Boolos and Jeffrey - HW5

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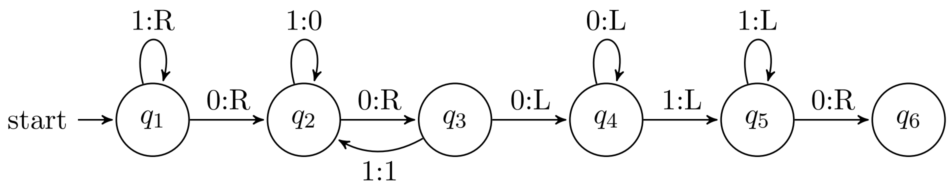
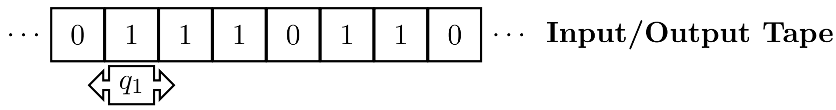
f(x,y) = x, when x=3 and y=2

Graphical representation of turing machine:





Translation into first order logic:

Our representation will take the form $\Delta \models H$, where Δ is a set of statements in first order logic and H is a single statement logically entailed from Δ. Our language has the domain $\mathbb{Z}$ of negative integers, 0 and positive integers. Special features of the language referred to in the following logical statements include the successor function ($'$), the constant $\mathbf{0}$ (which denotes the integer 0), the two-place ordering predicate $<$, the two-place state predicate $\mathbf{Q}$, and the two-place position predicate $\mathbf{S}$.

We first have to encode some mathematical facts about $'$ and $<$:

$$\forall z \exists x\, z = x' \wedge \forall z \forall x \forall y([z = x' \wedge z = y'] \rightarrow x = y) \tag{1}$$

and:

$$\forall x \forall y \forall z[(x < y \wedge y < z) \rightarrow x < z] \ \wedge \ \forall x \forall y(x' = y \rightarrow x < y) \\ \wedge \ \forall x \forall y(x < y \rightarrow x \neq y). \quad (2)$$

Next we need to encode the initial configuration of the tape:

$$\mathbf{0}Q_1\mathbf{0} \wedge \mathbf{0}S_1\mathbf{0} \wedge \mathbf{0}S_1\mathbf{0}' \wedge \mathbf{0}S_1\mathbf{0}'' \wedge \mathbf{0}S_1\mathbf{0}'''' \wedge \mathbf{0}S_1\mathbf{0}''''' \wedge \\ \forall y[(y \neq \mathbf{0} \wedge y \neq \mathbf{0}' \wedge y \neq \mathbf{0}'' \wedge y \neq \mathbf{0}'''' \wedge y \neq \mathbf{0}''''') \rightarrow \mathbf{0}S_0y] \quad (3)$$

Lastly we encode each of the state transitions (arrows in the flow graph) as logical statements: