Boolos and Jeffrey - HW1

David Maldonado, *david.m.maldonado@gmail.com*

August 28, 2014

1 A question about $\cap$

Premise:

The intersection of a finite set **S** and an enumerable set **T** is enumerable.

Proof:

Lemma 1.1. *Any finite set is enumerable.*

Let **S** be a finite set with n elements. Let $\mathbf{K} = \{1,2,\ldots,n\}$. Choose an element $\mathbf{s}$ in **S** and assign $f(n) = \mathbf{s}$. Set $\mathbf{S}'$ to **S** - $\{\mathbf{s}\}$. Choose an element $\mathbf{s}'$ in $\mathbf{S}'$ and assign $f(n-1) = \mathbf{s}'$. Repeat this procedure until **S** is exhausted. The resulting function $f : \mathbf{K} \rightarrow \mathbf{S}$ is an enumeration of **S**.

Theorem 1.1. *The intersection of two enumerable sets is enumerable.*

Let $\mathbf{f}_1$ represent a function to enumerate the first set and $\mathbf{f}_2$ represent a function to enumerate the second set. Let **A** be a set to store temporary values generated from $\mathbf{f}_1$ and $\mathbf{f}_2$. Let **B** be a set to store the final values. Run both functions in turn $(\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_1, \ldots)$ storing the output in **A**. If a value that is already contained in **A** is generated by either function copy it to **B**. If either of the sets are exhausted stop the procedure. The set **B** contains the intersection of the two sets enumerated by $\mathbf{f}_1$ and $\mathbf{f}_2$. $\square$

2 A harder question about $\cap$

Premise:

The intersection of an enumerable set of enumerable sets is itself enumerable.

Proof:

foo

3 It takes two...

Premise:

Let **F** be a set of *one to one* functions that both i) have a domain that's a subset of the positive integers, and ii) are *onto* a two element set {a,b}. **F** is enumerable.

Proof:

foo

4 Enumerate all the things!

Premise:

The set of all finite sequences of positive integers is enumerable.

Proof:

foo