

Boolos and Jeffrey - HW1

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1 A question about \cap

Premise:

The intersection of a finite set \mathbf{S} and an enumerable set \mathbf{T} is enumerable.

Lemma 1.1. *Any finite set is enumerable.*

Proof. Let \mathbf{S} be a finite set with n elements. Let $\mathbf{K} = \{1, 2, \dots, n\}$. Choose an element \mathbf{s} in \mathbf{S} and assign $f(1) = \mathbf{s}$. Set \mathbf{S}' to $\mathbf{S} - \{\mathbf{s}\}$. Choose an element \mathbf{s}' in \mathbf{S}' and assign $f(2) = \mathbf{s}'$. Repeat this procedure until \mathbf{S} is exhausted. The resulting function $f : \mathbf{K} \rightarrow \mathbf{S}$ is an enumeration of \mathbf{S} . \square

Theorem 1.1. *The intersection of two enumerable sets is enumerable.*

Proof. Let $f : \mathbb{N} \rightarrow \mathbf{A}$ represent a function that enumerates the first set. Let $g : \mathbb{N} \rightarrow \mathbf{B}$ represent a function that enumerates the second set. Let $h : \mathbb{N} \rightarrow \mathbf{A} \cap \mathbf{B}$ be a new function defined as follows:

$$h(x) = \begin{cases} f(x) & \text{if } f(x) \in \mathbf{B} \\ \text{undefined} & \text{if } f(x) \notin \mathbf{B}. \end{cases}$$

\square

Conclusion:

By **Lemma 1.1** the set \mathbf{S} is enumerable. By **Theorem 1.1** the intersection of \mathbf{S} and \mathbf{T} is enumerable.

2 A slightly harder question about \cap

Premise:

The intersection of an enumerable set of enumerable sets is itself enumerable.

Proof. Let \mathbf{S} be a enumerable set of enumerable sets. Pick a set \mathbf{A} from \mathbf{S} . Let \mathbf{B} be $\bigcap(\mathbf{S} - \mathbf{A})$. By **Theorem 1.1** we can define a function $h : \mathbb{N} \rightarrow \mathbf{A} \cap \mathbf{B}$ that enumerates $\bigcap \mathbf{S}$. \square

3 It takes two...

Premise:

Let \mathbf{F} be a set of *one to one* functions that both i) have a domain that's a subset of the positive integers, and ii) are *onto* a two element set $\{a,b\}$. \mathbf{F} is enumerable.

Conclusion:

(work in progress)

4 Enumerate all the things!

Premise:

The set of all finite sequences of positive integers is enumerable.

Theorem 4.1. *foo*

Conclusion:

(work in progress)