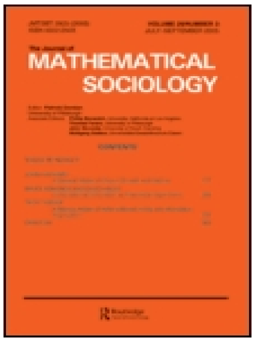


The Journal of Mathematical Sociology

ISSN: 0022-250X (Print) 1545-5874 (Online) Journal homepage: https://www.tandfonline.com/loi/gmas20

How Do Cultural Classes Emerge from Assimilation and Distinction? An Extension of the Cucker-Smale Flocking Model

JEONG-HAN KANG , SEUNG-YEAL HA , KYUNGKEUN KANG & EUNHEE JEONG

Journal of Mathematical Sociology, 38: 47–71, 2014

ISSN: 0022-250X print/1545-5874 online
DOI: 10.1080/0022250X.2011.629063

HOW DO CULTURAL CLASSES EMERGE FROM ASSIMILATION AND DISTINCTION? AN EXTENSION OF THE CUCKER-SMALE FLOCKING MODEL

Jeong-han Kang
Department of Sociology, Yonsei University, Seoul, South Korea

Seung-Yeal Ha
Department of Mathematical Sciences and Research Institute of Mathematics, Seoul National University, Seoul, South Korea

Kyungkeun Kang
Department of Mathematics, Yonsei University, Seoul, South Korea

Eunhee Jeong
Department of Mathematical Sciences, Seoul National University, Seoul, South Korea

When cultural tastes are not neutral but hierarchically matched to social status, people assimilate themselves to higher status by consuming cultural goods while distinguishing themselves from lower status by developing new tastes. Extending the Cucker-Smale model for mutual influence among agents, we examine when and how many cultural classes emerge from continuous distributions of tastes and what conditions those classes satisfy, through the assimilation-distinction mechanism. We simulate the models with different initial distributions of tastes (uniform, normal, and chi-square), given various ranges of 2 parameters: (a) the strength and (b) the range of distinction relative to assimilation. Tastes are flocking and cultural classes emerge when the range of assimilation is much larger than that of distinction. The number of classes increases with the strength of distinction, whereas the distance between classes equals the range of distinction. Some properties of emergent classes are mathematically proved. First, in a two-class system, the stronger distinction, the larger the upper class. Second, in a three-class system, the middle class is necessarily larger than the lower class and likely larger than the upper class. Third, a 3-class system cannot emerge if distinction is weaker than assimilation. These properties are universal and do not depend on the initial distribution of cultural tastes. This independence predicts homogeneous cultural classes emerging across different social conditions. Also, the cultural middle class as the largest group may explain why subjective class consciousness is often higher than objective position. Unless assimilating efforts can reach an infinite range, there emerges a cultural outcast at the lowest end of the cultural hierarchy.

Keywords: consumption, Cucker-Smale model, cultural class, distinction, emergence, stratification

Address correspondence to Jeong-han Kang, Department of Sociology, Yonsei University, 134 Shinchon-dong, Seodaemun-gu, Seoul 120-749, South Korea. E-mail: jhk55@yonsei.ac.kr

1. INTRODUCTION

The recent trends in globalization have abbreviated geographical distances, of which once-exotic cultures are easily accessed. However, studies have shown findings on the persistence of cultural diversity as opposed to global convergence (e.g., Elkins, 1997; Hermans, & Kemper, 1998). The possible causes for the persistence of cultural diversity, or local flocking of cultural traits, have been actively explored since Axelrod (1997) proposed an agent-based model for cultural assimilation. The principle of homophily for social interaction in Axelrod's model suppresses global homogeneity, on the grounds that two agents will increasingly push away from each other if the contrasts in their cultural traits are too great to share homophilous interactions. Flache and Macy (2011a, 2011b; Flache, Macy, & Takács, 2006) cumulatively extended Axelrod's approach and explored under what conditions cultural diversity is stable and robust. Most notably, Flache and Macy (2011b) employed repulsion as well as attraction between agents and demonstrated how "long-range ties" (p. 146) can foster rather than inhibit greater cultural polarization.

Our formal model builds on Flache and Macy's (2011) research stream and incorporates both attraction and repulsion as driving forces for local flocking of cultural traits. Our model also assumes endogenous processes of interactions where the likelihood of interactions for attraction or repulsion depends on the similarity of cultural traits between agents. Simultaneously, our model aims to contribute to the computational research of cultural diversity in several ways. First, our model will explore the hierarchical order among cultural traits and, therefore, examine the emergence of cultural classes. Second, we explore general properties of cultural classes independent of initial distributions of cultural traits. In other words, we extend the research question to include both the micro-level conditions for stable cultural classes to the macro-level general properties on those cultural classes. Third, we do not restrict our efforts to computational findings but mathematically derive analytic theorems on those general properties. To build a formal model for these goals, we can start by exploring research on cultural consumption and its relation to social classes.

Veblen (1899) identified a social class characterized by its consuming life-style for the first time. A new, wealthy class in the 19th century conspicuously consumed luxury goods to signify their wealth. Luxury consumption was a signal for the upper class then. In contemporary society, luxury goods are not necessarily consumed by the upper class anymore, but rather popularized to the middle and even lower classes. Consumption of luxury goods is not a sign of high social class anymore, but rather a sign of the consuming masses having a "love affair with luxury" (Twitchell, 2002). Luxury consumption has also diffused to developing countries. For example, East Asia emerged as a big market for luxury goods (Wong & Ahuvia, 1998). In short, different societies evidence surprisingly homogenous groups of consumers who share the same luxury tastes, even though those different societies must have different distributions of cultural tastes.

Observing those social classes emerging from cultural consumptions, various marketing research and sociological theories have explored why people consume luxury goods or, in general, "status goods" (Kapur, 2005), when cultural tastes are not neutral but hierarchically ordered. What is unknown yet is not why people consume status goods at the micro level but how those agents' behaviors at the micro level lead to the emergence of categorical cultural tastes at the macro level. In short, we do not know yet how cultural classes emerge from cultural behaviors in various societies where cultural tastes are hierarchical, nor what orders those cultural classes have.

Imagine a society where an initial distribution of hierarchical lifestyles is continuously differentiated. Then, imagine that lower class agents begin to copy lifestyles higher than their own, while higher class agents keep distinguishing themselves from lower class imitators by developing new styles. In this imaginary example, the overall distribution of styles will be constantly shifting toward the higher classes but would not necessarily collapse into several classes of tastes. If higher class agents develop new tastes faster than lower class agents copy their tastes, then the distribution of tastes is likely to become more differentiated than collapsed into categories.

In the real world, we often observe a few distinctive categories of lifestyles. For example, Katz-Gerro (1999), examining leisure activities and musical tastes of Americans, identified fours classes of lifestyles: popular lifestyle preferences, highbrow lifestyle, nature recreation culture, and a youth music type. Alderson, Junisbai, and Heacock (2007), in another study of American cultural consumptions, found three classes: omnivore class, paucivore class, and inactive class. In short, emerging social classes based on cultural tastes in the real world cannot be an trivial consequence of assimilation with higher agents and distinction from lower agents. Our article aims to identify when the distribution of cultural tastes evolves to categorical social classes rather than remains continuously differentiated. To put it another way, our article aims to answer under which circumstances and why people are locked in the class of popular tastes even if they constantly look for tastes differentiated from popular tastes.

In examining this paradox that people run away from the mass to stay in the mass, we view the formation of cultural classes as an *emergent* property: Agents flock around classes of homogeneous tastes without any preference for homophily. We explore conditions for the emergence and then examine how much those emerging categories are independent of initial distributions of lifestyles. We are interested in the independence from initial distribution because a prominent cultural class, involving, for example, popularized luxury consumption, may exist in various societies with their own cultural backgrounds.

2. THEORY AND MODEL OF ASSIMILATION-DISTINCTION

Social agents are status conscious. Laumann and Senter's (1976) comparative study of America and Germany, concerning subjective social distance, showed that competitive status consciousness was pervasive in both American and German communities, especially at the lower status levels. People at the lower status levels subjectively assimilate themselves with others at the higher statuses, driven by "prestige-seeking orientation" (Laumann & Senter, 1976, p. 1313). Consuming styles of higher status can be seen as a major effort for the assimilation.

Then, how would higher status people respond to lower status assimilation? According to Weber (1968), a status group keeps its identity by "social closure." Occupants of a high-status group protect their exclusive benefits from lower status intrusion. The cultural mechanism of Weberian social closure is explicated by

Bourdieu's (1984) "distinction." Bourdieu showed cultural tastes and lifestyles are so subtly reproduced in the family that simple economic successes cannot automatically bring higher class tastes. This is why the distinctive taste of higher classes is an effective mechanism to keep their class boundary from lower class occupants.

Even though Bourdieu (1984) showed how subtle cultural tastes are and how difficult they can be bought by money, people consume luxury culture as if they believe they can eventually buy those cultural subtleties by money, or as if their subjective assimilation by consumption has more importance than their economic status. Note that cultural consumption in the contemporary society is significantly influenced by "social status" (Alderson et al., 2007) and cultural values (Dubois & Duquesne, 1993). Consumer research and economic psychology report that popularized luxury consumption is influenced by social aspirations (Karlsson, Dellgran, Klingander, & Gärling, 2004) and psychological indulgence (e.g., Kivetz & Simonson, 2002).

In short, assimilation to higher styles and distinction from lower ones are two social forces behind cultural consumption. In order to observe how the two forces contribute to the emergence of cultural classes, we propose Cucker-Smale type flocking models (Cucker & Smale, 2007) for assimilation and distinction between agents. A first-order Cucker-Smale model has the form of:

$$\frac{d\zeta_i}{dt} = \frac{\lambda}{N}\sum_{j=1}^{N} a_{ji}(\zeta_j - \zeta_i), \quad t > 0, \quad 1 \le i \le N, \tag{1}$$

where N is the total number of agents in a social system, λ is a constant for the overall strength of mutual influences between agents, and more importantly, a_{ji} is an interaction weight between i and j. ζ_i is agent i's cultural taste or lifestyle for which a higher value for ζ implies the taste of higher status. The rate of change of ζ_i (i.e., $\frac{d\zeta_i}{dt}$) is a weighted sum of its interaction with all other agents js. Specifically for each j, ζ_i will move toward a higher taste (i.e., $\frac{d\zeta_i}{dt} > 0$) when both a_{ji} and $\zeta_j - \zeta_i$ are positive. In other words, for a taste ζ_j higher than ζ_i, a positive interaction weight, a_{ji}, should be assigned if i assimilate itself with j by modifying its taste toward j. By contrast, ζ_i will move away from a lower taste (i.e., $\frac{d\zeta_i}{dt} > 0$) when both a_{ji} and $\zeta_j - \zeta_i$ are negative. In other words, for a taste ζ_j lower than ζ_i, a negative a_{ji} should be assigned if i distinguishes itself from j by modifying its taste repulsively. In sum, the interaction weight in (1) reflects the mechanism of assimilation by positive values for higher tastes and that of distinction by negative values for lower tastes.

Employing ζ for cultural tastes in (1) seemingly assumes that cultural tastes are not only ordered but also quantifiable. Those assumptions, however, are questionable. First, not all the cultural tastes can be ordered according to social statuses. There certainly exist status-neutral tastes. For those tastes, our model is simply irrelevant. In other words, those tastes are not of our research interest because those tastes would not play significant roles in shaping cultural classes emerging from assimilation-distinction.

Second, it is clearly questionable if cultural tastes, though ordered, are quantifiable as a continuous variable ζ. It can be also asked how we can incorporate not a single but a collection of multiple products. We do not have to quantify each taste but conceive a vector of the quantity of n-different cultural products, $\mathbf{z} = \{z_1, z_2, \ldots, z_n\}$. $\mathbf{z}$ here denotes a total lifestyle defined by the consumption amount of n-different products. We define a status position ζ as a function of $\mathbf{z}$, or $\zeta = \zeta(\mathbf{z})$. $\zeta_j - \zeta_i$, then, is not the distance between two tastes or two lifestyles but rather the distance between two statuses linked to two respective lifestyles. In short, our variable ζ for tastes make sense to the degree that social status is quantifiable and linked to those cultural tastes.

Our way of operationalizing lifestyle as a collection of tastes is equivalent to the way Rosen (1974) incorporates quality in economics. Rosen does not quantify quality itself but rather defines quality as a vector of n-quantifiable characteristics of a product and then matches the vector to price. In this method, what matters is not how quality is directly measurable but rather how quality is linked to price. One theoretical advantage of our definition of $\zeta = \zeta(\mathbf{z})$ is its applicability to the omnivore theory (Peterson & Kenn, 1996) of cultural classes. The theory argues that the contemporary high cultural class does not enjoy exclusive tastes but rather has a wider range of cultural tastes than lower classes whose tastes are limited. This omnivore theory can be modeled by (1) when we define $\zeta(\mathbf{z})$ as the variance of $\mathbf{z} = \{z_1, z_2, \ldots, z_n\}$, namely as the range of cultural consumptions. An omnivore higher class will distinguish itself from the rest by adding a new element, z_{n+1}, which increases the range of the cultural tastes of the higher class. To the higher class the new element, z_{n+1}, is not necessarily new to the system. It can be an element of the lower class. In sum, our model can apply not only to "trickle down" cultural contagion from the higher class but also to "trickle round" feedback from the lower class (Trigg, 2001).

Although qualitative research is the most popular choice among sociological scholars in studying the formation of cultural classes, others have developed mathematical models. Kulkarni and Kumar (1989) developed one such model. They examined how socially ordered states are converging to one state or periodically repeating among multiple states. Lower states are attracted to higher ones but higher states do not regress to lower ones, according to their assumption of "progressive" trends. In this respect, our attention to assimilation-distinction (A-D hereafter) is in line with their assumption of progressive trends. Our approach is, however, different from Kulkarni and Kumar's proposal becausue we aim to explain the genesis of those states rather than observing movements between the given number of states. More recent and cumulative efforts have been made by Axelrod (1997) and its model resulted in subsequent extensions such as those that of Klemm, Eguiluz, Toral, and San Miguel (2003), Flache et al. (2006), and Flache and Macy (2011a). In Axelrod's (1997) agent-based model, culturally similar agents become even more similar to each other through homophilous interactions while dissimilar agents become segregated from each other (as when they share no cultural traits). His model shows how cultural categories can emerge from the principle of homophily. Axelrod's cultural categories, however, are not robust against a tiny proportion of cultural mutations (Klemm et al., 2003) and collapses into a single cultural trait. Inspired by this fragility of Axelrod's model, Flache and Macy (2011a; Flache et al., 2006) have been seeking extensions of Axelrod's original model which can keep cultural diversity from noises like cultural mutations and random interactions. Among their solutions are "bounded confidence" (Flache et al., 2006) and "social influence" (Flache & Macy, 2011a). The former sets a threshold for cultural similarity of agents, of which the agents must fulfill in order for interactions to occur. The latter restricts the object of cultural assimilation to modal traits of influential others rather than any trait of interacting others.

Our model shares similar academic interests and modeling features with that of Axelrod's and the subsequent models. Most of all, we share the same fundamental question: How can a collection of stable cultural clusters can emerge in between complete homogeneity and continuous differentiation? We are also concerned with the issue of robust boundaries and will show how cultural categories are stable across different initial conditions and noises. It will also become apparent that "status closure" in our model, which is essentially equivalent to Flache et al.'s (2006) bounded confidence, is a key condition for cultural clustering. More importantly, our model shares important elements of Flache and Macy's (2011b) model. Like Flache and Macy, our model incorporates not only attraction but also repulsion among agents simultaneously (i.e., A-D dynamics). In addition, our Cucker-Smale model assumes essentially the same dynamics of change (see Eq. (2), p. 153 in Flache and Macy, 2011b). In sum, our study significantly builds on the series of works cumulatively done by Flache and Macy.

Our model, however, is different from Axelrod's model and its variations in several aspects. Our theoretical interest lies in the mechanism of attraction and repulsion in a hierarchical order of cultural traits. If one agent is attracted to another who possesses higher cultural traits, the latter will necessarily distance itself from the former. Therefore, our concept of attraction to higher traits departs from the principle of homophily, or mutual attractions between agents sharing similar cultural traits, which has been a basic mechanism of Axelrod's and Flache and Macy's models. Our focus on the hierarchical cultural categories will yield implications for the theories of cultural classes such as Bourdieu's (1984) distinction and Peterson and Kenn's (1996) omnivore theory. In this sense, our theoretical scope is narrower while our methodological scope is broader than that of Axelrod or Flache and Macy.

Our model is specified by a system of differential equations while theirs are specified by a set of behavioral rules for agents. We consequently examine our model by analytic proofs, though aided by numeric simulations, while they mainly perform agent-based simulations. We focus on mathematically deriving general properties rather than exploring simulated outcomes on a range of parameters. We coin such properties as "general" because they do not depend on initial conditions for numeric simulations and, therefore, can apply to different social contexts. For example, one of our theorems can explain why the cultural middle class is inevitably larger than the lower and upper classes in different societies. In all, our analytic approach helps to identify general properties of cultural diversity.

In the following sections, we compare two classes of models of A-D dynamics developed from (1). The first class assumes localized interactions between neighboring statuses, while the second model assumes uniform interactions within a certain boundary of social closure.

2.1. Status-Neighborhood Model

In this part, we consider the interaction weight in (1) to be:

$$a_{ji} := \begin{cases} e^{\frac{-|\zeta_j - \zeta_i|}{k}}, & \zeta_j > \zeta_i, \\ -we^{\frac{-|\zeta_j - \zeta_i|}{kR}}, & \zeta_j \leq \zeta_i. \end{cases} \tag{2}$$

For an agent, i, The interaction weight in (2) represents assimilation in the case of $\zeta_j > \zeta_i$ and distinction in the case of $\zeta_j \leq \zeta_i$. Whether assimilation or distinction, the interaction weight decays exponentially in $|\zeta_j - \zeta_i|$. In other words, influence by tastes of distant statuses, whether higher or lower, decreases very fast and, therefore, agents' tastes are modified by tastes of close neighboring statuses. In this scenario, people do not have information on lifestyles of distant status groups, but rather simply influence and are influenced by adjacent status groups whom they easily interact with and observe in daily life. Presumably, this model may reflect A-D dynamics without cultural mass-marketing. For example, lower class people in this situation may have too little information on luxurious lifestyles to be stimulated by luxury consumption.

The fast-decaying weight may alternatively indicate budget constraints on cultural consumption. If agents of lower lifestyles do not have enough economic resources to purchase much higher status cultural goods, they can simply assimilate their styles to more accessible higher styles of the neighboring class. Given this economic constraint or fast-decaying weight for assimilation, a weight for distinction will also decay fast because agents at higher position must be rarely threatened by far-lower status agents.

A scaling factor, k, depends on the variance of ζ and standardizes $|\zeta_j - \zeta_i|$. Two parameters, w and R, in (2), respectively, denote the strength and range of distinction relative to those of assimilation. The strength and the range of assimilation are set to a unit, 1. $w = 1$ denotes the same strength of distinction as assimilation, $w > 1$ stronger distinction, and $w < 1$ weaker distinction than assimilation. In similar ways, $R = 1$ denotes the same speed of decay, $R > 1$ slower decay (i.e., wider range) of distinction than assimilation, and $R < 1$ narrower range. Note that the strength and the range of distinction are substantively different forces. The range determines how much distant styles affect i's style, while the strength determines how repulsively i's taste moves away from lower tastes within the range of distinction.

2.2. Status-Closure Model

In this part, we consider the interaction weight in (1) to be:

$$a_{ji} := \begin{cases} \mathbf{1}_{|\zeta_i - \zeta_j|} \leq R_a, & \zeta_j > \zeta_i, \\ -w\mathbf{1}_{|\zeta_i - \zeta_j|} \leq R_d, & \zeta_j \leq \zeta_i. \end{cases} \tag{3}$$

The interaction weight in (3) is a nondecaying, uniform weight when agent j is within the range of assimilation, R_a, or that of distinction, R_d. The weight is simply zero if j's tastes are outside those ranges. A parameter w again sets the strength of distinction relative to assimilation. This status-closure model assumes a threshold for social action, which is consistent with the tradition in Granovetter and Soong's (1983) model for collective action. Granovetter and Soong explored implications derived from different levels of threshold across individuals. In comparison, we set the thresholds or ranges for A-D constant across agents and focus on the endogenous processes of class-formation. How to randomize the ranges and what to observe from the randomization will be a separate research topic beyond the scope of this article.

We believe that, in the real world, the force of assimilation to higher status tends to reach farther than does the force of distinction from lower status. In short, $R_a > R_d$ can be a reasonable constraint to (3). Given the strong competitive status consciousness from lower status (Laumann & Senter, 1976) and the mass-marketing of cultural goods to the public in contemporary consumer society, people from most status groups share similar information on what higher status people are supposed to consume. By contrast, people do not care much about lower statuses' tastes unless these are becoming more alike their own. In this sense, assimilating forces, R_a, has a longer range, than distinguishing forces R_d has.

In the case of $R_a = \infty$, even lowest status agents copy high-end tastes. Therefore, we can set $a_{ji} = 1$ for $\zeta_j > \zeta_i$, regardless of the distance between agents i and j. We will mostly study this case of $R_a = \infty$ and later discuss how finite values of R_a make differences, including in cases of $R_a < R_d$.

2.2.1. A Two-Agent System. For the status-closure models (1) and (3), it is informative to examine an extreme case in which the system consists of two agents. This is not a realistic model but will help interpret the case of two cultural classes. First, assume the ranges of both assimilation and distinction are infinite, i.e., $R_a = R_d = \infty$. In this case, the difference, $\zeta = \zeta_2 - \zeta_1$, satisfies a single ordinal differential equation:

$$\frac{d\zeta}{dt} = -\frac{\lambda}{2}(a_{12} + a_{21})\zeta.$$

Note that

$$a_{12} + a_{21} = 1 - w.$$

Hence, we have

$$\frac{d\zeta}{dt} = -\frac{\lambda(1-w)}{2}\zeta,$$

which yields the explicit representation,

$$\zeta(t) = \zeta_0 \exp\left(-\frac{\lambda(1-w)t}{2}\right), \quad t \geq 0,$$

where $\zeta_0 := \zeta_{20} - \zeta_{10}$ is the initial difference of status between two cultural tastes:

Case 1 ($0 \leq w < 1$): Assimilation dominates, and the difference $\zeta := \zeta_2 - \zeta_1$ decays exponentially.

Case 2 ($w = 1$): Assimilation and distinction are balanced. Therefore, the difference remains constant.

Case 3 ($w > 1$): Distinction dominates, and the difference grows exponentially.

We next assume that the range of distinction is finite ($R_d < \infty$) and that the lower agent is out of the range of distinction of the higher agent, namely, $\zeta_0 > R_d$.

Case 1 ($0 \le w < 1$): Assimilation dominates. In the beginning, the higher agent will not move, whereas the lower agent moves toward the taste of the higher agent, and then, after some time, the lower one will be within the distinction range of the higher one. Hence, as in the previous situation, their tastes will be synchronized exponentially fast.

Case 2 ($w = 1$): Again, assimilation and distinction are balanced. Once the lower agent has moved inside the distinction range of the higher agent, the two agents will move with the same velocity, so that the difference does not change.

Case 3 ($w > 1$): Distinction dominates. Once the lower agent has moved inside the distinction range of the higher agent, the difference between the two tastes will grow exponentially until their distance becomes larger than R_d and then shrink again until the distance becomes smaller than R_d. In short, the two agents move close to and away from each other periodically.

In the case for which $R_a = \infty$ and $R_d < \infty$, due to the jump condition of the interaction weight, a_{ji}, for distinction, solutions may not be continuous. In other words, our above discussions for the three cases of $R_d < \infty$ are not mathematically proved yet. The same limitation can be applied to the three cases for a two-category system, which will be discussed in the following section.

For a mathematical investigation of this issue, a_{ji} for a finite R_d could be understood as the limiting case of a regularized version, for example,

$$a_{ji} := \begin{cases} 1, & \zeta_j > \zeta_i, \\ -w, & 0 \le \zeta_i - \zeta_j < R_d \\ \frac{-w}{\epsilon}(R_d + \epsilon - \zeta_i + \zeta_j), & R_d \le \zeta_i - \zeta_j < R_d + \epsilon \\ 0, & R_d + \epsilon \le \zeta_i - \zeta_j, \end{cases} \tag{4}$$

where ϵ is a given positive constant. Sending ϵ to zero would mean equivalence with our status-closure model, and the three cases discussed above can be understood by examining the asymptotic formation arising from this equation. Our numerical simulations, though not presented here, showed robust results for sufficiently small ϵ. Including a rigorous analysis of this issue, we are currently developing a separate paper on the mathematical dynamics of our model.

2.2.2. A Two-Category System. We next assume a system consisting of two categories of lifestyles, with multiple agents for $R_a = \infty$ and $R_d < \infty$. Examining this case will help to identify conditions on w for the formation of two stable cultural classes. Suppose that ζ_1 is a lower taste category with f_1 agents and ζ_2 a higher taste category with f_2 agents, where f_1 and f_2 are the numbers of agents. If a lower taste category is far distant from a higher taste category, then the higher taste category will not move until the lower taste one approaches, within the range of distinction. Therefore, it will be enough to examine when $\zeta_2 - \zeta_1 = R' \le R_d$. Then, by (1) and (3):

$$\frac{d\zeta_1}{dt} = f_2 \frac{\lambda}{f_1 + f_2} \cdot 1 \cdot R'$$

$$\frac{d\zeta_2}{dt} = f_1 \frac{\lambda}{f_1 + f_2} \cdot w \cdot R',$$

we set $\frac{d\zeta_1}{dt} = \frac{d\zeta_2}{dt}$ to obtain a balancing condition:

$$w = \frac{f_2}{f_1}. \tag{5}$$

Then we can interpret how the two categories of tastes behave in terms of the strength of distinction.

Case 1 ($0 \leq w < \frac{f_2}{f_1}$): Assimilation dominates. Like the two-agent system, the two categories will be synchronized exponentially fast.

Case 2 ($w = \frac{f_2}{f_1}$): Assimilation and distinction are balanced. Once the lower class has moved inside the distinction range of the higher class, the two classes will move with the same velocity, so that the distance R' does not change.

Case 3 ($w > \frac{f_2}{f_1}$): Distinction dominates. Once the lower class has moved inside the distinction range of the higher class, the difference between the two classes will grow exponentially until their distance becomes larger than R_d and then shrink again until the distance becomes smaller than R_d. In short, the two classes move close to and away from each other periodically.

In short, the strength of distinction should be equal to or larger than the odds of the higher class versus the lower class (i.e., Cases 2 and 3) in order for a higher class to keep its distance from a lower class. For example, if one third of the total population is clustered in the higher class and the other two thirds in the lower class, then the strength of distinction should be at least 0.5 to keep the cultural identity of the higher class distinct from that of the lower class chasing behind.

Theorem 1. *The larger size of the lower class, the smaller strength of distinction required to have two stable classes in a status-closure system. Specifically, if a status-closure system, involving (1) and (3), observes two categories of tastes not converging to each other, then the strength of distinction satisfies:*

$$w \geq \frac{f_2}{f_1} \tag{6}$$

for f_1, the number of agents in the lower class and f_2, the number of agents in the higher class.

Theorem 1 implies that an effective mechanism of distinction does not necessarily require strongly conscious effort by the higher class occupants. A larger number of lower class agents will simply make higher class couterparts more repulsive from lower ones because higher class agents simply observe more incidences of lower class ones chasing their tastes. It is, however, notable that Theorem 1 applies only to two-class, status-closure systems. If there are more than two classes of tastes, the dynamics of A-D becomes as complicated as those theorized below.

2.2.3. Multicategory System. In this part, we consider the dynamics of multiple categories of lifestyles with multiple agents for the infinite range of assimilation. Suppose that ζ_i is a ith category with f_i agents, where $i = 1, 2, \ldots, N$, so that

$$\zeta_i < \zeta_j \quad \text{if} \quad 1 \leq i < j \leq N.$$

As mentioned in the case of a two-category system, the ith category is always attracted to the higher tastes and, in comparison, the influence by lower tastes is taken into account only for the case in which lower tastes fall within the range of distinction. We denote by $R_{ji} = \zeta_j - \zeta_i$ the distance between the ith category and the jth category and by $\mathbf{1}_{ji} = \mathbf{1}_{\{0 \leq \zeta_j - \zeta_i \leq R\}}$ the characteristic function. Let $1 \leq \beta < \alpha \leq N$. By (1) and (3), we then have

$$\begin{aligned} \frac{d(\zeta_\alpha - \zeta_\beta)}{dt} = \frac{\lambda}{N} \Bigg(& \sum_{j=\alpha+1}^{N} f_j R_{j\alpha} - \sum_{j=\beta+1}^{N} f_j R_{j\beta} \\ & + \sum_{j=1}^{\beta-1} w f_j R_{j\beta} \mathbf{1}_{\beta j} - \sum_{j=1}^{\alpha-1} w f_j R_{j\alpha} \mathbf{1}_{\alpha j} \Bigg). \end{aligned} \tag{7}$$

As seen in (7), the A-D dynamics for multiple categories look more complicated than those for the two-category system. We will just generalize Theorem 1 to a three-category system by setting $\frac{d(\zeta_\alpha - \zeta_\beta)}{dt} = 0$ in (7) in the case $N = 3$, the proof of which is in the Appendix.

Theorem 2. *If a status-closure system, involving (1) and (3), observes three classes keeping distance from one another, then the strength of distinction satisfies*

$$w = \frac{f_3}{f_2 - f_1} = \frac{f_2 + f_3}{f_1} \tag{8}$$

for f_1, the number of agents in the lower class, f_2 in the middle class, and f_3 in the higher class.

From this theorem, we can infer the following propositions that can guide empirical studies of cultural classes. First, we can show $f_1 < f_2 < 2 \cdot f_1$ from (8), the proof of which is provided in the Appendix.

Proposition 1. *If a status-closure system, involving (1) and (3), observes three stable classes, then the middle class is larger than the lower class and smaller than twice the lower class.*

Proposition 1 sets a lower bound and an upper bound for the size of the middle class with respect to the lower class. The lower bound is particularly interesting because it implies that the cultural middle class is always larger than the lower class. The implication is that if cultural lifestyles are not tightly restricted by economic resources, a subgroup of people in the lower economic class have middle-class lifestyles because the economic middle class is normally smaller than its lower class counterpart in a society. Another implication is that people's subjective class identity tends to be higher than is expected by economic class position if class consciousness is significantly affected by consumption and lifestyles.

The empirical fitness of Proposition 1 can be tested. Alderson et al. (2007), following omnivore theory, identified three latent classes of cultural lifestyles by analyzing 2002 General Social Survey. The middle class, *paucivore*, showed the largest size although the higher class, *omnivore*, and the lower class, *inactive*, showed comparable sizes. Thus, Alderson et al.'s finding on the United States seems roughly consistent with our Proposition 1, which can be tested with other countries' cultural lifestyles.

Second, $w = \frac{f_2+f_3}{f_1}$ in (8) and $f_2 > f_1$ in Theorem 1 jointly imply that $w > 1$: the strength of distinction is larger than that of assimilation.

Proposition 2. *If the strength of distinction, w, is equal to or smaller than that of assimilation in a status-closure system, involving (1) and (3), then there cannot emerge a stable system of three classes.*

Proposition 2 suggests that three classes for lifestyles cannot emerge if the strength of distinction is weaker than that of assimilation. Note that distinction weaker than assimilation (i.e., $w < 1$) is possible for two-class systems, as was discussed in Theorem 1. As will be seen later in numerical analysis, the strength of distinction tends to affect the number of classes.

The most striking common property underlying all the theorems and propositions developed above is that those theorems contain no information on the initial distribution of cultural tastes. In other words, if stable cultural classes emerge, then the relative sizes of those classes are independent of the initial tastes profiles. This independence property will become apparent in our simulations below.

3. NUMERICAL SIMULATIONS

In this section, we provide various numerical simulation results for A-D models presented in the previous section. Examining the evolution of tastes over time in either status-neighborhood or status-closure models, we simulate 100 agents; that is, $N = 100$, for 100 or more time points. λ, the overall strength of mutual influence, is set to 1 throughout all of the simulations.

One hundred initial tastes are randomly generated from the three distinct distributions: uniform, normal, and chi-square. The uniform distribution of tastes assumes an even distribution of tastes from the top to the bottom positions of status hierarchy. The normal distribution assumes normative tastes in the middle, and deviant tastes at the both ends, of status ordering. The chi-square distribution of tastes is skewed, which presumably reflects fine-grained differentiation of cultural tastes at higher statuses. The standard deviation of the normal distribution is set to 10, and the uniform and chi-square distributions have variances comparable to those of the normal distribution. A scaling factor, k, in the status-neighborhood models is set to 10 and standardizes the normal distribution.

The strength of distinction relative to assimilation occurs at four different levels, $w \in \{0.5, 1, 3, 5\}$, covering those strengths weaker than, equal to, and stronger than assimilation. The range of distinction involves different values between status-neighborhood and status-closure models. For the former, the range involves two values, $R \in \{0.5, 2\}$, one for a smaller and the other for a larger range than that of assimilation. For the latter, we select two values, $R_d \in \{10, 20\}$, given the infinite range of assimilation and the variance ($=10$) of the normal distribution for initial tastes. For each combination of w and R, we simulate the randomly generated 100 tastes, which follow the behavioral rules in the status-neighborhood model, that is, (1) and (2), or in the status-closure model, that is, (1) and (3). Overlooking changing distributions of tastes over time, we examine how the distribution of the tastes evolves.

3.1. Status-Neighborhood Model

In this part, we fist present results for the local interaction model. Figure 1 presents results from a uniform distribution, given the combination of a low strength and a small range of distinction; that is, $w = 0.5$ and $R = 0.5$. The left panel shows how the initial distribution in a dotted line changes to a distribution in a solid line after 100 times, $t = 100$. The right panel shows continuous changes of density up to $t = 100$, with the blue-to-red spectrum denoting low-to-high density.

The overall distribution does not change much but simply shrinks to a smaller variance, because both low strength and a small range of distinction allow lower tastes to catch up with higher tastes. Increasing either the strength or the range of distinction leads to larger variance of evolving distribution and, thus, results in the finer differentiation of tastes. The strength and the range do not produce qualitatively different patterns of differentiation. This finer differentiation becomes apparent when both the strength and the range are increased in Figure 2. This finding is not counterintuitive: Higher status agents develop new tastes faster than

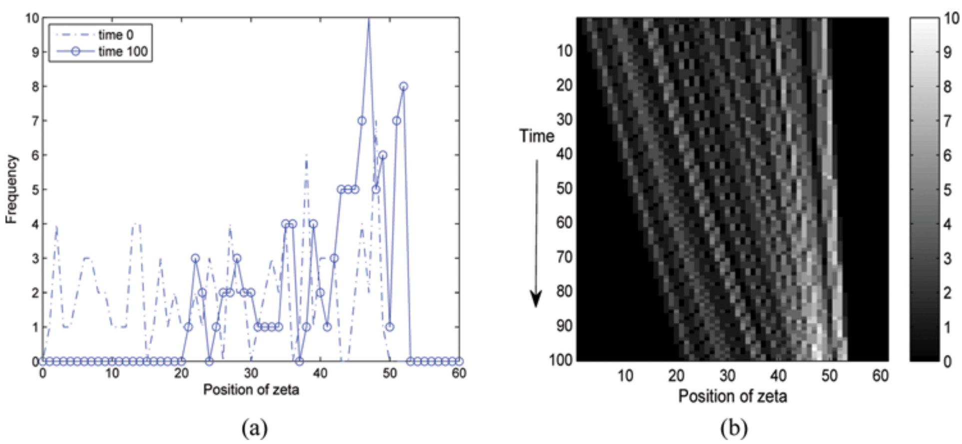


FIGURE 1 Status-neighborhood model with a uniform distribution ($w = 0.5$, $R = 0.5$) (color figure available online).

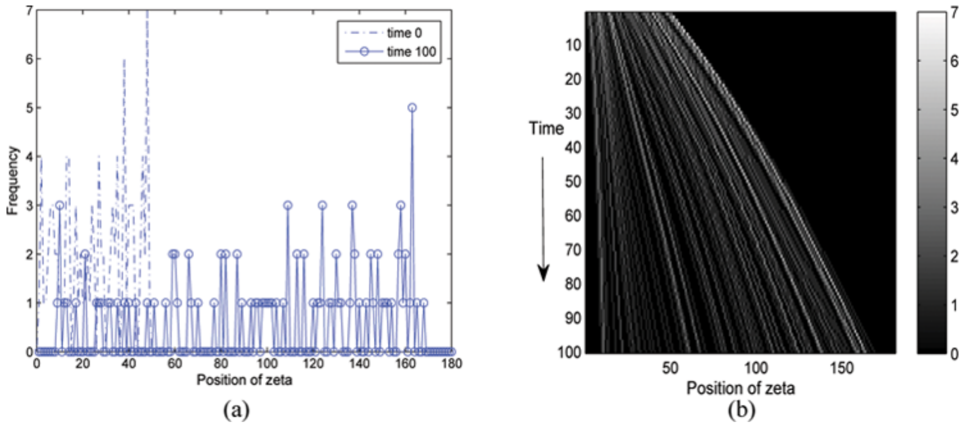


FIGURE 2 Status-neighborhood model with a uniform distribution ($w = 3$, $R = 2$) (color figure available online).

assimilating lower status agents when agents are more strongly stimulated by assimilating lower status agents farther behind.

Whether simulating from a normal distribution or from a chi-square distribution, we do not find much difference from the case of a uniform distribution in terms of the overall shrinking or differentiation of tastes, broken by parameter values of w and R. One notable difference, however, is found in the case of smaller parameter values (Figure 3) compared with those of uniform distribution (Figure 1). The distributions for smaller strength and range are not uniformly shrinking, but are rather merging from behind. Because there are few agents at the lowest end of a normal or chi-square distribution, agents at higher statuses are not so much stimulated by the lowest tastes that they keep their cultural distance from the lowest tastes, given a small range and a weak force of distinction. As a consequence, lifestyles of lower statuses converge upward.

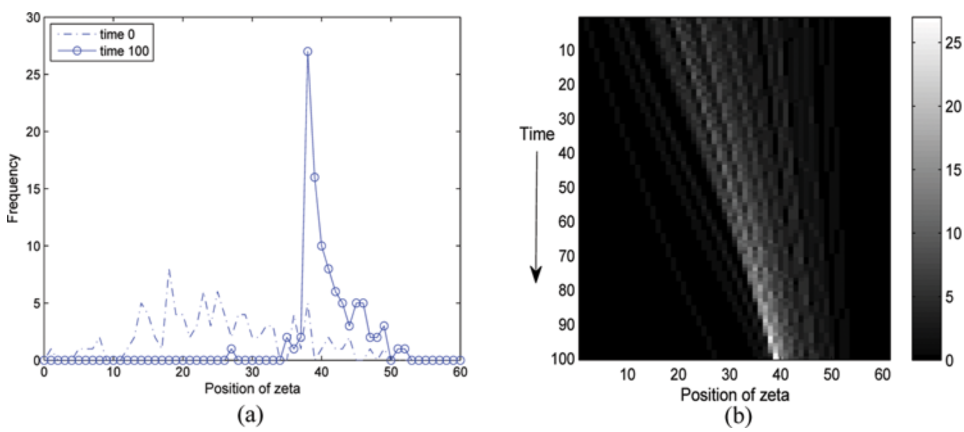


FIGURE 3 Status-neighborhood model with a normal distribution ($w = 0.5$, $R = 0.5$) (color figure available online).

In summary, status-neighborhood models do not yield emerging categories of cultural tastes. To the contrary, they show finer differentiations of tastes as distinction becomes stronger and wider. We will observe a fine-grained, full spectrum of hierarchical lifestyles when people conspicuously consume cultural goods, driven by a strong desire to distinguish themselves from those of a class, lower status.

Also, nonuniform distributions, in the case of weak and narrow distinction, begin collapsing from behind; therefore, we observe no taste at the lower level, suddenly a popular taste in the middle, and continuously decreasing frequencies at higher levels of taste. In this case, we can clearly observe a lower bound of cultural tastes, although not a categorical class, which is not characterized by lower class life styles but rather by the massive consumption of the middle-class life style. Regardless of their economic resources, agents in this social system never consume cultural goods under a certain threshold.

3.2. Status-Closure Model

In status-neighborhood models, we did not witness emerging categories of cultural classes. An emergence of lower bound taste was notable though. Simulating with status-closure models, in this section we will observe emerging classes of tastes.

Figure 4 shows results from a uniform distribution in the case of low strength ($w = 0.5$) and moderate range of distinction ($R_d = 10$). Tastes are converging to two classes and the emergence of two classes is consistent across different initial distributions. Figure 5 is from a chi-square distribution with the same parameter values. In both figures, the frequencies of the two categories roughly matches the two-to-one ratio, which was already predicted by Theorem 1. Robust patterns across different initial distributions were also implied in the theorem.

In Figure 6, both w and R_d are doubled. The increase of w to 1 results in the larger size of the higher class whose frequency is comparable to that of the lower class. The association between the stronger distinction, namely larger w, and more

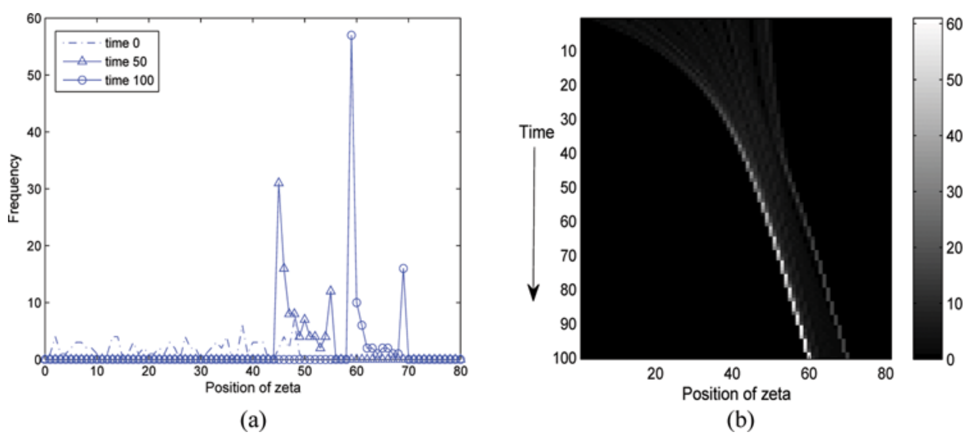
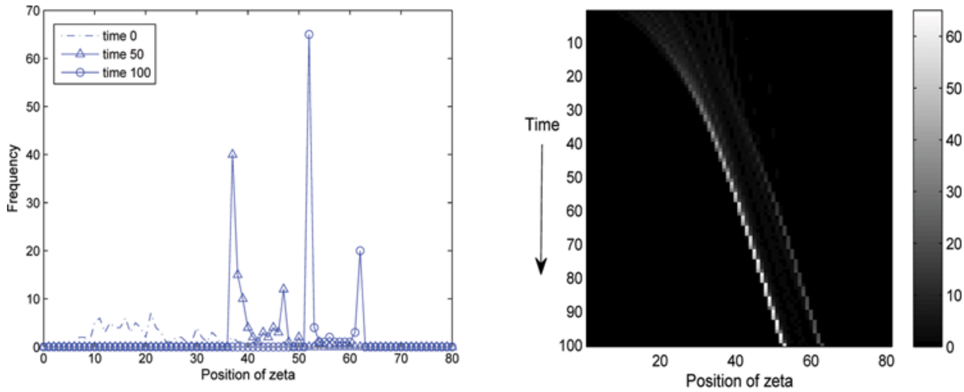


FIGURE 4 Status-closure model with a uniform distribution ($w = 0.5$, $R_d = 10$, $R_a = \infty$) (color figure available online).

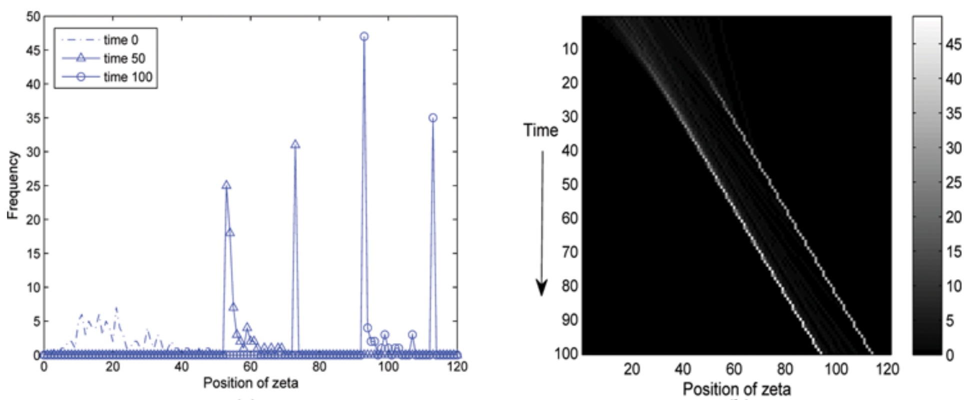


(a) (b)

FIGURE 5 Status-closure model with a chi-square distribution ($w = 0.5$, $R_d = 10$, $R_a = \infty$) (color figure available online).

agents in the high class was predicted in Theorem 1. The increased R_d results in the increased distance between the two classes. The distance between the two classes is roughly equal to R_d in all of the figures for the status-closure models.

Figure 7 triples the strength of the distinction w from the previous figure. With $w = 3$, we observe a three-class system, as was suggested by Proposition 2. The middle class in the figure is taller than the lower class, as was suggested by Proposition 1. Presumably, these observations remain the same for other initial distributions of tastes, although not shown here. The middle-class lifestyle forms the majority in a society, regardless of the initial distribution of lifestyles across social statuses. We occasionally observe four categories of tastes at $w = 5$ although the results are not presented here. The number of classes increasing with stronger distinction is the same pattern across different initial distributions.



(a) (b)

FIGURE 6 Status-closure model with a chi-square distribution ($w = 1$, $R_d = 20$, $R_a = \infty$) (color figure available online).

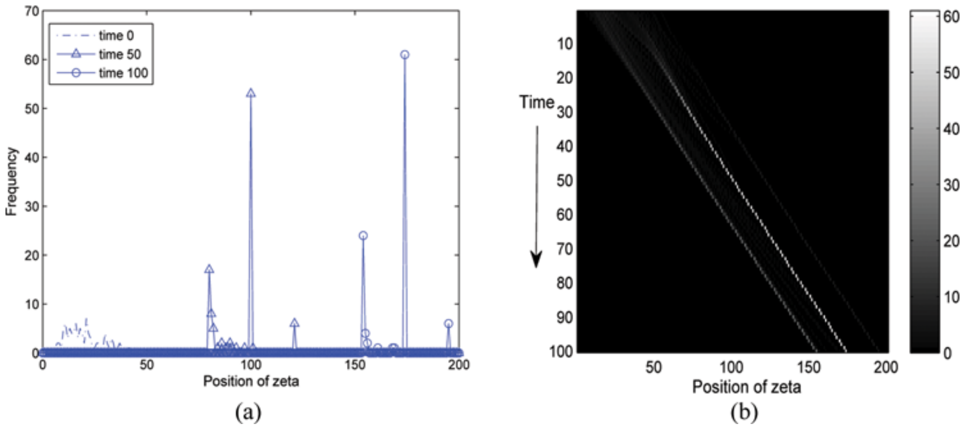


FIGURE 7 Status-closure model with a chi-square distribution ($w=3$, $R_d=20$, $R_a=\infty$) (color figure available online).

Note that status-closure models assume the infinite range of assimilation, which reflects mass marketing of high-class tastes and highly competitive status consciousness, and a discrete range, R_d, of distinction rather than continuous decay, which reflects conscious social closure. Under these assumptions, we observe a few categories of cultural tastes emerging from continuous distributions of categories. This is in sharp contrast with status-neighborhood models. We also observe that those classes do not depend on the initial distributions of cultural tastes. In sum, we observe homogeneous, easily identifiable lifestyle classes emerging across different societies.

3.3. Finite Range of Assimilation in the Status-Closure Model

Until now, we have examined and theorized class-formation in the status-closure model in the case of the infinite range of assimilation (i.e., $R_a=\infty$). In this section, we simulate and compare the cases of finite R_a to those of the infinite R_a. For these simulations, we fix the range of distinction at 10 ($R_d=10$) and try three values for that of assimilation, $R_a \in \{5, 10, 20\}$: smaller than, equal to, and larger than the range of distinction.

When the range of assimilation is smaller than or equal to that of distinction, simulations results are in between categorization and total differentiation. First, Figure 8 shows results from a range of assimilation smaller than that of distinction. Even for the weak strength of distinction (i.e., $w=0.5$), we can observe emerging categories with distances roughly equal to the range of distinction, 10. For larger values of w, these emerging categories become apparent (results not shown here). However, too many categories emerge to be called social classes. Instead, they appear to be differentiated status groups in a continuum.

Second, when the range of assimilation becomes equal to that of distinction (i.e., $R_a=10$), then emerging categories become less clear (results not shown here). As the range of assimilation becomes comparable to that of distinction, lower class

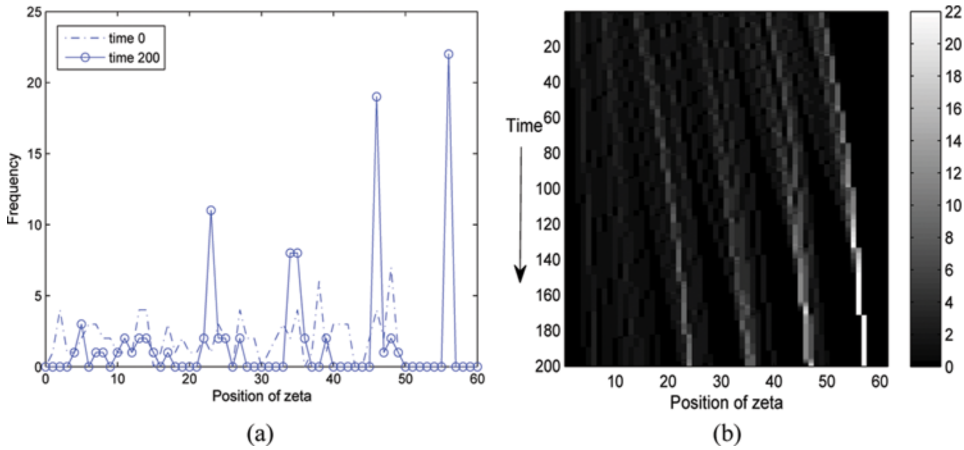


FIGURE 8 Status-closure model with a finite assimilation range and uniform distribution ($w = 0.5$, $R_d = 10$, $R_a = 5$) (color figure available online).

agents do not allow higher ones to make clear distinctions at will. When the range of assimilation becomes even larger than that of distinction, then numerous categories merge, resulting in fewer categories. Figure 9 for a large range of assimilation $R_a = 20$ looks very similar to that for the infinite range of assimilation. For $w = 0.5$, we clearly observe, two cultural classes whose ratio is roughly equal to that predicted by Theorem 1.

However, notable differences exist between the cultural classes emerging from the finite and the infinite ranges of assimilation. A group of cultural outcasts emerges, whose tastes lag far behind the two major classes. The distance between this outcast group and the major classes is much larger than the range of assimilation, 20, and the cultural outcast relinquishes assimilation to higher classes. Their lifestyles

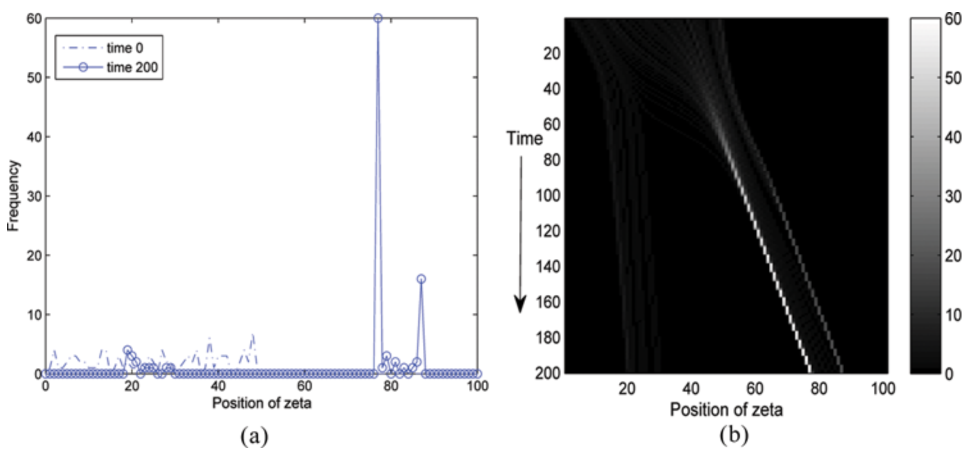


FIGURE 9 Status-closure model with a finite assimilation range and uniform distribution ($w = 0.5$, $R_d = 10$, $R_a = 20$) (color figure available online).

were initially at a continuum with those of higher statuses but have been increasingly diverging from the majority, as is clearly illustrated density changes in the right panel in Figure 9.

Flache et al. (2006) noticed that a threshold for homophilous interactions, or "bounded confidence," is not sufficient for robust cultural diversity and proposed to introduce repulsive interactions as a possible solution for stable diversity. Based on Bourdieu's theory of distinction in a hierarchical order of cultural traits, our model naturally incorporated repulsive interactions as well as attractive ones and subsequently employed two thresholds for those two types of interactions. A threshold for repulsive interactions arguably plays a more important role than that for attractive ones because the former sets the distance between cultural classes. In addition, we found in this section that the relative size between the two thresholds has impacts on the formation of cultural classes. However, in line with the observation by Flache and Macy, these thresholds are not sufficient to describe general properties for stable cultural classes. Those properties are rather determined by w, the relative strength between attractive and repulsive interactions, which was formally expressed in the two theorems.

3.4. Robustness of Cultural Clusters

How robust is the formation of cultural clusters we have observed in status-closure models? We already showed their robustness to the initial distributions of cultural tastes. We derived mathematical propositions that are independent of initial distributions and also observed emerging cultural categories in status-closure models across three different initial distributions. In addition to this robustness to initial distributions, our propositions and observations are also independent of the number of agents, namely, the size of a social system. In sum, we focused on finding general properties that are applicable to various societies of different resource distributions and sizes.

Other than these kinds of robustness to initial conditions, we need to check robustness to various kinds of noises to the model. One important type of noise could be stochastic noises to interaction patterns between agents. Adding a stochastic term to an interaction weight a_{ji} in (1) or (3) will generate these noises. As a quick and dirty check, we randomly increased the range of distinction by the probability of 0.01 from 10 to 15 given the initial conditions for Figure 4. These random noises allowed occasional interactions with agents outside the normal range, 10, but did not produce any notable difference from Figure 4. (Results can be provided by authors upon request.) Decreasing the range to 5 did not make any difference either. It will be worth to examine what if the distinction range really varies across agents as is sampled from a normal distribution. This modification, however, will not be merely adding noises to, but making a substantive elaboration to the model, which is beyond our current scope. In this article, we focus on discovering and interpreting general properties of our baseline model for cultural class formation.

Stochastic noises also can affect influence patterns, not interaction patterns. Adding a stochastic term to the right side of Eq. (1) will generate these noises. The robustness of the Cucker-Smale model to these noises is being lively studied in mathematics. For example, the Cucker-Smale system is proved to keep flocking in the presence of constant-strength noises (Ha, Lee, & Levy, 2009) and multiplicative noises (Ahn & Ha, 2010). It is not examined yet if those proofs hold intact for our status-closure model, and if not, what are sufficient conditions for robust flocking? We conclude that such questions are left open for future research topics requiring full attention.

4. DISCUSSION

In this study, we raised the issue of cultural classes beyond one of cultural studies to that of mathematical formalization and explored both analytic and numerical results. For a formalization, we started from the first-order Cucker-Smale-type model, which assumes mutual assimilation and results in the emergence of one class under certain conditions. We extended the model by assuming both assimilation and distinction (A-D), depending on the order between social statuses of cultural tastes and lifestyles. We compared two different interactions across tastes: interactions between adjacent statuses (status-neighborhood model), and long-reaching interactions between distant statuses within a boundary (status-closure model). In examining those models, we actively searched for analytic theorems and interpreted simulation results based on these.

Status-neighborhood models confirm that cultural classes cannot trivially emerge from A-D dynamics. Simulations showed patterns of either collapsing from behind or differentiating in a continuum. In comparison, status-closure models yielded the emergence of a few cultural classes through on A-D mechanism. Note that status-closure models assume a cognitive closure of cultural status in the agents' mind. Agents will move away from lower status styles if they approach within the closure boundary. In this respect, the emergence of categorized tastes as classes is not surprising. However, not all status-closure models witnessed emerging cultural classes. If the cognitive boundary for assimilation is smaller than or equal to that for distinction (i.e., $R_a \leq R_d$ in our model), then cultural classes do not emerge. More importantly, the number of classes and relations between the sizes of those classes cannot be trivially determined. Our analytical theorems formulate some of these emergent properties.

The most striking property is the relative sizes between emergent classes at a stable status independent of the initial distribution of tastes. This implies the emergence of homogeneous cultural classes from heterogeneous initial distributions of cultural tastes. Another notable property is the emergence of the cultural middle class as the majority class. Considering that the economic middle class is usually smaller than the lower class, this property implies that a considerable proportion of people belong to the economic lower class and the cultural middle class simultaneously. These people may be characterized by overconsumption, as their class consciousness, if strongly shaped by lifestyle, is higher than their objective, economic class. The prominence of overconsuming people with seemingly mistaken class consciousness might be inevitable in status-competitive, consumer societies to the extent that one's cultural consumption is not strongly restricted by budgets at hand but significantly affected by striving to be "on trend" (Dubois & Duquesne, 1993). We also confirmed that major findings under the assumption of the infinite range of assimilation also hold true for finite ranges of assimilation which are considerably larger than those of distinction. We, however, observed the emergence of a cultural outcast group when the range of, or say aspiration for, assimilation is finite.

Our model has a few limitations. First, our theorems and subsequent propositions do not hold if cultural consumptions are tightly restricted by economic budget constraints. Those general theorems and propositions were analytically derived from the status-closure model whereas economic constraint was reflected in the interaction weight of the status-neighborhood model. The behaviors of the status-neighborhood model were briefly explored in this article, and under specific conditions (see Figure 3), we were able to observe the emergence of one massive lifestyle as a lower bound in combination with continuously differentiated higher status styles.

Our status-closure model and derived properties are valid to the extent that cultural classes are decoupled from economic resource distributions. It has been observed that spending on cultural goods is less ruled by economic budget constraints than by necessities, often justified by precommitment to indulgence (Kivetz & Simonson, 2002), and significantly affected by cultural attitudes when controlling for income (Dubois & Duquesne, 1993). People from all status positions are constantly stimulated and encouraged to consume symbolic goods in a consumer society. As is implied by the omnivore theory (Peterson & Kenn, 1996), it is often not the high price but rather the breadth of cultural consumption that matters to those copying high-class lifestyles. In other words, what matters is not monetary resources at hand but information costs to search for relevant cultural goods. Marketing efforts and consumer magazines, which are dedicated to informing mass consumers of highbrow styles, reduce the information cost to the public and enable their assimilation. In general, the relationship between economic and cultural classes is intriguing as did Bourdieu (1984) elegantly show how they structure each other to reproduce the entire social class system. Modeling the coevolution of socioeconomic and cultural structures will be a challenging task (see Klüver & Schmidt, 2003 as a recent effort).

Another limitation is that we have not yet explored the properties of more than three classes in the status-closure model. We proved a negative association between the strength of distinction and the relative size of the lower class in a stable two-class system (see Theorem 1). As the strength increases, a stable status takes the form of three classes satisfying Theorem 2. Implications from this theorem were derived in the subsequent two propositions but not totally exploited yet. Finding more implications in Theorem 2 and exploring stable statuses with more than three classes will clarify how dynamics between three or more classes are qualitatively different from two-class dynamics (Simmel, 1964).

The Cucker-Smale model is recently proposed and has been widely adopted by various disciplines (e.g., Ha & Levy, 2009; Park, Kim, & Ha, 2010; Perea, Elosegui, & Gòmez, 2009) but not yet introduced to sociology, despite of its rich social implications (see Flache & Macy 2011b, as a notable exception). In principle, the model is a simple but dynamic characterization of interactions and mutual influences among social actors. Our effort in this article could be a first step to incorporate this model into sociology with analytic proofs. Our effort, however, was not a mere application, but rather an extension of the model in three respects. First, we modeled not only attraction (i.e., assimilation) but also repulsion (i.e., distinction) depending on relative positions in a hierarchical distribution. Second, the interaction weight,

a_{ij}, is not externally given but endogenously determined by distance between the outcome variable, ζ. Third, we actively derived analytic theorems and propositions having social implications. These theorems and propositions may provide a firm basis for empirical hypotheses and tests, which is often absent in simulation-based findings.

The simplicity and flexibility of the Cucker-Smale model has great potential to apply to various fields of sociology. While this article applied the model to the formation of cultural classes, another promising area of research could be with the dynamics of opinion formation (e.g., Friedkin & Johnsen, 1999; Baldassarri & Bearman, 2007). The Cucker-Smale model can specify how agents influence one another in both positive and negative ways on multiple dimensions of political and social issues and, therefore, may clarify when and how either opinion consensus or polarization occurs. It is certainly true that both cultural formation and opinion dynamics have been actively examined by agent-based simulations (e.g., Axelrod, 1997; Flache et al., 2006; Baldassarri & Bearman, 2007). A significant advantage of the Cucker-Smale model, however, is its mathematical nature, which allows generation of general properties, as we have proved theorems on the relative sizes of cultural classes. Generating those theorems independent of initial conditions can provide a firmer ground to simulation-based findings and increase our confidence in the computational model of the society.

FUNDING

J.-H. Kang's work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2010-332-B00228). S.-Y. Ha's work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No.2009-0083521). K. Kang's work was partially supported by NRF-2009-0088692.

REFERENCES

Ahn, S. M., & Ha, S.-Y. (2010). Stochastic floccking dynamics of the Cucker-Smale model with multiplicative white noises. *Journal of Mathematical Physics*, *51*, Article 103301.

Alderson, A. S., Junisbai, A., & Heacock, I. (2007). Social status and cultural consumption in the United States. *Poetics*, *35*, 191–212.

Axelrod, R. (1997). The dissemination of culture. *Journal of Conflict Resolution*, *41*, 203–226.

Baldassarri, D., & Bearman, P. (2007). Dynamics of political polarization. *American Sociological Review*, *72*, 784–811.

Bourdieu, P. (1984). *Distinction: A social critique of the judgment of taste*. Cambridge, MA: Harvard University Press.

Cucker, F., & Smale, S. (2007). Emergent behavior in flocks. *IEEE Transactions on Automatic Control*, *52*, 852–862.

Dubois, B., & Duquesne, P. (1993). The market for luxury goods: Income versus culture. *European Journal of Marketing*, *27*, 35–44.

Flache, A., & Macy, M. W. (2011a). Local convergence and global diversity: From interpersonal to social influence. *Journal of Conflict Resolution*, *55*, 968–993.

Flache, A., & Macy, M. W. (2011b). Small worlds and cultural polarization. *Journal of Mathematical Sociology*, *35*, 146–176.

Flache, A., Macy, M. W., & Takács, K. (2006). What sustains stable cultural diversity and what undermines it? Axelrod and beyond. In S. Takahashi, D. Sallach, & J. Rouchier (Eds.), *Proceedings of the First World Congress on Social Simulation, Kyoto, Japan* (Vol. 2, pp. 9–16). Tokyo, Japan: Springer.

Friedkin, N. E., & Johnsen, E. C. (1999). Social influence networks and opinion change. *Advances in Group Processes*, *16*, 1–29.

Granovetter, M., & Soong, R. (1983). Threshold models of diffusion and collective behavior. *Journal of Mathematical Sociology*, *9*, 165–179.

Ha, S.-Y., & Levy, D. (2009). Particle, kinetic and fluid models for phototaxis. *Discrete and Continuous Dynamical System B*, *12*, 77–108.

Ha, S.-Y., Lee, K., & Levy, D. (2009). Emergence of time-asymptotic flocking in a stochastic Cucker-Smale system. *Communications in Mathematical Sciences*, *7*, 453–469.

Hermans, H., & Kemper, H. (1998). Moving cultures. *American Psychologist*, *53*, 1111–1120.

Kapur, B. K. (2005). Can faster income growth reduce well-being? *Social Choice and Welfare*, *25*, 155–171.

Karlsson, N., Dellgran, P., Klingander, B., & Gärling, T. (2004). Household consumption: Influences of aspiration level, social comparison, and money management. *Journal of Economic Psychology*, *25*, 753–769.

Katz-Gerro, T. (1999). Cultural consumption and social stratification: Leisure activities, musical tastes, and social location. *Sociological Perspectives*, *42*, 627–646.

Kivetz, R., & Simonson, I. (2002). Self-control for the righteous: Toward a theory of precommitment to indulgence. *Journal of Consumer Research*, *29*, 199–217.

Klemm, K., Eguiluz, V. M., Toral, R., & San Miguel, M. (2003). Global culture: A noise induced transition in finite systems. *Physical Review E*, *67*, Article 045101(R).

Klüver, J., & Schmidt, J. (2003). Historical evolution and mathematical models: A sociocultural algorithm. *Journal of Mathematical Sociology*, *27*, 53–83.

Kulkarni, M. S., & Kumar, A. (1989). Interactive Markovian models of "progressive" trends. *Journal of Mathematical Sociology*, *14*, 45–65.

Laumann, E. O., & Senter, R. (1976). Subjective social distance, occupational stratification, and forms of status and class construction: A cross-national replication and extension. *American Journal of Sociology*, *81*, 1304–1338.

Lewis, O. (1968). *Study of slum culture: Backgrounds for La Vida*. New York, NY: Random House.

Macy, M., Kitts, J., Flache, A., & Benard, S. (2003). Polarization in dynamic networks: A Hopfield model of emergent structure. In R. Breiger & K. Carley (Eds.), *Dynamic social network modelling and analysis: Workshop summary and papers* (pp. 162–173). Washington, DC: The National Academies Press.

Park, J., Kim, H. J., & Ha, S.-Y. (2010). Cucker-Smale flocking with inter-particle bonding forces. *IEEE Transations on Automatic Control*, *55*, 2617–2623.

Perea, L., Elosegui, P., & Gómez, G. (2009). Extension of the Cucker-Smale control law to space flight formation. *Journal of Guidance, Control, and Dynamics*, *32*, 527–537.

Peterson, R., & Kenn, R. M. (1996). Changing highbrow taste: From snob to omnivore. *American Sociological Review*, *61*, 900–907.

Rosen, S. (1974). Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, *82*, 34–55.

Simmel, G. (1964). Triad. In K. Wolff (Ed.), *The sociology of Georg Simmel* (pp. 145–169). New York, NY: Free Press.

Trigg, A. B. (2001). Veblen, Bourdieu, and conspicuous consumption. *Journal of Economic Issues*, *35*, 99–115.

Twitchell, J. B. (2002). *Living it up: America's love affair with luxury*. New York, NY: Columbia University Press.

Veblen, T. (1899). *The theory of leisure class*. New York, NY: Macmillan.

Wilson, W. J. (1987). *The truly disadvantaged*. Chicago, IL: The University of Chicago Press.
Wong, N. Y., & Ahuvia, A. C. (1998). Personal taste and family face: Luxury consumption in confucian and western societies. *Psychology and Marketing*, *15*, 423–441.

APPENDIX: PROOFS OF THEOREM 2 AND PROPOSITION 1

In this section, we present the proofs of Theorem 2 and Proposition 1 in separate subsections.

Proof of Theorem 2. We set R to be

$$R := R_{21} = R_{32}.$$

Then, we have

$$R_{31} = 2\,R = -R_{13}.$$

We again set

$$\frac{d(\zeta_2 - \zeta_1)}{dt} = \frac{d(\zeta_3 - \zeta_2)}{dt} = 0$$

to find

$$\begin{aligned} \frac{d(\zeta_2 - \zeta_1)}{dt} &= \frac{\lambda}{f_1 + f_2 + f_3}(Rf_3 - Rf_2 - 2Rf_3 + wRf_1) = 0, \\ \frac{d(\zeta_3 - \zeta_2)}{dt} &= \frac{\lambda}{f_1 + f_2 + f_3}(wRf_2 - wRf_1 - Rf_3) = 0. \end{aligned}$$

Then, the above equations yield

$$w = \frac{f_3 + f_2}{f_1} = \frac{f_3}{f_2 - f_1}. \qquad \square$$

Proof of Proposition 1. Let $f_2 = \alpha f_1$. Then, it follows from $w = \frac{f_3}{f_2 - f_1}$ in (8) that

$$w = \frac{f_3}{(\alpha - 1)f_1} > 0.$$

Hence, we have

$$\alpha > 1.$$

On the other hand, we use $\frac{f_3 + f_2}{f_1} = \frac{f_3}{f_2 - f_1}$ in (8) to find

$$(2 - \alpha)f_1 f_3 = \alpha(\alpha - 1)f_1^2.$$

Since $f_1 > 0$ and $f_3 > 0$, we can also obtain

$$f_3 = \frac{\alpha(\alpha - 1)}{2 - \alpha} f_1 = \frac{\alpha - 1}{2 - \alpha} f_2 > 0.$$

Therefore, we obtain $2 - \alpha > 0$, and hence, we have

$$1 < \alpha < 2,$$

as claimed in Proposition 1. $\square$