Assignment 7

Quantum Ising model

December 4th 2023

QUANTUM ISING MODEL

Consider N spin-1/2 particles on a one-dimensional lattice, described by the Hamiltonian:

$$\hat{H} = \lambda \sum_{i}^{N} \sigma_i^z + \sum_{i}^{N-1} \sigma_i^x \sigma_{i+1}^x$$

where σ_x and σ_z are the Pauli matrices and λ is the strength of the external field.

1) Write a program to compute the $2N \times 2N$ matrix representation of the Hamiltonian $\hat{H}$ for different size N.

2) Diagonalize $\hat{H}$ for different $N = 1,...,N_{max}$ and $\lambda \in [0, -3]$. What is the largest is N_{max} you can reach

3) Plot the first k levels as a function of λ for different N and comment on the spectrum.

Quantum Ising model is one of the simplest non trivial many-body quantum systems. Let's consider a linear chain of interacting spin-1/2 in the presence of an external field of intensity λ.

The two pieces of the Hamiltonian do not commute:

$$[\mathbb{I}_1 \otimes \sigma_2^z, \sigma_1^x \otimes \sigma_2^x] \neq 0$$

Remember that:

$$\sigma_i^z = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \ldots \otimes \sigma_i^z \otimes \mathbb{I}_{i+1} \ldots \otimes \mathbb{I}_N$$

$$\sigma_i^x \otimes \sigma_i^x = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \ldots \otimes \sigma_i^x \otimes \sigma_{i+1}^x \otimes \mathbb{I}_{i+2} \ldots \otimes \mathbb{I}_N$$

So the overall size is?

CODE DEVELOPMENT

Matrix representation of the operator $A \otimes B$, with $A = (M_r, M_c) \quad B = (N_r, N_c)$, labeled as:

$$A \otimes B = (M_r \times N_r, M_c \times N_c)$$

$$((ii-1)N_r + kk, (jj-1)N_c + mm) \text{ with } \begin{aligned} ii &\in [1, M_r] \\ jj &\in [1, M_c] \\ kk &\in [1, N_r] \\ mm &\in [1, N_c] \end{aligned}$$

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1M_c}B \\ A_{21}B & A_{22}B & \dots & A_{2M_c}B \\ \vdots & \vdots & \dots & \vdots \\ A_{M_r1}B & A_{M_r2}B & \dots & A_{M_rM_c}B \end{bmatrix}$$