Homework 02

CS 624, 2022 Fall

Review the course homework policies before you start!

1. Exercise 6.5-8 (page 166) on HEAP-DELETE.
2. Exercise 6.5-9 (page 166) on merging k sorted lists.
3. Exercise 6.1 in Lecture 3 handout on selecting k smallest elements.
4. Exercise 7.3-2 (page 180) on the number of calls to RANDOM.
5. Problem 7-4 (page 188) on TAIL-RECURSIVE-QUICKSORT.
6. Assume that $c \geq 0$, and assume you had some kind of super-hardware that, when given two lists of length n that are sorted, merges them into one sorted list, and takes only n^c steps.
   (a) Write down a recursive algorithm that uses this hardware to sort lists of length n.
   (b) Write down a recurrence to describe the run time.
   (c) For what values of c does this algorithm perform substantially better than $O(n \log n)$? Why is it highly implausible that this kind of super-hardware could exist for these values of c?
7. In a binary tree, a *leaf node* is a node whose left and right children are both NIL. The *depth* of the tree is the maximum number of edges between the root node and any leaf node.

   Show that if a binary tree has depth n, then it has at most 2^n leaf nodes.

1. 6.5-8
The operation HEAP-DELETE(A; i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in O(lg n) time for an n-element max-heap.

6.5-9
Give an O.n lg k/-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (Hint: Use a minheap for k-way merging.)

6.1 Exercise Show that there is an algorithm that produces the k smallest elements of an unsorted set of n elements in time O(n + k log n).
Be careful: To do this problem correctly, you have to do two things:
1. State the algorithm carefully and prove that it does what it is supposed to do. (The proof can be very simple.)
2. Prove that the algorithm runs in time O(n + k log n)

7.3-2
When RANDOMIZED-QUICKSORT runs, how many calls are made to the randomnumber generator RANDOM in the worst case? How about in the best case? Give your answer in terms of ,-notation.