Boolos and Jeffrey - HW2

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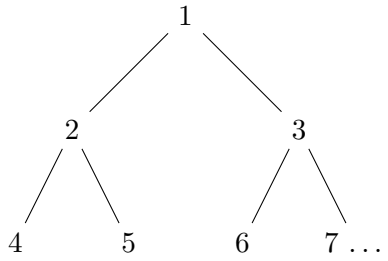
1 All nodes lead to Rome.

Proposition:

The set of nodes of an infinite binary tree is enumerable.

Conclusion:

Proof. Starting from the single origin node at the first level $d = 1$ the amount of nodes on each level is 2^d. The nodes can be counted simply starting at the origin like so:



□

2 What a long, strange trip it's been.

Proposition:

The set of infinite paths beginning at the origin down an infinite binary tree is *not* enumerable.

Conclusion:

Proof. Let each path p_n from a particular node be represented by 0 and 1. With this encoding each path beginning from the origin can be represented as a binary string of 0's and 1's. We can arrange the paths in a two dimensional grid:

p_1	0	1	0	0	1	$\dots$
p_2	1	0	0	0	0	$\dots$
p_3	1	1	0	1	1	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

We can create a new path not contained in our representation by taking the converse of each binary digit along the diagonal $(1, 1, 1, \dots)$. Therefore by diagonalization we have shown the paths are *not* enumerable. □

3 $\mathbb{N}$ *into* $\mathbb{N}$

Proposition:

Where $\mathbb{N}$ is the set of positive integers, prove that the set of all *one-to-one*, *total* functions from $\mathbb{N}$ *into* $\mathbb{N}$ is not enumerable.

Conclusion:

(in progress)

4 $\mathbb{N}$ *onto* $\mathbb{N}$

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