

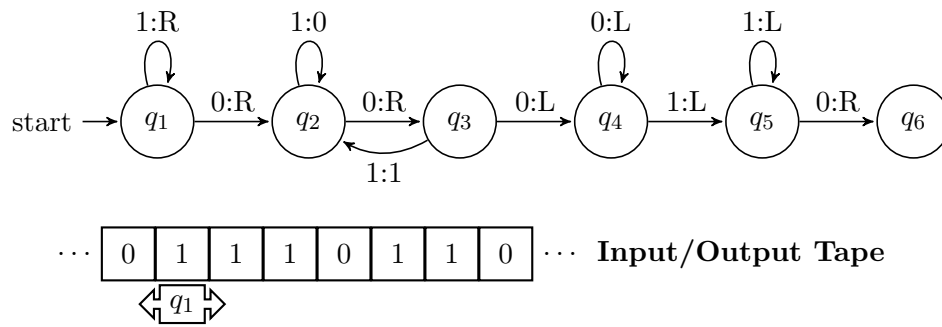
# Boolos and Jeffrey - HW5

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**f(x,y) = x, when x=3 and y=2**

**Graphical representation of turing machine:**



**Translation into first order logic:**

Our representation will take the form  $\Delta \models H$ , where  $\Delta$  is a set of statements in first order logic and  $H$  is a single statement logically entailed from  $\Delta$ . Our language has the domain  $\mathbb{Z}$  of negative integers, 0 and positive integers. Special features of the language referred to in the following logical statements include the successor function ( $'$ ), the constant  $\mathbf{0}$  (which denotes the integer 0), the two-place ordering predicate  $<$ , the two-place state predicate  $\mathbf{Q}$ , and the two-place "scanning" predicate  $\mathbf{S}$ .

We first have to encode some mathematical facts about  $'$  and  $<$  :

$$\forall z \exists x z = x' \wedge \forall z \forall x \forall y ([z = x' \wedge z = y'] \rightarrow x = y) \quad (1)$$

and:

$$\begin{aligned} \forall x \forall y \forall z [(x < y \wedge y < z) \rightarrow x < z] \wedge \forall x \forall y (x' = y \rightarrow x < y) \\ \wedge \forall x \forall y (x < y \rightarrow x \neq y). \end{aligned} \quad (2)$$

Next we need to encode the initial configuration of the tape:

$$\begin{aligned} 0Q_10 \wedge 0S_10 \wedge 0S_10' \wedge 0S_10'' \wedge 0S_10''' \wedge 0S_10'''' \wedge \\ \forall y [(y \neq 0 \wedge y \neq 0' \wedge y \neq 0'' \wedge y \neq 0''' \wedge y \neq 0'''' ) \rightarrow 0S_0y] \end{aligned} \quad (3)$$

To finish  $\Delta$  we encode each of the state transitions (arrows in the flow graph) as logical statements:

$$\forall t \forall x [(tQ_1x \wedge tS_1x) \rightarrow (t'Q_1x' \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (4)$$

$$\forall t \forall x [(tQ_1x \wedge tS_0x) \rightarrow (t'Q_2x' \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (5)$$

$$\begin{aligned} \forall t \forall x [(tQ_2x \wedge tS_1x) \rightarrow \\ ([t'Q_2x \wedge t'S_0x] \wedge \forall y [(y \neq x) \rightarrow ([tS_0y \rightarrow t'S_0y] \wedge [tS_1y \rightarrow t'S_1y])])] \end{aligned} \quad (6)$$

$$\forall t \forall x [(tQ_2x \wedge tS_0x) \rightarrow (t'Q_3x' \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (7)$$

$$\forall t \forall x [(tQ_3x' \wedge tS_0x') \rightarrow (t'Q_4x \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (8)$$

$$\begin{aligned} \forall t \forall x [(tQ_3x \wedge tS_1x) \rightarrow \\ ([t'Q_2x \wedge t'S_1x] \wedge \forall y [(y \neq x) \rightarrow ([tS_0y \rightarrow t'S_0y] \wedge [tS_1y \rightarrow t'S_1y])])] \end{aligned} \quad (9)$$

$$\forall t \forall x [(tQ_4x' \wedge tS_0x') \rightarrow (t'Q_4x \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (10)$$

$$\forall t \forall x [(tQ_4x' \wedge tS_1x') \rightarrow (t'Q_5x \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (11)$$

$$\forall t \forall x [(tQ_5x' \wedge tS_1x') \rightarrow (t'Q_5x \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (12)$$

$$\forall t \forall x [(tQ_5x \wedge tS_0x) \rightarrow (t'Q_6x' \wedge \forall y [(tS_0y \rightarrow t'S_0y) \wedge (tS_1y \rightarrow t'S_1y)])] \quad (13)$$

The last statement to encode is  $H$ , which is the disjunction of all sentences stating the machine is in a potential halting state:

$$\begin{aligned} & \exists t \exists x (tQ_1x \wedge tS_0x) \vee \exists t \exists x (tQ_1x \wedge tS_1x) \vee \\ & \exists t \exists x (tQ_2x \wedge tS_0x) \vee \exists t \exists x (tQ_2x \wedge tS_1x) \vee \\ & \exists t \exists x (tQ_3x \wedge tS_0x) \vee \exists t \exists x (tQ_3x \wedge tS_1x) \vee \\ & \exists t \exists x (tQ_4x \wedge tS_0x) \vee \exists t \exists x (tQ_4x \wedge tS_1x) \vee \\ & \exists t \exists x (tQ_5x \wedge tS_0x) \vee \exists t \exists x (tQ_5x \wedge tS_1x) \vee \\ & \exists t \exists x (tQ_6x \wedge tS_0x) \vee \exists t \exists x (tQ_6x \wedge tS_1x) \vee \end{aligned} \quad (14)$$

(is halting symbol separate from blank and mark?)