UNIVERSIDADE FEDERAL DO ABC

Tabela de Derivadas, Integrais e Identidades Trigonométricas

Derivadas

Regras de Derivação

- $(cf(x))' = cf'(x)$
- Derivada da Soma
$$(f(x) + g(x))' = f'(x) + g'(x)$$
- Derivada do Produto
$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$
- Derivada do Quociente
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
- Regra da Cadeia
$$(f(g(x))' = (f'(g(x))g'(x)$$

Funções Simples

- $\frac{d}{dx}c = 0$
- $\frac{d}{dx}x = 1$
- $\frac{d}{dx}cx = c$
- $\frac{d}{dx}x^c = cx^{c-1}$
- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -x^{-2} = -\frac{1}{x^2}$
- $\frac{d}{dx}\left(\frac{1}{x^c}\right) = \frac{d}{dx}\left(x^{-c}\right) = -\frac{c}{x^{c+1}}$
- $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},$

Funções Exponenciais e Logarítmicas

- $\frac{d}{dx}e^x = e^x$
- $\frac{d}{dx}\ln(x) = \frac{1}{x}$
- $\frac{d}{dx}a^x = a^x \ln(a)$

Funções Trigonométricas

- $\frac{d}{dx}\operatorname{sen} x = \cos x$
- $\frac{d}{dx}\cos x = -\operatorname{sen} x,$
- $\frac{d}{dx}\operatorname{tg} x = \sec^2 x$
- $\frac{d}{dx}\sec x = \operatorname{tg} x \sec x$
- $\frac{d}{dx}\operatorname{cotg} x = -\operatorname{cossec}^2 x$
- $\frac{d}{dx}\operatorname{cossec} x = -\operatorname{cossec} x \operatorname{cotg} x$

Funções Trigonométricas Inversas

- $\frac{d}{dx}\operatorname{arcsen} x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\operatorname{arctg} x = \frac{1}{1+x^2}$
- $\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\operatorname{arccotg} x = \frac{-1}{1+x^2}$
- $\frac{d}{dx}\operatorname{arccossec} x = \frac{-1}{|x|\sqrt{x^2-1}}$

Funções Hiperbólicas

- $\frac{d}{dx}\operatorname{senh} x = \cosh x = \frac{e^x+e^{-x}}{2}$
- $\frac{d}{dx}\cosh x = \operatorname{senh} x = \frac{e^x-e^{-x}}{2}$
- $\frac{d}{dx}\operatorname{tgh} x = \operatorname{sech}^2 x$
- $\frac{d}{dx}\operatorname{sech} x = -\operatorname{tgh} x \operatorname{sech} x$
- $\frac{d}{dx}\operatorname{cotgh} x = -\operatorname{cossech}^2 x$

Funções Hiperbólicas Inversas

- $\frac{d}{dx}\operatorname{csch} x = -\coth x \operatorname{cossech} x$
- $\frac{d}{dx}\operatorname{arcsenh} x = \frac{1}{\sqrt{x^2+1}}$
- $\frac{d}{dx}\operatorname{arccosh} x = \frac{1}{\sqrt{x^2-1}}$
- $\frac{d}{dx}\operatorname{arctgh} x = \frac{1}{1-x^2}$
- $\frac{d}{dx}\operatorname{arcsech} x = \frac{-1}{x\sqrt{1-x^2}}$
- $\frac{d}{dx}\operatorname{arccoth} x = \frac{1}{1-x^2}$
- $\frac{d}{dx}\operatorname{arccossech} x = \frac{-1}{|x|\sqrt{1+x^2}}$

Integrais

Regras de Integração

- $\int cf(x)\,dx = c\int f(x)\,dx$
- $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
- $\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$

Funções Racionais

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \qquad \text{para } n \neq -1$
- $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + c$
- $\displaystyle\int \frac{du}{1+u^2} = \operatorname{arctg} u + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\operatorname{arctg}(x/a) + c$
- $\displaystyle\int \frac{du}{1-u^2} = \begin{cases} \operatorname{arctgh} u + c, & \text{se } |u| < 1 \\ \operatorname{arccotgh} u + c, & \text{se } |u| > 1 \end{cases} = \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| + c$

Funções Logarítmicas

- $\int \ln x\,dx = x\ln x - x + c$
- $\int \log_a x\,dx = x\log_a x - \frac{x}{\ln a} + c$

Funções Irracionais

- $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \operatorname{arcsen} u + c$
- $\displaystyle\int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec} u + c$
- $\displaystyle\int \frac{du}{\sqrt{1+u^2}} = \operatorname{arcsenh} u + c = \ln|u+\sqrt{u^2+1}| + c$
- $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \operatorname{arccosh} u + c = \ln|u+\sqrt{u^2-1}| + c$
- $\displaystyle\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{arcsech} |u| + c$
- $\displaystyle\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{arccosech} |u| + c$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \operatorname{arcsen}\frac{x}{a} + c$
- $\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \arccos\frac{x}{a} + c$

Funções Trigonométricas

- $\int \cos x\,dx = \operatorname{sen} x + c$
- $\int \operatorname{sen} x\,dx = -\cos x + c$
- $\int \operatorname{tg} x\,dx = \ln|\sec x| + c$
- $\int \csc x\,dx = \ln|\csc x - \cot x| + c$
- $\int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + c$
- $\int \cot x\,dx = \ln|\operatorname{sen} x| + c$
- $\int \sec x \operatorname{tg} x\,dx = \sec x + c$
- $\int \csc x \cot x\,dx = -\csc x + c$
- $\int \sec^2 x\,dx = \operatorname{tg} x + c$
- $\int \csc^2 x\,dx = -\cot x + c$
- $\int \operatorname{sen}^2 x\,dx = \frac{1}{2}(x - \operatorname{sen} x\cos x) + c$
- $\int \cos^2 x\,dx = \frac{1}{2}(x + \operatorname{sen} x\cos x) + c$

Funções Hiperbólicas

- $\int \sinh x\,dx = \cosh x + c$
- $\int \cosh x\,dx = \sinh x + c$
- $\int \operatorname{tgh} x\,dx = \ln(\cosh x) + c$
- $\int \operatorname{csch} x\,dx = \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c$
- $\int \operatorname{sech} x\,dx = \operatorname{arctg}(\sinh x) + c$
- $\int \coth x\,dx = \ln|\sinh x| + c$

Identidades Trigonométricas

1. $\operatorname{sen}(90^{\circ} - \theta) = \cos\theta$
2. $\cos(90^{\circ} - \theta) = \operatorname{sen}\theta$
3. $\dfrac{\operatorname{sen}\theta}{\cos\theta} = \operatorname{tg}\theta$
4. $\operatorname{sen}^2\theta + \cos^2\theta = 1$
5. $\sec^2\theta - \operatorname{tg}^2\theta = 1$
6. $\csc^2\theta - \cot^2\theta = 1$
7. $\operatorname{sen} 2\theta = 2\operatorname{sen}\theta\cos\theta$
8. $\cos 2\theta = \cos^2\theta - \operatorname{sen}^2\theta = 2\cos^2\theta - 1$
9. $\operatorname{sen} 2\theta = 2\operatorname{sen}\theta\cos\theta$
10. $\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$
11. $\cos(\alpha \pm \beta) = \cos\alpha\operatorname{sen}\beta \pm \operatorname{sen}\alpha\cos\beta$
12. $\operatorname{tg}(\alpha \pm \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$
13. $\operatorname{sen}\alpha \pm \operatorname{sen}\beta = 2\operatorname{sen}\dfrac{1}{2}(\alpha \pm \beta)\cos\dfrac{1}{2}(\alpha \pm \beta)$
14. $\cos\alpha + \cos\beta = 2\cos\dfrac{1}{2}(\alpha + \beta)\cos\dfrac{1}{2}(\alpha - \beta)$
15. $\cos\alpha - \cos\beta = 2\operatorname{sen}\dfrac{1}{2}(\alpha + \beta)\operatorname{sen}\dfrac{1}{2}(\alpha - \beta)$