Homework - Week 2

Adam Collado

Contents

Question 7:

3.1.1: Set membership and subsets - true or false.

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}$
$B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}$
$C = \{4, 5, 9, 10\}$
$D = \{2, 4, 11, 14\}$
$E = \{3, 6, 9\}$
$F = \{4, 6, 16\}$

An integer x is a perfect square if there is an integer y such that $x = y^2$.

A) $27 \in A$ **is True**

$27/3 = 9$, making 27 an integer multiple of 3, and therefore True.

B) $27 \in B$ **is False**

$5^2 = 25$ and $6^2 = 36$. Therefore, 27 is not a perfect square.

C) $100 \in B$ **is True**

$10^2 = 100$, therefore making 100 a perfect square.

D) $E \subseteq C$ **or** $C \subseteq E$ **is False**

A subset request a set to contain all elements of another set.

Set C does not contain the element 3, and set E does not contain the element 4, meaning C nor E can be subsets of the other.

E) $E \subseteq A$ **is True**

The elements 3, 6, 9 are all multiples of 3, therefore making them a subset of set A.

F) $A \subset E$ **is False**

E only has 3 elements, while set A includes all integers that are multiple of 3, thus making it impossible for A to be a subset of E.

G) $E \in A$ **False**

Set A will contain elements that are multiples of 3, whereas E is a set that is not in Set A.

3.1.2: Set membership and subsets - true or false, cont.

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}$
$B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}$
$C = \{4, 5, 9, 10\}$
$D = \{2, 4, 11, 14\}$
$E = \{3, 6, 9\}$
$F = \{4, 6, 16\}$

An integer x is a perfect square if there is an integer y such that $x = y^2$.

A) $15 \subset A$ **is False**

An element cannot be a proper subset of a set.

B) $\{15\} \subset A$ **True**

Unlike the previous question, this is a set, which can be a proper set of a set, and is a proper subset of A.

C) $\varnothing \subset A$ **True**

All sets contain empty sets, therefore an empty set is a subset of A.

D) $A \subseteq A$ **True**

Since set A is equal to set A, and contains the same elements, then A is a subset of A

E) $\varnothing \in B$ **False**

While an empty set is a subset of B, it is not an element of B, as an empty set is not an element.

3.1.5: Expressing sets in set builder notation.

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

B) $\{3, 6, 9, 12, ...\}$

$\{x \in \mathbb{N} : 0 < x$ and x is a natural number and a multiple of $3\}$

$|B|$ is infinite

D) $\{0, 10, 20, 30, ..., 1000\}$

$\{x \in \mathbb{N} : 0 \leq x \leq 1000$ and x is a natural number and a multiple of $10\}$

$|D| = 101$

3.2.1: Sets of sets - true or false.

Let $X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$. Which statements are true?

A) $2 \in X$ **is True**

B) $\{2\} \subseteq X$ **is True**

C) $\{2\} \in X$ **is False**

D) $3 \in X$ **is False**

E) $\{1, 2\} \in X$ **is True**

F) $\{1, 2\} \subseteq X$ **is True**

G) $\{2, 4\} \subseteq X$ **is True**

H) $\{2, 4\} \in X$ **is False**

I) $\{2, 3\} \subseteq X$ **is False**

J) $\{2, 3\} \in X$ **False**

K) $|X| = 7$ **False**

Question 8:

3.2.4: A subset of a power set.

B) Let $A = \{1, 2, 3\}$. What is $\{X \in P(A) : 2 \in X\}$?

This is stating that X is the subsets of all sets in the $P(A)$ where 2 is an element in that subset.

In which case, the subsets would be $X = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

Question 9:

3.3.1: Unions and intersections of sets.

Define the sets A, B, C, and D as follows:

$$
\begin{aligned}
A &= \{-3, 0, 1, 4, 17\} \\
B &= \{-12, -5, 1, 4, 6\} \\
C &= \{x \in \mathbb{Z} : \text{ x is odd}\} \\
D &= \{x \in \mathbb{Z} : \text{ x is positive}\}
\end{aligned}
$$

C) $A \cap C$

$A \cap C = \{-3, 1, 17\}$

D) $A \cup (B \cap C)$

$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

E) $A \cap B \cap C$

$A \cap B \cap C = \{1\}$

3.3.3: Unions and intersections of sequences of sets, part 2

Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations. For each definition, $i \in \mathbb{Z}^+$

$$
\begin{aligned}
A_i &= \{i^0, i^1, i^2\} \\
B_i &= \{x \in \mathbb{R} : -i \leq x \leq 1/i\} \\
C_i &= \{x \in \mathbb{R} : -1/i \leq x \leq 1/i\}
\end{aligned}
$$

A) $\bigcap_{i=2}^{5} Ai$

$$
\begin{aligned}
\bigcap_{i=2}^{5} Ai &= \{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\} \\
&= \{1\}
\end{aligned}
$$

B) $\bigcup_{i=2}^{5} Ai$

$$\begin{aligned}\bigcup_{i=2}^{5} Ai &= \{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\} \\ &= \{1, 2, 3, 4, 5, 9, 16, 25\}\end{aligned}$$

E) $\bigcap_{i=1}^{100} Ci$

$$\bigcap_{i=1}^{100} Ci = \{x \in \mathbb{R} : -1/100 \leq x \leq 1/100\}$$

F) $\bigcup_{i=1}^{100} Ci$

$$\bigcup_{i=1}^{100} Ci = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$$

3.3.4: Power sets and set operations.

User the set definitions $A = \{a, b\}$ and $B = \{b, c\}$ to express each set below. Use roster notation in your solutions.

B) $P(A \cup B)$

$$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

D) $P(A) \cup P(B)$

$$\begin{aligned}P(A) &= \{\emptyset, a, b, \{a, b\}\} \\ P(B) &= \{\emptyset, b, c, \{b, c\}\} \\ P(A) \cup P(B) &= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\end{aligned}$$

Question 10:

3.5.1: Cartesian product of three small sets.

The sets A, B, and C are defined as follows:

$A = \{tall, grande, venti\}$
$B = \{foam, no - foam\}$
$C = \{non - fat, whole\}$

Use the definitions of A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

B) Write an element from the set $B \times A \times C$

$(foam, tall, non - fat)$

C) Write the set $B \times C$ **using roster notation.**

$\{(foam, non{-}fat), (foam, whole\}, \{no{-}foam, non{-}fat\}, \{no{-}foam, whole\}\}$

3.5.3: Cartesian product - true or false.

Indicate which of the following statements are true.

B) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$ **is True**

C) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$ **is True**

E) For any three sets, A, B, and C, if $A \subseteq B$**, then** $A \times C \subseteq B \times C$ **is True**

3.5.6: Roster notation for sets defined using set builder notation and the Cartesian product

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

D) $\{xy :$ **where** $x \in \{0\} \cup \{0\}^2$ **and** $y \in \{1\} \cup \{1\}^2\}$

$\{01, 011, 001, 0011\}$

E) $\{xy : x \in \{aa, ab\}$ **and** $y \in \{a\} \cup \{a\}^2\}$

$\{aaa, aba, aaaa, abaa\}$

3.5.7: Cartesian products, power sets, and set operations.

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

- $A = \{a\}$
- $B = \{b, c\}$
- $C = \{a, b, d\}$

C) $(A \times B) \cup (A \times C)$

$(A \times B) = \{ab, ac\}$
$(A \times C) = \{aa, ab, ad\}$
$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

F) $P(A \times B)$

$P(A \times B) = \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

G) $P(A) \times P(B)$. **Use ordered pair notation for elements of the Cartesian product.**

$P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}, (\emptyset\{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Question 11:

3.6.2: Proving set identities.

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

B) $(B \cup A) \cap (\overline{B} \cup A) = A$

Proof		
1.	$(B \cup A) \cap (\overline{B} \cup A)$	Start
2.	$(A \cup B) \cap (\overline{B} \cup A)$	Commutative Law
2.	$(A \cup B) \cap (A \cup \overline{B})$	Commutative Law
3.	$A \cap (B \cup \overline{B})$	Distributive Law
4.	$A \cap \mathbb{U}$	Complement Law
5.	A	Identity Law

C) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

Proof		
1.	$(\overline{A \cap \overline{B}})$	Start
2.	$\overline{A} \cup \overline{\overline{B}}$	DeMorgan's Law
3.	$\overline{A} \cup B$	Double Complement Law

3.6.3: Showing set equations that are not identities.

A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

Show that each set equation given below is not a set identity.

B) $A - (B \cap A) = A$

Lets assume $A = \{1\}$ and $B = \{1\}$.

$B \cap A = \{1\}$
$A - (B \cap A) = \{\emptyset\}$

Therefore $A - (B \cap A) \neq A$ and is not a set identity.

D) $(B - A) \cup A = A$

Lets assume $A = \{1\}$ and $B = \{2, 3\}$:

$B - A = \{2, 3\}$
$(B - A) \cup A = \{1, 2, 3\}$

Therefore $(B - A) \cup A \neq A$ and is not a set identity.

3.6.4: Proving set identities with the set difference operation.

The set subtraction law states that $A - B = A \cap \overline{B}$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

B) $A \cap (B - A) = \emptyset$

Proof		
1.	$A \cap (B - A)$	Start
2.	$A \cap (B \cap \overline{A})$	Set Subtraction Law
3.	$(A \cap B) \cap \overline{A}$	Associative Law
4.	$(B \cap A) \cap \overline{A}$	Commutative Law
5.	$B \cap (A \cap \overline{A})$	Associative Law
6.	$B \cap \emptyset$	Complement Law
7.	$\emptyset$	Domination Law

C) $A \cup (B - A) = A \cup B$

Proof		
1.	$A \cup (B - A)$	Start
2.	$A \cup (B \cap \overline{A})$	Set Subtraction Law
3.	$(A \cup B) \cap (A \cup \overline{A})$	Distributive Law
4.	$(A \cup B) \cap \mathbb{U}$	Complement Law
5.	$A \cup B$	Identity Law