Numerical methods for nonlinear nonlocal water wave models

Daulet Moldabayev

Dissertation for the degree of Philosophiae Doctor (PhD)

Department of Mathematics
University of Bergen
Norway

April 2017

Preface

This dissertation is submitted as a partial fulfillment of the requirements for the degree Doctor of Philosophy (PhD) at the Department of Mathematics, University of Bergen. The research was supported by the Research Council of Norway on grant no. 213474/F20.

The advisory committee has consisted of Henrik Kalisch (University of Bergen, Bergen, Norway), Dmitry Pelinovsky (McMaster University, Hamilton, Ontario, Canada) and Angel Durán (Universidad de Valladolid, Valladolid, Spain).

Acknowledgements

First of all I would like to thank Henrik Kalisch and Magnus Svärd who carried out the selection process of candidates for the PhD position. I am sincerely thankful to Professor Henrik Kalisch, my main supervisor, for sharing knowledge and guiding me, for his enthusiasm, support and encouragement. The thesis would not have been possible without you.

In the course of the doctorate studies, I have had fruitful collaboration with Olivier Verdier, Evgueni Dinvay, Denys Dutykh, Ben Segal and others. I would like to thank them for productive discussions, advices on research questions and some inspiring ideas we came up with.

As a PhD Candidate I have been lucky to meet my colleagues and fellow students at the Department of Mathematics. They have always provided a friendly and supportive environment. I appreciate everything we have experienced during the time of our studies.

I would like to express my gratitude to my family for their continuous support and encouragement over the years of my studies. I am also thankful for all of my teachers and professors from schools and universities I studied at.

I am heartily grateful for my wife Assem. You have always been there for me. Without your love, support and understanding I would not have been able to go that far.

Outline

This thesis is organised in the following way. Part I contains general theoretical background on nonlinear wave models as well as description of methods used to solve the equations involved. Some properties of solutions to the equations and a summary of results are also given in the first part. Part 2 consists of the research papers that present scientific results in detail.

List of reseach papers included in Part II

Paper A:
Moldabayev, D., Kalisch, H., Dutykh, D.: *The Whitham equation as a model for surface water waves*, Phys. D **309**, 99–107 (2015),
http://dx.doi.org/10.1016/j.physd.2015.07.010.

Paper B:
Dinvay, E., Moldabayev, D., Dutykh, D., Kalisch, H.: *The Whitham equation with surface tension*, Nonlinear Dynamics, 1–14 (2017),
http://dx.doi.org/10.1007/s11071-016-3299-7.

Paper C:
Henrik Kalisch, Daulet Moldabayev, Olivier Verdier: *A numerical study of nonlinear dispersive wave models with SpecTraVVave*, specify status of the paper.

Paper D:
Benjamin Segal, Daulet Moldabayev, Henrik Kalisch, Bernard Deconinck: *Explicit solutions for a long-wave model with constant vorticity*, submitted to European Journal of Mechanics - B/Fluids.

Contents

Part I

Background

Chapter 1

Introduction

This is the introduction [1]. . .

Chapter 2

Summary of results

This chapter provides an overview of the results achieved in the course of research work. Detailed

2.1 A numerical study of nonlinear dispersive wave models with SpecTraVVave

Bibliography

[1] DINVAY, E., MOLDABAYEV, D., DUTYKH, D., AND KALISCH, H. The whitham equation with surface tension. *Nonlinear Dynamics* (2017), 1–14, doi: 10.1007/s11071-016-3299-7. 1