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# ComplexInfinity

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#### **Notations**

#### **Traditional name**

The complex quantity with infinite magnitude but indeterminate phase

#### **Traditional notation**

 $\tilde{\infty}$ 

#### **Mathematica** StandardForm notation

ComplexInfinity

## **Primary definition**

 $\tilde{\infty}$  represents an infinite numerical quantity whose direction in the complex plane is unknown (undetermined).

### **General characteristics**

 $\tilde{\infty}$  is a special symbol. On the Riemann sphere it is the north pole. In the projective complex plane it is the line at infinity.

### **Limit representations**

$$\tilde{\infty} = \lim_{z \to 0} \frac{1}{z}$$

### **Transformations**

### Products, sums, and powers of the direct function

#### **Products involving the direct function**

02.12.16.0001.01  

$$0 \tilde{\infty} = i$$

$$02.12.16.0002.01$$

$$a \tilde{\infty} = \tilde{\infty} /; a \neq 0$$

02.12.16.0003.01

$$\frac{\infty}{\tilde{\infty}} = i$$

#### Sums of the direct function

$$02.12.16.0004.01$$
 
$$\tilde{\infty} + \tilde{\infty} = \zeta$$
 
$$02.12.16.0005.01$$
 
$$\tilde{\infty} - \tilde{\infty} = \zeta$$

#### **Related transformations**

02.12.16.0006.01  $\tilde{\infty}^0 = \zeta$  02.12.16.0007.01  $1^{\tilde{\infty}} = \zeta$  02.12.16.0008.01  $(\tilde{\infty})^{\infty} = \tilde{\infty}$ 

# **Identities**

### **Functional identities**

$$\begin{array}{c} 02.12.17.0001.01 \\ \tilde{\infty} == \, \tilde{\infty} \, \infty \end{array}$$

# **Complex characteristics**

### Real part

$$02.12.19.0001.01$$
 
$$\mathrm{Re}(\tilde{\infty}) == \mathcal{L}$$

### **Imaginary part**

$$02.12.19.0002.01$$
 
$$Im(\tilde{\infty}) == \xi$$

#### **Absolute value**

$$02.12.19.0003.01$$
 
$$|\tilde{\infty}| = \infty$$

### Argument

$$\begin{array}{c} \textbf{02.12.19.0004.01} \\ \arg(\tilde{\infty}) \in (-\pi,\,\pi] \end{array}$$

# Conjugate value

$$\frac{02.12.19.0005.01}{\widetilde{\infty} = \widetilde{\infty}}$$

### **Differentiation**

### Low-order differentiation

$$\frac{02.12.20.0001.01}{\partial \tilde{\omega}} = 0$$

# Integration

# Indefinite integration

$$\int \tilde{\infty} \, dz = z \, \infty$$

### **Summation**

### **Finite summation**

02.12.23.0001.01 
$$\sum_{k=0}^{m} \tilde{\infty} = \tilde{\infty}$$
 02.12.23.0002.01 
$$\tilde{\infty} - \tilde{\infty} = \mathbf{i}$$

# **Integral transforms**

### Fourier exp transforms

$$02.12.22.0001.01$$

$$\mathcal{F}_t[\tilde{\infty}](z) = \tilde{\infty}$$

### **Inverse Fourier exp transforms**

$$02.12.22.0002.01$$
 
$$\mathcal{F}_t^{-1}[\tilde{\infty}](z) = \tilde{\infty}$$

#### Fourier cos transforms

$$02.12.22.0003.01$$

$$\mathcal{F}c_t[\tilde{\infty}](z) = \tilde{\infty}$$

#### Fourier sin transforms

$$02.12.22.0004.01$$
 
$$\mathcal{F}s_t[\tilde{\infty}](z) = \tilde{\infty}$$

### Laplace transforms

02.12.22.0005.01 
$$\mathcal{L}_t[\tilde{\infty}](z) = \tilde{\infty}$$

### **Inverse Laplace transforms**

$$02.12.22.0006.01$$

$$\mathcal{L}_t^{-1}[\tilde{\infty}](z) = \tilde{\infty}$$

# Representations through more general functions

## Through other functions

$$02.12.26.0001.01$$
 
$$\tilde{\infty}=\xi \infty$$
 
$$02.12.26.0002.01$$
 
$$\tilde{\infty}=(0 \infty)$$

## Representations through equivalent functions

$$\tilde{\infty} = \frac{1}{0}$$

# **History**

- –John Wallis (1655) introduced the sign ∞ to signify infinite number
- K. Weierstrass (1876) used symbol  $\infty$  to represent an actual infinity, which is prototype of symbol ComplexInfinity  $\tilde{\infty}$  in Mathematica

The symbol  $\infty$  is encountered often in mathematics and the natural sciences.

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