

EXTENDS *TLC, Integers, FiniteSets, Sequences, Reals*

CONSTANTS *PRODUCTS, APPS, IDs, GATEAPPS*

ASSUME *Cardinality(APPS) > 0*

$PT \triangleq$ INSTANCE *PT*

$set ++ item \triangleq set \cup \{item\}$

$set -- item \triangleq set \setminus \{item\}$

In shopping list, the product is in fact the identifier. Any item could have an information for how much of a product one wants to buy (not relevant in this specification).

$ShopyItems \triangleq [id : PRODUCTS, bought : BOOLEAN]$

$ADD_ACTION \triangleq$ "add"

$RM_ACTION \triangleq$ "rm"

$SET_BOUGHT_ACTION \triangleq$ "set_bought"

$REQ_SYNC_ACTION \triangleq$ "req_sync"

$RESP_SYNC_ACTION \triangleq$ "resp_sync"

$END_SYNC_ACTION \triangleq$ "end_sync"

Actions is the set of all possible actions in the system.

$Actions \triangleq \{$
 $ADD_ACTION,$
 $RM_ACTION,$
 $SET_BOUGHT_ACTION,$
 $REQ_SYNC_ACTION,$
 $RESP_SYNC_ACTION,$
 END_SYNC_ACTION
 $\}$

$SyncActions \triangleq \{REQ_SYNC_ACTION, RESP_SYNC_ACTION\}$

SyncMsgs is the set of all possible messages sent for synchronisation of shopy lists.

$SyncMsgs \triangleq$
 $[id : IDs,$
 $app : APPS,$
 $list : SUBSET ShopyItems,$
 $mergedList : SUBSET ShopyItems,$
 $type : SyncActions]$

Messages sent for joining the network.

$JoinRespMsgs \triangleq$
 $[app : APPS,$
 $knownHosts : PT!SeqOf(APPS, Cardinality(APPS))]$

$$\text{JoinReqMsgs} \triangleq \\ [app : APPS]$$

Messages sent for notifications about new joiners in the network.

$$\text{JoinNotifMsgs} \triangleq \\ [app : APPS]$$

The spec now depicts a shopping-list *app* where the server *app* manages several users and hence multiple lists of items that synch eventually.

The list contains unique items, thus we use a set.

--algorithm *OptiShopyList*

variable

whether an *app* is a gate
 $isGate = [a \in APPS \mapsto a \in GATEAPPS],$
 one shopping list for all *APPS*
 $shopyList = [a \in APPS \mapsto \{\}],$
 sync *shopyList* requests/responses
 $syncReqQueue = [a \in APPS \mapsto \langle \rangle],$
 $syncRespQueue = [a \in APPS \mapsto \langle \rangle],$
 join to network requests/responses
 $joinReqQueue = [a \in APPS \mapsto \langle \rangle],$
 $joinRespQueue = [a \in APPS \mapsto \langle \rangle],$
 new joiner notifications
 $newJoinerNotif = [a \in APPS \mapsto \langle \rangle],$
 set of taken *IDs*
 $takenIDs = \{\};$

define

A couple of helpers for shopy-list items

$\text{NewShopyItem}(list) \triangleq$
 $[id \mapsto (\text{CHOOSE } x \in PRODUCTS : \neg \exists i \in list : x = i.id),$
 $bought \mapsto \text{FALSE}]$

$\text{ExistingShopyItem}(list) \triangleq \text{CHOOSE } x \in list : \text{TRUE}$

$\text{ExistingNotBoughtShopyItem}(list) \triangleq \text{CHOOSE } x \in list : x.bought = \text{FALSE}$

Helpers for *Sync* messages request/response.

$\text{NewSyncMsg}(id, a, l, ml, t) \triangleq$
 $[id \mapsto id,$
 $app \mapsto a,$
 $list \mapsto l,$
 $mergedList \mapsto ml,$

$$\begin{aligned}
& type \mapsto t] \\
NewSyncReqMsg(a, l, ml, t) & \triangleq \\
& NewSyncMsg(\\
& \quad (CHOOSE \ i \in \ IDs : \forall \ ti \in \ takenIDs : i \neq ti), \\
& \quad a, l, ml, t \\
&) \\
NewSyncReq(app) & \triangleq \\
& NewSyncReqMsg(app, shopyList[app], \{\}, REQ_SYNC_ACTION) \\
NewSyncResp(app, mergeResult, id) & \triangleq \\
& NewSyncMsg(id, app, shopyList[app], mergeResult, RESP_SYNC_ACTION)
\end{aligned}$$

Helpers for the decentralized network features.

$$\begin{aligned}
NewJoinReqMsg(app) & \triangleq [app \mapsto app] \\
NewJoinRespMsg(app, hosts) & \triangleq \\
& [app \mapsto app, \\
& \quad knownHosts \mapsto hosts] \\
GateApps & \triangleq \{a \in APPS : isGate[a]\} \\
NewJoinerNotifReq(app) & \triangleq [app \mapsto app]
\end{aligned}$$

Amongst 'knownApps', wisely choose an element different than 'app'.

$$\begin{aligned}
PickGossipFriends(app, knownApps) & \triangleq \\
\text{LET} & \\
Opposit & \triangleq \\
\text{LET } I & \triangleq PT!Index(knownApps, app) \text{ IN} \\
& I - (Len(knownApps) \div 2) + \\
& (IF \ I < Len(knownApps) \div 2 \text{ THEN } Len(knownApps) \text{ ELSE } 0) \\
KnownApps(exclude) & \triangleq \\
PT!ReduceSeq(& \\
\text{LAMBDA } a, acc : & IF \ a \in exclude \text{ THEN } acc \text{ ELSE } Append(acc, a), \\
knownApps, \langle \rangle &) \\
Between(val, min, max) & \triangleq \\
PT!Max(min, PT!Min(max, val)) & \\
FirstGossip & \triangleq \text{LET } ka \triangleq KnownApps(\{app\}) \text{ IN} \\
& \{ka[Between(Opposit - 1, 1, Len(ka))]\} \\
SecondGossip & \triangleq \text{LET } ka \triangleq KnownApps(\{app\} \cup FirstGossip) \text{ IN} \\
& IF \ Len(ka) > 0 \\
& \text{THEN } \{ka[Between(Opposit, 1, Len(ka))]\}
\end{aligned}$$

```

ELSE {}

IN FirstGossip  $\cup$  SecondGossip

Merge two lists of apps together without duplicating equal elements.
MergeKnownApps(apps1, apps2)  $\triangleq$ 
  LET AppSeq(n)  $\triangleq$  PT!SeqOf(APPS, n)

  Contains(appSeq, appItem)  $\triangleq$ 
    Cardinality(PT!Matching(appSeq, appItem)) > 0

  f[args  $\in$  AppSeq(Len(apps1))
     $\times$  AppSeq(Len(apps2))
     $\times$  AppSeq(Len(apps1) + Len(apps2))]  $\triangleq$ 

  LET l1  $\triangleq$  args[1]
    l2  $\triangleq$  args[2]
    acc  $\triangleq$  args[3]

    PickFromL1  $\triangleq$  f[(
      Tail(l1),
      l2,
      Append(acc, Head(l1)))]

    SkipOneL1  $\triangleq$  f[(
      Tail(l1),
      l2,
      acc)]

    PickFromL2  $\triangleq$  f[(
      l1,
      Tail(l2),
      Append(acc, Head(l2)))]

    SkipOneL2  $\triangleq$  f[(
      l1,
      Tail(l2),
      acc)]

  IN
    IF Len(l2) = 0
    THEN IF Len(l1) = 0
    THEN acc
    ELSE IF Contains(acc, Head(l1)) THEN SkipOneL1 ELSE PickFromL1
    ELSE IF Head(l1)  $\neq$  Head(l2)
    THEN IF Contains(acc, Head(l2)) THEN SkipOneL2 ELSE PickFromL2
    ELSE IF Contains(acc, Head(l1)) THEN SkipOneL1 ELSE PickFromL1

```

```

      IN  $f[\langle apps1, apps2, \langle \rangle \rangle]$ 
end define ;

macro notify(apps, newJoiner)
begin
  newJoinerNotif := [ $a \in APPS \mapsto$ 
    IF  $a \in apps$ 
    THEN Append(newJoinerNotif[ $a$ ], NewJoinerNotifReq(newJoiner))
    ELSE newJoinerNotif[ $a$ ];
end macro ;

```

This process represents a shopy-list running in one of the several network clients.

Since Opti-shopylist is a decentralized program, the user creates a network of connected instances of Opti-shopylist.

We assume that every client *app* has only one shopy-list.

```

fair + process ClientApp  $\in APPS$ 
variables

```

On network outage, an *app* can't pull new messages.

```

networkOutage = FALSE,
joined = FALSE,
gossipFriends = {},
knownApps =  $\langle self \rangle$ ;

```

```

begin AppLoop:
  while TRUE do

```

MANAGE DECENTRALIZED NETWORK

either

SEND JOIN REQUEST

A client *app* might send a join request to any available gate *app*.

await $\neg joined$;

with $a \in (GateApps -- self)$

do

joinReqQueue[a] := *Append*(*joinReqQueue*[a], *NewJoinReqMsg*(*self*));

end with ;

or

NETWORK OUTAGE

networkOutage := TRUE ;

or

RESPOND TO JOIN REQUEST

Any gate *app* receiving a join request will :

- respond with currently known joined *apps*,
- pick new gossip friends,
- *notify* its gossip friends.

```

if isGate[self] then
  await  $\neg$ networkOutage ;
  await joinReqQueue[self]  $\neq \langle \rangle$  ;
  with joinRequest = Head(joinReqQueue[self]),
      updatedKnownApps = Append(knownApps, joinRequest.app)
  do
    PULL FROM REQ QUEUE
    joinReqQueue[self] := Tail(joinReqQueue[self]) ;

    RESPOND TO REQUESTER
    joinRespQueue[joinRequest.app] := Append(
      joinRespQueue[joinRequest.app],
      NewJoinRespMsg(self, SelectSeq(knownApps,
        LAMBDA app : app  $\neq$  joinRequest.app))) ;

    UPDATE KNOWN APPS AND PICK NEW GOSSIP FRIENDS
    if joinRequest.app  $\notin$  PT!Range(knownApps)
    then
      knownApps := updatedKnownApps ;
      gossipFriends := PickGossipFriends(self, knownApps) ;
    end if ;

    NOTIFY NETWORK OF NEW JOINER
    notify(gossipFriends — joinRequest.app, joinRequest.app) ;

    SET STATUS TO JOINED
    joined := TRUE ;
  end with ;
end if ;
or
  RECEIVE JOIN RESPONSE
  Upon receiving a join response from a gate app, an app will have to update its known
  apps and pick gossip friends.
  await joinRespQueue[self]  $\neq \langle \rangle$  ;
  await  $\neg$ networkOutage ;
  with joinResponse = Head(joinRespQueue[self])
  do
    PULL FROM RESP QUEUE
    joinRespQueue[self] := Tail(joinRespQueue[self]) ;

    UPDATE KNOWN APPS
    knownApps := MergeKnownApps(knownApps, joinResponse.knownHosts) ;

    PICK NEW GOSSIP FRIENDS
    gossipFriends := PickGossipFriends(self, knownApps) ;

    SET STATUS TO JOINED

```

```

    joined := TRUE ;
  end with ;
or
  RECEIVE JOIN NOTIFICATION
  Upon receiving a join notification from any other app, an app updates its known
  apps as well as its gossips.
  await newJoinerNotif[self] ≠ ⟨⟩ ;
  await ¬networkOutage ;
  with joinNotifMsg = Head(newJoinerNotif[self])
  do
    PULL FROM NOTIF QUEUE
    newJoinerNotif[self] := Tail(newJoinerNotif[self]) ;

    UPDATE KNOWN APPS
    knownApps := MergeKnownApps(knownApps, ⟨joinNotifMsg.app⟩) ;

    PICK NEW GOSSIP FRIENDS
    gossipFriends := PickGossipFriends(self, knownApps) ;
  end with ;

```

Following are the actions applying to the shopy-list managed by the *app*. We need to abstract actions down to one single add action in order not to have infinite loops between adding, removing and so on. User behavior must not be constrained because it cannot be controlled.

```

or
  ADD
  await Cardinality(shopyList[self]) < Cardinality(PRODUCTS) ;
  shopyList[self] := shopyList[self] ++ NewShopyItem(shopyList[self]) ;

```

Below actions manage the synchronization of the list.

```

or
  SEND SYNC REQUEST
  with  $a \in (PT!Range(knownApps) -- self)$ ,
    req = NewSyncReq(self)
  do
    takenIDs := takenIDs ++ req.id ;
    syncReqQueue[a] := Append(syncReqQueue[a], req) ;
  end with ;
or
  RECEIVE SYNC REQUEST
  await syncReqQueue[self] ≠ ⟨⟩ ;
  await ¬networkOutage ;
  with syncRequest = Head(syncReqQueue[self]),
    mergeResult = shopyList[self] ∪ syncRequest.list,
    newResp = NewSyncResp(self, mergeResult, syncRequest.id)
  do

```

```

    syncReqQueue[self] := Tail(syncReqQueue[self]);
    merge from request app
    shopyList[self] := mergeResult;
    syncRespQueue[syncRequest.app] := Append(syncRespQueue[syncRequest.app], newResp);
  end with ;
or
  RECEIVE SYNC RESPONSE
  await syncRespQueue[self] ≠ ⟨⟩ ;
  await ¬networkOutage ;
  with syncResponse = Head(syncRespQueue[self]),
       mergeResult = shopyList[self] ∪ syncResponse.list
  do
    shopyList[self] := mergeResult ;
    syncRespQueue[self] := Tail(syncRespQueue[self]);
  end with ;
end either ;
end while ;
end process ;

```

end algorithm ;

BEGIN TRANSLATION ($chksum(pcal) = \text{"7a780ec0"} \wedge chksum(tla) = \text{"e46ebe0c"}$)
 VARIABLES $isGate$, $shopyList$, $syncReqQueue$, $syncRespQueue$, $joinReqQueue$,
 $joinRespQueue$, $newJoinerNotif$, $takenIDs$

define statement

$NewShopyItem(list) \triangleq$
 $[id \mapsto (\text{CHOOSE } x \in PRODUCTS : \neg \exists i \in list : x = i.id),$
 $bought \mapsto \text{FALSE}]$

$ExistingShopyItem(list) \triangleq \text{CHOOSE } x \in list : \text{TRUE}$

$ExistingNotBoughtShopyItem(list) \triangleq \text{CHOOSE } x \in list : x.bought = \text{FALSE}$

$NewSyncMsg(id, a, l, ml, t) \triangleq$
 $[id \mapsto id,$
 $app \mapsto a,$
 $list \mapsto l,$
 $mergedList \mapsto ml,$
 $type \mapsto t]$

$NewSyncReqMsg(a, l, ml, t) \triangleq$
 $NewSyncMsg($
 $(\text{CHOOSE } i \in IDs : \forall ti \in takenIDs : i \neq ti),$

$$\begin{aligned}
& a, l, ml, t \\
&) \\
NewSyncReq(app) & \triangleq \\
& NewSyncReqMsg(app, shopyList[app], \{\}, REQ_SYNC_ACTION) \\
NewSyncResp(app, mergeResult, id) & \triangleq \\
& NewSyncMsg(id, app, shopyList[app], mergeResult, RESP_SYNC_ACTION) \\
\\
NewJoinReqMsg(app) & \triangleq [app \mapsto app] \\
NewJoinRespMsg(app, hosts) & \triangleq \\
& [app \mapsto app, \\
& \quad knownHosts \mapsto hosts] \\
GateApps & \triangleq \{a \in APPS : isGate[a]\} \\
NewJoinerNotifReq(app) & \triangleq [app \mapsto app] \\
\\
PickGossipFriends(app, knownApps) & \triangleq \\
\text{LET} & \\
\quad Opposit & \triangleq \\
\quad \quad \text{LET } I & \triangleq PT!Index(knownApps, app) \text{ IN} \\
\quad \quad I - (Len(knownApps) \div 2) + & \\
\quad \quad (\text{IF } I < Len(knownApps) \div 2 \text{ THEN } Len(knownApps) \text{ ELSE } 0) & \\
\quad KnownApps(exclude) & \triangleq \\
\quad \quad PT!ReduceSeq(& \\
\quad \quad \quad \text{LAMBDA } a, acc : \text{IF } a \in exclude \text{ THEN } acc \text{ ELSE } Append(acc, a), & \\
\quad \quad \quad knownApps, \langle \rangle) & \\
\quad Between(val, min, max) & \triangleq \\
\quad \quad PT!Max(min, PT!Min(max, val)) & \\
\quad FirstGossip & \triangleq \text{LET } ka \triangleq KnownApps(\{app\}) \text{ IN} \\
\quad \quad \{ka[Between(Opposit - 1, 1, Len(ka))]\} & \\
\quad SecondGossip & \triangleq \text{LET } ka \triangleq KnownApps(\{app\} \cup FirstGossip) \text{ IN} \\
\quad \quad \text{IF } Len(ka) > 0 & \\
\quad \quad \text{THEN } \{ka[Between(Opposit, 1, Len(ka))]\} & \\
\quad \quad \text{ELSE } \{\} & \\
\text{IN } & FirstGossip \cup SecondGossip
\end{aligned}$$

```

MergeKnownApps(apps1, apps2)  $\triangleq$ 
  LET AppSeq( $n$ )  $\triangleq$  PT!SeqOf(APPS,  $n$ )

  Contains(appSeq, appItem)  $\triangleq$ 
    Cardinality(PT!Matching(appSeq, appItem)) > 0

  f[ $args \in$  AppSeq(Len(apps1))
     $\times$  AppSeq(Len(apps2))
     $\times$  AppSeq(Len(apps1) + Len(apps2))]  $\triangleq$ 

    LET  $l1 \triangleq args[1]$ 
     $l2 \triangleq args[2]$ 
     $acc \triangleq args[3]$ 

    PickFromL1  $\triangleq$  f[⟨
      Tail( $l1$ ),
       $l2$ ,
      Append( $acc$ , Head( $l1$ )))⟩]

    SkipOneL1  $\triangleq$  f[⟨
      Tail( $l1$ ),
       $l2$ ,
       $acc$ ⟩]

    PickFromL2  $\triangleq$  f[⟨
       $l1$ ,
      Tail( $l2$ ),
      Append( $acc$ , Head( $l2$ )))⟩]

    SkipOneL2  $\triangleq$  f[⟨
       $l1$ ,
      Tail( $l2$ ),
       $acc$ ⟩]

  IN
    IF Len( $l2$ ) = 0
    THEN IF Len( $l1$ ) = 0
    THEN  $acc$ 
    ELSE IF Contains( $acc$ , Head( $l1$ )) THEN SkipOneL1 ELSE PickFromL1
    ELSE IF Head( $l1$ )  $\neq$  Head( $l2$ )
    THEN IF Contains( $acc$ , Head( $l2$ )) THEN SkipOneL2 ELSE PickFromL2
    ELSE IF Contains( $acc$ , Head( $l1$ )) THEN SkipOneL1 ELSE PickFromL1
  IN f[⟨apps1, apps2,  $\langle \rangle$ ⟩]

VARIABLES networkOutage, joined, gossipFriends, knownApps

vars  $\triangleq$  ⟨isGate, shopyList, syncReqQueue, syncRespQueue, joinReqQueue,

```

$joinRespQueue, newJoinerNotif, takenIDs, networkOutage, joined,$
 $gossipFriends, knownApps\}$

$ProcSet \triangleq (APPS)$

$Init \triangleq$ Global variables

$\wedge isGate = [a \in APPS \mapsto a \in GATEAPPS]$
 $\wedge shopyList = [a \in APPS \mapsto \{\}]$
 $\wedge syncReqQueue = [a \in APPS \mapsto \langle \rangle]$
 $\wedge syncRespQueue = [a \in APPS \mapsto \langle \rangle]$
 $\wedge joinReqQueue = [a \in APPS \mapsto \langle \rangle]$
 $\wedge joinRespQueue = [a \in APPS \mapsto \langle \rangle]$
 $\wedge newJoinerNotif = [a \in APPS \mapsto \langle \rangle]$
 $\wedge takenIDs = \{\}$
Process *ClientApp*
 $\wedge networkOutage = [self \in APPS \mapsto \text{FALSE}]$
 $\wedge joined = [self \in APPS \mapsto \text{FALSE}]$
 $\wedge gossipFriends = [self \in APPS \mapsto \{\}]$
 $\wedge knownApps = [self \in APPS \mapsto \langle self \rangle]$

$ClientApp(self) \triangleq \wedge \vee \wedge \neg joined[self]$
 $\wedge \exists a \in (GateApps -- self) :$
 $joinReqQueue' = [joinReqQueue \text{ EXCEPT } ![a] = Append(joinReqQueue[a], NewJoinerNotif[a])]$
 $\wedge \text{UNCHANGED } \langle shopyList, syncReqQueue, syncRespQueue, joinRespQueue, newJoinerNotif \rangle$
 $\vee \wedge networkOutage' = [networkOutage \text{ EXCEPT } ![self] = \text{TRUE}]$
 $\wedge \text{UNCHANGED } \langle shopyList, syncReqQueue, syncRespQueue, joinReqQueue, joinRespQueue \rangle$
 $\vee \wedge \text{IF } isGate[self]$
 $\quad \text{THEN } \wedge \neg networkOutage[self]$
 $\quad \wedge joinReqQueue[self] \neq \langle \rangle$
 $\quad \wedge \text{LET } joinRequest \triangleq Head(joinReqQueue[self]) \text{ IN}$
 $\quad \text{LET } updatedKnownApps \triangleq Append(knownApps[self], joinRequest.app)$
 $\quad \wedge joinReqQueue' = [joinReqQueue \text{ EXCEPT } ![self] = Tail(joinReqQueue)]$
 $\quad \wedge joinRespQueue' = [joinRespQueue \text{ EXCEPT } ![joinRequest.app] = Append(joinRespQueue[joinRequest.app], NewJoinerNotif[joinRequest.app])]$
 $\quad \wedge \text{IF } joinRequest.app \notin PT!Range(knownApps[self])$
 $\quad \quad \text{THEN } \wedge knownApps' = [knownApps \text{ EXCEPT } ![self] = updatedKnownApps]$
 $\quad \quad \wedge gossipFriends' = [gossipFriends \text{ EXCEPT } ![self] = Append(gossipFriends[self], joinRequest.app)]$
 $\quad \quad \text{ELSE } \wedge \text{TRUE}$
 $\quad \quad \wedge \text{UNCHANGED } \langle gossipFriends, knownApps \rangle$
 $\wedge newJoinerNotif' = [a \in APPS \mapsto \text{IF } a \in (gossipFriends'[self] -- joinRequest.app) \text{ THEN } Append(newJoinerNotif[a], NewJoinerNotif[joinRequest.app]) \text{ ELSE } newJoinerNotif[a]]$

```

       $\wedge \text{joined}' = [\text{joined} \text{ EXCEPT } ![self] = \text{TRUE}]$ 
ELSE  $\wedge \text{TRUE}$ 
       $\wedge \text{UNCHANGED } \langle \text{joinReqQueue},$ 
         $\text{joinRespQueue},$ 
         $\text{newJoinerNotif}, \text{joined},$ 
         $\text{gossipFriends}, \text{knownApps} \rangle$ 
 $\wedge \text{UNCHANGED } \langle \text{shopyList}, \text{syncReqQueue}, \text{syncRespQueue}, \text{takenIDs}, \text{networkOutage} \rangle$ 
 $\vee \wedge \text{joinRespQueue}[self] \neq \langle \rangle$ 
 $\wedge \neg \text{networkOutage}[self]$ 
 $\wedge \text{LET } \text{joinResponse} \triangleq \text{Head}(\text{joinRespQueue}[self]) \text{IN}$ 
   $\wedge \text{joinRespQueue}' = [\text{joinRespQueue} \text{ EXCEPT } ![self] = \text{Tail}(\text{joinRespQueue}[self])]$ 
   $\wedge \text{knownApps}' = [\text{knownApps} \text{ EXCEPT } ![self] = \text{MergeKnownApps}(\text{knownApps}[self], \text{joinResponse})]$ 
   $\wedge \text{gossipFriends}' = [\text{gossipFriends} \text{ EXCEPT } ![self] = \text{PickGossipFriends}(\text{self}, \text{knownApps})]$ 
   $\wedge \text{joined}' = [\text{joined} \text{ EXCEPT } ![self] = \text{TRUE}]$ 
 $\wedge \text{UNCHANGED } \langle \text{shopyList}, \text{syncReqQueue}, \text{syncRespQueue}, \text{joinReqQueue}, \text{newJoinerNotif} \rangle$ 
 $\vee \wedge \text{newJoinerNotif}[self] \neq \langle \rangle$ 
 $\wedge \neg \text{networkOutage}[self]$ 
 $\wedge \text{LET } \text{joinNotifMsg} \triangleq \text{Head}(\text{newJoinerNotif}[self]) \text{IN}$ 
   $\wedge \text{newJoinerNotif}' = [\text{newJoinerNotif} \text{ EXCEPT } ![self] = \text{Tail}(\text{newJoinerNotif}[self])]$ 
   $\wedge \text{knownApps}' = [\text{knownApps} \text{ EXCEPT } ![self] = \text{MergeKnownApps}(\text{knownApps}[self], \text{joinNotifMsg})]$ 
   $\wedge \text{gossipFriends}' = [\text{gossipFriends} \text{ EXCEPT } ![self] = \text{PickGossipFriends}(\text{self}, \text{knownApps})]$ 
 $\wedge \text{UNCHANGED } \langle \text{shopyList}, \text{syncReqQueue}, \text{syncRespQueue}, \text{joinReqQueue}, \text{joinRespQueue} \rangle$ 
 $\vee \wedge \text{Cardinality}(\text{shopyList}[self]) < \text{Cardinality}(\text{PRODUCTS})$ 
 $\wedge \text{shopyList}' = [\text{shopyList} \text{ EXCEPT } ![self] = \text{shopyList}[self] ++ \text{NewShopyItem}(\text{shopyList}[self])]$ 
 $\wedge \text{UNCHANGED } \langle \text{syncReqQueue}, \text{syncRespQueue}, \text{joinReqQueue}, \text{joinRespQueue}, \text{newJoinerNotif} \rangle$ 
 $\vee \wedge \exists a \in (PT! \text{Range}(\text{knownApps}[self]) -- self) :$ 
   $\text{LET } \text{req} \triangleq \text{NewSyncReq}(\text{self}) \text{IN}$ 
     $\wedge \text{takenIDs}' = \text{takenIDs} ++ \text{req.id}$ 
     $\wedge \text{syncReqQueue}' = [\text{syncReqQueue} \text{ EXCEPT } ![a] = \text{Append}(\text{syncReqQueue}[a], \text{req})]$ 
 $\wedge \text{UNCHANGED } \langle \text{shopyList}, \text{syncRespQueue}, \text{joinReqQueue}, \text{joinRespQueue}, \text{newJoinerNotif} \rangle$ 
 $\vee \wedge \text{syncReqQueue}[self] \neq \langle \rangle$ 
 $\wedge \neg \text{networkOutage}[self]$ 
 $\wedge \text{LET } \text{syncRequest} \triangleq \text{Head}(\text{syncReqQueue}[self]) \text{IN}$ 
   $\text{LET } \text{mergeResult} \triangleq \text{shopyList}[self] \cup \text{syncRequest.list} \text{IN}$ 
     $\text{LET } \text{newResp} \triangleq \text{NewSyncResp}(\text{self}, \text{mergeResult}, \text{syncRequest.id}) \text{IN}$ 
       $\wedge \text{syncReqQueue}' = [\text{syncReqQueue} \text{ EXCEPT } ![self] = \text{Tail}(\text{syncReqQueue}[self])]$ 
       $\wedge \text{shopyList}' = [\text{shopyList} \text{ EXCEPT } ![self] = \text{mergeResult}]$ 
       $\wedge \text{syncRespQueue}' = [\text{syncRespQueue} \text{ EXCEPT } ![syncRequest.app] = \text{Append}(\text{syncRespQueue}[self], \text{newResp})]$ 
 $\wedge \text{UNCHANGED } \langle \text{joinReqQueue}, \text{joinRespQueue}, \text{newJoinerNotif}, \text{takenIDs}, \text{networkOutage} \rangle$ 
 $\vee \wedge \text{syncRespQueue}[self] \neq \langle \rangle$ 
 $\wedge \neg \text{networkOutage}[self]$ 
 $\wedge \text{LET } \text{syncResponse} \triangleq \text{Head}(\text{syncRespQueue}[self]) \text{IN}$ 
   $\text{LET } \text{mergeResult} \triangleq \text{shopyList}[self] \cup \text{syncResponse.list} \text{IN}$ 
     $\wedge \text{shopyList}' = [\text{shopyList} \text{ EXCEPT } ![self] = \text{mergeResult}]$ 
     $\wedge \text{syncRespQueue}' = [\text{syncRespQueue} \text{ EXCEPT } ![self] = \text{Tail}(\text{syncRespQueue}[self])]$ 

```

$$\begin{aligned} & \wedge \text{UNCHANGED } \langle \text{syncReqQueue}, \text{joinReqQueue}, \text{joinRespQueue}, \text{newJoinerNotif}, \text{takeReqQueue} \rangle \\ & \wedge \text{UNCHANGED } \text{isGate} \end{aligned}$$

$$\text{Next} \triangleq (\exists \text{self} \in \text{APPS} : \text{ClientApp}(\text{self}))$$

$$\begin{aligned} \text{Spec} \triangleq & \wedge \text{Init} \wedge \Box[\text{Next}]_{\text{vars}} \\ & \wedge \forall \text{self} \in \text{APPS} : \text{SF}_{\text{vars}}(\text{ClientApp}(\text{self})) \end{aligned}$$

END TRANSLATION

There's no duplicated items in the sequence 'seq'.

$$\begin{aligned} \text{NoDuplicates}(\text{seq}) \triangleq & \\ & \forall i, j \in \text{DOMAIN } \text{seq} : \\ & i \neq j \Rightarrow \text{seq}[i] \neq \text{seq}[j] \end{aligned}$$

All *apps* in 'joinedApps' that joined the network.

$$\text{JoinedApps}(\text{joinedF}, \text{apps}) \triangleq \{j \in \text{apps} : \text{joinedF}[j]\}$$

The count of gossips for an *app* 'a' is the number of other joined *apps* that lists 'a' in its gossip friends set.

$$\begin{aligned} \text{CountGossipOf}(\text{app}, \text{gossips}, \text{joinedApps}) \triangleq & \\ & \text{PT!ReduceSet}(\text{LAMBDA } a, \text{acc} : \text{acc} + (\text{IF } \text{app} \in \text{gossips}[a] \wedge \text{joinedApps}[a] \\ & \text{THEN } 1 \text{ ELSE } 0), \\ & \text{APPS}, 0) \end{aligned}$$

The average of gossips is the average count over all joined *apps* of the count of gossips for an *app*.

$$\begin{aligned} \text{AverageGossipOf}(\text{gossips}, \text{joinedApps}) \triangleq & \\ & \text{PT!ReduceSet}(\text{LAMBDA } a, \text{acc} : \text{acc} + \text{CountGossipOf}(a, \text{gossips}, \text{joinedApps}), \\ & \text{APPS}, 0) \\ & \div \\ & \text{Cardinality}(\text{JoinedApps}(\text{joinedApps}, \text{APPS})) \end{aligned}$$

$$\begin{aligned} \text{ExistsRoute}(\text{from}, \text{to}, \text{gossipFriends}) \triangleq & \\ \text{LET } f[\langle \text{app}, \text{visited} \rangle \in \text{APPS} \times \text{SUBSET } \text{APPS}] \triangleq & \\ & \text{to} \in \text{gossipFriends}[\text{app}] \\ & \vee \exists a \in (\text{gossipFriends}[\text{app}] \setminus \text{visited}) : f[a, \text{visited} ++ \text{app}] \\ \text{IN } \text{from} = \text{to} \vee f[\langle \text{from}, \{\} \rangle] \end{aligned}$$

$$\begin{aligned} \text{TypeOK} \triangleq & \\ & \wedge \forall a \in \text{APPS} : \end{aligned}$$

Checking on variables' domains.

$$\begin{aligned} & \wedge \text{shopyList}[a] \subseteq \text{ShopyItems} \\ & \wedge \text{PT!Range}(\text{syncReqQueue}[a]) \subseteq \text{SyncMsgs} \\ & \wedge \text{PT!Range}(\text{syncRespQueue}[a]) \subseteq \text{SyncMsgs} \\ & \text{The queue for join requests is only for gate apps} \end{aligned}$$

$$\begin{aligned}
& \wedge \text{IF } isGate[a] \\
& \quad \text{THEN } PT!Range(joinReqQueue[a]) \subseteq JoinReqMsgs \\
& \quad \quad \wedge \forall req \in PT!Range(joinReqQueue[a]) : req.app \neq a \\
& \quad \text{ELSE } joinReqQueue[a] = \langle \rangle \\
& \wedge PT!Range(joinRespQueue[a]) \subseteq JoinRespMsgs \\
& \wedge PT!Range(newJoinerNotif[a]) \subseteq JoinNotifMsgs \\
& \quad \text{knownApps is a collection of unique, ordered apps.} \\
& \wedge PT!Range(knownApps[a]) \subseteq APPS \\
& \wedge NoDuplicates(knownApps[a]) \\
& \quad \text{Apps don't gossip themselves} \\
& \wedge gossipFriends[a] \subseteq (APPS - a) \\
& \wedge takenIDs \subseteq IDs \\
GossipInvariants & \triangleq \\
& \wedge \forall a \in APPS : \\
& \quad \text{Choose more than one gossip if have more than one known app.} \\
& \wedge Cardinality(PT!Range(knownApps[a]) - a) > 1 \Rightarrow Cardinality(gossipFriends[a]) > 1 \\
& \quad \text{Invariant when we're connected or not.} \\
& \wedge gossipFriends[a] \neq \{\} \\
& \quad \equiv knownApps[a] \neq \langle a \rangle \\
& \quad \text{We verify that for every joined app, the count of any other joined app gossiping the new} \\
& \quad \text{joiner is more or less in the average.} \\
& \wedge \forall ja \in JoinedApps(joined, APPS) : \\
& \quad \vee (\exists n \in JoinedApps(joined, APPS) : newJoinerNotif[n] \neq \langle \rangle) \\
& \quad \vee CountGossipOf(ja, gossipFriends, joined) = 0 \\
& \quad \vee (CountGossipOf(ja, gossipFriends, joined) \geq AverageGossipOf(gossipFriends, joined) - 1 \\
& \quad \quad \wedge CountGossipOf(ja, gossipFriends, joined) \leq AverageGossipOf(gossipFriends, joined) + 1 \\
& \quad \quad) \\
Liveness & \triangleq \\
& \quad \text{At some point, someone has joined and gossips have been assigned.} \\
& \wedge \Diamond (\forall ja \in JoinedApps(joined, APPS) : joined[ja] \wedge CountGossipOf(ja, gossipFriends, joined) > 0) \\
& \wedge \forall a \in APPS : \\
& \quad \text{Joining leads to having a route from every other joined app to the joiner.} \\
& \wedge joined[a] \\
& \quad \leadsto \forall a2 \in JoinedApps(joined, APPS - a) : \\
& \quad \quad \text{ExistsRoute}(a2, a, gossipFriends) \\
& \quad \text{Joining leads to every other joined app adding the new joiner to its list of known apps.} \\
& \wedge joined[a] \\
& \quad \leadsto \forall a2 \in JoinedApps(joined, APPS - a) : \\
& \quad \quad a \in PT!Range(knownApps[a2])
\end{aligned}$$

\ * Modification History

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