Homework 9

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1. $C = \{0,9\} \cup \{1,8\} \cup \{2,7\} \cup \{3,6\} \cup \{4,5\}$

   Since you have to pick 6 integers, and there are 5 "pigeonholes" (pairs of numbers that sum to 9), at least one pair of numbers in the 6 chosen has to sum to 9.

2. If f is 1to1,
   Suppose f is not onto,
   then there exists a $b \in B$ s.t. $f(x) \neq b$ for any $x \in A$.
   Since $|A| = |B|$, $\longrightarrow \longleftarrow$, thus f is onto.

3.

a)

$|B| = \binom{5}{3} = 10$
$|A| = 5$

$A \longrightarrow B$, # of functions $= |B|^{|A|}$

$10^5 = 100{,}000$

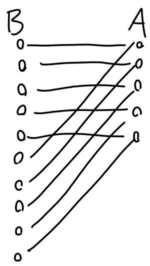
b) $P(10,5) = \dfrac{10!}{(10-5)!} = \dfrac{10!}{5!} = 30{,}240$

c)

There are none, because since $|A| < |B|$, once all elements in A have a corresponding element in B, there are still 5 elements in B left unhit, no matter the function from $A \longrightarrow B$.

d) $B \longrightarrow A$, $|A|^{|B|} = 5^{10} = 9{,}765{,}625$

e)



There are no 1to1 functions from $B \to A$, b/c There has to be at least one element in the codomain (A) that gets hit more than once if $|A| < |B|$.

f)

$$9\,765\,625 - \binom{5}{1}4^{10} + \binom{5}{2}3^{10} - \binom{5}{3}2^{10} + \binom{5}{4}1^{10} - \binom{5}{5}0^{10}$$

$$= 5{,}103{,}000$$

g)

$$\frac{n!}{(n-n)!} \qquad \frac{5!}{0!} = 5! = 120$$

4.

A: 1999

B: $\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1} = 256$

C: $\binom{9}{1}\binom{8}{1}\binom{7}{1}\binom{6}{1} = 3{,}024$

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$(1999 + 256 + 3024) - \underbrace{64}_{1\cdot 4\cdot 4\cdot 4} - \underbrace{1008}_{\binom{2}{1}\binom{9}{1}\binom{8}{1}\binom{7}{1}} - \underbrace{96}_{\binom{4}{1}\binom{4}{1}\binom{3}{1}\binom{2}{1}} + \underbrace{24}_{1\binom{4}{1}\binom{3}{1}\binom{2}{1}} = 4135$$

5. $f: S \longrightarrow \mathbb{Z}^-$, $f(x) = -\sqrt{x}$

1to1: Let $x, y \in S$

$$
\begin{aligned}
f(x) &= f(y) \\
-\sqrt{x} &= -\sqrt{y} \\
\sqrt{x} &= \sqrt{y} \\
x &= y \ \checkmark
\end{aligned}
$$

onto: Let $x \in S$, and $y \in \mathbb{Z}^-$

$$
\begin{aligned}
y &= -\sqrt{x} \\
-y &= \sqrt{x} \\
x &= (-y)^2
\end{aligned}
$$

for $\forall y \in \mathbb{Z}^-$, there exists an $x \in S$. $\checkmark$

$f(x) = -\sqrt{x}$ is a bijection, $f: S \longrightarrow \mathbb{Z}^-$