	Rx7	Rx8	Solution
z (max)	0.00	0.00	0.00	60.00	121.21	blocked	blocked	blocked	2160.00
x1	1.00	0.00	0.00	0.00	1.01	0.33	-0.33	-1.01	3.00
x2	0.00	1.00	0.00	0.50	-1.52	-0.50	1.00	1.52	10.50
x3	0.00	0.00	1.00	-1.00	-3.03	0.00	0.00	3.03	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n	n				

Here we see that x_2 is a non-integer therefore we add another Gomory's cut in the second table to solve it further.

Gomory's cut for x_2 is found to be:

$$-\frac{1}{2}s_2 + \frac{13}{25}s_3 + s_4 = -\frac{1}{2}$$

Now we give an input in Tora:

	x1	x2	x3	x4	x5	x6	Enter <, >, or =	R.H.S.
Var. Name	x1	x2	s1	s2	s3	s4		
Maximize	300.00	120.00	0.00	0.00	0.00	0.00		
Constr 1	6.00	2.00	1.00	0.00	0.00	0.00	=	40.00
Constr 2	3.00	2.00	0.00	1.00	0.00	0.00	=	30.00
Constr 3	0.00	0.00	-0.33	0.33	1.00	0.00	=	-0.33
Constr 4	0.00	0.00	0.00	-0.50	0.52	1.00	=	-0.50
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n		

Output:

Iteration 1 and Iteration 2:

Phase 1 (Iter 1	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	9.00	4.00	1.33	1.17	-1.52	-1.00	0.00	0.00	0.00	0.00	70.83
Rx7	6.00	2.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	40.00
Rx8	3.00	2.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	30.00
Rx9	0.00	0.00	0.33	-0.33	-1.00	0.00	0.00	0.00	1.00	0.00	0.33
Rx10	0.00	0.00	0.00	0.50	-0.52	-1.00	0.00	0.00	0.00	1.00	0.50
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Phase 1 (Iter 2	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	0.00	1.00	-0.17	1.17	-1.52	-1.00	-1.50	0.00	0.00	0.00	10.83
x1	1.00	0.33	0.17	0.00	0.00	0.00	0.17	0.00	0.00	0.00	6.67
Rx8	0.00	1.00	-0.50	1.00	0.00	0.00	-0.50	1.00	0.00	0.00	10.00
Rx9	0.00	0.00	0.33	-0.33	-1.00	0.00	0.00	0.00	1.00	0.00	0.33
Rx10	0.00	0.00	0.00	0.50	-0.52	-1.00	0.00	0.00	0.00	1.00	0.50
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Iteration 3 and Iteration 4:

Phase 1 (Iter 3	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	0.00	1.00	-0.17	0.00	-0.30	1.34	-1.50	0.00	0.00	-2.34	9.66
x1	1.00	0.33	0.17	0.00	0.00	0.00	0.17	0.00	0.00	0.00	6.67
Rx8	0.00	1.00	-0.50	0.00	1.04	2.00	-0.50	1.00	0.00	-2.00	9.00
Rx9	0.00	0.00	0.33	0.00	-1.34	-0.66	0.00	0.00	1.00	0.66	0.66
x4	0.00	0.00	0.00	1.00	-1.04	-2.00	0.00	0.00	0.00	2.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Phase 1 (Iter 4	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	0.00	0.33	0.17	0.00	-1.00	0.00	-1.17	-0.67	0.00	-1.00	3.63
x1	1.00	0.33	0.17	0.00	0.00	0.00	0.17	0.00	0.00	0.00	6.67
x6	0.00	0.50	-0.25	0.00	0.52	1.00	-0.25	0.50	0.00	-1.00	4.50
Rx9	0.00	0.33	0.17	0.00	-1.00	0.00	-0.17	0.33	1.00	0.00	3.63
x4	0.00	1.00	-0.50	1.00	0.00	0.00	-0.50	1.00	0.00	0.00	10.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Iteration 5 and Iteration 6:

Phase 1 (Iter 5	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	0.00	0.00	0.33	0.00	-1.34	-0.66	-1.00	-1.00	0.00	-0.34	0.66
x1	1.00	0.00	0.33	0.00	-0.35	-0.67	0.33	-0.33	0.00	0.67	3.67
x2	0.00	1.00	-0.50	0.00	1.04	2.00	-0.50	1.00	0.00	-2.00	9.00
Rx9	0.00	0.00	0.33	0.00	-1.34	-0.66	0.00	0.00	1.00	0.66	0.66
x4	0.00	0.00	0.00	1.00	-1.04	-2.00	0.00	0.00	0.00	2.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					
Phase 1 (Iter 6	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	-1.00	0.00
x1	1.00	0.00	0.00	0.00	1.01	0.00	0.33	-0.33	-1.01	0.00	3.00
x2	0.00	1.00	0.00	0.00	-1.00	1.00	-0.50	1.00	1.52	-1.00	10.00
x3	0.00	0.00	1.00	0.00	-4.07	-2.00	0.00	0.00	3.03	2.00	2.00
x4	0.00	0.00	0.00	1.00	-1.04	-2.00	0.00	0.00	0.00	2.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Iteration 7 (Optimal):

Phase 2 (Iter 7	x1	x2	s1	s2	s3	s4					
Basic	x1	x2	x3	x4	x5	x6	Rx7	Rx8	Rx9	Rx10	Solution
z (max)	0.00	0.00	0.00	0.00	183.61	120.00	blocked	blocked	blocked	blocked	2100.00
x1	1.00	0.00	0.00	0.00	1.01	0.00	0.33	-0.33	-1.01	0.00	3.00
x2	0.00	1.00	0.00	0.00	-1.00	1.00	-0.50	1.00	1.52	-1.00	10.00
x3	0.00	0.00	1.00	0.00	-4.07	-2.00	0.00	0.00	3.03	2.00	2.00
x4	0.00	0.00	0.00	1.00	-1.04	-2.00	0.00	0.00	0.00	2.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n	n	n	n					

Now we get an optimal solution with both integers.

Final Solution: Maximized profit = 2100, if 3 reel-to-reel (x_1) and 10 cassette (x_2) recorders are made.

Ques 4.

ABC manufacturing company faces the following problem: Should they make or buy each of their several products? The company policies specify that they will either make or buy the whole lot of each product in a complete lot. The company has four products to make or buy with six machines involved in making these products if they are made in the shop. The time per unit (in hours) required are as follows:

Product	Machine					
	A	B	C	D	E	F
1	0.04	0.02	0.02	0	0.03	0.06
2	0	0.01	0.05	0.15	0.09	0.06
3	0.02	0.06	0	0.06	0.02	0.02
4	0.06	0.04	0.15	0	0	0.05

Forty hours are available of each machine. One hundred ten units of each product are needed. The costs of making the products are listed below:

Products:	1	2	3	4
Cost/unit (in Rs.):	2.25	2.22	4.50	1.90

The costs to buy the products are:

Products:	1	2	3	4
Cost/unit (in Rs.):	3.10	2.60	4.75	2.25

Formulate this problem as a zero-one integer programming problem.

Solution:

Let x = cost of making product and y = cost of buying product.

So, the formulated problem is:

$$Min\ Z = 247.5x_1 + 244.2x_2 + 495x_3 + 209x_4 + 341y_1 + 286y_2 + 522.5y_3 + 247.5y_4$$

Subject to:

i)
$$x_1 + y_1 = 1$$
$$x_2 + y_2 = 1$$
$$x_3 + y_3 = 1$$
$$x_4 + y_4 = 1$$

ii)
$$4.4x_1 + 2.2x_3 + 6.6x_4 \leq 40$$
$$2.2x_1 + 1.1x_2 + 6.6x_3 + 4.4x_4 \leq 40$$
$$2.2x_1 + 5.5x_2 + 16.5x_4 \leq 40$$
$$16.5x_2 + 6.6x_3 \leq 40$$
$$3.3x_1 + 9.9x_2 + 2.2x_3 \leq 40$$
$$6.6x_1 + 6.6x_2 + 2.2x_3 + 5.5x_4 \leq 40$$

All $x'_{ij}s\ and\ y'_{ij}s = 0,1$

INPUT:

	x1	x2	x3	x4	x5	x6	x7	x8	Enter <, >, or =	R.H.S.
Var. Name	x1	x2	x3	x4	y1	y2	y3	y4		
Minimize	247.50	244.20	495.00	209.00	341.00	286.00	522.50	247.50		
Constr 1	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	=	1.00
Constr 2	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	=	1.00
Constr 3	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	=	1.00
Constr 4	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	=	1.00
Constr 5	4.40	0.00	2.20	6.60	0.00	0.00	0.00	0.00	<=	40.00
Constr 6	2.20	1.10	6.60	4.40	0.00	0.00	0.00	0.00	<=	40.00
Constr 7	2.20	5.50	0.00	16.50	0.00	0.00	0.00	0.00	<=	40.00
Constr 8	0.00	16.50	6.60	0.00	0.00	0.00	0.00	0.00	<=	40.00
Constr 9	3.30	9.90	2.20	0.00	0.00	0.00	0.00	0.00	<=	40.00
Constr 10	6.60	6.60	2.20	5.50	0.00	0.00	0.00	0.00	<=	40.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y	y	y		

OUTPUT:

Subproblem N10 -- Best Bound

Variable	x1	x2	x3	x4	x5	x6	x7	x8
Var. Name	x1	x2	x3	x4	y1	y2	y3	y4
Value	1	1	1	1	0	0	0	0
Integer(y/n)?	y	y	y	y	y	y	y	y

Min Z = 1195.70

With the values

$$x_1 = x_2 = x_3 = x_4 = 1$$

$$y_1 = y_2 = y_3 = y_4 = 0$$

Ques 5.

Consider the following production data:

Product	Profit Per Unit (Rs.)	Direct Labor Requirement (in Hours)
1	8	15
2	10	14
3	7	17

Fixed Cost	Direct Labor Requirement
10,000	Up to 20,000 hours
20,000	20,000-40,000 hours
30,000	40,000-70,000 hours

Formulate an integer programming problem to determine the production schedule so as to maximize the total net profit.

Solution:

Let the profit for product 1, 2 and 3 be denoted by $x_1,\ x_2\ and\ x_3$ respectively.

Let the fixed variables for product 1, 2 and 3 be denoted by $y_1,\ y_2\ and\ y_3$ respectively.

The formulated problem is:

$$Max\ Z = 8x_1 + 10x_2 +\ 7x_3 - 10{,}000y_1, -20{,}000y_2 - 30{,}000y_3$$

$$Subject\ to:$$

$$15x_1 + 14x_2 + 17x_3 \leq 20{,}000y_1 + 40{,}000y_2 + 70{,}000y_3$$

$$y_1 + y_2 + y_3 = 1$$

$$x_1,\ x_2,\ x_3,\ y_1,\ y_2 and\ y_3 \geq 0$$

Input in Tora:

	x1	x2	x3	x4	x5	x6	Enter <, >, or =	R.H.S.
Var. Name				y1	y2	y3		
Maximize	8.00	10.00	7.00	-10000.00	-20000.00	-30000.00		
Constr 1	15.00	14.00	17.00	-20000.00	-40000.00	-70000.00	<=	0.00
Constr 2	0.00	0.00	0.00	1.00	1.00	1.00	=	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y		

Output:

N10
z= 20000.00
integer
Best LBound

Detailed values:

Variable	x1	x2	x3	x4	x5	x6
Var. Name				y1	y2	y3
Value	0	5000	0	0	0	1
Integer(y/n)?	y	y	y	y	y	y

Final Solution: Maximized profit = 20000, if 5000 product 2 (x_2) units are made.

Ques 6.

Solve the following problem for profit maximization:

Model	Quantity	Plant A	Plant B	Plant C	Capacity	Selling Price	Plant
Standard	450	8	7.95	8.10	800	14.95	A
Deluxe	1050	8.50	8.60	8.45	600	18.85	B
New Deluxe	600	9.25	9.20	9.30	700	21.95	C

Solution:

New profit matrix after subtracting selling price from variable cost:

Model	Plant A	Plant B	Plant C	Quantity
Standard	6.95	7	6.85	450
Deluxe	10.35	10.25	10.40	1050
New Deluxe	12.70	12.75	12.65	600
Capacity	800	600	700	

Now forming an Integer Programming Problem:

$$Max\ Z = 6.95x_{11} + 7x_{12} + 6.85x_{13} + 10.35x_{21} + 10.25x_{22} + 10.40x_{23} + 12.70x_{31} + \ 12.75x_{32} + 12.65x_{33}$$

$$Subject\ to:$$

$$x_{11} + x_{21} + x_{31} = 800$$

$$x_{12} + \ x_{22} + x_{32} = 600$$

$$x_{13} \ + x_{23} + x_{33} = 700$$

$$x_{11} + x_{12} + x_{13} = 450$$

$$x_{21} + x_{22} + x_{23} = 1050$$

$$x_{31} + \ x_{32} + x_{33} = 600$$

$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0\ and\ integers.$$

Input in Tora:

	x1	x2	x3	x4	x5	x6	x7	x8	x9	Enter <, >, or =	R.H.S.
Var. Name	x11	x12	x13	x21	x22	x23	x31	x32	x33		
Maximize	6.95	7.00	6.85	10.35	10.25	10.40	12.70	12.75	12.65		
Constr 1	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	=	800.00
Constr 2	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	=	600.00
Constr 3	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	=	700.00
Constr 4	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	=	450.00
Constr 5	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	=	1050.00
Constr 6	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	=	600
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y	y	y	y		

Output:

z= 21680.00
integer
Best LBound

Detailed output with all the values:

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9
Var. Name	x11	x12	x13	x21	x22	x23	x31	x32	x33
Value	0	450	0	350	0	700	450	150	0
Integer(y/n)?	y	y	y	y	y	y	y	y	y

Final Solution: Following allocations would make the profit maximized to Rs. 21,680:

Model	**Plant A**	**Plant B**	**Plant C**	**Quantity**
Standard	0	450	0	450
Deluxe	350	0	700	1050
New Deluxe	450	150	0	600
Capacity	800	600	700	

Ques 7.

Find the minimum cost for covering all zones:

	i	**ii**	**iii**	**iv**	**v**	**vi**
A	73	91	87	82	78	80
B	81	85	69	76	74	85
C	75	72	83	84	78	91
D	93	96	86	91	83	82
E	90	91	79	89	89	76

Solution:

New balanced Matrix:

	i	**ii**	**iii**	**iv**	**v**	**vi**
A	73	91	87	82	78	80
B	81	85	69	76	74	85
C	75	72	83	84	78	91
D	93	96	86	91	83	82
E	90	91	79	89	89	76
F	0	0	0	0	0	0

Formulated IPP:

$$\begin{aligned} Min\ Z = {} & 73x_{11} + 91x_{12} + 87x_{13} + 82x_{14} + 78x_{15} + 80x_{16} + 81x_{21} + 85x_{22} + 69x_{23} \\ & + 76x_{24} + 74x_{25} + 85x_{26} + 75x_{31} + 72x_{32} + 83x_{33} + 84x_{34} + 78x_{35} + 91x_{36} \\ & + 93x_{41} + 96x_{42} + 86x_{43} + 91x_{44} + 83x_{45} + 82x_{46} + 90x_{51} + 91x_{52} + 79x_{53} \\ & + 89x_{54} + 89x_{55} + 76x_{56} + 0x_{61} + 0x_{62} + 0x_{63} + 0x_{64} + 0x_{65} + 0x_{66} \end{aligned}$$

$$Subject\ to:$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} = 1$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$$

$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$$

$$All\ x_{ij}'s \geq 0$$

Input in Tora:

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
Var. Name	x11	x12	x13	x14	x15	x16	x21	x22	x23	x24	x25	x26	x31	x32	x33
Maximize	73.00	91.00	87.00	82.00	78.00	80.00	81.00	85.00	69.00	76.00	74.00	85.00	75.00	72.00	83.00
Constr 1	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 2	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 3	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 4	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 6	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 7	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 8	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00
Constr 9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
Constr 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30
Var. Name	x34	x35	x36	x41	x42	x43	x44	x45	x46	x51	x52	x53	x54	x55	x56
Maximize	84.00	78.00	91.00	93.00	96.00	86.00	91.00	83.00	82.00	90.00	91.00	79.00	89.00	89.00	76.00
Constr 1	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 3	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 4	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 5	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 6	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 9	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 10	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00
Constr 12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

x31	x32	x33	x34	x35	x36	Enter <, >, or =	R.H.S.
x61	x62	x63	x64	x65	x66		
0.00	0.00	0.00	0.00	0.00	0.00		
1.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	1.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	1.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	1.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	1.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	1.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00	<=	1.00
1.00	1.00	1.00	1.00	1.00	1.00	<=	1.00
0.00	0.00	0.00	0.00	0.00	0.00		
infinity	infinity	infinity	infinity	infinity	infinity		
n	n	n	n	n	n		
y	y	y	y	y	y		

Output:

N10
z= 445.00
integer
Best LBound

Detailed output with all the values:

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17
Var. Name	x11	x12	x13	x14	x15	x16	x21	x22	x23	x24	x25	x26	x31	x32	x33	x34	x35
Value	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0

x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31	x32	x33	x34	x35	x36
x36	x41	x42	x43	x44	x45	x46	x51	x52	x53	x54	x55	x56	x61	x62	x63	x64	x65	x66
1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

Final Solution: Following allocations would make the minimum cost for covering all zones Rs.445:

	i	**ii**	**iii**	**iv**	**v**	**vi**
A	0	0	1	0	0	0
B	0	1	0	0	0	0
C	0	0	0	0	0	1
D	1	0	0	0	0	0
E	0	0	0	1	0	0
F	0	0	0	0	0	0

Ques 8. A manufacturer has distribution centers at Agra, Allahabad and Kolkata. These centers have availability of 40, 20 and 40 units of his product respectively. His retail outlets at A , B , C , D , E require 25 , 10, 20 , 30 and 15 units of the products respectively. The transport cost (in rupees) per unit between each center outlet is given below: -

Distribution Centre	**Retail Outlets**				
	A	**B**	**C**	**D**	**E**
Agra	55	30	40	50	40
Allahabad	35	30	100	45	60
Kolkata	40	60	95	35	30

Determine the optimal distribution so as to minimize the cost of transportation.

Solution: Formulated IPP:

$$Max\ Z = 55x_{11} + 30x_{12} + 40x_{13} + 50x_{14} + 40x_{15} + 35x_{21} + 30x_{22} + 100x_{23} + 45x_{24} + 60x_{25} + 40x_{31} + 60x_{32} + 95x_{33} + 35x_{34} + 30x_{35}$$

Subject to: -

$$x_{11} + x_{21} + x_{31} = 25$$

$$x_{12} + x_{22} + x_{32} = 10$$

$$x_{13} + x_{23} + x_{33} = 20$$

$$x_{14} + x_{24} + x_{34} = 30$$

$$x_{15} + x_{25} + x_{35} = 15$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 40$$

$$x_{21} + x_{22} + x_{23} + x_{14} + x_{15} = 20$$

$$x_{31} + x_{32} + x_{33} + x_{14} + x_{15} = 40$$

$$All\ x'_{ij}s \geq 0$$

Input in Tora:

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
Var. Name	x11	x12	x13	x14	x15	x21	x22	x23	x24	x25	x31	x32	x33	x34	x35
Maximize	55.00	30.00	40.00	50.00	40.00	35.00	30.00	100.00	45.00	60.00	40.00	60.00	95.00	35.00	30.00
Constr 1	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 2	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 3	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 4	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00
Constr 7	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 8	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

Output:

N10
z= 6100.00
integer
Best LBound

Detailed Output with all the values:-

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
Var. Name	x11	x12	x13	x14	x15	x21	x22	x23	x24	x25	x31	x32	x33	x34	x35
Value	15	0	0	25	0	0	0	0	5	15	10	10	20	0	0
Integer(y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

Final Solution: Following allocations would make to minimize the cost of transportation Rs.6100:

Distribution Centre	Retail Outlets				
	A	B	C	D	E
Agra	15	0	0	25	0
Allahabad	0	0	0	5	15
Kolkata	10	10	20	0	0

Ques 9. Find minimum production cost.

	Jan	Feb	Mar	Apr	May
Production cost	24	27	32	50	34
Demand	200	250	150	80	120
Capacity	250	225	250	200	225

Inventory carrying cost: 5 per unit per month

Solution:

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Supply**
Jan	24	29	34	39	44	250
Feb	-	27	32	37	42	225
Mar	-	-	32	37	42	250
Apr	-	-	-	50	55	200
May	-	-	-	-	34	225
Demand	200	250	150	80	120	

Rim condition not satisfied, adding a dummy variable.

	Jan	Feb	Mar	Apr	May	Dummy	Supply
Jan	24	29	34	39	44	0	250
Feb	-	27	32	37	42	0	225
Mar	-	-	32	37	42	0	250
Apr	-	-	-	50	55	0	200
May	-	-	-	-	34	0	225
Demand	200	250	150	80	120	350	

This will generate the following integer programming problem:

$$\begin{aligned} Minimize\ Z = 24x_{11} + 29x_{12} + 34x_{13} + 39x_{14} + 44x_{15} + 0x_{16} + 1000x_{21} + 27x_{22} \\ + 32x_{23} + 37x_{24} + 42x_{25} + 0x_{26} + 1000x_{31} + 1000x_{32} + 32x_{33} + 37x_{34} \\ + 42x_{35} + 0x_{36} + 1000x_{41} + 1000x_{42} + 1000x_{43} + 50x_{44} + 55x_{45} + 0x_{46} \\ + 1000x_{51} + 1000x_{52} + 1000x_{53} + 1000x_{54} + 34x_{55} + 0x_{56} \end{aligned}$$

$$Subject\ to$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 250$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 225$$

$$x_{31} + x_{32} + 32x_{33} + 37x_{34} + 42x_{35} + 0x_{36} = 250$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 200$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 225$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 200$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 250$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 150$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 80$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 120$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} = 350$$

$$All\ x_{ij}'s \geq 0$$

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
Var. Name	x11	x12	x13	x14	x15	x16	x21	x22	x23	x24	x25	x26	x31	x32	x33
Minimize	24.00	29.00	34.00	39.00	44.00	0.00	1000.00	27.00	32.00	37.00	42.00	0.00	1000.00	1000.00	32.00
Constr 1	1.00	1.00	34.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 6	1	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 7	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 8	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 9	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 10	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 11	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

	x13	x14	x15	x16	x17	x18	x19	x20	x21	x22	x23	x24
Var. Name	x31	x32	x33	x34	x35	x36	x41	x42	x43	x44	x45	x46
Minimize	1000.00	1000.00	32.00	37.00	42.00	0.00	1000.00	1000.00	1000.00	50.00	55.00	0.00
Constr 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 3	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 6	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 7	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 8	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 9	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 10	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 11	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y

	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	Enter <, >, or =	R.H.S.
Var. Name	x36	x41	x42	x43	x44	x45	x46	x51	x52	x53	x54	x55	x56		
Minimize	0.00	1000.00	1000.00	1000.00	50.00	55.00	0.00	1000.00	1000.00	1000.00	1000.00	34.00	0.00		
Constr 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	250.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	225.00
Constr 3	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	250.00
Constr 4	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	=	200.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	=	225.00
Constr 6	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	=	200.00
Constr 7	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	=	250.00
Constr 8	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	=	150.00
Constr 9	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	=	80.00
Constr 10	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	=	120.00
Constr 11	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	=	350.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y		

Output:

N10
z= 23440.00
integer
Best UBound

Detailed output with all the values:

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18
Var. Name	x11	x12	x13	x14	x15	x16	x21	x22	x23	x24	x25	x26	x31	x32	x33	x34	x35	x36
Value	200	25	0	0	0	25	0	225	0	0	0	0	0	0	150	80	0	20

x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30
x41	x42	x43	x44	x45	x46	x51	x52	x53	x54	x55	x56
0	0	0	0	0	200	0	0	0	0	120	105

Final Solution: Following allocations would make the minimum production cost to Rs.23440:

	Jan	Feb	Mar	Apr	May	Dummy
Jan	200	25	0	0	0	25
Feb	0	225	0	0	0	0
Mar	0	0	150	80	0	20
Apr	0	0	0	0	0	200
May	0	0	0	0	120	105

Ques 10. Find minimum production cost.

Month	Max Production	Demand	Production Cost	Inventory Cost
Jan	120	100	60	15
Feb	120	130	60	15
March	150	160	55	20
April	150	160	55	20
May	150	140	50	20
June	150	140	50	20

Solution:

	Jan	**Feb**	**Mar**	**Apr**	**May**	**June**	**Supply**
Jan	60	75	90	110	130	150	120
Feb	-	60	75	95	115	135	120
Mar	-	-	55	75	95	115	150
Apr	-	-	-	55	75	95	150
May	-	-	-	-	50	70	150
June						50	150
Demand	100	130	160	160	140	140	

Supply not equal to demand, we add a dummy variable

	Jan	**Feb**	**Mar**	**Apr**	**May**	**June**	**Dummy**	**Supply**
Jan	60	75	90	110	130	150	0	120
Feb	-	60	75	95	115	135	0	120
Mar	-	-	55	75	95	115	0	150
Apr	-	-	-	55	75	95	0	150
May	-	-	-	-	50	70	0	150
June						50	0	150
Demand	100	130	160	160	140	140	10	

$$\begin{aligned}
Minimize\ Z = {} & 60x_{11} + 75x_{12} + 90x_{13} + 110x_{14} + 130x_{15} + 150x_{16} + 0x_{17} + 1000x_{21} \\
& + 60x_{22} + 75x_{23} + 95x_{24} + 115x_{25} + 135x_{26} + 0x_{27} + 1000x_{31} + 1000x_{32} \\
& + 55x_{33} + 75x_{34} + 95x_{35} + 115x_{36} + 0x_{37} + 1000x_{41} + 1000x_{42} + 1000x_{43} \\
& + 55x_{44} + 75x_{45} + 95x_{46} + 0x_{47} + 1000x_{51} + 1000x_{52} + 1000x_{53} + 1000x_{54} \\
& + 50x_{55} + 70x_{56} + \ 0x_{57} + 1000x_{61} + 1000x_{62} + 1000x_{63} + 1000x_{64} \\
& + 1000x_{65} + 50x_{66} + 0x_{67}
\end{aligned}$$

$$Subject\ to$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} = 120$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} = 120$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} = 150$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} = 150$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} = 150$$

$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} = 150$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 100$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 130$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 160$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} = 160$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 140$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 140$$

$$x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} = 10$$

$$All\ x'_{ij}s \geq 0$$

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
Var. Name	x11	x12	x13	x14	x15	x16	x17	x21	x22	x23	x24	x25	x26	x27	x31
Minimize	60.00	75.00	90.00	110.00	130.00	150.00	0.00	1000.00	60.00	75.00	95.00	115.00	135.00	0.00	1000.00
Constr 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 7	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 8	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 9	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 10	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 11	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 12	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 13	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30
Var. Name	x32	x33	x34	x35	x36	x37	x41	x42	x43	x44	x45	x46	x47	x51	x52
Minimize	1000.00	55.00	75.00	95.00	115.00	0.00	1000.00	1000.00	1000.00	55.00	75.00	95.00	0.00	1000.00	1000.00
Constr 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 3	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
Constr 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 7	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Constr 8	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 9	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 10	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 11	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 12	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 13	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

	x30	x31	x32	x33	x34	x35	x36	x37	x38	x39	x40	x41	x42	Enter <, >, or =	R.H.S.
Var. Name	x52	x53	x54	x55	x56	x57	x61	x62	x63	x64	x65	x66	x67		
Minimize	1000.00	1000.00	1000.00	50.00	70.00	0.00	1000.00	1000.00	1000.00	1000.00	1000.00	50.00	0.00		
Constr 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	120.00
Constr 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	120.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	150.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	150.00
Constr 5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	150.00
Constr 6	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	=	150.00
Constr 7	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	=	100.00
Constr 8	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	=	130.00
Constr 9	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	=	160.00
Constr 10	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	=	160.00
Constr 11	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	=	140.00
Constr 12	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	=	140.00
Constr 13	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	=	10.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y		

Output:

N10
z= 55350.00
integer
Best UBound

Detailed output:

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18
Var. Name	x11	x12	x13	x14	x15	x16	x17	x21	x22	x23	x24	x25	x26	x27	x31	x32	x33	x34
Value	100	10	10	0	0	0	0	0	120	0	0	0	0	0	0	0	150	0

x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31	x32	x33	x34	x35	x36	x37
x35	x36	x37	x41	x42	x43	x44	x45	x46	x47	x51	x52	x53	x54	x55	x56	x57	x61	x62
0	0	0	0	0	0	150	0	0	0	0	0	0	0	140	0	10	0	0

x38	x39	x40	x41	x42
x63	x64	x65	x66	x67
0	10	0	140	0

Final Solution: Following allocations would make the minimum production cost to Rs.55350**:**

	Jan	**Feb**	**Mar**	**Apr**	**May**	**June**	**Dummy**
Jan	100	10	10	0	0	0	0
Feb	0	120	0	0	0	0	0
Mar	0	0	150	0	0	0	0
Apr	0	0	0	150	0	0	0
May	0	0	0	0	140	0	10
June	0	0	0	10	0	0	140

Ques 11.

ABC Company wishes to develop a monthly production schedule for the next three months depending upon the sales commitments, the company can keep the production constant, allowing fluctuations in inventory or inventories can be maintained at constant level, with fluctuating production. Fluctuating production necessitates, working overtime, the cost of which is estimated to be double the normal production cost of 12 Rupee per unit. Fluctuating inventories result in inventory carrying of 2 Rupee per unit per month. If the company fails to fulfill its sales commitment it incurs a shortage cost of 4 Rupee per unit per month. The production capacities for the next three month are shown in table: -

Production Capacity

Month	Regular	Overtime	Sales
1	50	30	60
2	50	0	120
3	60	50	40

Determine optimal production schedule.

Solution:

	1	2	D	3	
R1	12	14	0	16	50
O1	24	26	0	28	30
R2	16	12	0	14	50
R3	20	16	0	12	60
O3	32	28	0	24	50
	60	120	20	40	240

$$Minimize\ Z = 12x_{11} + 14x_{12} + 0x_{13} + 16x_{14} + 24x_{21} + 36x_{22} + 0x_{23} + 28x_{24} + 16x_{31} + 12x_{32} + 0x_{33} + 14x_{34} + 20x_{41} + 16x_{42} + 0x_{43} + 12x_{44} + 32x_{51} + 28x_{52} + 0x_{53} + 24x_{54}$$

$$Subject\ to$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 50$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 30$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 50$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 60$$

$$x_{51} + x_{52} + x_{53} + x_{54} = 50$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 120$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 40$$

$$All\ x_{ij}'s \geq 0$$

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
Var. Name	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41
Minimize	12.00	14.00	0.00	16.00	24.00	36.00	0.00	28.00	16.00	12.00	0.00	14.00	20.00
Constr 1	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 6	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00
Constr 7	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 8	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 9	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y

	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	Enter <, >, or =	R.H.S.
Var. Name	x24	x31	x32	x33	x34	x41	x42	x43	x44	x51	x52	x53	x54		
Minimize	28.00	16.00	12.00	0.00	14.00	20.00	16.00	0.00	12.00	32.00	28.00	0.00	24.00		
Constr 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	50.00
Constr 2	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	30.00
Constr 3	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	=	50.00
Constr 4	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	=	60.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	=	50.00
Constr 6	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	=	60
Constr 7	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	=	120.00
Constr 8	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	=	20.00
Constr 9	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	=	40.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n	n	n		
Integer (y/n)?	y	y	y	y	y	y	y	y	y	y	y	y	y		

Output:

N10
z= 3600.00
integer
Best UBound

Detailed output:

Variable	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14
Var. Name	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41	x42
Value	30	20	0	0	30	0	0	0	0	50	0	0	0	20

x15	x16	x17	x18	x19	x20
x43	x44	x51	x52	x53	x54
0	40	0	30	20	0

Final allocations will be:

	1	2	D	3
R1	30	20	0	0
O1	21	0	0	0
R2	0	50	0	0
R3	0	20	40	0
O3	0	30	20	0

Ques 12.

Consider the game with the following payoff table:

	Player B	
Player A	B1	B2
A1	2	6
A2	-2	λ

i) Show that the game is strictly determinable, whatever λ may be.

ii) Determine the value of the game.

Solution:

Solving the above game using excel:

	Player B		
Player A	B1	B2	Row Minima
A1	2	6	2
A2	-2	λ	-2
Column Maxima	2	6	
Maximin	2		
Minimax	2		

i) Ignoring the value of λ, the maximin = minimax = 2.
Thus, the game is strictly determinable.

ii) The value of game,
V = 2

Ques 13.

Determine which of the following two-person zero sum games are strictly determinable and fair. Give the optimum strategies for each player in the case of strictly determinable

a)

	Player B	
Player A	B1	B2
A1	1	2
A2	4	-3

b)

	Player B	
Player A	B1	B2
A1	-5	2
A2	-7	-4

Solution:

a)		Player B		
	Player A	B1	B2	Row Minima
	A1	1	2	1
	A2	4	-3	-3
	Column Maxima	4	2	
	Maximin	1		
	Minimax	2		

Since, Maximin != Minimax

Therefore, game (a) does not have a saddle point. So, game a is neither strictly determinable nor fair.

b)		Player B		
	Player A	B1	B2	Row Minima
	A1	-5	2	-5
	A2	-7	-4	-7
	Column Maxima	-5	2	
	Maximin	-5		
	Minimax	-5		

Here, Minimax = Maximin. So, the value of the game is -5.

This game is strictly determinable but not fair.

Player A chooses strategy A1, and Player B chooses strategy B1.

Ques 14.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below: Include in your answer:

iii) Strategy selection for each player
iv) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	1	7	3	4
A2	5	6	4	5
A3	7	2	0	3

	Player B				
Player A	B1	B2	B3	B4	Row Minima
A1	1	7	3	4	1
A2	5	6	4	5	4
A3	7	2	0	3	0
Column Maxima	7	7	4	5	
Maximin	4				
Minimax	4				

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is 4.

This game is strictly determinable.

Player A chooses strategy A2, and Player B chooses strategy B3.

Ques 15.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below: Include in your answer:

iii) Strategy selection for each player
iv) The value of the game to each player

	Player B				
Player A	B1	B2	B3	B4	B5
A1	-2	0	0	5	3
A2	3	2	1	2	2
A3	-4	-3	0	-2	6
A4	5	3	-4	2	6

Solution:

			Player B			
Player A	B1	B2	B3	B4	B5	Row Minima
A1	-2	0	0	5	3	-2
A2	3	2	1	2	2	1
A3	-4	-3	0	-2	6	-4
A4	5	3	-4	2	6	-4
Column Maxima	5	3	1	5	6	
Maximin	1					
Minimax	1					

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is 4.

This game is strictly determinable.

Player A chooses strategy A2, and Player B chooses strategy B3.

Ques 16.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below: Include in your answer:

iii) Strategy selection for each player
iv) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	3	-5	0	6
A2	-4	-2	1	2
A3	5	4	2	3

Solution:

	Player B				
Player A	B1	B2	B3	B4	Row Minima
A1	3	-5	0	6	-5
A2	-4	-2	1	2	-4
A3	5	4	2	3	2
Column Maxima	5	4	2	6	
Maximin	2				
Minimax	2				

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is 2.This game is strictly determinable.

Player A chooses strategy A3, and Player B chooses strategy B3.

Ques 17.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below: Include in your answer:

i) Strategy selection for each player
ii) The value of the game to each player

	Player B		
Player A	B1	B2	B3
A1	-2	15	-2
A2	-5	-6	-4
A3	-5	20	-8

Solution:

	Player B			
Player A	B1	B2	B3	Row Minima
A1	-2	15	-2	-2
A2	-5	-6	-4	-6
A3	-5	20	-8	-8
Column Maxima	-2	20	-2	
Maximin	-2			
Minimax	-2			

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is -2.

This game is strictly determinable.

There are two points of optimal strategies (A1, B1) & (A1, B3).

Ques 18.

Solve the following game by using maximin(minimax) principle whose payoff matrix are given below: Include in your answer:

iii) Strategy selection for each player
iv) The value of the game to each player

Does the game have saddle point?

	Player B			
Player A	B1	B2	B3	B4
A1	-5	3	1	10
A2	5	5	4	6
A3	4	-2	0	-5

Solution:

	Player B				
Player A	B1	B2	B3	B4	Row Minima
A1	-5	3	1	10	-5
A2	5	5	4	6	4
A3	4	-2	0	-5	-5
Column Maxima	5	5	4	10	
Maximin	4				
Minimax	4				

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is 4.

This game is strictly determinable.

Player A chooses strategy A2, and Player B chooses strategy B3.

Ques 19.

Two competitive manufacturers are producing a new toy under license from a patent holder. In order to meet the demand, they have the option of running the plant for 8, 16 or 24 hours a day. As the length of production increases so does the cost. One of the manufacturers, say A, has set up the matrix given below. He uses the matrix to estimate the percentage of the market that he could capture and maintain the different production schedules:

	Manufacturer B		
Manufacturer A	C1: 8 hrs.	C2:16 hrs.	C3:24 hrs.
S1:8 hrs.	60%	56%	34%
S2:16 hrs.	63%	60%	55%
S3:24 hrs.	83%	72%	60%

Solution:

	Manufacturer B			
Manufacturer A	C1: 8hrs	C2: 16hrs	C3: 24hrs	Row Minima
S1: 8hrs	60	56	34	34
S2: 16hrs	63	60	55	55
S3: 24hrs	83	72	60	60
Column Maxima	83	72	60	
Maximin	60			
Minimax	60			

Here, Minimax = Maximin. Therefore, it has a saddle point and the value of the game is 60.

This game is strictly determinable.

Optimal Strategy: (S3, C3), i.e. both should produce at the level of 24hours per day.

Ques 20.

Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	2	1	0	-2
A2	1	0	3	2

Solution:

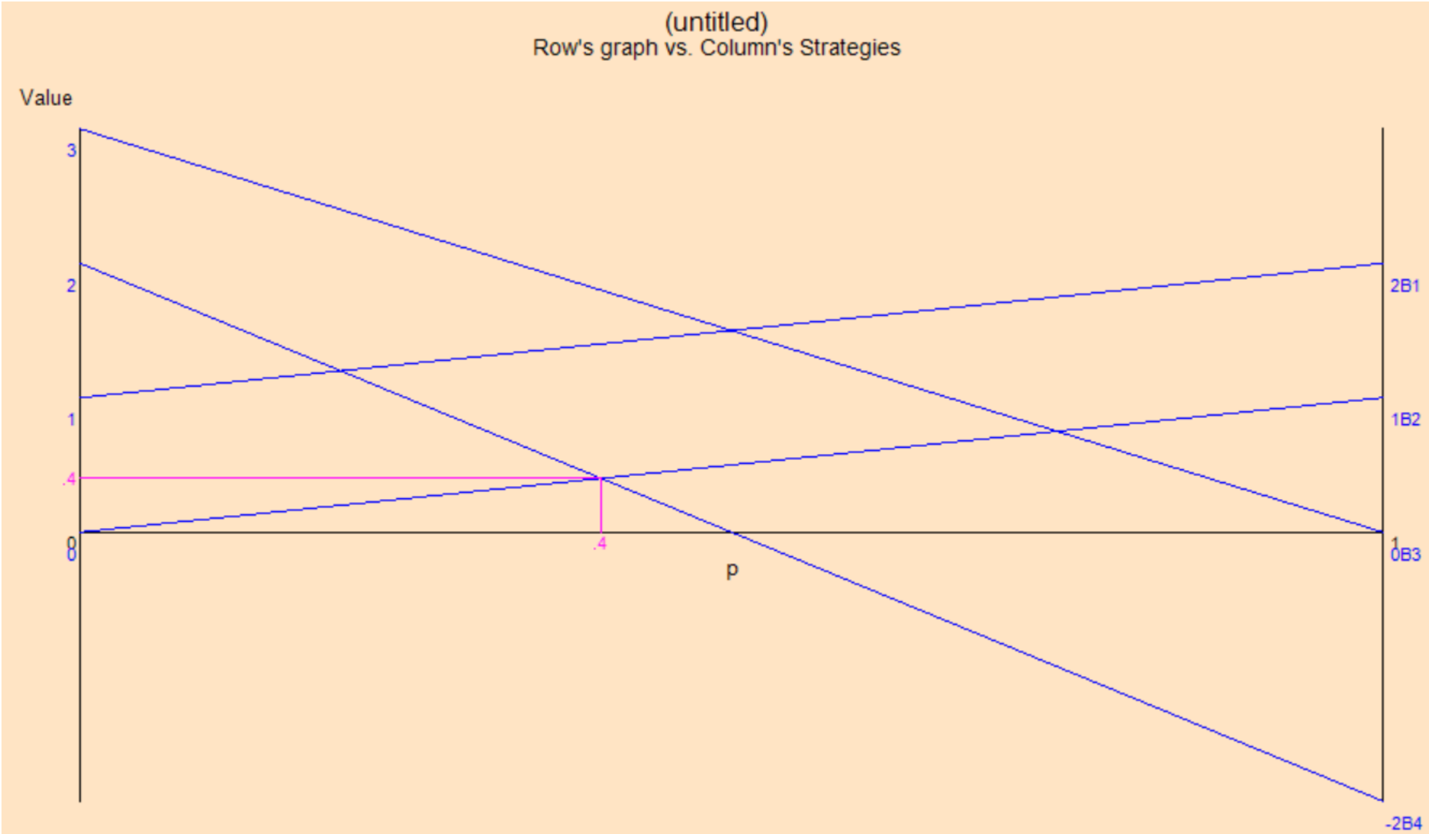
Since, it is a 2×n game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in TORA using graphical method:

INPUT:

	B1	B2	B3	B4
A1	2	1	0	-2
A2	1	0	3	2

OUTPUT:



Game Theory Results

(untitled) Solution

	B1	B2	B3	B4	Row Mix
A1	2	1	0	-2	.4
A2	1	0	3	2	.6
Column Mix--->	0	.8	0	.2	
Value of game (to row)	.4				

Here, the optimal strategy of player A and B are respectively:

$$||0.4, 0.6|| \text{ and } ||0, 0.8, 0, 0.2||$$

And, the value of game is 0.4.

Ques 21.

Solve graphically, the rectangular game, whose payoff matrix is:

	Player B	
Player A	B1	B2
A1	1	-3
A2	3	5
A3	-1	6
A4	4	1
A5	2	2
A6	-5	0

Solution:

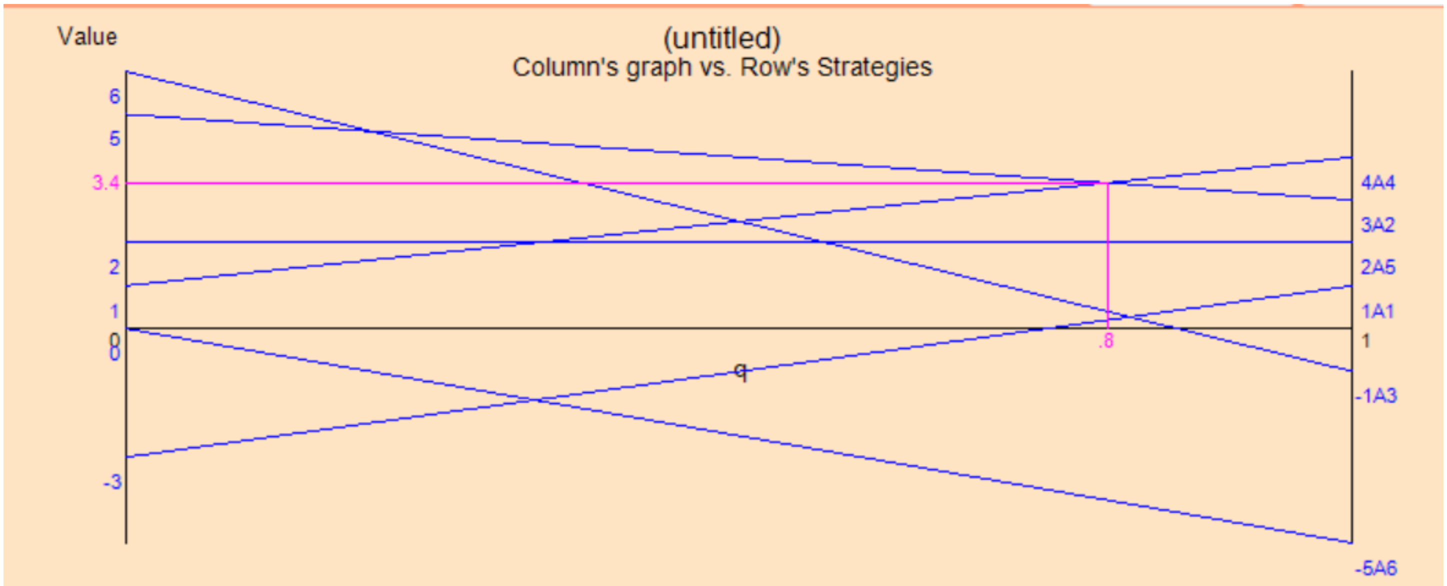
Since, it is a m×2 game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in POM QM using graphical method:

INPUT:

	B1	B2
A1	1	-3
A2	3	5
A3	-1	6
A4	4	1
A5	2	2
A6	-5	0

OUTPUT:



Game Theory Results

(untitled) Solution

	B1	B2	Row Mix
A1	1	-3	0
A2	3	5	.6
A3	-1	6	0
A4	4	1	.4
A5	2	2	0
A6	-5	0	0
Column Mix--->	.8	.2	
Value of game (to row)	3.4		

Here, the optimal strategy of player A and B are respectively:

||0, 0.6, 0, 0.4, 0, 0|| and ||0.8, 0.2||

And, the value of game is 3.4.

Ques 22.

Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	6	5	2	3
A2	1	2	6	3

Solution:

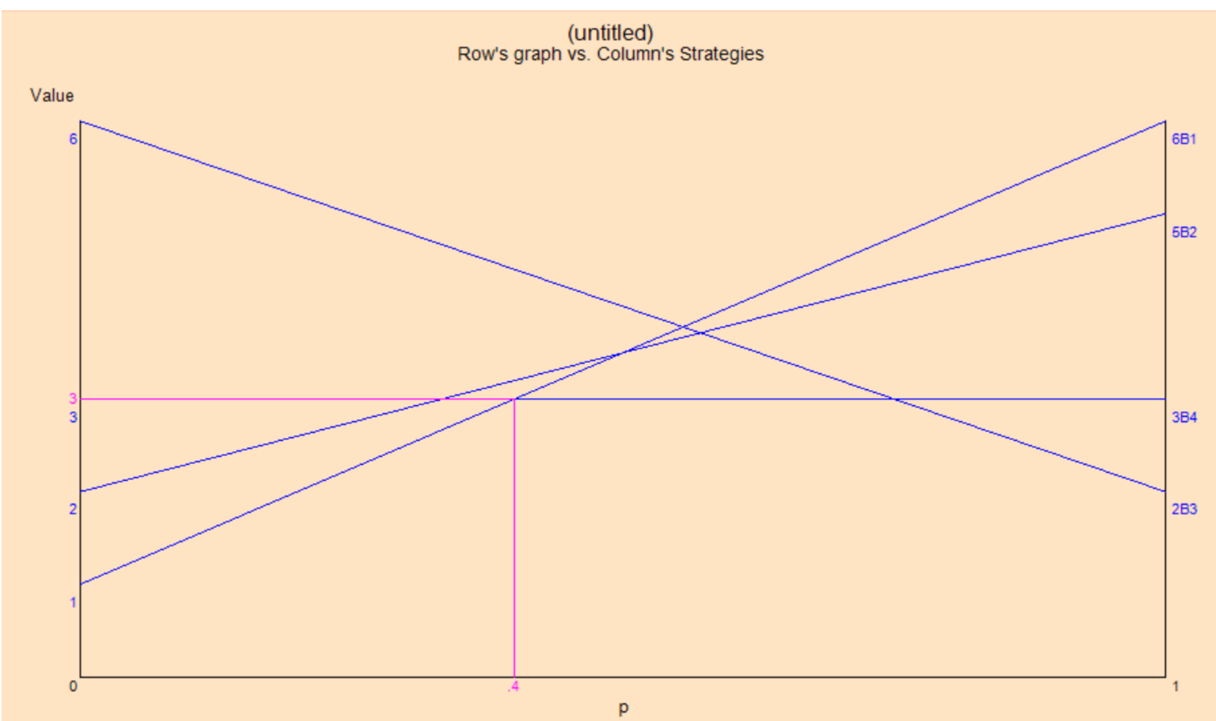
Since, it is a 2×n game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in POM QM using graphical method:

INPUT:

	B1	B2	B3	B4
A1	6	5	2	3
A2	1	2	6	3

OUTPUT:



Game Theory Results

(untitled) Solution

	B1	B2	B3	B4	Row Mix
A1	6	5	2	3	.4
A2	1	2	6	3	.6
Column Mix--->	0	0	0	1	
Value of game (to row)	3				

Here, the optimal strategy of player A and B are respectively:

$$||0.4, 0.6|| \text{ and } ||0, 0, 0, 1||$$

And, the value of game is 3.

Ques 23.

Solve graphically, the rectangular game, whose payoff matrix is:

	Player B			
Player A	B1	B2	B3	B4
A1	3	1	0	-2
A2	1	0	3	2

Solution:

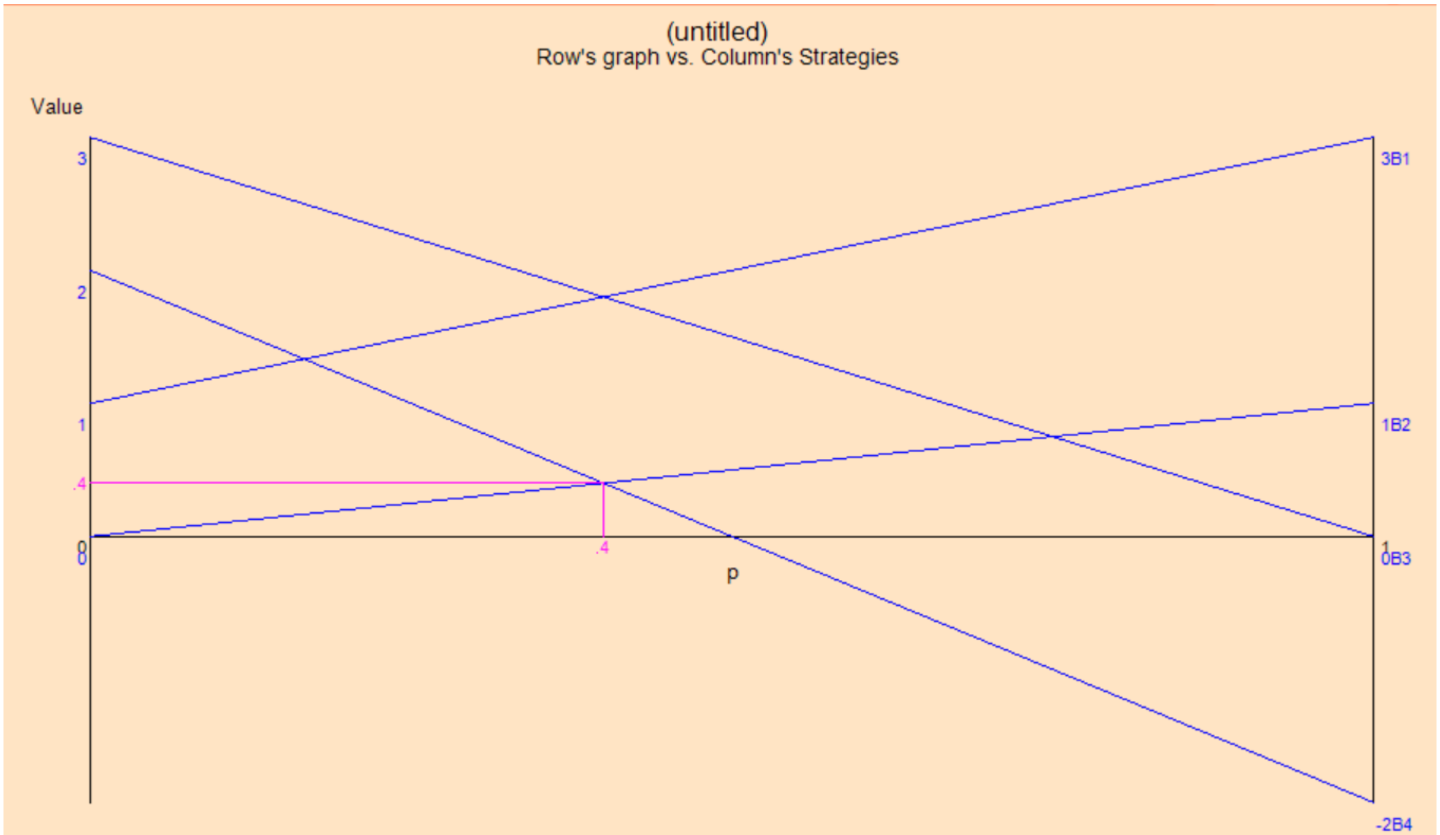
Since, it is a 2×n game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in POM QM using graphical method:

INPUT:

	B1	B2	B3	B4
A1	3	1	0	-2
A2	1	0	3	2

OUTPUT:



Game Theory Results

(untitled) Solution

	B1	B2	B3	B4	Row Mix
A1	3	1	0	-2	.4
A2	1	0	3	2	.6
Column Mix--->	0	.8	0	.2	
Value of game (to row)	.4				

Here, the optimal strategy of player A and B are respectively:

||0.4, 0.6|| and ||0, 0.8, 0, 0.2||

And, the value of game is 0.4.

Ques 24.

Two computer manufacturers A and B are attempting to sell computer systems to two banks 1 and 2. Company A has 4 salesmen; company B only has 3 salesmen available. The computer companies must decide upon how many salesmen to assign to sell computer to each bank. Thus, company A can assign 4 salesmen to bank 1 and none to bank 2 or three to bank 1 and one to bank 2, etc.

Each bank will buy one computer system. The probability that a bank will buy from a particular computer company is directly related to the number of salesmen calling from that company, relative to the total salesmen calling. Thus, if company A assigns three salesmen to bank 1 and company B assigns two salesmen, the odds would be three out of five that bank 1 would purchase company A's computer system. (If none calls from either company the odds are one-half for buying either computer.)

Let the payoff be the expected number of computer systems that company A sells. (2 minus this payoff is the expected number company B sells).

What strategy would company A use in allocating its salesmen? What strategy should company B use? What is the value of the game to company A? What is the meaning of the value of the game in this problem?

Solution:

Payoff Matrix:

Player A		B1	B2	B3	B4
		0	1	2	3
A1	0	1/2	0	0	0
A2	1	1	1/2	1/3	1/4
A3	2	1	2/3	1/2	2/5
A4	3	1	3/4	3/5	1/2
A5	4	1	4/5	2/3	4/7

Formula used to calculate payoff probabilities:

$$P(e) = \frac{n(A)}{n(A) + n(B)}$$

Final solution:

Player A		B1	B2	B3	B4	B5	Min
		0	1	2	3		
A1	0	1/2	0	0	0		0
A2	1	1	1/2	1/3	1/4		1/4
A3	2	1	2/3	1/2	2/5		2/5
A4	3	1	3/4	3/5	1/2		1/2
A5	4	1	4/5	2/3	4/7		4/7
Max		1	4/5	2/3	4/7		

Minimax		4/7
Maximin		4/7

This is a pure strategy, with saddle point at 4/7.

Strategy used by A-> A5

Strategy used by B-> B4

Ques 25.

The firms A and B have for years been selling's a competitive product which forms a part of both firms' total sales. The marketing executive of firm A raised the question, "What should be the firm's strategies in terms of advertising product in question?" The market research team of firm A developed the following data for varying degrees of advertising:

V) No advertising, medium advertising, and large advertising for both firms will result in equal market shares.

VI) Firm A with no advertising: 40% of market with medium advertising by firm B and 28% of the market with large advertising by firm B

VII) Firm A using medium advertising: 70% of the market with no advertising by firm B and 45% of the market with large advertising by firm B

VIII) Firm A using large advertising: 75% of the market with no advertising by firm B and 47.5% of the market with medium advertising by firm B

c) Based upon their foregoing information, answer the marketing executive's questions.
d) What advertising policy should firm A pursue when consideration is given to the above factors: selling price Rs. 4 per unit: variable cost of product Rs. 2.5 per unit; annual volume of 30,000 units for firm A; cost of annual medium advertising Rs. 5,000 and cost of annual large advertising Rs. 15,000? What contribution before other fixed costs is available to the firm?

Solution:

Player A	B1	B2	B3	Row Minima
A1	0.5	0.4	0.28	0.28
A2	0.7	0.5	0.45	0.45
A3	0.75	0.48	0.5	0.48
Column Maxima	0.75	0.5	0.5	
Maximin	0.48			
Minimax	0.5			

Since, Maximin != Minimax.

Therefore, it is an impure strategy.

Using rule of dominance our new pay off matrix will be:

	B1	B2
A1	0.5	0.45
A2	0.48	0.50

Now as we know,

Probability of firm A = p1+p2 = 1

where,

P1 is when strategy A1 is chooses by firm A.

P2 is when strategy A2 is chooses by firm A.

Probability of firm B = q1+q2 = 1

where, q1 is when strategy B1 is chosen by firm B.

and, q2 is when strategy B2 is chosen by firm B.

Now,

p1	0.333333
q1	0.666667
p2	0.666667
q2	0.333333
game	0.483333

Quantity table:

Player A		B1	B2	B3	MIN	Max
A1		22500.00	18000.00	12600.00	12600.00	26500.00
A2		26500.00	17500.00	15250.00	15250.00	18000.00
A3		18750.00	6375.00	7500.00	6375.00	15250.00
MAXMIN	15250.00					
MINMAX	15250.00					
A2,B2 is the strategy and (a2,b3) is our saddle point						

Profit table:

Player A		B1	B2	B3	MIN	Max
A1		22500.00	18000.00	12600.00	12600.00	26500.00
A2		26500.00	17500.00	15250.00	15250.00	18000.00
A3		18750.00	6375.00	7500.00	6375.00	15250.00
MAXMIN	15250.00					
MINMAX	15250.00					
A2,B2 is the strategy and (a2,b3) is our saddle point						

Firm A should adopt medium strategy and spend Rs 5000.

Ques 26.

Solve the given payoff matrix. Transfer the zero sum two-person game into equivalent linear programming problem. Solve using Simplex Method.

	B1	B2	B3
A1	**5**	**3**	**7**
A2	**7**	**9**	**1**
A3	**10**	**6**	**2**

Solution:

The given payoff matrix will result in following LPP:

$$Min\ Z\ =\ x_1 + x_2 + x_3$$

$$Subject\ to,$$

$$a_{11}x_1 + a_{21}x_2 + a_{31}x_3\ >=\ 1$$
$$a_{12}x_1 + a_{22}x_2 + a_{32}x_3\ >=\ 1$$
$$a_{13}x_1 + a_{23}x_2 + a_{33}x_3\ >=\ 1$$

$$Min\ Z\ =\ x_1 + x_2 + x_3$$
$$Subject\ to,$$
$$5x_1 + 7x_2 + 10x_3\ >=\ 1$$
$$3x_1 + 9x_2 + 6x_3\ >=\ 1$$
$$7x_1 + 1x_2 + 2x_3\ >=\ 1$$
$$x_1, x_2, x_3 \geq 0$$

$$where, x_i = \frac{p_i}{v}\ and\ \sum_{i=1}^{3} x_i = \frac{1}{v}$$
$$v = value\ of\ the\ game$$

INPUT IN TORA:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name	x1	x2	x3		
Minimize	1.00	1.00	1.00		
Constr 1	5.00	7.00	10.00	>=	1.00
Constr 2	3.00	9.00	6.00	>=	1.00
Constr 3	7.00	1.00	2.00	>=	1
Lower Bound					
Upper Bound					
Unrestr'd (y/n)?					

OUTPUT:

Phase 1 (Iter 1										
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	15.00	17.00	18.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	3.00
Rx7	5.00	7.00	10.00	-1.00	0.00	0.00	1.00	0.00	0.00	1.00
Rx8	3.00	9.00	6.00	0.00	-1.00	0.00	0.00	1.00	0.00	1.00
Rx9	7.00	1.00	2.00	0.00	0.00	-1.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n							
Phase 1 (Iter 2										
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	6.00	4.40	0.00	0.80	-1.00	-1.00	-1.80	0.00	0.00	1.20
x3	0.50	0.70	1.00	-0.10	0.00	0.00	0.10	0.00	0.00	0.10
Rx8	0.00	4.80	0.00	0.60	-1.00	0.00	-0.60	1.00	0.00	0.40
Rx9	6.00	-0.40	0.00	0.20	0.00	-1.00	-0.20	0.00	1.00	0.80
Lower Bound	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n							
Phase 1 (Iter 3										
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	0.00	4.80	0.00	0.60	-1.00	0.00	-1.60	0.00	-1.00	0.40
x3	0.00	0.73	1.00	-0.12	0.00	0.08	0.12	0.00	-0.08	0.03
Rx8	0.00	4.80	0.00	0.60	-1.00	0.00	-0.60	1.00	0.00	0.40
x1	1.00	-0.07	0.00	0.03	0.00	-0.17	-0.03	0.00	0.17	0.13
Lower Bound	0.00	0.00	0.00							

Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	0.00	0.00	-6.55	1.36	-1.00	-0.55	-2.36	0.00	-0.45	0.18
x2	0.00	1.00	1.36	-0.16	0.00	0.11	0.16	0.00	-0.11	0.05
Rx8	0.00	0.00	-6.55	1.36	-1.00	-0.55	-1.36	1.00	0.55	0.18
x1	1.00	0.00	0.09	0.02	0.00	-0.16	-0.02	0.00	0.16	0.14
Lower Bound	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n							
Phase 1 (Iter 5										
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	0.00
x2	0.00	1.00	0.60	0.00	-0.12	0.05	0.00	0.12	-0.05	0.07
Sx4	0.00	0.00	-4.80	1.00	-0.73	-0.40	-1.00	0.73	0.40	0.13
x1	1.00	0.00	0.20	0.00	0.02	-0.15	0.00	-0.02	0.15	0.13
Lower Bound	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n							
Phase 2 (Iter 6										
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Rx7	Rx8	Rx9	Solution
z (min)	0.00	0.00	-0.20	0.00	-0.10	-0.10	blocked	blocked	blocked	0.20
x2	0.00	1.00	0.60	0.00	-0.12	0.05	0.00	0.12	-0.05	0.07
Sx4	0.00	0.00	-4.80	1.00	-0.73	-0.40	-1.00	0.73	0.40	0.13
x1	1.00	0.00	0.20	0.00	0.02	-0.15	0.00	-0.02	0.15	0.13
Lower Bound	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity							

Value of Z = 0.20 therefore V = 5, since Z = 1/v

$$p_1 = x_1 * v = 0.13 * 5 = 0.65$$

$$p_2 = x_2 * v = 0.07 * 5 = 0.35$$

$$p_3 = x_3 * v = 0 * 5 = 0$$

Ques 27.

For the following payoff matrix, transform the zero sum two-person game into an equivalent linear programming problem and solve it by simplex method.

	Player B		
Player A	B1	B2	B3
A1	9	1	4
A2	0	6	3
A3	5	2	8

Solution:

We first add a suitable number so that the value of the game is strictly positive. Adding 1 to all the entries the problem becomes:

	Player B		
Player A	B1	B2	B3
A1	10	2	5
A2	1	7	4
A3	6	3	9

The above payoff matrix will result in the following LPP:

The game for player A is to maximize v,

Subject to $10x_1 + x_2 + 6x_3 \geq v$

$$2x_1 + 7x_2 + 3x_3 \geq v$$

$$5x_1 + 4x_2 + 9x_3 \geq v$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1,\ x_2,\ x_3 \geq 0$$

The game for player B is to minimize v,

Subject to $10y_1 + 2y_2 + 5y_3 \le v$

$$1y_1 + 7y_2 + 4y_3 \le v$$

$$6y_1 + 3y_2 + 9y_3 \le v$$

$$y_1 + y_2 + y_3 = 1$$

$$y_1,\ y_2,\ y_3 \ge 0$$

Now, dividing each of the constraints of player A and B by v and writing $\frac{x_i}{v} = x_i'$ and $\frac{y_j}{v} = y_j'$ the constraint can be written as:

$$10x_1' + x_2' + 6x_3' \ge 1$$

$$2x_1' + 7x_2' + 3x_3' \ge 1$$

$$5x_1' + 4x_2' + 9x_3' \ge 1$$

$$x_1' + x_2' + x_3' = 1/v$$

$$x_1',\ x_2',\ x_3' \ge 0$$

and,

$$10y_1' + 2y_2' + 5y_3' \le 1$$

$$1y_1' + 7y_2' + 4y_3' \le 1$$

$$6y_1' + 3y_2' + 9y_3' \le 1$$

$$y_1' + y_2' + y_3' = 1/v$$

$$y_1',\ y_2',\ y_3' \ge 0$$

A wishes to maximize v that is he wishes to

Minimize $\frac{1}{v} = x_1' + x_2' + x_3'$

Subject to $10x_1' + x_2' + 6x_3' \ge 1$

$$2x_1' + 7x_2' + 3x_3' \ge 1$$

$$5x_1' + 4x_2' + 9x_3' \ge 1$$

$$x_1',\ x_2',\ x_3' \ge 0$$

B wishes to minimize v that is he wishes to

Maximize $\frac{1}{v} = y_1' + y_2' + y_3'$

Subject to $10y_1' + 2y_2' + 5y_3' \leq 1$

$$1y_1' + 7y_2' + 4y_3' \leq 1$$

$$6y_1' + 3y_2' + 9y_3' \leq 1$$

$$y_1',\ y_2',\ y_3' \geq 0$$

We can see that the previous two LPP's are primal dual pair. So, if we solve any one, the solution to the other may be obtained from the optimal table of the considered problem. So, we'll solve the last problem by simplex method using TORA:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name	y1	y2	y3		
Maximize	1.00	1.00	1.00		
Constr 1	10.00	2.00	5.00	<=	1.00
Constr 2	1.00	7.00	4.00	<=	1.00
Constr 3	6.00	3.00	9.00	<=	1.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Iteration1 and Iteration2:

Iteration 1	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
sx4	10.00	2.00	5.00	1.00	0.00	0.00	1.00
sx5	1.00	7.00	4.00	0.00	1.00	0.00	1.00
sx6	6.00	3.00	9.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				
Iteration 2	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	-0.80	-0.50	0.10	0.00	0.00	0.10
x1	1.00	0.20	0.50	0.10	0.00	0.00	0.10
sx5	0.00	6.80	3.50	-0.10	1.00	0.00	0.90
sx6	0.00	1.80	6.00	-0.60	0.00	1.00	0.40
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Iteration3:

Iteration 3	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	0.00	-0.09	0.09	0.12	0.00	0.21
x1	1.00	0.00	0.40	0.10	-0.03	0.00	0.07
x2	0.00	1.00	0.51	-0.01	0.15	0.00	0.13
sx6	0.00	0.00	5.07	-0.57	-0.26	1.00	0.16
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Iteration 4(Optimal):

Iteration 4	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	0.00	0.00	0.08	0.11	0.02	0.21
x1	1.00	0.00	0.00	0.15	-0.01	-0.08	0.06
x2	0.00	1.00	0.00	0.04	0.17	-0.10	0.12
x3	0.00	0.00	1.00	-0.11	-0.05	0.20	0.03
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Value of Z = 0.21 therefore, V = 4.76, since Z = 1/v

$y_1 = y_1' \times v = 0.06 \times 4.76 = 0.28$ $\quad x_1 = x_1' \times v = 0.08 \times 4.76 = 0.38$

$y_2 = y_2' \times v = 0.12 \times 4.76 = 0.57$ $\quad x_2 = x_2' \times v = 0.11 \times 4.76 = 0.52$

$y_3 = y_3' \times v = 0.03 \times 4.76 = 0.14$ $\quad x_3 = x_3' \times v = 0.02 \times 4.76 = 0.09$

Since, we have added 1 to all the entries therefore, value of game, $v = 4.76 - 1 = 3.76$

$$v = 3.76$$

Therefore, optimal strategy of the two players A and B are respectively;

$$||0.38, 0.52, 0.09|| \text{ and } ||0.28, 0.57, 0.14||$$

And the value of game $v = 3.76$

Ques 28.

For the following payoff matrix, transform the zero sum two-person game into an equivalent linear programming problem and solve it by simplex method.

	Company A		
Company B	A1	A2	A3
B1	2	-2	3
B2	-3	5	-1

Solution:

Assuming that Company A is gainer so, the new matrix will be:

	Company B	
Company A	B1	B2
A1	-2	3
A2	2	-5
A3	-3	1

We first add a suitable number so that the value of the game is strictly positive. Adding 6 to all the entries the problem becomes:

	Company B	
Company A	B1	B2
A1	4	9
A2	8	1
A3	3	7

The above payoff matrix will result in the following LPP:

The game for Company A is to maximize v,

Subject to $4x_1 + 8x_2 + 3x_3 \geq v$

$$9x_1 + 1x_2 + 7x_3 \geq v$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1,\ x_2,\ x_3 \geq 0$$

The game for Company B is to minimize v,

Subject to $4y_1 + 9y_2 \leq v$

$$8y_1 + 1y_2 \leq v$$

$$3y_1 + 7y_2 \leq v$$

$$y_1 + y_2 = 1$$

$$y_1,\ y_2\ \geq 0$$

Now, dividing each of the constraints of Company A and B by v and writing $\frac{x_i}{v} = x_i'$ and $\frac{y_j}{v} = y_j'$ the constraint can be written as:

$$4x_1' + 8x_2' + 3x_3' \geq 1$$

$$9x_1' + 1x_2' + 7x_3' \geq 1$$

$$x_1' + x_2' + x_3' = 1/v$$

$$x_1',\ x_2',\ x_3' \geq 0$$

and,

$$4y_1' + 9y_2' \leq 1$$

$$8y_1' + 1y_2' \leq 1$$

$$3y_1' + 7y_2' \leq 1$$

$$y_1' + y_2' = 1/v$$

$$y_1',\ y_2'\ \geq 0$$

A wishes to maximize v that is he wishes to

Minimize $\frac{1}{v} = x_1' + x_2' + x_3'$

Subject to $4x_1' + 8x_2' + 3x_3' \geq 1$

$9x_1' + 1x_2' + 7x_3' \geq 1$

$x_1',\ x_2',\ x_3' \geq 0$

B wishes to minimize v that is he wishes to

Maximize $\frac{1}{v} = y_1' + y_2'$

Subject to $4y_1' + 9y_2' \leq 1$

$8y_1' + 1y_2' \leq 1$

$3y_1' + 7y_2' \leq 1$

$y_1',\ y_2' \geq 0$

We can see that the previous two LPP's are primal dual pair. So, if we solve any one, the solution to the other may be obtained from the optimal table of the considered problem. So, we'll solve the last problem by simplex method using TORA:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name	y1	y2		
Maximize	1.00	1.00		
Constr 1	4.00	9.00	<=	1.00
Constr 2	8.00	1.00	<=	1.00
Constr 3	3.00	7.00	<=	1.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Iteration1:

Iteration 1	y1	y2				
Basic	x1	x2	sx3	sx4	sx5	Solution
z (max)	-1.00	-1.00	0.00	0.00	0.00	0.00
sx3	4.00	9.00	1.00	0.00	0.00	1.00
sx4	8.00	1.00	0.00	1.00	0.00	1.00
sx5	3.00	7.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Iteration2:

Iteration 2	y1	y2				
Basic	x1	x2	sx3	sx4	sx5	Solution
z (max)	0.00	-0.88	0.00	0.13	0.00	0.13
sx3	0.00	8.50	1.00	-0.50	0.00	0.50
x1	1.00	0.13	0.00	0.13	0.00	0.13
sx5	0.00	6.63	0.00	-0.38	1.00	0.63
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Iteration 3(Optimal):

Iteration 3	y1	y2				
Basic	x1	x2	sx3	sx4	sx5	Solution
z (max)	0.00	0.00	0.10	0.07	0.00	0.18
x2	0.00	1.00	0.12	-0.06	0.00	0.06
x1	1.00	0.00	-0.01	0.13	0.00	0.12
sx5	0.00	0.00	-0.78	0.01	1.00	0.24
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Value of Z = 0.18 therefore, V = 5.55, since Z = 1/v

$y_1 = y_1' \times v = 0.12 \times 5.55 = 0.66$

$y_2 = y_2' \times v = 0.06 \times 5.55 = 0.33$

$x_1 = x_1' \times v = 0.10 \times 5.55 = 0.55$

$x_2 = x_2' \times v = 0.07 \times 5.55 = 0.38$

$x_3 = x_3' \times v = 0 \times 5.55 = 0$

Since, we have added 6 to all the entries therefore, value of game, $v = 5.55 - 6 = 0.45$

$$v = 0.45$$

Therefore, optimal strategy of the two Company A and B are respectively;

$$||0.55, 0.38, 0|| \text{ and } ||0.66, 0.33||$$

And the value of game $v = 0.45$

Ques 29.

A soft drink company calculated the market share of two of its products against its major competitor, which has three products. The company found out the impact of additional advertisement in any one of its products against the other.

	Company B		
Company A	B1	B2	B3
A1	6	7	15
A2	20	12	10

What is the best strategy for the company as well as the competitor? What is the payoff obtained by the company and the competitor in the long run? Use the graphical method to obtain the solution.

Solution:

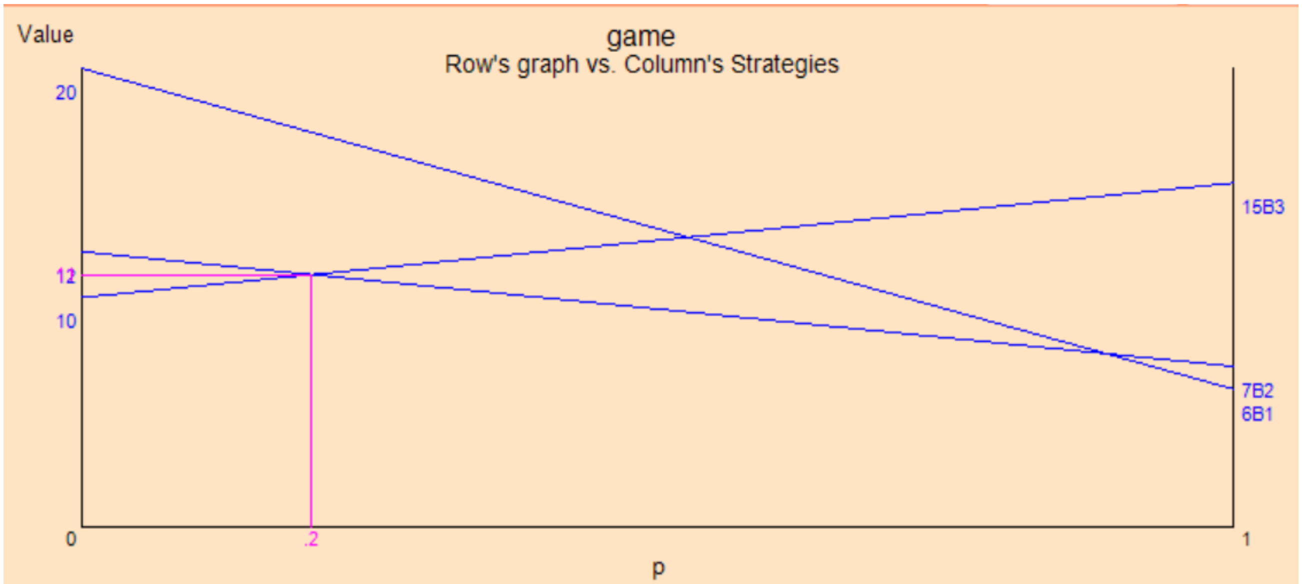
Since, it is a 2×n game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in POM QM using graphical method:

INPUT:

	B1	B2	B3
A1	6	7	15
A2	20	12	10

OUTPUT:



Game Theory Results

game solution

	B1	B2	B3	Row Mix
A1	6	7	15	.2
A2	20	12	10	.8
Column Mix--->	0	.5	.5	
Value of game (to row)	11			

Here, the optimal strategy of player A and B are respectively:

$$||0.2, 0.8|| \text{ and } ||0, 0.5, 0, 0.5||$$

And, the value of game is 11.

Ques 30.

Two Firms A and B make color and black & white television sets. Firm A can make either 150 color sets in a week or an equal number of black and white sets, and make a profit of Rs 400 per color set and Rs 300 per black & white set. Firm B can, on the other hand, make either 300 color sets, or 150 color and 150 black & white sets, or 300 black & white sets per week. It also has the same profit margin on the two sets as A. Each week there is a market of 150 color sets and 300 black & white sets. The manufacturers would share market depending upon the proportion in which they manufacture a particular type of set.

Write the payoff matrix of A per week. Obtain, graphically, A's and B's optimal strategies and the value of the game.

Solution:

Let,

A1: 150 colour sets;

A2: 150 black & white sets

B1: 300 colour sets;

B2: 150 colour and 150 black & white sets;

B3: 300 black & white sets.

The payoff matrix of A per week will be:

	Firm B		
Firm A	B1	B2	B3
A1	20,000	30,000	60,000
A2	45,000	45,000	30,000

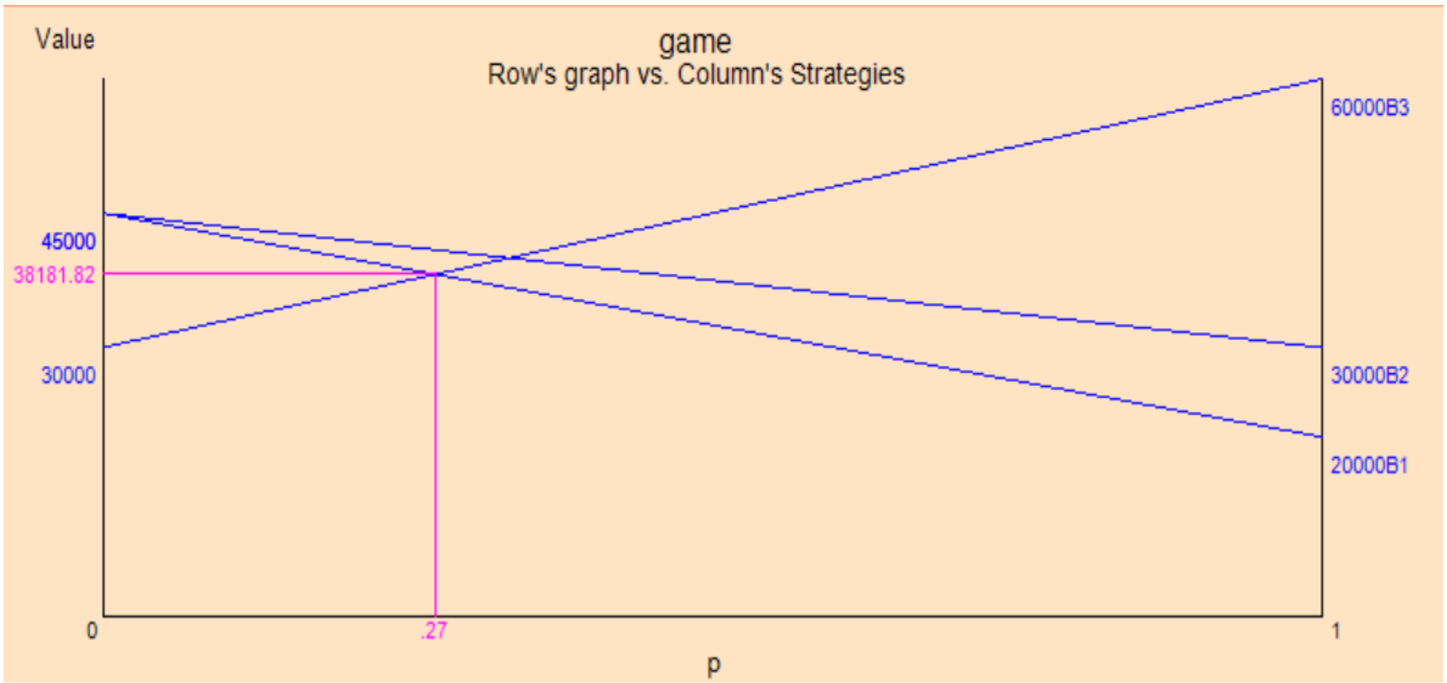
Since, it is a 2×n game therefore, we can solve the given rectangular game using graphical method.

Solving the given game in POM QM using graphical method:

INPUT:

	B1	B2	B3
A1	20000	30000	60000
A2	45000	45000	30000

OUTPUT:



Game Theory Results

game solution

	B1	B2	B3	Row Mix
A1	20000	30000	60000	.27
A2	45000	45000	30000	.73
Column Mix--->	.55	0	.45	
Value of game (to row)	38181.82			

Here, the optimal strategy of player A and B are respectively:

||0.27, 0.73|| and ||0.55, 0, 0.45||

And, the value of game is 38181.82.

Ques 31.

In a town there are only two discount stores ABC and XYZ. Both stores run annual pre-Diwali sales. Sales are advertised through local newspapers with the aid of an advertising firm. ABC stores constructed following payoff in units of Rs 1,00,000. Find the optimal strategies for both stores and the value of the game:

	Store XYZ		
Store ABC	B1	B2	B3
A1	1	-2	1
A2	-1	3	2
A3	-1	-2	3

Solution:

We first add a suitable number so that the value of the game is strictly positive. Adding 3 to all the entries the problem becomes:

	Store XYZ		
Store ABC	B1	B2	B3
A1	4	1	4
A2	2	6	5
A3	2	1	6

The above payoff matrix will result in the following LPP:

The game for player A is to maximize v,

Subject to $4x_1 + 2x_2 + 2x_3 \geq v$

$$x_1 + 6x_2 + x_3 \geq v$$

$$4x_1 + 5x_2 + 6x_3 \geq v$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1,\ x_2,\ x_3 \geq 0$$

The game for player B is to minimize v,

Subject to $4y_1 + 1y_2 + 4y_3 \leq v$

$$2y_1 + 6y_2 + 5y_3 \leq v$$

$$2y_1 + 1y_2 + 6y_3 \leq v$$

$$y_1 + y_2 + y_3 = 1$$

$$y_1,\ y_2,\ y_3 \geq 0$$

Now, dividing each of the constraints of player A and B by v and writing $\frac{x_i}{v} = x_i'$ and $\frac{y_j}{v} = y_j'$ the constraint can be written as:

$$4x_1' + 2x_2' + 2x_3' \geq 1$$

$$x_1' + 6x_2' + x_3' \geq 1$$

$$4x_1' + 5x_2' + 6x_3' \geq 1$$

$$x_1' + x_2' + x_3' = 1/v$$

$$x_1',\ x_2',\ x_3' \geq 0$$

and,

$$4y_1' + y_2' + 4y_3' \leq 1$$

$$2y_1' + 6y_2' + 5y_3' \leq 1$$

$$2y_1' + y_2' + 6y_3' \leq 1$$

$$y_1' + y_2' + y_3' = 1/v$$

$$y_1',\ y_2',\ y_3' \geq 0$$

A wishes to maximize v that is he wishes to

Minimize $\frac{1}{v} = x_1' + x_2' + x_3'$

Subject to $4x_1' + 2x_2' + 2x_3' \geq 1$

$$x_1' + 6x_2' + x_3' \geq 1$$

$$4x_1' + 5x_2' + 6x_3' \geq 1$$

$$x_1',\ x_2',\ x_3' \geq 0$$

B wishes to minimize v that is he wishes to

Maximize $\frac{1}{v} = y_1' + y_2' + y_3'$

Subject to $4y_1' + y_2' + 4y_3' \leq 1$

$$2y_1' + 6y_2' + 5y_3' \leq 1$$

$$2y_1' + y_2' + 6y_3' \leq 1$$

$$y_1',\ y_2',\ y_3' \geq 0$$

We can see that the previous two LPP's are primal dual pair. So, if we solve any one, the solution to the other may be obtained from the optimal table of the considered problem. So, we'll solve the last problem by simplex method using TORA:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name	y1	y2	y3		
Maximize	1.00	1.00	1.00		
Constr 1	4.00	1.00	4.00	<=	1.00
Constr 2	2.00	6.00	5.00	<=	1.00
Constr 3	2.00	1.00	6.00	<=	1.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Iteration1 and Iteration2:

Iteration 1	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
sx4	4.00	1.00	4.00	1.00	0.00	0.00	1.00
sx5	2.00	6.00	5.00	0.00	1.00	0.00	1.00
sx6	2.00	1.00	6.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				
Iteration 2	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	-0.75	0.00	0.25	0.00	0.00	0.25
x1	1.00	0.25	1.00	0.25	0.00	0.00	0.25
sx5	0.00	5.50	3.00	-0.50	1.00	0.00	0.50
sx6	0.00	0.50	4.00	-0.50	0.00	1.00	0.50
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Iteration3 (Optimal):

Iteration 3	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	0.00	0.41	0.18	0.14	0.00	0.32
x1	1.00	0.00	0.86	0.27	-0.05	0.00	0.23
x2	0.00	1.00	0.55	-0.09	0.18	0.00	0.09
sx6	0.00	0.00	3.73	-0.45	-0.09	1.00	0.45
Lower Bound	0.00	0.00	0.00				

Value of Z = 0.32 therefore, V = 3.125, since Z = 1/v

$y_1 = y_1' \times v = 0.23 \times 3.125 = 0.71$ $x_1 = x_1' \times v = 0.18 \times 3.125 = 0.56$

$y_2 = y_2' \times v = 0.09 \times 3.125 = 0.28$ $x_2 = x_2' \times v = 0.14 \times 3.125 = 0.44$

$y_3 = y_3' \times v = 0 \times 3.125 = 0$ $x_3 = x_3' \times v = 0 \times 3.125 = 0$

Since, we have added 3 to all the entries therefore, value of game, $v = 3.125 - 3 = 0.125$

$$v = 0.125 \approx 0$$

Therefore, optimal strategy of the two players A and B are respectively;

$$||0.56, 0.44, 0|| \text{ and } ||0.71, 0.28, 0||$$

And the value of game $v = 0(approx.)$

Ques 32.

Assume that the two firms are competing for market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share for Firm A and the decrease in market share for Firm B. Determine the optimal strategies for each firm.

	Firm B		
Firm A	No Promotion	Moderate Promotion	Much Promotion
No Promotion	5	0	-10
Moderate Promotion	10	6	2
Much Promotion	20	15	10

Formulate a suitable linear programming model of the game, with respect to minimizing player B's losses and derive the optimal strategy for B.

Solution:

We first add a suitable number so that the value of the game is strictly positive. Adding 11 to all the entries the problem becomes:

	Firm B		
Firm A	No Promotion	Moderate Promotion	Much Promotion
No Promotion	16	11	1
Moderate Promotion	21	17	13
Much Promotion	31	26	21

The above payoff matrix will result in the following LPP:

The game for player A is to maximize v,

Subject to $16x_1 + 21x_2 + 31x_3 \geq v$

$11x_1 + 17x_2 + 26x_3 \geq v$

$1x_1 + 13x_2 + 21x_3 \geq v$

$x_1 + x_2 + x_3 = 1$

$x_1,\ x_2,\ x_3 \geq 0$

The game for player B is to minimize v,

Subject to $16y_1 + 11y_2 + 1y_3 \leq v$

$21y_1 + 17y_2 + 13y_3 \leq v$

$31y_1 + 26y_2 + 21y_3 \leq v$

$y_1 + y_2 + y_3 = 1$

$y_1,\ y_2,\ y_3 \geq 0$

Now, dividing each of the constraints of player A and B by v and writing $\frac{x_i}{v} = x_i'$ and $\frac{y_j}{v} = y_j'$ the constraint can be written as:

$$16x_1' + 21x_2' + 31x_3' \geq 1$$

$$11x_1' + 17x_2' + 26x_3' \geq 1$$

$$1x_1' + 13x_2' + 21x_3' \geq 1$$

$$x_1' + x_2' + x_3' = 1/v$$

$$x_1',\ x_2',\ x_3' \geq 0$$

and,

$$16y_1' + 11y_2' + 1y_3' \leq 1$$

$$21y_1' + 17y_2' + 13y_3' \leq 1$$

$$31y_1' + 26y_2' + 21y_3' \leq 1$$

$$y_1' + y_2' + y_3' = 1/v$$

$$y_1',\ y_2',\ y_3' \geq 0$$

A wishes to maximize v that is he wishes to

Minimize $\frac{1}{v} = x_1' + x_2' + x_3'$

Subject to $16x_1' + 21x_2' + 31x_3' \geq 1$

$$11x_1' + 17x_2' + 26x_3' \geq 1$$

$$1x_1' + 13x_2' + 21x_3' \geq 1$$

$$x_1',\ x_2',\ x_3' \geq 0$$

B wishes to minimize v that is he wishes to

Maximize $\frac{1}{v} = y_1' + y_2' + y_3'$

Subject to $16y_1' + 11y_2' + 1y_3' \leq 1$

$$21y_1' + 17y_2' + 13y_3' \leq 1$$

$$31y_1' + 26y_2' + 21y_3' \leq 1$$

$$y_1',\ y_2',\ y_3' \geq 0$$

We can see that the previous two LPP's are primal dual pair. So, if we solve any one, the solution to the other may be obtained from the optimal table of the considered problem. So, we'll solve the last problem by simplex method using TORA:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name	y1	y2	y3		
Maximize	1.00	1.00	1.00		
Constr 1	16.00	11.00	1.00	<=	1.00
Constr 2	21.00	17.00	13.00	<=	1.00
Constr 3	31.00	26.00	21.00	<=	1.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Iteration1 and Iteration2:

Iteration 1	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
sx4	16.00	11.00	1.00	1.00	0.00	0.00	1.00
sx5	21.00	17.00	13.00	0.00	1.00	0.00	1.00
sx6	31.00	26.00	21.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				
Iteration 2	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	-0.16	-0.32	0.00	0.00	0.03	0.03
sx4	0.00	-2.42	-9.84	1.00	0.00	-0.52	0.48
sx5	0.00	-0.61	-1.23	0.00	1.00	-0.68	0.32
x1	1.00	0.84	0.68	0.00	0.00	0.03	0.03
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Iteration3 (Optimal):

Iteration 3	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.48	0.24	0.00	0.00	0.00	0.05	0.05
sx4	14.52	9.76	0.00	1.00	0.00	-0.05	0.95
sx5	1.81	0.90	0.00	0.00	1.00	-0.62	0.38
x3	1.48	1.24	1.00	0.00	0.00	0.05	0.05
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Value of Z = 0.05 therefore, V = 20, since Z = 1/v

$y_1 = y_1' \times v = 0 \times 20 = 0$ $x_1 = x_1' \times v = 0 \times 20 = 0$

$y_2 = y_2' \times v = 0 \times 20 = 0$ $x_2 = x_2' \times v = 0 \times 20 = 0$

$y_3 = y_3' \times v = 0.05 \times 20 = 1$ $x_3 = x_3' \times v = 0.05 \times 20 = 1$

Since, we have added 11 to all the entries therefore, value of game, $v = 20 - 11 = 9$

$$v = 9$$

Therefore, optimal strategy of the two players A and B are respectively;

$$||0, 0, 1|| \text{ and } ||0, 0, 1||$$

And the value of game $v = 9$

Ques 33.

Firm X is fighting for its life against the determination of firm Y to drive it out of the industry. Firm X has the choice of increasing the price, leaving it unchanged, or lowering it. Firm Y has the same three options. Firm X's gross sales in the event of each of the pairs of choices are shown below:

Firm Y's Pricing Strategies	Firm Y's Pricing Strategies		
	Increase Price	Do not change	Reduce Price
Increase Price	90	80	110
Do not change	110	100	90
Reduce Price	120	70	80

Assuming firm X as the maximizing one, formulate and solve the problem as a linear programming problem.

Solution:

Firm Y's Pricing Strategies	Firm Y's Pricing Strategies		
	Increase Price	Do not change	Reduce Price
Increase Price	90	80	110
Do not change	110	100	90
Reduce Price	120	70	80

The above payoff matrix will result in the following LPP:

The game for player A is to maximize v,

Subject to $90x_1 + 110x_2 + 120x_3 \geq v$

$80x_1 + 100x_2 + 70x_3 \geq v$

$110x_1 + 90x_2 + 80x_3 \geq v$

$x_1 + x_2 + x_3 = 1$

$x_1,\ x_2,\ x_3 \geq 0$

The game for player B is to minimize v,

Subject to $90y_1 + 80y_2 + 110y_3 \leq v$

$110y_1 + 100y_2 + 90y_3 \leq v$

$120y_1 + 70y_2 + 80y_3 \leq v$

$y_1 + y_2 + y_3 = 1$

$y_1,\ y_2,\ y_3 \geq 0$

Now, dividing each of the constraints of player A and B by v and writing $\frac{x_i}{v} = x_i'$ and $\frac{y_j}{v} = y_j'$ the constraint can be written as:

$$90x_1' + 110x_2' + 120x_3' \geq 1$$

$$80x_1' + 100x_2' + 70x_3' \geq 1$$

$$110x_1' + 90x_2' + 80x_3' \geq 1$$

$$x_1' + x_2' + x_3' = 1/v$$

$$x_1',\ x_2',\ x_3' \geq 0$$

and,

$$90y_1' + 80y_2' + 110y_3' \leq 1$$

$$110y_1' + 100y_2' + 90y_3' \leq 1$$

$$120y_1' + 70y_2' + 80y_3' \leq 1$$

$$y_1' + y_2' + y_3' = 1/v$$

$$y_1',\ y_2',\ y_3' \geq 0$$

A wishes to maximize v that is he wishes to

Minimize $\frac{1}{v} = x_1' + x_2' + x_3'$

Subject to $90x_1' + 110x_2' + 120x_3' \geq 1$

$$80x_1' + 100x_2' + 70x_3' \geq 1$$

$$110x_1' + 90x_2' + 80x_3' \geq 1$$

$$x_1',\ x_2',\ x_3' \geq 0$$

B wishes to minimize v that is he wishes to

Maximize $\frac{1}{v} = y_1' + y_2' + y_3'$

Subject to $90y_1' + 80y_2' + 110y_3' \leq 1$

$$110y_1' + 100y_2' + 90y_3' \leq 1$$

$$120y_1' + 70y_2' + 80y_3' \leq 1$$

$$y_1',\ y_2',\ y_3' \geq 0$$

We can see that the previous two LPP's are primal dual pair. So, if we solve any one, the solution to the other may be obtained from the optimal table of the considered problem. So, we'll solve the last problem by simplex method using TORA:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name	y1	y2	y3		
Maximize	1.00	0.00	1.00		
Constr 1	90.00	80.00	110.00	<=	1.00
Constr 2	110.00	100.00	90.00	<=	1.00
Constr 3	120.00	70.00	80.00	<=	1.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Iteration1 and Iteration2:

Iteration 1	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	-1.00	0.00	-1.00	0.00	0.00	0.00	0.00
sx4	90.00	80.00	110.00	1.00	0.00	0.00	1.00
sx5	110.00	100.00	90.00	0.00	1.00	0.00	1.00
sx6	120.00	70.00	80.00	0.00	0.00	1.00	1.00
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				
Iteration 2	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	0.58	-0.33	0.00	0.00	0.01	0.01
sx4	0.00	27.50	50.00	1.00	0.00	-0.75	0.25
sx5	0.00	35.83	16.67	0.00	1.00	-0.92	0.08
x1	1.00	0.58	0.67	0.00	0.00	0.01	0.01
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Iteration3 (Optimal):

Iteration 3	y1	y2	y3				
Basic	x1	x2	x3	sx4	sx5	sx6	Solution
z (max)	0.00	0.77	0.00	0.01	0.00	0.00	0.01
x3	0.00	0.55	1.00	0.02	0.00	-0.02	0.01
sx5	0.00	26.67	0.00	-0.33	1.00	-0.67	0.00
x1	1.00	0.22	0.00	-0.01	0.00	0.02	0.01
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Value of Z = 0.01 therefore, V = 100, since Z = 1/v

$y_1 = y_1' \times v = 0.01 \times 100 = 1$

$y_2 = y_2' \times v = 0 \times 100 = 0$

$y_3 = y_3' \times v = 0.01 \times 100 = 1$

$x_1 = x_1' \times v = 0.01 \times 100 = 1$

$x_2 = x_2' \times v = 0 \times 100 = 0$

$x_3 = x_3' \times v = 0 \times 100 = 0$

Therefore, optimal strategy of the two players A and B are respectively;

$$||1, 0, 1|| \text{ and } ||1, 0, 0||$$

And the value of game $v = 100$