

# Groups

## Examples of Groups

Axioms for groups. Examples from geometry, symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios; preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius map and iterations. [4]

### Lagrange's theorem

Cosets. Lagrange theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view [5]

## Group actions

Group actions, orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes.

Cauchy theorem. [4]

## Quotient groups

Normal ~~groups~~ Subgroups, quotient groups and the isomorphism theorem

## Matrix groups

The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in  $\mathbb{R}^3$ ) that every element of the orthogonal group is the product of reflections and every rotation in  $\mathbb{R}^3$  has an axis. Basis change as an example of conjugation

[3]

## Appropriate Books

M.A. Armstrong Groups and Symmetry Springer-Verlag 1988

† Alan F. Beardon Algebra and Geometry CUP 2005

R.P. Burn Groups, a Path to Geometry. CUP 1987

J.A. Green Sets and Groups: a first Course in Algebra CRC 1988

W. Lederman Introduction to Group theory. Longman 1976

Nathan Carter Visual Group theory. MAA Textbooks