Modelling the Hidden Flexibility of Clustered Unit Commitment: Addendum

This file contains the full description of the optimization models described in [1].

I. Clustered UC Formulation

A. Nomenclature

Indexes and Sets:

$g \in \mathcal{G}$ Generating units within a cluster, running from 1 to G

$t \in \mathcal{T}$ Hourly periods, running from 1 to T hours

Individual Unit Parameters:

RD Ramp-down capability [MW/h]

RU Ramp-up capability [MW/h]

SD Shutdown ramping capability [MW/h]

SU Startup ramping capability [MW/h]

TD Minimum down time [h]

TU Minimum up time [h]

Continuous Non-negative Variables:

$\tilde{p}_{gt}$ Power output above minimum output of unit g [MWh]

$\tilde{r}^+_{gt}, \tilde{r}^-_{gt}$ Reserve up/down of unit g [MWh]

p_t Cluster power output above minimum output [MWh]

r^+_t, r^-_t Cluster secondary reserve up/down [MWh]

$\widehat{p}_t$ Total cluster power output [MWh]

Binary/Integer Variables:

$\tilde{u}_{gt}$ Binary variable which is equal to 1 if the unit g is producing above minimum output and 0 otherwise

u_t Integer variable indicating the number of units producing above minimum output

y_t Integer variable indicating how many units start up

z_t Integer variable indicating how many units shut down

B. Classic Clustered UC Formulation

This section shows the classic CUC formulation, which is simply a scaled 1-unit formulation. Now the variables u_t, y_t, z_t take integer values $\{1, 2, \ldots, G\}$ instead of binary values. For the sake of brevity, here we show the formulation for one cluster, hence we drop the index for different clusters. The objective function minimizes the total system operational cost (i.e., fixed and variable generation cost, startup/shutdown cost, renewable curtailment cost) and is also subject to system-wide constraints, such as demand balance, transmission limits, and total up/down reserve requirements, see Section II. The commitment, startup/shutdown logic and the minimum up/down times are guaranteed with

$$u_t - u_{t-1} = y_t - z_t \quad \forall t \tag{1}$$

$$\sum_{i=t-TU+1}^{t} y_i \le u_t \quad \forall t \in [TU, T] \tag{2}$$

$$\sum_{i=t-TD+1}^{t} z_i \le G - u_t \quad \forall t \in [TD, T]. \tag{3}$$

The following constraint ensures that the clustered unit operates within its power capacity limits for the case $TU \geq 2$:

$$p_t + r^+_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) y_t - \left(\overline{P} - SD\right) z_{t+1} \quad \forall t \tag{4}$$

and when $TU = 1$, the following constraints should be used instead:

$$p_t + r^+_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) z_{t+1} - \max\left(SD - SU, 0\right) y_t \quad \forall t \tag{5}$$

$$p_t + r^+_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) y_t - \max\left(SU - SD, 0\right) z_{t+1} \quad \forall t. \tag{6}$$

The minimum output and the total energy production are obtained as follows:

$$p_t - r^-_t \ge 0 \quad \forall t \tag{7}$$

$$\widehat{p}_t = \underline{P} u_t + p_t \quad \forall t. \tag{8}$$

Finally, the ramping limits are also written as a scaled version of the 1-unit constraint:

$$\left(p_t + r^+_t\right) - p_{t-1} \le RU \cdot u_t \quad \forall t \tag{9}$$

$$-\left(p_t - r^-_t\right) + p_{t-1} \le RD \cdot u_{t-1} \quad \forall t. \tag{10}$$

C. Proposed Individual Unit's Constraints for CUC

Constraints (11)-(13) aim to order the commitment of the units in such a way that unit 1 is committed first and successively unit G is committed last.

$$\tilde{u}_{1t} \le 1 \quad \forall t \tag{11}$$

$$\tilde{u}_{g+1,t} \le \tilde{u}_{gt} \quad \forall g \in [1, G), t \tag{12}$$

$$\tilde{u}_{Gt} \ge 0 \quad \forall t. \tag{13}$$

The production of a single unit is limited by

$$\tilde{p}_{gt} + \tilde{r}^+_{gt} \le \left(\overline{P} - \underline{P}\right) \tilde{u}_{gt} \quad \forall g, t \tag{14}$$

$$\tilde{p}_{gt} - \tilde{r}^-_{gt} \ge 0 \quad \forall g, t. \tag{15}$$

The total commitment, up reserve, down reserve, and production (above $\underline{P}$) of the cluster are given by

$$u_t = \sum_{g \in \mathcal{G}} \tilde{u}_{gt} \quad \forall t \tag{16}$$

$$r^+_t = \sum_{g \in \mathcal{G}} \tilde{r}^+_{gt} \quad \forall t \tag{17}$$

$$r^-_t = \sum_{g \in \mathcal{G}} \tilde{r}^-_{gt} \quad \forall t \tag{18}$$

$$p_t = \sum_{g \in \mathcal{G}} \tilde{p}_{gt} \quad \forall t. \tag{19}$$

To correctly model the startup and shutdown unit's capabilities, (14) should be replaced by (20) and (21) or by (22): for the case $TU \geq 2$, capacity limits are ensured with

$$\tilde{p}_{gt} + \tilde{r}^{+}_{gt} \leq \left(SU - \underline{P}\right) \tilde{u}_{gt} + \left(\overline{P} - SU\right) \tilde{u}_{g,t-1} \quad \forall g, t \quad (20)$$

$$\tilde{p}_{gt} + \tilde{r}^{+}_{gt} \leq \left(SD - \underline{P}\right) \tilde{u}_{gt} + \left(\overline{P} - SD\right) \tilde{u}_{g,t+1} \quad \forall g, t \in [1, T) \quad (21)$$

and when $TU = 1$, the following constraints should be used:

$$\tilde{p}_{gt} + \tilde{r}^{+}_{gt} \leq \left(SU - \overline{P} + SD - \underline{P}\right) \tilde{u}_{gt} + \left(\overline{P} - SU\right) \tilde{u}_{g,t-1} + \left(\overline{P} - SD\right) \tilde{u}_{g,t+1} \quad \forall g, t \in [1, T). \quad (22)$$

It is important to highlight that (20)-(22) overcome the startup/shutdown capability overestimation.

The ramping limits for individual units are guaranteed with

$$\left(\tilde{p}_{gt} + \tilde{r}^{+}_{gt}\right) - \tilde{p}_{g,t-1} \leq RU \cdot \tilde{u}_{gt} \quad \forall g, t \quad (23)$$

$$-\left(\tilde{p}_{gt} - \tilde{r}^{-}_{gt}\right) + \tilde{p}_{g,t-1} \leq RD \cdot \tilde{u}_{g,t-1} \quad \forall g, t. \quad (24)$$

II. UC formulation

This section presents the UC formulation, based on [2] and [3], which applies for both the individual UC and the CUC. For the case of the individual UC, the unit's index $j \in \mathcal{J}$ stands for each unit, and for the case of CUC, j stands for each cluster. Here, we introduce additional notations when needed beyond those presented in Section I.

The UC seeks to minimize all system costs [2], [4], [5]:

$$\min \sum_{t \in \mathcal{T}} \left(\sum_{j \in \mathcal{J}} \left[C^{\mathrm{NL}}_{j} u_{jt} + C^{\mathrm{LV}}_{j} \left(\underline{P}_{j} u_{jt} + p_{jt}\right) + C^{\mathrm{SU}}_{j} y_{jt} + C^{\mathrm{SD}}_{j} z_{jt} + C^{+}_{j} r^{+}_{jt} + C^{-}_{j} r^{-}_{jt} \right] + \sum_{b \in \mathcal{B}^{\mathrm{W}}} C^{\mathrm{C}}_{b} w^{\mathrm{C}}_{bt} \right) \quad (25)$$

where the parameter C^{LV}_{j} is the unit's linear variable production cost, C^{NL}_{j} is the non-load cost, C^{SD}_{j}/C^{SU}_{j} are the shutdown/startup costs, C^{+}_{j}/C^{-}_{j} are the costs of providing up/down reserves, C^{C}_{b} is the penalization of wind curtailment w^{C}_{bt} at bus b, and $\mathcal{B}^{\mathrm{W}}$ is the subset of buses in $\mathcal{B}$ with wind power injection.

The demand-balance and up/down reserve requirements are satisfied with

$$\sum_{j \in \mathcal{J}} \left(\underline{P}_{j} u_{jt} + p_{jt}\right) = \sum_{b \in \mathcal{B}} D_{bt} - \sum_{b \in \mathcal{B}^{\mathrm{W}}} \left(P^{\mathrm{W}}_{bt} - w^{\mathrm{C}}_{bt}\right) \quad \forall t \quad (26)$$

$$\sum_{j \in \mathcal{J}} r^{+}_{jt} \geq D^{+}_{t} \quad \forall t \quad (27)$$

$$\sum_{j \in \mathcal{J}} r^{-}_{jt} \geq D^{-}_{t} \quad \forall t \quad (28)$$

where P^{W}_{bt} is the expected wind production, D_{bt} is the energy demand, and D^{+}_{t}/D^{-}_{t} are the up/down reserve requirements.

The transmission limits are ensured with

$$-\overline{F}_{l} \leq \sum_{j \in \mathcal{J}} \Gamma^{\mathrm{G}}_{lj} \left(\underline{P}_{j} u_{jt} + p_{jt}\right) + \sum_{b \in \mathcal{B}^{\mathrm{W}}} \Gamma_{lb} \left(P^{\mathrm{W}}_{bt} - w^{\mathrm{C}}_{bt}\right) - \sum_{\forall b \in \mathcal{B}} \Gamma_{lb} D_{bt} \leq \overline{F}_{l} \quad \forall l, t \quad (29)$$

where $\overline{F}_{l}$ is the power flow limit on transmission line $l \in \mathcal{L}$, and Γ_{lb}/Γ^{G}_{lj} are the shift factors associated with bus-b/unit-j.

For the classical individual-UC and CUC, the units' constraints are guaranteed by (1)-(10), which are replicated for each generation unit or cluster j.

For the clustered formulation proposed in this paper PCUC, the units' constraints are guaranteed by (1)–(6), (8) and (11)-(24), as discussed in section I-C.

III. Summary

Here we sumarize the five UC formulations in [1]: IUC, the individual UC; CCUC, the classic CUC; PCUC-S, the proposed CUC without SU/SD constraints (20)-(22); PCUC-R, the proposed CUC without ramping constraints (23)-(24); and PCUC, the complete proposed CUC, including both ramping and SU/SD constraints of individual units. Table I shows the complete definition of all models that were implemented. Notice that u_t could be defined as a continuous variable in PCUC-R, PCUC-S, and PCUC models since (16) guarantees the integer value of the variable.

TABLE I
Unit Commitment Models

Equations	IUC	CCUC	PCUC-S	PCUC-R	PCUC
Objective function	(25)	(25)	(25)	(25)	(25)
System constraints	(26)-(29)	(26)-(29)	(26)-(29)	(26)-(29)	(26)-(29)
UC constraints	(1)-(10)	(1)-(10)	(1)-(10)	(1)-(10)	(1)-(10)
Additional constraints	–	–	(11)-(19) (23)-(24)	(11)-(22)	(11)-(24)
Integer variables	–	u_{jt}, y_{jt}, z_{jt} *			
Binary variables	u_{jt}, y_{jt}, z_{jt} †	–	$\tilde{u}_{gt}$ ‡		

† Here, j is the index for each individual unit, see Section II

* Here, j is the index for each cluster, see Section II

‡ Here, $g \in \mathcal{G}_j$ is the index for the units within cluster j

References

[1] G. Morales-Espana and D. A. Tejada-Arango, "Modelling the Hidden Flexibility of Clustered Unit Commitment," *arXiv e-prints*, p. arXiv:1811.02622, Nov. 2018.

[2] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013.

[3] C. Gentile, G. Morales-España, and A. Ramos, "A tight MIP formulation of the unit commitment problem with start-up and shut-down constraints," *EURO J Comput Optim*, vol. 5, no. 1, pp. 177–201, Apr. 2016.

[4] M. Carrion and J. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, 2006.

[5] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, "An MIP Formulation for Joint Market-Clearing of Energy and Reserves Based on Ramp Scheduling," *IEEE Trans. Power Syst.*, vol. 29, no. 1, pp. 476–488, 2014.