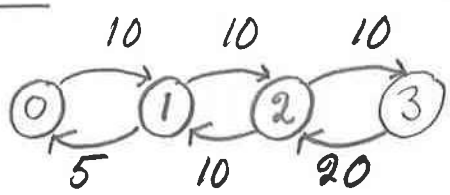
Kösystem 28 maj 2018

Uppgift 1

a)



$$\Rightarrow \begin{cases} p_0 = \frac{1}{6} \\ p_1 = \frac{2}{6} \\ p_2 = \frac{2}{6} \\ p_3 = \frac{1}{6} \end{cases}$$

$$P(\text{spärr}) = p_3 = \frac{1}{6}$$

b) $\lambda_{serv} = 5 \cdot p_1 + 10 \cdot (p_2 + p_3) = \frac{46}{6}\ s^{-1} =$

$= \frac{40}{6} \cdot 3600\ h^{-1} =$ **24 000 h^{-1}**

c) $E(N) = 1 \cdot p_1 + 2 \cdot p_2 + 3 \cdot p_3 = \frac{9}{6}$

$\lambda_{eff} = 10 \cdot (1 - p_3) = \frac{50}{6}$

$E(T) = \frac{E(N)}{\lambda_{eff}} = \frac{9}{50} =$ **0,18 s**

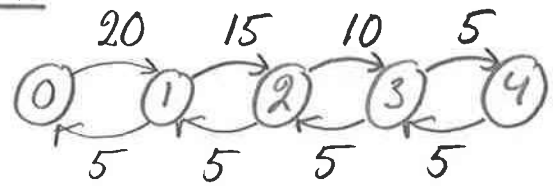
d) $\frac{\lambda_{serv}}{\lambda_{eff}} =$ $\frac{4}{5} = 0{,}8$

e) Markovhedjan blir:

Medeltiden från 3 till 0 är $\frac{1}{20} + \frac{1}{10} + \frac{1}{5} =$ **0,35 s**

Uppgift 2

a)



$$\Rightarrow \begin{cases} p_0 = 1/65 \\ p_1 = 4/65 \\ p_2 = 12/65 \\ p_3 = 24/65 \\ p_4 = 24/65 \end{cases}$$

$$E(N) = \sum_{i=1}^{4} i \cdot p_i = \boxed{\frac{196}{65} \approx 3{,}0}$$

b) $\lambda_{eff} = 20p_0 + 15p_1 + 10p_2 + 5p_3 = \frac{320}{65}$

$$E(T) = \frac{E(N)}{\lambda_{eff}} = \frac{196}{320} = \frac{49}{80}$$

E(tid i buffert) = E(T) - E(betjäningstid) = 33/80

c) Anhomshinteniket = 20 i tillstånd 0 $\Rightarrow$ $\boxed{E(\text{Idle}) = \frac{1}{20}\ s}$

d) $\frac{E(\text{Idle})}{E(\text{Busy}) + E(\text{Idle})} = p_0 \Rightarrow E(\text{busy}) = \frac{1-p_0}{p_0} E(\text{Idle}) =$

$$= \frac{64/65}{1/65} \cdot \frac{1}{20} = \boxed{\frac{64}{20} = 3{,}2\ s}$$

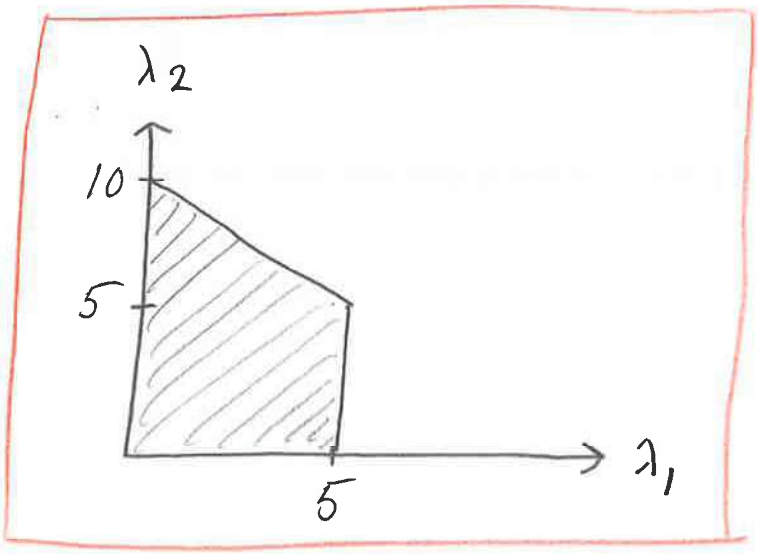
e) Fler buffertplatser än kunder $\Rightarrow$ $P(\text{spärr}) = 0$

Uppgift 3

a) Villkoren:

$$\begin{cases} \lambda_1 < 5 \\ \lambda_2 < 10 \\ 0{,}5\lambda_1 < 10 \\ 0{,}5\lambda_1 + \lambda_2 < 10 \\ \lambda_1 + \lambda_2 < 10 \end{cases}$$

ger



b) $E(N_1) = \dfrac{\lambda_1}{\mu_1 - \lambda_1} = \dfrac{4}{5-4} = 4$

$E(N_2) = \dfrac{5}{10-5} = 1$

$E(N_3) = \dfrac{2}{10-2} = \dfrac{1}{4}$

$E(N_4) = \dfrac{7}{10-7} = \dfrac{7}{3}$

$E(N_5) = \dfrac{9}{10-9} = 9$

$E(T) = \dfrac{\sum E(N_i)}{\lambda_1 + \lambda_2} \approx \boxed{1{,}84\ s}$

Uppgift 3 forts.

c) Två vägar $1 \to 4 \to 5$ och $2 \to 4 \to 5$

$$E(T_{145}) = \frac{N_1}{\lambda_1} + \frac{N_4}{\lambda_4} + \frac{N_5}{\lambda_5} = \frac{7}{3}$$

$$E(T_{245}) = \frac{N_2}{\lambda_2} + \frac{N_4}{\lambda_4} + \frac{N_5}{\lambda_5} = \frac{23}{15}$$

$\lambda_{145} = 2$
$\lambda_{245} = 5$

Medeltiden blir $= \frac{\lambda_{145}}{\lambda_{145} + \lambda_{245}} \cdot E(T_{145}) +$

$+ \frac{\lambda_{245}}{\lambda_{145} + \lambda_{245}} \cdot E(T_{245}) = \boxed{\frac{37}{21} \approx 1{,}76 \text{ s}}$

d) T_b = tid i buffert, T_s = tid i betjäning

$E(T_b) = E(T) - E(T_s) =$

$= E(T) - \frac{\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5}{\lambda_1 + \lambda_2} \approx \boxed{1{,}50 \text{ s}}$

Uppgift 4

a) $$\begin{cases} \lambda_1 = 10 + 0{,}75\lambda_4 \\ \lambda_2 = \lambda_1 + \lambda_3 \\ \lambda_3 = 0{,}5\lambda_2 \\ \lambda_4 = 0{,}5\lambda_2 \end{cases} \Rightarrow \begin{cases} \lambda_1 = 40 \\ \lambda_2 = 80 \\ \lambda_3 = 40 \\ \lambda_4 = 40 \end{cases}$$

Använder man att $E(N) = \frac{\lambda}{\mu - \lambda}$ så får man

$$\begin{cases} E(N_1) = 4 \\ E(N_2) = 2 \\ E(N_3) = 2 \\ E(N_4) = 2 \end{cases}$$

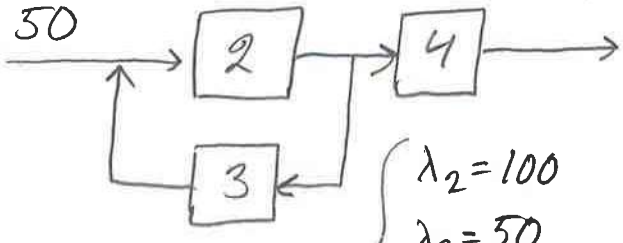
b) $$E(T_2) = \frac{E(N_2)}{\lambda_2} = \frac{2}{80} = \frac{1}{40}$$

Totala tiden i nod 2 $= \frac{\lambda_2}{\lambda} \cdot E(T_2) = \frac{80}{10} \cdot \frac{1}{40} = 0{,}25$

c) $E(\text{antal besök i nod } k) = \frac{\lambda_k}{\lambda} \Rightarrow$

$$E(\text{antal beställningar}) = \sum_{i=1}^{4} \frac{\lambda_i}{\lambda} = 20$$

d) Nod 1 överbelastas och blir en källa med utströmningstakt 50 s^{-1}. Nätet ser ut så här:



$$\begin{cases} \lambda_2 = 100 \\ \lambda_3 = 50 \\ \lambda_4 = 50 \end{cases} \Rightarrow$$

$$E(N_2) = \frac{100}{120-100} = 5$$

$$E(N_3) = \frac{50}{60-50} = 5$$

$$E(N_4) = E(N_3) = 5$$

Uppgift 5

a) Man kan se detta som ett M/G/1-system där tiden för betjäning har täthetsfunktionen:

$$f(t) = 0{,}5\,\delta(t-0{,}02) + 0{,}5\,\delta(t-0{,}06)$$

$$E(x) = 0{,}5 \cdot 0{,}02 + 0{,}5 \cdot 0{,}06 = 0{,}04 \Rightarrow \rho = \lambda E(x) = 0{,}8$$
$$E(x^2) = 0{,}5 \cdot 0{,}02^2 + 0{,}5 \cdot 0{,}06^2 = 0{,}002$$

$$E(N) = \rho + \frac{\lambda^2 E(x^2)}{2(1-\rho)} = 0{,}8 + \frac{400 \cdot 0{,}002}{2(1-0{,}8)} = \boxed{2{,}8}$$

b) $E(\text{tid i system för kund av typ A}) =$

$= E(\text{betjäningstid för kund av typ A}) + E(\text{tid i buffert}) =$

$$= 0{,}02 + \frac{E(N_q)}{\lambda} = 0{,}02 + \frac{E(N)-\rho}{\lambda} =$$

$$= 0{,}02 + \frac{2}{20} = \boxed{0{,}12\,s}$$

c) $E(\text{Idle}) = \frac{1}{20}$ (dvs $\frac{1}{\lambda}$)

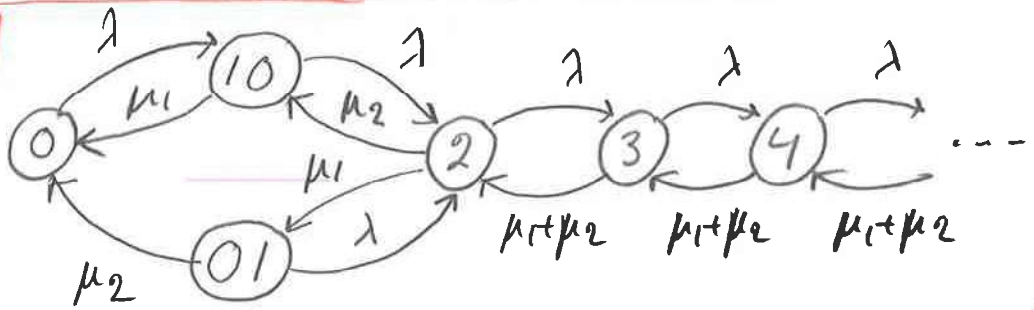
$$\frac{E(\text{Busy})}{E(\text{Busy}) + E(\text{Idle})} = \rho \Rightarrow E(\text{Busy}) = \frac{0{,}8}{1-0{,}8} E(\text{Idle}) =$$

$$= 4 \cdot \frac{1}{20} = \boxed{0{,}2\,s}$$

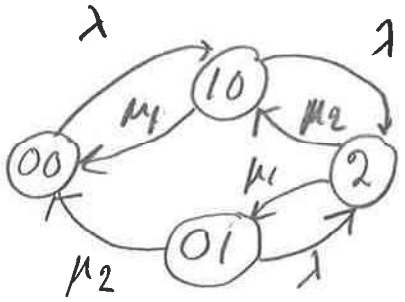
Uppgift 6

a) $\lambda < \mu_1 + \mu_2$

b)



c) Markovkedjan blir



$$\Rightarrow \begin{cases} p_{00} = 7/22 \\ p_{10} = 5/22 \\ p_{01} = 4/22 \\ p_2 = 6/22 \end{cases}$$

I processor 1: $\mu_1 p_{10} + \mu_1 p_2 = 1$

I processor 2: $\mu_2 p_{01} + \mu_2 p_2 = \frac{10}{22} \approx 0{,}45$

d) $E(N) = 1 \cdot (p_{10} + p_{01}) + 2 \cdot p_2 = \frac{21}{22}$

$$\lambda_{eff} = \lambda (1 - p_2) = \frac{32}{22}$$

$$E(T) = \frac{E(N)}{\lambda_{eff}} = \frac{21}{32} \approx 0{,}66$$

Uppgift 6 forts.

e) T_i = E (tiden innan bägge arbetar för första gången om man är i tillstånd i)

$$\begin{cases} T_{00} = \frac{1}{\lambda} + T_{10} \\ T_{10} = \frac{1}{\lambda+\mu_1} + \frac{\mu_1}{\lambda+\mu_1} T_{00} + \frac{\lambda}{\lambda+\mu_1} T_2 \\ T_2 = 0 \end{cases}$$

$$\Rightarrow$$

$$\boxed{T_0 = \frac{3}{2} = 1{,}5 \text{ s}}$$