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Research paper

Marshall–Olkin Power Lomax distribution for modeling of wind speed data

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ABSTRACT

Accurate collection of wind speed records is significant for numerous wind power applications. The present investigation aims to highlight the use of the Marshall–Olkin Power Lomax (MOPLx) distribution for wind speed data. We examine the actual wind speed records gathered from three stations Bahawalpur, Gwadar, and Haripur. The dataset is demonstrated by using MOPLx distribution and compare its modeling performance with renowned probability distributions, for example, Weibull–Lomax, power Lomax, Weibull, power Lindley, Lindley, and Lomax. Findings indicate that MOPLx distribution gives the best fitting as per model evaluation criteria, Akaike information criterion (AIC), Bayesian information criterion (BIC), Kolmogorov Smirnov test (KS), coefficient of determination (R^2) and root mean square error (RMSE). Overall, the results demonstrate the feasibility, precision, and effectiveness of the MOPLx distribution for portraying wind speed modeling. It is also observed that the MOPLx model is ideal in terms of the power density error (PDE) criterion.

1. Introduction

Energy is the key to sustainable development in countries. Wind energy is a maintainable source of energy, free of pollution and carbon and worldwide its growth is increasing in many areas. In reality, the environmental benefits of using wind energy together with modern scientific and profitable enhancements build wind power a significant supplier for the present and future mix of worldwide energy. Although the new generation of wind turbines has permitted monetarily successful development in places where wind power is low, it is imperative to investigate the characteristics of wind energy at potential locations. Most nations utilized fossil fuels, also known as conventional energy sources, for heating, production, shipping, etc. Arslan et al. (2017).

Providing reliable wind velocity and wind power supply information is essential to describe wind velocity performance, as well as to assess the potential of wind energy and the performance of wind power organizations at prospective locations. In this sense, probability density functions (PDFs) have developed and employed to describe wind speed and wind power distribution in various areas of the globe (Lo Brano et al., 2011; Zhou et al., 2010; Irwanto et al., 2014; Ouarda et al., 2015; Alavi et al., 2016). The precision of these distributions, which are qualified by their capacity to comply with experimental figures have an important impact on the exactness and ambiguity of the expected wind production at a specified area.

Exploring the literature, some eminent PDFs have been employed to model the wind speed distribution. For instance an exponential (Alavi et al., 2016), Weibull and Rayleigh (Pishgar-komleh et al., 2015), Extreme Value (Sarkar et al., 2011), Generalized Extreme Value (Kollu et al., 2012), Gamma (Morgan et al., 2011), Logistic (Eneroy et al., 1996), Inverse Burr (Chiodo and De Falco, 2016), Inverse Log–logistic (Chiodo et al., 2018), Nakagami (Alavi et al., 2016), generalized Lindley and Power Lindley (Arslan et al., 2017), Birnbaum–Saunders (Mohammadi et al., 2017) for more details of probability models reader can see Johnson et al. (1994). Weibull (2 parameters) and Rayleigh distributions are most commonly applied to the purpose of Wind speed. Weibull distribution is considered the most preferable distribution due to its flexible and variable forms of distribution and hazard rate curves. Mahmood et al. (2019) used the Weibull distribution to analyze the wind speed data of Al-Salman city Iraq.

Regardless of the information that Weibull distribution is the traditional model and offers many advantages, such as a system with elevated percentages of null wind accelerates and bimodal

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Table 1
Probability density function and cummulative distribution function for selected distributions.

Model	PDF	CDF
Weibull (WD)	$f_{WD}(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right)$	$F_{WD}(x)=1-\exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right)$
Lomax (LxD)	$f_{LxD}(x)=\frac{\alpha\beta^{\alpha}}{(\beta+x)^{\alpha+1}}$	$F_{LxD}(x)=1-\frac{\beta^{\alpha}}{(\beta+x)^{\alpha}}$
Lindley (LD)	$f_{LD}(x)=\frac{\alpha^2}{\alpha+1}(1+x)e^{-\alpha x}$	$F_{LD}(x)=1-\left(\frac{1+\alpha+\alpha x}{1+\alpha}\right)e^{-\alpha x}$
Power Lindley (PLD)	$f_{PLD}(x)=\frac{\beta\alpha^2}{\alpha+1}\left(1+x^{\beta}\right)\upsilon^{\beta-1}e^{-\alpha x^{\beta}}$	$F_{PLD}(x)=1-\left(1+\frac{\alpha}{1+\alpha}x^{\beta}\right)e^{-\alpha x^{\beta}}$
Power Lomax (PLxD)	$f_{PLxD}(x)=\frac{\alpha\beta\lambda^{\alpha}x^{\beta-1}}{\left(\lambda+x^{\beta}\right)^{\alpha+1}}$	$F_{PLxD}(x)=1-\frac{\lambda^{\alpha}}{\left(\lambda+x^{\beta}\right)^{\alpha}}$
Weibull–Lomax (WLx)	$f_{WLxD}(x)=\frac{\alpha\beta\gamma}{\lambda}\left(1+\frac{x}{\lambda}\right)^{\beta\gamma-1}\left\{1-\left(1+\frac{x}{\lambda}\right)^{-\gamma}\right\}^{\beta-1}$ $\times\exp\left\{-\alpha\left\{\left(1+\frac{x}{\lambda}\right)^{\gamma}-1\right\}^{\beta}\right\}$	$F_{WLxD}(x)=1-\exp\left\{-\alpha\left\{\left(1+\frac{x}{\lambda}\right)^{\gamma}-1\right\}^{\beta}\right\}$

distributions. Therefore, numerous new and extended forms of the distributions have been proposed and studied. These distributions are commonly derived by add shape(s) parameters and a mixture of two distributions. For example, 3-parameter Weibull, generalized Lindley (Nadarajah et al., 2011), power Lindley (Ghitany et al., 2013).

The Lomax distribution (also known as Pareto Type II distribution) is well-known distributions used to model the actuarial sciences, business failure, size of cities, medical and biological sciences, income and wealth inequality, engineering, lifetime and reliability datasets. Lomax distribution has been considered as a more skewed alternative to the exponential, Weibull and gamma distributions (Bryson, 1974). It is also associated with the Burr family of distributions (Tadikamalla, 1980). For failure times, the Lomax distribution is related to the family of a non-increasing failure rate (Chahkandi and Ganjali, 2009). Lomax distribution has been used for modeling income and wealth data (Atkinson and Harrison, 1978), for firm size data (Corbellini et al., 2010), for reliability and life testing (Harris, 1968), receiver operating characteristic (ROC) curve analysis (Campbell and Ratnaparkhi, 1993) and Hirsch-related statistics (Glänzel, 1987).

It is obvious from the literature that the utilization of renowned Weibull distribution in wind speed modeling may lead to erroneous results (Akgül and Şenoğlu, 2019). This experience is at hand when the distribution of the wind speed has more height or data incorporates extreme wind speeds on the right tail. In such cases, the Weibull distribution lower guesses energy production and critics should find alternative distributions that capture the more height and positively skewed. In the current study, MOPLx distribution is proposed as an alternative wind speed distribution.

The motivation behind this work to propose the use of the Marshall–Olkin Power Lomax (MOPLx) distribution for an alternative to study the wind speed data. MOPLx distribution is more flexible with three shape and one scale parameters. Considering this potential MOPLx and some well-known wind speed models, Weibull (W), Lomax (Lx), Lindley (L), Power Lindley (PL), Power-Lomax (POLx), and Weibull–Lomax (WLx) distributions are compared on wind speed data from different regions of Pakistan, Bahawalpur, Gawadar and Haripur.

2. Modeling methodology

2.1. The MOPLx distribution

Four-parameter Marshall–Olkin Power Lomax distribution proposed by Haq et al. (2020) with the cumulative distribution function (CDF) is given by

$$F(x)=\frac{1-\lambda^{\alpha}\left(\lambda+x^{\beta}\right)^{-\alpha}}{1-(1-\gamma)\left(\lambda^{\alpha}\left(\lambda+x^{\beta}\right)^{-\alpha}\right)},\quad x\geq 0. \tag{1}$$

The corresponding probability density function (PDF) is

$$f(x)=\frac{\gamma\alpha\beta\lambda^{\alpha}x^{\beta-1}\left(\lambda+x^{\beta}\right)^{-\alpha-1}}{\left[1-(1-\gamma)\lambda^{\alpha}\left(\lambda+x^{\beta}\right)^{-\alpha}\right]^2},\quad x\geq 0, \tag{2}$$

where $\alpha,\beta,\gamma,\lambda$ are positive parameters. Significant probability distributions are special cases from the MOPLx distribution such as the Marshall–Olkin Lomax ($\beta=1$), power Lomax ($\gamma=1$) distribution. The PDF and CDF curves for MOPLx distribution are presented in Fig. 1.

The PDF and CDF of the competitor probability distributions are given in Table 1.

3. Model evaluation

The selection of the best fit model for wind speed analysis is carried out using renowned and reliable goodness-of-fit criteria. The goodness-of-fit measures are the Akaike information criterion (AIC), Bayesian information criterion (BIC). Furthermore, we also consider the coefficient of determination (R^2) and Root mean square error (RMSE) are as follows:

$$R^2=1-\frac{\sum_{i=1}^{n}\left(\widehat{F}\left(X_{(i)}\right)-\frac{i}{n+1}\right)^2}{\sum_{i=1}^{n}\left(\widehat{F}\left(X_{(i)}\right)-\overline{\widehat{F}}\left(X_{(i)}\right)\right)^2} \tag{3}$$

$$RMSE=\left[\frac{1}{n}\sum_{i=1}^{n}\left(\widehat{F}_i-\frac{i}{n+1}\right)^2\right]^{\frac{1}{2}} \tag{4}$$

$$AIC=2K-2\ln L \tag{5}$$

$$KS=\max_{1\leq i\leq n}\left|\widehat{F}_i-\frac{i}{n+1}\right| \tag{6}$$

where $\widehat{F}_i$ is estimated cdf for the ith ordered observation, $\overline{\widehat{F}}=(1/n)\sum_{i=1}^{n}\widehat{F}\left(X_{(i)}\right)$, n is the sample size, k is the number of parameters, L is the likelihood value.

Moreover, Power Density Error (PDE) can also be measured to decide on the best model and it is given by

$$PDE=\left|\frac{P_{ref}-PDE}{P_{ref}}\right|\times 100 \tag{7}$$

where P_{ref} is the mean wind power density depends on observations and PDE is the mean power density based on the

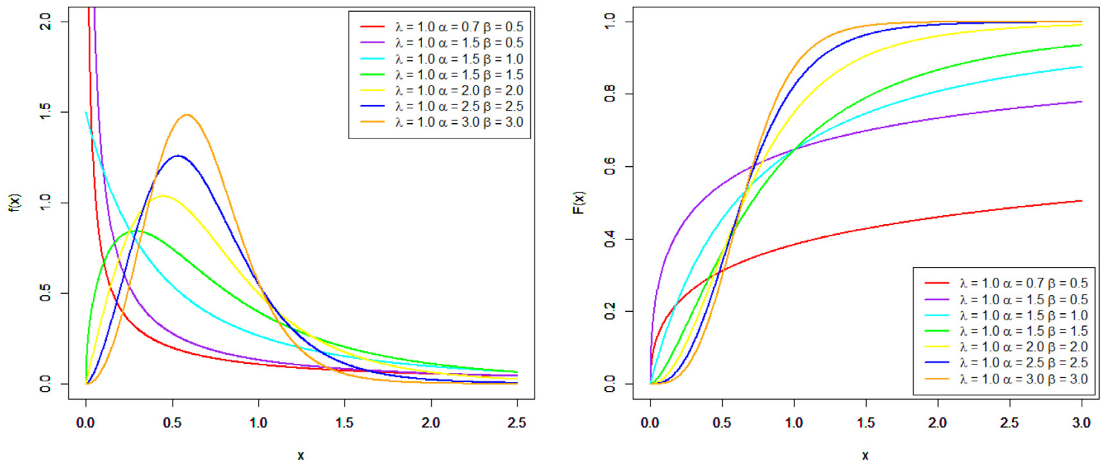

Fig. 1. PDF and CDF curves for selected parameter values.

Table 2
Location coordinates and elevations for the Bahawalpur, Gwadar and Haripur stations.

Stations	Latitude	Longitude	Altitude	Time interval	Height
Bahawalpur	29.3544°N	71.6911°E	214 m	10-min	20 m
Gwadar	25.2460°N	62.2861°E	7 m	10-min	20 m
Haripur	33.9946°N	72.9106°E	520 m	10-min	20 m

Table 3
Descriptive statistics of wind speed data for the Bahawalpur, Gwadar and Haripur stations.

Station	Maximum (m/s)	Mean	Variance	Skewness	Kurtosis	n
Bahawalpur	20.073	3.5049	2.8053	1.1347	6.0846	68 769
Gwadar	14.560	3.8650	5.8548	0.9309	3.3810	69 831
Haripur	17.909	2.8253	2.0746	1.4330	7.9588	69 836

distributional model. P_{ref} and PDE are computed by using the following formula as:

$$P_{ref} = \frac{1}{2}\rho A \frac{1}{n}\sum_{i=1}^{n} x_i^3 \tag{8}$$

and

$$PDE = \frac{1}{2}\rho A \int_0^{\infty} x^3 f_D(x)\, dx \tag{9}$$

Here, ρ is air density (kg/m^3) and A is the wind turbine blade sweep area (m^2). The indicator D stands for the distribution of interest, i.e. MOPLx or WLx, etc.

4. Results and discussion

4.1. Description of wind speed data

In the present investigation the wind speed data collected at 20 m height for the period of 2017–18, is obtained from the Pakistan State Meteorological Service at the heights of 10 m, on an hourly basis. It comprises wind speeds gained from 3 meteorological stations situated at Bahawalpur, Gwadar, and Haripur in Pakistan. The geological data about these stations is given in Table 2. Descriptive statistics for these data are arranged in Table 3.

To compare the modeling performances of MOPLx, WLx, POLx, PL, W, Lx and L distributions. Firstly, the model parameters are estimated using the maximum likelihood approach. Table 4 provides the estimated value for MOPLx distribution and competitor models for the wind speed data of Bahawalpur, Gwadar and Haripur stations respectively. Then goodness-of-fit measures, R^2, RMSE, AIC, BIC and -2LogLik calculated for each model and are tabulated in Table 5.

According to the tables, wind speed data for Gwadar and Haripur sites, the goodness of fit measures emphasize that the MOPLx distribution provides a better fit as compared to the well-known Weibull model and other competitors' models. Besides, Figs. 2–4 show the fitting ability of distributions for Bahawalpur, Gwadar and Haripur sites. It can be also seen the figures that MOPLx distribution is a better fit than the other distributions for three of the stations.

Probability density errors (PDE) values, demonstrated in Table 6 for all distributions, can also be used to evaluate the goodness of distributions. It is observed that MOPLx and POLx models are more preferable than other models. However, in all three stations, MOPLx distribution is the most appropriate model in terms of the PDE values.

Besides, Figs. 2–4 show the fitting ability of distributions for Bahawalpur, Gwadar and Haripur sites. It can be also seen the figures that MOPLx distribution has better fitting than the other distributions for three of the stations. Overall, the results indicate that using MOPLx distribution is more desirable than using competitor distributions to model the wind speed data.

5. Conclusion

The determination of wind speed characteristics and evaluation of wind energy potential highly depends greatly on appropriate wind speed distribution. In this work, we proposed the use of the Marshall–Olkin Power Lomax (MOPLx) distribution as a wind speed model. The estimation of parameters is computed by the maximum likelihood approach. Further, we used MOPLx distribution to describe wind speed data obtained from Bahawalpur, Gwadar, and Haripur. This study showed that the MOPLx distribution is more appropriate to model such wind speed data than many important distributions that are commonly used in the literature. These results are based on the AIC, BIC, RMSE, and R^2 values. Overall, the MOPLx distribution provides a better fit and can be considered a good model for a skewed dataset and can be considered as a wind speed distribution. In nutshell, we can wind up that MOPLx distribution is good alternatives to the Weibull–Lomax, Weibull, Power Lindley, Power Lomax, Lomax distributions due to the flexibility and superiority on modeling the more height and positively skewed of the wind speed distribution. Based on the present study, it is recommended that the MOPLx distribution can be applied to forecast and estimate the wind speed at wind stations. The present study can be extended using neutrosophic statistics as future research.

Table 4
Estimates of parameters for all stations.

Stations		MOPLx	WLx	POLx	W	Lx	PL	L
Bahawalpur	α	13.8796	248.399	3.66695	2.15546	345.5677	0.200540	0.478314
	β	0.82320	2.54761	2.61412	3.94342	1208.045	1.670651	–
	λ	5.60046	6.92208	106.809	–	–	–	–
	γ	225.274	0.24356	–	–	–	–	–
Gwadar	α	34.7046	251.552	8.59040	1.676791	362.5150	0.286999	0.438615
	β	0.43359	2.44417	1.78863	4.342821	1398.404	1.297042	–
	λ	8.27739	2.57858	108.530	–	–	–	–
	γ	627.602	0.10305	–	–	–	–	–
Haripur	α	7.916748	307.4208	2.651855	2.064102	449.1585	0.288043	0.578211
	β	0.840692	3.632763	2.679724	3.193989	1267.336	1.634536	–
	λ	2.073689	0.918593	43.91258	–	–	–	–
	γ	329.9499	0.127180	–	–	–	–	–

Table 5
Goodness-of-fit measures for all three stations.

Station	Model	-2LogLik	AIC	BIC	R^2	RMSE	Sum of ranks	Rank
Bahawalpur	MOPLx	254 939.0$^{(1)}$	254 947.0$^{(1)}$	254 983.6$^{(1)}$	0.999561$^{(1)}$	0.006005$^{(1)}$	5	1
	WLx	260 583.8$^{(4)}$	260 591.8$^{(4)}$	260 628.4$^{(4)}$	0.988851$^{(4)}$	0.028539$^{(4)}$	20	4
	POLx	258 945.4$^{(2)}$	258 951.4$^{(2)}$	258 978.8$^{(2)}$	0.995230$^{(2)}$	0.019194$^{(2)}$	10	2
	W	261 367.8$^{(5)}$	261 371.8$^{(5)}$	261 390.1$^{(5)}$	0.984532$^{(5)}$	0.033173$^{(5)}$	25	5
	Lx	310 187.2$^{(7)}$	310 191.2$^{(7)}$	310 209.5$^{(7)}$	0.509283$^{(6)}$	0.164958$^{(7)}$	34	7
	PL	258 970.4$^{(3)}$	258 974.4$^{(3)}$	258 992.7$^{(3)}$	0.990063$^{(3)}$	0.025999$^{(3)}$	15	3
	L	289 559.4$^{(6)}$	289 563.4$^{(6)}$	289 581.7$^{(6)}$	0.706956$^{(5)}$	0.124187$^{(5)}$	28	6
Gwadar	MOPLx	303 238.1$^{(1)}$	303 246.0$^{(1)}$	303 282.6$^{(1)}$	0.998118$^{(1)}$	0.012840$^{(1)}$	5	1
	WLx	306 520.2$^{(5)}$	306 528.2$^{(5)}$	306 564.8$^{(5)}$	0.996269$^{(3)}$	0.018120$^{(3)}$	21	4
	POLx	303 833.4$^{(2)}$	303 839.4$^{(2)}$	303 866.9$^{(2)}$	0.996407$^{(2)}$	0.017596$^{(2)}$	10	2
	W	304 005.9$^{(3)}$	304 010.0$^{(3)}$	304 028.3$^{(3)}$	0.994939$^{(4)}$	0.020684$^{(4)}$	17	3
	Lx	328 585.8$^{(7)}$	328 589.8$^{(7)}$	328 608.1$^{(7)}$	0.807745$^{(7)}$	0.104025$^{(7)}$	28	6
	PL	304 573.2$^{(4)}$	304 577.2$^{(4)}$	304 595.5$^{(4)}$	0.994882$^{(5)}$	0.020917$^{(5)}$	22	5
	L	313 626.2$^{(6)}$	313 630.2$^{(6)}$	313 648.5$^{(6)}$	0.952928$^{(6)}$	0.054669$^{(6)}$	30	7
Haripur	MOPLx	234 070.3$^{(1)}$	234 076.2$^{(1)}$	234 103.7$^{(1)}$	0.999947$^{(1)}$	0.003176$^{(1)}$	5	1
	WLx	235 152.8$^{(4)}$	235 160.8$^{(4)}$	235 197.4$^{(4)}$	0.998941$^{(2)}$	0.009158$^{(2)}$	16	4
	POLx	235 060.1$^{(2)}$	235 068.2$^{(2)}$	235 104.8$^{(2)}$	0.998427$^{(4)}$	0.011626$^{(4)}$	14	2
	W	238 532.6$^{(5)}$	238 536.6$^{(5)}$	238 554.9$^{(4)}$	0.994115$^{(5)}$	0.020881$^{(5)}$	24	5
	Lx	284 853.4$^{(7)}$	284 857.4$^{(7)}$	284 875.8$^{(7)}$	0.555853$^{(7)}$	0.153511$^{(7)}$	35	7
	PL	237 739.5$^{(3)}$	237 743.6$^{(3)}$	237 761.9$^{(3)}$	0.9963423$^{(3)}$	0.0167476$^{(3)}$	15	3
	L	266 678.2$^{(6)}$	266 682.2$^{(6)}$	266 700.5$^{(6)}$	0.746358$^{(5)}$	0.114491$^{(5)}$	28	6

Table 6
Power density error for wind speed data of Bahawalpur, Gwadar and Haripur stations.

	P_{ows}	P_{MOPLx}	P_{WLx}	P_{POLx}	P_W	P_{Lx}	P_{PL}	P_L
Bahawalpur	599.2	593.5	582.0	589.2	582. 9	2006.6	574.3	1277. 8
Gwadar	1067.7	1050.1	1152.2	1094.3	1046.8	2693.9	1116. 7	1687.7
Haripur	341.7	334.8	323.2	371.4	322.7	1050.8	316.5	692. 7
		PDE_{MOPLx}	PDE_{WLx}	PDE_{POLx}	PDE_W	PDE_{Lx}	PDE_{PL}	PDE_L
Bahawalpur		0.9440	2.8646	1.6595	2.7164	234.90	4.1411	273.94
Gwadar		1.6487	7.9108	2.4861	1.9615	152.30	2.0466	58.067
Haripur		2.0274	5.4307	8.6904	5.5762	207.53	7.3863	102.70

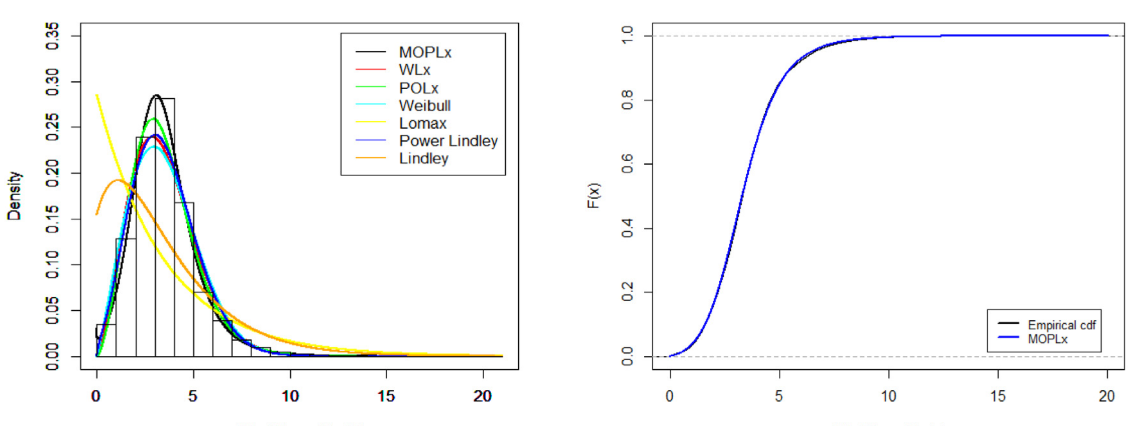

Fig. 2. The histogram and fitted density of models for wind speed data at Bahawalpur station.

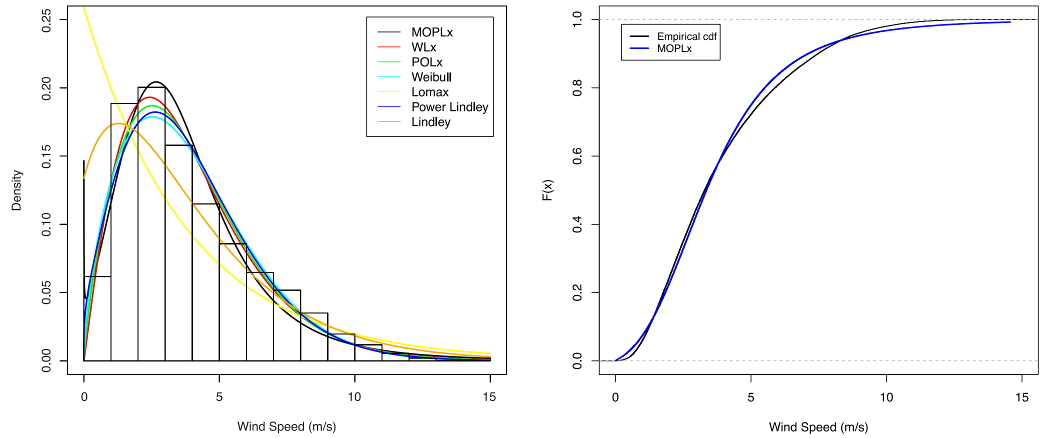

Fig. 3. The histogram and fitted density of models for wind speed data at Gwadar station.

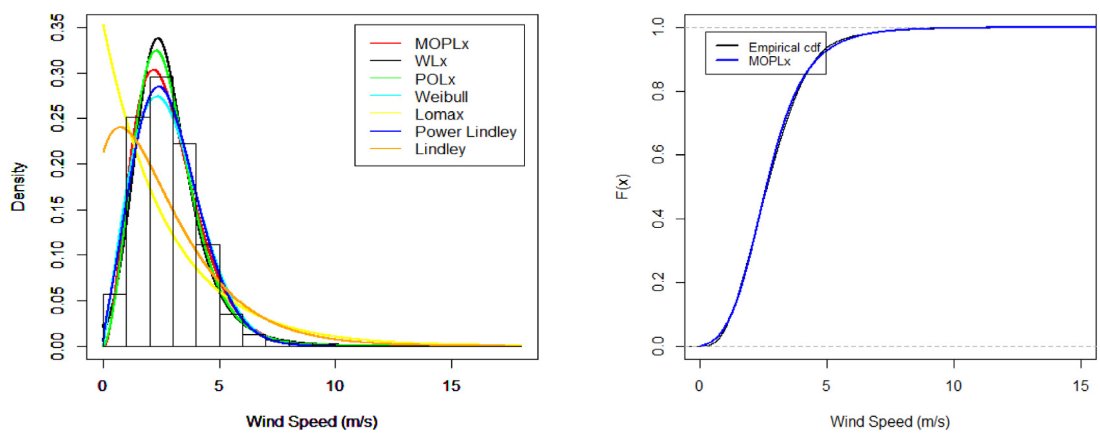

Fig. 4. The histogram and fitted density of models for wind speed data at Haripur station.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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