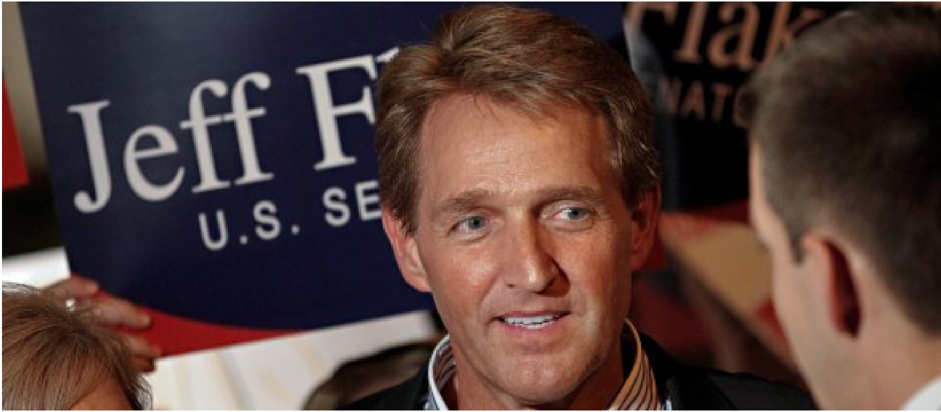
Unit 5: Clustering

Florida State Summer Methods Workshop

Dr. Rochelle Terman

Department of Political Science
University of Chicago

May 2019


Jeff F
U.S. SE
NAT

Today: Cluster press releases

Goal: partition documents such that:

- similar documents are together
- dissimilar documents are apart

Method: Clustering methods

Game Plan:

1) What makes two data points (i.e. documents) similar?
2) How do we find a good partition?
3) How do we interpret the clusters?

Key Terms:

- (Multidimensional) Space
- Distance
- Euclidean Distance
- Cosine Distance
- Cluster Analysis / Clustering
- K-means
- Centroid

K-Means Clustering

K-means clustering is popular method to partition a data set into K distinct, non-overlapping clusters.

K-Means Clustering

K-means clustering is popular method to partition a data set into K distinct, non-overlapping clusters.

Inputs

1. A document term matrix (or any multidimensional dataset)
2. K: the desired number of clusters.

K-Means Clustering

K-means clustering is popular method to partition a data set into K distinct, non-overlapping clusters.

Inputs

1. A document term matrix (or any multidimensional dataset)
2. K: the desired number of clusters.

Then the K-means algorithm will assign each observation into exactly one of the K clusters.

K-Means Clustering

K-means clustering is popular method to partition a data set into K distinct, non-overlapping clusters.

Inputs

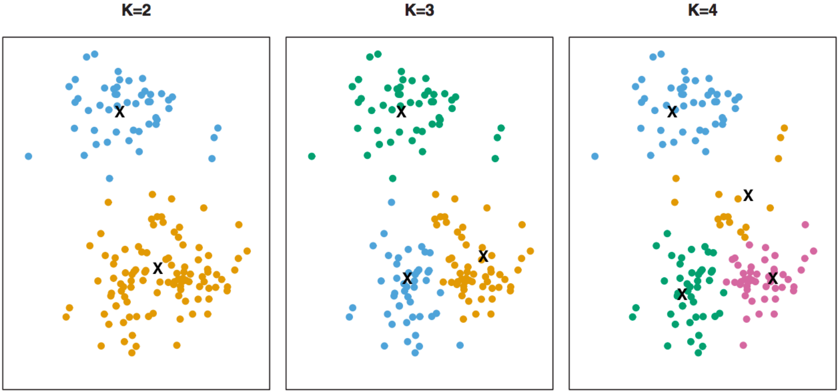
1. A document term matrix (or any multidimensional dataset)
2. K: the desired number of clusters.

Then the K-means algorithm will assign each observation into exactly one of the K clusters.

Outputs

1. C_k: The set of observations assigned to each cluster.
2. μ_k: The mean for each K – a vector representing the average values of all observations in that cluster. Also called centroid.

K-Means Clustering



K-Means Clustering: Outputs

Centroid (μ_k): The mean for each K – a vector representing the average values of all observations in that cluster.

K-Means Clustering: Outputs

Centroid (μ_k): The mean for each K – a vector representing the average values of all observations in that cluster.

Consider the following cluster with two vectors:

$$\begin{aligned} \boldsymbol{X_1} &= [1, 0, 3] \\ \boldsymbol{X_2} &= [0, 4, 1] \end{aligned}$$

K-Means Clustering: Outputs

Centroid (μ_k): The mean for each K – a vector representing the average values of all observations in that cluster.

Consider the following cluster with two vectors:

$$\begin{aligned} \boldsymbol{X_1} &= [1, 0, 3] \\ \boldsymbol{X_2} &= [0, 4, 1] \end{aligned}$$

Then its mean is:

$$\mu = [\text{mean}(X_{1,1}, X_{2,1}), \text{mean}(X_{1,2}, X_{2,2}), \text{mean}(X_{1,3}, X_{2,3})]$$

K-Means Clustering: Outputs

Centroid (μ_k): The mean for each K – a vector representing the average values of all observations in that cluster.

Consider the following cluster with two vectors:

$$\begin{aligned} \boldsymbol{X_1} &= [1, 0, 3] \\ \boldsymbol{X_2} &= [0, 4, 1] \end{aligned}$$

Then its mean is:

$$\begin{aligned} \mu &= [\text{mean}(X_{1,1}, X_{2,1}), \text{mean}(X_{1,2}, X_{2,2}), \text{mean}(X_{1,3}, X_{2,3})] \\ \mu &= [0.5, 2, 2] \end{aligned}$$

K-Means Clustering: Outputs

Centroid (μ_k): The mean for each K – a vector representing the average values of all observations in that cluster.

Consider the following cluster with two vectors:

$$\begin{aligned} \boldsymbol{X_1} &= [1, 0, 3] \\ \boldsymbol{X_2} &= [0, 4, 1] \end{aligned}$$

Then its mean is:

$$\begin{aligned} \mu &= [\mathrm{mean}(X_{1,1}, X_{2,1}), \mathrm{mean}(X_{1,2}, X_{2,2}), \mathrm{mean}(X_{1,3}, X_{2,3})] \\ \mu &= [0.5, 2, 2] \end{aligned}$$

The K-means algorithm will assign each observation to the cluster with the closest mean.

K-Means Clustering: Example

Goal: Cluster the following documents:

- `I like to eat broccoli and bananas.`
- `I eat a banana smoothie for breakfast.`
- `Hamsters and kittens are cute.`
- `She adopted a cute kitten.`

K-Means Clustering: Example

Inputs

1. A document term matrix

	adopt	banana	breakfast	broccoli	cute	eat	hamster	kitten	like	smoothi
1	0	1	0	1	0	1	0	0	1	0
2	0	1	1	0	0	1	0	0	0	1
3	0	0	0	0	1	0	1	1	0	0
4	1	0	0	0	1	0	0	1	0	0

K-Means Clustering: Example

Inputs

1. A document term matrix

	adopt	banana	breakfast	broccoli	cute	eat	hamster	kitten	like	smoothi
1	0	1	0	1	0	1	0	0	1	0
2	0	1	1	0	0	1	0	0	0	1
3	0	0	0	0	1	0	1	1	0	0
4	1	0	0	0	1	0	0	1	0	0

2. K: 2

K-Means Clustering: Example

Inputs

1. A document term matrix

	adopt	banana	breakfast	broccoli	cute	eat	hamster	kitten	like	smoothi
1	0	1	0	1	0	1	0	0	1	0
2	0	1	1	0	0	1	0	0	0	1
3	0	0	0	0	1	0	1	1	0	0
4	1	0	0	0	1	0	0	1	0	0

2. K: 2

Outputs

1. C_k: Cluster assignment:
   - $\boldsymbol{C_1}$: [1, 2]
   - $\boldsymbol{C_2}$: [3, 4]

K-Means Clustering: Example

Inputs

1. A document term matrix

	adopt	banana	breakfast	broccoli	cute	eat	hamster	kitten	like	smoothi
1	0	1	0	1	0	1	0	0	1	0
2	0	1	1	0	0	1	0	0	0	1
3	0	0	0	0	1	0	1	1	0	0
4	1	0	0	0	1	0	0	1	0	0

2. K: 2

Outputs

1. C_k: Cluster assignment:
   - $\boldsymbol{C_1}$: [1, 2]
   - $\boldsymbol{C_2}$: [3, 4]
2. μ_k: Cluster means / centroids:

	adopt	banana	breakfast	broccoli	cute	eat	hamster	kitten	like	smoothi
μ_1	0.0	1.0	0.5	0.5	0.0	1.0	0.0	0.0	0.5	0.5
μ_2	0.5	0.0	0.0	0.0	1.0	0.0	0.5	1.0	0.0	0.0

K-Means Clustering

A chicken and egg problem:

K-Means Clustering

A chicken and egg problem:

- Means $\rightsquigarrow$ Assignments

K-Means Clustering

A chicken and egg problem:

- Means $\rightsquigarrow$ Assignments
- Assignments $\rightsquigarrow$ Means

K-Means Clustering

A chicken and egg problem:

- Means $\rightsquigarrow$ Assignments
- Assignments $\rightsquigarrow$ Means

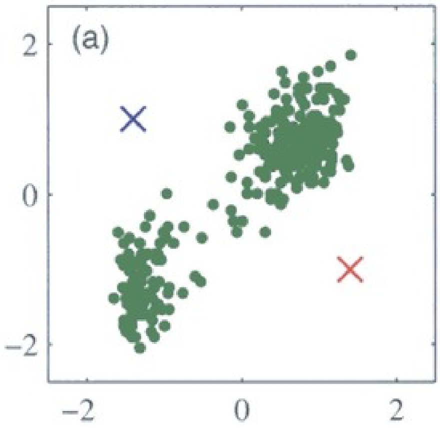
How do we find a good partition?

K-Means Clustering: Algorithm

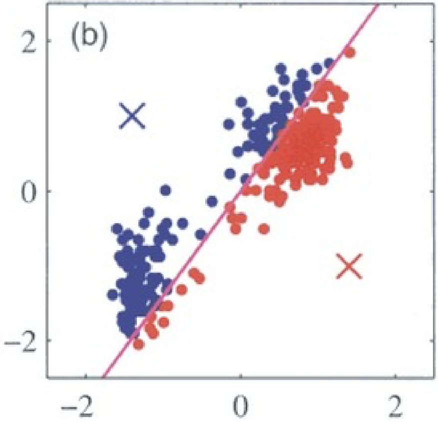
1) Randomly initialize K cluster centroids $(\mu_1, \mu_2, \cdots, \mu_k)$ in random locations.
2) Repeat:
   - Assignment: Assign each observation $\boldsymbol{X}$ to cluster with closest mean μ_k.
   - Update: Calculate new centroids μ_k by averaging all points assigned to each cluster.

   Stop when cluster assignments stop changing.

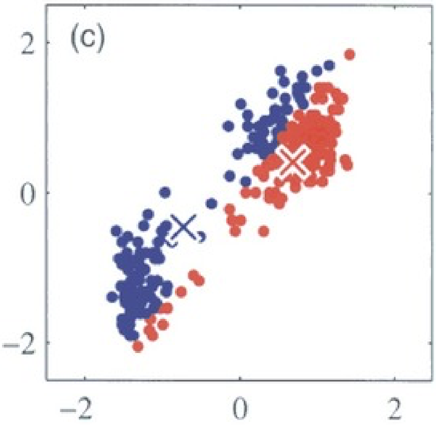
K-Means Clustering: Algorithm



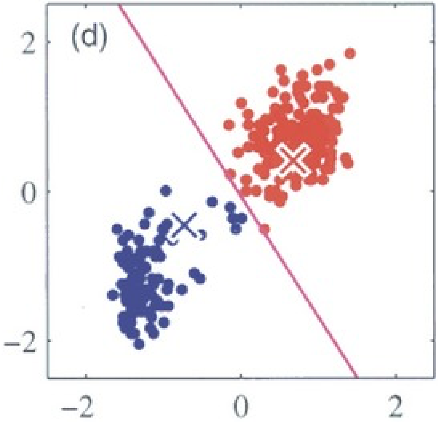
K-Means Clustering: Algorithm



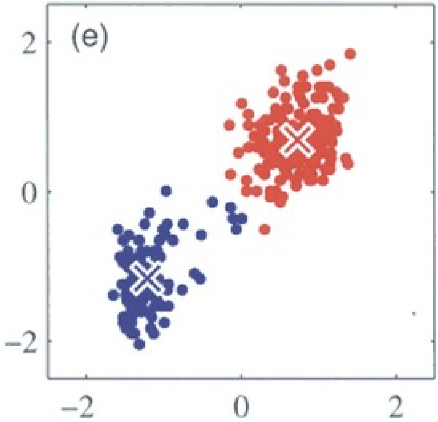
K-Means Clustering: Algorithm



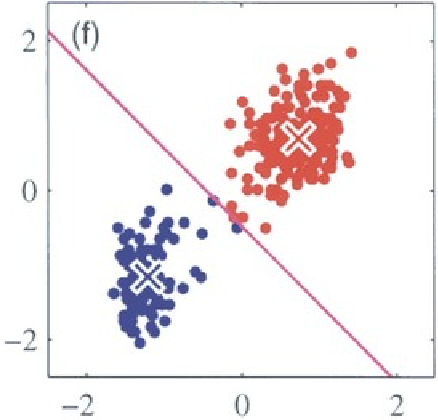
K-Means Clustering: Algorithm



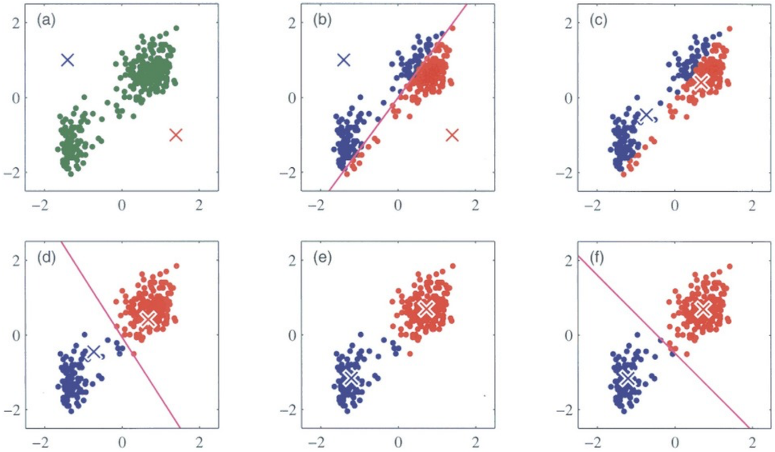
K-Means Clustering: Algorithm



K-Means Clustering: Algorithm



K-Means Clustering: Algorithm



K-Means Clustering: Decisions

Small Decisions with Big Consequences:

K-Means Clustering: Decisions

Small Decisions with Big Consequences:

1) How should we preprocess the data?
   - k-means are very sensitive to feature scaling / weighting.
   - Common to normalize the DTM in some way, e.g. by dividing each vector by the vector length.

K-Means Clustering: Decisions

Small Decisions with Big Consequences:

1) How should we preprocess the data?
   - k-means are very sensitive to feature scaling / weighting.
   - Common to normalize the DTM in some way, e.g. by dividing each vector by the vector length.
2) How to chose K?
   - User must assign the number of clusters (K)
   - Different values of K will lead to different partitions.

K-Means Clustering: Decisions

Small Decisions with Big Consequences:

1) How should we preprocess the data?
   - k-means are very sensitive to feature scaling / weighting.
   - Common to normalize the DTM in some way, e.g. by dividing each vector by the vector length.
2) How to chose K?
   - User must assign the number of clusters (K)
   - Different values of K will lead to different partitions.
3) Random starting values!
   - Results will depend on the initial (random) cluster centroid assignment (in step 1).
   - Important to run the algorithm multiple times from different random starting values.

K-Means Clustering: Decisions

Small Decisions with Big Consequences:

1) How should we preprocess the data?
   - k-means are very sensitive to feature scaling / weighting.
   - Common to normalize the DTM in some way, e.g. by dividing each vector by the vector length.
2) How to chose K?
   - User must assign the number of clusters (K)
   - Different values of K will lead to different partitions.
3) Random starting values!
   - Results will depend on the initial (random) cluster centroid assignment (in step 1).
   - Important to run the algorithm multiple times from different random starting values.

How do we decide?

K-Means Clustering: How do we decide?

What makes a good partition?

K-Means Clustering: How do we decide?

What makes a good partition?

Two kinds of validation criteria:

K-Means Clustering: How do we decide?

What makes a good partition?

Two kinds of validation criteria:

1. Quantitative evaluation:
   - A good clustering is one for which the within-cluster variation is as small as possible.

K-Means Clustering: How do we decide?

What makes a good partition?

Two kinds of validation criteria:

1. Quantitative evaluation:
   - A good clustering is one for which the within-cluster variation is as small as possible.
2. Qualitative evaluation:
   - A good clustering is one for which clusters are substantially / semantically interpretable.

Quantitative evaluation: within-cluster variation is as small as possible.

- Within-cluster variation: a measure of the amount by which the observations within a cluster differ from each other.
- Common metric: Sum of Squared Euclidean Distance

For a given document $\boldsymbol{X}$ in cluster k, the squared Euclidean distance is:

$$D(\boldsymbol{X}, \boldsymbol{\mu_k})^2 = \sum_{p=1}^{P} (x_p - \mu_{kp})^2$$

For a given document $\boldsymbol{X}$ in cluster k, the squared Euclidean distance is:

$$D(\boldsymbol{X}, \boldsymbol{\mu_k})^2 = \sum_{p=1}^{P} (x_p - \mu_{kp})^2$$

The within-cluster sum of squared distances for a given cluster C_k is:

$$W(C_k) = \sum_{i \in C_k} D(\boldsymbol{X}_i, \boldsymbol{\mu_k})^2$$

For a given document $\boldsymbol{X}$ in cluster k, the squared Euclidean distance is:

$$D(\boldsymbol{X}, \boldsymbol{\mu_k})^2 = \sum_{p=1}^{P} (x_p - \mu_{kp})^2$$

The within-cluster sum of squared distances for a given cluster C_k is:

$$W(C_k) = \sum_{i \in C_k} D(\boldsymbol{X}_i, \boldsymbol{\mu_k})^2$$

Thus our goal is to minimize the total within-cluster sum of squares:

$$\sum_{k=1}^{K} W(C_k)$$

Qualitative evaluation: clusters are substantially / semantically interpretable.

How do we interpret the clusters?

Qualitative evaluation: clusters are substantially / semantically interpretable.

How do we interpret the clusters?

1. Manual identification
    - Sample set of documents from same cluster
    - Read documents
    - Assign cluster "label" by hand
        - I like to eat broccoli and bananas. $\rightsquigarrow$ "food"
        - Hamsters and kittens are cute. $\rightsquigarrow$ "pets"

Qualitative evaluation: clusters are substantially / semantically interpretable.

How do we interpret the clusters?

1. Manual identification
    - Sample set of documents from same cluster
    - Read documents
    - Assign cluster "label" by hand
        - I like to eat broccoli and bananas. $\rightsquigarrow$ "food"
        - Hamsters and kittens are cute. $\rightsquigarrow$ "pets"
2. Automatic identification
    - Use methods to identify separating words between clusters
    - Use these to help infer differences across clusters

Qualitative evaluation: clusters are substantially / semantically interpretable.

How do we interpret the clusters?

1. Manual identification
   - Sample set of documents from same cluster
   - Read documents
   - Assign cluster "label" by hand
     - I like to eat broccoli and bananas. $\rightsquigarrow$ "food"
     - Hamsters and kittens are cute. $\rightsquigarrow$ "pets"
2. Automatic identification
   - Use methods to identify separating words between clusters
   - Use these to help infer differences across clusters
3. Be Transparent
   - Provide documents + code
   - Detail labeling procedures
   - Acknowledge ambiguity

To the `R` code!