

Boolos and Jeffrey - HW6

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1 The story so far...

We've previously established via the *Church-Turing thesis* that there cannot be a decision procedure for validity. This means there cannot be **both** a positive test and a negative test for validity. At this point in the book it is revealed that there is in fact a positive test for validity. A proof of this fact will have to show the implication in both directions. The first implication that *If the test says "yes", the formula is valid.* is known as *Soundness*. The second implication that *If the formula is valid, the test says "yes".* is known as *Completeness*. Chapter 11 presents a proof of Soundness for the positive validity test.

2 a few refutations

2.1

argument: $\{\exists x(Fx \wedge Gx), \forall x(Gx \rightarrow \neg Hx)\} \vdash \exists x[x = x \wedge (Fx \wedge \neg Hx)]$

$\Delta : \{\exists x(Fx \wedge Gx), \forall x(Gx \rightarrow \neg Hx), \forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)]\}$

refutation of Δ :

$\exists x(Fx \wedge Gx)$	Δ
$\forall x(Gx \rightarrow \neg Hx)$	Δ
$\forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)]$	Δ
$Fa \wedge Ga$	1
$Ga \rightarrow \neg Ha$	2
$a \neq a \vee \neg(Fa \wedge \neg Ha)$	3
$\not\vdash$	4,5,6

2.2

argument: $\{\exists x Lbx \rightarrow \forall x Lxb, \neg Lbb\} \vdash \neg Lba$

$\Delta : \{\forall x \forall y (Lbx \rightarrow Lyb), \neg Lbb, Lba\}$

refutation of Δ :

$\forall x \forall y (Lbx \rightarrow Lyb)$	Δ
$\neg Lbb$	Δ
Lba	Δ
$\forall y (Lba \rightarrow Lyb)$	1
$Lba \rightarrow Lbb$	4
$\not\vdash$	2,3,5

2.3

argument: $\{\exists y (Gy \wedge \forall z Kyz), \forall y (Fy \rightarrow \neg \forall z Kyz)\} \vdash \exists y (Gy \wedge \neg Fy)$

$\Delta : \{\exists y \forall z (Gy \wedge Kyz), \forall y \exists z (Fy \rightarrow \neg Kyz), \forall y \neg (Gy \wedge \neg Fy)\}$

refutation of Δ :

$\exists y \forall z (Gy \wedge Kyz)$	Δ
$\forall y \exists z (Fy \rightarrow \neg Kyz)$	Δ
$\forall y \neg (Gy \wedge \neg Fy)$	Δ
$\forall z (Ga \wedge Kaz)$	1
$\exists z (Fa \rightarrow \neg Kaz)$	2
$Fa \rightarrow \neg Kab$	5
$Ga \wedge Kab$	4
$\neg (Ga \wedge \neg Fa)$	3
$\not\vdash$	6,7,8