Probability

Basic Concepts

Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling's formula (asymptotics for log n! proved)

Axiomatic approach

Axiom (countable case). Probability spaces. Inclusion-exclusion formula. Continuity and subadditivity of probability measures. Independence. Binomial, Poission and geometric distributions. Relations between Poission and binomial distributions. Conditional probability, Bayes' formula. Examples, including Simpson's paradox.

Discrete Random Variables

Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions; sum of independent random variables, random sum formula, moments.

Conditional expetation. Random walks: gambler's ruin, recurrence relations. Difference equations and their solution. Mean time to absorption. Branching processes: Generating functions and extinction probability. Combinatorial applications of generating functions.

Continuous random variables

Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of the exponential distribution

Joint distributions: transformation of random variables (including Jacobians), examples. Simulation: generating continuous random variables, Box-Muller transform, rejection sampling. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation cofficients, bivariate normal random variables.

Inequalities and Limits

Markov's inequality, Chebychev's inequality. Weak law of large numbers. Convexity: Jensen's inequality for general random variables. AM/GM inequality.
Moment generating functions and statement (no proof) of continuity theorem. Statement of Central limit theorem and sketch of proof. Examples, including sampling.

<u>Appropriate books</u>

W. Feller An Introduction to Probability Theory and its Applications (Vol 1) 1968

† G. Grimmett and D Welsh Probability: An Introduction OUP 2nd Edition 2014

S Ross A First Course in Probability (Pearson 2004)

D.R Stirzaker Elementary Probability (CUP 1994/2003)

Vector Calculus (Part IA)

Curves in $\mathbb{R}^3$

Parameterised curves and arc length, tangents and normals to the curve in $\mathbb{R}^3$, curvature and torsion

Integration $\mathbb{R}^2$ and $\mathbb{R}^3$

Line integrals, surface and volume integrals; definitions; ~~intepretation as normal~~ examples using cartesian, cylinderical and spherical coordinates, change of variables

Vector Operators

Directional dirative, The gradient of a real-valued function; definition; interpretation as normal to level surfaces, examples including the use of cylinderical, spherical and general orthogonal curvilinear coordinates

Divergence, curl and ∇^2 in Cartesian coordinates, examples, formulae for these operators (statement only) in cylinderical, spherical and general orthogonal curvilinear coordinates. Solenoidal fields, ~~irrational~~ irrotational fields and conservative fields, scalar potentials. Vector derative identities

Integration theorems

Divergence theorem, Greer's theorem, Strokes theorem, Green's second theorem; statements; informal proofs, examples, applications to fluid dynamics, and to electromagnetics including statement of Maxwell equations.

Laplace's equation

Laplace's equation in $\mathbb{R}^2$ and $\mathbb{R}^3$, uniqueness theorem, maximum principle. Solution of Poission's equation by Gauss method (for spherical and cylinderical symmetry) and as an integral.

Cartesian tensors in $\mathbb{R}^3$

Tensor transformation laws, addition, multiplication, contradiction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revisions of principal axes and diagonalisation. Quotient theorem. Examples including inertia and conductivity.

Appropiate books

B. Anton Calculus Wiley Student Edition(2000)

T. M Apostol Calculus Wiley student Edition(1975)

M.L. Boas Mathematical Methods in the Physical Sciences Wiley 1983

D.E Bourne and P.C Kendall Vector Analysis and Cartesian tensors 3rd Edition, Nelson Thornes 1999

E Kreyszig Advanced Engineering Mathematics . Wiley International Edition, 1999

J E Martsen den and A. J Tromba Vector Calculus Freeman 1996

P.C matthews Vector Calculus SUMS(Springer Undergraduate Math series)

K.F. Riley and M.P Hobson Mathematical Methods for Physics and Engineering CUP 2002

H.M Schey Div, grad curl and all that, an informal text to vector calculus Norton 1996

M.R Spiegal Schaum Outline for Vector Calculus (Analysis) Mcgraw Hill 1974

Differential Equations

Basic calculus

Informal treatment of differentation as a limit, the chain rule. Leibnitz rule, Taylor series, informal treatment of O and o notation and l'Hôpital's rule; integration as an area, fundmental theorem of calculus, integration by substitution and parts

Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter.

First-order linear differential equations

Equations with constant cofficients: expoential growth, comparsion with discrete equations, series solutions; modelling examples of radioactive decay.

Equations with non-constant cofficients: Solution by integrating factor.

Non linear- first order equations

Separable equations. Exact equations. Sketching solution trajectories. Equilibrium solutions; stability by ~~equilibrium~~ perturbation; examples; including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic map.

Higher order differential equations (linear)

Complementary function and particular integral, linear independence. Wronksian (for second-order equations), Abel's theorem. Equations with constant cofficients and examples including radioactive sequences, comparsion in simple cases with difference equations, reduction of order, resonance, transsients, damping. Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution.

Multivariate functions: applications

Directional deratives and gradient vector. Statement of Taylor series for functions on $\mathbb{R}^n$. Local extrema of real functions. Classification using the Hessian matrix. Coupled first order systems; equivalence to single higher order equations; solution by matrix methods. Non-degenerate phase portraits local to equilibrium points; stability.

Simple examples of first- and second-order partial differential equations. Solution of the wave equation in the form $f(x+ct)+g(x-ct)$.

Appropiate Books

J. Robinson An Introduction to Differential Equations CUP 2004

W.E Boyce and R.C DiPrima Elementary Differential Equations and Boundary-Value Problems (and associated web site: google Boyce DiPrima). Wiley, 2004

G.F Simmons Differential Equation (with applications and historial notes) Mc-Graw Hill 1991

D.G Zill and M.R Cullen Differential Equations with Boundary Value Problems Broolces/Cole 2001

Dynamics and Relativity

Basic concepts

Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz

Newtonian dynamics of a single particle

Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; Stable equilibria and small oscillations; effect of damping

Angular velocity, angular momentum, torque
Orbits; the $u(\theta)$ equation; escape velocity; Kepler's laws; Stability of orbits; motion in a repulsive potential (Rutherford scattering)

Rotating frames: centrifugal and Coriolis forces.
* Brief discussion of Foucault pendulum *

~~Newtonian dynamics of system of particles~~ (Rigid bodies

Momentum, angular momentum and energy of a rigid body. Parallel axis theorem. Simple examples of motion involving both rotation and translation (eg rolling)

~~Rigid bodies~~ (Newtonian dynamics of system of particles)

~~Moments of inertia, angular momentum and energy of a rigid body.~~ Momentum, angular momentum and energy. Motion relative to the ~~mass~~ centre of mass; the two-body problem. Variable mass problems; the rocket equation.

Special relativity

The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in (1+1)-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for (1+1)-dimensional spacetime.

Lorentz transformations in (3+1) ~~dimensional~~ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4 momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit

Appropiate books

† D. Gregory Classical Mechanics Cambridge University Press 2006

G.F.R Ellis and R.M Williams Flat and Curved Space-times. Oxford University Press 2000

A.P French and M.G Ebison Introduction to Classical Mechanics Kluwer 1986

M.A Lunn A First Course in Mechanics. Oxford University Press 1991

P.J. O'Donnell Essential Dynamics and Relativity CRC Press 2015

W. Rindler Introduction to Special Relativity Oxford University Press 1991

E.F Taylor and J.A Wheeler Spacetime Physics: Introduction to Special relativity. Freeman 1992

Groups

Examples of Groups

Axioms for groups. Examples from geometry, symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The mödius group; cross-ratios; perservation of circles, the point at infinity. Conjugation. Fixed points of Mobius map and iterations. [4]

Lagrange's theorem

Cosets. Lagrange theorem. Groups of small order (up to order 8). Quaterions. Fermat-Euler theorem from the group-theoretic point of view [5]

Group actions

Group actions, orbits and stabilizers. Oribit-stabilizer theorem. Cayley's theolem (every group is ismorphic to a subgroup of a permutation group). Conjugacy classes. Cacuchy theorem. [4]

Quotient groups

Normal ~~groups~~ Subgroups, quotient groups and the isomorphism theorem

Matrix groups

The general and special linear groups; relation with the mobius group. The orthogonal and special orthogonal groups. Proof (in $\mathbb{R}^3$) that every element of the orthogonal group is the product of reflections and every rotation in $\mathbb{R}^3$ has an axis. Basis change as an example of conjugation

[3]

Appropriate Books

M.A Armstrong Groups and Symmetry Springer-Verlag 1988

+Alan F Beardon Algerbra and Geometry CUP 2005

R.P. Burn Groups, a Path to Geometry. CUP 1987

J.A Green Sets and Groups: a first Course in Algerbra CRC 1988

W. Lederman Introduction to Group theory. Longman 1976

Nathan Carter Visual Group theory. MAA Textbooks

NUMBERS AND SETS

Introduction to numbers systems and logic

Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebric and transcendental numbers. Brief discussion of complex numbers, statement of the fundmental theorem of Algebra

Ideas of axiomatic systems and proof with mathematics, the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction.

Sets, relations and functions

Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities: Functions injections; surjections; and bijections. Relations and equivalence relations. Counting the combinations and permutations of a set. The Inclusion-Exclusion Principle

Hence do not write with an arm, ugly! ☹

The integers

The natural numbers; mathematical induction and the well-ordering principle. Examples, including the Binomial theorem

Elementary Number theory

Prime numbers: existence and uniqueness of prime factorisation into primes. Highest common factors and Least common multiples. Euclid proof of the infinity of primes. Eculid algorithm. Solution in integers of $ax+by=c$

Modular arithmetic (congruences). Units modulo n. Chinese remainder theorem. Wilson theorem; the Fermats - Euler theorem. Public-key Crypetography and the RSA algorithms

The real numbers

Least upper bounds; simple examples. Least upper bound axioms. Sequences and series; convergence of bounded monotonic sequences. Irrationality of e and $\sqrt{2}$. Decimal expansions. Contstmction of a transcendental number.

Countability and uncountability

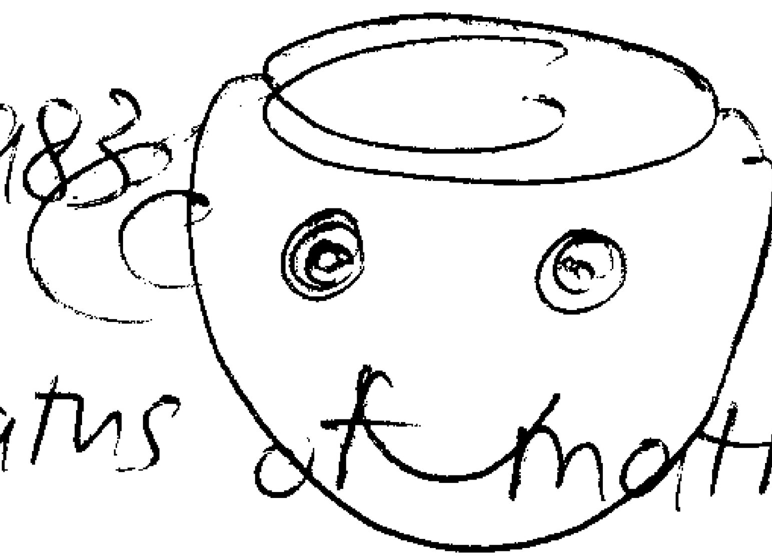
Definitions of finite, infinite, countable and uncountable sets. A countable set is countable. ($\{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$) Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its powers set. Indirect proof of existence of transcendental numbers

Appropriate Books

R. B. J. T Allenby Numbers and Proofs. Butterworth-Heinemann 1997

R. P Burn Numbers and Functions: Steps into Analysts CUP 2000

H Daveport The higher arithmetic. CUP 1999 CUP 1983



A G Hamilton Numbers, sets and axioms: the apparatus of mathematics.

C Schumacher Chapter Zero: Fundmental Notions of Abstract Mathematics. Addision-Wesley (Pearson) 2001

I Steward and D Tall The Foundations of Mathematics (OUP 1977)

Analysis I (I am Sleeply) (Part IA)

Limits and Convergence

Sequences and series in $\mathbb{R}$ and $\mathbb{C}$. Sums, products and quotients. Absolute convergence. Absolute convergence implies convergence. The Bolazono Weierstrass theorem and applications (the general principle of convergence). Comparsion and ratio tests, alternating series test.

Continuity

Continuity of real and complex valued functions defined on subsets on $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds

Differentiability

Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. ~~Dre~~ Deraivative of the inverse function. A Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor theorem for $\mathbb{R}$ to $\mathbb{R}$; Lagrange's form of the reminder. Complex differentation

Power Series

Complex power series and radius ~~and~~ of convergence. Expotential, trigonometric and hyperbolic functions and relations between them.

"Direct proof of differentiability of a power series within it circle of convergence"

Integration

Definition and basic propteries of the Riemann integral. A non-integrable function, integrability of monotone functions. Intecgrability of piecewise-continuous functions. The fundmental theorem of calculus. Differentation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor theorem. Improper integrals.

Appropriate Books

T.M Apostol Calculus, vol I Wiley 1967-69

JC Burkkill A First Course in Mathematical Analysis CUP 1978

DJH Garling A Course in Mathematical Analysis (Vol I) CUP 2013

J.B Reade Introduction to Mathematical Analysis (OUP)

M Spivak Calculus Addison-Wesley (Pearson)

David M Bressoud A Radical Approach to Real Analysis MAA Textbooks

Vectors and Matrices

Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus, and Argand diagram. Informal treatment of complex logarithm, n-th roots and complex powers.

de Moivre's theorem

Vectors

Review of elementary algebra of vectors in $\mathbb{R}^3$, including scalar product. Brief discussion of vectors in $\mathbb{R}^n$ an $\mathbb{C}^n$; scalar product and the ~~Cach~~ Cauchy-Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δ_{ij} and ϵ_{ijk}. Vector product and triple product: definition and geometrical interpretation. Solution of linear vectors equations. Applications of vectors to geometry, including equations of lines, planes and spheres.

Matrices

Elementary algerbra of 3X3 matrices, including determinants. Extension to nxn complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears, kernal and image, rank-nullity theorem

Simultaneous linear equations; matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination.

Symmetric, anti-symmetric, orthogonal, ~~herf~~ hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts.

Eigenvalues & Eigenvectors

Eigenvalues and Eigenvectors; geometric Significane

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2X2 matrices.

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms.

Rotation matrices and Lorentz transformation as transformation groups.

Appropiate books

Alan F Beardon Algerbra and geometry. CUP 2005

Gilbert Strang Linear Algerbra and its Applications Thomson Brooks/Cole 2006

Richard Kaye and Robert Wilson Linear Algerbra (Oxford Science 1998)

D.E Bourne and P.C Kendall Vector Analysis and Cartesian Tensors, (Nelson Thornes)

E. Sernesi Linear Algebra: A Geometric Approach CRC Press 1993

James J. Callahan The Geometry of spacetime: An Introduction to Special and General Relativity. Springer 2000.