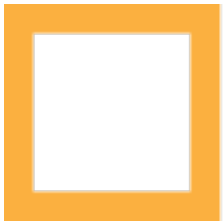
Algorithms and Data Structures
Searching Abstract Data Structures, Symbol Tables

Jacob Trier Frederiksen & Anders Kalhauge



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Spring 2019

Outline

Arrays

Symbol Tables
- List-based
- Array-based

Expandable arrays

Arrays are by nature of fixed length. How can we make them expandable and still have direct memory access?

When adding a new element to a full array we could:

1. Copy the array to an array one bigger
2. Copy the array to an array m elements bigger
3. Copy the array to an array of double size

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Average time for adding an element will be:

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2.
3.

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2.
3.

Expandable arrays

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2. $O(n)$ - all elements are copied each m^{th} time $O(\frac{n}{m}) = O(n)$
3. $O(1)$ - how can that be?

Expandable arrays

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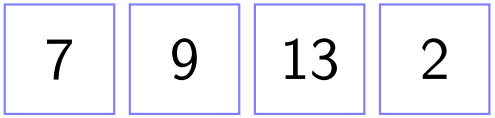
Money in the bank

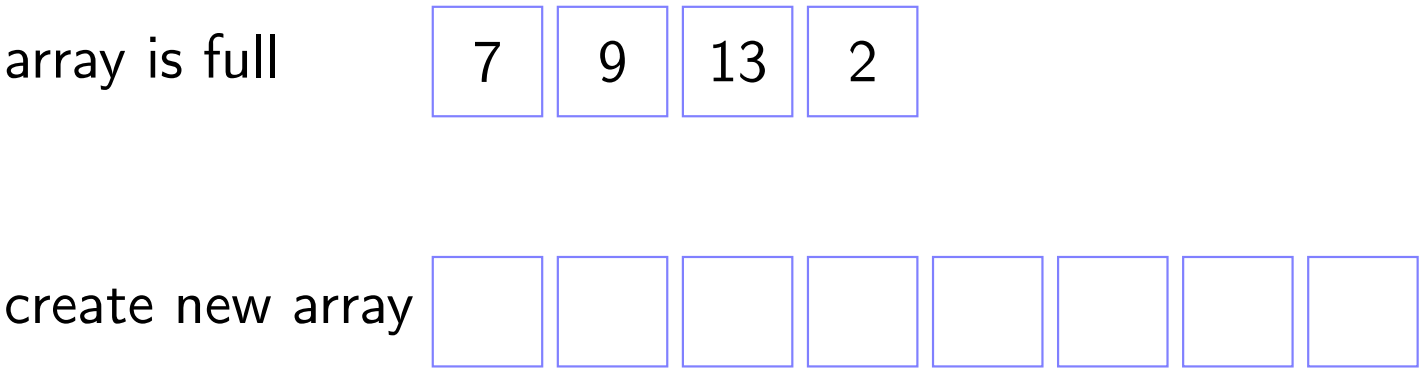
Balance: 4 we hope that is enough to pay for future expansions

array is full



Money in the bank

Balance: $4 - 0 = 4$ (creating new array considered free here)



Expandable arrays

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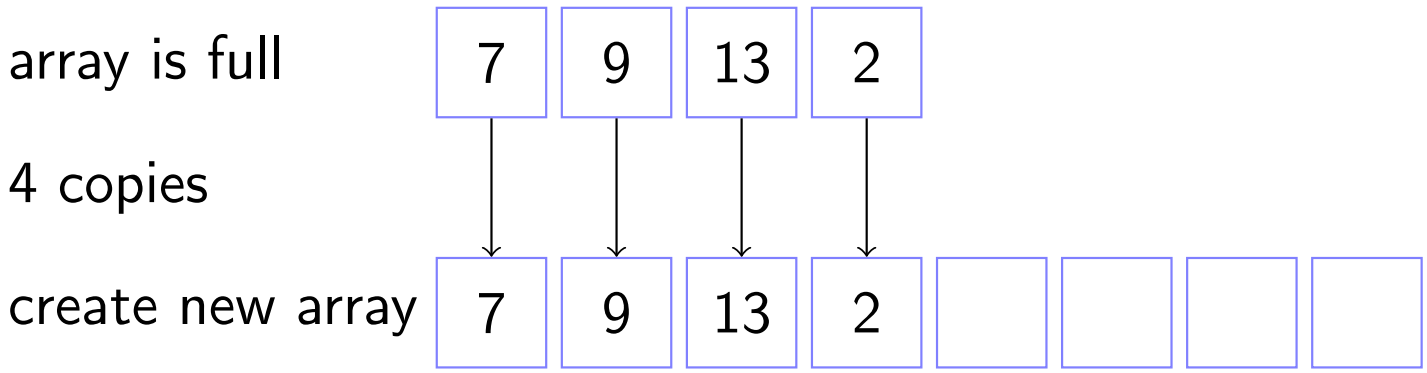
Money in the bank

Balance: $4 - 4 = 0$ (using 1 per copy)



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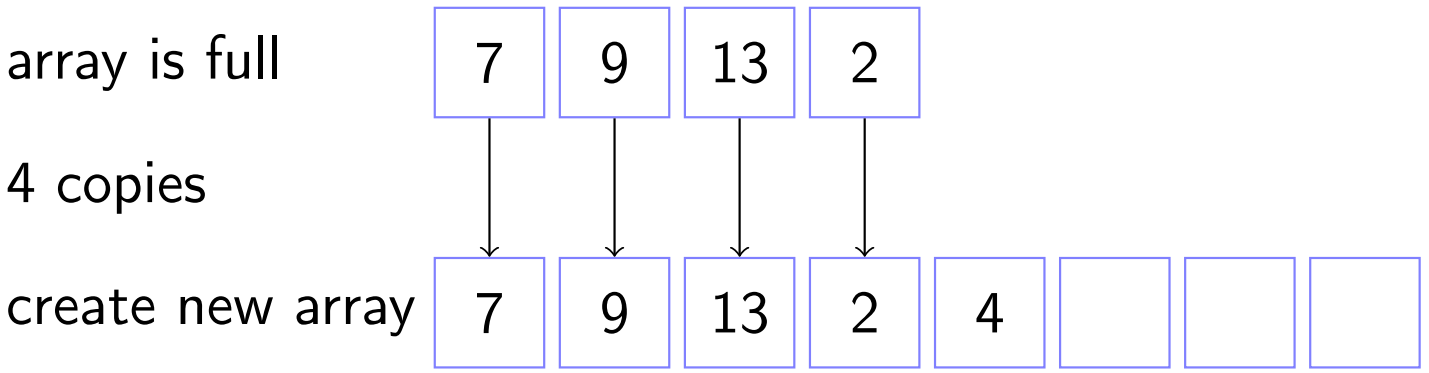
Money in the bank

Balance: $0 + 3 - 1 = 2$ (charging 3 for an insert, using 1)



add element 4

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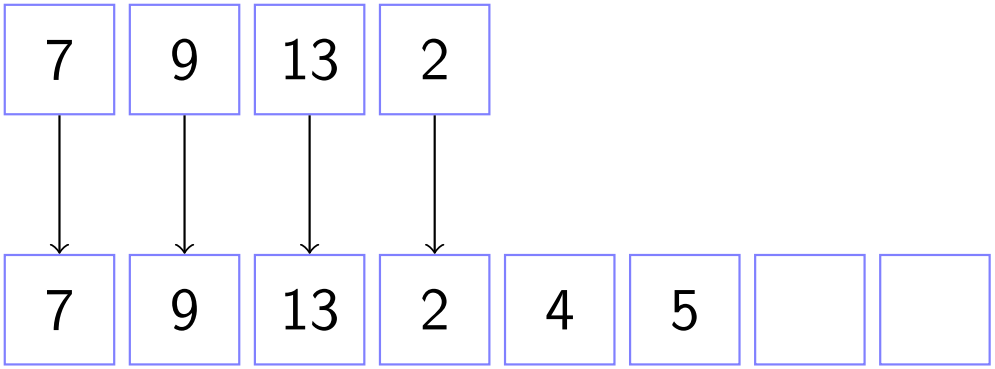
Money in the bank

Balance: $2 + 3 - 1 = 4$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

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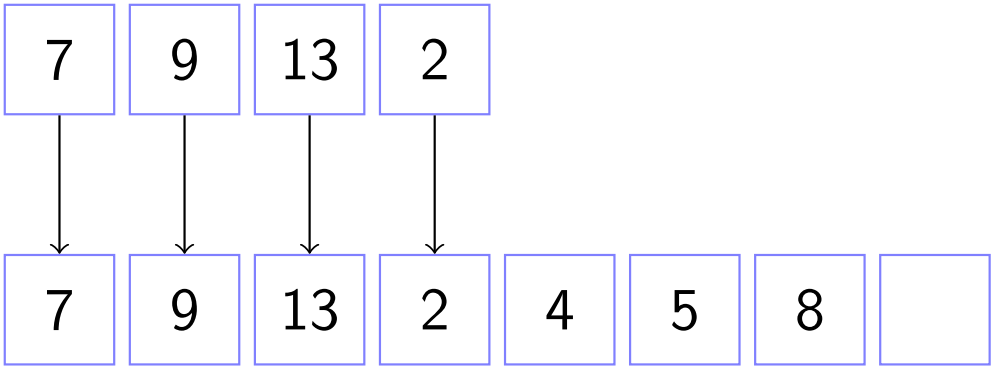
Money in the bank

Balance: $4 + 3 - 1 = 6$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

add element 8

Expandable arrays

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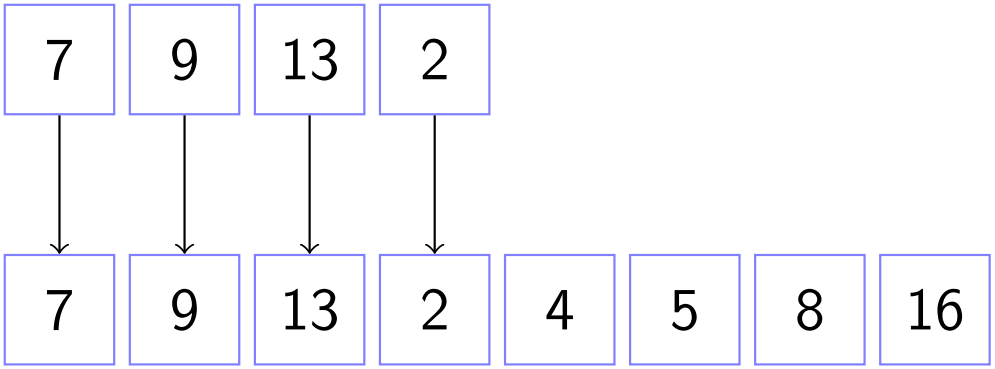
Money in the bank

Balance: $6 + 3 - 1 = 8$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

add element 8

add element 16

Expandable arrays

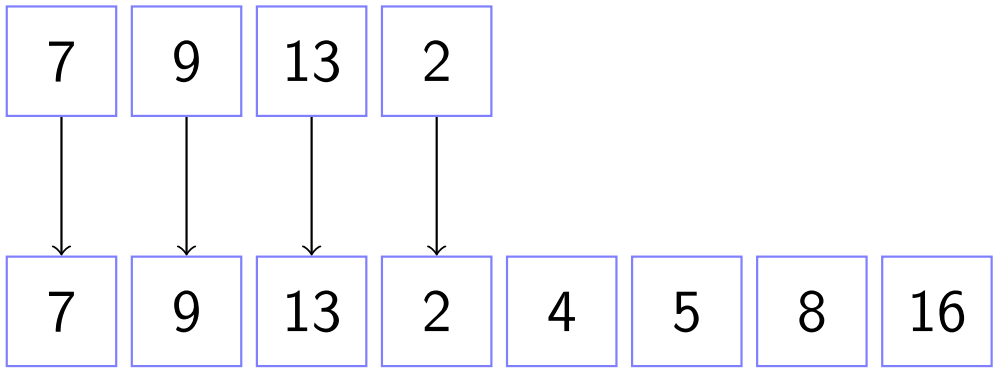
Money in the bank

Balance: 8 enough to pay for 8 copies

array is full

4 copies

create new array



add element 4

add element 5

add element 8

add element 16

$$O(3) = O(1)$$

Expandable arrays

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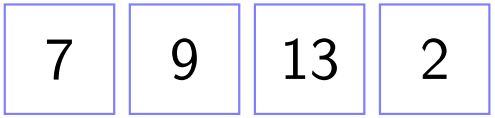
Constant payload

Payload: 0

array is full



Expandable arrays

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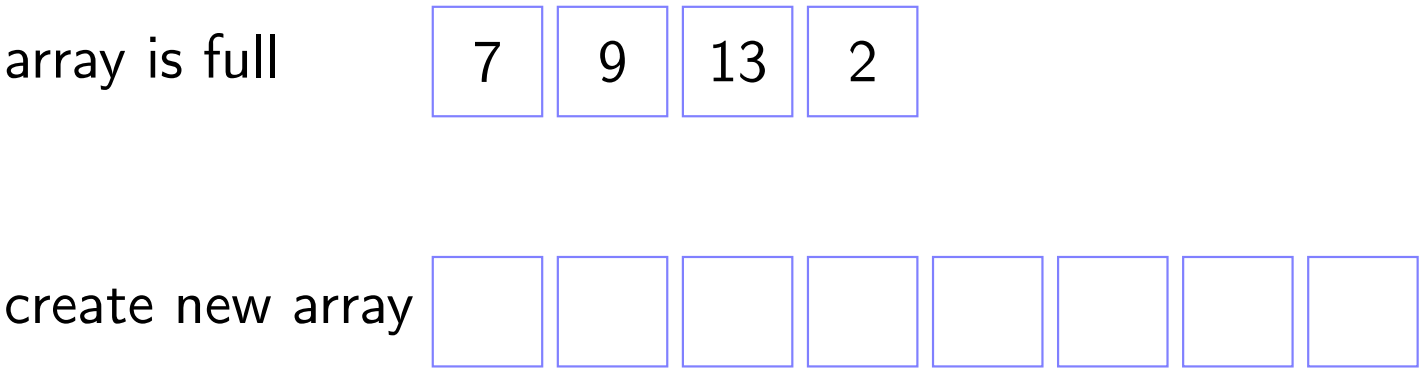
Constant payload

Payload: 0 (creating new array considered free here)



Expandable arrays

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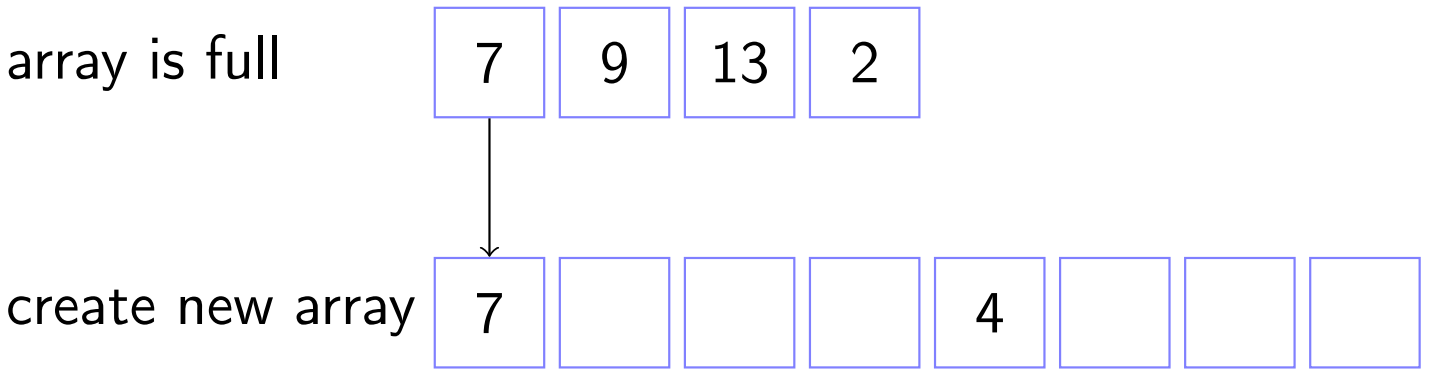
Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)



Expandable arrays

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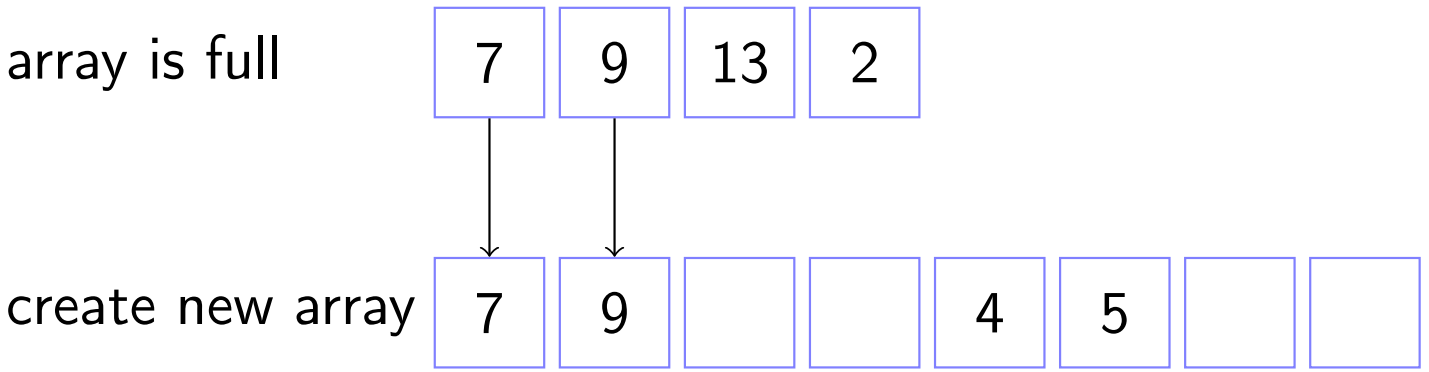


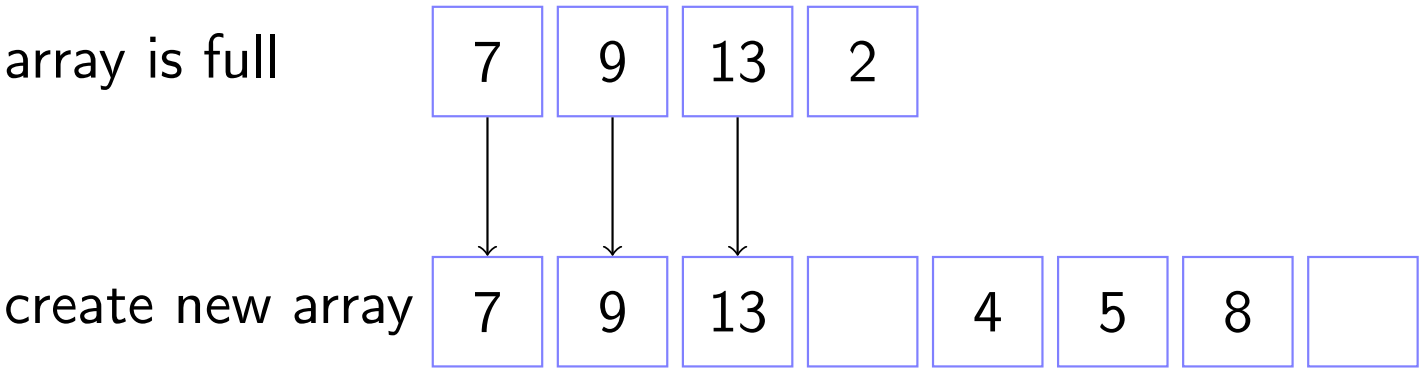
copy 7 insert 4

copy 9 insert 5

Expandable arrays

Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

Expandable arrays

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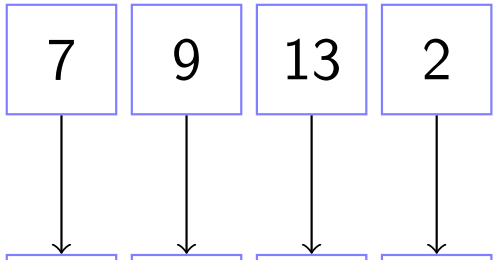
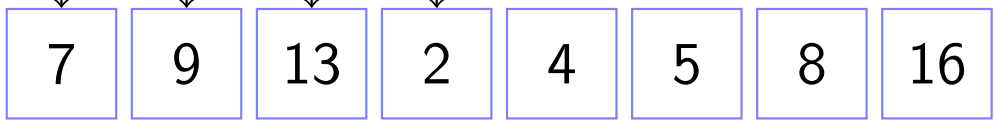
Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)

array is full



create new array



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

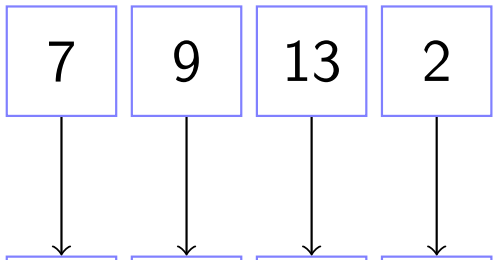
copy 2 insert 16

Expandable arrays

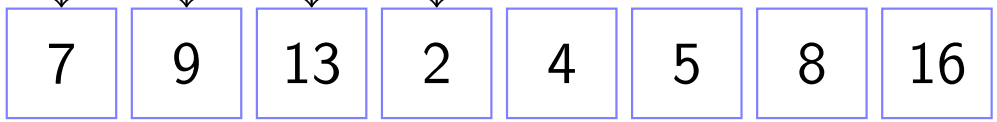
Constant payload

Payload: 8 in total for 4 insertions

array is full



create new array



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

copy 2 insert 16

$$O(2) = O(1)$$

Exercise 1 - Complexity

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- What would the complexity (big-O) be if we:
  - Triple the array size instead of doubling it?
  - Only made the new array 50% bigger?
- Bearing in mind that most modern memory is paged[1], consider why doubling the array size is not such a bad idea?

[1] typically in 2^n sized pages

1. Create a Java class `FlexibleArray` that uses the "Constant payload" algorithm.

```
public class FlexibleArray<T> {
   ...
  public T get(int index) { ... }
  public void set(int index, T element) { ... }
  public void add(T element) { ... }
  public int size() { ...}
  }
```

Note that to create a new array of type `T` you must:

```
private T[] arrayOfT = (T[])new Object[1000];
```

2. Measure the time it takes to add 10.000, 100.000, and 1.000.000 elements.
3. Measure Javas build-in `ArrayList` with the same data.

Arrays

Symbol Tables

- List-based
- Array-based

Symbol tables

`Map` in Java

- Lookup a value based on a key
- No `null` keys
- No `null` values
- No duplicate keys

```
public interface SymbolTable<K,V> {
  void put(K key, V value);
  V get(K key);
  int size();
  Iterable<K> keys();
  default void delete(K key) { put(key, null); }
  default boolean contains(K key) {
    return get(key) != null;
    }
  default boolean isEmpty() { return size() == 0; }
  }
```

Ordered Symbol Tables

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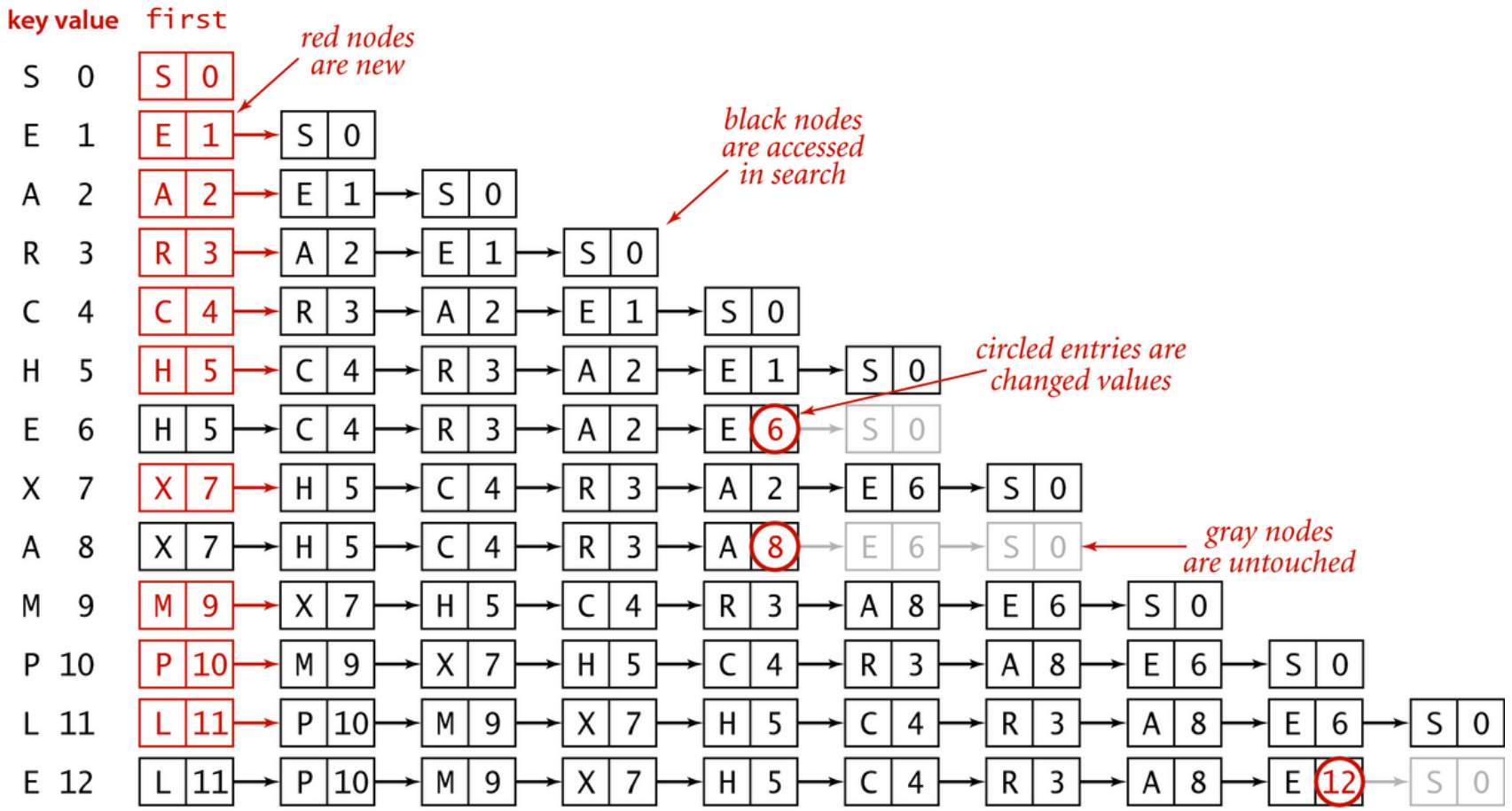
`OrderedMap` in Java

```
public interface
    OrderedSymbolTable<K extends Comparable<K>, V>
    extends SymbolTable<K,V> {
  K min();
  K max();
  K floor(K key);
  K ceiling(K key);
  int rank(K key);
  K select(int rank);
  void deleteMin();
  void deleteMax();
  int size(K low, K high);
  Iterable<K> keys(K low, K high);
  }
```

List-based Symbol Tables



Trace of linked-list ST implementation for standard indexing client

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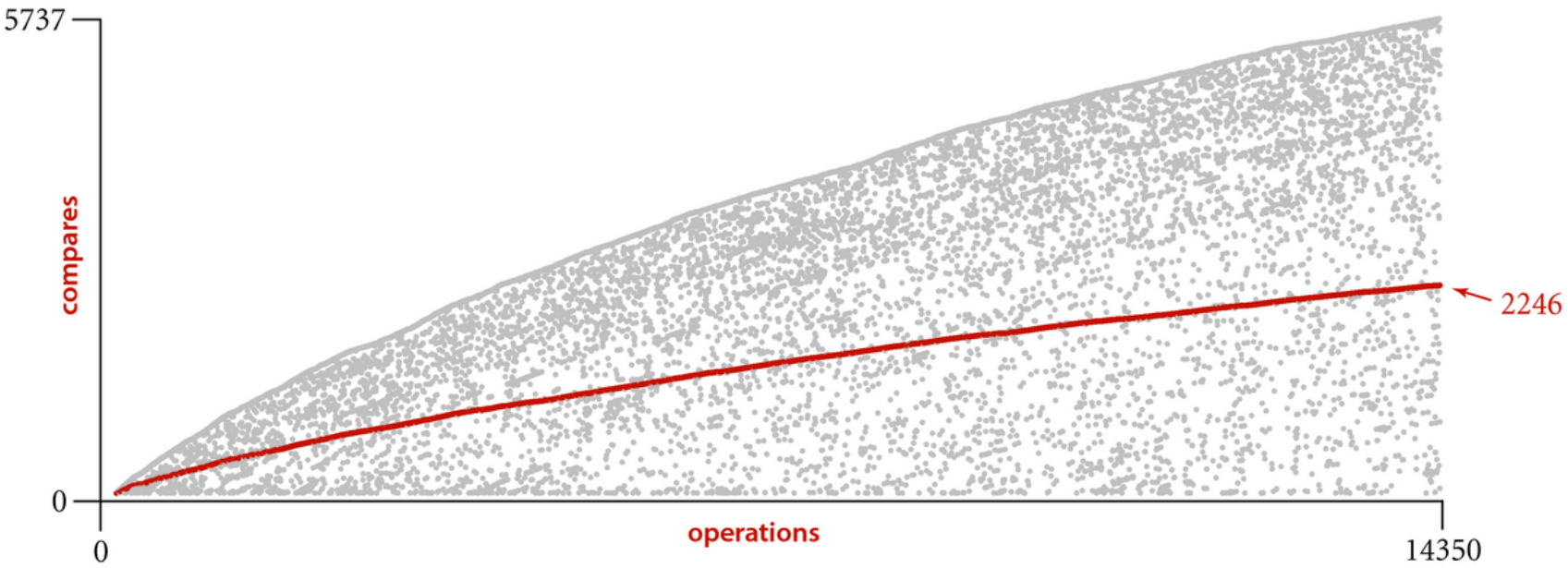
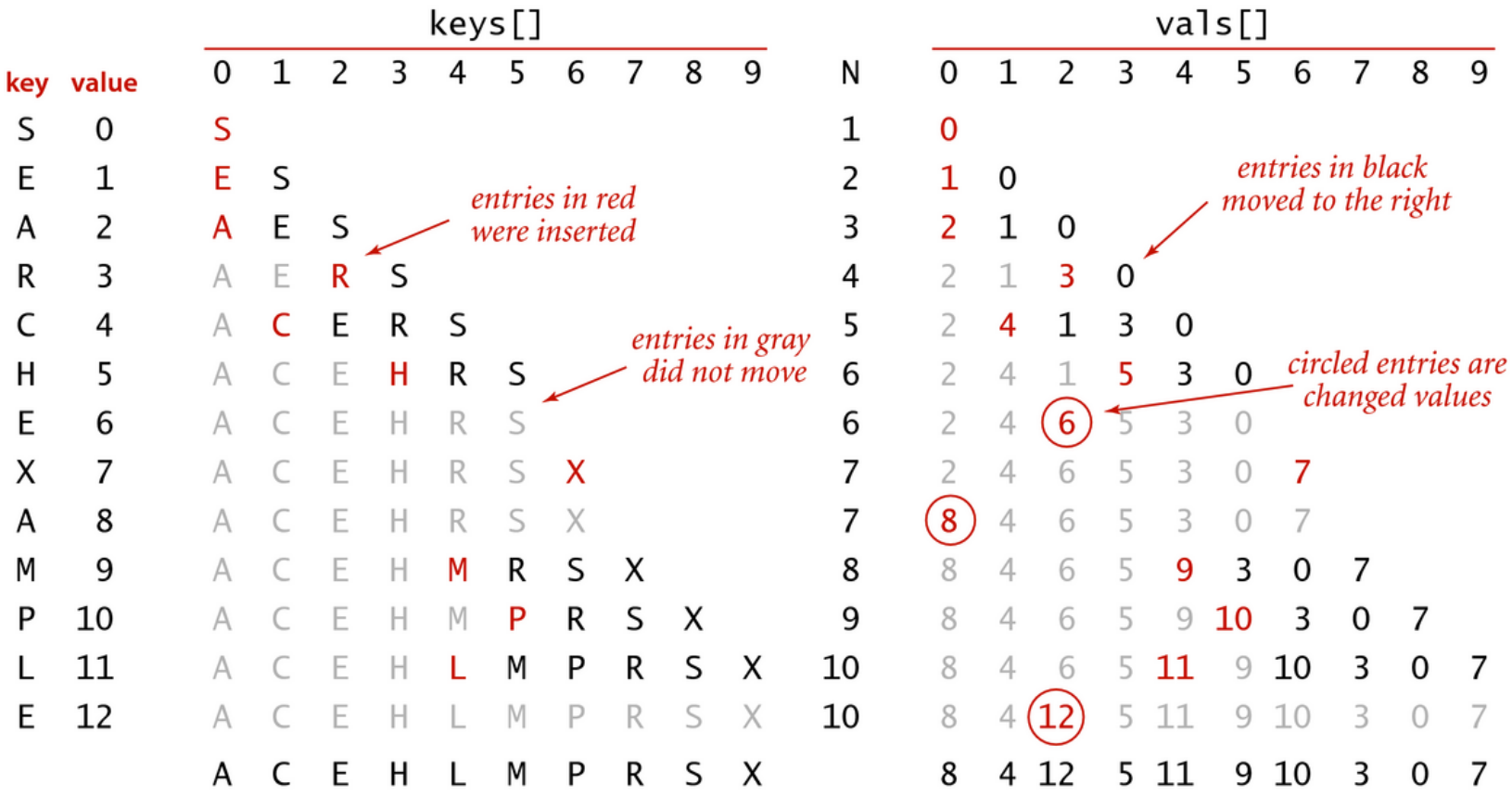
Costs



Costs for `java FrequencyCounter 8 < tale.txt` **using** `SequentialSearchST`

Q: go to the book (Ch.3.1, subsect. 'Performance Client') and figure out what the input argument '8' does here? Discuss in Plenum.

Array-based Symbol Tables



key	value	0	1	2	3	4	5	6	7	8	9	N	0	1	2	3	4	5	6	7	8	9
S	0	S										1	0									
E	1	E	S									2	1	0								
A	2	A	E	S								3	2	1	0							
R	3	A	E	R	S							4	2	1	3	0						
C	4	A	C	E	R	S						5	2	4	1	3	0					
H	5	A	C	E	H	R	S					6	2	4	1	5	3	0				
E	6	A	C	E	H	R	S					6	2	4	6	5	3	0				
X	7	A	C	E	H	R	S	X				7	2	4	6	5	3	0	7			
A	8	A	C	E	H	R	S	X				7	8	4	6	5	3	0	7			
M	9	A	C	E	H	M	R	S	X			8	8	4	6	5	9	3	0	7		
P	10	A	C	E	H	M	P	R	S	X		9	8	4	6	5	9	10	3	0	7	
L	11	A	C	E	H	L	M	P	R	S	X	10	8	4	6	5	11	9	10	3	0	7
E	12	A	C	E	H	L	M	P	R	S	X	10	8	4	12	5	11	9	10	3	0	7
		A	C	E	H	L	M	P	R	S	X		8	4	12	5	11	9	10	3	0	7

Trace of ordered-array ST implementation for standard indexing client

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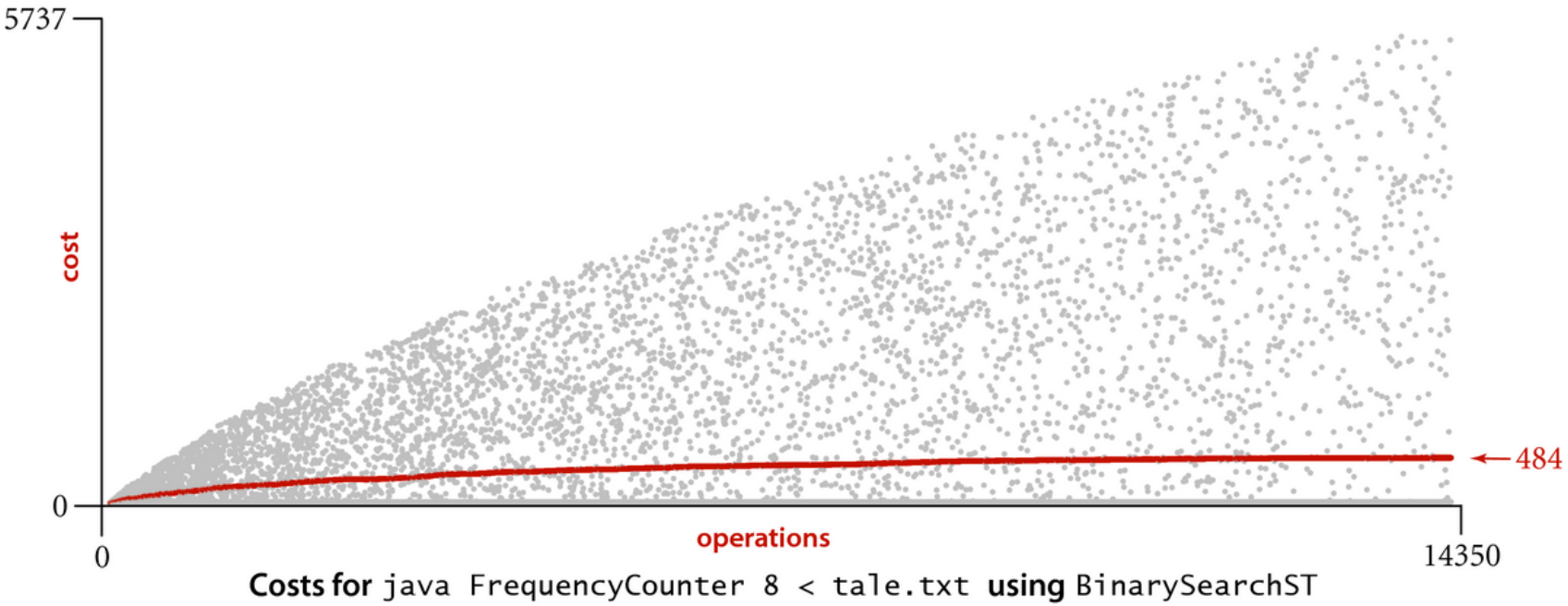
Costs



Q: What happened here? Look at that decrease in cost... Discuss in plenum.

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Costs Comparison

Unordered sequential search *vs.* Ordered array binary search.

algorithm (data structure)	worst-case cost (after N inserts) search	worst-case cost (after N inserts) insert	average-case cost (after N random inserts) search hit	average-case cost (after N random inserts) insert	efficiently support ordered operations?
sequential search (unordered linked list)	N	N	$N/2$	N	no
binary search (ordered array)	$\lg N$	$2N$	$\lg N$	N	yes

Cost summary for basic symbol-table implementations

Q: What about insert (or `put()`)? What is *its* complexity? And can we achieve better performance? Discuss in plenum, 'tales.txt' and '.

Array-based Symbol Tables

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Pros & Cons in Searching

underlying data structure	implementation	pros	cons
linked list (sequential search)	`SequentialSearchST`	best for tiny STs	slow for large STs
ordered array (binary search)	`BinarySearchST`	optimal search and space, order-based ops	slow insert
binary search tree	`BST`	easy to implement, order-based ops	no guarantees space for links
balanced BST	`RedBlackBST`	optimal search and insert, order-based ops	space for links
hash table	`SeparateChainingHashST` `LinearProbingHashST`	fast search/insert for common types of data	need hash for each type no order-based ops space for links/empty

Pros and cons of symbol-table implementations

Q: how can we get better e.g. `put()` performance? Need more than linked lists and binary searches=> Datastructures.

Exercise 3 - Sequential Search

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NB: These last exercises are optional, but will definitely help you to understand the linked listst, search complexity issues better.

- Add to the `Algorithm 3.1` in the Book (seq. search in unordered list), the methods mentioned in the text, namely
  - `size()`,
  - `keys()`,

  so essentially exercise 3.1.5, yet, without the *eager* `delete()`.
- Exercise 3.1.6 of the Book.
- (ex. 3.1.10 of the book) What's frequent most word of 10+ letters in *'Tale of two cities'* (ex. 3.1.10 of the book)?