Quantum Information and Computing - Assignment 5

For this assignment, it was required to write an algorithm to numerically solve the time-dependent Schrödinger equation with the Hamiltonian of a harmonic oscillator that has its center moving linearly in time.

First thing, I have tweaked the harmonic_eig function defined for assignment 4 to better suit the needs of this usecase, removing the final part for checking the solution against the exact one and changing a bit the function signature to accept all the parameters needed.

The evolve function takes as arguments two functions for K and V, so that it might be used for different cases; K must be a function of the momentum, while V must be a function of time that returns a function of the position (eg. V=lambda t: lambda x: x**2/t would give an harmonic potential that widens over time); all the reference values for time and space discretization, error tolerance and sampling frequency may also be changed via the arguments.

The time evolution function then proceeds with the following steps:

- generate the grid in both real and momentum space
- define the operator that applies time evolution for the kinetic part; since this does not depend on time, it can be defined outside the evolution loop
- start looping over time; for each time step:
    - calculate the operator that applies time evolution for the potential part, already defined with dt/2 as needed in the Trotter decomposition
    - apply the Trotter-Suzuki decomposition for the timestep
    - check normalization, within a tolerance
    - every sample_t steps, save the wavefunction value
- finally, the saved values are returned

The overall algorithm uses these two functions and plots the results using an animation provided by matplotlib.

The results are close to expectation: with a faster movement of the potential, we see a more accentuated oscillation of the wavefunction’s center with respect to the potential center, while with a slower movement the oscillation becomes less and less evident; the wavefunction in momentum space oscillates around the value corresponding to the velocity used.