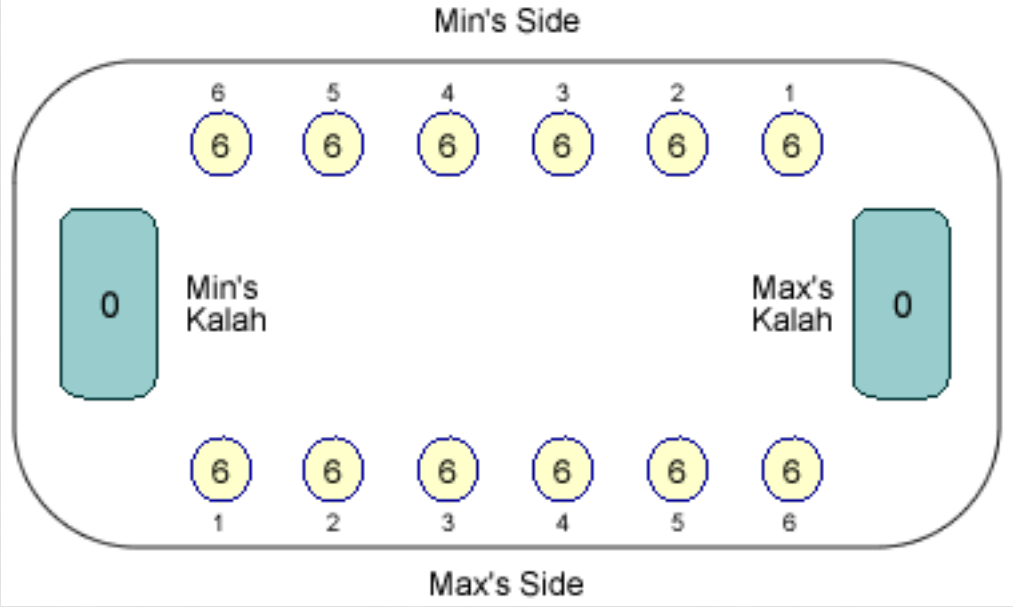
CASE STUDY - KALAH

JEFFREY L. POPYACK

KALAH

Kalah, also known as *Mancala*, *Wari*, or *Owari*, originated in Africa.

- Two players (Max & Min)
- Six *pits* for each player and larger pit (*Kalah*) on their right.



Game begins with each small pit filled with some number *k* of stones (e.g., *k=6*)

Excerpted from “Etudes for Programmers”, Charles Wetherell, Prentice Hall, 1978.

KALAH

On player's turn:

- Remove all stones from one of the player's small pits,
- Sow them into other pits counterclockwise around the board.

Sowing begins to the right of the source pit

- includes player's own Kalah
- includes opponent's small pits,
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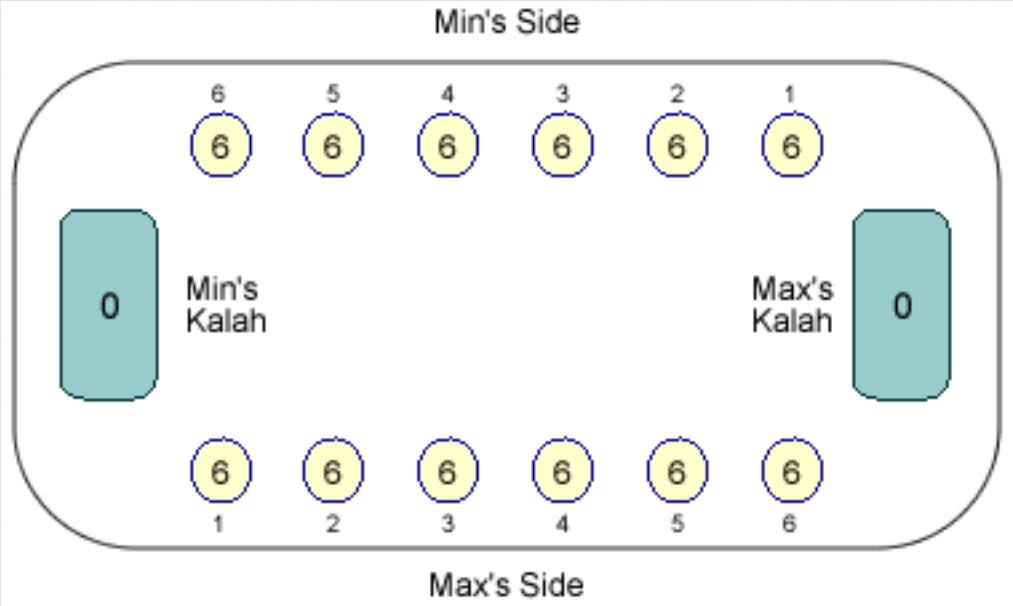

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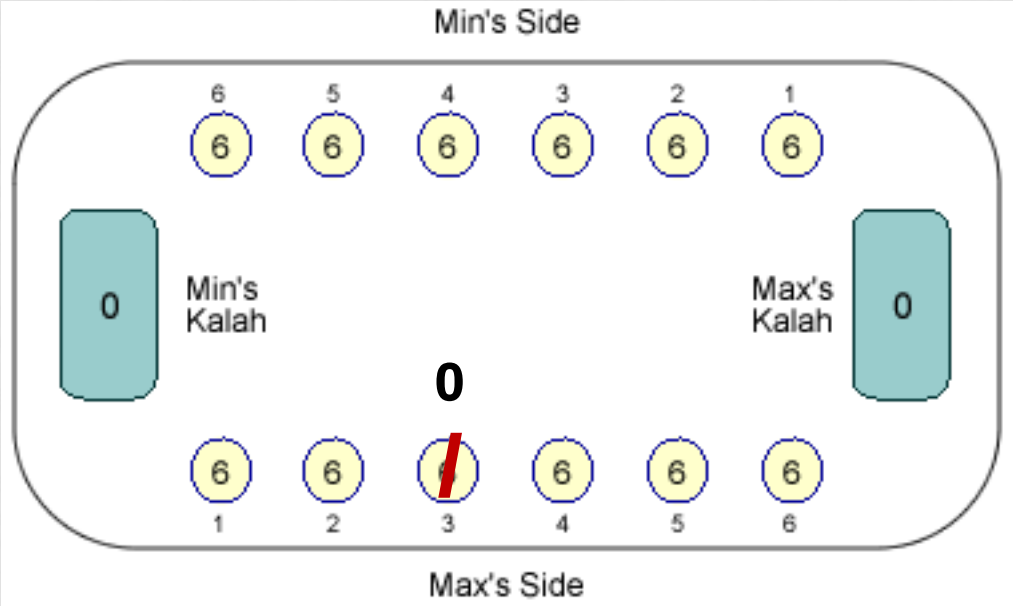

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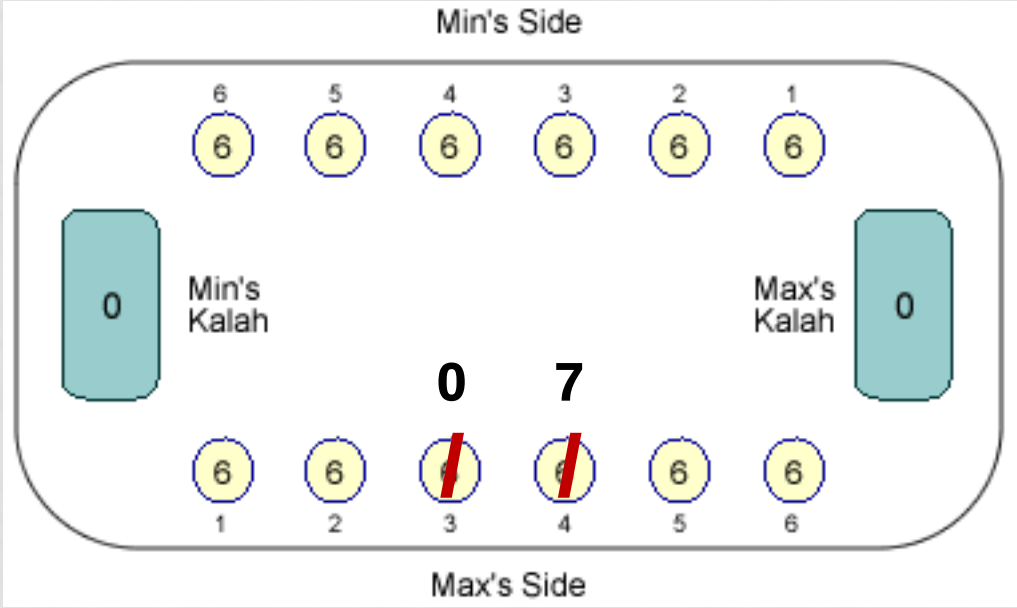

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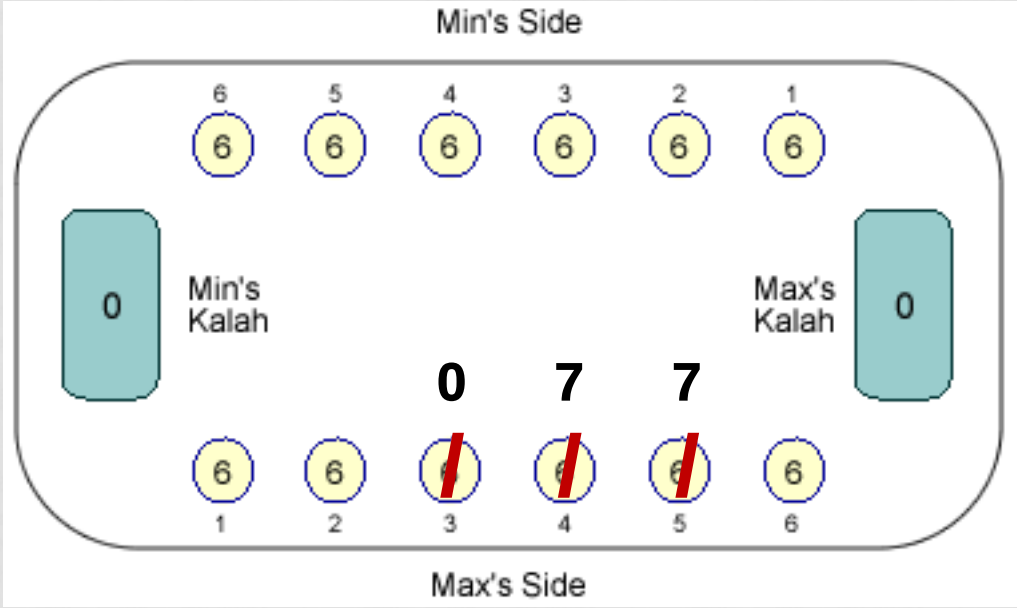

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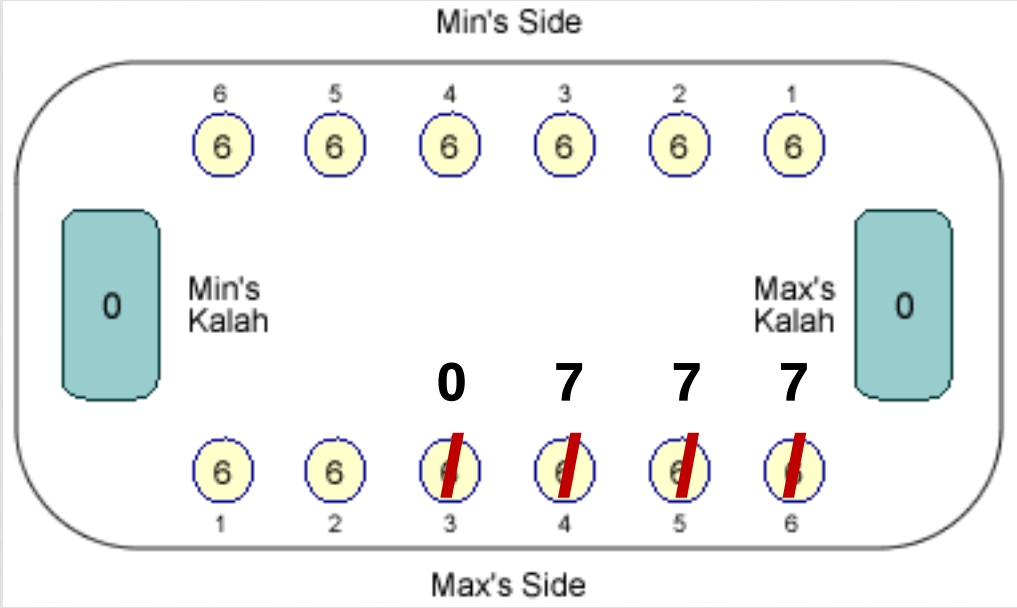

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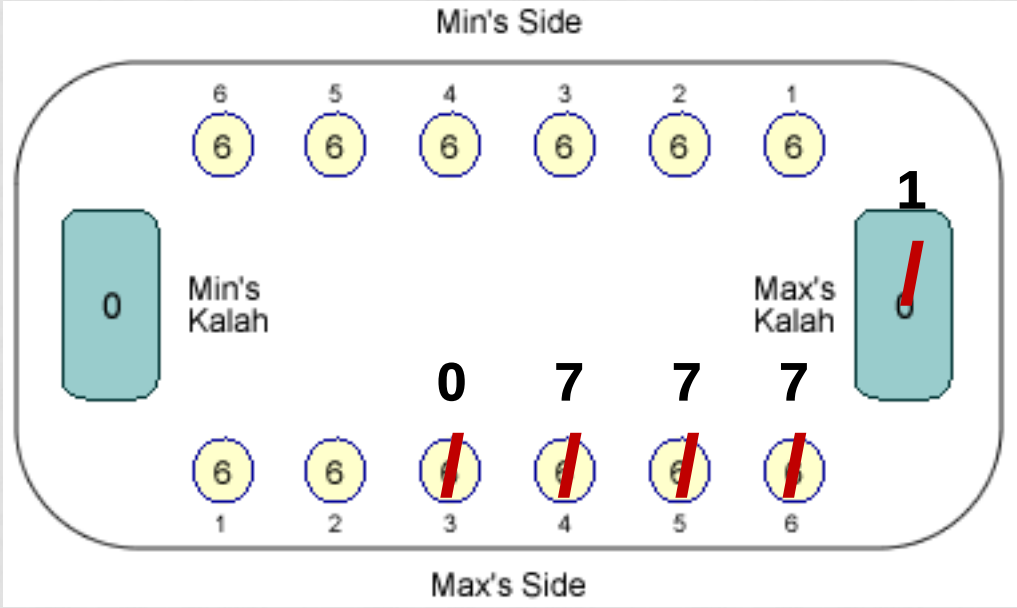

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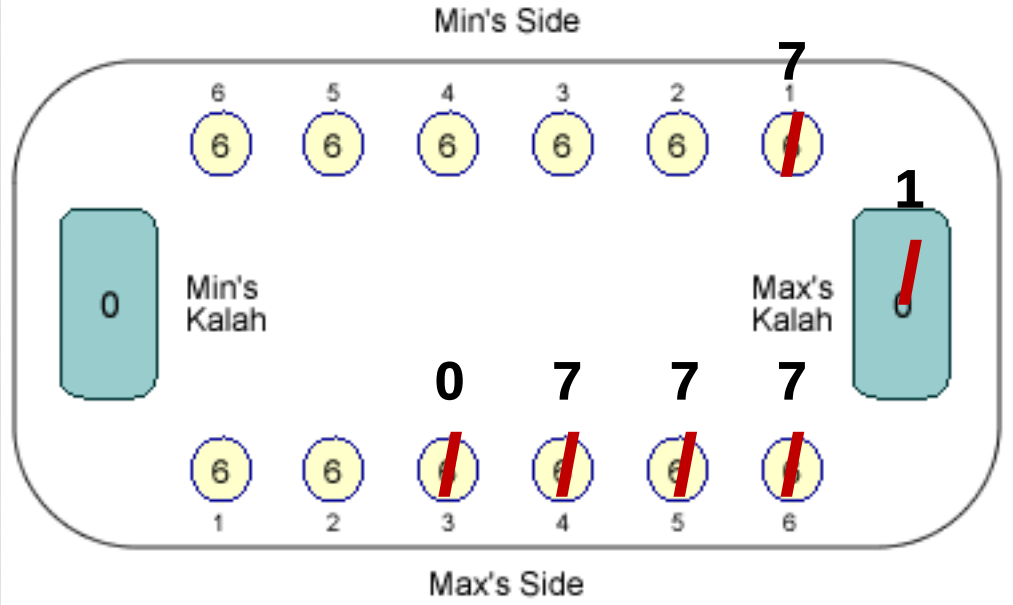

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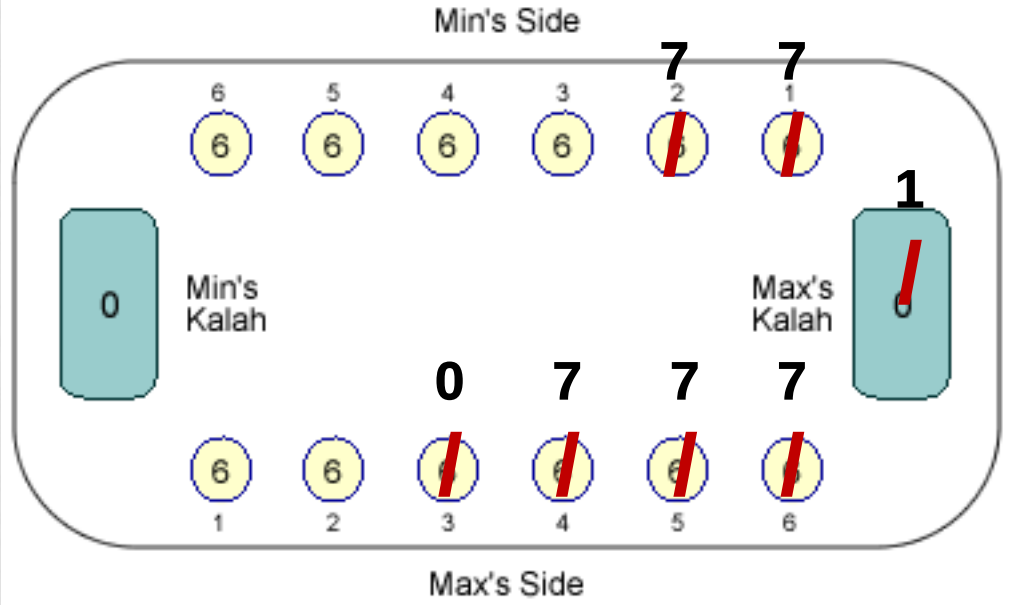

KALAH – LOOPING MOVE

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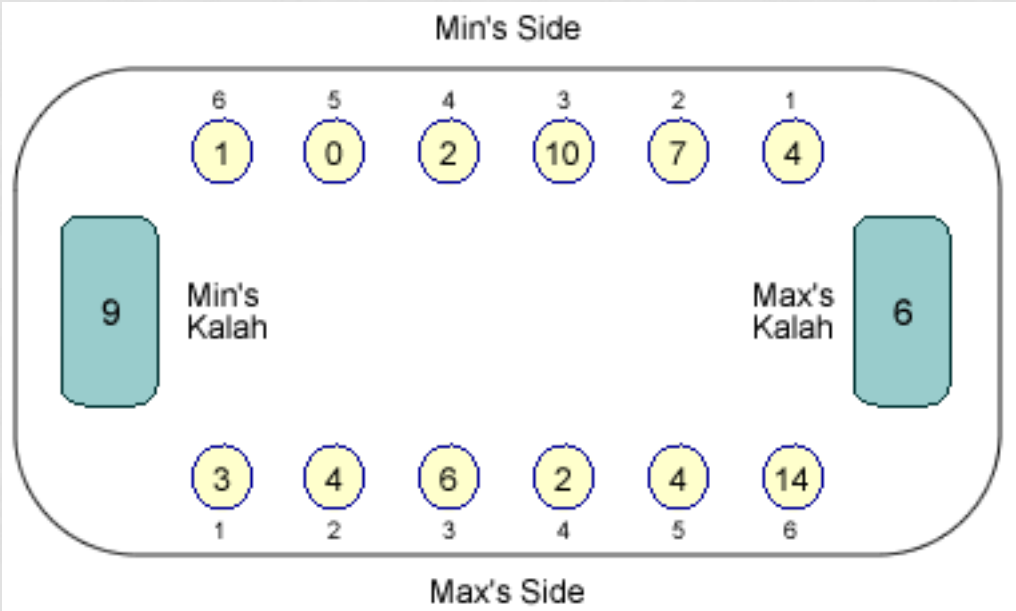

In this configuration, it is Max's turn to move.
Max will choose to move from Pit 6, performing a Looping Move.

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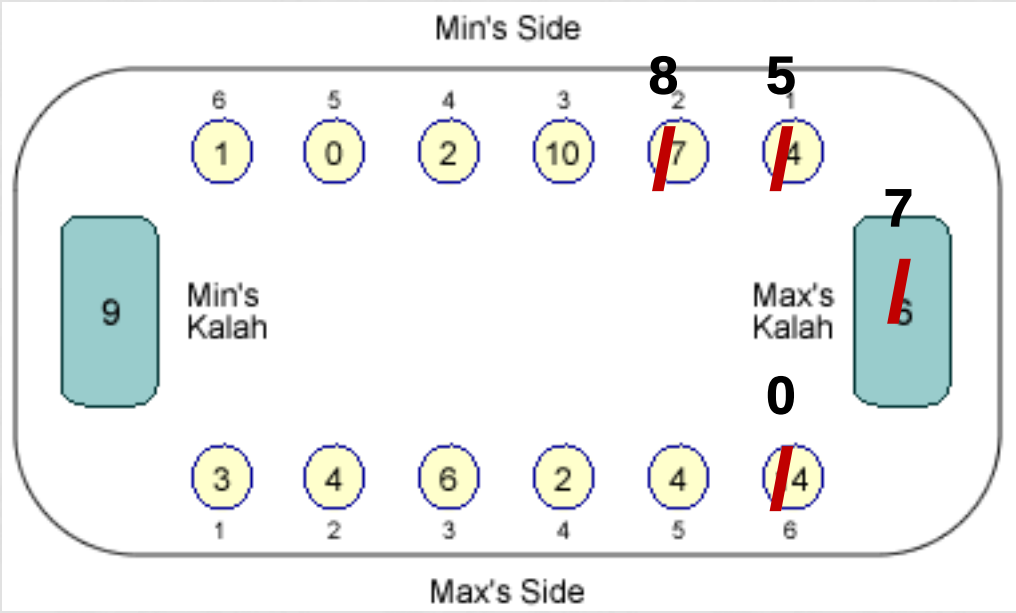

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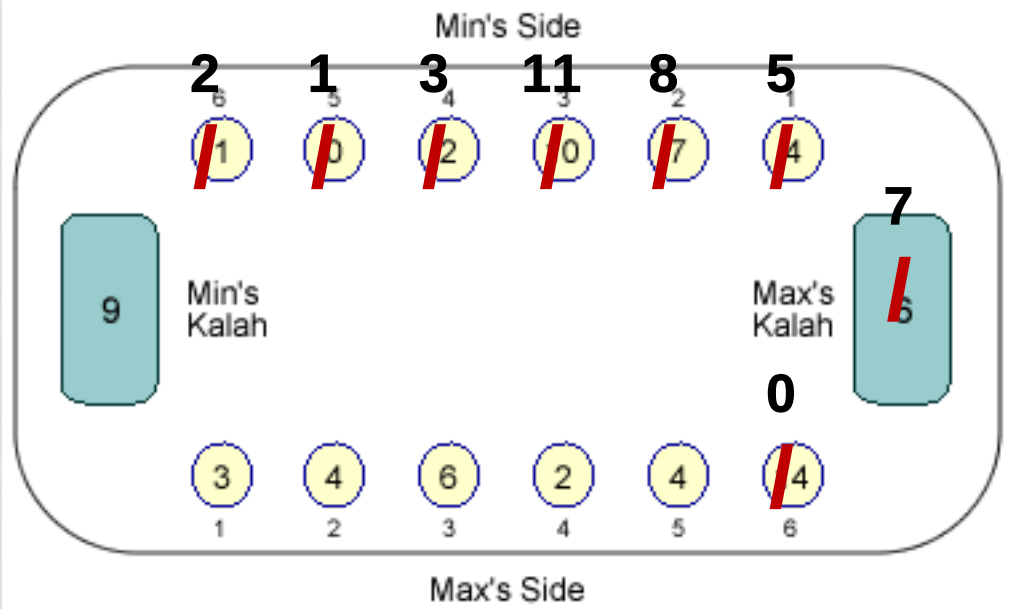

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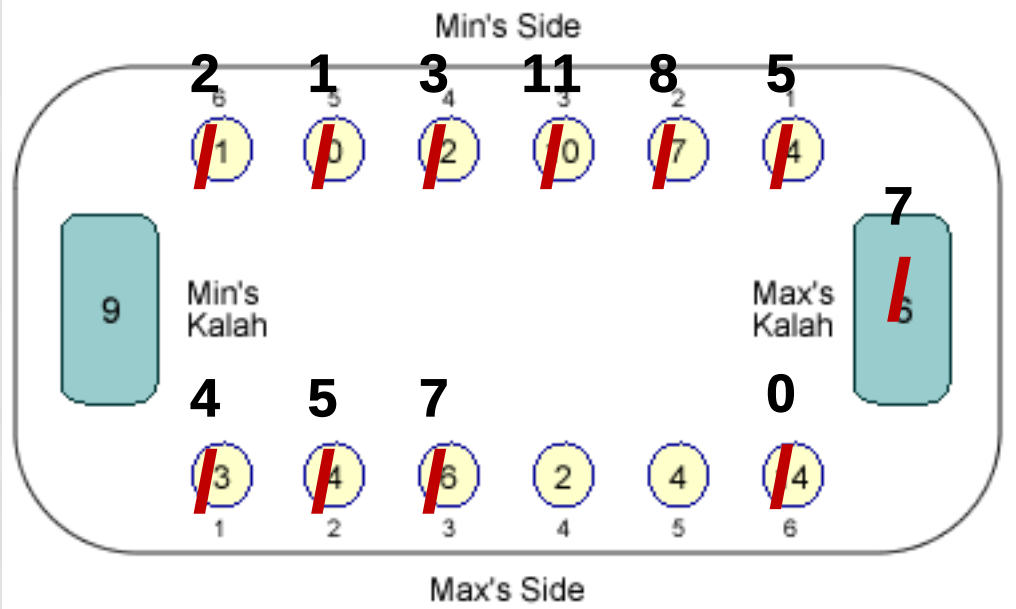

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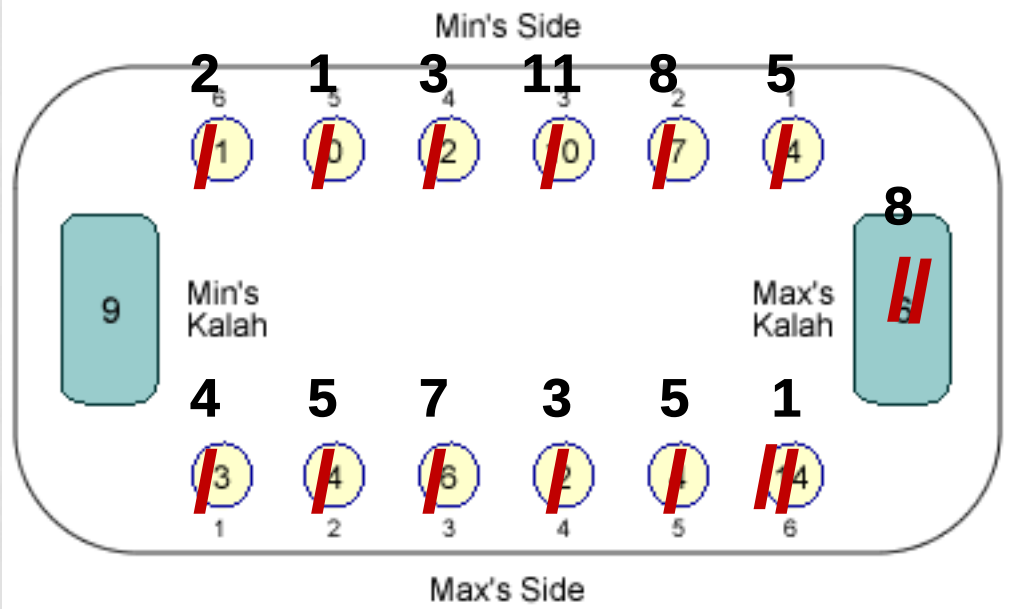

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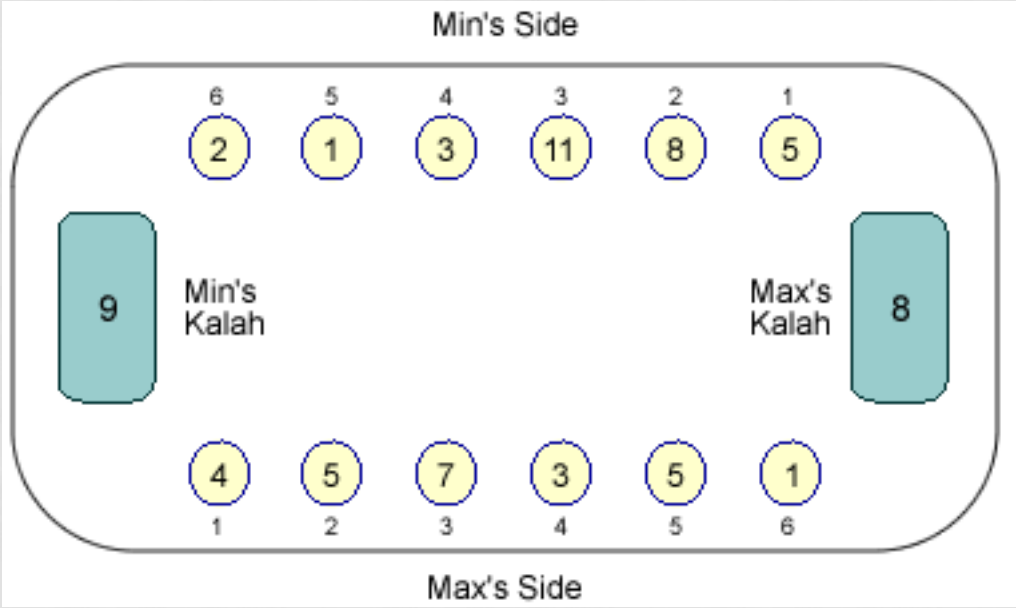


Max has completed the Looping Move. Max's Kalah has been sown twice; Min's Kalah has not been sown.

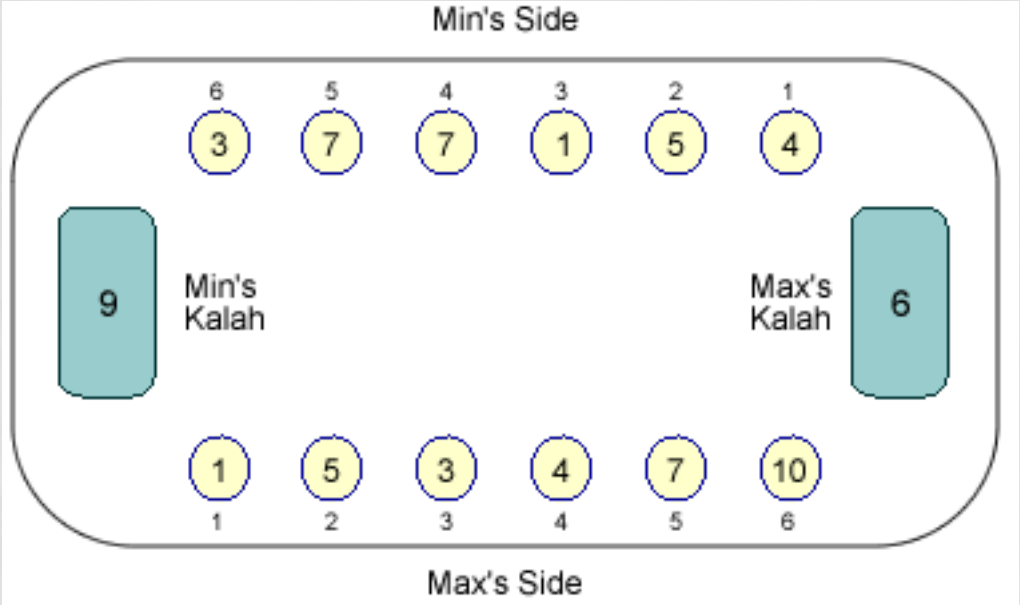
VARIATION #1 – *GO-AGAIN MOVE*

If last stone falls into one of moving player's own small *nonempty* pits;

and

Stones were played on the opponent's side during sowing:

- Stones in final pit are used to start a *Go-Again Move*, just like the original move. A player can have an arbitrarily long chain of go-agains.



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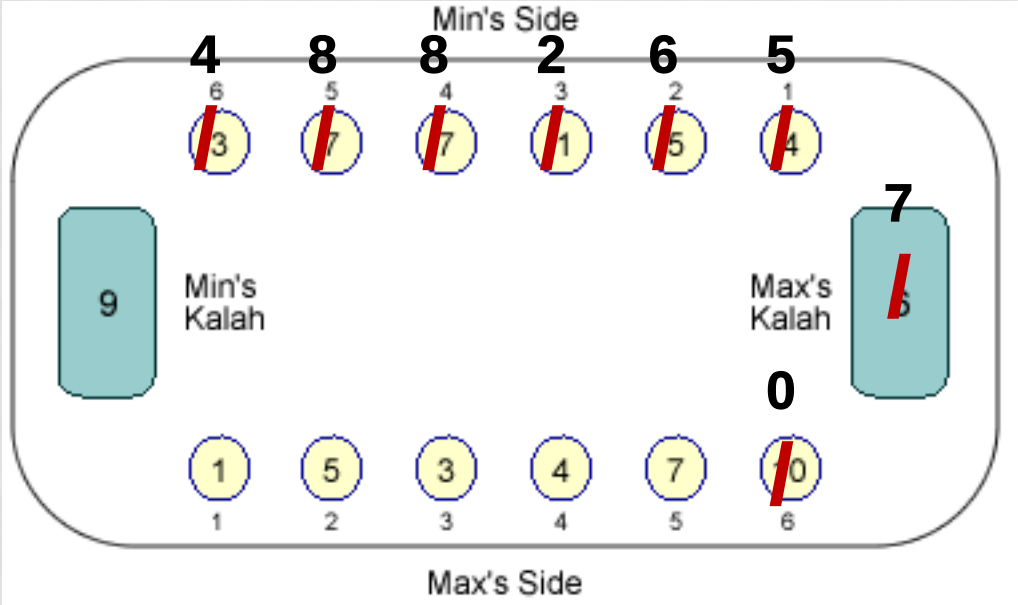

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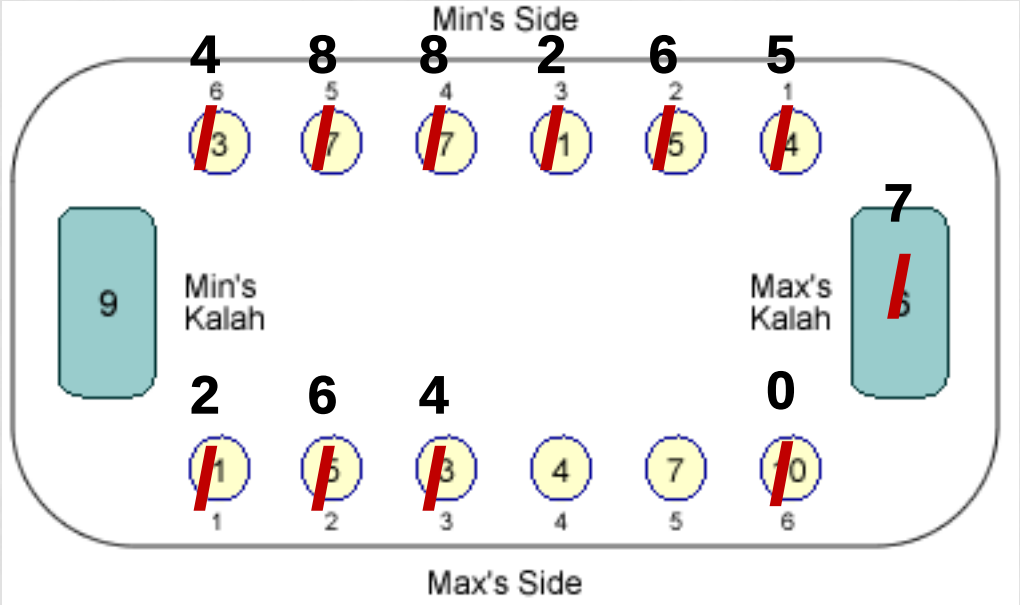

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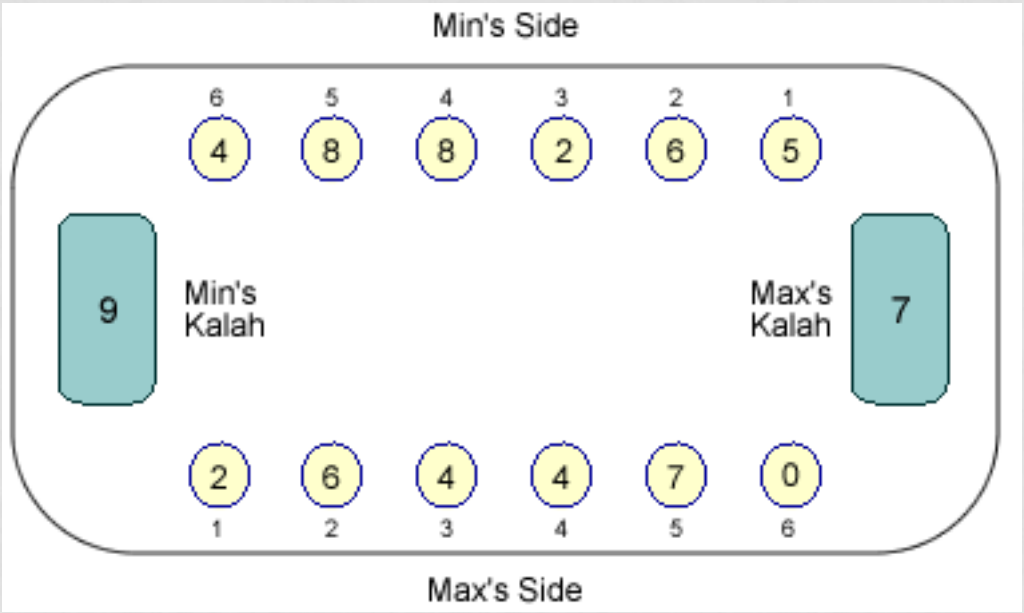

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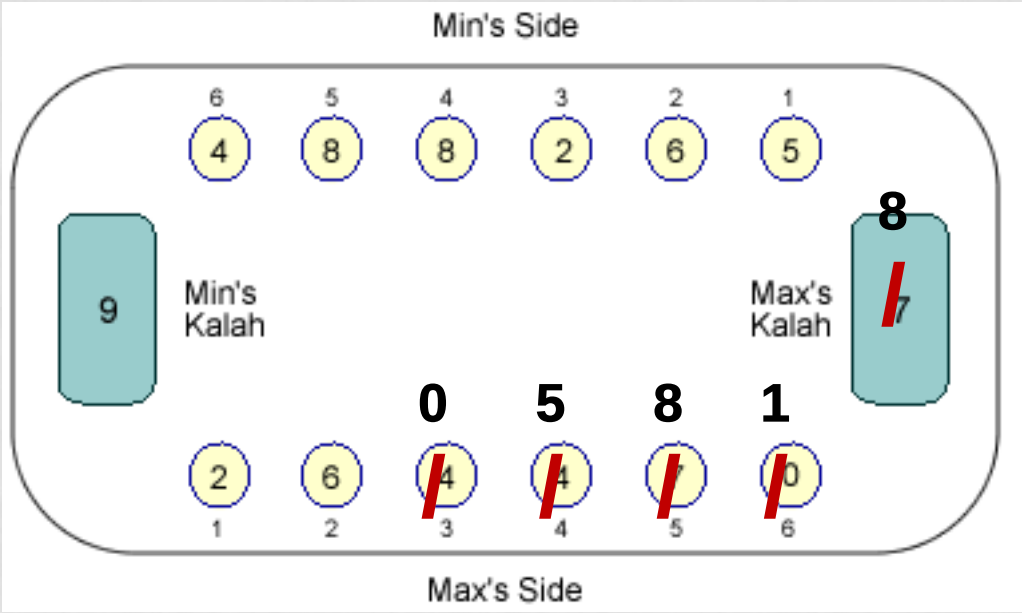

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there are either <u>two or three stones</u> in the pit:

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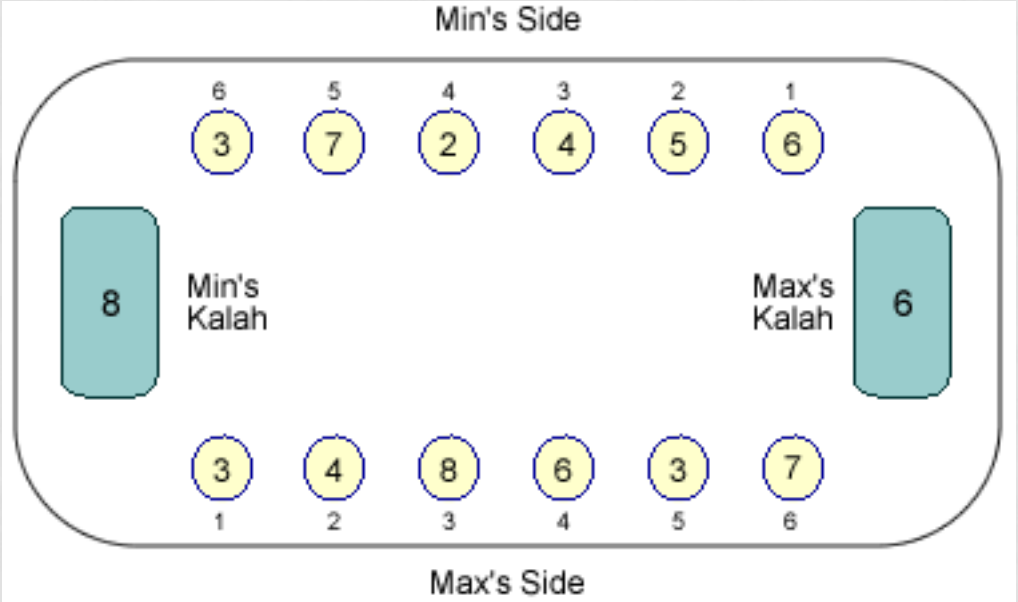

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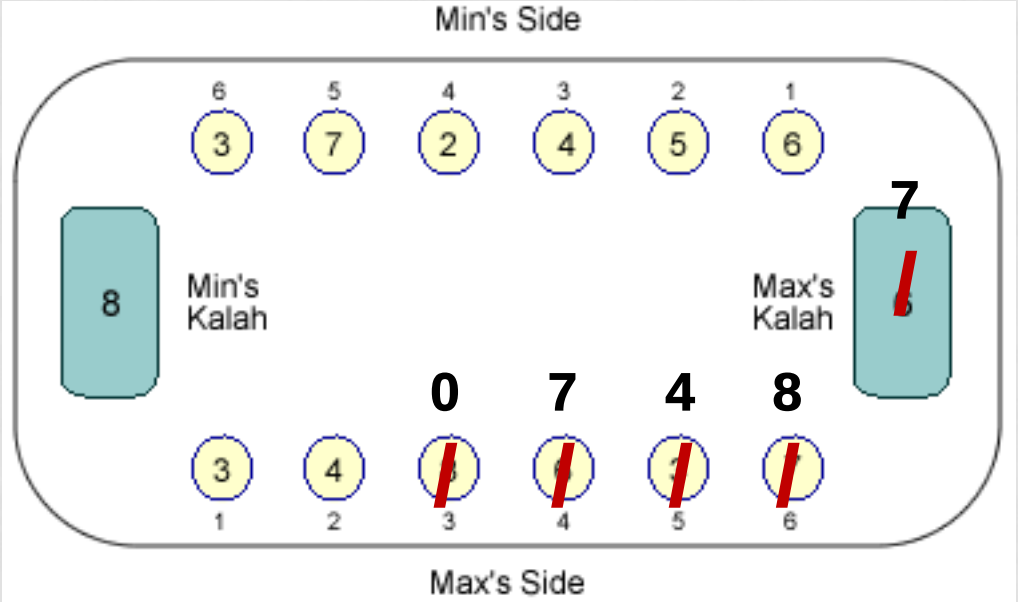

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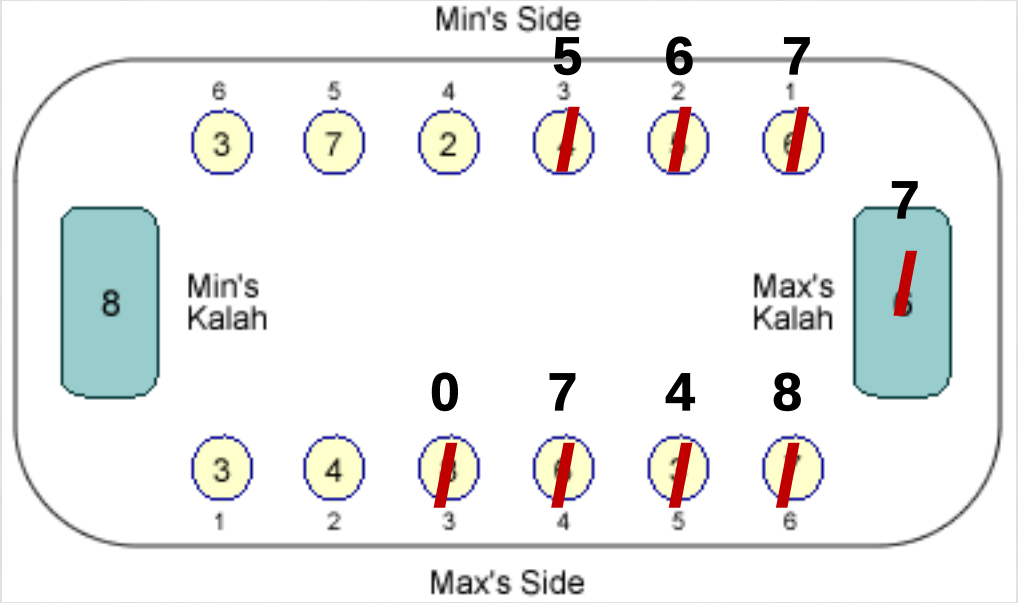

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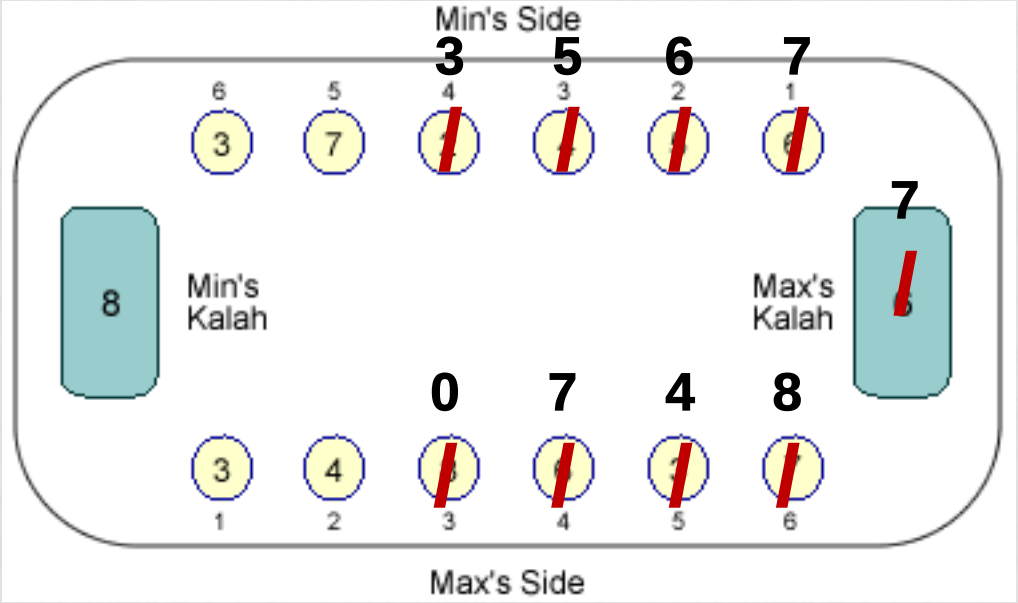

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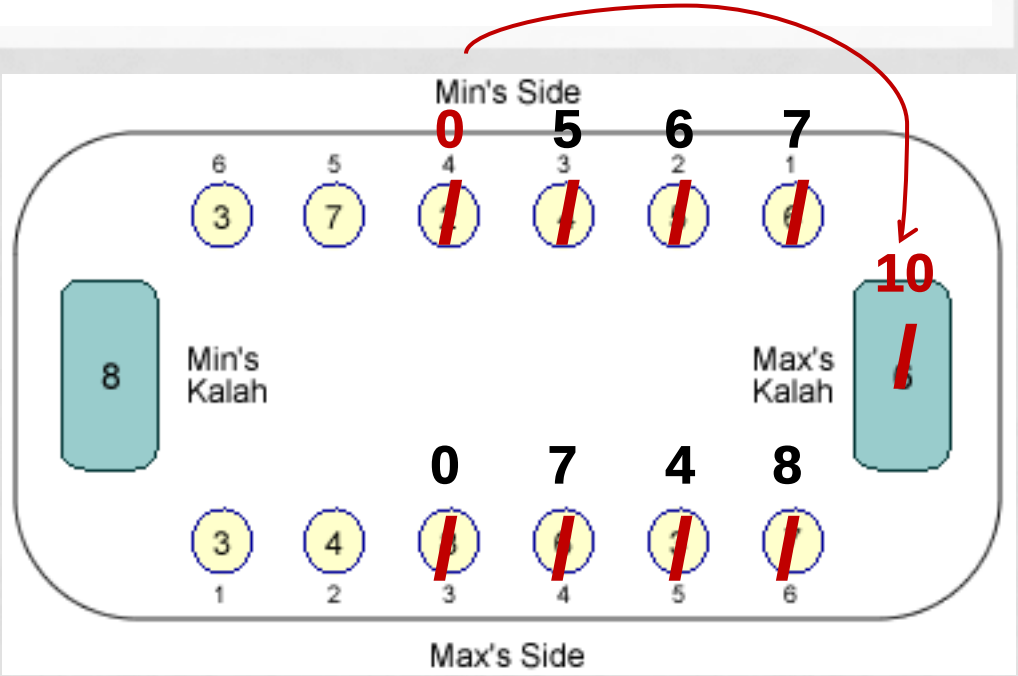

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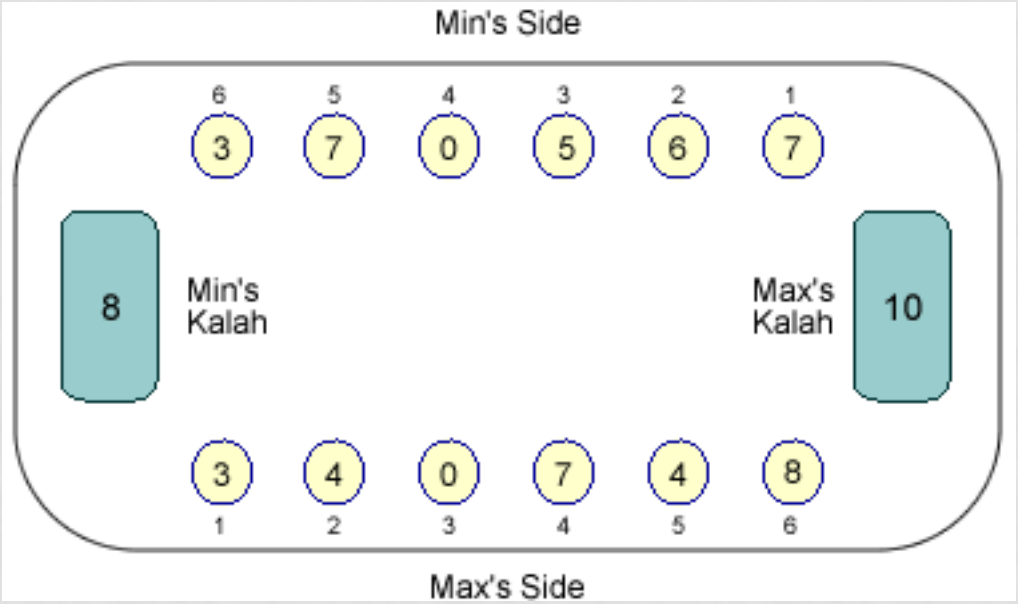

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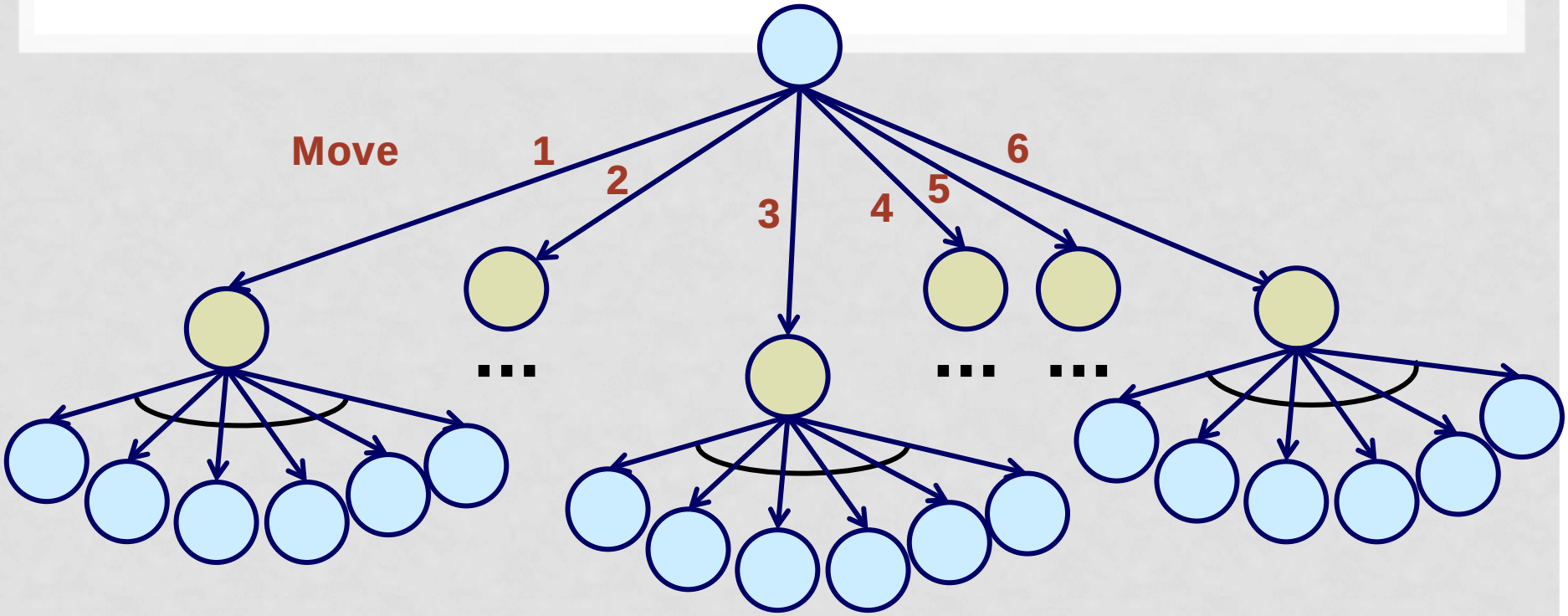
KALAH – GAME OVER

The game is over as soon as more than half the stones are in one player's *Kalah*.

(notice that once a stone enters a Kalah, it can never leave)

Excerpted from "Etudes for Programmers", Charles Wetherell, Prentice Hall, 1978.

MINIMAX AND KALAH



Move *i* : Player removes stones from Pit *i*, and sows them accordingly.
Precondition: Pit *i* is nonempty

For 1 move lookahead, ~42 node expansions, 36 node evaluations.

MINIMAX AND KALAH

IDEA: Let *h(node)* = heuristic that evaluates board quality at bottom level, used by Minimax to backup value

Let *s(node)* = heuristic used to evaluate internal nodes for perceived quality when deciding what order to evaluate them in.

e.g.

h() = (#stones in our Kalah) - (#stones in opponent's Kalah) + α [(#stones in our 6 pits) - (#stones in opponent's pits))

for some α

MINIMAX AND KALAH

Another idea:

As # stones in Kalah increases, perhaps the value should increase more than linearly --

There are 72 total stones in game, so need 36+1=37 to win.

So if we have 13 and opponent has 11, h() = 13-11 = 2.

But if we have 33 and opponent has 31, h() = 33-31 = 2.

Is this value more **urgent** as values get closer to 36?

MINIMAX AND KALAH

Idea for *s(board)* = "rank" $\in$ {0, 1, 2, 3}

Rank(i) = **0** if #stones is not sufficient to cause a wraparound - or a stone in our Kalah
[NumInPit(i) + i $\leq$ 6]

Rank(i) = **2** if #stones causes wraparound to one of our own pits or beyond –
[NumInPit(i) + i > 2*6+1 = 13]
Kalah increases, potential go-again

Rank(i) = **3** if we end up in opponent's pit
potential capture of opponent's pit, more

MINIMAX AND KALAH

Slightly improved ranking system:

Rank(i) = **0** if **[NumInPit(i) + i ≤ 6]** *(stay in our own pits)*

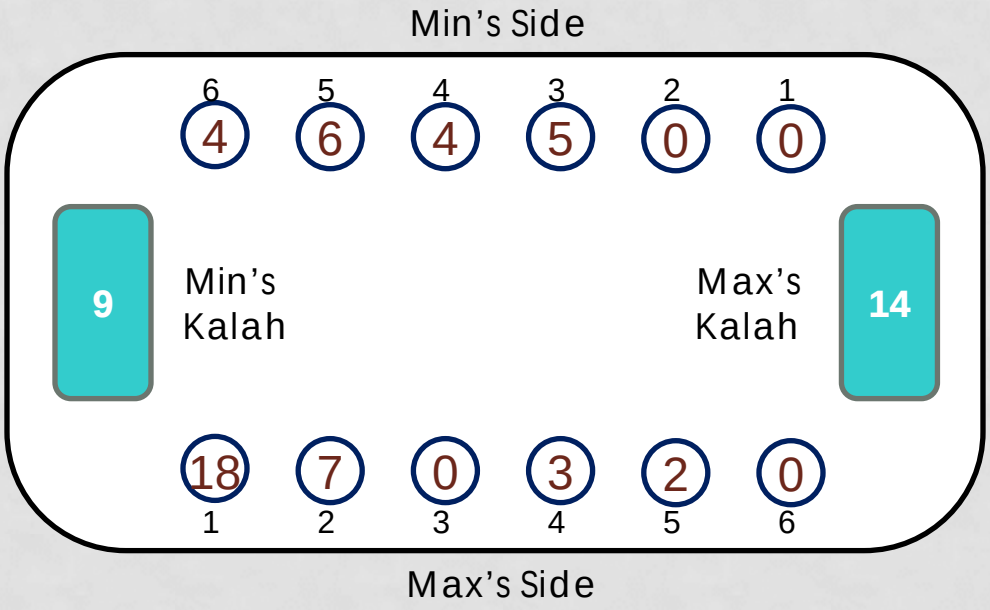
Rank(i) = **1** if **[NumInPit(i) + i = 7]** *(land in our Kalah)*

Rank(i) = **(NumInPit(i) + i)/13** *(int division)*
if **[NumInPit(i)%13 + i ≤ 6]**
*(#stones added to our Kalah if we finish move in one of our own pits – accounts for double wraparounds – will be **1** or **2** or **3**…)*

Rank(i) = **3** otherwise *(land in opponent's pits)*

Recall that the order in which moves are considered is important when doing α-β pruning

EXAMPLE



Our choices: **1**, **2**, **4**, **5**

Move(1): **Rank=2**
Move(2): **Rank=3**
Move(4): **Rank=1**
Move(5): **Rank=1**

Evaluate in order **2**, **1**, **4**, **5**

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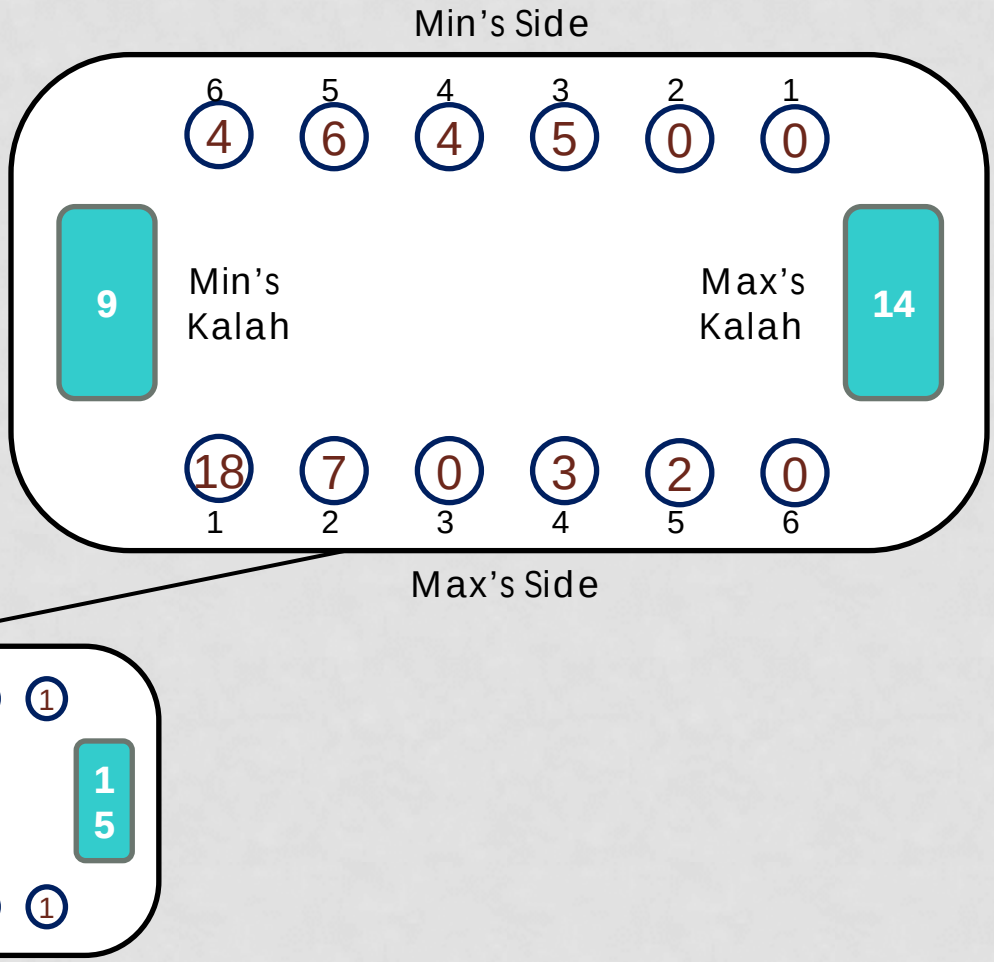
Min's Side
6 5 4 3 2 1
4 6 4 5 0 0
9
Min's Kalah
Max's Kalah
14
18 7 0 3 2 0
1 2 3 4 5 6
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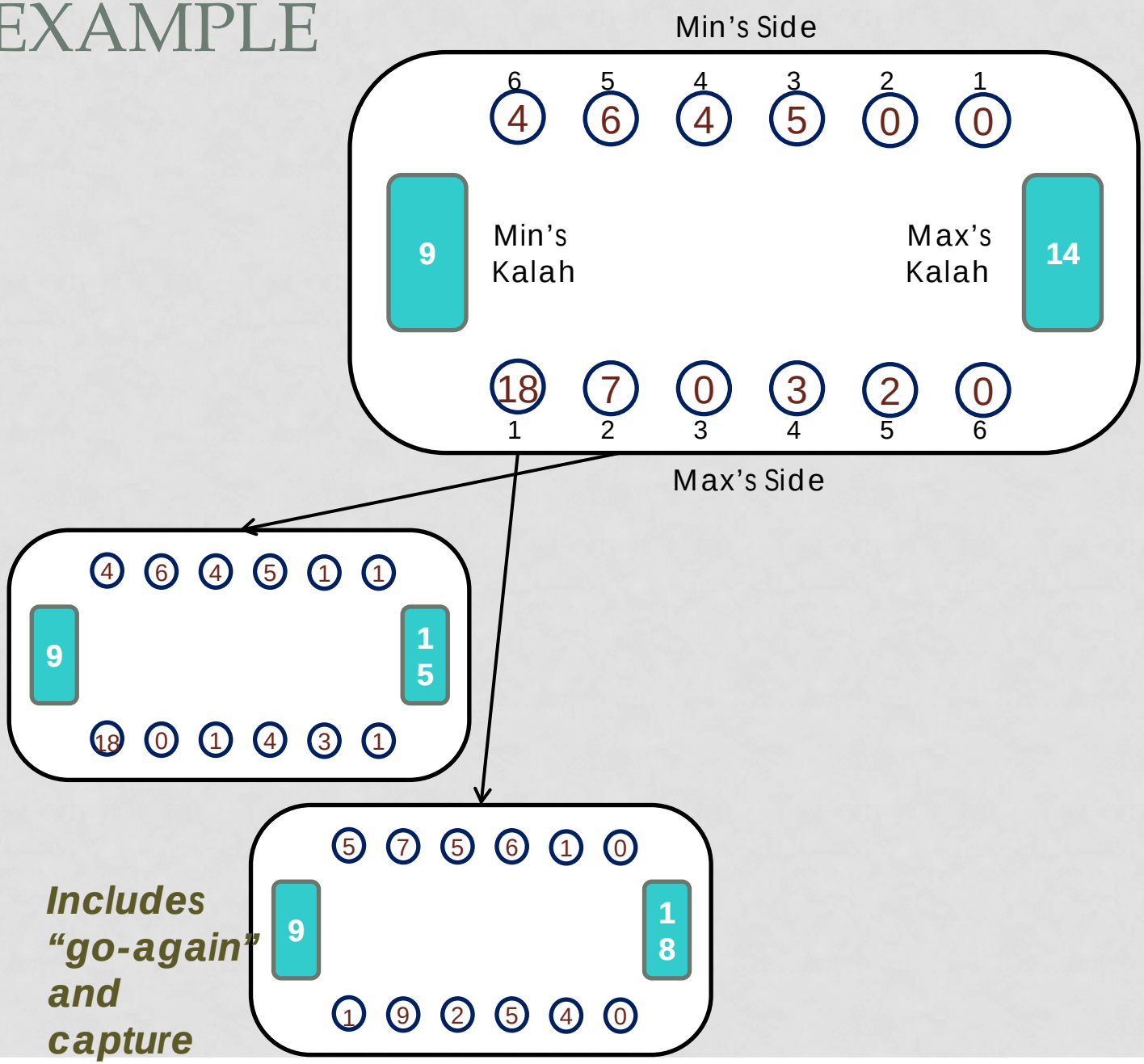
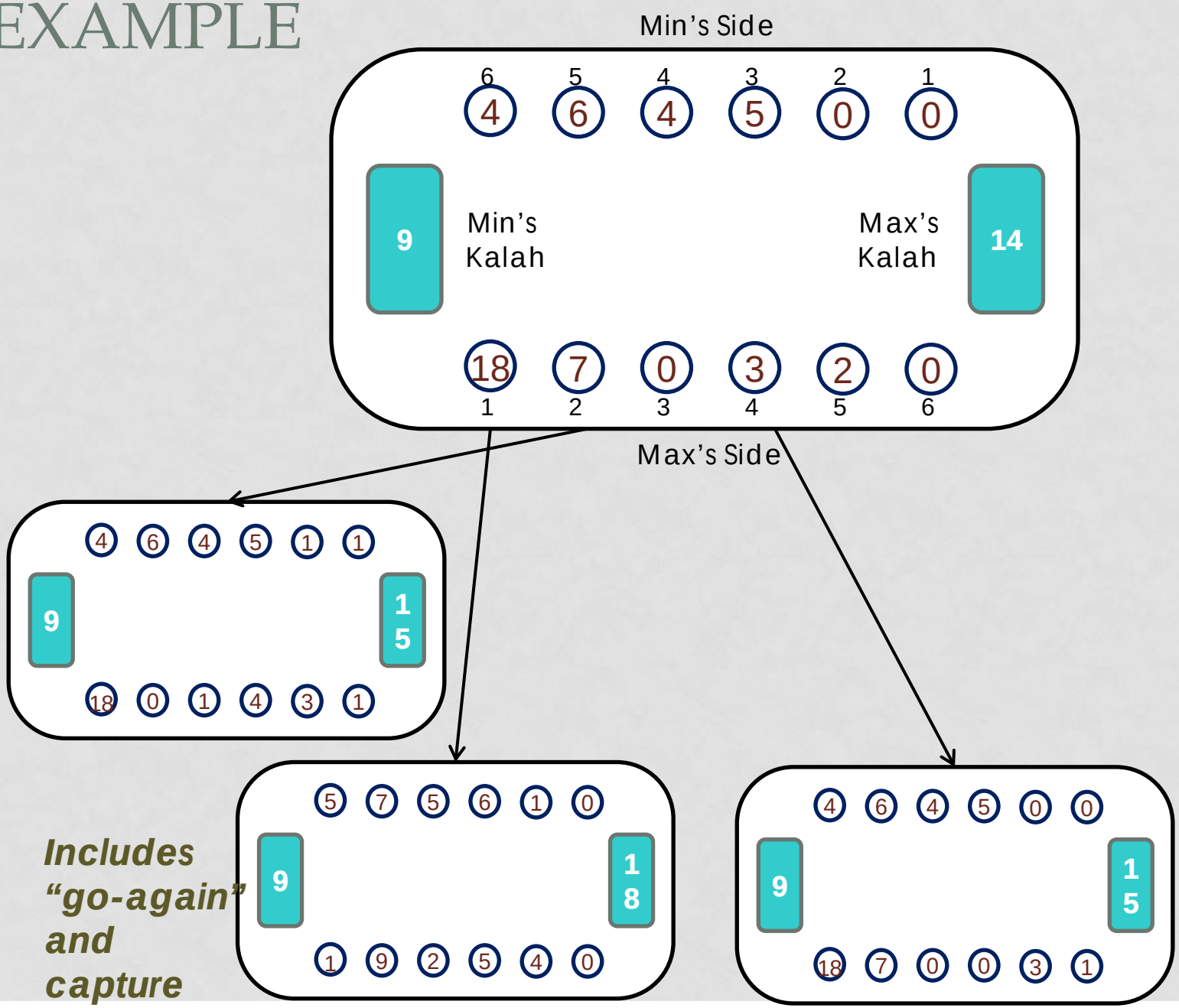
9
15
18 0 1 4 3 1

EXAMPLE



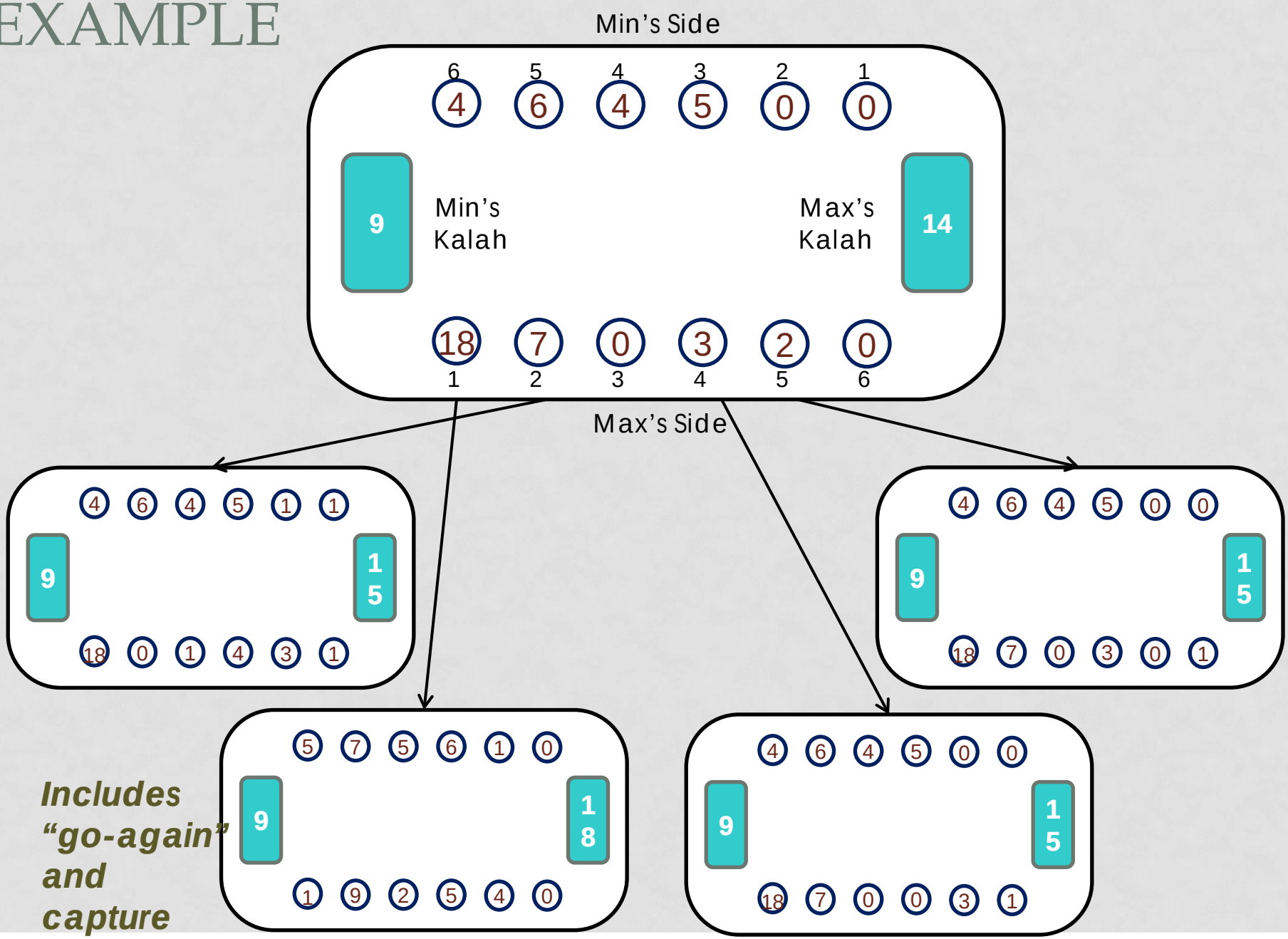
Includes "go-again" and capture

EXAMPLE



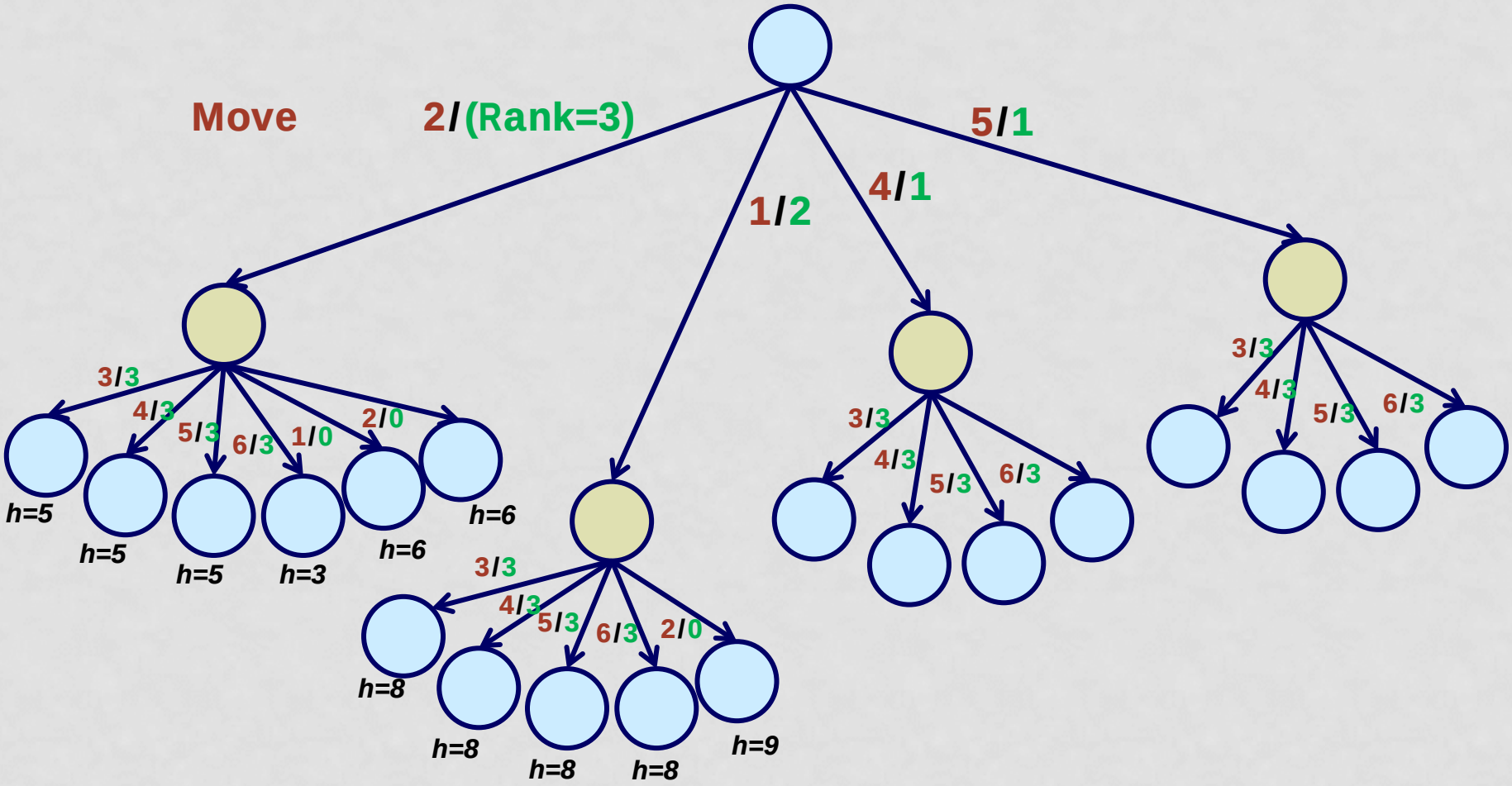
Includes "go-again" and capture

EXAMPLE

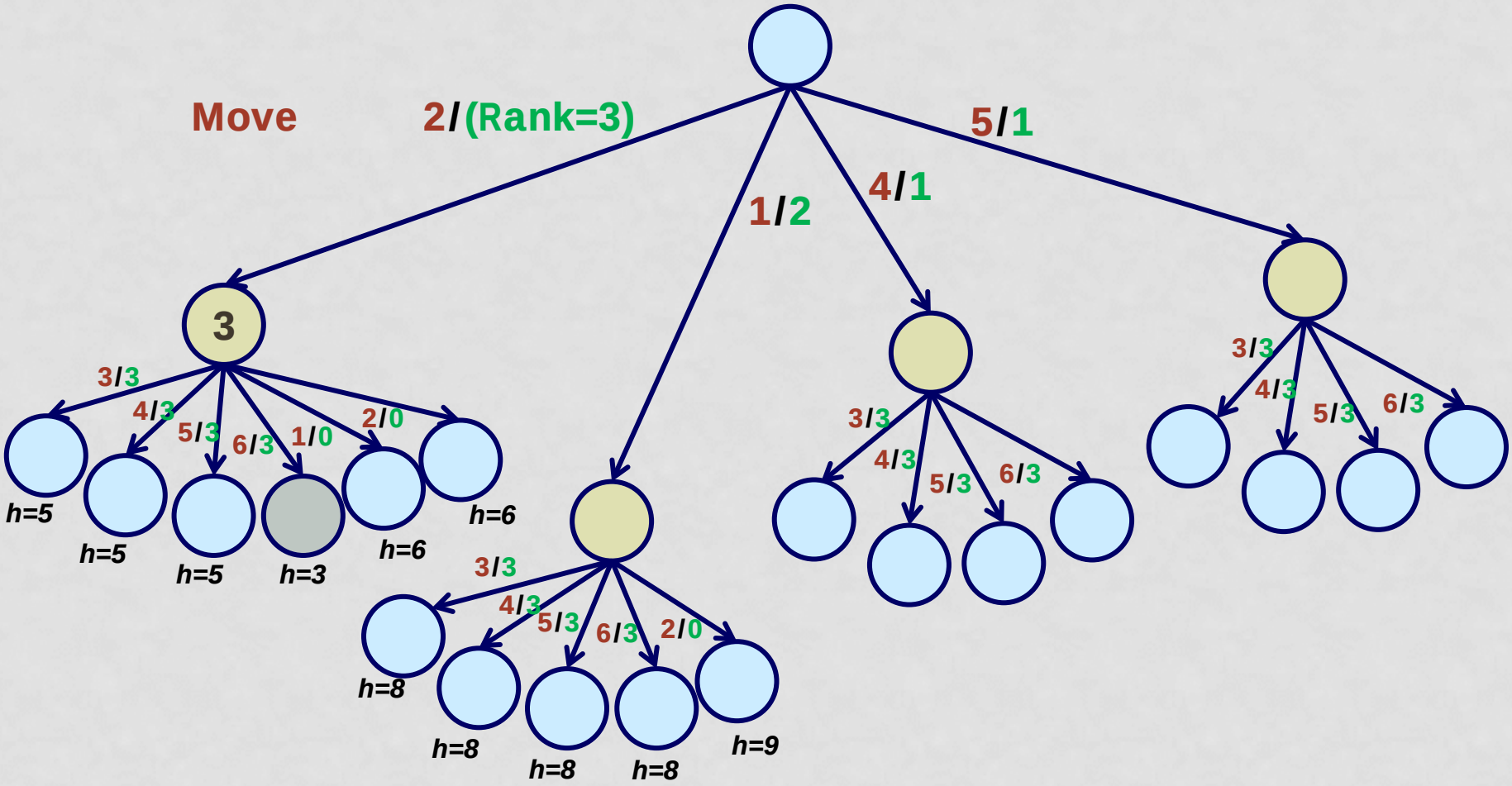


Includes "go-again" and capture

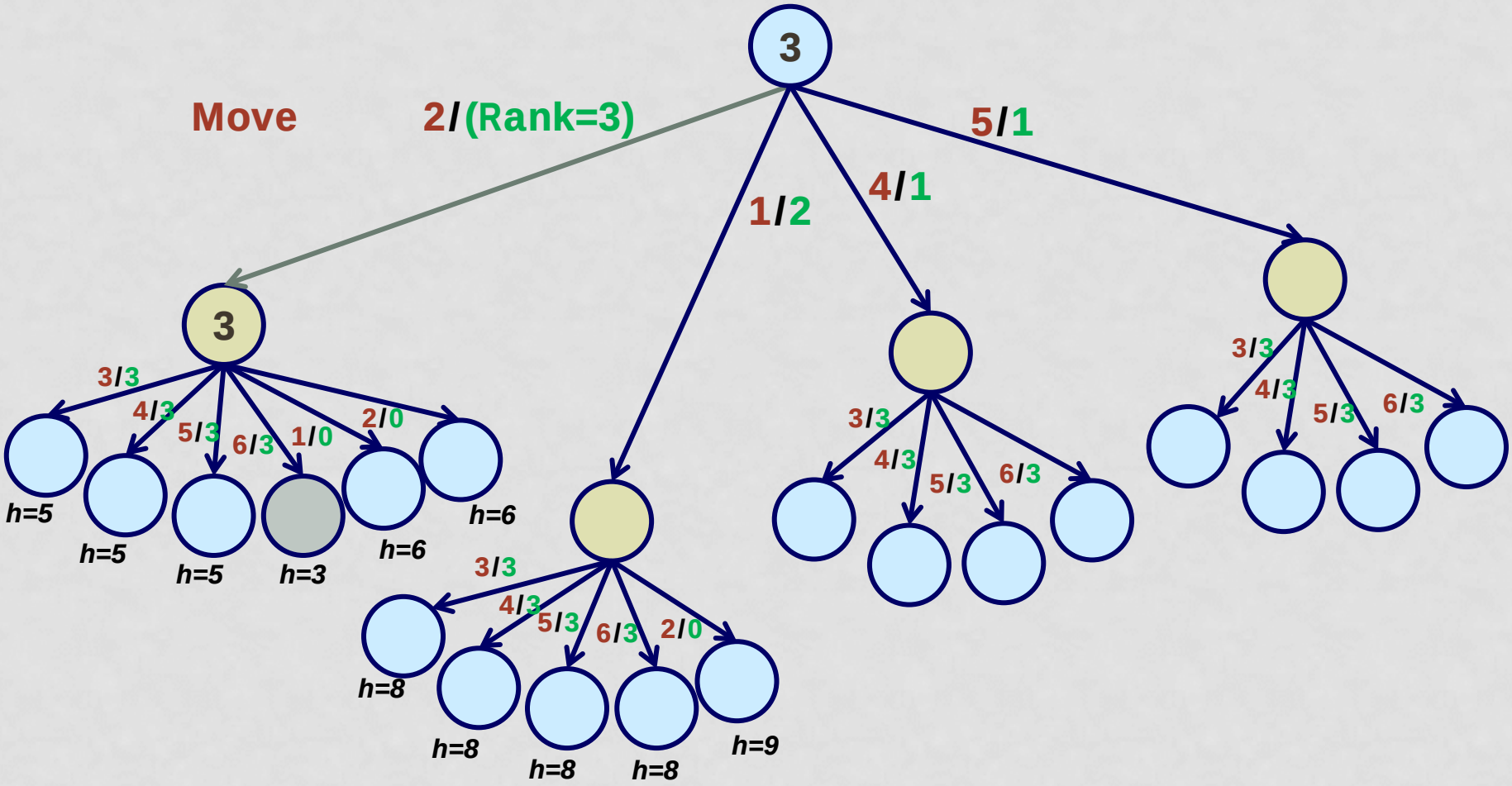
EXAMPLE


Move
2/(Rank=3)
1/2
4/1
5/1
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
3/3
4/3
5/3
6/3

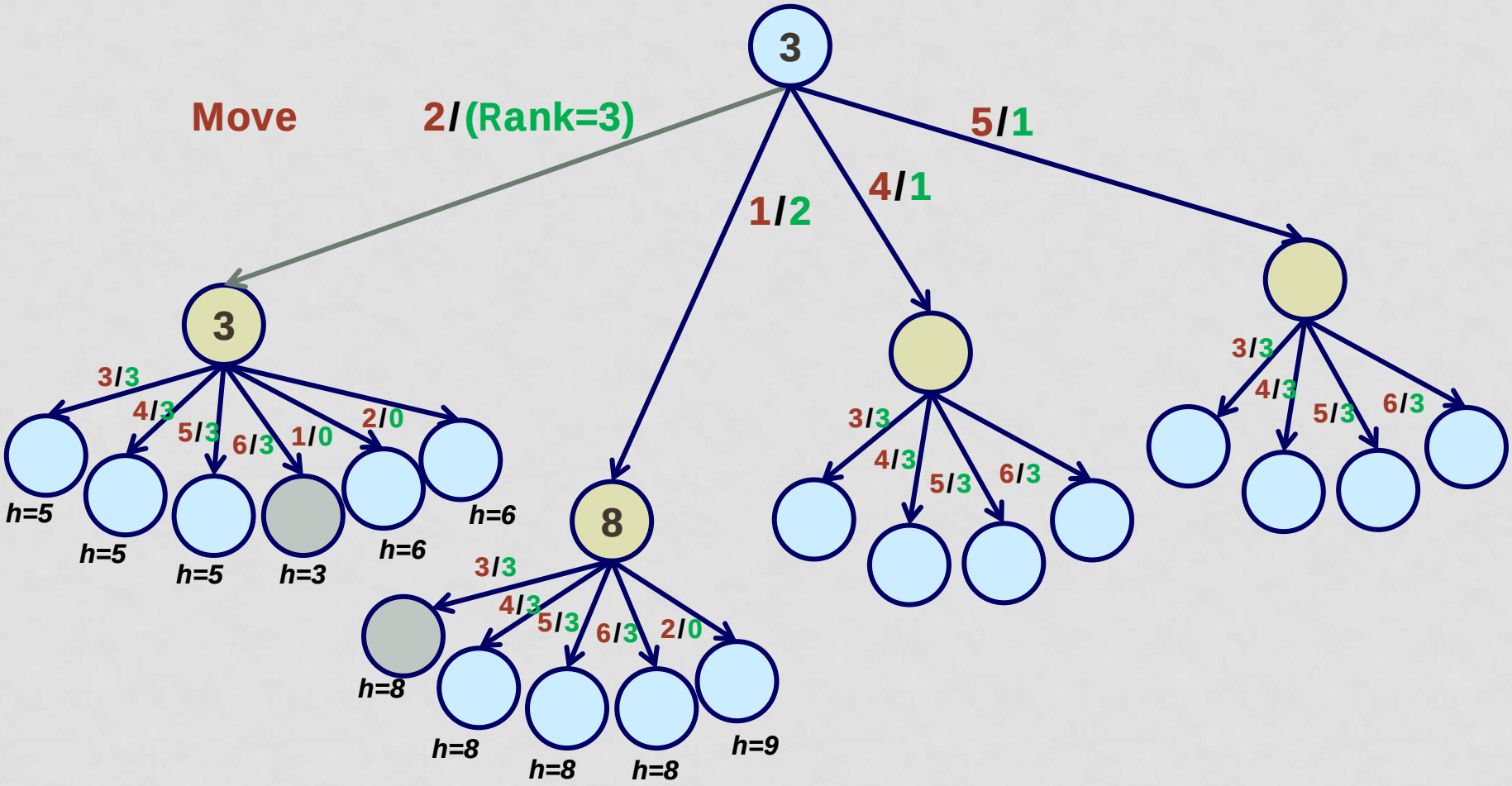
EXAMPLE


Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
3/3
4/3
5/3
6/3

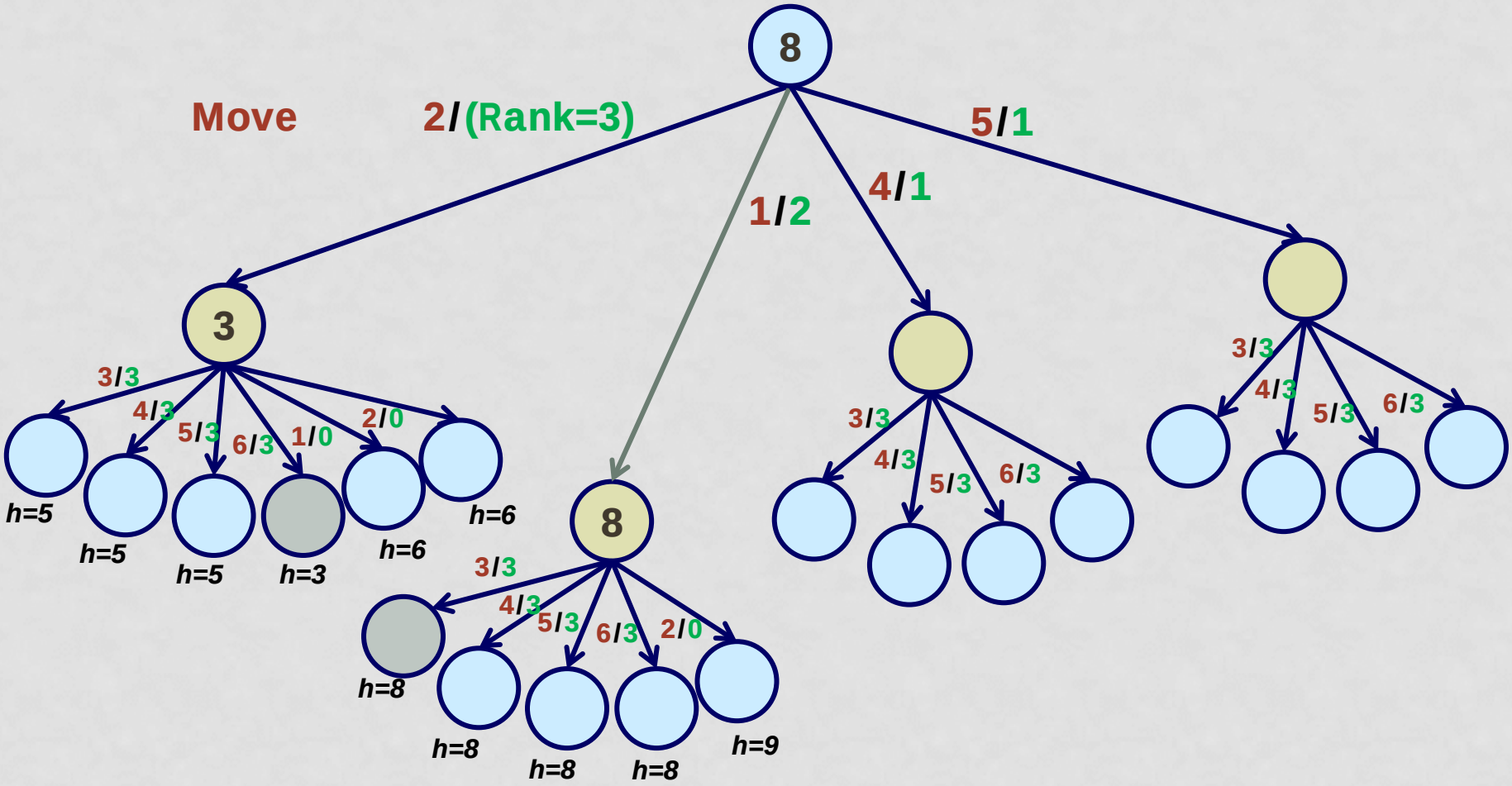
EXAMPLE


3
Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
3/3
4/3
5/3
6/3
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9

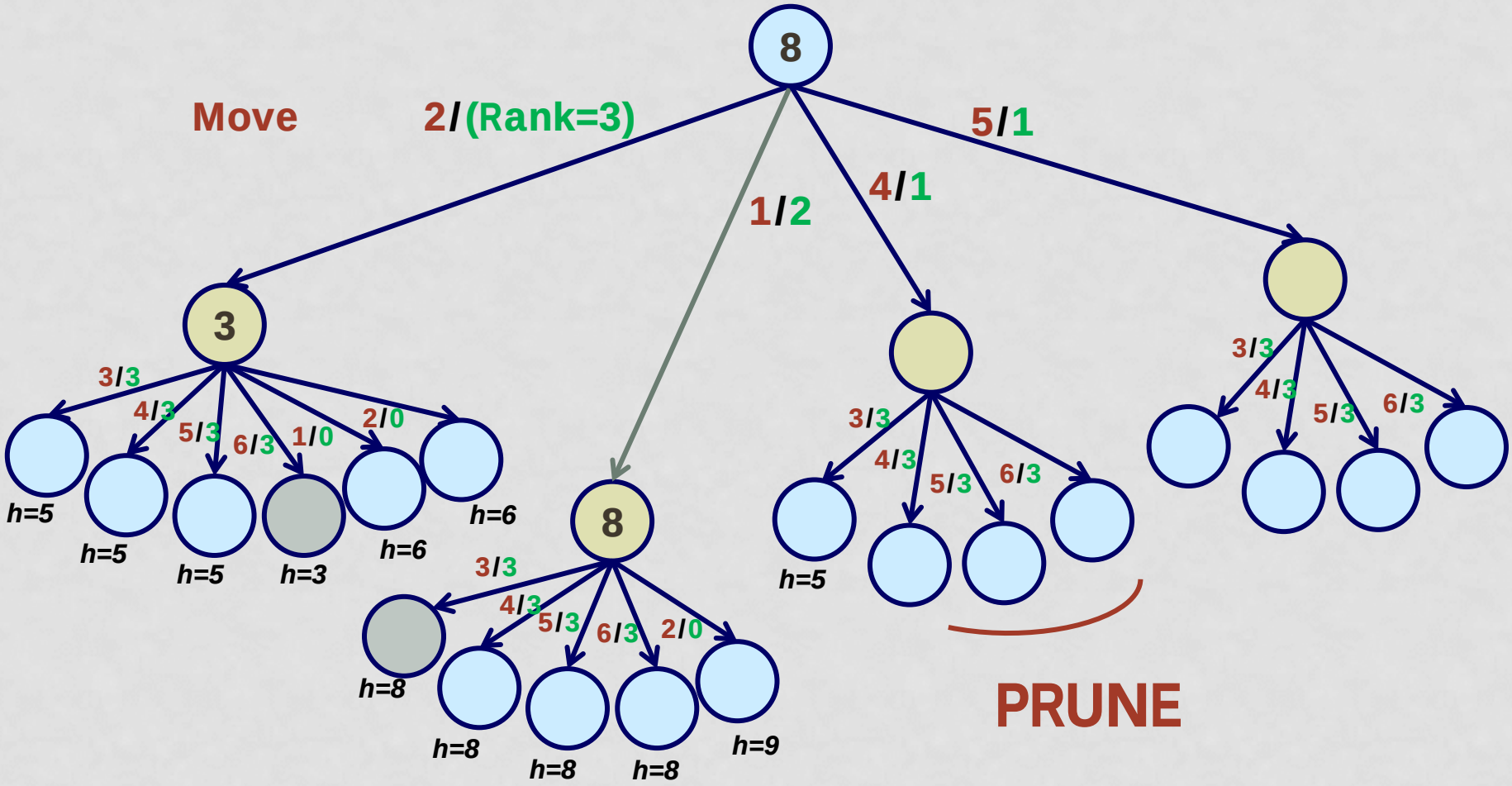
EXAMPLE


3
Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
8
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
3/3
4/3
5/3
6/3

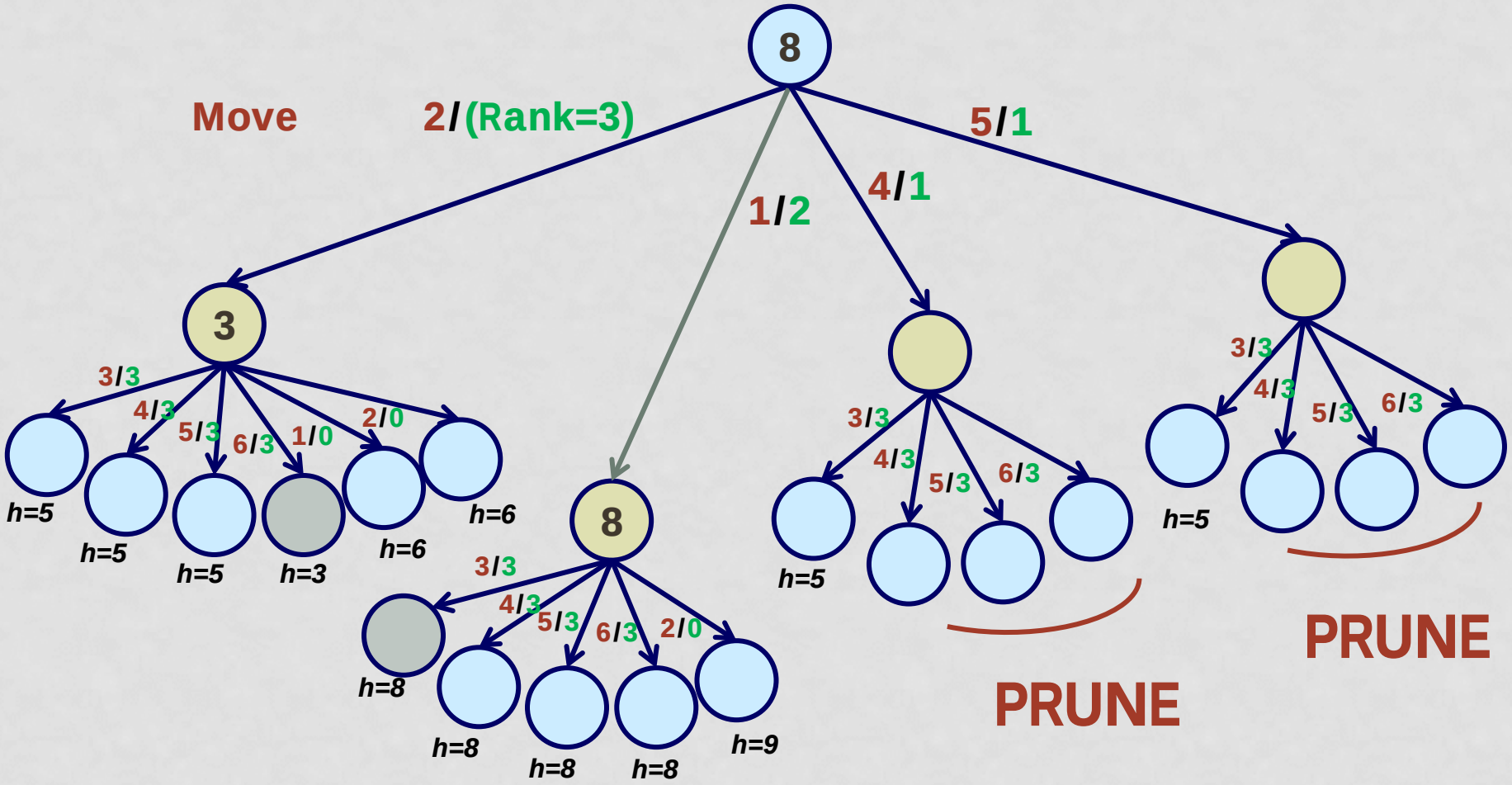
EXAMPLE


8
Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
8
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
3/3
4/3
5/3
6/3

EXAMPLE


8
Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
8
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
h=5
3/3
4/3
5/3
6/3
PRUNE

EXAMPLE


8
Move
2/(Rank=3)
1/2
4/1
5/1
3
3/3
4/3
5/3
6/3
1/0
2/0
h=5
h=5
h=5
h=3
h=6
h=6
8
3/3
4/3
5/3
6/3
2/0
h=8
h=8
h=8
h=8
h=9
3/3
4/3
5/3
6/3
h=5
PRUNE
3/3
4/3
5/3
6/3
h=5
PRUNE

SAMPLE GAME

Level of Recursion=1
(1move per player)

Player Move	Computer Move	Backed Up Value	Nodes Evaluated	Node evals w/Pruning
2	1	-1	41	17
1	2	1	41	17
3	3	1	39	18
4	4	0	40	18
5	6	1	39	23
6	1	1	43	18
1	1	0	41	29
2	2	0	39	18
3	3	0	41	18
4	5	0	40	18
5	6	0	39	18
6	5	0	33	16
2	1	0	29	14
3	4	3	30	24
4	2	4	27	13
5	6	4	23	20
6	3	∞	26	2
		TOTALS	611	301

SAMPLE GAME

Level of Recursion=2
(2 moves per player)

Player Move	Computer Move	Backed Up Value	Nodes Evaluated	Node evals w/Pruning
2	1	-2	1246	375
1	2	1	12898	144
3	4	1	1151	288
4	3	0	1170	343
5	5	1	1107	198
6	6	1	1227	308
1	1	1	1093	217
2	5	3	1022	313
3	4	2	1017	247
4	2	7	682	189
5	5	10	879	219
6	3	14	660	143
3	4	∞	468	84
4	6	∞	131	54
		TOTALS	13142	3122

RESULTS FROM 2 SAMPLE GAMES

Level of recursion = 1

- total # nodes eval (w/o pruning) - 611
- (w/ pruning) - 356
- (w/re-ordering & pruning) - 301
- >50% fewer node evals with this function and α-β pruning

Level of recursion = 2

- total # nodes eval (w/o pruning) - 13142
- (w/ pruning) - 3122
- (w/re-ordering & pruning) - 3002
- >75% fewer nodes