Boolos and Jeffrey - HW6

David Maldonado, *david.m.maldonado@gmail.com*

November 18, 2014

1 The story so far...

We've previously established via the *Church-Turing thesis* that there cannot be a decision procedure for validity. This means there cannot be **both** a positive test and a negative test for validity. At this point in the book it is revealed that there is in fact a positive test for validity. A proof of this fact will have to show the implication in both directions. The first implication that *If the test says "yes", the formula is valid.* is known as *Soundness.* The second implication that *If the formula is valid, the test says "yes".* is known as *Completeness.* Chapter 11 presents a proof of Soundness for the positive validity test.

2 a few refutations

2.1

argument: $\{\exists x(Fx \wedge Gx), \forall x(Gx \to \neg Hx)\} \vdash \exists x[x = x \wedge (Fx \wedge \neg Hx)]$

$\Delta : \{\exists x(Fx \wedge Gx), \forall x(Gx \to \neg Hx), \forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)]\}$

refutation of Δ :

$$
\begin{array}{rr}
\exists x(Fx \wedge Gx) & \Delta \\
\forall x(Gx \to \neg Hx) & \Delta \\
\forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)] & \Delta \\
Fa \wedge Ga & 1 \\
Ga \to \neg Ha & 2 \\
a \neq a \vee \neg(Fa \wedge \neg Ha) & 3 \\
\neg Fa \vee Ha & 6
\end{array}
$$

2.2

argument: $\{\exists x Lbx \to \forall x Lxb, \neg Lbb\} \vdash \neg Lba$

$\Delta : \{\}$

refutation of Δ :

2.3

argument: $\{\exists y(Gy \wedge \forall z Kyz), \forall y(Fy \to \neg \forall z Kyz)\} \vdash \exists y(Gy \wedge \neg Fy)$

$\Delta : \{\}$

refutation of Δ :