Probability

Basic Concepts

Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling's formula (asymptotics for log n! proved)

Axiomatic approach

Axiom (countable case). Probability spaces. Inclusion-exclusion formula. Continuity and subadditivity of probability measures. Independence. Binomial, Poission and geometric distributions. Relations between Poission and binomial distributions. Conditional probability, Bayes' formula. Examples, including Simpson's paradox.

Discrete Random Variables

Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions; sum of independent random variables, random sum formula, moments.

Conditional expetation. Random walks: gambler's ruin, recurrence relations. Difference equations and their solution. Mean time to absorption. Branching processes: Generating functions and extinction probability. Combinatorial applications of generating functions.

Continuous random variables

Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of the exponential distribution

Joint distributions: transformation of random variables (including Jacobians), examples, Simulation: generating continuous random variables, Box-Muller transform, rejection sampling. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation cofficients, bivariate normal random variables.

Inequalities and Limits

Markov's inequality, Chebychev's inequality. Weak law of large numbers. Convexity: Jensen's inequality for general random variables. AM/GM inequality.

Moment generating functions and statement (no proof) of continuity theorem. Statement of Central limit theorem and sketch of proof. Examples, including sampling.

Appropriate books

W. Feller An Introduction to Probability Theory and its Applications (Vol I) 1968

† G. Grimmett and D Welsh Probability: An Introduction OUP 2nd Edition 2014

S Ross A First Course in Probability (Pearson 2004)

D.R Stirzaker Elementary Probability (CUP 1994/2003)