Quantum Information and Computing - Assignment 7

In this assignment we were required to calculate the Hamiltonian and find the eigenvalues of the Ising model in 1D.

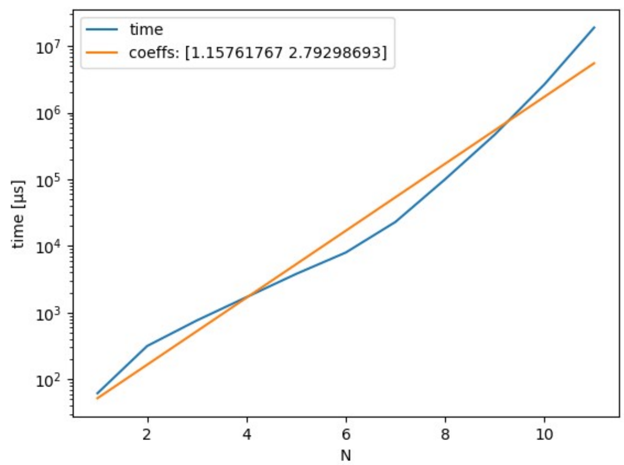
Firs of all, we were required to write the Hamiltonian $H=\lambda\sum_{i=0}^{N}\sigma_i^z+\sum_{i=0}^{N-1}\sigma_i^x\sigma_{i+1}^x$ in matrix form.

To achieve this, in the ising_hamiltonian function I summed up the various terms, using the pad function developed for the last assignment to find their full matrix form over the complete space. The function takes N and λ and returns the matrix; λ is called “l”, since “lambda” is a syntax term in python.

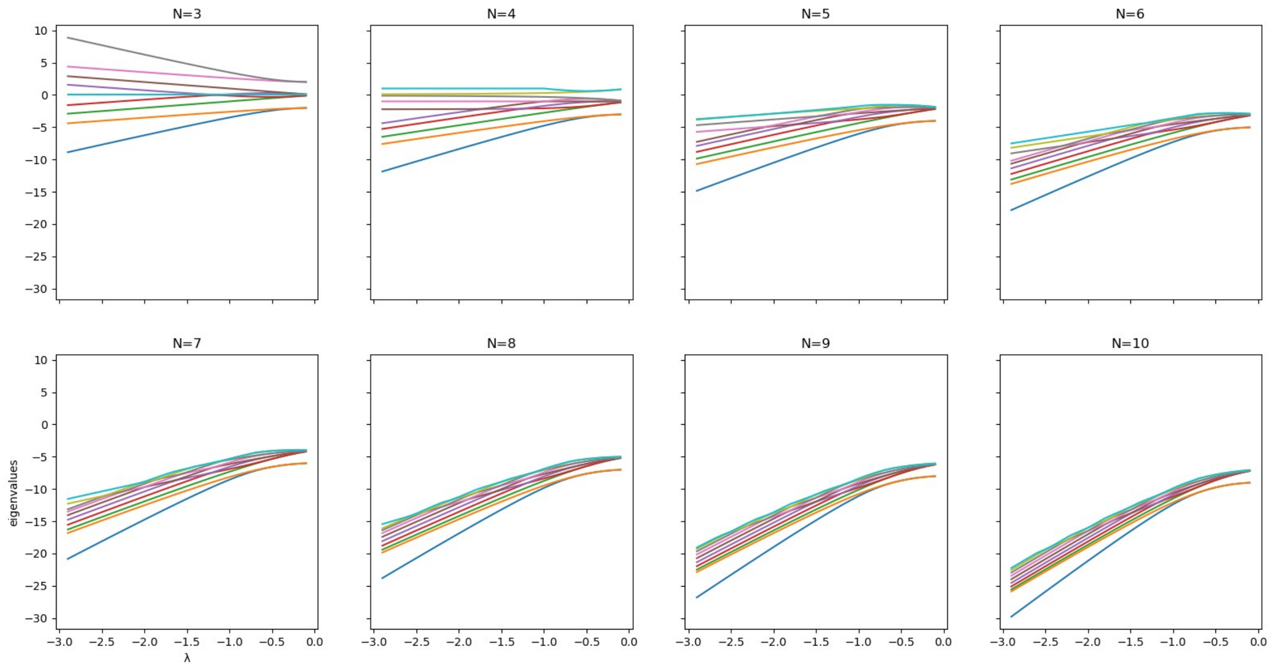
The diagonalization is done using the eigh function provided by the numpy.linalg module.

In terms of memory storage, N=14 is the last value for which the matrix can be stored on my laptop if the numbers are saved as complex128, while N=15 can be reached using float32, which can be done in this case as the Hamiltonian only has real entries. Both cases amount to the same amount of memory, 32 gigabits, approximately 4GB out of the 16GB I have available; it would seem possible to go an additional step further, however, accounting for system memory etc means there is not enough memory practically available.

To characterize diagonalization times I only took values up to N=11, due to the very long computation times. The exponential behavior is quite evident; on the right is a lin-log plot of the computation times for creation + diagonalization.



The orange line is the fit for the exponential, with law $t=e^{c_0 N+c_1}$; c_0 and c_1 are the coefficients printed in the legend.



Above are the plots of the first 10 eigenvalues for varying lambda, for different system sizes.

It can be seen that for large enough λ the ground state eigenvalue goes as N * λ, as expected.

For smaller system sizes, the eigenvalues are well spaced and distinct, since they are spanning a considerable portion of the whole spectrum (in the N=3 case, the whole spectrum); as N grows, the number of possible configurations with low energy increases, and we can see an ever greater number of values close together, though separated from the ground state.