MODULE *TigerHumanSheep*

This module specifies what a man, accompanied by a sheep, some food and a tiger, can do in order to cross a river with a boat and according to the following rules:

- The boat can only fit for the man and one of the other elements (food, sheep, tiger).
- If the sheep is left alone on one side of the river with the food, the sheep will eat the man's food.
- If the tiger is left alone with the sheep, the tiger will eat the sheep.
- The puzzle is completed once the man, the sheep, the tiger and the food have crossed the river.
- The tiger does not eat the food the man caries since it's only vegs.

EXTENDS *Integers*

Variable *bRSide* represents the side of the river on which an animal of the *BEINGS* set stands on (the initial side is 0 and the other side is 1).

VARIABLES *bRSide*
CONSTANTS *SHEEP*, *FOOD*, *TIGER*, *MAN*, *BEINGS*

$haveNotCrossed(being1,\ being2) \triangleq$
  $\land\ bRSide[being1] = 0$
  $\land\ bRSide[being2] = 0$
$haveCrossed(being1,\ being2) \triangleq$
  $\land\ bRSide[being1] = 1$
  $\land\ bRSide[being2] = 1$

$changeBeingsSide(being1,\ being2) \triangleq$
  $\land\ bRSide[being1] = bRSide[being2]$
  $\land\ bRSide' = [bRSide \text{ EXCEPT } ![being1] = 1 - bRSide[being1],$
  $\qquad ![being2] = 1 - bRSide[being2]]$

For each the four elements of the enigma : -0 means that it has not yet crossed or has come back to the original
border of the river.
-1 means that it has crossed the river and is now on the other side.

$TypeOK \triangleq bRSide \in [BEINGS \to 0\,..\,1]$

First, all beings are on the same river *side*!

$Init \triangleq bRSide = [b \in BEINGS \mapsto 0]$

Some invariants :
– The sheep cannot be left with the food without man's presence.
- The tiger cannot be left with the sheep without man's presence.

$SheepNotWithFood \triangleq$
  IF $bRSide[SHEEP] \neq bRSide[MAN]$ THEN $bRSide[SHEEP] \neq bRSide[FOOD]$ ELSE TRUE
$TigerNotWithSheep \triangleq$
  IF $bRSide[TIGER] \neq bRSide[MAN]$ THEN $bRSide[TIGER] \neq bRSide[SHEEP]$ ELSE TRUE
$BeingsInvariants \triangleq$

$\lor\ Init$
$\lor\ \land\ SheepNotWithFood \land TigerNotWithSheep$

Man takes sheep over. Behind him, eventually, tiger and food could be left alone.

$TakeSheepOver\ \triangleq$
$\land\ haveNotCrossed(MAN,\ SHEEP)$
$\land\ changeBeingsSide(MAN,\ SHEEP)$

Man takes tiger over and takes care not to leave sheep and food alone!

$TakeTigerOver\ \triangleq$
$\land \neg(bRSide[SHEEP] = bRSide[TIGER] \land bRSide[FOOD] = bRSide[SHEEP])$
$\land\ haveNotCrossed(MAN,\ TIGER)$
$\land\ changeBeingsSide(MAN,\ TIGER)$

Man takes the food over and takes care not to leave sheep with tiger.

$TakeFoodOver\ \triangleq$
$\land \neg(bRSide[TIGER] = bRSide[FOOD] \land bRSide[SHEEP] = bRSide[TIGER])$
$\land\ haveNotCrossed(MAN,\ FOOD)$
$\land\ changeBeingsSide(MAN,\ FOOD)$

Man takes the sheep back, but tiger could stay with food.

$TakeSheepBack\ \triangleq$
$\land\ haveCrossed(MAN,\ SHEEP)$
$\land\ changeBeingsSide(MAN,\ SHEEP)$

Man takes the tiger back and takes care not to leave the food with the sheep alone!

$TakeTigerBack\ \triangleq$
$\land \neg(bRSide[SHEEP] = bRSide[TIGER] \land bRSide[FOOD] = bRSide[SHEEP])$
$\land\ haveCrossed(MAN,\ TIGER)$
$\land\ changeBeingsSide(MAN,\ TIGER)$

When the man takes the food back, he shall not let the tiger with the sheep behind him!

$TakeFoodBack\ \triangleq$
$\land \neg(bRSide[TIGER] = bRSide[FOOD] \land bRSide[SHEEP] = bRSide[TIGER])$
$\land\ haveCrossed(MAN,\ FOOD)$
$\land\ changeBeingsSide(MAN,\ FOOD)$

When the man travels alone, he shall not let the tiger with the sheep or the sheep with the food behind him!

$ManTravelsAlone\ \triangleq$
$\land \neg(bRSide[MAN] = bRSide[SHEEP] \land bRSide[SHEEP] = bRSide[FOOD])$
$\land \neg(bRSide[MAN] = bRSide[TIGER] \land bRSide[TIGER] = bRSide[SHEEP])$
$\land\ bRSide' = [bRSide\ \text{EXCEPT}\ ![MAN] = 1 - bRSide[MAN]]$

$NotAllCrossed\ \triangleq\ \exists\, b \in BEINGS : bRSide[b] \neq 1$
$AllCrossed\ \triangleq\ \forall\, b \in BEINGS : bRSide[b] = 1$

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129 Next ≜
130     ∨ TakeSheepOver
131     ∨ TakeSheepBack
132     ∨ TakeTigerOver
133     ∨ TakeTigerBack
134     ∨ TakeFoodOver
135     ∨ TakeFoodBack
136     ∨ ManTravelsAlone

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