Vectors and Matrices

Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus, and Argand diagram. Informal treatment of complex logarithm, n-th roots and complex powers.

de Moivre's theorem

Vectors

Review of elementary algebra of vectors in $\mathbb{R}^3$, including scalar product. Brief discussion of vectors in $\mathbb{R}^n$ an $\mathbb{C}^n$; scalar product and the ~~fact~~ Cauchy-Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δ_{ij} and ϵ_{ijk}. Vector product and triple product: definition and geometrical interpretation. Solution of linear vectors equations. Applications of vectors to geometry, including equations of lines, planes and spheres.

Matrices

Elementary algerbra of 3X3 matrices, including determinants. Extension to nxn complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears, kernal and image, rank-nullity theorem

Simultaneous linear equations; matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination.

Symmetric, anti-symmetric, orthogonal, ~~herf~~ hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts.

Eigenvalues & Eigenvectors

Eigenvalues and Eigenvectors; geometric Significane

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2x2 matrices.

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms.

Rotation matrices and Lorentz transformations as transformation groups.

Appropiate books

Alan F Beardon Algebra and geometry. CUP 2005

Gilbert Strang Linear Algebra and its Applications Thomson Brooks/Cole 2006

Richard Kaye and Robert Wilson Linear Algebra (Oxford Science 1998)

D.E Bourne and P.C Kendall Vector Analysis and Cartesian Tensors (Nelson Thornes)

E. Sernesi Linear Algebra: A Geometric Approach CRC Press 1993

James J. Callahan The Geometry of spacetime: An Introduction to Special and General Relativity. Springer 2000.