Eksamens

Renats Jakubovskis

May 2019

Bringing these two terms together we obtain the mean number of customers waiting in the queue as

$$L_q = \frac{p_0}{c!}\left[\frac{(\rho c)^{c+1}}{c}\right]\left\{\ \tfrac{1}{(1-\rho)^2} + \tfrac{c/\rho}{1-\rho} - \tfrac{c/\rho}{1-\rho}\ \right\}$$

and thus

$$L_q = \frac{(\rho c)^{c+1}/c}{c!(1-\rho)^2}p_0$$

or, alternatively

$$L_q = \frac{(\lambda/\mu)^c\lambda\mu}{(c-1)!(c\mu-)^2}p_0$$

Having computed L_q, we are now in a position to find other performance measures, either by means of Little's law or from simple relationships among the measures. To find W_q, the mean time spent waiting prior to service; W, the mean time spent in the system, and L, the mean number of customers in the system, we proceed as follows:

1. Use $L_q = \lambda W_q$ to find W_q.
2. Use $W = W_q + 1/\mu$ to find W.
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$$W_q = \left[\frac{(\lambda/\mu)^c\mu}{(c-1)!(c\mu-)^2}\right]p_0,$$

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The probability that an arriving customer is forced to wait in the queue, which means that there is no server available, leads to the "Erlang-C formula." It is the probability that all servers are busy and is given by

$$\begin{aligned} Prob\,\{queueing\} &= \sum_{n=c}^{\infty} p_n = p_0 \sum_{n=c}^{\infty} \frac{c^c}{c!}\rho^n = p_0\frac{c^c}{c!}\left[\frac{\rho^c}{1-\rho}\right] \\ &= \frac{(c\rho)^c}{c!(1-\rho)p_0} = \frac{(\lambda/\mu)^c\mu}{(c-1)!(c\mu-\lambda)}p_0. \end{aligned}$$

This then is the Erlang-C formula. It is denoted by $C(c, \lambda/\mu)$. Observe that mean queue/system lengths and mean waiting times can all be written in terms of this formula. We have

$$L_q = \frac{(\rho c)^{c+1}/c}{c!(1-\rho)^2}p_0 = \frac{(\rho c)^c}{c!(1-\rho)}p_0 \times \frac{\rho}{(1-\rho)} = \frac{\rho}{(1-\rho)}C(c, \lambda/\mu) = \frac{\lambda}{c\lambda - \lambda}C(c, \lambda/\mu).$$

$$W_q = \frac{1}{\lambda}\frac{\rho C(c, \lambda/\mu)}{(1-\rho)} = \frac{C(c, \lambda/\mu)}{c\mu - \lambda},$$

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```latex
    \documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{geometry}
\usepackage{manfnt}
\usepackage{wasysym}
\usepackage{amssymb}
\usepackage{graphicx}
\geometry{verbose,a4paper,tmargin=2cm,bmargin=2cm,lmargin=3cm,rmargin=3cm}

\title{Eksamens}
\author{Renats Jakubovskis}
\date{May 2019}

\begin{document}

\maketitle
\pagestyle{empty}

\textbf{422 Elementary Queueing Theory}


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either by means of Little‘s law or from simple relationships among the measures.
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\begin{verbatim}