

# Algorithms & Data Structures

## Advanced Sorting

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## Tools & Housekeeping

- Quick Polls

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## Last Week: Exercise Follow-up

## Sorting

- Divide and Conquer

- Merge Sort

- Quick Sort

- Stability – what it means

- Sorting – which one to choose?

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Sorting – which one to choose?

Go to [www.menti.com](https://www.menti.com).

**Comfort Zone:** how do you feel about the course at this early point in time?

**Workshop Hours:** we need to find the most optimal place for face time.

To complete this course successfully, it is vital that communication and tooling is in place.

*You should check that you're ready!*

### Are you up-to-speed on:

**Java:** do you have functioning Java development environment, either on your machine or in the cloud?

**Moodle:** can you access the course on Moodle? Receiving emails from Moodle?

**Github:** are you familiar with GitHub. Can you clone, pull, push, etc.?

**Peergrade:** can you achieve access to the grading site (after having received the site code from me? Go to [www.peergrade.io/join](http://www.peergrade.io/join).

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### Sorting

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Sorting – which one to choose?

Create a class called `SortingAlgorithms`. This class should have an array of integer as a datafield and array size. The constructor should be used to create an array of given size.

`SortingAlgorithms` class should have three methods.

- ❑ One method for filling the array with random integers which you can call from the constructor.
- ❑ One method for implementing Insertion Sort,
- ❑ One method for Selection Sort.
- ❑ One method for an improved Three-Sum-Zero algorithm (book, `ThreeSumFast`, p.190), and time it, comparing with the `ThreeSum` for various sized arrays.

In the main method you create three objects with array sizes of  $10^2$ ,  $10^3$ ,  $10^4$ ,  $10^5$ , and  $10^6$ . Time your code using `Stopwatch`<sup>1</sup> class.

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<sup>1</sup>see page 175 in Algorithms book

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### Sorting

Divide and Conquer

Merge Sort

Quick Sort

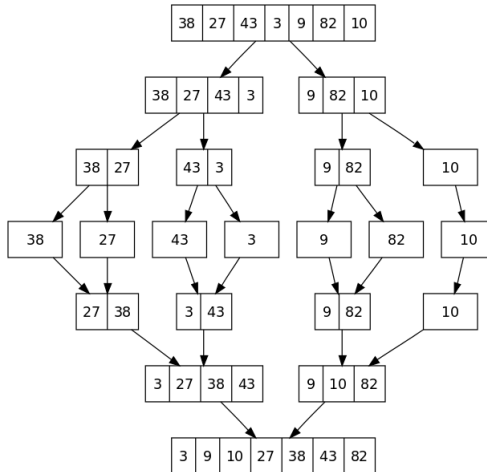
Stability – what it means

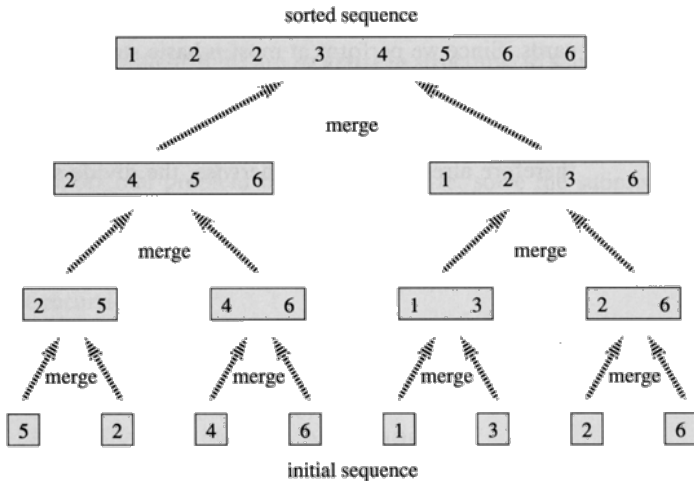
Sorting – which one to choose?



- Divide and break
  - Break the problem in to smaller sub-problem recursively
  - Sub-problem should represent a part of the original problem
  - Keep on dividing until no more division is possible
- Conquer/Solve
  - Smaller sub-problem are solved
  - Solutions of all the sub-problems are merged
- Merge/Combine
  - Combines small solutions to the big solutions

## Divide and Conquer





- Pros:**
- Time: complexity is robustly  $N \cdot \text{Lg}(N)$ .
  - Stability: the algorithm is stable.
- Cons:**
- Space: memory overhead proportional to  $N$ .

Quick sort doesn't use auxiliary space

1. Choose a pivot  $p$ , e.g: 1<sup>st</sup> element of array
2. Bring all elements less than the pivot to left end of the array and all elements greater to the other.
3. Place the pivot in between.
4. Sort items left of the pivot
5. Sort items right of the pivot

**Classroom exercise (need two volunteers):** on the whiteboard, sort the following arrays, using QS:

**Team 1:** [2, 4, 1, 3, 5],

**Team 2:** [1, 2, 3, 4, 5].

			a[]															
	i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial values	0	16	K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
scan left, scan right	1	12	K	<u>R</u>	A	T	E	L	E	P	U	I	M	Q	<u>C</u>	X	O	S
exchange	1	12	K	C	A	T	E	L	E	P	U	I	M	Q	R	X	O	S
scan left, scan right	3	9	K	C	<u>A</u>	<u>T</u>	E	L	E	P	U	<u>I</u>	<u>M</u>	<u>Q</u>	R	X	O	S
exchange	3	9	K	C	A	I	E	L	E	P	U	T	M	Q	R	X	O	S
scan left, scan right	5	6	K	C	A	I	<u>E</u>	<u>L</u>	<u>E</u>	<u>P</u>	<u>U</u>	T	M	Q	R	X	O	S
exchange	5	6	K	C	A	I	E	E	L	P	U	T	M	Q	R	X	O	S
scan left, scan right	6	5	K	C	A	I	E	<u>E</u>	<u>L</u>	P	U	T	M	Q	R	X	O	S
final exchange	6	5	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
result		5	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S

Partitioning trace (array contents before and after each exchange)

- Pros:**
- Space: no (significant) overhead:  $\sim \lg(N)$
  - Time: complexity is  $\sim N \cdot \lg(N)$  (best-to-avg case).
- Cons:**
- Time: complexity is  $\sim N^2$  (worst case).
  - Stability: the algorithm is not stable.

**Q: how to circumvent the  $N^2$  worst case of Quick Sort?**

### So, what does it mean, Stability?

- Team 1: Sort half a deck of cards of spades ♠ (ranks 2 through 7) and diamonds ◇ (ranks 2 through 7), first by rank, then by suit. NB: Random shuffle cards well to begin with. Use QUICK SORT.
- Team 2: Do the same on the half-deck of hearts ♥ and clubs ♣, instead. NB: Random shuffle cards well to begin with. Use MERGE SORT.



sorted by time

Chicago	09:00:00
Phoenix	09:00:03
Houston	09:00:13
Chicago	09:00:59
Houston	09:01:10
Chicago	09:03:13
Seattle	09:10:11
Seattle	09:10:25
Phoenix	09:14:25
Chicago	09:19:32
Chicago	09:19:46
Chicago	09:21:05
Seattle	09:22:43
Seattle	09:22:54
Chicago	09:25:52
Chicago	09:35:21
Seattle	09:36:14
Phoenix	09:37:44

sorted by location (not stable)

Chicago	09:25:52
Chicago	09:03:13
Chicago	09:21:05
Chicago	09:19:46
Chicago	09:19:32
Chicago	09:00:00
Chicago	09:35:21
Chicago	09:00:59
Houston	09:01:10
Houston	09:00:13
Phoenix	09:37:44
Phoenix	09:00:03
Phoenix	09:14:25
Seattle	09:10:25
Seattle	09:36:14
Seattle	09:22:43
Seattle	09:10:11
Seattle	09:22:54

*no  
longer  
sorted  
by time*

sorted by location (stable)

Chicago	09:00:00
Chicago	09:00:59
Chicago	09:03:13
Chicago	09:19:32
Chicago	09:19:46
Chicago	09:21:05
Chicago	09:25:52
Chicago	09:35:21
Houston	09:00:13
Houston	09:01:10
Phoenix	09:00:03
Phoenix	09:14:25
Phoenix	09:37:44
Seattle	09:10:11
Seattle	09:10:25
Seattle	09:22:43
Seattle	09:22:54
Seattle	09:36:14

*still  
sorted  
by time*

Stability when sorting on a second key

Which sorting algorithm should I pick?

algorithm	stable?	in place?	order of growth to sort $N$ items		notes
			running time	extra space	
<i>selection sort</i>	no	yes	$N^2$	1	
<i>insertion sort</i>	yes	yes	between $N$ and $N^2$	1	depends on order of items
<i>shellsort</i>	no	yes	$N \log N$ ? $N^{6/5}$ ?	1	
<i>quicksort</i>	no	yes	$N \log N$	$\lg N$	probabilistic guarantee
<i>3-way quicksort</i>	no	yes	between $N$ and $N \log N$	$\lg N$	probabilistic, also depends on distribution of input keys
<i>mergesort</i>	yes	no	$N \log N$	$N$	
<i>heapsort</i>	no	yes	$N \log N$	1	

Performance characteristics of sorting algorithms

Re-design your `SortingAlgorithms` class. This time do the following:

1. Implement a Bottom-Up method for Merge Sort
2. Implement one method for Quick Sort

For these methods, check running time (optionally max memory consumption) for three cases:

1. Randomized integer arrays of sizes  $10^1$ ,  $10^3$ ,  $10^5$ ,  $10^7$
2. Forward sorted integer arrays of sizes  $10^1$ ,  $10^3$ ,  $10^5$ ,  $10^7$
3. Reverse sorted integer arrays of sizes  $10^1$ ,  $10^3$ ,  $10^5$ ,  $10^7$

For the sake of simplicity you may assume that the integer arrays have only unique values (no multiplets).