

1. a. Given we have $k > 1$ bags and a range of $n > 0$ possible tolerance values that is $\{0, \dots, (n-1)\}$

so, we have $k > 1$ bags which means we have more than 1 bag.

Now suppose we run a test with m books.

We have two cases: test either succeeds or fails

Here we also know that when we test a bag with m books, either the bag passes, or it fails. Also, if a bag succeeds, we can use the same bag to test the tolerance by considering other cases.

If we want test to be successful, at least one bag must pass so that we can find the maximum weight each bag can hold.

So, we might have to run the tests from m number of books and check whether it passes.

If it passes, we don't need to consider the books less than the m number of books. Because it succeeds for any lesser number of books.

Then we can consider increasing books count by $m+1$ and check if the bag pass or fail. If passes, add another book test up to the range $n-1$ until the bag fails.

So we have the tolerance range $\{m+1, m+2, \dots, n-1\}$.

So, the number of tests we need to do is decreased by half.

If the test fails with m books, it also fails for any greater number of books. So, we consider taking values lesser than m until the point where the bag passes the test. If the bag passes the test, we don't need to consider the values lesser than that point. so we decrement the m count until tolerance range is $\{0, 1, \dots, m-1\}$.

So, the number of tests we need to do is also decreased by half.

b.

So, if m is the best case to run, then $T(k, n)$ will change as follows:

If test succeeds, considering m the best case to run, we will increment the count to the range $n-1$ until the bag fails.

So, we take $T(k, m)$ are the minimum number of tests we need to run.

If test fails, considering m the best case to run, we will decrement the count from m to $n=0$. so, we take $T(k-1, n-m)$ are the minimum number of tests we need to run to find the point until the bag passes the test.

c.

To find out which m is the best case to run a test, we need to take minimum value from both the cases where the bag passes or fails the test which will help us determine m value that minimizes the number of tests we can run.

If test passes, the number of tests we need to do is decreased by half.

And if test fails then also the number of tests we need to do is decreased by half.

So taking the minimum value from the both cases we can determine best m value.

So we will take the capacity $1 + \min\{T(k, m), T(k-1, n-m)\}$.

d.

The recursive equation for $T(k, n)$ where $k > 1$ and $n > 0$

if test succeeds:

$T(k, n) = T(k, n^*)$ where n^* is the range of values which are greater than n—since we increment the values.

If test fails:

$T(k, n) = T(k^*, n^*)$ where k^* is the range of values which are greater than k and n^* is the range of values which are less than n

Recursive equation will be $1 + \min\{T(k, n^*), T(k^*, n^*)\}$

2. a. so there are guests with $n > 0$ and $k > 0$ tables to handle and decided to seat the last g guests that is guests labelled $\{n - g + 1, \dots, n\}$ together at a table. Happiness of that table can be computed as

$$\sum_{i=n-(g+1)}^n h(i, g)$$

The i values are computes as $n - (g + 1)$

b. The last g guests $\{n-g+1, \dots, n\}$ are seated in the one table and the remaining n-g guests are seated at the remaining k-1 tables so that it maximizes their total happiness.

Thus, Maximum happiness of the rest of the guests is $H(n, k) = H(n - g, k - 1)$.

c. recursive equation for $H(n, k)$ where $k > 0$ and $n > 0$ is

$$H(n, k) = \max (\text{happiness of table} + H(n - g, k - 1))$$

$$H(n, k) = \max \left(\sum_i^n h(i, g) + H(n - g, k - 1) \right) \\ \text{for } i = n - g + 1 \text{ and } g = 1 \text{ to } n$$