

Boolos and Jeffrey - HW2

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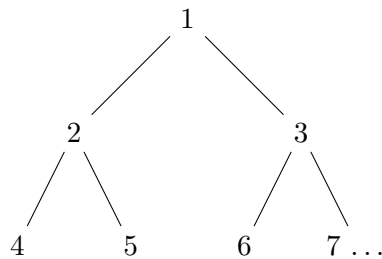
1 All nodes lead to Rome.

Proposition:

The set of nodes of an infinite binary tree is enumerable.

Conclusion:

Proof. Starting from the single origin node at the first level $d = 1$ the amount of nodes on each level is 2^d . The nodes can be counted simply starting at the origin like so:



□

2 What a long, strange trip it's been.

Proposition:

The set of infinite paths beginning at the origin down an infinite binary tree is *not* enumerable.

Conclusion:

Proof. Let each pair of paths from a particular node be represented by 0 and 1. With this encoding each path p_n , beginning from the origin, can be represented as a binary string of 0's and 1's. We can arrange the paths in a two dimensional grid:

$$\begin{array}{c|cccccc}
 p_1 & 0 & 1 & 0 & 0 & 1 & \dots \\
 p_2 & 1 & 0 & 0 & 0 & 0 & \dots \\
 p_3 & 1 & 1 & 0 & 1 & 1 & \dots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
 \end{array}$$

We can create a new path not contained in our representation by taking the converse of each binary digit along the diagonal $(1, 1, 1, \dots)$. Therefore by diagonalization we have shown the paths are *not* enumerable. \square

3 \mathbb{N} *into* \mathbb{N} **Proposition:**

Where \mathbb{N} is the set of positive integers, prove that the set of all *one-to-one, total* functions from \mathbb{N} *into* \mathbb{N} is not enumerable.

Conclusion:

(in progress)

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