NUMBERS AND SETS

Introduction to numbers systems and logic

Overview of the natural numbers, integers, real numbers, rational and irrational numbers algerbric and transcendental numbers. Brief discussion of complex numbers, statement of the fundmental theorem of Algebra

Ideas of axiomatic systems and proof with mathematics, the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction.

Sets, relations and functions

Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions injections; surjections; and bijections. Relations and equivalence relations. Counting the combinations and permutations of a set. The Inclusion-Exclusion Principle

Hence do not write with an arm, ugly! ☹

The integers

The natural numbers; mathematical induction and the well-ordering principle. Examples, including the Binomial theorem

Elementary Number theory

Prime numbers: existence and uniqueness of prime factorisation into primes. Highest common factors and Least common multiples. Euclid proof of the infinity of primes. Eculid algorithm. Solution in integers of $ax+by=c$

Modular arithmetic (congruences). Units modulo n. Chinese remainder theorem. Wilson theorem; the Fermats - Euler theorem. Public-key Crypetography and the RSA algorithms

The real numbers

Least upper bounds; simple examples. Least upper bound axioms. Sequences and series; convergence of bounded monotonic sequences. Irrationality of e and $\sqrt{2}$. Decimal expansions. Contstruction of a transcendental number

Countability and uncountability

Definitions of finite, infinite, countable and uncountable sets. A countable set is countable. ($\{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$) Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its powers set. Indirect proof of existence of transcendental numbers

Appropriate Books

R.B.J.T Allenby Numbers and Proofs. Butterworth-Heinemann 1997

R.P Burn Numbers and Functions: Steps into Analysts CUP 2000

H Daveport The higher arithmetic. CUP 1999 CUP 1988

A G Hamilton Numbers, sets and axioms: the apparatus of mathematics.

C Schumacher Chapter Zero: Fundmental Notions of Abstract Mathematics. Addision-Wesley (Pearson) 2001

I Steward and D Tall The Foundations of Mathematics (OUP 1977)