Homework - Week 2

Adam Collado

Contents

Question 5:

1.12.2

Use the rules of inference and the laws of propositional logic to prove that each argument is valid. Number each line of your argument and label each line of your proof “Hypothesis” or with the name of the rule of inference used at that line. If a rule of inference is used, then include the numbers of the previous lines to which the rule is applied.

B)

$$\begin{array}{ll} & \text{Argument} \\ \hline & p \to (q \land r) \\ & \neg q \\ \hline \therefore & \neg p \end{array}$$

1.	$\neg q$	Hypothesis
2.	$\neg q \lor \neq r$	Addition 1
3.	$\neg (q \land r)$	De Morgans Law 2
4.	$p \to (q \land r)$	Hypothesis
5.	$\neg p$	Modus Tollens 3, 4

E)

$$\begin{array}{ll} & \text{Argument} \\ \hline & p \lor q \\ & \neg p \lor r \\ & \neg q \\ \hline \therefore & r \end{array}$$

1.	$\neg q$	Hypothesis
2.	$p \lor q$	Hypothesis
3.	p	Disjunctive Syllogism 1, 2
4.	$\neg p \lor r$	Hypothesis
5.	r	Disjunctive Syllogism 3, 4

1.12.3

Some of the rules of inference can be proven using the other rules of inference and the laws of propositional logic.

C) One of the rules of inference is Disjunctive Syllogism:

$$\begin{array}{ll} & \text{Argument} \\ \hline & p \to q \\ & p \\ \hline \therefore & q \end{array}$$

Prove that Disjunctive syllogism is valid using the laws of propositional logic and any of the other rules of inference besides Disjunctive syllogism. (Hint: you will need one of the conditional identities from the laws of propositional logic).

1.	$p \to q$	Hypothesis
2.	$\neg p \lor q$	Conditional Identity 1
3.	q	Hypothesis
4.	$q \lor \neg p$	Addition 3
5.	$\neg p \lor q$	Commutative Law 4
6.	$q \lor q$	Resolution 2, 5
7.	q	Idempotent Law 6

1.12.5

Give the form of each argument. Then prove whether the argument is valid or invalid. For valid arguments, use the rules of inference to prove validity.

C)

	Argument
	I will buy a new car and a new house only if I get a job
	I am not going to get a job
$\therefore$	I will not buy a new car

c: I will buy a new car

h: I will get a new house

j: I am getting a job

	Argument Form
	$(c \land h) \to j$
	$\neg j$
$\therefore$	$\neg c$

c	h	j	$(c \land h) \to j$	$\neg j$	$\neg c$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

When $c = T$, $h = F$, and $j = F$, hypotheses are True, but our conclusion is False.

Therefore, this argument is **invalid** when $c = T$, $h = F$, and $j = F$.

D)

	Argument
	I will buy a new car and a new house only if I get a job
	I am not going to get a job
	I will buy a new house
$\therefore$	I will not buy a new car

c: I will buy a new car

h: I will get a new house

j: I am getting a job

	Argument Form
	$(c \wedge h) \to j$
	$\neg j$
	h
$\therefore$	$\neg c$

c	h	j	$(c \wedge h) \to j$	$\neg j$	$\neg c$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

When $c = F$, $h = T$, and $j = F$, all hypotheses are True and our conclusion is true. This is the only row in which all hypotheses are shown to be valid.

Therefore, this argument is **valid**.

Proof		
1.	$(c \wedge h) \to j$	Hypothesis
2.	$\neg j$	Hypothesis
3.	$\neg(c \wedge h)$	Modus Tollens 1, 2
4.	$\neg c \vee \neg h$	De Morgans Law 3
5.	h	Hypothesis
6.	$\neg c$	Disjunctive Syllogism

1.13.3

Show that the given argument is invalid by giving values for the predicates P and Q over the domain {a, b}.

B)

	Argument Form
	$\exists x(P(x) \lor Q(x))$
	$\exists \neg Q(x)$
$\therefore$	$\exists x P(x)$

	p	q
a	F	F
b	F	T

We can see from the truth table that when $P(x)$ is false and $Q(x)$ is true, $\exists x(P(x) \lor Q(x))$ evaluates to true. When $P(x)$ is false and $Q(x)$ is false, $\exists \neg Q(x)$ evaluates to true. However, this results in $\exists x P(x)$ evaluating to false.

Therefore, this argument is **invalid**.

1.13.5

Prove whether each argument is valid or invalid. First find the form of the argument by defining predicates and expressing the hypotheses and the conclusion using the predicates. If the argument is valid, then use the rules of inference to prove that the form is valid. If the argument is invalid, give values for the predicates you defined for a small domain that demonstrate the argument is invalid.

The domain for each problem is the set of students in a class.

D)

	Argument
	Every student who missed class got a detention.
	Penelope is a student in the class.
	Penelope did not miss class.
$\therefore$	Penelope did not get a detention.

Definitions:

- $C(x)$: x is a student who missed class
- $D(x)$: x got detention

	Argument Form
	$\forall x(C(x) \to D(x))$
	Penelope, a student in class
	$\neg C(Penelope)$
$\therefore$	$\neg D(Penelope)$

	C	D
Penelope	F	T

When $C(x)$ is false and $D(x)$ is true, both hypothesis both evaluate to true. However, $\neg D(x)$ evaluates to false.

Therefore, this argument is **invalid**.

E)

	Argument
	Every student who missed class or got detention did not get an A.
	Penelope is a student in the class.
	Penelope got an A.
$\therefore$	Penelope did not get a detention.

Definitions:

- $C(x)$: x is a student who missed class
- $D(x)$: x got detention
- $A(x)$: x got an A

	Argument Form
	$\forall x((C(x) \lor D(x)) \to \neg A(x))$
	Penelope, a student in class
	$A(Penelope)$
$\therefore$	$\neg D(Penelope)$

Proof		
1.	Penelope, a student in class	Hypothesis
2.	$\forall x((C(x) \lor D(x)) \to \neg A(x))$	Hypothesis
3.	$C(Penelope) \lor D(Penelope) \to \neg A(Penelope)$	Universal Instantiation 1, 2
4.	$A(Penelope)$	Hypothesis
5.	$\neg(C(Penelope) \lor D(Penelope))$	Modus Tollens 3, 4
6.	$\neg C(Penelope) \land \neg D(Penelope)$	De Morgans Law 5
7.	$\neg D(Penelope)$	Simplification 6

Question 6:

2.4.1: Proving statements about odd and even integers with direct proofs.

Each statement below involves odd and even integers. An odd integer is an integer that can be expressed as $2k + 1$, where k is an integer. An even integer is an integer that can be expressed as $2k$, where k is an integer.

Prove each of the following statements using a direct proof.

D) The product of two odd integers is an odd integer. Let x and y be odd integers. We shall prove that $x * y$ is an odd integer.

Since x and y are odd integers, $x = 2k + 1$ and $y = 2j + 1$ where k and j are integers and x, and y are not equal to 0.

Plugging in $x = 2k + 1$ and $y = 2j + 1$ for $x * y$ gives:

$$\begin{aligned} x * y &= (2k + 1)(2j + 1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1 \end{aligned}$$

Since k and j are integers, $2(2kj + k + j) + 1$ is also an integer.

Therefore, the product of $x*y$ is can be expressed as $2l+1$ where $l = 2kj+k+j+1$ is an integer.

We can conclude from this that the product of odd integers x and y is an odd integer. ■

2.4.3: Proving Algebraic Statements with Direct Proof

Prove each of the following statements using a direct proof.

B) If x is a real number and $x \geq 3$, then $12 - 7x + x^2 \geq 0$. Let x be a real number and $x \geq 3$. Since $x \geq 3$, we can show that $x - 3 \geq 0$

Since x is a real number, then $12 - 7x + x^2$ is a real number. And since $x - 3 \geq 0$, we can say that $12 - 7x + x^2 \geq x - 3$

Thus:

$$\begin{aligned}
12 - 7x + x^2 &\geq x - 3 \\
(x-3)(x-4) &\geq x - 3 \\
x - 4 &\geq 1 \\
x - 3 &\geq 0 \\
x &\geq 3
\end{aligned}$$

■

Question 7:

2.5.1: Proof by contrapositive of statements about odd and even integers.

D) For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

2.5.4: Proof by contrapositive of algebraic statements.

A) For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$. Let x and y be real numbers. Assume $x > y$. We will prove that $x^3 + xy^2 > x^2y + y^3$.

Thus:

$$x^3 + xy^2 > x^2y + y^3$$
$$x(x^2 + y^2) > y(x^2 + y^2)$$
$$\frac{x(x^2 + y^2)}{(x^2 + y^2)} > \frac{y(x^2 + y^2)}{(x^2 + y^2)}$$
$$x > y$$

Since $x^3 + xy^2 > x^2y + y^3$ can be expressed as $x(p) > y(p)$, it follows that x is always greater than y.

Therefore, $x^3 + xy^2 > x^2y + y^3$. ■

B) For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$. Let x and y be real numbers, $x \leq 10$ and $y \leq 10$. We will prove that $x + y \leq 20$.

Since x is a real number, we can say $x \leq 10$ is a real number and show it as $x - 10 \leq 0$.

Since y is a real number, we can say $y \leq 10$ is a real number and show it as $0 \leq 10 - y$.

Thus, we can say the following: $x - 10 \leq 0 \leq 10 - y$

$$x - 10 \leq 10 - y$$
$$x + y \leq 10 + 10$$
$$x + y \leq 20$$

2.5.5: Proving statements using direct proof or by contrapositive.

C) For every non-zero real number x, if x is irrational, then $1/x$ is also irrational Let x be a non-zero real number, and $1/x$ is rational. We will be prove that x is rational.

Since x is a real number and $x \neq 0$, we can assume that $1/x$ is a real number and show it as the integers p and q such that p/q where $q \neq 0$.

Thus $1/x = p/q$.

Since $1/x \neq 0$, then $p \neq 0$.

$$
\begin{aligned}
x &= \frac{1}{1/x} \\
&= \frac{1}{p/q} \\
&= \frac{q}{p}
\end{aligned}
$$

Since x can be shown to be the quotient of two integers with a non-zero denominator, it follows that x must be a rational number. ∎

Question 8:

2.6.6

C)

D)

Question 9:

2.7.2

B)