Boolos and Jeffrey - HW4

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1 some equivalence proofs...

Theorem 1.1. $\neg QvF \cong Q'v\neg F$

Proof. We'll begin with the first case:

$$\neg\forall vF \cong \exists v\neg F \tag{1}$$

The implication $\neg\forall vF \implies \exists v\neg F$ is proven simply by noting that if we assume $\neg\forall vF$ to be **true** that means there exists at least one term in a model that makes $\neg F$ **true**, which is precisely the statement on the right-hand side.

The converse implication $\neg\forall vF \impliedby \exists v\neg F$ is proven in the same way by assuming $\exists v\neg F$ to be **true**. It follows directly that because there is at least one term in a model that makes $\neg F$ **true** not all models make F true which is the statement on the left-hand side.

For the second case:

$$\neg\exists vF \cong \forall v\neg F \tag{2}$$

The implication $\neg\exists vF \implies \forall v\neg F$ is proven by first assuming $\neg\exists vF$ is **true**. With this assumption we can say that there does not exist a model where F is **true**, this is essentially the statement on the right-hand side ☐

2 proof of prenex normal form

Theorem 2.1. *Where **prenex normal form** is a formula where all the quantifiers are written as a string at the front and range over the quantifier-free portion, every formula in first-order logic has an equivalent prenex normal form.*

Proof. We will proceed by induction. Let us first agree on the following equivalences:

$$\neg QvF \cong Q'v\neg F \tag{1}$$

$\square$