Boolos and Jeffrey - HW6

David Maldonado, *david.m.maldonado@gmail.com*

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1 The story so far...

We've previously established via the *Church-Turing thesis* that there cannot be a decision procedure for validity. This means there cannot be **both** a positive test and a negative test for validity. At this point in the book it is revealed that there is in fact a positive test for validity. A proof of this fact will have to show the implication in both directions. The first implication that *If the test says "yes", the formula is valid.* is known as *Soundness.* The second implication that *If the formula is valid, the test says "yes".* is known as *Completeness.* Chapter 11 presents a proof of Soundness for the positive validity test.

2 a few refutations

2.1

argument: $\{\exists x(Fx \wedge Gx), \forall x(Gx \rightarrow \neg Hx)\} \vdash \exists x[x = x \wedge (Fx \wedge \neg Hx)]$

$\Delta : \{\exists x(Fx \wedge Gx), \forall x(Gx \rightarrow \neg Hx), \forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)]\}$

refutation of Δ :

$\exists x(Fx \wedge Gx)$	Δ
$\forall x(Gx \rightarrow \neg Hx)$	Δ
$\forall x[x \neq x \vee \neg(Fx \wedge \neg Hx)]$	Δ
$Fa \wedge Ga$	1
$Ga \rightarrow \neg Ha$	2
$a \neq a \vee \neg(Fa \wedge \neg Ha)$	3
$\not\models$	4,5,6

2.2

argument: $\{\exists x Lbx \to \forall x Lxb, \neg Lbb\} \vdash \neg Lba$

$\Delta : \{\forall x \forall y (Lbx \to Lyb), \neg Lbb, Lba\}$

refutation of Δ :

$$\begin{array}{rr}
\forall x \forall y (Lbx \to Lyb) & \Delta \\
\neg Lbb & \Delta \\
Lba & \Delta \\
\forall y (Lba \to Lyb) & 1 \\
Lba \to Lbb & 4 \\
\not\models & 2,3,5
\end{array}$$

2.3

argument: $\{\exists y (Gy \wedge \forall z Kyz), \forall y (Fy \to \neg \forall z Kyz)\} \vdash \exists y (Gy \wedge \neg Fy)$

$\Delta : \{\exists y \forall z (Gy \wedge Kyz), \forall y \exists z (Fy \to \neg Kyz), \forall y \neg (Gy \wedge \neg Fy)\}$

refutation of Δ :

$$\begin{array}{rr}
\exists y \forall z (Gy \wedge Kyz) & \Delta \\
\forall y \exists z (Fy \to \neg Kyz) & \Delta \\
\forall y \neg (Gy \wedge \neg Fy) & \Delta \\
\not\models & ?
\end{array}$$