Quantum Information and Computing

Assignment 6 (due in two weeks)

November 28, 2023

1. **Density Matrices** Consider a quantum system composed by N subsystems (spins, atoms, particles etc..) each described by a wave function $\psi_i \in \mathcal{H}^D$ where $\mathcal{H}^D$ is a D-dimensional Hilbert space. How do you write the total wave function of the system $\Psi(\psi_1, \psi_2, ..., \psi_N)$?

   (a) Write a code (Fortran or Python) to describe the composite system in the case of N-body non interacting, separable pure state;

   (b) and in the case of a general N-body pure wave function $\Psi \in \mathcal{H}^{DN}$;

   (c) Comment and compare their efficiency;

   (d) Given N=2, write the density matrix of a general pure state Ψ, $\rho = |\Psi\rangle\langle\Psi|$;

   (e) Given a generic density matrix of dimension D^N x D^N compute the reduced density matrix of either the left or the right system, e.g. $\rho_1 = \text{Tr}_2\rho$.

   (f) Test the functions described before (and all others needed) on two-spin one-half (qubits) with different states.