```python
def update_nodes_mapping(star_membership, new_sm, first_iter):
    """After a collapse step, update the mapping between node
    indices in the new graph and original ones."""
    if first_iter:
        star_membership = new_sm
    else:
        new_mm = {}
        for orig_nodes, previous_star in star_membership.items():
            new_mm[orig_nodes] = new_sm[previous_star]
        star_membership = new_mm
    original_basis = defaultdict(set)
    for orig_id, star_id in star_membership.items():
        original_basis[star_id].add(orig_id)
    return star_membership, original_basis
```