Алгоритм двухуровневой оптимизации

1 Введение

Таблица 1: Travel time functions and improvement cost functions for the network

Route	Nodes	(Flow, Capacity)	Travel time function	Improvement cost function
1	1, 2, 6	(x_1, c_1)	$t_1(x_1, c_1) = 20 + 0.15\left(\frac{x_1}{c_1}\right)^4$	$g_1(c_1) = 100(c_1 - 50)^2$
2	1, 3, 6	(x_2, c_2)	$t_2(x_2, c_2) = 25 + 0.15\left(\frac{x_2}{c_2}\right)^4$	$g_2(c_2) = 100(c_2 - 80)^2$
3	1, 4, 6	(x_3, c_3)	$t_3(x_1, c_3) = 18 + 0.15\left(\frac{x_3}{c_3}\right)^4$	$g_3(c_3) = 100(c_3 - 70)^2$
4	1, 5, 6	(x_4, c_4)	$t_4(x_1, c_4) = 28 + 0.15\left(\frac{x_4}{c_4}\right)^4$	$g_4(c_4) = 100(c_4 - 40)^2$

Constraints on capacities of available routes are given in Table 2.

Таблица 2: Lower and upper bounds of available capacities for the network

$i = \overline{1,4}$	c_1	c_2	c_3	c_4
Lower bound (l_i)	50	80	70	40
Upper bound (u_i)	80	100	95	70

Therefore, the manager faces the following bilevel optimization problem:

$$\min_{c} \sum_{i=1}^{4} t_i(x_i, c_i)x_i + 0.01\sum_{i=1}^{4} g_i(c_i) \tag{1}$$

subject to

$$\sum_{i=1}^{4} g_i(c_i) \leq 200000, \tag{2}$$

$$50 \leq c_1 \leq 80, \tag{3}$$

$$80 \leq c_2 \leq 100, \tag{4}$$

$$70 \leq c_3 \leq 95, \tag{5}$$

$$40 \leq c_4 \leq 70, \tag{6}$$

where

$$x = \arg\min_{x} \sum_{i=1}^{4} \int_{0}^{x_i} t_i(u, c_i) du, \tag{7}$$

under constraints

$$\sum_{i=1}^{4} x_i = 1000, \tag{8}$$

$$x_i \geq 0 \quad \forall i = \overline{1,4}. \tag{9}$$