



StruveL



Notations

Traditional name

Struve function **L**

Traditional notation

L_n

Mathematica StandardForm notation

StruveL[n, z]

Primary definition

03.10.02.0001.01

$$L_n(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(k + \frac{3}{2}n + \frac{3}{2})} \frac{z^{2k+1}}{2^{2k+1}}$$

Specific values

Specialized values

For fixed n

03.10.03.0001.01

$$L_n(0) = 0; \operatorname{Re}(n) > -1$$

03.10.03.0002.01

$$L_n(z) \sim \frac{z}{2}; \operatorname{Re}(n) < -1$$

03.10.03.0003.01

$$L_n(z) \sim \frac{z}{2}; \operatorname{Re}(n) = -1$$

For fixed z

Explicit rational n

03.10.03.0008.01

$$L_{-\frac{11}{2}}(z) = \frac{\sqrt{\frac{2}{\pi}} (z^4 I_2(z^4 + 105z^2 + 945) \cosh(z) - 15z^4 + 28z^2 + 63) \sinh(z)}{z^{11/2}}$$

03.10.03.0009.01

$$L_{-\frac{9}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{\sqrt{\frac{2}{p}} \left(\text{I} z^4 + 45 z^2 + 105 \text{M} \sinh \text{H} \text{L} - 5 z \text{I} z^2 + 21 \text{M} \cosh \text{H} \text{L} \right)}{z^{9 \cdot 2}}$$

03.10.03.0010.01

$$L_{-\frac{7}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{\sqrt{\frac{2}{p}} \left(\text{I} z \text{I} z^2 + 15 \text{M} \cosh \text{H} \text{L} - 3 \text{I} z^2 + 5 \text{M} \sinh \text{H} \text{L} \right)}{z^{7 \cdot 2}}$$

03.10.03.0011.01

$$L_{-\frac{5}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{\sqrt{\frac{2}{p}} \left(\text{I} z^2 + 3 \text{M} \sinh \text{H} \text{L} - 3 z \cosh \text{H} \text{L} \right)}{z^{5 \cdot 2}}$$

03.10.03.0012.01

$$L_{-\frac{3}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{\sqrt{\frac{2}{p}} \left(\text{H} \cosh \text{H} \text{L} - \sinh \text{H} \text{L} \right)}{z^{3 \cdot 2}}$$

03.10.03.0005.01

$$L_{-\frac{1}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \sqrt{\frac{2}{p} z} \sinh \text{H} \text{L}$$

03.10.03.0004.01

$$L_{\frac{1}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \sqrt{\frac{2}{p} z} \left(\text{H} \cosh \text{H} \text{L} - \text{I} \text{L} \right)$$

03.10.03.0013.01

$$L_{\frac{3}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} - \frac{z^2 - 2 \sinh \text{H} \text{L} z + 2 \cosh \text{H} \text{L} - 2}{\sqrt{2 p} z^{3 \cdot 2}}$$

03.10.03.0014.01

$$L_{\frac{5}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{-z^4 + 4 z^2 - 24 \sinh \text{H} \text{L} z + 8 \text{I} z^2 + 3 \text{M} \cosh \text{H} \text{L} - 24}{4 \sqrt{2 p} z^{5 \cdot 2}}$$

03.10.03.0015.01

$$L_{\frac{7}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{-z^6 + 6 z^4 - 72 z^2 + 48 \text{I} z^2 + 15 \text{M} \sinh \text{H} \text{L} z - 144 \text{I} z^2 + 5 \text{M} \cosh \text{H} \text{L} + 720}{24 \sqrt{2 p} z^{7 \cdot 2}}$$

03.10.03.0016.01

$$L_{\frac{9}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{1}{192 \sqrt{2 p} z^{9 \cdot 2}} \left(\text{I} - z^8 + 8 z^6 - 144 z^4 + 2880 z^2 - 1920 \text{I} z^2 + 21 \text{M} \sinh \text{H} \text{L} z + 384 \text{I} z^4 + 45 z^2 + 105 \text{M} \cosh \text{H} \text{L} - 40320 \text{M} \right)$$

03.10.03.0017.01

$$L_{\frac{11}{2}} \text{H} \tilde{\text{L}} \tilde{\text{S}} \frac{1}{1920 \sqrt{2 p} z^{11 \cdot 2}} \left(\text{I} - z^{10} + 10 z^8 - 240 z^6 + 7200 z^4 - 201600 z^2 + 3840 \text{I} z^4 + 105 z^2 + 945 \text{M} \sinh \text{H} \text{L} z - 57600 \text{I} z^4 + 28 z^2 + 63 \text{M} \cosh \text{H} \text{L} + 3628800 \text{M} \right)$$

Symbolic rational n

03.10.03.0006.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} = \frac{1}{\sqrt{z}} \tilde{a}^{\frac{1}{2} p \tilde{a} j n + \frac{1}{2} N} \sqrt{\frac{2}{p}} \left(\sinh \left(\frac{1}{2} \tilde{a} p \left(n + \frac{1}{2} \right) - z \right) \tilde{a} \frac{f^{-\frac{1}{4}} \mathbb{H} n + 1 l y}{k=0} \frac{J 2 k - n - \frac{1}{2} N!}{\mathbb{H} k L! J - 2 k - n - \frac{1}{2} N! \mathbb{H} z L^{2 k}} + \right. \\ \left. \cosh \left(\frac{1}{2} \tilde{a} p \left(n + \frac{1}{2} \right) - z \right) \tilde{a} \frac{f^{-\frac{1}{4}} \mathbb{H} n + 3 l y}{k=0} \frac{J 2 k - n + \frac{1}{2} N! \mathbb{H} z L^{-2 k - 1}}{\mathbb{H} k + 1 L! J - 2 k - n - \frac{3}{2} N!} \right); - n - \frac{1}{2} \hat{I} N$$

03.10.03.0007.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} = \frac{2^{1-n} z^{n-1}}{\sqrt{p} J n - \frac{1}{2} N!} \tilde{a}^{\frac{n-1}{2}} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{z^2}{4} \right)^{-k} + \\ - \frac{1}{\sqrt{z}} \tilde{a}^{\frac{1}{2} p \tilde{a} j n + \frac{1}{2} N} \sqrt{\frac{2}{p}} \left(\sinh \left(\frac{1}{2} \tilde{a} p \left(n + \frac{1}{2} \right) - z \right) \tilde{a} \frac{f^{-\frac{1}{4}} \mathbb{H} n \alpha - 1 l y}{k=0} \frac{J 2 k + n \alpha - \frac{1}{2} N!}{\mathbb{H} k L! J n \alpha - 2 k - \frac{1}{2} N! \mathbb{H} z L^{2 k}} + \right. \\ \left. \cosh \left(\frac{1}{2} \tilde{a} p \left(n + \frac{1}{2} \right) - z \right) \tilde{a} \frac{f^{-\frac{1}{4}} \mathbb{H} n \alpha - 3 l y}{k=0} \frac{J 2 k + n \alpha + \frac{1}{2} N! \mathbb{H} z L^{-2 k - 1}}{\mathbb{H} k + 1 L! J n \alpha - 2 k - \frac{3}{2} N!} \right); n - \frac{1}{2} \hat{I} Z$$

Values at fixed points

03.10.03.0018.01

$$L_{-1} \mathbb{H} \mathbb{L} \tilde{S} = \frac{2}{p}$$

General characteristics

Domain and analyticity

$L_n \mathbb{H} \mathbb{L}$ is an analytical function of n and z which is defined over C^2 .

03.10.04.0001.01

$$\mathbb{H} * z L^{\mathbb{T} M} L_n \mathbb{H} \mathbb{L} > \mathbb{H} \tilde{A} C L^{\mathbb{T} M} C$$

Symmetries and periodicities

Parity

03.10.04.0002.01

$$L_n \mathbb{H} z L \tilde{S} = \mathbb{H} z L^n z^{-n} L_n \mathbb{H} \mathbb{L}$$

Mirror symmetry

03.10.04.0003.02

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \overline{L_n \mathbb{H} \mathbb{L}}; z \tilde{I} \mathbb{H} \mathbb{Y}, 0 L$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed n , the function $L_n H_L$ has an essential singularity at $z = \infty$. At the same time, the point $z = \infty$ is a branch point for generic n .

03.10.04.0004.01

$\text{Sing}_z L_n H_L \sim \infty, \infty$

With respect to n

For fixed z , the function $L_n H_L$ has only one singular point at $n = \infty$. It is an essential singular point.

03.10.04.0005.01

$\text{Sing}_n L_n H_L \sim \infty, \infty$

Branch points

With respect to z

For fixed noninteger n , the function $L_n H_L$ has two branch points: $z = 0$, $z = \infty$. At the same time, the point $z = \infty$ is an essential singularity.

For integer n , the function $L_n H_L$ does not have branch points.

03.10.04.0006.01

$\text{BP}_z L_n H_L \sim 0, \infty; n \in \mathbb{Z}$

03.10.04.0007.01

$\text{BP}_z L_n H_L \sim \infty; n \in \mathbb{Z}$

03.10.04.0008.01

$R_z L_n H_L \sim \log \infty; n \in \mathbb{Q}$

03.10.04.0009.01

$R_z \left(L_{\frac{p}{q}} H_L, 0 \right) \sim q \infty; p \in \mathbb{Z}, q - 1 \in \mathbb{N}^+, \gcd(p, q) = 1$

03.10.04.0010.01

$R_z L_n H_L \sim \log \infty; n \in \mathbb{Q}$

03.10.04.0011.01

$R_z \left(L_{\frac{p}{q}} H_L, \infty \right) \sim q \infty; p \in \mathbb{Z}, q - 1 \in \mathbb{N}^+, \gcd(p, q) = 1$

With respect to n

For fixed z , the function $L_n H_L$ does not have branch points.

03.10.04.0012.01

$\text{BP}_n L_n H_L \sim \infty$

Branch cuts

With respect to z

When n is an integer, $L_n(z)$ is an entire function of z . For fixed noninteger n , it has one infinitely long branch cut. For fixed noninteger n , the function $L_n(z)$ is a single-valued function on the z -plane cut along the interval $[-1, 0]$, where it is continuous from above.

03.10.04.0013.01

$$\text{BC}_z L_n(z) \text{ is } \Re z \in [-1, 0], -\infty < \Im z < \infty; n \notin \mathbb{Z}$$

03.10.04.0014.01

$$\text{BC}_z L_n(z) \text{ is } \Re z < -1; n \in \mathbb{Z}$$

03.10.04.0015.01

$$\lim_{x \rightarrow 0^+} L_n(x) + \frac{1}{2} \ln x = L_n(x) + \frac{1}{2} \ln x; x < 0$$

03.10.04.0016.01

$$\lim_{x \rightarrow 0^+} L_n(x) - \frac{1}{2} \ln x = \frac{1}{2} \ln x - \frac{1}{2} \ln x; x < 0$$

With respect to n

For fixed z , the function $L_n(z)$ is an entire function of n and does not have branch cuts.

03.10.04.0017.01

$$\text{BC}_n L_n(z) \text{ is } \Re z < -1$$

Series representations

Generalized power series

Expansions at generic point $z \notin [-1, 0]$

For the function itself

03.10.06.0017.01

$$L_n(z) \mu \left(\frac{1}{z_0} \right)^n \left[\frac{\arg(z - z_0)}{2\pi} \right] \left[\frac{\arg(z - z_0)}{2\pi} \right] \left(L_n(z_0) + \left(L_{n-1}(z_0) - \frac{n L_n(z_0)}{z_0} \right) (z - z_0) + \frac{1}{2 z_0^2} \left(z_0 \left(\frac{2^{1-n} z_0^n}{\sqrt{p} \Gamma(n + \frac{1}{2})} - L_{n-1}(z_0) \right) + L_n(z_0) \ln^2 + n + z_0^2 \right) (z - z_0)^2 + \frac{1}{4} \right); \Re z \notin [-1, 0]$$

03.10.06.0018.01

$$L_n(z) \mu \left(\frac{1}{z_0} \right)^n \left[\frac{\arg(z - z_0)}{2\pi} \right] \left[\frac{\arg(z - z_0)}{2\pi} \right] \left(L_n(z_0) + \left(L_{n-1}(z_0) - \frac{n L_n(z_0)}{z_0} \right) (z - z_0) + \frac{1}{2 z_0^2} \left(z_0 \left(\frac{2^{1-n} z_0^n}{\sqrt{p} \Gamma(n + \frac{1}{2})} - L_{n-1}(z_0) \right) + L_n(z_0) \ln^2 + n + z_0^2 \right) (z - z_0)^2 + O((z - z_0)^3) \right)$$

03.10.06.0019.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \sqrt{p} \mathbb{G} \mathbb{H} + 2 \mathbb{L} \mathbb{K} \frac{z_0^{n+1}}{4} \left(\frac{1}{z_0} \right)^n \left[\frac{\arg(z-z_0)N}{2p} \right] \frac{n}{z_0} \left[\frac{\arg(z-z_0)N}{2p} \right]$$

$$\hat{a} \frac{2^k z_0^{-k}}{k!} {}_3F_4 \left(1, \frac{n}{2} + 1, \frac{n+3}{2}; \frac{3}{2}, \frac{n-k}{2} + 1, \frac{1}{2} \mathbb{H} k + n + 3 \mathbb{L}, n + \frac{3}{2}; \frac{z_0^2}{4} \right) \mathbb{H} - z_0 \mathbb{L}^k$$

03.10.06.0020.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \left(\frac{1}{z_0} \right)^n \left[\frac{\arg(z-z_0)N}{2p} \right] \frac{n}{z_0} \left[\frac{\arg(z-z_0)N}{2p} \right]$$

$$\hat{a} \left(\frac{1}{k!} z_0^{-k} \hat{a} \mathbb{H} 1 \mathbb{L}^{k+m} \binom{k}{m} \mathbb{H} n \mathbb{L}_{k-m} \hat{a} \frac{m}{p=0} \frac{\mathbb{H} 1 \mathbb{L}^{p-1} 2^{2p-m} \mathbb{H} m \mathbb{L}_2 \mathbb{H} m-p \mathbb{L} \mathbb{H} \mathbb{L}_p}{\mathbb{H} m-p \mathbb{L}!} \left(\frac{1}{2} z_0 \hat{a} \frac{p-1}{j! \mathbb{H} 2j+p-1 \mathbb{L}! \mathbb{H} p-n+1 \mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}} \right. \right. \\ \left. \left. \left(-\frac{z_0^2}{4} \right)^j L_{n-1} \mathbb{H} \mathbb{L}_0 \mathbb{L} - \hat{a} \frac{p}{j! \mathbb{H} p-2j \mathbb{L}! \mathbb{H} p-n+1 \mathbb{L}_j \mathbb{H} \mathbb{L}_j} \left(-\frac{z_0^2}{4} \right)^j L_n \mathbb{H} \mathbb{L}_0 \mathbb{L} \right) + \right. \\ \left. \frac{2^{-n} z_0^{-k+n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N} k!} \hat{a} \hat{a} \mathbb{H} 1 \mathbb{L}^{i+m} \binom{i}{m} \mathbb{H} n \mathbb{L}_{i-m} \hat{a} \frac{m}{p=0} \frac{\mathbb{H} 1 \mathbb{L}^{p-1} 2^{2p-m} \mathbb{H} m \mathbb{L}_2 \mathbb{H} m-p \mathbb{L} \mathbb{H} \mathbb{L}_p}{\mathbb{H} m-p \mathbb{L}!} \right. \\ \left. \hat{a} \frac{p-1 \mathbb{H} 1 \mathbb{L}^j 2^{-2j} \mathbb{H} j+p-1 \mathbb{L}! \mathbb{H} j-k+n+2 \mathbb{L}_{i+k-1} z_0^{2j}}{j! \mathbb{H} 2j+p-1 \mathbb{L}! \mathbb{H} p-n+1 \mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}} \right) \mathbb{H} - z_0 \mathbb{L}^k$$

03.10.06.0021.01

$$L_n \mathbb{H} \mathbb{L} \mu \left(\frac{1}{z_0} \right)^n \left[\frac{\arg(z-z_0)N}{2p} \right] \frac{n}{z_0} \left[\frac{\arg(z-z_0)N}{2p} \right] L_n \mathbb{H} \mathbb{L}_0 \mathbb{L} \mathbb{H} + O \mathbb{H} - z_0 \mathbb{L}$$

Expansions on branch cuts

For the function itself

03.10.06.0022.01

$$L_n \mathbb{H} \mathbb{L} \mu$$

$$\hat{a}^{2n p \hat{a} \hat{g} \frac{\arg \mathbb{H} - x \mathbb{L}}{2p} - w} \left(L_n \mathbb{H} \mathbb{L} + \left(L_{n-1} \mathbb{H} \mathbb{L} - \frac{n L_n \mathbb{H} \mathbb{L}}{x} \right) \mathbb{H} - x \mathbb{L} + \frac{1}{2 x^2} \left(x \left(\frac{2^{1-n} x^n}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} - L_{n-1} \mathbb{H} \mathbb{L} \right) + \mathbb{L} x^2 + n^2 + n \mathbb{M} L_n \mathbb{H} \mathbb{L} \right) \mathbb{H} - x \mathbb{L}^2 + \frac{1}{4} \right) \bullet;$$

$$\mathbb{H} \otimes x \mathbb{L} \mathbb{B} x \hat{\mathbb{I}} \mathbb{R} \mathbb{B} x < 0$$

03.10.06.0023.01

$$L_n \mathbb{H} \mathbb{L} \mu \hat{a}^{2n p \hat{a} \hat{g} \frac{\arg \mathbb{H} - x \mathbb{L}}{2p} - w} \left(L_n \mathbb{H} \mathbb{L} + \left(L_{n-1} \mathbb{H} \mathbb{L} - \frac{n L_n \mathbb{H} \mathbb{L}}{x} \right) \mathbb{H} - x \mathbb{L} + \right.$$

$$\left. \frac{1}{2 x^2} \left(x \left(\frac{2^{1-n} x^n}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} - L_{n-1} \mathbb{H} \mathbb{L} \right) + \mathbb{L} x^2 + n^2 + n \mathbb{M} L_n \mathbb{H} \mathbb{L} \right) \mathbb{H} - x \mathbb{L}^2 + O \mathbb{H} - x \mathbb{L}^3 \mathbb{M} \right) \bullet; x \hat{\mathbb{I}} \mathbb{R} \mathbb{B} x < 0$$

03.10.06.0024.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \sqrt{p} \mathbb{G} \mathbb{H} + 2 \mathbb{L} \mathbb{K} - \frac{x^{n+1}}{4} \tilde{a}^{2np} \tilde{a}^{\frac{\arg \mathbb{H} - x \mathbb{L}}{2p} w} \tilde{a}^{\frac{2^k x^{-k}}{k!}} {}_3F_4 \left(1, \frac{n}{2} + 1, \frac{n+3}{2}; \frac{3}{2}, \frac{n-k}{2} + 1, \frac{1}{2} \mathbb{H} k + n + 3 \mathbb{L}, n + \frac{3}{2}; \frac{x^2}{4} \right) \mathbb{H} - x \mathbb{L}^k \bullet;$$

$$x \hat{\mathbb{I}} \mathbb{R} \mathbb{B} x < 0$$

03.10.06.0025.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \tilde{a}^{2p} \tilde{a}^{\frac{\arg \mathbb{H} - x \mathbb{L}}{2p} w} \tilde{a}^{\frac{1}{k!} x^{-k}} \tilde{a}^{\frac{\mathbb{H} - 1 \mathbb{L}^{k+m}}{m}} \binom{k}{m} \mathbb{H} n \mathbb{L}_{k-m} \tilde{a}^{\frac{m}{p=0} \frac{\mathbb{H} - 1 \mathbb{L}^{p-1} 2^{2p-m} \mathbb{H} m \mathbb{L}_{2\mathbb{H}-p \mathbb{L}} \mathbb{H} \mathbb{L}_p}{\mathbb{H} n - p \mathbb{L}!}} \left(\frac{1}{2} x \tilde{a}^{\frac{p-1}{j=0} \frac{\mathbb{H} j + p - 1 \mathbb{L}!}{j! \mathbb{H} 2j + p - 1 \mathbb{L}! \mathbb{H} p - n + 1 \mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}}} \right. \\ \left. \left(- \frac{x^2}{4} \right)^j L_{n-1} \mathbb{H} \mathbb{L} - \tilde{a}^{\frac{p}{j=0} \frac{\mathbb{H} p - j \mathbb{L}!}{j! \mathbb{H} p - 2j \mathbb{L}! \mathbb{H} p - n + 1 \mathbb{L}_j \mathbb{H} \mathbb{L}_j}} \left(- \frac{x^2}{4} \right)^j L_n \mathbb{H} \mathbb{L} \right) + \\ \frac{2^{-n} x^{-k+n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N} k!} \tilde{a}^{\frac{k-1}{i=1} \frac{i}{m=0} \mathbb{H} 1 \mathbb{L}^{i+m}} \binom{i}{m} \mathbb{H} n \mathbb{L}_{i-m} \tilde{a}^{\frac{m}{p=0} \frac{\mathbb{H} - 1 \mathbb{L}^{p-1} 2^{2p-m} \mathbb{H} m \mathbb{L}_{2\mathbb{H}-p \mathbb{L}} \mathbb{H} \mathbb{L}_p}{\mathbb{H} n - p \mathbb{L}!}} \\ \left. \tilde{a}^{\frac{p-1}{j=0} \frac{\mathbb{H} 1 \mathbb{L}^j 2^{-2j} \mathbb{H} j + p - 1 \mathbb{L}! \mathbb{H} j - k + n + 2 \mathbb{L}_{i+k-1} z^{2j}}{j! \mathbb{H} 2j + p - 1 \mathbb{L}! \mathbb{H} p - n + 1 \mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}}} \right) \mathbb{H} - x \mathbb{L}^k \bullet; x \hat{\mathbb{I}} \mathbb{R} \mathbb{B} x < 0$$

03.10.06.0026.01

$$L_n \mathbb{H} \mathbb{L} \mu \tilde{a}^{2np} \tilde{a}^{\frac{\arg \mathbb{H} - x \mathbb{L}}{2p} w} L_n \mathbb{H} \mathbb{L} \mathbb{H} + O \mathbb{H} - x \mathbb{L} \bullet; x \hat{\mathbb{I}} \mathbb{R} \mathbb{B} x < 0$$

Expansions at $z \tilde{S} 0$

For the function itself

General case

03.10.06.0001.02

$$L_n \mathbb{H} \mathbb{L} \mu \frac{2^{-n} z^{n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}} \left(1 + \frac{z^2}{3 \mathbb{H} n + 3 \mathbb{L}} + \frac{z^4}{15 \mathbb{H} n + 3 \mathbb{L} \mathbb{H} n + 5 \mathbb{L}} + \frac{1}{4} \right) \bullet; \mathbb{H} \mathbb{R} 0 \mathbb{L}$$

03.10.06.0027.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{2^{-n} z^{n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}} \left(1 + \frac{z^2}{3 \mathbb{H} n + 3 \mathbb{L}} + \frac{z^4}{15 \mathbb{H} n + 3 \mathbb{L} \mathbb{H} n + 5 \mathbb{L}} + O(z^6) \right)$$

03.10.06.0002.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \mathbb{K} - \frac{z^{n+1}}{2} \tilde{a}^{\frac{1}{k=0} \frac{\mathbb{H} - 1 \mathbb{L}^{k+1}}{\mathbb{G} \mathbb{J} k + \frac{3}{2} \mathbb{N} \mathbb{G} \mathbb{J} k + n + \frac{3}{2} \mathbb{N}}} \mathbb{K} - \frac{z^{2k}}{2}$$

03.10.06.0028.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \frac{2}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}} \mathbb{K} - \frac{z^{n+1}}{2} \tilde{a}^{\frac{1}{k=0} \frac{\mathbb{H} - 1 \mathbb{L}^{k+1}}{4^k \mathbb{J}_{\frac{3}{2} \mathbb{N}} \mathbb{J} n + \frac{3}{2} \mathbb{N}_k}} \frac{z^{2k}}{2}$$

03.10.06.0029.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \frac{2}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}} \mathbb{K} \check{0} \frac{z^{n+1}}{2} {}_1F_2 \left(1; \frac{3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right)$$

03.10.06.0003.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \mathbb{K} \check{0} \frac{z^{n+1}}{2} {}_1\check{F}_2 \left(1; \frac{3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right)$$

03.10.06.0004.02

$$L_n \mathbb{H} \mathbb{L} \mu \frac{2^{-n} z^{n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}} + O(\mathbb{L} z^{n+3} \mathbb{M}; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N})$$

03.10.06.0030.01

$$L_n \mathbb{H} \mathbb{L} \check{S} F_{\check{\mathbb{N}}} \mathbb{H}, n \mathbb{L};$$

$$\left(\left(F_n \mathbb{H}, n \mathbb{L} \check{S} \mathbb{K} \check{0} \frac{z^{n+1}}{2} \hat{\mathbb{a}} \frac{\mathbb{I} \frac{z}{2} \mathbb{M}^{2k}}{\mathbb{G} \mathbb{J} k + \frac{3}{2} \mathbb{N} \mathbb{G} \mathbb{J} k + n + \frac{3}{2} \mathbb{N}} \check{S} L_n \mathbb{H} \mathbb{L} - \frac{1}{\mathbb{G} \mathbb{J} n + \frac{5}{2} \mathbb{N} \mathbb{G} \mathbb{J} n + n + \frac{5}{2} \mathbb{N}} \mathbb{K} \check{0} \frac{z^{2n+n+3}}{2} {}_1F_2 \left(1; n + \frac{5}{2}, n + n + \frac{5}{2}; \frac{z^2}{4} \right) \right) \hat{\mathbb{I}} \right. \\ \left. n \hat{\mathbb{I}} \mathbb{N} \right)$$

Summed form of the truncated series expansion.

Special cases

03.10.06.0031.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{1}{\mathbb{G} \mathbb{H} - n \mathbb{L}} \mathbb{K} \check{0} \frac{z^{-n}}{2} \left(1 + \frac{z^2}{4 \mathbb{H} - n \mathbb{L}} + \frac{z^4}{32 \mathbb{H} - n \mathbb{L} \mathbb{H} - n \mathbb{L}} + \frac{1}{4} \right) \bullet; \mathbb{H} \mathbb{L} \mathbb{O} \mathbb{L} \hat{\mathbb{I}} - n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}^+$$

03.10.06.0032.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{1}{\mathbb{G} \mathbb{H} - n \mathbb{L}} \mathbb{K} \check{0} \frac{z^{-n}}{2} \left(1 + \frac{z^2}{4 \mathbb{H} - n \mathbb{L}} + \frac{z^4}{32 \mathbb{H} - n \mathbb{L} \mathbb{H} - n \mathbb{L}} + O(\mathbb{L} z^6 \mathbb{M}) \right) \bullet; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}^+$$

03.10.06.0033.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \hat{\mathbb{a}} \frac{\mathbb{I} \frac{z}{2} \mathbb{M}^{2k-n}}{\mathbb{G} \mathbb{H} - n + 1 \mathbb{L} k!} \bullet; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}$$

03.10.06.0034.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \frac{1}{\mathbb{G} \mathbb{H} - n \mathbb{L}} \mathbb{K} \check{0} \frac{z^{-n}}{2} \hat{\mathbb{a}} \frac{z^{2k}}{4^k \mathbb{H} - n \mathbb{L}_k k!} \bullet; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}$$

03.10.06.0035.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \frac{1}{\mathbb{G} \mathbb{H} - n \mathbb{L}} \mathbb{K} \check{0} \frac{z^{-n}}{2} {}_0F_1 \left(; 1 - n; \frac{z^2}{4} \right) \bullet; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}$$

03.10.06.0036.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \mathbb{K} \check{0} \frac{z^{-n}}{2} {}_0\check{F}_1 \left(; 1 - n; \frac{z^2}{4} \right) \bullet; -n - \frac{3}{2} \hat{\mathbb{I}} \mathbb{N}$$

03.10.06.0037.01

$$L_n \mathbb{H} \tilde{L} \check{S} \quad L_n \mathbb{H} L \bullet; -n - \frac{3}{2} \hat{I} \quad N$$

03.10.06.0005.02

$$L_n \mathbb{H} L \mu \frac{2^n z^{-n}}{\Gamma(n)} + O(z^{2-n} M); -n - \frac{3}{2} \hat{I} \quad N$$

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z^{\otimes} \mathbb{Y}$

In exponential form ||| In exponential form

03.10.06.0038.01

$$L_n \mathbb{H} L \mu \frac{1}{\sqrt{2p}z} \left(\tilde{a}^z \left(1 + \frac{1-4n^2}{8z} + \frac{9-40n^2+16n^4}{128z^2} + \frac{1}{4} \right) + \tilde{a}^{-z-\tilde{a}pn} \tilde{a} \left(1 - \frac{1-4n^2}{8z} + \frac{9-40n^2+16n^4}{128z^2} + \frac{1}{4} \right) \right) -$$

$$\frac{2^{1-n} z^{n-1}}{\sqrt{p} \Gamma(n + \frac{1}{2}N)} \left(1 + \frac{1-2n}{z^2} + \frac{3I_3 - 8n + 4n^2 M}{z^4} + \frac{1}{4} \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \hat{I} \quad \mathbb{H} z^{\otimes} \mathbb{Y} L$$

03.10.06.0039.01

$$L_n \mathbb{H} L \mu \frac{1}{\sqrt{2p}z} \left(\tilde{a}^z \left(\hat{a} \frac{\binom{n}{Jn + \frac{1}{2}N} \frac{J^{\frac{1}{2}} - nN}{k!} \left(\frac{1}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) + \tilde{a}^{-z-\tilde{a}pn} \tilde{a} \left(\hat{a} \frac{\binom{n}{Jn + \frac{1}{2}N} \frac{J^{\frac{1}{2}} - nN}{k!} \left(-\frac{1}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) \right) -$$

$$\frac{2^{1-n} z^{n-1}}{\sqrt{p} \Gamma(n + \frac{1}{2}N)} \left(\hat{a} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O\left(\frac{1}{z^{2n+2}} \right) \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \hat{I} \quad \mathbb{H} z^{\otimes} \mathbb{Y} L$$

03.10.06.0040.01

$$L_n \mathbb{H} L \mu \frac{1}{\sqrt{2p}z} \left(\tilde{a}^z {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; \frac{1}{2z} \right) + \tilde{a}^{-z-\tilde{a}pn} \tilde{a} {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; -\frac{1}{2z} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \Gamma(n + \frac{1}{2}N)} {}_3F_0 \left(1, \frac{1}{2}, \frac{1}{2} - n; ; \frac{4}{z^2} \right) \bullet;$$

$$-p < \arg \mathbb{H} L < \frac{p}{2} \hat{I} \quad \mathbb{H} z^{\otimes} \mathbb{Y} L$$

03.10.06.0041.01

$$L_n \mathbb{H} L \mu \frac{1}{\sqrt{2p}z} \left(\tilde{a}^z \left(1 + O\left(\frac{1}{z} \right) \right) + \tilde{a}^{-z-\tilde{a}pn} \tilde{a} \left(1 + O\left(\frac{1}{z} \right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \Gamma(n + \frac{1}{2}N)} \left(1 + O\left(\frac{1}{z^2} \right) \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \hat{I} \quad \mathbb{H} z^{\otimes} \mathbb{Y} L$$

03.10.06.0006.02

$$L_n \mathbb{H} L \mu \frac{\tilde{a}^z}{\sqrt{2p}z} \left(1 + O\left(\frac{1}{z} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \Gamma(n + \frac{1}{2}N)} \left(1 + O\left(\frac{1}{z^2} \right) \right) \bullet; \operatorname{Re} \mathbb{H} L \geq 0 \cap \mathbb{H} z^{\otimes} \mathbb{Y} L$$

In hyperbolic form ||| In hyperbolic form

03.10.06.0042.01

$$L_n \mathbb{H} L \mu \sqrt{\frac{2}{-z^{n+1} I - z^2 M^{\frac{2n+3}{4}}}} \left(\sin \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 L p}{4} \right) \left(1 + \frac{16 n^4 - 40 n^2 + 9}{128 z^2} + \frac{256 n^8 - 5376 n^6 + 31584 n^4 - 51664 n^2 + 11025}{98304 z^4} + \frac{1}{4} \right) + \frac{4 n^2 - 1}{8 \sqrt{-z^2}} \cos \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 L p}{4} \right) \left(1 + \frac{16 n^4 - 136 n^2 + 225}{384 z^2} + \frac{256 n^8 - 10496 n^6 + 137824 n^4 - 656784 n^2 + 893025}{491520 z^4} + \frac{1}{4} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(1 - \frac{2n-1}{z^2} + \frac{3I4n^2-8n+3M}{z^4} + \frac{1}{4} \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \text{ i } \mathbb{H} z^{\mathbb{H}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0043.01

$$L_n \mathbb{H} L \mu \frac{\sqrt{2} \tilde{a}^{-\frac{p\tilde{a}}{4} \mathbb{H} + 2nL}}{\sqrt{p} \sqrt{z}} \left(\sinh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) \left(\hat{a} \frac{{}_n J_{\frac{1}{4}} \mathbb{H} - 2n \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} - 2n \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} n + 1 \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} N {}_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{1}{2}} N {}_k!} \right) + \frac{1-4n^2}{8z} \cosh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) \left(\hat{a} \frac{{}_n J_{\frac{1}{4}} \mathbb{H} - 2n \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} - 2n \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} N {}_k J_{\frac{1}{4}} \mathbb{H} n + 5 \mathbb{I} N {}_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{3}{2}} N {}_k!} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(\hat{a} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \text{ i } \mathbb{H} z^{\mathbb{H}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0044.01

$$L_n \mathbb{H} L \mu \frac{\sqrt{2} \tilde{a}^{-\frac{p\tilde{a}}{4} \mathbb{H} + 2nL}}{\sqrt{p} \sqrt{z}} \left(\sinh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) {}_4F_1 \left(\frac{1-2n}{4}, \frac{3-2n}{4}, \frac{2n+1}{4}, \frac{2n+3}{4}; \frac{1}{2}; \frac{1}{z^2} \right) + \frac{1-4n^2}{8z} \cosh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) {}_4F_1 \left(\frac{3-2n}{4}, \frac{5-2n}{4}, \frac{3+2n}{4}, \frac{5+2n}{4}; \frac{3}{2}; \frac{1}{z^2} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} {}_3F_0 \left(\frac{1}{2}, 1, \frac{1}{2} - n; ; \frac{4}{z^2} \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \text{ i } \mathbb{H} z^{\mathbb{H}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0045.01

$$L_n \mathbb{H} L \mu \frac{\sqrt{2} \tilde{a}^{-\frac{p\tilde{a}}{4} \mathbb{H} + 2nL}}{\sqrt{p} \sqrt{z}} \left(\sinh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) + \frac{1-4n^2}{8z} \cosh \left(z + \frac{p\tilde{a}}{4} \mathbb{H} n + 1L \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(1 + O \left(\frac{1}{z^2} \right) \right) \bullet; -p < \arg \mathbb{H} L < \frac{p}{2} \text{ i } \mathbb{H} z^{\mathbb{H}} \mathbb{R} \mathbb{Y} L$$

Containing Bessel functions

03.10.06.0046.01

$$L_n \mathbb{H} L - \frac{z}{\sqrt{z^2}} I_n \mathbb{H} L \mu - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(1 - \frac{2n-1}{z^2} + \frac{3 \operatorname{I4} n^2 - 8n + 3M}{z^4} + \frac{1}{4} \right) \bullet; \arg \mathbb{H} L^{\mathbb{H}^1} \frac{p}{2} \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0047.01

$$L_n \mathbb{H} L - \frac{z}{\sqrt{z^2}} I_n \mathbb{H} L \mu - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(\hat{\mathbf{a}} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O\left(\frac{1}{z^{2n+2}} \right) \right) \bullet; \arg \mathbb{H} L^{\mathbb{H}^1} \frac{p}{2} \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0048.01

$$L_n \mathbb{H} L - \frac{z}{\sqrt{z^2}} I_n \mathbb{H} L \mu - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} {}_3F_0 \left(\frac{1}{2}, \frac{1}{2} - n, 1; ; -\frac{4}{z^2} \right) \bullet; \arg \mathbb{H} L^{\mathbb{H}^1} \frac{p}{2} \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0049.01

$$L_n \mathbb{H} L - \frac{z}{\sqrt{z^2}} I_n \mathbb{H} L \mu - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(1 + O\left(\frac{1}{z^2} \right) \right) \bullet; \arg \mathbb{H} L^{\mathbb{H}^1} \frac{p}{2} \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

Expansions containing $z^{\mathbb{H}^1} - \mathbb{H}$

In exponential form ||| In exponential form

03.10.06.0050.01

$$L_n \mathbb{H} L \mu - \frac{\hat{\mathbf{a}}}{\sqrt{-2p}z} \left(\hat{\mathbf{a}}^z \left(1 + \frac{1-4n^2}{8z} + \frac{16n^4-40n^2+9}{128z^2} + \frac{1}{4} \right) - \hat{\mathbf{a}} \hat{\mathbf{a}}^{-z+\hat{\mathbf{a}}pn} \left(1 - \frac{1-4n^2}{8z} + \frac{16n^4-40n^2+9}{128z^2} + \frac{1}{4} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(1 - \frac{2n-1}{z^2} + \frac{3 \operatorname{I4} n^2 - 8n + 3M}{z^4} + \frac{1}{4} \right) \bullet; 0 < \arg \mathbb{H} L^{\mathbb{H}^1} p \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0051.01

$$L_n \mathbb{H} L \mu - \frac{\hat{\mathbf{a}}}{\sqrt{-2p}z} \left(\hat{\mathbf{a}}^z \left(\hat{\mathbf{a}} \frac{{}^n \operatorname{Jn} + \frac{1}{2} N \operatorname{J} \frac{1}{2} - nN}{k!} \left(\frac{1}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) - \hat{\mathbf{a}} \hat{\mathbf{a}}^{-z+\hat{\mathbf{a}}pn} \left(\hat{\mathbf{a}} \frac{{}^n \operatorname{Jn} + \frac{1}{2} N \operatorname{J} \frac{1}{2} - nN}{k!} \left(-\frac{1}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(\hat{\mathbf{a}} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O\left(\frac{1}{z^{2n+2}} \right) \right) \bullet; 0 < \arg \mathbb{H} L^{\mathbb{H}^1} p \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0052.01

$$L_n \mathbb{H} L \mu - \frac{\hat{\mathbf{a}}}{\sqrt{-2p}z} \left(\hat{\mathbf{a}}^z {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; \frac{1}{2z} \right) - \hat{\mathbf{a}} \hat{\mathbf{a}}^{-z+\hat{\mathbf{a}}pn} {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; -\frac{1}{2z} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} {}_3F_0 \left(\frac{1}{2}, \frac{1}{2} - n, 1; ; \frac{4}{z^2} \right) \bullet; 0 < \arg \mathbb{H} L^{\mathbb{H}^1} p \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

03.10.06.0053.01

$$L_n \mathbb{H} L \mu - \frac{\hat{\mathbf{a}}}{\sqrt{-2p}z} \left(\hat{\mathbf{a}}^z \left(1 + O\left(\frac{1}{z} \right) \right) - \hat{\mathbf{a}} \hat{\mathbf{a}}^{\hat{\mathbf{a}}pn-z} \left(1 + O\left(\frac{1}{z} \right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} N} \left(1 + O\left(\frac{1}{z^2} \right) \right) \bullet; 0 < \arg \mathbb{H} L^{\mathbb{H}^1} p \mathbb{H} z^{\mathbb{H}^1} \mathbb{H} L$$

In hyperbolic form ||| In hyperbolic form

03.10.06.0054.01

$$L_n \mathbb{H} \mathbb{L} \mu \sqrt{-\frac{2}{p z}} \tilde{a}^{\frac{ap}{4}} \mathbb{H}^{n-1} \mathbb{L} \left(\sinh \left(z - \frac{ap}{4} \mathbb{H} + 2 n \mathbb{L} \right) \right. \\ \left. \left(1 + \frac{16 n^4 - 40 n^2 + 9}{128 z^2} + \frac{256 n^8 - 5376 n^6 + 31584 n^4 - 51664 n^2 + 11025}{98304 z^4} + \frac{1}{4} \right) + \frac{1 - 4 n^2}{8 z} \cosh \left(z - \frac{ap}{4} \mathbb{H} + 2 n \mathbb{L} \right) \right. \\ \left. \left(1 + \frac{16 n^4 - 136 n^2 + 225}{384 z^2} + \frac{256 n^8 - 10496 n^6 + 137824 n^4 - 656784 n^2 + 893025}{491520 z^4} + \frac{1}{4} \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(1 - \frac{2 n - 1}{z^2} + \frac{3 \mathbb{I} 4 n^2 - 8 n + 3 \mathbb{M}}{z^4} + \frac{1}{4} \right) \bullet; \text{Im} \mathbb{H} \mathbb{L}^3 \mathbb{O} \mathbb{B} \mathbb{H} z^{\mathbb{R}} \mathbb{N} \mathbb{L}$$

03.10.06.0055.01

$$L_n \mathbb{H} \mathbb{L} \mu \sqrt{-\frac{2}{p z}} \tilde{a}^{\frac{p a}{4}} \mathbb{H}^{n-1} \mathbb{L} \left(\sinh \left(z - \frac{p a}{4} \mathbb{H} n + 1 \mathbb{L} \right) \left(\hat{a} \frac{{}_n J_{\frac{1}{4}} \mathbb{H} - 2 n \mathbb{I} n J_{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} n J_{\frac{1}{4}} \mathbb{H} n + 1 \mathbb{I} n J_{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} n}{J_{\frac{1}{2}} n k!} \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) + \right. \\ \left. \frac{1 - 4 n^2}{8 z} \cosh \left(z - \frac{p a}{4} \mathbb{H} n + 1 \mathbb{L} \right) \left(\hat{a} \frac{{}_n J_{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} n J_{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} n J_{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} n J_{\frac{1}{4}} \mathbb{H} n + 5 \mathbb{I} n}{J_{\frac{3}{2}} n k!} \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(\hat{a} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \bullet; \text{Im} \mathbb{H} \mathbb{L}^3 \mathbb{O} \mathbb{B} \mathbb{H} z^{\mathbb{R}} \mathbb{N} \mathbb{L}$$

03.10.06.0007.02

$$L_n \mathbb{H} \mathbb{L} \mu \sqrt{-\frac{2}{p z}} \tilde{a}^{\frac{1}{4} ap} \mathbb{H}^{n-1} \mathbb{L} \left(\sinh \left(z - \frac{2 n + 1}{4} ap \right) {}_4 F_1 \left(\frac{1}{4} \mathbb{H} - 2 n \mathbb{L}, \frac{1}{4} \mathbb{B} - 2 n \mathbb{L}, \frac{1}{4} \mathbb{H} n + 1 \mathbb{L}, \frac{1}{4} \mathbb{H} n + 3 \mathbb{L}; \frac{1}{2}; \frac{1}{z^2} \right) + \right. \\ \left. \frac{1 - 4 n^2}{8 z} \cosh \left(z - \frac{2 n + 1}{4} ap \right) {}_4 F_1 \left(\frac{1}{4} \mathbb{B} - 2 n \mathbb{L}, \frac{1}{4} \mathbb{B} - 2 n \mathbb{L}, \frac{1}{4} \mathbb{H} n + 3 \mathbb{L}, \frac{1}{4} \mathbb{H} n + 5 \mathbb{L}; \frac{3}{2}; \frac{1}{z^2} \right) - \right. \\ \left. \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} {}_3 F_0 \left(\frac{1}{2}, 1, \frac{1}{2} - n; ; \frac{4}{z^2} \right) \right) \bullet; \text{Im} \mathbb{H} \mathbb{L}^3 \mathbb{O} \mathbb{B} \mathbb{H} z^{\mathbb{R}} \mathbb{N} \mathbb{L}$$

03.10.06.0008.02

$$L_n \mathbb{H} \mathbb{L} \mu \sqrt{-\frac{2}{p z}} \tilde{a}^{\frac{1}{4} ap} \mathbb{H}^{n-1} \mathbb{L} \left(\sinh \left(z - \frac{2 n + 1}{4} ap \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) + \frac{1 - 4 n^2}{8 z} \cosh \left(z - \frac{2 n + 1}{4} ap \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(1 + O \left(\frac{1}{z^2} \right) \right) \bullet; \text{Im} \mathbb{H} \mathbb{L}^3 \mathbb{O} \mathbb{B} \mathbb{H} z^{\mathbb{R}} \mathbb{N} \mathbb{L}$$

The general formulas

03.10.06.0009.01

$$L_n \mathbb{H} \mathbb{L} \mu \mathbb{K} \frac{z^{n+1}}{2} \mathbb{A}_F^{\frac{1}{2}} \left(\frac{3}{2}, n + \frac{3}{2}; \frac{z^2}{4}, \frac{z}{4}, \frac{z}{4} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{N} \mathbb{L}$$

03.10.06.0010.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{z^{n+1}}{2} \left(A_F^{\text{power}} \left(\frac{1}{\frac{3}{2}, n + \frac{3}{2}}; \frac{z^2}{4}, \frac{\tilde{z}}{4}, \mathbb{Y} > \right) + A_F^{\text{rigl}} \left(\frac{1}{\frac{3}{2}, n + \frac{3}{2}}; \frac{z^2}{4}, \frac{\tilde{z}}{4}, \mathbb{Y} > \right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} \mathbb{L}$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.10.06.0056.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{z^{n+1}}{\sqrt{2p}} I - z^2 \tilde{M}^{\frac{1}{4}} \mathbb{R} n + 3 \mathbb{L} \left(\tilde{a}^{-\tilde{a} \sqrt{-z^2} + \frac{1}{4}} \mathbb{R} n + 3 \mathbb{L} p \tilde{a} \left(1 - \frac{\tilde{a} I 4 n^2 - 1 M}{8 \sqrt{-z^2}} + \frac{16 n^4 - 40 n^2 + 9}{128 z^2} + \frac{1}{4} \right) + \right. \\ \left. \tilde{a}^{\tilde{a} \sqrt{-z^2} - \frac{1}{4}} \mathbb{R} n + 3 \mathbb{L} p \tilde{a} \left(1 + \frac{\tilde{a} I 4 n^2 - 1 M}{8 \sqrt{-z^2}} + \frac{16 n^4 - 40 n^2 + 9}{128 z^2} + \frac{1}{4} \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(1 - \frac{2n-1}{z^2} + \frac{3 I 4 n^2 - 8 n + 3 M}{z^4} + \frac{1}{4} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} \mathbb{L}$$

03.10.06.0057.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{1}{\sqrt{2p}} z^{n+1} I - z^2 \tilde{M}^{\frac{2n+3}{4}} \left(\tilde{a}^{-\tilde{a} \sqrt{-z^2} + \frac{2n+3}{4}} p \tilde{a} \left(\hat{a} \frac{\mathbb{J} n + \frac{1}{2} \mathbb{N} \frac{1}{2} - n \mathbb{N}}{k!} \left(\frac{\tilde{a}}{2 \sqrt{-z^2}} \right)^k + O \left(\frac{1}{z^{n+1}} \right) \right) + \right. \\ \left. \tilde{a}^{\tilde{a} \sqrt{-z^2} - \frac{2n+3}{4}} p \tilde{a} \left(\hat{a} \frac{\mathbb{J} n + \frac{1}{2} \mathbb{N} \frac{1}{2} - n \mathbb{N}}{k!} \left(- \frac{\tilde{a}}{2 \sqrt{-z^2}} \right)^k + O \left(\frac{1}{z^{n+1}} \right) \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(\hat{a} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} \mathbb{L}$$

03.10.06.0058.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{z^{n+1}}{\sqrt{2p}} I - z^2 \tilde{M}^{\frac{2n+3}{4}} \left(\tilde{a}^{-\tilde{a} \sqrt{-z^2} + \frac{2n+3}{4}} p \tilde{a} {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; \frac{\tilde{a}}{2 \sqrt{-z^2}} \right) + \tilde{a}^{\tilde{a} \sqrt{-z^2} - \frac{2n+3}{4}} p \tilde{a} {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; - \frac{\tilde{a}}{2 \sqrt{-z^2}} \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} {}_3F_0 \left(\frac{1}{2}, \frac{1}{2} - n, 1; ; \frac{4}{z^2} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} \mathbb{L}$$

03.10.06.0059.01

$$L_n \mathbb{H} \mathbb{L} \mu \frac{1}{\sqrt{2p}} z^{n+1} I - z^2 \tilde{M}^{\frac{2n+3}{4}} \left(\tilde{a}^{-\tilde{a} \sqrt{-z^2} + \frac{2n+3}{4}} p \tilde{a} \left(1 + O \left(\frac{1}{z} \right) \right) + \tilde{a}^{\tilde{a} \sqrt{-z^2} - \frac{2n+3}{4}} p \tilde{a} \left(1 + O \left(\frac{1}{z} \right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(1 + O \left(\frac{1}{z^2} \right) \right) \bullet; \\ \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} \mathbb{L}$$

Using exponential function with branch cut-free arguments

03.10.06.0060.01

$$L_n \mathbb{H} L_\mu \frac{1}{2\sqrt{2p}} z^{n+1} I_- z^2 \tilde{M}^{\frac{3+2n}{4}} \left(\tilde{a}^{-\frac{2n+3}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(1 + \frac{\tilde{a} I_4 n^2 - 1M}{8\sqrt{-z^2}} + \frac{16n^4 - 40n^2 + 9}{128z^2} + \frac{1}{4} \right) + \right. \\ \left. \tilde{a}^{\frac{2n+3}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(1 - \frac{\tilde{a} I_4 n^2 - 1M}{8\sqrt{-z^2}} + \frac{16n^4 - 40n^2 + 9}{128z^2} + \frac{1}{4} \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} GJn + \frac{1}{2}N} \left(1 - \frac{2n-1}{z^2} + \frac{3I_4 n^2 - 8n + 3M}{z^4} + \frac{1}{4} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0061.01

 $L_n \mathbb{H} L_\mu$

$$\frac{1}{2\sqrt{2p}} z^{n+1} I_- z^2 \tilde{M}^{\frac{3+2n}{4}} \left(\tilde{a}^{\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(\hat{\tilde{a}} \frac{Jn + \frac{1}{2}N J_{\frac{1}{2}} - nN}{k!} \left(\frac{\tilde{a}}{2\sqrt{-z^2}} \right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) + \right. \\ \left. \tilde{a}^{-\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(\hat{\tilde{a}} \frac{Jn + \frac{1}{2}N J_{\frac{1}{2}} - nN}{k!} \left(-\frac{\tilde{a}}{2\sqrt{-z^2}} \right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} GJn + \frac{1}{2}N} \left(\hat{\tilde{a}} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right) \left(\frac{4}{z^2} \right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0062.01

$$L_n \mathbb{H} L_\mu \frac{1}{2\sqrt{2p}} z^{n+1} I_- z^2 \tilde{M}^{\frac{3+2n}{4}} \left(\tilde{a}^{\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; \frac{\tilde{a}}{2\sqrt{-z^2}} \right) + \right. \\ \left. \tilde{a}^{-\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) {}_2F_0 \left(n + \frac{1}{2}, \frac{1}{2} - n; ; -\frac{\tilde{a}}{2\sqrt{-z^2}} \right) \right) - \\ \frac{2^{1-n} z^{n-1}}{\sqrt{p} GJn + \frac{1}{2}N} {}_3F_0 \left(\frac{1}{2}, \frac{1}{2} - n, 1; ; \frac{4}{z^2} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} L$$

03.10.06.0063.01

$$L_n \mathbb{H} L_\mu \frac{1}{2\sqrt{2p}} z^{n+1} I_- z^2 \tilde{M}^{\frac{3+2n}{4}} \left(\tilde{a}^{\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(1 + O\left(\frac{1}{z}\right) \right) + \right. \\ \left. \tilde{a}^{-\frac{3+2n}{4}} \tilde{a}^p \left(\tilde{a}^z \left(1 + \frac{\tilde{a}\sqrt{-z^2}}{z} \right) + \tilde{a}^{-z} \left(1 - \frac{\tilde{a}\sqrt{-z^2}}{z} \right) \right) \left(1 + O\left(\frac{1}{z}\right) \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} GJn + \frac{1}{2}N} \left(1 + O\left(\frac{1}{z^2}\right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \mathbb{R} \mathbb{Y} L$$

Expansions for any z in trigonometric and hyperbolic forms

Using trigonometric functions with branch cut-containing arguments

03.10.06.0064.01

$$\begin{aligned}
& L_n \mathbb{H} \mathbb{L} \mu \sqrt{\frac{2}{p}} z^{n+1} \mathbb{I} - z^2 \bar{\mathbb{M}}^{\frac{2n+3}{4}} \\
& \left(\sin \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 \mathbb{L} p}{4} \right) \left(1 + \frac{16 n^4 - 40 n^2 + 9}{128 z^2} + \frac{256 n^8 - 5376 n^6 + 31584 n^4 - 51664 n^2 + 11025}{98304 z^4} + \frac{1}{4} \right) + \right. \\
& \quad \frac{4 n^2 - 1}{8 \sqrt{-z^2}} \cos \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 \mathbb{L} p}{4} \right) \\
& \quad \left. \left(1 + \frac{16 n^4 - 136 n^2 + 225}{384 z^2} + \frac{256 n^8 - 10496 n^6 + 137824 n^4 - 656784 n^2 + 893025}{491520 z^4} + \frac{1}{4} \right) \right) - \\
& \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(1 - \frac{2 n - 1}{z^2} + \frac{3 \mathbb{I} 4 n^2 - 8 n + 3 \mathbb{M}}{z^4} + \frac{1}{4} \right) \bullet; \mathbb{H} z^{\mathfrak{A}} \mathbb{R} \mathbb{Y} \mathbb{L}
\end{aligned}$$

03.10.06.0065.01

$$\begin{aligned}
& L_n \mathbb{H} \mathbb{L} \mu \sqrt{\frac{2}{p}} \mathbb{I} - z^2 \bar{\mathbb{M}}^{\frac{2n+3}{4}} z^{n+1} \\
& \left(\sin \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 \mathbb{L} p}{4} \right) \left(\hat{\mathbb{A}} \frac{{}_n J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{H} - 2 n \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{H} n + 1 \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} N_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{1}{2}}^{\frac{1}{2}} N_k k!} \right) + \frac{4 n^2 - 1}{8 \sqrt{-z^2}} \right. \\
& \quad \left. \cos \left(\sqrt{-z^2} - \frac{\mathbb{H} n + 1 \mathbb{L} p}{4} \right) \left(\hat{\mathbb{A}} \frac{{}_n J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{B} - 2 n \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{H} n + 3 \mathbb{I} N_k J_{\frac{1}{4}}^{\frac{1}{4}} \mathbb{H} n + 5 \mathbb{I} N_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{1}{2}}^{\frac{1}{2}} N_k k!} \right) \right) - \\
& \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} \left(\hat{\mathbb{A}} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \bullet; \mathbb{H} z^{\mathfrak{A}} \mathbb{R} \mathbb{Y} \mathbb{L}
\end{aligned}$$

03.10.06.0011.01

$$\begin{aligned}
& L_n \mathbb{H} \mathbb{L} \mu \sqrt{\frac{2}{p}} z^{n+1} \mathbb{I} - z^2 \bar{\mathbb{M}}^{\frac{2n+3}{4}} \\
& \left(\sin \left(\sqrt{-z^2} - \frac{2 n + 1}{4} p \right) {}_4 F_1 \left(\frac{1 - 2 n}{4}, \frac{3 - 2 n}{4}, \frac{2 n + 1}{4}, \frac{2 n + 3}{4}; \frac{1}{2}; \frac{1}{z^2} \right) + \frac{4 n^2 - 1}{8 \sqrt{-z^2}} \cos \left(\sqrt{-z^2} - \frac{2 n + 1}{4} p \right) \right. \\
& \quad \left. {}_4 F_1 \left(\frac{3 - 2 n}{4}, \frac{5 - 2 n}{4}, \frac{3 + 2 n}{4}, \frac{5 + 2 n}{4}; \frac{3}{2}; \frac{1}{z^2} \right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} {}_3 F_0 \left(\frac{1}{2}, 1, \frac{1}{2} - n; ; \frac{4}{z^2} \right) \bullet; \mathbb{H} z^{\mathfrak{A}} \mathbb{R} \mathbb{Y} \mathbb{L}
\end{aligned}$$

03.10.06.0012.01

$$L_n \mathbb{H} L \mu \sqrt{\frac{2}{p}} z^{n+1} I - z^2 \bar{M}^{\frac{2n+3}{4}} \left(\sin \left(\sqrt{-z^2} - \frac{2n+1}{4} p \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) + \frac{4n^2 - 1}{8 \sqrt{-z^2}} \cos \left(\sqrt{-z^2} - \frac{2n+1}{4} p \right) \left(1 + O \left(\frac{1}{z^2} \right) \right) \right) -$$

$$\frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(1 + O \left(\frac{1}{z^2} \right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \otimes \mathbb{Y} L$$

Using hyperbolic functions with branch cut-free arguments

03.10.06.0066.01

$$L_n \mathbb{H} L \mu \sqrt{\frac{2}{p}} z^{n+1} I - z^2 \bar{M}^{\frac{2n+3}{4}} \left(- \left(\frac{z}{\sqrt{-z^2}} \cos \left(\frac{2n+1}{4} p \right) \sinh \mathbb{H} L + \sin \left(\frac{2n+1}{4} p \right) \cosh \mathbb{H} L \right) \right.$$

$$\left(1 + \frac{16n^4 - 40n^2 + 9}{128z^2} + \frac{256n^8 - 5376n^6 + 31584n^4 - 51664n^2 + 11025}{98304z^4} + \frac{1}{4} \right) +$$

$$\frac{4n^2 - 1}{8} \left(\frac{1}{\sqrt{-z^2}} \cos \left(\frac{2n+1}{4} p \right) \cosh \mathbb{H} L + \frac{1}{z} \sin \left(\frac{2n+1}{4} p \right) \sinh \mathbb{H} L \right)$$

$$\left. \left(1 + \frac{16n^4 - 136n^2 + 225}{384z^2} + \frac{256n^8 - 10496n^6 + 137824n^4 - 656784n^2 + 893025}{491520z^4} + \frac{1}{4} \right) \right) -$$

$$\frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(1 - \frac{2n-1}{z^2} + \frac{3I4n^2 - 8n + 3M}{z^4} + \frac{1}{4} \right) \bullet; \mathbb{H} z^{\mathbb{R}} \otimes \mathbb{Y} L$$

03.10.06.0067.01

$$L_n \mathbb{H} L \mu \sqrt{\frac{2}{p}} z^{n+1} I - z^2 \bar{M}^{\frac{2n+3}{4}} \left(- \left(\frac{z}{\sqrt{-z^2}} \cos \left(\frac{2n+1}{4} p \right) \sinh \mathbb{H} L + \sin \left(\frac{2n+1}{4} p \right) \cosh \mathbb{H} L \right) \right.$$

$$\left(\hat{\mathbf{a}}_{k=0}^n \frac{J_{\frac{1}{4}}^1 \mathbb{H} L - 2n I N_k J_{\frac{1}{4}}^1 \mathbb{B} - 2n I N_k J_{\frac{1}{4}}^1 \mathbb{D} n + 1 I N_k J_{\frac{1}{4}}^1 \mathbb{D} n + 3 I N_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{1}{2}}^1 N_k k!} \right) +$$

$$\frac{4n^2 - 1}{8} \left(\frac{1}{\sqrt{-z^2}} \cos \left(\frac{2n+1}{4} p \right) \cosh \mathbb{H} L + \frac{1}{z} \sin \left(\frac{2n+1}{4} p \right) \sinh \mathbb{H} L \right)$$

$$\left. \left(\hat{\mathbf{a}}_{k=0}^n \frac{J_{\frac{1}{4}}^1 \mathbb{B} - 2n I N_k J_{\frac{1}{4}}^1 \mathbb{B} - 2n I N_k J_{\frac{1}{4}}^1 \mathbb{D} n + 3 I N_k J_{\frac{1}{4}}^1 \mathbb{D} n + 5 I N_k \left(\frac{1}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right)}{J_{\frac{3}{2}}^1 N_k k!} \right) \right) -$$

$$\frac{2^{1-n} z^{n-1}}{\sqrt{p} G J n + \frac{1}{2} N} \left(\hat{\mathbf{a}}_{k=0}^n \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{4}{z^2} \right)^k + O \left(\frac{1}{z^{2n+2}} \right) \right) \bullet; \mathbb{H} z^{\mathbb{R}} \otimes \mathbb{Y} L$$

03.10.06.0068.01

 $L_n \mathbb{H} L \mu$

$$\sqrt{\frac{2}{p}} z^{n+1} I_- z^2 \bar{M}^{\frac{2n+3}{4}} \left(- \left(\frac{z}{\sqrt{-z^2}} \cos\left(\frac{2n+1}{4} p\right) \sinh \mathbb{H} L + \sin\left(\frac{2n+1}{4} p\right) \cosh \mathbb{H} L \right) {}_4F_1\left(\frac{1}{4} \mathbb{H} - 2nL, \frac{1}{4} \mathbb{H} - 2nL, \frac{1}{4} \mathbb{H} n + 1L, \frac{1}{4} \mathbb{H} n + 3L; \frac{1}{2}; \frac{1}{z^2}\right) + \frac{4n^2 - 1}{8} \left(\frac{1}{\sqrt{-z^2}} \cos\left(\frac{2n+1}{4} p\right) \cosh \mathbb{H} L + \frac{1}{z} \sin\left(\frac{2n+1}{4} p\right) \sinh \mathbb{H} L \right) {}_4F_1\left(\frac{1}{4} \mathbb{H} - 2nL, \frac{1}{4} \mathbb{H} - 2nL, \frac{1}{4} \mathbb{H} n + 3L, \frac{1}{4} \mathbb{H} n + 5L; \frac{3}{2}; \frac{1}{z^2}\right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} n + \frac{1}{2} N} {}_3F_0\left(1, \frac{1}{2}, \frac{1}{2} - n; ; \frac{4}{z^2}\right) \bullet; \mathbb{H} z^{\mathbb{H}} \otimes \mathbb{Y} L$$

03.10.06.0069.01

$$L_n \mathbb{H} L \mu \sqrt{\frac{2}{p}} z^{n+1} I_- z^2 \bar{M}^{\frac{2n+3}{4}} \left(- \left(\frac{z}{\sqrt{-z^2}} \cos\left(\frac{2n+1}{4} p\right) \sinh \mathbb{H} L + \sin\left(\frac{2n+1}{4} p\right) \cosh \mathbb{H} L \right) \left(1 + O\left(\frac{1}{z^2}\right) \right) + \frac{4n^2 - 1}{8} \left(\frac{1}{\sqrt{-z^2}} \cos\left(\frac{2n+1}{4} p\right) \cosh \mathbb{H} L + \frac{1}{z} \sin\left(\frac{2n+1}{4} p\right) \sinh \mathbb{H} L \right) \left(1 + O\left(\frac{1}{z^2}\right) \right) - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{G} n + \frac{1}{2} N} \left(1 + O\left(\frac{1}{z^2}\right) \right) \right) \bullet; \mathbb{H} z^{\mathbb{H}} \otimes \mathbb{Y} L$$

Residue representations

03.10.06.0013.01

$$L_n \mathbb{H} L \check{S} - p \csc\left(\frac{p n}{2}\right) z^{n-1} I_z^2 \bar{M}^{\frac{1-n}{2}} \mathbb{A} \operatorname{res}_s \left(\frac{\mathbb{G} J^{\frac{1-n}{2}} - s N J^{\frac{1-n}{2}} \bar{N}^{-s}}{\mathbb{G} J s + \frac{1}{2} N \mathbb{G} J^{\frac{1}{2}} - s N \mathbb{G} I 1 + \frac{n}{2} - s M \mathbb{G} I 1 - \frac{n}{2} - s M} G\left(\frac{n+1}{2} + s\right) \right) \left(-\frac{n+1}{2} - j \right)$$

03.10.06.0014.01

$$L_n \mathbb{H} L \check{S} - p \csc\left(\frac{p n}{2}\right) \mathbb{A} \operatorname{res}_s \left(\frac{\mathbb{G} J^{\frac{1-n}{2}} - s N J^{\frac{1-n}{2}} \bar{M}^{2s}}{\mathbb{G} J s + \frac{1}{2} N \mathbb{G} J^{\frac{1}{2}} - s N \mathbb{G} I 1 + \frac{n}{2} - s M \mathbb{G} I 1 - \frac{n}{2} - s M} G\left(\frac{n+1}{2} + s\right) \right) \left(-\frac{n+1}{2} - j \right)$$

Other series representations

03.10.06.0015.01

$$L_0 \mathbb{H} L \check{S} \frac{4}{p} \mathbb{A} \sum_{k=0}^{\infty} \frac{I_{2k+1} \mathbb{H} L}{2k+1}$$

03.10.06.0016.01

$$L_1 \mathbb{H} L \check{S} \frac{2}{p} \mathbb{H}_0 \mathbb{H} L - 1L + \frac{4}{p} \mathbb{A} \sum_{k=1}^{\infty} \frac{I_{2k} \mathbb{H} L}{4k^2 - 1}$$

Integral representations

On the real axis

Of the direct function

03.10.07.0001.01

$$L_n \text{H} \tilde{S} \frac{2^{1-n} z^n}{\sqrt{p} \Gamma(n + \frac{1}{2})} \hat{a}^1 \text{I} 1 - t^2 \tilde{M}^{\frac{1}{2}} \sinh \text{H} z \hat{L} \hat{a} t \bullet; \operatorname{Re} \text{H} \text{L} > -\frac{1}{2}$$

03.10.07.0002.01

$$L_n \text{H} \tilde{S} \frac{2^{1-n} z^n}{\sqrt{p} \Gamma(n + \frac{1}{2})} \hat{a}^{\frac{p}{2}} \sin^{2n} \text{H} \sinh \text{H} \cos \text{H} \hat{L} \hat{a} t \bullet; \operatorname{Re} \text{H} \text{L} > -\frac{1}{2}$$

03.10.07.0003.01

$$L_n \text{H} \tilde{S} L_{-n} \text{H} \tilde{L} - \frac{2^{1-n} z^n}{\sqrt{p} \Gamma(n + \frac{1}{2})} \hat{a}^{\infty} \sin \text{H} z \text{L} t^2 + 1 \tilde{M}^{\frac{1}{2}} \hat{a} t \bullet; z > 0 \text{ i } \operatorname{Re} \text{H} \text{L} < \frac{1}{2}$$

Contour integral representations

03.10.07.0004.01

$$L_n \text{H} \tilde{S} - p \csc\left(\frac{p n}{2}\right) z^{n-1} z^2 \frac{1}{2 p \hat{a}} \hat{a}^L \frac{\Gamma \frac{n+1}{2} + s \Gamma \frac{1-n}{2} - s \Gamma \frac{n}{2}}{\Gamma s + \frac{1}{2} \Gamma \frac{1}{2} - s \Gamma 1 + \frac{n}{2} - s \Gamma 1 - \frac{n}{2} - s \Gamma} \left(\frac{z^2}{4}\right)^{-s} \hat{a} s$$

03.10.07.0005.01

$$L_n \text{H} \tilde{S} - p \csc\left(\frac{p n}{2}\right) \frac{1}{2 p \hat{a}} \hat{a}^L \frac{\Gamma \frac{n+1}{2} + s \Gamma \frac{1-n}{2} - s \Gamma \frac{n}{2}}{\Gamma s + \frac{1}{2} \Gamma \frac{1}{2} - s \Gamma 1 + \frac{n}{2} - s \Gamma 1 - \frac{n}{2} - s \Gamma} \frac{z^{-2s}}{2} \hat{a} s$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.10.13.0001.01

$$w^{\text{cc}} \text{H} \tilde{L} z^2 + w^{\text{c}} \text{H} \tilde{L} z - \text{I} z^2 + n^2 \text{M} w \text{H} \tilde{L} \tilde{S} \frac{4}{\sqrt{p} \Gamma(n + \frac{1}{2})} \frac{z^{n+1}}{2} \hat{a}^{\text{K}-0} \bullet; w \text{H} \tilde{L} \tilde{S} c_1 I_n \text{H} \tilde{L} + c_2 K_n \text{H} \tilde{L} + L_n \text{H} \tilde{L}$$

03.10.13.0002.01

$$W_z \text{H} \text{H} \tilde{L} K_n \text{H} \tilde{L} \tilde{S} - \frac{1}{z}$$

03.10.13.0003.01

$$w^{\text{cc}} \text{H} \tilde{L} z^2 + w^{\text{c}} \text{H} \tilde{L} z - \text{I} z^2 + n^2 \text{M} w \text{H} \tilde{L} \tilde{S} \frac{4}{\sqrt{p} \Gamma(n + \frac{1}{2})} \frac{z^{n+1}}{2} \hat{a}^{\text{K}-0} \bullet; w \text{H} \tilde{L} \tilde{S} c_1 I_n \text{H} \tilde{L} + c_2 L_{-n} \text{H} \tilde{L} + L_n \text{H} \tilde{L} \hat{a}^{\text{K}-0} \bullet$$

03.10.13.0004.01

$$W_z \text{H} \text{H} \tilde{L} L_{-n} \text{H} \tilde{L} \tilde{S} - \frac{2 \sin \text{H} p n \text{L}}{p z}$$

03.10.13.0005.01

$$z^3 w^{\text{bl}} \text{H} \tilde{L} - \text{H} \text{H} - 2 \text{L} z^2 w^{\text{cc}} \text{H} \tilde{L} - \text{I} z^2 + n^2 + n \text{M} w^{\text{c}} \text{H} \tilde{L} + \text{I} \text{H} \text{H} - 1 \text{L} z^2 + n^2 \text{H} \text{H} + 1 \text{I} \text{M} w \text{H} \tilde{L} \hat{a}^{\text{K}-0} \bullet; w \text{H} \tilde{L} \hat{a}^{\text{K}-0} \bullet L_n \text{H} \tilde{L} c_1 + c_2 I_n \text{H} \tilde{L} + c_3 K_n \text{H} \tilde{L}$$

03.10.13.0006.01

$$W_z \mathbb{H}_n \mathbb{H}_L, I_n \mathbb{H}_L, K_n \mathbb{H}_L \tilde{S} - \frac{2^{1-n} z^{n-2}}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} \operatorname{N}}$$

03.10.13.0007.01

$$w^{\mathbb{H}^L} \mathbb{H}_L - \frac{\mathbb{H} \mathbb{h} - 2 \operatorname{L} g^{\mathbb{H}} \mathbb{H} \mathbb{B} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}} w^{\mathbb{C}} \mathbb{H}_L + \left(- \frac{n \mathbb{H} + 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{g^{\mathbb{H}} \mathbb{L}^2} - g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2 + \frac{3 g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2} + \frac{\mathbb{H} \mathbb{h} - 2 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \mathbb{L}} - \frac{g^{\mathbb{H}^L} \mathbb{H} \mathbb{L}}{g^{\mathbb{C}} \mathbb{H} \mathbb{L}} \right) w^{\mathbb{C}} \mathbb{H}_L +$$

$$\left(\frac{\mathbb{H} \mathbb{h} - 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{g^{\mathbb{H}} \mathbb{L}} + \frac{n^2 \mathbb{H} \mathbb{h} + 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{g^{\mathbb{H}} \mathbb{L}^3} \right) w^{\mathbb{H}} \mathbb{L} \nmid 0 \bullet; w^{\mathbb{H}} \mathbb{L} \nmid c_1 \operatorname{L}_n \mathbb{H} \mathbb{B} \mathbb{H} \mathbb{L} + c_2 I_n \mathbb{H} \mathbb{B} \mathbb{H} \mathbb{L} + c_3 K_n \mathbb{H} \mathbb{B} \mathbb{H} \mathbb{L}$$

03.10.13.0008.01

$$W_z \mathbb{H}_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L}, I_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L}, K_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L} \tilde{S} - \frac{2^{1-n} g^{\mathbb{H}} \mathbb{L}^{n-2} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} \operatorname{N}}$$

03.10.13.0009.01

$$w^{\mathbb{H}^L} \mathbb{H}_L - \left(\frac{\mathbb{H} \mathbb{h} - 2 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \mathbb{L}} + \frac{3 h^{\mathbb{C}} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \mathbb{L}} + \frac{3 g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{C}} \mathbb{H} \mathbb{L}} \right) w^{\mathbb{C}} \mathbb{H}_L +$$

$$\left(- \frac{n \mathbb{H} + 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{g^{\mathbb{H}} \mathbb{L}^2} - g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2 + \frac{2 \mathbb{H} \mathbb{h} - 2 \operatorname{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \operatorname{L} h^{\mathbb{H}} \mathbb{L}} + \frac{6 h^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{h^{\mathbb{H}} \mathbb{L}^2} + \frac{3 g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2} + \frac{6 h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}} + \frac{\mathbb{H} \mathbb{h} - 2 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \mathbb{L}} - \frac{3 h^{\mathbb{C}} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \mathbb{L}} - \frac{g^{\mathbb{H}^L} \mathbb{H} \mathbb{L}}{g^{\mathbb{C}} \mathbb{H} \mathbb{L}} \right)$$

$$w^{\mathbb{C}} \mathbb{H}_L + \left(\frac{\mathbb{H} \mathbb{h} - 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{g^{\mathbb{H}} \mathbb{L}} + \frac{n^2 \mathbb{H} \mathbb{h} + 1 \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{g^{\mathbb{H}} \mathbb{L}^3} + \frac{n \mathbb{H} + 1 \operatorname{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{g^{\mathbb{H}} \mathbb{L}^2 h^{\mathbb{H}} \mathbb{L}} - \frac{2 \mathbb{H} \mathbb{h} - 2 \operatorname{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L}^2 g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \operatorname{L} h^{\mathbb{H}} \mathbb{L}^2} + \frac{6 h^{\mathbb{C}} \mathbb{H} \mathbb{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \mathbb{L}^2} + \right.$$

$$\frac{3 g^{\mathbb{C}} \mathbb{H} \mathbb{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L} + h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{H}^L} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}} + \frac{g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2 h^{\mathbb{C}} \mathbb{H} \mathbb{L} - h^{\mathbb{H}^L} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \mathbb{L}} - \frac{\mathbb{H} \mathbb{h} - 2 \operatorname{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L} - g^{\mathbb{C}} \mathbb{H} \mathbb{L} h^{\mathbb{C}} \mathbb{H} \mathbb{L}}{g^{\mathbb{H}} \operatorname{L} h^{\mathbb{H}} \mathbb{L}} - \frac{6 h^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{h^{\mathbb{H}} \mathbb{L}^3} -$$

$$\left. \frac{6 h^{\mathbb{C}} \mathbb{H} \mathbb{L}^2 g^{\mathbb{C}} \mathbb{H} \mathbb{L}}{h^{\mathbb{H}} \mathbb{L}^2 g^{\mathbb{C}} \mathbb{H} \mathbb{L}} - \frac{3 h^{\mathbb{C}} \mathbb{H} \mathbb{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2}{h^{\mathbb{H}} \operatorname{L} g^{\mathbb{C}} \mathbb{H} \mathbb{L}^2} \right) w^{\mathbb{H}} \mathbb{L} \nmid 0 \bullet; w^{\mathbb{H}} \mathbb{L} \nmid c_1 h^{\mathbb{H}} \operatorname{L}_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L} + c_2 h^{\mathbb{H}} \operatorname{L}_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L} + c_3 h^{\mathbb{H}} \operatorname{L}_n K_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L}$$

03.10.13.0010.01

$$W_z \mathbb{H}_{\mathbb{B}} \mathbb{H} \operatorname{L}_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L}, h^{\mathbb{H}} \operatorname{L}_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L}, h^{\mathbb{H}} \operatorname{L}_n K_n \mathbb{H}_{\mathbb{B}} \mathbb{H} \mathbb{L} \tilde{S} - \frac{2^{1-n} g^{\mathbb{H}} \mathbb{L}^{n-2} h^{\mathbb{H}} \mathbb{L}^3 g^{\mathbb{C}} \mathbb{H} \mathbb{L}^3}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} \operatorname{N}}$$

03.10.13.0011.01

$$z^3 w^{\mathbb{H}^L} \mathbb{H}_L - \mathbb{H} \mathbb{h} r + r + 3 s - 3 \operatorname{L} z^2 w^{\mathbb{C}} \mathbb{H}_L + \operatorname{I} - \operatorname{I} a^2 z^{2r} + n^2 \operatorname{M}^2 + \operatorname{B} s - 1 \operatorname{L} \mathbb{H} \mathbb{h} + 1 \operatorname{L} r + 3 \mathbb{H} - 1 \operatorname{L} s + 1 \operatorname{M} w^{\mathbb{C}} \mathbb{H}_L +$$

$$\operatorname{I} a^2 \mathbb{H} \mathbb{h} - 1 \operatorname{L} z^{2r} + n^2 \mathbb{H} \mathbb{h} + 1 \operatorname{I} \operatorname{M}^3 + s \operatorname{I} a^2 z^{2r} + n^2 \operatorname{M}^2 - s^2 \mathbb{H} \mathbb{h} + 1 \operatorname{L} r - s^3 \operatorname{M} w^{\mathbb{C}} \mathbb{H}_L \nmid 0 \bullet;$$

$$w^{\mathbb{H}} \mathbb{L} \nmid c_1 z^s \operatorname{L}_n \mathbb{H} z^r \mathbb{L} + c_2 z^s I_n \mathbb{H} z^r \mathbb{L} + c_3 z^s K_n \mathbb{H} z^r \mathbb{L}$$

03.10.13.0012.01

$$W_z \mathbb{H}^s \operatorname{L}_n \mathbb{H} z^r \mathbb{L}, z^s I_n \mathbb{H} z^r \mathbb{L}, z^s K_n \mathbb{H} z^r \mathbb{L} \tilde{S} - \frac{2^{1-n} a r^3 z^{r+3s-3} \mathbb{H} z^r \mathbb{L}^n}{\sqrt{p} \operatorname{GJn} + \frac{1}{2} \operatorname{N}}$$

03.10.13.0013.01

$$w^{\mathbb{H}^L} \mathbb{H}_L + \mathbb{H} \mathbb{h} + 1 \operatorname{L} \log \mathbb{H} \mathbb{L} - 3 \log \mathbb{H} \mathbb{L} w^{\mathbb{C}} \mathbb{H}_L + \operatorname{I} - \operatorname{I} a^2 r^{2z} + n^2 \operatorname{M} \log^2 \mathbb{H} \mathbb{L} + 2 \mathbb{H} \mathbb{h} + 1 \operatorname{L} \log \mathbb{H} \mathbb{L} \log \mathbb{H} \mathbb{L} + 3 \log^2 \mathbb{H} \operatorname{I} \operatorname{M} w^{\mathbb{C}} \mathbb{H}_L +$$

$$\operatorname{I} a^2 \mathbb{H} \mathbb{h} - 1 \operatorname{L} r^{2z} + n^2 \mathbb{H} \mathbb{h} + 1 \operatorname{I} \operatorname{M} \log^3 \mathbb{H} \mathbb{L} + \operatorname{I} a^2 r^{2z} + n^2 \operatorname{M} \log \mathbb{H} \mathbb{L} \log^2 \mathbb{H} \mathbb{L} - \mathbb{H} \mathbb{h} + 1 \operatorname{L} \log^2 \mathbb{H} \mathbb{L} \log \mathbb{H} \mathbb{L} - \log^3 \mathbb{H} \operatorname{I} \operatorname{M} w^{\mathbb{C}} \mathbb{H}_L \nmid 0 \bullet;$$

$$w^{\mathbb{H}} \mathbb{L} \nmid c_1 s^z \operatorname{L}_n \mathbb{H} r^z \mathbb{L} + c_2 s^z I_n \mathbb{H} r^z \mathbb{L} + c_3 s^z K_n \mathbb{H} r^z \mathbb{L}$$

03.10.13.0014.01

$$W_z \mathbb{H}^z L_n \mathbb{H} r^z \mathbb{L}, s^z I_n \mathbb{H} r^z \mathbb{L}, s^z K_n \mathbb{H} r^z \mathbb{L} \check{S} - \frac{2^{1-n} \mathbb{H} r^z \mathbb{L}^{n+1} s^3 z \log^3 \mathbb{H} \mathbb{L}}{\sqrt{p} \text{GJn} + \frac{1}{2} \mathbb{N}}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.10.16.0001.01

$$L_n \mathbb{H} z \mathbb{L} \check{S} - \mathbb{H} z \mathbb{L}^n z^{-n} L_n \mathbb{H} \mathbb{L}$$

03.10.16.0002.01

$$L_n \mathbb{H} z \mathbb{L} \check{S} \hat{a} \mathbb{H} z \mathbb{L}^n z^{-n} H_n \mathbb{H} \mathbb{L}$$

03.10.16.0003.01

$$L_n \mathbb{H} \hat{a} z \mathbb{L} \check{S} - \hat{a} \mathbb{H} \hat{a} z \mathbb{L}^n z^{-n} H_n \mathbb{H} \mathbb{L}$$

03.10.16.0004.01

$$L_n \left(\sqrt{z^2} \right) \check{S} z^{-n-1} \mathbb{L} z^{\frac{n+1}{2}} \mathbb{L} L_n \mathbb{H} \mathbb{L}$$

03.10.16.0005.01

$$L_n \mathbb{H} \mathbb{H} z^n \mathbb{L}^m \mathbb{L} \check{S} \frac{\mathbb{H} \mathbb{H} z^n \mathbb{L}^m \mathbb{L}^{n+1}}{\mathbb{H} d^m z^{mn} \mathbb{L}^{n+1}} L_n \mathbb{H} d^m z^{mn} \mathbb{L} \bullet; 2m \hat{\mathbb{I}} \mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

03.10.17.0001.01

$$L_n \mathbb{H} \mathbb{L} \check{S} \frac{2 \mathbb{H} + 1 \mathbb{L}}{z} L_{n+1} \mathbb{H} \mathbb{L} + L_{n+2} \mathbb{H} \mathbb{L} + \frac{2^{-n-1} z^{n+1}}{\sqrt{p} \text{GJn} + \frac{5}{2} \mathbb{N}}$$

03.10.17.0002.01

$$L_n \mathbb{H} \mathbb{L} \check{S} - \frac{2 \mathbb{H} - 1 \mathbb{L}}{z} L_{n-1} \mathbb{H} \mathbb{L} + L_{n-2} \mathbb{H} \mathbb{L} - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \text{GJn} + \frac{1}{2} \mathbb{N}}$$

Distant neighbors

Increasing

03.10.17.0012.01

$$L_n \mathbb{H} L \dagger 2^{n-1} z^{-n} \mathbb{H} + 1 L_{n-1}$$

$$\left(2 \mathbb{H} + n L \hat{\mathbb{A}} \frac{\frac{f_2^n v}{2} \mathbb{H} - 1 L^k \mathbb{H} - k L!}{k! \mathbb{H} - 2 k L! \mathbb{H} - n - n L_k \mathbb{H} + 1 L_k} \left(\frac{z^2}{4} \right)^k L_{n+n} \mathbb{H} L + z \hat{\mathbb{A}} \frac{\frac{f_2^{n-1} v}{2} \mathbb{H} - 1 L^k \mathbb{H} - k - 1 L!}{k! \mathbb{H} - 2 k - 1 L! \mathbb{H} - n - n L_k \mathbb{H} + 1 L_k} \left(\frac{z^2}{4} \right)^k L_{n+n+1} \mathbb{H} L \right) +$$

$$\frac{1}{\sqrt{p}} \frac{K-0}{2} \hat{\mathbb{A}} \frac{z^{-n+1} n^{-1} \mathbb{H} + 1 L_j}{j=0 \text{ GJ} j + n + \frac{5}{2} N} \frac{\frac{f_2^j v}{2} \mathbb{H} - 1 L^k \mathbb{H} j - k L!}{k! \mathbb{H} j - 2 k L! \mathbb{H} j - n L_k \mathbb{H} + 1 L_k} \left(\frac{z^2}{4} \right)^k \bullet; n \hat{\mathbb{I}} N$$

Brychkov Yu.A. (2005)

03.10.17.0013.01

$$L_n \mathbb{H} L \dagger 2^{n-1} \mathbb{H} + 1 L_{n-1} \left(2 \mathbb{H} + n L_3 F_4 \left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, -n-n, n+1; z^2 \right) L_{n+n} \mathbb{H} L + \right.$$

$$\left. z {}_3 F_4 \left(1, \frac{1-n}{2}, 1 - \frac{n}{2}; 1, 1-n, 1-n-n, n+1; z^2 \right) L_{n+n+1} \mathbb{H} L \right) z^{-n} +$$

$$\frac{2^{-n-1} z^{n+1} n^{-1} \mathbb{H} + 1 L_j}{\sqrt{p}} \hat{\mathbb{A}} \frac{z^{n+1} n^{-1} \mathbb{H} + 1 L_j}{j=0 \text{ GJ} j + n + \frac{5}{2} N} {}_3 F_4 \left(1, \frac{1-j}{2}, -\frac{j}{2}; 1, -j, -j-n, n+1; z^2 \right) \bullet; n \hat{\mathbb{I}} N$$

Brychkov Yu.A. (2005)

03.10.17.0004.01

$$L_n \mathbb{H} L \tilde{S} \frac{2^{-n-2} \mathbb{H} n + 7 L z^{n+1}}{\sqrt{p} \text{ GJ} n + \frac{7}{2} N} + \left(\frac{4 \mathbb{H} + 1 L \mathbb{H} + 2 L}{z^2} + 1 \right) L_{n+2} \mathbb{H} L + \frac{2 \mathbb{H} + 1 L L_{n+3} \mathbb{H} L}{z}$$

03.10.17.0005.01

$$L_n \mathbb{H} L \tilde{S} \frac{2^{-n-3} I z^2 + 12 n^2 + 54 n + 57 M^{n+1}}{\sqrt{p} \text{ GJ} n + \frac{9}{2} N} + \frac{4 \mathbb{H} + 2 L I z^2 + 2 n^2 + 8 n + 6 M L_{n+3} \mathbb{H} L}{z^3} + \left(\frac{4 \mathbb{H} + 1 L \mathbb{H} + 2 L}{z^2} + 1 \right) L_{n+4} \mathbb{H} L$$

03.10.17.0006.01

$$L_n \mathbb{H} L \tilde{S} \frac{2^{-n-4} I 32 n^3 + 264 n^2 + 688 n + z^2 \mathbb{H} n + 17 L + 561 M^{n+1}}{\sqrt{p} \text{ GJ} n + \frac{11}{2} N} +$$

$$\left(\frac{4 \mathbb{H} + 1 L \mathbb{H} + 2 L}{z^2} + \frac{8 \mathbb{H} + 4 L I z^2 + 2 n^2 + 8 n + 6 M \mathbb{H} + 2 L}{z^4} + 1 \right) L_{n+4} \mathbb{H} L + \frac{4 \mathbb{H} + 2 L I z^2 + 2 n^2 + 8 n + 6 M L_{n+5} \mathbb{H} L}{z^3}$$

03.10.17.0007.01

$$L_n \mathbb{H} L \tilde{S} \frac{2 \mathbb{H} + 3 L I 3 z^4 + 16 I n^2 + 6 n + 8 M^2 + 16 I n^4 + 12 n^3 + 49 n^2 + 78 n + 40 M L_{n+5} \mathbb{H} L}{z^5} +$$

$$\left(1 + \frac{4 \mathbb{H} + 1 L \mathbb{H} + 2 L}{z^2} + \frac{8 \mathbb{H} + 4 L I z^2 + 2 n^2 + 8 n + 6 M \mathbb{H} + 2 L}{z^4} \right) L_{n+6} \mathbb{H} L +$$

$$\frac{2^{-n-5} z^{n+1} I z^4 + I 24 n^2 + 160 n + 259 M^2 + 5 I 16 n^4 + 208 n^3 + 968 n^2 + 1898 n + 1311 M}{\sqrt{p} \text{ GJ} n + \frac{13}{2} N}$$

03.10.17.0014.01

$$L_n \mathbb{H} \tilde{S} C_n \mathbb{H}, z L_{n+n} \mathbb{H} L + C_{n-1} \mathbb{H}, z L_{n+n+1} \mathbb{H} L + \frac{1}{\sqrt{p}} \hat{a} \frac{1}{Gj + n + \frac{5}{2}N} \mathbb{K} - 0 \frac{z^{j+n+1}}{2} C_j \mathbb{H}, z L \bullet;$$

$$C_0 \mathbb{H}, z L \tilde{S} 1 \dot{\mathbb{I}} C_1 \mathbb{H}, z L \tilde{S} \frac{2 \mathbb{H} + 1 L}{z} \dot{\mathbb{I}} C_n \mathbb{H}, z L \tilde{S} \frac{2 \mathbb{H} + n L}{z} C_{n-1} \mathbb{H}, z L + C_{n-2} \mathbb{H}, z L \dot{\mathbb{I}} n \hat{\mathbb{I}} N^+$$

03.10.17.0015.01

$$L_n \mathbb{H} L \dot{\mathbb{I}} C_n \mathbb{H}, z L_{n+n} \mathbb{H} L + C_{n-1} \mathbb{H}, z L_{n+n+1} \mathbb{H} L + \frac{1}{\sqrt{p}} \mathbb{K} - 0 \frac{z^{n+1} \frac{n-1}{2} \mathbb{H} + 1 L_j}{Gj + \frac{5}{2}N} \hat{a} \frac{1}{j + \frac{5}{2}N_j} {}_2F_3 \left(\frac{1-j}{2}, -\frac{j}{2}; n+1, -j, -j-n; z^2 \right) \bullet;$$

$$C_n \mathbb{H}, z L \dot{\mathbb{I}} 2^n z^{-n} \mathbb{H} + 1 L_n {}_2F_3 \left(\frac{1-n}{2}, -\frac{n}{2}; n+1, -n, -n-n; z^2 \right) \dot{\mathbb{I}} n \hat{\mathbb{I}} N^+$$

Brychkov Yu.A. (2005)

Decreasing

03.10.17.0016.01

$$L_n \mathbb{H} L \dot{\mathbb{I}} 2^{n-1} z^{-n} \mathbb{H} - n L_{n-1}$$

$$\left(2 \mathbb{H} - n L \hat{a} \frac{f_2^n}{k! \mathbb{H} - 2 k L! \mathbb{H} - n L_k \mathbb{H} - n L_k} \left(\frac{z^2}{4} \right)^k L_{n-n} \mathbb{H} L + z \hat{a} \frac{f_2^{n-1}}{k! \mathbb{H} - 2 k - 1 L! \mathbb{H} - n L_k \mathbb{H} - n + 1 L_k} \left(\frac{z^2}{4} \right)^k L_{n-n-1} \mathbb{H} L \right) -$$

$$\frac{1}{\sqrt{p}} \mathbb{K} - 0 \frac{z^{n-1} \frac{n-1}{2} \mathbb{H} - n L_j}{Gj - j + \frac{1}{2}N} \hat{a} \frac{f_2^j}{k! \mathbb{H} - 2 k L! \mathbb{H} - n L_k \mathbb{H} - j L_k} \frac{z^{\frac{j}{2}}}{4} N \bullet; n \hat{\mathbb{I}} N$$

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03.10.17.0017.01

$$L_n \mathbb{H} L \dot{\mathbb{I}} 2^{n-1} z^{-n} \mathbb{H} - n L_{n-1} \left({}_3F_4 \left(1, \frac{1-n}{2}, 1 - \frac{n}{2}; 1, 1-n, 1-n, -n+n+1; z^2 \right) L_{n+n-1} \mathbb{H} L + \right.$$

$$\left. 2 \mathbb{H} - n L {}_3F_4 \left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, 1-n, n-n; z^2 \right) L_{n-n} \mathbb{H} L \right) -$$

$$\frac{2^{1-n} z^{n-1} \frac{n-1}{2} 4^j z^{-2j} \mathbb{H} - n L_j}{\sqrt{p}} \hat{a} \frac{1}{Gj - j + \frac{1}{2}N} {}_3F_4 \left(1, \frac{1-j}{2}, -\frac{j}{2}; 1, -j, 1-n, n-j; z^2 \right) \bullet; n \hat{\mathbb{I}} N$$

Brychkov Yu.A. (2005)

03.10.17.0008.01

$$L_n \mathbb{H} \tilde{S} \frac{2^{1-n} \mathbb{I} - z^2 + 4 n^2 - 6 n + 2 M^{n-3}}{\sqrt{p} Gj + \frac{1}{2}N} + \left(\frac{4 \mathbb{H} - 2 L \mathbb{H} - 1 L}{z^2} + 1 \right) L_{n-2} \mathbb{H} L - \frac{2 \mathbb{H} - 1 L L_{n-3} \mathbb{H} L}{z}$$

03.10.17.0009.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \left(\frac{4 \mathbb{H} - 2 \mathbb{L} \mathbb{H} - 1 \mathbb{L}}{z^2} + 1 \right) L_{n-4} \mathbb{H} \mathbb{L} - \frac{I^4 \mathbb{H} - 2 \mathbb{L} I z^2 + 2 n^2 - 8 n + 6 \mathbb{M} \mathbb{L}_{n-3} \mathbb{H} \mathbb{L}}{z^3} -$$

$$\frac{2^{1-n} z^{n-5} I z^4 + \mathbb{H} - 2 n \mathbb{L} z^2 + 4 \mathbb{H} - 2 \mathbb{L} \mathbb{H} - 1 \mathbb{L} \mathbb{H} n - 3 \mathbb{L} \mathbb{H} n - 1 \mathbb{M}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}}$$

03.10.17.0010.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \left(\frac{4 \mathbb{H} - 2 \mathbb{L} \mathbb{H} - 1 \mathbb{L}}{z^2} + \frac{8 \mathbb{H} - 4 \mathbb{L} \mathbb{H} - 2 \mathbb{L} I z^2 + 2 n^2 - 8 n + 6 \mathbb{M}}{z^4} + 1 \right) L_{n-4} \mathbb{H} \mathbb{L} -$$

$$\frac{4 \mathbb{H} - 2 \mathbb{L} I z^2 + 2 n^2 - 8 n + 6 \mathbb{M} \mathbb{L}_{n-5} \mathbb{H} \mathbb{L}}{z^3} - \frac{1}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}} I^{2^{1-n}} z^{n-7}$$

$$I z^6 + \mathbb{H} - 2 n \mathbb{L} z^4 - 4 I^4 n^4 - 32 n^3 + 83 n^2 - 82 n + 24 \mathbb{M}^2 - 8 I^8 n^6 - 84 n^5 + 350 n^4 - 735 n^3 + 812 n^2 - 441 n + 90 \mathbb{M} \mathbb{M}$$

03.10.17.0011.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \left(\frac{4 \mathbb{H} - 2 \mathbb{L} \mathbb{H} - 1 \mathbb{L}}{z^2} + \frac{8 \mathbb{H} - 4 \mathbb{L} \mathbb{H} - 2 \mathbb{L} I z^2 + 2 n^2 - 8 n + 6 \mathbb{M}}{z^4} + 1 \right) L_{n-6} \mathbb{H} \mathbb{L} -$$

$$\frac{I^2 \mathbb{H} - 3 \mathbb{L} I z^4 + 16 I n^2 - 6 n + 8 \mathbb{M}^2 + 16 I n^4 - 12 n^3 + 49 n^2 - 78 n + 40 \mathbb{M} \mathbb{M} \mathbb{L}_{n-5} \mathbb{H} \mathbb{L}}{z^5} - \frac{1}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} \mathbb{N}}$$

$$I^{2^{1-n}} z^{n-9} I z^8 + \mathbb{H} - 2 n \mathbb{L} z^6 + 3 I^4 n^2 - 8 n + 3 \mathbb{M}^4 + 4 I^3 n^6 - 456 n^5 + 2540 n^4 - 7050 n^3 + 10163 n^2 - 7029 n + 1710 \mathbb{M}^2 +$$

$$16 I^6 n^8 - 288 n^7 + 2184 n^6 - 9072 n^5 + 22449 n^4 - 33642 n^3 + 29531 n^2 - 13698 n + 2520 \mathbb{M} \mathbb{M}$$

03.10.17.0018.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} C_n \mathbb{H}, z L L_{n-n} \mathbb{H} \mathbb{L} + C_{n-1} \mathbb{H}, z L L_{n-n-1} \mathbb{H} \mathbb{L} - \frac{1}{\sqrt{p}} \hat{a} \frac{1}{j=0} \frac{1}{\mathbb{G} \mathbb{J} n + \frac{1}{2} - j \mathbb{N}} \mathbb{K} \frac{z^{n-j-1}}{2} C_j \mathbb{H}, z \mathbb{L} \bullet;$$

$$C_0 \mathbb{H}, z \mathbb{L} \tilde{S} 1 \hat{=} C_1 \mathbb{H}, z \mathbb{L} \tilde{S} - \frac{2 \mathbb{H} - 1 \mathbb{L}}{z} \hat{=} C_n \mathbb{H}, z \mathbb{L} \tilde{S} - \frac{2 \mathbb{H} - n \mathbb{L}}{z} C_{n-1} \mathbb{H}, z \mathbb{L} + C_{n-2} \mathbb{H}, z \mathbb{L} \hat{=} n \hat{=} \mathbb{N}^+$$

03.10.17.0019.01

$$L_n \mathbb{H} \mathbb{L} \ddagger C_n \mathbb{H}, z L L_{n-n} \mathbb{H} \mathbb{L} + C_{n-1} \mathbb{H}, z L L_{n-n-1} \mathbb{H} \mathbb{L} - \frac{1}{\sqrt{p}} \mathbb{K} \frac{z^{n-1} n^{-1}}{2} \hat{a} \frac{\mathbb{H} - n \mathbb{L}_j}{j=0 \mathbb{G} \mathbb{J} n - j + \frac{1}{2} \mathbb{N} \frac{z^2}{4} \mathbb{N}} {}_2F_3 \left(\frac{1-j}{2}, -\frac{j}{2}; 1-n, -j, n-j; z^2 \right) \bullet;$$

$$C_n \mathbb{H}, z \mathbb{L} \ddagger 2^n z^{-n} \mathbb{H} - n \mathbb{L}_n {}_2F_3 \left(\frac{1-n}{2}, -\frac{n}{2}; 1-n, -n, n-n; z^2 \right) \bullet; n \hat{=} \mathbb{N}^+$$

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Functional identities

Relations between contiguous functions

03.10.17.0003.01

$$L_n \mathbb{H} \mathbb{L} \tilde{S} \frac{z}{2n} \mathbb{H} \mathbb{L}_{n-1} \mathbb{H} \mathbb{L} - L_{n+1} \mathbb{H} \mathbb{L} - \frac{2^{-n-1} z^{n+1}}{\sqrt{p} n \mathbb{G} \mathbb{J} n + \frac{3}{2} \mathbb{N}}$$

Differentiation

Low-order differentiation

With respect to n

03.10.20.0001.01

$$L_n^{(0)} \mathbb{H} \tilde{S} \log \frac{z}{2} \mathbb{K} - \frac{z^{n+1}}{2} \frac{1}{\hat{a}} \frac{1}{\sum_{k=0}^{\infty} \mathbb{G} \mathbb{J} k + \frac{3}{2} \mathbb{N} \mathbb{G} \mathbb{J} k + n + \frac{3}{2} \mathbb{N}} y \left(k + n + \frac{3}{2} \right) \mathbb{K} - \frac{z^{2k}}{2}$$

03.10.20.0014.01

$$L_n^{(0)} \mathbb{H} \tilde{S} \mathbb{H} - 1 \mathbb{L}^n \mathbb{K}_n \mathbb{H} \mathbb{L} + \frac{\mathbb{H} - 1 \mathbb{L}^{n+1} 2^{n-1}}{z^n p^2} G_{2,4}^{4,2} \left(\frac{z}{2}, \frac{1}{2} \left| \begin{array}{c} \frac{1}{2}, \frac{1}{2} \\ n, 0, \frac{1}{2}, \frac{1}{2} \end{array} \right. \right) -$$

$$\frac{1}{p} \frac{n-1}{\hat{a}} \frac{\mathbb{H} - 1 \mathbb{L}^k}{\mathbb{K} - \frac{z}{2} \mathbb{K} - \frac{1}{2} \mathbb{N}} \mathbb{K} - \frac{z^{n-2k-1}}{2} \left(\log \mathbb{K} - \frac{z}{2} \mathbb{K} - y \left(n - k + \frac{1}{2} \right) \right) + \frac{1}{2} \frac{n-1}{n!} \frac{1}{\hat{a}} \frac{1}{k! \mathbb{H} - k \mathbb{L}} \mathbb{K} - \frac{z^{k-n}}{2} L_{-k} \mathbb{H} \mathbb{L}; n \hat{\mathbb{I}} \mathbb{N}$$

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03.10.20.0015.01

$$L_n^{(0)} \mathbb{H} \tilde{S} \mathbb{H} - 1 \mathbb{L}^n \mathbb{K}_n \mathbb{H} \mathbb{L} + \frac{\mathbb{H} - 1 \mathbb{L}^{n+1} 2^{n-1}}{z^n p^2} G_{2,4}^{4,2} \left(\frac{z}{2}, \frac{1}{2} \left| \begin{array}{c} \frac{1}{2}, \frac{1}{2} \\ n, 0, \frac{1}{2}, \frac{1}{2} \end{array} \right. \right) - \frac{n!}{2} \frac{n-1}{\hat{a}} \frac{1}{k! \mathbb{H} - k \mathbb{L}} \mathbb{K} - \frac{z^{k-n}}{2} L_{-k} \mathbb{H} \mathbb{L}; n \hat{\mathbb{I}} \mathbb{N}$$

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03.10.20.0016.01

$$L_{n+\frac{1}{2}}^{(0)} \mathbb{H} \tilde{S} \frac{\mathbb{H} - 1 \mathbb{L}^n n!}{2 \sqrt{p}} \mathbb{K} - \frac{z^{-n-\frac{1}{2}}}{2} \frac{1}{\hat{a}} \frac{\mathbb{J}_{\frac{1}{2}} \mathbb{N}_k}{k! \mathbb{H} - k \mathbb{L}} - \frac{1}{n! \sqrt{p}} \log \mathbb{K} - \frac{z}{2} \mathbb{K} - \frac{z^{n-\frac{1}{2}}}{2} {}_3F_0 \left(-n, \frac{1}{2}, 1; ; \frac{4}{z^2} \right) +$$

$$\frac{1}{\sqrt{p}} \mathbb{K} - \frac{z}{2} \mathbb{K} - \frac{z^{n-\frac{1}{2}}}{2} \frac{1}{\hat{a}} \frac{\mathbb{J}_{\frac{1}{2}} \mathbb{N}_k y \mathbb{H} k + n + 1 \mathbb{L}}{\mathbb{H} - k \mathbb{L}} \left(-\frac{4}{z^2} \right)^k - \frac{\mathbb{H} - 1 \mathbb{L}^n \sqrt{p} n!}{2} \mathbb{K} - \frac{z^{\frac{1}{2}-n}}{2}$$

$$\frac{n-1}{\hat{a}} \frac{1}{k! \mathbb{H} - k \mathbb{L}} \mathbb{K} - \frac{z}{2} \mathbb{K} - \frac{z^k}{2} \frac{1}{\hat{a}} \frac{1}{p!} \mathbb{K} - \frac{z^p}{2} \left(\left(2 I_{p-\frac{1}{2}} \mathbb{H} \mathbb{L} - 2^{p+\frac{1}{2}} I_{p-\frac{1}{2}} \mathbb{H} \mathbb{L} \right) I_{-k-\frac{1}{2}} \mathbb{H} \mathbb{L} - \left(2 I_{\frac{1}{2}-p} \mathbb{H} \mathbb{L} - 2^{p+\frac{1}{2}} I_{\frac{1}{2}-p} \mathbb{H} \mathbb{L} \right) I_{k+\frac{1}{2}} \mathbb{H} \mathbb{L} \right) +$$

$$I_{-n-\frac{1}{2}} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} - \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} + \frac{\mathbb{H} - 1 \mathbb{L}^{n+1}}{2 p} \mathbb{K} - \frac{z^{-n-\frac{1}{2}}}{2} \mathbb{G} \left(n + \frac{1}{2} \right) \left(\log \mathbb{H} \mathbb{L} + y \left(\frac{1}{2} - n \right) + 3 y \right) +$$

$$I_{n+\frac{1}{2}} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} - 2 \mathbb{H} \mathbb{L} \mathbb{H} \mathbb{L} + \frac{\mathbb{H} - 1 \mathbb{L}^n n!}{2} \left(\frac{2}{z} \right)^{n-1} \frac{1}{\hat{a}} \frac{1}{k! \mathbb{H} - k \mathbb{L}} \mathbb{K} - \frac{z^k}{2} I_{-k-\frac{1}{2}} \mathbb{H} \mathbb{L}; n \hat{\mathbb{I}} \mathbb{N}$$

Brychkov Yu.A. (2005)

03.10.20.0017.01

$$L_{-n-\frac{1}{2}}^{\text{H},0\text{L}} \text{H}\tilde{\text{L}} \tilde{\text{S}} I_{n+\frac{1}{2}} \text{H}\tilde{\text{L}} \text{H}\tilde{\text{L}} \text{Chi}\tilde{\text{L}} - \text{Chi}\tilde{\text{L}} \text{H}\tilde{\text{L}} + I_{-n-\frac{1}{2}} \text{H}\tilde{\text{L}} \text{H}\tilde{\text{L}} \text{Shi}\tilde{\text{L}} \text{H}\tilde{\text{L}} - 2 \text{Shi}\tilde{\text{L}} \text{H}\tilde{\text{L}} - \frac{1}{2} n! \hat{\text{a}} \frac{1}{k! \text{H} - k\text{L}} \text{K} - \frac{z^{k-n}}{2} I_{k+\frac{1}{2}} \text{H}\tilde{\text{L}} - \frac{n! \sqrt{\text{p}} z}{2}$$

$$\hat{\text{a}} \frac{1}{\text{H} - k\text{L}} \left(-\frac{2}{z} \right)^k \frac{\text{H} z^p}{p!} \left(\left(2^{\frac{1}{2}-p} I_{p-\frac{1}{2}} \text{H}\tilde{\text{L}} - I_{p-\frac{1}{2}} \text{H}\tilde{\text{L}} \right) I_{n-k+\frac{1}{2}} \text{H}\tilde{\text{L}} - \left(2^{\frac{1}{2}-p} I_{\frac{1}{2}-p} \text{H}\tilde{\text{L}} - I_{\frac{1}{2}-p} \text{H}\tilde{\text{L}} \right) I_{k-n-\frac{1}{2}} \text{H}\tilde{\text{L}} \right) \bullet; n \hat{\text{I}} \text{N}$$

Brychkov Yu.A. (2005)

03.10.20.0002.01

$$L_n^{\text{H},0\text{L}} \text{H}\tilde{\text{L}} \tilde{\text{S}} - \frac{2^{-n} z^{n+3}}{3 \sqrt{\text{p}} \text{H} n + 3\text{LGJn} + \frac{5}{2}\text{N}} F_3^{1,1,1} \left(\begin{matrix} 2; 1; 1, n + \frac{3}{2}; \\ 2, \frac{5}{2}, n + \frac{5}{2}; n + \frac{5}{2}; \end{matrix} \frac{z^2}{4}, \frac{z^2}{4} \right) + \left(\log \text{H}\tilde{\text{L}} - \log \text{H}\tilde{\text{L}} - y \left(n + \frac{3}{2} \right) \right) L_n \text{H}\tilde{\text{L}}$$

With respect to z

03.10.20.0003.01

$$\frac{\text{H} L_n \text{H}\tilde{\text{L}}}{\text{H} z} \tilde{\text{S}} L_{n-1} \text{H}\tilde{\text{L}} - \frac{n}{z} L_n \text{H}\tilde{\text{L}}$$

03.10.20.0004.01

$$\frac{\text{H} L_n \text{H}\tilde{\text{L}}}{\text{H} z} \tilde{\text{S}} \frac{2^{-n} z^n}{\sqrt{\text{p}} \text{GJn} + \frac{3}{2}\text{N}} + L_{n+1} \text{H}\tilde{\text{L}} + \frac{n}{z} L_n \text{H}\tilde{\text{L}}$$

03.10.20.0005.01

$$\frac{\text{H} L_n \text{H}\tilde{\text{L}}}{\text{H} z} \tilde{\text{S}} \frac{1}{2} \left(\frac{2^{-n} z^n}{\sqrt{\text{p}} \text{GJn} + \frac{3}{2}\text{N}} + L_{n-1} \text{H}\tilde{\text{L}} + L_{n+1} \text{H}\tilde{\text{L}} \right)$$

03.10.20.0006.01

$$\frac{\text{H}^2 L_n \text{H}\tilde{\text{L}}}{\text{H} z^2} \tilde{\text{S}} \frac{1}{z^2} \text{I} z^2 L_{n-2} \text{H}\tilde{\text{L}} + \text{H} - 2 z n L_{n-1} \text{H}\tilde{\text{L}} + n \text{H} + 1 L_n \text{H}\tilde{\text{L}}$$

03.10.20.0007.01

$$\frac{\text{H}^2 L_n \text{H}\tilde{\text{L}}}{\text{H} z^2} \tilde{\text{S}} \frac{1}{4} \text{H} L_{n-2} \text{H}\tilde{\text{L}} + L_{n+2} \text{H}\tilde{\text{L}} + 2 L_n \text{H}\tilde{\text{L}} + \frac{2^{-n-1} \text{I} z^2 + 8 n^2 + 14 n + 3 \text{M}^{n-1}}{\sqrt{\text{p}} \text{H} n \text{H} + 2\text{L} + 3\text{LGJn} + \frac{1}{2}\text{N}}$$

03.10.20.0008.01

$$\frac{\text{H}^n L_n \text{H}\tilde{\text{L}}}{\text{H} z} \tilde{\text{S}} z^n L_{n-1} \text{H}\tilde{\text{L}}$$

03.10.20.0009.01

$$\frac{\text{H}^{-n} L_n \text{H}\tilde{\text{L}}}{\text{H} z} \tilde{\text{S}} z^{-n} L_{n+1} \text{H}\tilde{\text{L}} + \frac{2^{-n}}{\sqrt{\text{p}} \text{GJn} + \frac{3}{2}\text{N}}$$

Symbolic differentiation

With respect to z

03.10.20.0018.01

$$\frac{{}_n L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^n} \tilde{S} \frac{n!}{1 - \frac{z}{2}} \hat{a} \frac{f_2^{n-v}}{2^{2k} k! \mathbb{H} - 2k\mathbb{L}} \frac{\mathbb{H} 1\mathbb{L}^k}{2^{2k} k! \mathbb{H} - 2k\mathbb{L}} \frac{n-k}{p=0} \binom{n-k}{p} \binom{n}{2} \frac{z^p}{2} {}_0 L_{n-p} \mathbb{H} \mathbb{L} \bullet; n \hat{\mathbb{I}} N$$

Brychkov Yu.A. (2005)

03.10.20.0019.01

$$\frac{{}_n L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^n} \tilde{S} \frac{n!}{1 - \frac{z}{2}} \hat{a} \frac{f_2^{n-v}}{2^{2k} k! \mathbb{H} - 2k\mathbb{L}} \frac{\mathbb{H} 1\mathbb{L}^k}{2^{2k} k! \mathbb{H} - 2k\mathbb{L}} \frac{n-k}{p=0} \binom{n-k}{p} \binom{n}{2} \frac{z^p}{2} {}_0 L_{n-p} \mathbb{H} \mathbb{L} + \frac{1}{p} \frac{z^{2p+n-1}}{2} \frac{\mathbb{H} 1\mathbb{L}^k \mathbb{G} \mathbb{J} r + \frac{1}{2} N}{\mathbb{G} \mathbb{J} p - r + n + \frac{1}{2} N} \frac{z^{-2r}}{2} \frac{K-0}{2} \bullet; n \hat{\mathbb{I}} N$$

Brychkov Yu.A. (2005)

03.10.20.0020.01

$$\frac{{}_n L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^n} \tilde{S} z^{-n} \hat{a} \frac{\mathbb{H} 1\mathbb{L}^{m+n}}{m!} \binom{n}{m} \mathbb{H} n\mathbb{L}_{n-m} \hat{a} \frac{m}{k=0} \frac{\mathbb{H} 1\mathbb{L}^{k-1} 2^{2k-m} \mathbb{H} m\mathbb{L}_2 \mathbb{H}_{n-k\mathbb{L}} \mathbb{H} \mathbb{L}_k}{\mathbb{H} n - k\mathbb{L}!}$$

$$\left(\frac{z^{k-1}}{2} \hat{a} \frac{\mathbb{H} - j - 1\mathbb{L}! \mathbb{J} - \frac{z^2}{4} N}{j! \mathbb{H} - 2j - 1\mathbb{L}! \mathbb{H} - k - n\mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}} {}_{n-1} \mathbb{H} \mathbb{L} - \hat{a} \frac{\mathbb{H} - j\mathbb{L}! \mathbb{J} - \frac{z^2}{4} N}{j! \mathbb{H} - 2j\mathbb{L}! \mathbb{H} - k - n\mathbb{L}_j \mathbb{H} \mathbb{L}_j} {}_n \mathbb{H} \mathbb{L} \right) + \frac{2^{-n} z^{n+1}}{\sqrt{p} \mathbb{G} \mathbb{J} n + \frac{1}{2} N}$$

$$\hat{a} \hat{a} \frac{n-1}{i=1} \frac{i}{m=0} \mathbb{H} 1\mathbb{L}^{i+m} \binom{i}{m} \mathbb{H} n\mathbb{L}_{i-m} \hat{a} \frac{m}{k=0} \frac{\mathbb{H} 1\mathbb{L}^{k-1} 2^{2k-m} \mathbb{H} m\mathbb{L}_2 \mathbb{H}_{n-k\mathbb{L}} \mathbb{H} \mathbb{L}_k}{\mathbb{H} n - k\mathbb{L}!} \hat{a} \frac{n-1}{j=0} \frac{j}{\mathbb{H} - j - 1\mathbb{L}! \mathbb{H} - k - n\mathbb{L}_j \mathbb{H} \mathbb{L}_{j+1}} \frac{z^{2j}}{2} \bullet; n \hat{\mathbb{I}} N$$

03.10.20.0010.02

$$\frac{{}_n L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^n} \tilde{S} 2^{n-2n-2} \sqrt{p} z^{n+1} \mathbb{G} \mathbb{H} + 2\mathbb{L}_3 \tilde{F}_4 \left(1, \frac{n}{2} + 1, \frac{n+3}{2}; \frac{3}{2}, \frac{n-n}{2} + 1, \frac{n-n+3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right) \bullet; n \hat{\mathbb{I}} N$$

Fractional integro-differentiation

With respect to z

03.10.20.0011.01

$$\frac{{}_a L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^a} \tilde{S} 2^{a-2n-2} \sqrt{p} z^{1-a+n} \mathbb{G} \mathbb{H} + 2\mathbb{L}_3 \tilde{F}_4 \left(1, \frac{n}{2} + 1, \frac{n+3}{2}; \frac{3}{2}, \frac{n-a}{2} + 1, \frac{3+n-a}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right) \bullet; -n \hat{\mathbb{I}} N^+$$

03.10.20.0012.01

$$\frac{{}_a L_n \mathbb{H} \mathbb{L}}{\mathbb{I} z^a} \tilde{S} \mathbb{H} 1\mathbb{L} f_2^{n+1-v} 2^{a-2\mathbb{H}+1\mathbb{L}+4f_2^{n+1-v}} \sqrt{p} \mathbb{G} \left(n-2 \left\lfloor \frac{n+1}{2} \right\rfloor + 2 \right) {}_3 \tilde{F}_4 \left(1, \frac{1}{2} \left(n-2 \left\lfloor \frac{n+1}{2} \right\rfloor + 2 \right), \frac{1}{2} \left(n-2 \left\lfloor \frac{n+1}{2} \right\rfloor + 3 \right); \right.$$

$$\left. \frac{3}{2} - \left\lfloor \frac{n+1}{2} \right\rfloor, \frac{1}{2} \left(n-a-2 \left\lfloor \frac{n+1}{2} \right\rfloor + 2 \right), \frac{1}{2} \left(n-a-2 \left\lfloor \frac{n+1}{2} \right\rfloor + 3 \right), n - \left\lfloor \frac{n+1}{2} \right\rfloor + \frac{3}{2}; \frac{z^2}{4} \right) z^{n-a-2f_2^{n+1-v}+1} +$$

$$\frac{-f_2^{n+3-v}}{\hat{a}} \frac{\mathbb{H} 1\mathbb{L}^p 2^{-2k-n-1} z^{2k-a+n+1} \mathbb{H} \mathbb{G} \mathbb{H} \mathbb{L} + y \mathbb{H} 2k-n-1\mathbb{L} - y \mathbb{H} 2k-a+n+2\mathbb{L}}{\mathbb{H} 2k-n-2\mathbb{L}! \mathbb{G} \mathbb{J} k + \frac{3}{2} \mathbb{N} \mathbb{G} \mathbb{J} k + n + \frac{3}{2} \mathbb{N} \mathbb{G} \mathbb{H} 2k-a+n+2\mathbb{L}} \bullet; -n \hat{\mathbb{I}} N^+$$

03.10.20.0013.01

$$\frac{\sum_{k=0}^a L_n H_k z^k}{z^a} \tilde{S} \hat{a} \frac{{}_2F_3^{\text{HL}} \left(\begin{matrix} - \\ 2k+n+1 \end{matrix} ; \begin{matrix} 1 \\ 2 \end{matrix} ; \begin{matrix} 2k-n-1 \\ 2 \end{matrix} \right) \exp \left(\frac{2k-n-1}{2} \right)}{GJk + \frac{3}{2}NGJk + n + \frac{3}{2}N}$$

Integration

Indefinite integration

Involving only one direct function

03.10.21.0001.01

$$\hat{a} L_n H_L \hat{a} z \tilde{S} \frac{2^{-n} z^{n+2}}{\sqrt{p} H + 2LGJn + \frac{3}{2}N} {}_2F_3 \left(1, \frac{n}{2} + 1; \frac{3}{2}, \frac{n}{2} + 2, n + \frac{3}{2}; \frac{z^2}{4} \right)$$

Involving one direct function and elementary functions

Involving power function

03.10.21.0002.01

$$\hat{a} z^{a-1} L_n H_L \hat{a} z \tilde{S} \frac{2^{-n} z^{a+n+1}}{\sqrt{p} H + n + 1LGJn + \frac{3}{2}N} {}_2F_3 \left(1, \frac{a+n+1}{2}; \frac{3}{2}, \frac{a+n+3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right)$$

03.10.21.0003.01

$$\hat{a} z^{1-n} L_n H_L \hat{a} z \tilde{S} z^{1-n} L_{n-1} H_L - \frac{2^{1-n} z}{\sqrt{p} GJn + \frac{1}{2}N}$$

03.10.21.0004.01

$$\hat{a} z^n L_n H_L \hat{a} z \tilde{S} 2^{-n-2} a z^{n+2} H_L z^L \left(\frac{1}{2} H + n + 2L \right) {}_2F_3 \left(1, \frac{1}{2} H + n + 2L; n + \frac{3}{2}, \frac{3}{2}, \frac{1}{2} H + n + 4L; \frac{a^2 z^2}{4} \right)$$

03.10.21.0005.01

$$\hat{a} z^{1-n} L_n H_L \hat{a} z \tilde{S} \frac{z^{1-n} \left(L_{n-1} H_L z^L - \frac{2^{1-n} H_L z^L}{\sqrt{p} GJn + \frac{1}{2}N} \right)}{a}$$

03.10.21.0006.01

$$\hat{a} z^{n+1} L_n H_L \hat{a} z \tilde{S} \frac{z^{n+1} L_{n+1} H_L z^L}{a}$$

Involving exponential function and a power function

03.10.21.0007.01

$$\hat{a} z^n \tilde{a}^{-z} L_n H_L \hat{a} z \tilde{S} \frac{1}{2n+1} \left(\tilde{a}^{-z} H_L H_L + L_{n+1} H_L z^{n+1} + \frac{2^{-n} GJn + 2, z^L}{\sqrt{p} GJn + \frac{3}{2}N} \right)$$

03.10.21.0008.01

$$\partial_z^{-n} \tilde{a}^{-z} L_n H_L \hat{a} z \tilde{S} \frac{\tilde{a}^{-z}}{2n-1} \left(-H_{n-1} H_L + L_n H_L z^{1-n} - \frac{2^{1-n}}{\sqrt{p} \Gamma(n + \frac{1}{2})} \right)$$

03.10.21.0009.01

$$\partial_z^n \tilde{a}^z L_n H_L \hat{a} z \tilde{S} \frac{z^n}{2n+1} \left(\frac{2^{-n} z^n \Gamma(n+2, -z L H_L z^{1-2n})}{\sqrt{p} \Gamma(n + \frac{3}{2})} + \tilde{a}^z z H_n H_L - L_{n+1} H_L \right)$$

03.10.21.0010.01

$$\partial_z^{-n} \tilde{a}^z L_n H_L \hat{a} z \tilde{S} \frac{\tilde{a}^z}{2n-1} \left(z^{1-n} H_{n-1} H_L - L_n H_L - \frac{2^{1-n}}{\sqrt{p} \Gamma(n + \frac{1}{2})} \right)$$

Involving direct function and Bessel-, Airy-, Struve-type functions

Involving Bessel functions

Involving Bessel I and power

03.10.21.0011.01

$$\partial_z^n H_n H_L - L_n H_L z L \hat{a} z \tilde{S} 2^{-n-2} z^{n+1} H_L z L^n \left(2^{2n+1} G\left(\frac{1}{2}, H_L - n + 1L\right) {}_1\tilde{F}_2\left(\frac{1}{2}, H_L - n + 1L; 1 - n, \frac{1}{2} H_L - n + 3L; \frac{a^2 z^2}{4}\right) - \right. \\ \left. a z H_L z L^{2n} G\left(\frac{1}{2}, H_L + n + 2L\right) {}_2\tilde{F}_3\left(1, \frac{1}{2} H_L + n + 2L; n + \frac{3}{2}, \frac{3}{2}, \frac{1}{2} H_L + n + 4L; \frac{a^2 z^2}{4}\right) \right)$$

03.10.21.0012.01

$$\partial_z^n H_n H_L - L_n H_L z L \hat{a} z \tilde{S} 2^{-n-2} z^{n+1} H_L z L^n \left(2 G\left(\frac{1}{2}, H_L + n + 1L\right) {}_1\tilde{F}_2\left(\frac{1}{2}, H_L + n + 1L; n + 1, \frac{1}{2} H_L + n + 3L; \frac{a^2 z^2}{4}\right) - \right. \\ \left. a z G\left(\frac{1}{2}, H_L + n + 2L\right) {}_2\tilde{F}_3\left(1, \frac{1}{2} H_L + n + 2L; n + \frac{3}{2}, \frac{3}{2}, \frac{1}{2} H_L + n + 4L; \frac{a^2 z^2}{4}\right) \right)$$

Definite integration

Involving the direct function

03.10.21.0013.01

$$\int_0^\infty t^{a-1} \tilde{a}^{-at} L_n H_L t L \hat{a} t \tilde{S} \frac{2^{-n} a^{-a-n-1} b^{n+1} \Gamma(a+n+1L)}{\sqrt{p} \Gamma(n + \frac{3}{2})} {}_3F_2\left(1, \frac{1}{2} H_L + n + 2L, \frac{1}{2} H_L + n + 1L; n + \frac{3}{2}, \frac{3}{2}; \frac{b^2}{a^2}\right) \bullet;$$

$$\operatorname{Re} H_L + nL > -1 \text{ i } \operatorname{Re} H_L > \operatorname{Re} H_L$$

Integral transforms

Laplace transforms

03.10.22.0001.01

$$L_n \text{H} \text{L} \check{S} \frac{2^{-n} z^{-n-2} \text{GH} + 2\text{L}}{\sqrt{p} \text{GJn} + \frac{3}{2}\text{N}} {}_3F_2 \left(1, \frac{n+3}{2}, \frac{n+2}{2}; n + \frac{3}{2}, \frac{3}{2}; \frac{1}{z^2} \right); \bullet; \text{ReH} > -2$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

03.10.26.0001.01

$$L_n \text{H} \text{L} \check{S} \frac{z^{-n+1}}{2} {}_1F_2 \left(1; \frac{3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right)$$

03.10.26.0010.01

$$L_n \text{H} \text{L} \check{S} \frac{z^{-n}}{2} {}_0F_1 \left(; 1 - n; \frac{z^2}{4} \right); \bullet; -n - \frac{3}{2} \hat{I} \text{ N}$$

Involving ${}_pF_q$

03.10.26.0002.01

$$L_n \text{H} \text{L} \check{S} \frac{z^{n+1}}{2^n \sqrt{p} \text{GJn} + \frac{3}{2}\text{N}} {}_1F_2 \left(1; \frac{3}{2}, n + \frac{3}{2}; \frac{z^2}{4} \right); \bullet; -n - \frac{3}{2} \hat{I} \text{ N}$$

03.10.26.0011.01

$$L_n \text{H} \text{L} \check{S} \frac{1}{\text{GH} - n\text{L}} \frac{z^{-n}}{2} {}_0F_1 \left(; 1 - n; \frac{z^2}{4} \right); \bullet; -n - \frac{3}{2} \hat{I} \text{ N}$$

Through Meijer G

Classical cases for the direct function itself

03.10.26.0003.01

$$L_n \text{H} \text{L} \check{S} - p \csc \left(\frac{p}{2} \right) z^{n-1} \text{Iz}^2 \text{M}^2 G_{2,4}^{1,1} \left(\frac{z^2}{4} \left| \begin{matrix} \frac{n+1}{2}, \frac{1}{2} \\ \frac{n+1}{2}, \frac{1}{2}, -\frac{n}{2}, \frac{n}{2} \end{matrix} \right. \right)$$

03.10.26.0004.01

$$L_n \text{H} \text{L} \check{S} - p \csc \left(\frac{p}{2} \right) G_{2,4}^{1,1} \left(\frac{z^2}{4} \left| \begin{matrix} \frac{n+1}{2}, \frac{1}{2} \\ \frac{n+1}{2}, \frac{1}{2}, -\frac{n}{2}, \frac{n}{2} \end{matrix} \right. \right); \bullet; \text{ReH} > 0$$

03.10.26.0005.01

$$L_n \text{I} \sqrt{z} \text{N} \check{S} - p \csc \left(\frac{p}{2} \right) G_{2,4}^{1,1} \left(\frac{z}{4} \left| \begin{matrix} \frac{n+1}{2}, \frac{1}{2} \\ \frac{n+1}{2}, \frac{1}{2}, -\frac{n}{2}, \frac{n}{2} \end{matrix} \right. \right)$$

Classical cases involving Bessel I

03.10.26.0006.01

$$I_n \sqrt{z} N - L_n \sqrt{z} N \tilde{S} - \frac{1}{p} G_{1,3}^{2,1} \left(\frac{z}{4} \left| \begin{matrix} \frac{n+1}{2} \\ \frac{n+1}{2}, \frac{n}{2}, -\frac{n}{2} \end{matrix} \right. \right)$$

Generalized cases for the direct function itself

03.10.26.0007.01

$$L_n \mathbb{H} \tilde{S} - p \csc \left(\frac{pn}{2} \right) G_{2,4}^{1,1} \left(\frac{z}{2}, \frac{1}{2} \left| \begin{matrix} \frac{n+1}{2}, \frac{1}{2} \\ \frac{n+1}{2}, \frac{1}{2}, -\frac{n}{2}, \frac{n}{2} \end{matrix} \right. \right)$$

Generalized cases involving Bessel I

03.10.26.0008.01

$$I_n \mathbb{H} L - L_n \mathbb{H} \tilde{S} - \frac{1}{p} G_{1,3}^{2,1} \left(\frac{z}{2}, \frac{1}{2} \left| \begin{matrix} \frac{n+1}{2} \\ \frac{n+1}{2}, \frac{n}{2}, -\frac{n}{2} \end{matrix} \right. \right)$$

Through other functions

03.10.26.0009.01

$$L_n \mathbb{H} \tilde{S} z \csc \mathbb{H} p n L \left(\frac{\sqrt{p}}{G \mathbb{H} - n L G \mathbb{H} n + \frac{1}{2} N} I_n \mathbb{H} L_1 F_2 \left(\frac{1}{2}; \frac{3}{2}, 1 - n; \frac{z^2}{4} \right) - \frac{z^{2n}}{G \mathbb{H} \mathbb{H} n + 1 L} L_n \mathbb{H} L_1 F_2 \left(n + \frac{1}{2}; n + 1, n + \frac{3}{2}; \frac{z^2}{4} \right) \right)$$

Representations through equivalent functions

With related functions

03.10.27.0001.01

$$L_n \mathbb{H} z \tilde{S} - \mathbb{H} z L^n z^{-n} H_n \mathbb{H} L$$

03.10.27.0002.01

$$L_n \mathbb{H} - \mathbb{H} z \tilde{S} - \mathbb{H} z L^n z^{-n} H_n \mathbb{H} L$$

03.10.27.0003.01

$$L_n \mathbb{H} \tilde{S} L_n \mathbb{H} L - \frac{2^{1-n} z^{n-1}}{\sqrt{p} \mathbb{H} n - \frac{1}{2} N!} \hat{a} \left(\frac{1}{2} \right)_k \left(\frac{1}{2} - n \right)_k \left(\frac{z^2}{4} \right)^{-k} \bullet; n - \frac{1}{2} \hat{I} Z$$

03.10.27.0004.01

$$L_n \mathbb{H} \tilde{S} L_n \mathbb{H} L \bullet; -n - \frac{1}{2} \hat{I} N$$

Inequalities

03.10.29.0001.01

$$L_n \mathbb{H} L \geq 0 \bullet; x \geq 0 \mathbb{H} n \hat{I} R$$

Theorems

The Coulomb potential

The Coulomb potential, including the order quantum electrodynamical correction, is given by

$$V_{\text{HL}} = \frac{1}{r} \left(1 + \alpha \frac{1}{72 p^2} \left(p \left(3 + \frac{r^2}{l_C^2} \right) \frac{r}{l_C} - 2 \left(4 + \frac{r^2}{l_C^2} \right) \frac{r}{l_C} K_1 \left(\frac{r}{l_C} \right) - \right. \right. \\ \left. \left. 2 \left(\frac{r^4}{l_C^4} + \frac{2 r^2}{l_C^2} - 6 \right) K_0 \left(\frac{r}{l_C} \right) + p \frac{r^2}{l_C^2} \left(3 + \frac{r^2}{l_C^2} \right) \left(K_1 \left(\frac{r}{l_C} \right) L_0 \left(\frac{r}{l_C} \right) - K_0 \left(\frac{r}{l_C} \right) L_1 \left(\frac{r}{l_C} \right) \right) \right) \right).$$

Here, α is the fine structure constant and l_C is the Compton wavelength.

History

–J. W. Nicholson (1911)

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