Boolos and Jeffrey - HW1

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1 A question about $\cap$

Premise:

The intersection of a finite set $\mathbf{S}$ and an enumerable set $\mathbf{T}$ is enumerable.

Lemma 1.1. *Any finite set is enumerable.*

Proof. Let $\mathbf{S}$ be a finite set with n elements. Let $\mathbf{K} = \{1,2,\ldots,n\}$. Choose an element $\mathbf{s}$ in $\mathbf{S}$ and assign $f(n) = \mathbf{s}$. Set $\mathbf{S}'$ to $\mathbf{S}$ - $\{\mathbf{s}\}$. Choose an element $\mathbf{s}'$ in $\mathbf{S}'$ and assign $f(n-1) = \mathbf{s}'$. Repeat this procedure until $\mathbf{S}$ is exhausted. The resulting function $f : \mathbf{K} \to \mathbf{S}$ is an enumeration of $\mathbf{S}$. $\square$

Theorem 1.1. *The intersection of two enumerable sets is enumerable.*

Proof. Let $f : \mathbb{N} \to \mathbf{A}$ represent a function that enumerates the first set. Let $g : \mathbb{N} \to \mathbf{B}$ represent a function that enumerates the second set. Let $h : \mathbb{N} \to \mathbf{A} \cap \mathbf{B}$ be a new function defined as follows:

$$h(x) = \begin{cases} f(x) & \text{if } x \in \mathbf{B} \\ undefined & \text{if } x \notin \mathbf{B}. \end{cases}$$

$\square$

Conclusion:

By **Lemma 1.1** the set $\mathbf{S}$ is enumerable. By **Theorem 1.1** the intersection of $\mathbf{S}$ and $\mathbf{T}$ is enumerable.

2 A slightly harder question about $\cap$

Premise:

The intersection of an enumerable set of enumerable sets is itself enumerable.

Proof. Let $\mathbf{S}$ be a enumerable set of enumerable sets. Pick a set $\mathbf{A}$ from $\mathbf{S}$. Let $\mathbf{B}$ be $\bigcap \mathbf{S} - \mathbf{A}$. By **Theorem 1.1** we can define a function $h : \mathbb{N} \to \mathbf{A} \cap \mathbf{B}$ that enumerates $\bigcap \mathbf{S}$. $\square$

3 It takes two...

Premise:

Let $\mathbf{F}$ be a set of *one to one* functions that both i) have a domain that's a subset of the positive integers, and ii) are *onto* a two element set $\{$a,b$\}$. $\mathbf{F}$ is enumerable.

Conclusion:

foo

4 Enumerate all the things!

Premise:

The set of all finite sequences of positive integers is enumerable.

Theorem 4.1. *foo*

Conclusion:

foo