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Ví dụ 1:

$$T(n) = \begin{cases} 2T(n-1) - 1 & \text{if } n > 0 \\ 1 & \text{otherwise} \end{cases}$$

$T(n) = 2T(n-1) - 1$

$= 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2^1 - 1$

$= \dots$

$= 2^n T(1) - 2^{n-1} - 2^{n-2} - \dots - 2 - 1$

$= 2^n - (2^{n-1} + 2^{n-2} + \dots + 2 + 1)$

$= 2^n - \frac{u_1(1 - q^n)}{1 - q}$

$= 2^n - \frac{1 - 2^n}{-1} = 1$

$\Rightarrow T(n) = O(1)$

VD 2:

$$T(n) = \begin{cases} 3T(n-1) & \text{if } n > 0 \\ 1 & \text{otherwise} \end{cases}$$

$T(n) = 3T(n-1)$

$= 3^2 T(n-2)$

$= \dots = 3^n T(0) = O(3^n)$

Ví dụ 3:

1. $$T(n) = \begin{cases} 2T(n/2) + 6n - 1 & \text{otherwise} \\ 1 & n = 1 \end{cases}$$

Sd Định lý chính

$a = 2, b = 2, f(n) = 6n + 1$

Ta có: $n^{\log_b a} = n$

$T(n) = 2T\left(\frac{n}{2}\right) + 6n - 1$

$= 2\left[T\left(\frac{n}{n}\right) + 3n - 1\right] + 6n - 1$

KLONG

$= 2^2 T\left(\frac{n}{4}\right) + 12n - 2 + 6n - 1$

$= 2^2 T\left(\frac{n}{4}\right) + 12n - 3$

$= \dots$

$= 2^k T\left(\frac{n}{2^k}\right) + (6 + 12 + 18 + \dots + 6 \cdot 2^{k-1})n - (1 + 2 + 3 + \dots + k)$

$= 2^k T\left(\frac{n}{2^k}\right) + (6 \cdot (2^k - 1))n - \frac{k(k+1)}{2}$

Ta có $T(1) = 1 \rightarrow \frac{n}{2^k} = 1$

$\Rightarrow k = \log_2 n$

Thay $k = \log_2 n$ vào:

$T(n) = 2^{\log_2 n} T(1) + (6 \cdot (2^{\log_2 n} - 1))n - \frac{(\log_2 n)(\log_2 n + 1)}{2}$

$= n \cdot 1 + (6 \cdot (n-1))n - \frac{\log_2 n (\log_2 n + 1)}{2}$

$= n + 6n^2 - 6n - \frac{\log_2 n (\log_2 n + 1)}{2}$

$\rightarrow O(n^2)$

2. $T(n) = \begin{cases} 2 \\ 4T\left(\frac{n}{3}\right) + 3n - 5 \end{cases}$

$T(n) = 4T\left(\frac{n}{3}\right) + 3n - 5$

$= 4\left[4T\left(\frac{n}{9}\right) + n - 5\right] + 3n - 5$

$= 4^2 T\left(\frac{n}{9}\right) + 4n - 4 \cdot 5 + 3n - 5$

$= 4^2 T\left(\frac{n}{9}\right) + (3n + 4n) - (5 + 20)$

$= \dots$

$= 4^k T\left(\frac{n}{3^k}\right) + (3 + 12 + 48 + \dots + 4^k \cdot 3)n - (5 + 10 + 15 + \dots + 5 \cdot 4^{k-1})$

$= 4^k T\left(\frac{n}{3^k}\right) + \left(3 \cdot \frac{(4^k - 1)}{3}\right)n - \left(5 \cdot \frac{(4^k - 1)}{3}\right)$

KI.ÔNG

$T(n) = 2$

$\Rightarrow \frac{n}{3^k} = 2 \rightarrow n = 2 3^k \rightarrow k = \log_3 \frac{n}{2}$

$T(n) = 4^{\log_3 \frac{n}{2}} T(1) + \left(3.\left(\frac{4^{\log_3 \frac{n}{2}} - 1}{3}\right)\right) n - 5.\frac{14^{\log_3 \frac{n}{2}} - 1}{3}$

$T(n) = 2n + (3.(n-1))n - (5.(n-1))$

$= 3n^2 - 6n + 5$

$O(n^2)$.

3. $T(n) = \begin{cases} 3T(\frac{n}{2}) + n^2 - n \\ 1 \end{cases}$

Định lý chính

$a = 3, \ b = 2 \qquad f(n) = n^2 - n$

~~n=~~ log Ta có:

$n^{\log_2 3 + \varepsilon} = n^2$ với $\varepsilon = 1$

$T(n) = O(n^2 - n)$

Bài tập 1:

1. a) $x(n) = 2x(n-3), \ n > 1 \quad x(1) = 1$

$x(n) = 2x(n-3)$

$= 2^2 x(n-6) = 2^3 x(n-9)$

$= \ldots = 2^{n/3} x(1)$

$\Rightarrow x(n) = O(2^{n/3})$

b) $x(n) = x(n-2) - 2 \qquad n > 1 \quad x(1) = 0$

$= x(n-4) - 4 = \ldots = x(n-2k) - 2k$

$n - 2k = 1 \Rightarrow k = \frac{n-1}{2}$

$x(n) = x(1) - 2\left(\frac{n-1}{2}\right) = -n + 1$

2. $T(n) = 3T(n-1) + 2$

$= 3^2 T(n-2) + 3.2 + 2$

$= \ldots$

$= 3^{n-1} T(1) + 3^{n-2}.2 + 3^{n-3}.2 + \ldots + 3.2 + 2$

$= 3^{n-1} + 2(3^{n-2} + 3^{n-3} + \ldots + 3 + 1)$

$= 3^{n-1} + 2\left(\frac{3^{n-1} - 1}{3-1}\right)$

KI.ONG

$= 3^{n-1} + 2\left(\frac{3^{n-1}}{2} - \frac{1}{2}\right)$

$= 3^{n-1} + 3^n - 1$

$T(n) = 3^n + 3^{n-1} - 1$

$O(3^n)$

- $T(1) = 3 \; ; \; T(n) = T(n-1) + 2n - 3$

$T(n) = T(n-1) + 2n - 3$

$= T(n-2) + 2(n-1) + 2n - 6$

$= T(n-2) + 4n - 4$

$= T(n-3) + 6n - 7$

$= \ldots$

$= 2n^2 + 5n + 1$

$O(n^2)$

- $T(n) = 2\,T(n-1)\,n - 1 \; ; \; T(1) = 1$

$= 2^2 T(n-2) + 3n - 3$

$= 2^3 T(n-3) + 7n - 7$

$= 2^k T(n-k) + (2^0 + 2^1 + \ldots + 2^{k-1})\,n$

$- (1 + 2 + 4 + \ldots + 2^{k-1})$

$= 2^k T(n-k) + (2^k - 1)\,n - (2^k - 1)$

$T(1) = 1 \rightarrow n - k = 1 \rightarrow k = n - 1$

$T(n) = 2^{n-1} n - n + 1$

$O(2^n)$