Boolos and Jeffrey - HW2

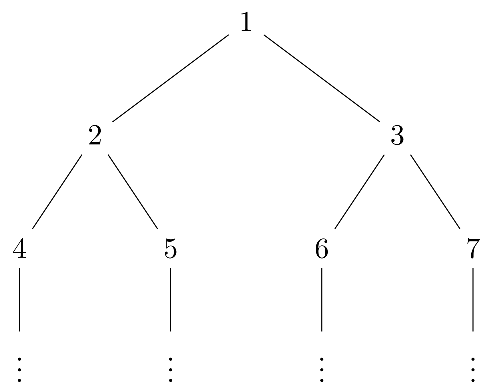
David Maldonado, *david.m.maldonado@gmail.com*

September 8, 2014

1 All nodes lead to Rome.

Theorem 1.1. *The set of nodes of an infinite binary tree is enumerable.*

Proof. Starting from the single origin node at the first level $d = 1$ the amount of nodes on each level is 2^d. The nodes can simply be counted by starting at the origin and moving left to right at each level:



$\square$

2 What a long, strange trip it's been.

Theorem 2.1. *The set of infinite paths beginning at the origin down an infinite binary tree is not enumerable.*

Proof. Let each pair of paths from a particular node be represented by 0 and 1. With this encoding each path p_n, beginning from the origin, can be represented as a binary string of 0's and 1's. We can arrange the paths in a two dimensional grid:

$$\begin{array}{c|ccccc} p_1 & 0 & 1 & 0 & 0 & \dots \\ p_2 & 1 & 0 & 0 & 0 & \dots \\ p_3 & 1 & 1 & 0 & 1 & \dots \\ p_3 & 0 & 0 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

We can create a new path not contained in our representation by taking the converse of each binary digit along the diagonal $(1, 1, 1, 0, \dots)$. Therefore by diagonalization we have shown the paths are *not* enumerable. □

3 ℕ *into* ℕ

Theorem 3.1. *Where* $\mathbb{N}$ *is the set of positive integers, the set of all one-to-one, total functions from* $\mathbb{N}$ *into* $\mathbb{N}$ *is not enumerable.*

Proof. (in progress) □

4 ℕ *onto* ℕ

Theorem 4.1. *Where* $\mathbb{N}$ *is the set of positive integers, the set of all one-to-one, total functions from* $\mathbb{N}$ *onto* $\mathbb{N}$ *is not enumerable.*

Proof. (in progress) □