

Differential Equations

Basic calculus

Informal treatment of differentiation as a limit, the chain rule - Leibnitz rule, Taylor series, informal treatment of ∂ and ∇ notation and I'Hôpital's rule; integration as an area, fundamental theorem of calculus, integration by substitution and parts

Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentiation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter. First-order linear differential equations

Equations with constant coefficients: exponential growth, comparison with discrete equations, series solutions; modelling examples of radioactive decay.

Equations with non-constant coefficients: solution by integrating factor.

Nonlinear - first order equations

Separable equations. Exact equations. Sketching solution trajectories. Equilibrium solutions; stability by ~~equilibrium~~ perturbation; examples; including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic map.

Higher order differential equations (linear)

Complementary function and particular integral

linear independence. Wronskian (for second-order equations, Abel's theorem. Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping. Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution.

Multivariate functions: applications

Directional derivatives and gradient vector. Statement of Taylor series for functions on \mathbb{R}^n . Local extrema of real functions. Classification using the Hessian matrix. Coupled first order systems: equivalence to single higher order equations; solution by matrix methods. Non-degenerate phase portraits local to equilibrium points; stability.

Simple examples of first - and second - order partial differential equations. Solution of the wave equation in the form $f(x+ct) + g(x-ct)$.

Appropriate Books

J. Robinson An Introduction to Differential Equations CUP 2004
W-E Boyce and R.C DiPrima Elementary Differential Equations and
Boundary-Value Problems (and associated web site: google Boyce DiPrima).
Wiley, 2004

G-F Simmons Differential Equation (with applications and historical
notes) Mc-Graw Hill 1991

D.G Zill and M.R Cullen Differential Equations with Boundary Value
Problems Brooks/Cole 2001