Homework - Week 6

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Contents

Question 5:

Use the definitnion of Θ in order to show the following:

A) $5n^3 + 2n^2 + 3n = \Theta(n^3)$

In order to prove that $5n^3+2n^2+3n = \Theta(n^3)$ we will show that the upper-bound is $5n^3 + 2n^2 + 3n = O(n^3)$ and the lower bound is $5n^3 + 2n^2 + 3n = \Omega(n^3)$.

First we will find the $O(n^3)$:

$$\begin{aligned} 5n^3 + 2n^2 + 3n &<= 5n^3 + 2n^3 + 3n^3 \\ &= 10n^3 \\ &= c_1 n \end{aligned}$$

We can see that $5n^3 + 2n^2 + 3n = O(n^3)$ when $c = 10$ and $n_0 = 1$.

Now let us find $\Omega(n^3)$:

$$\begin{aligned} 5n^3 + 2n^2 + 3n &>= 5n^3 \\ &= 5n^3 \\ &= cn \end{aligned}$$

We can see that $5n^3 + 2n^2 + 3n = \Omega(n^3)$ when $c = 5$ and $n_0 = 1$.

Since $T(N) = O(n^3)$ and $T(N) = \Omega(n^3)$, it follows that $5n^3 + 2n^2 + 3n = \Theta(n^3)$.

B) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

In order to prove that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$ we will show that the upper-bound $\sqrt{7n^2 + 2n - 8} = O(n)$ is and the lower bound is $\sqrt{7n^2 + 2n - 8} = \Omega(n)$.

First we will find the $O(n)$:

$$\begin{aligned} \sqrt{7n^2 + 2n - 8} &<= \sqrt{7n^2 + 2n - 8} \\ &= \sqrt{7n^2 + 2n} \\ &= \sqrt{7n^2 + 2n^2} \\ &= \sqrt{9n^2} \\ &= 3n \\ &= cn \end{aligned}$$

We can see that $\sqrt{7n^2 + 2n - 8} = O(n)$ when $c = 3$ and $n_0 = 1$.

Now let us find $\Omega(n)$.

$$
\begin{aligned}
\sqrt{7n^2 + 2n - 8} >&= \sqrt{7n^2 + 2n - 8} \\
&= \sqrt{7n^2 + 2n^2 - 8n^2} \\
&= \sqrt{7n^2 - 6n^2} \\
&= \sqrt{n^2} \\
&= n \\
&= cn
\end{aligned}
$$

We can see that $\sqrt{7n^2 + 2n - 8} = \Omega(n)$ when $c = 1$ and $n_0 = 1$.

Since $T(N) = O(n)$ and $T(N) = \Omega(n)$, it follows that $\sqrt{7n^2 + 2n^2 - 8n^2} = \Theta(n)$.