

# Homework 05

CS 624, 2022 Fall

Review the course homework policies before you start!

1. Problem 26.1-7 (p714)

Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is, each vertex  $v$  has a limit  $l(v)$  on how much flow can pass through  $v$ . Show how to transform a flow network  $G = (V, E)$  with vertex capacities into an equivalent flow network  $G' = (V', E')$  without vertex capacities, such that a maximum flow in  $G'$  has the same value as a maximum flow in  $G$ . How many vertices and edges does  $G'$  have?

2. Exercise 1.4 ([aux16] p7)

Prove that all problems that are NP-complete are polynomially equivalent, in the sense that if  $A$  and  $B$  are NP-complete, then  $A \leq_P B$  and  $B \leq_P A$ .

3. Exercise 2.1 ([aux16] p9)

*Summary:* Prove that the problem of satisfiability for expressions in *disjunctive normal form* is in P.

4. Exercise 3.6 ([aux16] p16)

Prove that the following are equivalent:

- (a)  $V_1$  is a vertex cover of  $G$ .
- (b)  $V - V_1$  is an independent set in  $G$ .

and, continuing, prove that the following are equivalent:

- (a)  $V_2$  is an independent set in  $G$ .
- (b)  $V_2$  is a clique in  $G^c$ .