Boolos and Jeffrey - HW1

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1 A question about $\cap$

Premise:

The intersection of a finite set **S** and an enumerable set **T** is enumerable.

Lemma 1.1. *Any finite set is enumerable.*

Proof. Let **S** be a finite set with n elements. Let $\mathbf{K} = \{1,2,\ldots,n\}$. Choose an element **s** in **S** and assign $f(n) = \mathbf{s}$. Set $\mathbf{S}'$ to **S** - {**s**}. Choose an element $\mathbf{s}'$ in $\mathbf{S}'$ and assign $f(n-1) = \mathbf{s}'$. Repeat this procedure until **S** is exhausted. The resulting function $f : \mathbf{K} \to \mathbf{S}$ is an enumeration of **S**. □

Theorem 1.1. *The intersection of two enumerable sets is enumerable.*

Proof. Let $\mathbf{K} \subseteq \mathbb{N}$. Let $f : \mathbf{K} \to \mathbf{A}$ represent a function that enumerates the first set. Let $g : \mathbf{K} \to \mathbf{B}$ represent a function that enumerates the second set. Let $h : \mathbf{K} \to \mathbf{A} \cap \mathbf{B}$ be a new function defined as follows:

$$h(x) = \begin{cases} f(x) & \text{if } x \in \mathbf{B} \\ undefined & \text{if } x \notin \mathbf{B}. \end{cases}$$

□

Conclusion:

By **Lemma 1.1** the set **S** is enumerable. By **Theorem 1.1** the intersection of **S** and **T** is enumerable.

2 A slightly harder question about ∩

Premise:

The intersection of an enumerable set of enumerable sets is itself enumerable.

Conclusion:

foo

3 It takes two...

Premise:

Let **F** be a set of *one to one* functions that both i) have a domain that's a subset of the positive integers, and ii) are *onto* a two element set {a,b}. **F** is enumerable.

Conclusion:

foo

4 Enumerate all the things!

Premise:

The set of all finite sequences of positive integers is enumerable.

Conclusion:

foo