ASSIGNMENT-1

Que

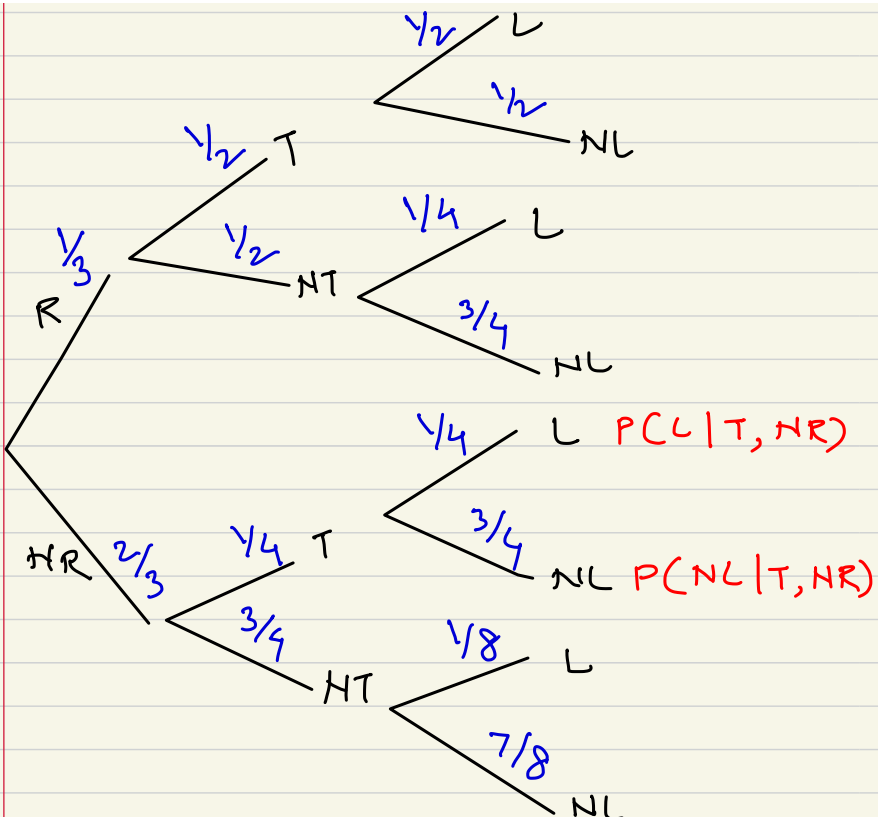
5. In my town, it's rainy for one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25, 0.25. You pick a random day.

a) What is the probability that it's not raining and there is heavy traffic and I am not late?

b) What is the probability that I am late?

c) Given that I arrived late at work, what is the probability that it rained that day?

Ans



a) $P(NR) \cap P(\text{heavy traffic}) \cap P(\text{Not late}) =$

$P(NR)\, P(\text{heavy traffic} \mid NR)\, P(NL \mid NR, \text{heavy traffic})$

$= \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4_2} = \frac{3^1}{24_8} = \frac{1}{8}$

b) $P(L) =$

$\left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{4}\right)$

$+ \left(\frac{2}{3} \times \frac{3}{4} \times \frac{1}{8}\right)$

$= \frac{1}{12} + \frac{1}{24} + \frac{2}{48} + \frac{6}{96}$

$= \frac{1}{12_4} + \frac{1}{24_{(2)}} + \frac{1}{24_{(2)}} + \frac{1}{16_3}$

$= \frac{4+2+2+3}{48}$

$= \frac{11}{48}$

c) $P(R|L) = \dfrac{P(L|R) \times P(R)}{P(L)}$

$= \dfrac{P(L|R,T) + P(L|R,HT)}{P(L)}$

$= \dfrac{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{1}{3}\right)}{11/48}$

$= \dfrac{\frac{1}{12} + \frac{1}{24}}{11/48} = \dfrac{3}{24} \times \dfrac{48}{11} = \dfrac{6}{11}$