Time Value of Money Refresher

F305 Intermediate Corporate Finance

Troy Adair

Slide Set A2 - TVM

Fall 2025

Overview

Time Value of Money

- Excel Functions and Speadsheet Setup
- Different Kinds of “Patterned” Cash Flows
  - Perpetuities
  - Annuities
- (Looking Ahead) Sets of Cash Flows with No Pattern
- Effect of Growth
- Converting Rates
- Amortizing Loans

Excel Functions and Speadsheet Setup

Key to Solving Time Value Problems

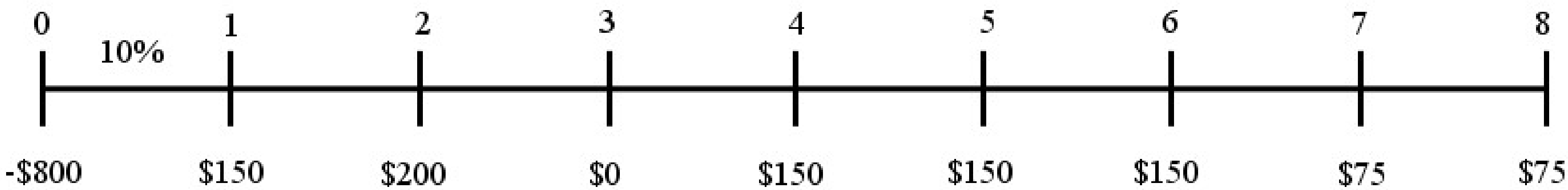
Use a timeline (either an actual line or a row in Excel)

Remind yourself what question you're trying to answer

Identify patterns in the cash flows

- Are the CFs growing at a constant rate?
- At different rates?
- How long do the CFs last? Forever?

Time Lines and Terminology



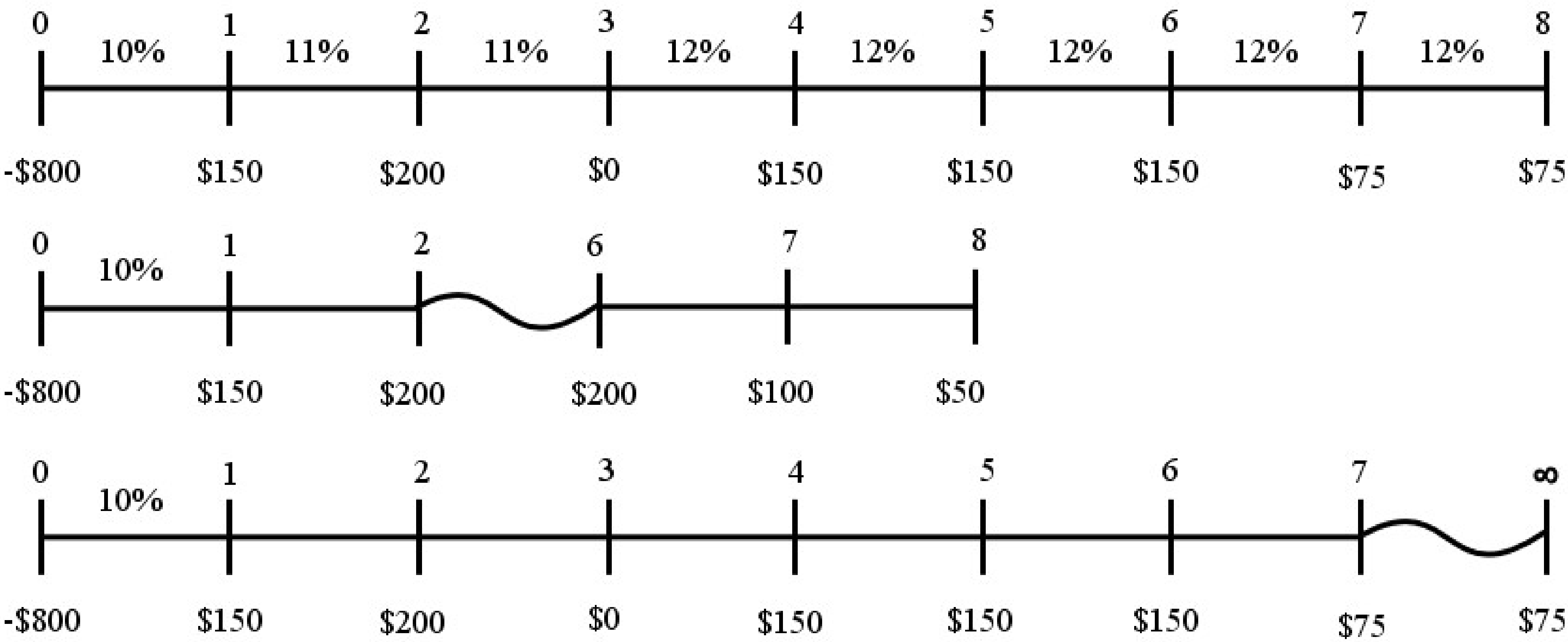
Present Value – earlier money on a time line or “discounted value”

Future Value – later money on a time line or “compounded value”

Interest rate – “exchange rate” between earlier money and later money

- Discount rate
- Cost of capital
- Opportunity cost of capital
- Required return

Time Lines and Terminology



Different Kinds of “Patterned” Cash Flows: Perpetuities

Perpetuity

Three things have to hold true:

- Cash flows have to remain the same *amount* forever
- Cash flows have to have the same
- Interest rate has to remain the same forever

Perpetuities

We can buy a vacation home that will produce $30,000 in rent per year (the lease is a one-year, single payment contract). The first rent payment will come at the end of the year. It will cost $5,000 per year for maintenance/upkeep. We plan on holding the property forever. Our required rate of return (our discount rate) is 20%.

What's the timeline, the formula, and what is the house worth to us today?

Perpetuity Example 1

	A	B	C	D	E	F	G
1	r	20%					
2							
3	Period	0	1	2	3	...	∞
4	Cash In		$ 30,000	$ 30,000	$ 30,000		$ 30,000
5	Cash Out		$ (5,000)	$ (5,000)	$ (5,000)		$ (5,000)
6	Net Cash Flow		$ 25,000	$ 25,000	$ 25,000		$ 25,000
7			=C4+C5	=D4+D5	=E4+E5		=G4+G5
8							
9	PV	$ 125,000	=C6/B1				

Perpetuity – Example 2

What is the PV of a cash flow stream that pays $1,000 every year forever, starting 4 years from now? Assume an annual interest rate of 10%.

Hints:

- Set up the cash flows
- What kind of perpetuity is this?

Perpetuity – Example 2 (cont.)

	A	B	C	D	E	F	G	H	I	J
1	r	10%								
2										
3	Period	0	1	2	3	4	5	6	...	∞
4	Net Cash Flow		$ -	$ -	$ -	$ 1,000	$ 1,000	$ 1,000		$ 1,000
5										

5 Basic TVM Functions in Excel

FV(rate,nper,pmt,pv,type)	Returns the future value of an investment
NPER(rate,pmt,pv,fv,type)	Returns the number of periods for an investment
PMT(rate,nper,pv,fv,type)	Returns the periodic payment for an annuity
PV(rate,nper,pmt,fv,type)	Returns the present value of an investment
RATE(nper,pmt,pv,fv,type,guess)	Returns the interest rate per period of an annuity

where *type* = 0 if the annuity portion is an ordinary annuity (where *pmt* occurs at the **end** of each period), 1 if the annuity is an "annuity due" (where *pmt* occurs at the **beginning** of each period), and where *guess* is an optional value at which to initiate an iterative search for the *rate*.

1 Important Thing That Matters Whether You Do TVM by Hand, with Calculator, or in Excel

Any variables that measure or are pertain to time all have to agree on how long a "time period" is.

- For PV and FV of lump sums problems, *nper* and *rate* have to be "thinking" about time that same way.
- For PV and FV annuity-type problems, *nper*, *rate* and *pmt* all have to think of time the same way.
- For all PV and NPV types of problems with payments, the formula/function will give you an answer "1 length of time between payments in front of the first payment"

Perpetuities - Special Types

- Normal perpetuity: CF stream ***starts 1 period from today***
- Perpetuity ***Due:*** CF stream ***starts today***
- ***Delayed*** Perpetuity: CF stream starts later than a normal perpetuity

The $PV_{perpetuity}$ formula needs to be adjusted to apply to perpetuities due and delayed

- Growing Perpetuity: Where cash flows grow forever at a constant rate (think: growing dividends)

Different Kinds of “Patterned” Cash Flows: Annuities

Annuities

The Vacation Home Revisited:

- Annual Rent = $30,000 per year
- First payment comes at the end of the year
- Annual Maint/Upkeep = $5,000 per year
- *We plan on holding the property for 4 years*
- *Sell the home for $130,000*
- Our required rate of return (our discount rate) is 20%.

What is the vacation house worth today?

Annuity Example 1

The Math

$$PV_{annuity} = C \times \frac{1}{r} \times \left(1 - \frac{1}{(1+r)^t}\right)$$

$$PVA_0(CFS_1) = \frac{30{,}000}{20\%} \times \left[1 - \frac{1}{(1+20\%)^4}\right]$$

$$PVA_0(CFS_2) = \frac{-5{,}000}{20\%} \times \left[1 - \frac{1}{(1+20\%)^4}\right] \qquad PV_0(CF_3) = \frac{130{,}000}{(1+20\%)^4}$$

$$PV(House) = PVA_0(CFS_1) + PVA_0(CFS_2) + PV(CF_3)$$

Annuity Example 1

	A	B	C	D	E	F
1	r	20%				
2						
3	Period	0	1	2	3	4
4	Rent		$ 30,000	$ 30,000	$ 30,000	$ 30,000
5	Sale of House					$ 130,000
6	Cash Out		$ (5,000)	$ (5,000)	$ (5,000)	$ (5,000)
7	Net Cash Flow		$ 25,000	$ 25,000	$ 25,000	$ 155,000
8			=C4+C5+C6	=D4+D5+D6	=E4+E5+E6	=F4+F5+F6
9						
10	PV	$ 127,411	=-PV(B1,F3,C4+C6,F5)			

Annuity – Exercise 2

If you put up $40,000 today into a 6.75%, 12-year annuity, what will the annual cash flow be?

A. $8,977.81

B. $3,333.33

C. $4,705.91

D. $5,239.25

E. $4,969.19

Said Differently:
What is the level payment over 12 years that produces a PV of $40,000, given a discount rate of 6.75%

Annuity – Exercise 2

	A	B	C
1	CF_0	$ 40,000	
2	r	6.75%	
3	t	12	
4			
5	PMT	$4,969.19	=-PMT(B2,B3,B1)

Annuity Due Exercise

Today and annually thereafter for 4 more years, Gary is putting $1,000 into an account that will earn 5% per year. What is the Future Value of those deposits?

	A	B	C
1	PMT	$ 1,000	
2	r	5.00%	
3	t	5	
4			
5	FV	$5,801.91	=-FV(B2,B3,B1,,1)
6	or		
7	FV	$5,801.91	=-FV(B2,1,,-FV(B2,B3,B1))

(Looking Ahead) Sets of Cash Flows with No Pattern

Multiple Cash Flows

$$NPV = \frac{CF_0}{(1+r)^0} + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^{T} CF_t \times \frac{1}{(1+r)^t}$$

Net Present Value

because we will be both adding and subtracting at some point

NPV = the sum of all the discounted cash flows

Dealing with Uneven Cash Flows

Two basic choices:

- Use TVM functions on each different-sized cash flow separately.
- Use NPV(**rate, value1,** value2, ...) function. $NPV = \sum_{j=1}^{n} \frac{values_j}{(1+rate)^j}$

	A	B	C	D	E	F	G
1	Net Present Value						
2							
3	Inputs						
4	Interest Rate	7.34%					
5	Period	0	1	2	3	4	5
6	Cash Flows	-$10,000	$3,500	$5,600	$5,600	$4,000	$1,000
7							
8	Net Present Value	$6,363.82	=NPV(B4,C6:G6)+B6				
9		Result	Formula				
10							

Example: Using Excel to Compute NPV

You’re considering investing in a project with the following cash flows when the appropriate interest rate is 7.23%:

• Initial investment:	$625,000
• Cash flows, years 1-5	$200,000
• Cash flows, years 6-9	$130,000
• Cash flow, year 10	$145,000

Top Hat 3

What is the NPV?

- $532,644.61
- $571,154.82

Effect of Growth

Time Value & Growth

Back to the Vacation Home:

- *Annual Rent = $15,000 in yr 1, growing at 10%/year for 3 years afterword*
- First payment comes at the end of the year
- Annual Maint/Upkeep = $5,000/yr
- We plan on holding the property for 4 years
- Sell the home for $130,000
- Our required rate of return (our discount rate) is 20%.

What is the home worth today?

Growth of Cash Flows – Example

An annuity in which cash flows grow at some constant rate ***r>g*** each period

$$PV_{\text{growing annuity}} = \frac{CF_1}{r-g} \times \left[1 - \left(\frac{1+g}{1+r} \right)^t \right]$$

Growth of Cash Flows – Example

$$PV_{\text{growing annuity}} = \frac{CF_1}{r-g} \times \left[1 - \left(\frac{1+g}{1+r}\right)^t\right]$$

$$PVA_0(CFS_1) = \frac{15{,}000}{20\% - 10\%} \times \left[1 - \frac{(1+10\%)^4}{(1+20\%)^4}\right]$$

$$PVA_0(CFS_2) = \frac{-5{,}000}{20\%} \times \left[1 - \frac{1}{(1+20\%)^4}\right] \quad PV_0(CF_3) = \frac{130{,}000}{(1+20\%)^4}$$

= $93,839.22

Growth of Cash Flows – Example

	A	B	C	D	E	F	G	H	I
1	r	20%							
2	g	10%							
3									
4	Period	0	1	2		3		4	
5	Rent		$ 15,000	$ 16,500	=FV($B2,1,,-C5)	$ 18,150	=FV($B2,1,,-D5)	$ 19,965	=FV($B2,1,,-F5)
6	Sale of House							$ 130,000	
7	Cash Out		$ (5,000)	$ (5,000)		$ (5,000)		$ (5,000)	
8	Net Cash Flow	$ -	$ 10,000	$ 11,500		$ 13,150		$ 144,965	
9		=SUM(B5:B7)	=SUM(C5:C7)	=SUM(D5:D7)		=SUM(F5:F7)		=SUM(H5:H7)	
10									
11	NPV	$ 93,839	=NPV(B1,C8,D8,F8,H8)+B8						

Growing Perpetual Cash Flows

Two-Stage Growth

Back to the Vacation Home:

- *We plan on holding the property indefinitely*
- Annual Rent = $15,000 in yr 1, growing at 10%/year for 3 years, *then growth at 5%/year thereafter*
- First payment comes at the end of the year
- Annual Maint/Upkeep = $5,000/yr
- Our required rate of return (our discount rate) is 20%.

What is the home worth today?

Growing Perpetual Cash Flows - Example

	A	B	C	D	E	F	G	H	I	J	K
1	r	20%									
2	g_1	10%									
3	g_2	5%									
4	Period	0	1	2		3		4		5	...
5	Rent		$ 15,000	$ 16,500	=FV($B2,1,,-C5)	$ 18,150	=FV($B2,1,,-D5)	$ 19,965	=FV($B2,1,,-F5)	$20,963.25	
6	Sale of House										
7	Cash Out		$ (5,000)	$ (5,000)		$ (5,000)		$ (5,000)		$ (5,000)	
8	Net Cash Flow	$ -	$ 10,000	$ 11,500		$ 13,150		$ 14,965		$15,963.25	
9		=SUM(B5:B7)	=SUM(C5:C7)	=SUM(D5:D7)		=SUM(F5:F7)		=SUM(H5:H7)			
10											
11	Pass 1		$ 10,000	$ 11,500		$ 13,150		$ 129,720			
12			=C8	=D8		=F8		=J5/(B1-B3)+J7/B1+H8			
13											
14	NPV	$ 86,487	=NPV(B1,C11,D11,F11,H11)								

Converting Rates

APR vs. EAR

$$EAR = \left(1 + \frac{APR}{m}\right)^m - 1$$

$$EAR = \left(1 + \frac{Quoted\ Rate}{m}\right)^m - 1$$

$$r_{long} = (1 + r_{short})^m - 1$$

EARs and APRs

Truth-in-lending laws in the U.S. require lenders to disclose an APR on virtually all consumer loans

- **Annual percentage rate (APR)** is the interest rate charged per period multiplied by the number of periods per year.
- Is APR an EAR? In other words, if a bank quotes a car loan at 12% APR, is the consumer actually paying 12% interest? No!
- For example, an APR of 12% on a loan calling for monthly payments is really 1% per month
- However, the EAR on such a loan is the following:

$$EAR = \left[1 + \left(\mathrm{APR}/12\right)\right]^{12} - 1 = 1.01^{12} - 1 = .126825 \text{ or } 12.6825$$

- There are also truth-in-saving laws that require banks and other borrowers to quote an "annual percentage yield," or APY, on things like savings accounts
- Unlike APR, an APY is an EAR; rates quoted to borrowers (APR's) and those quoted to savers (AP Y's) are not computed the same way

APR vs. EAR Example

	A	B	C
1	APR	10.50%	
2			
3	EAR	11.02%	=EFFECT(B1,12)
4			
5	nominal rate	10.50%	=NOMINAL(B3,12)
6			

APR vs. EAR Example 2

	A	B	C	D
1	Stock	Stock Price	$ 75.00	
2		Dividends throughout Year	$ 3.50	
3		Quoted Dividend Yield	4.6667%	=C2/C1
4				
5	Bond	Bond Price	$ 753.00	
6		Coupon	3.50%	
7		Current Yield	4.6481%	=C6*1000/C5

A Note about Continuous Compounding

There is no upper limit to the number of times your money could be compounded during the year.

We have seen daily compounding, but compounding (in theory) could occur every hour or minute or second.

Compounding Period	Number of Times Compounded	Effective Annual Rate
Year	1	10.00000%
Quarter	4	10.38129
Month	12	10.47131
Week	52	10.50648
Day	365	10.51558
Hour	8,760	10.51703
Minute	525,600	10.51709

A Note about Continuous Compounding 2

How high will the EAR get?

- There is an upper limit to the EAR
- If we let q stand for the quoted rate, then as the number of times interest is compounded gets extremely large, EAR approaches:

$$\mathrm{EAR} = e^{q} - 1$$

What's the Law?

At one time, commercial banks and savings and loan associations (S&Ls) were restricted in the interest rates they could offer on savings accounts. Under what was known as Regulation Q, S&Ls were allowed to pay at most 5.5 percent, and banks were not allowed to pay more than 5.25 percent (the idea was to give the S&Ls a competitive advantage; it didn't work). The law did not say how often these rates could be compounded, however. Under Regulation Q, then, what were the maximum allowed interest rates?

The maximum allowed rates occurred with continuous, or instantaneous, compounding. For the commercial banks, 5.25 percent compounded continuously would be:

$$
\begin{aligned}
EAR &= e^{.0525} - 1 \\
&= 2.71828^{.0525} - 1 \\
&= 1.0539026 - 1 \\
&= .0539026 \text{ or } 5.39026\%
\end{aligned}
$$

This is what banks could actually pay. Check for yourself to see that S&Ls could effectively pay 5.65406 percent.

Amortizing Loans

Loan Types and Loan Amortization: Pure Discount Loans

Pure discount loans are those in which the borrower receives money today and repays a single lump sum at some time in the future.

- Simplest form of loan.
- Common when the loan term is short – say a year or less.

What kind of TVM problem is this kind of loan?

Treasury Bills

When the U.S. government borrows money on a short-term basis (a year or less), it does so by selling what are called Treasury bills, or *T-bills* for short. A T-bill is a promise by the government to repay a fixed amount at some time in the future—for example, in 3 months or 12 months.

Treasury bills are pure discount loans

Loan Types and Loan Amortization: Interest Only Loans

Interest-only loans call for the borrower to pay interest each period and to repay the entire principal (the original loan amount) at some point in the future.

- Most corporate bonds have general form of an interest-only loan
- For example, with a three-year, 10 percent, interest-only loan of $1,000, the borrower would pay $1,000 × .10 = $100 in interest at the end of the first and second years.
- At the end of the third year, the borrower would return the $1,000 along with another $100 in interest for that year.

Interest-Only Loans In Excel

	A	B	C	D	E
1	r	10%			
2					
3	Period	0	1	2	3
4	Net Cash Flow	1000	-100	-100	-1100
5					
6	Rate	10.00%	=RATE(3,-100,1000,-1000)		

Loan Types and Loan Amortization: Amortized Loans

If the lender requires the borrower to repay parts of the loan over time, it is an amortized loan; process of providing for a loan to be paid off by making regular principal reductions is called amortizing the loan

Simple way of amortizing a loan is to have the borrower pay the interest each period plus some fixed amount

Loan Types and Loan Amortization: Amortized Loans

Suppose a business takes out a $5,000, 5-year loan at 9%. The loan agreement calls for the borrower to pay the interest on the loan balance each year and to reduce the loan balance each year by $1,000. Because the loan amount declines by $1,000 each year, it is fully paid in 5 years.

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$ 5,000	$ 1,450	$ 450	$ 1,000	$ 4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		$ 6,350	$ 1,350	$ 5,000	

Loan Types and Loan Amortization: Amortized Loans

Most common way of amortizing a loan is to have the borrower make a single, fixed payment every period.

What kind of TVM problem does this constitute?

Amortization Table

	A	B	C	D	E	F
1	Loan Amount	$ 300,000				
2	Term (Years)	30				
3	Rate (APR)	6.375%				
4						
5	Payment	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
6	1	$ 300,000	$1,871.61	$ 1,593.75	$277.86	$ 299,722.14
7	2	$ 299,722.14				

Partial Amortization, Or “Bite The Bullet”

With partially amortizing loans, payment is set on one (longer) amortization period, but loan must be “paid off early”, with the payoff equal to the remaining PV of the unpaid cash flows.

Partial Amortization, Or “Bite The Bullet” 3

$$\text{Loan balance} = \$1101.09 \times \left[\left(1 - 1/1.01^{180}\right)/.01\right]$$

$$= \$1101.09 \times 83.321$$

$$= \$91744.33$$

The balloon payment is a substantial $91,744.33. Why is it so large? To get an idea, consider the first payment on the mortgage. The interest in the first month is $100,000 × .01 = $1,000. Your payment is $1,101.09, so the loan balance declines by only $101.09. Because the loan balance declines so slowly, the cumulative “pay down” over five years is not great.

Up Next

Cash Flow!

Chapter 2