# CS 624: Notes 05

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September 21, 2022

*These notes contain an outline of what I said in lecture (but only an outline), and they also contain interactive questions and exercises. The corresponding slides are in* `slides03.pdf`.

## 1 Administrative

- Homework 01 questions on Piazza
- Midterm exam(s):
  - probably two midterms
  - first probably in second half of October

## 2 From last time

Calculating asymptotic bounds from recurrence equations

- recursion trees
- the master method / theorem

Introduced heaps

- preheap: binary tree with each level filled except maybe last, last is filled from left
- heap: a preheap, plus:
  each node's key is greater than its children's keys

## 3 Building a heap

(resume at slide 14)

## 3.1 Slide 15: Heapify

Why must we exchange with the larger child node? That is, why can't we pick either child to exchange and recur on?

## 3.2 Slide 18: Heapify run time

Q1: Why $2n/3$?

if the right subtree has $2^k$ nodes
left subtree can have at most $2^{k+1}$
total is $2^k + 2^{k+1} = 3 * 2^k$

Q2: Does it matter?

$p = \log_{2/3} 1 = 0$
In general, $\log_b 1 = 0$

### 3.2.1 Heapify correctness

`Heapify(A,i)` correctness conditions:

- Preconditions:
  - The tree rooted at `Left(i)`, if it exists, is a heap.
  - The tree rooted at `Right(i)`, if it exists, is a heap.
- Postcondition:
  - The tree rooted at `i` is a heap.

## 3.3 Slide 19: Building a heap

Alternate version with simpler loop bounds:

```
BuildHeap(A) :=

heapsize[A] ← length[A]
for i ← length[A] to 1 do
  Heapify(A,i)
end for
```

Why is this the same?

## 3.4 Slide 21: Running time for BuildHeap

See Appendix A, specifically A.8 for more information about the convergence of the infinite series.