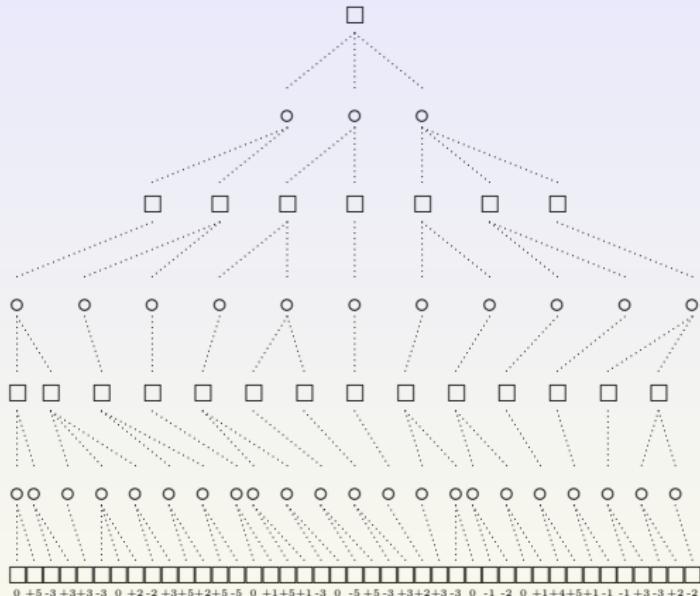


MiniMax Search by Example

Evan A. Sultanik

Drexel University

2006-10-18

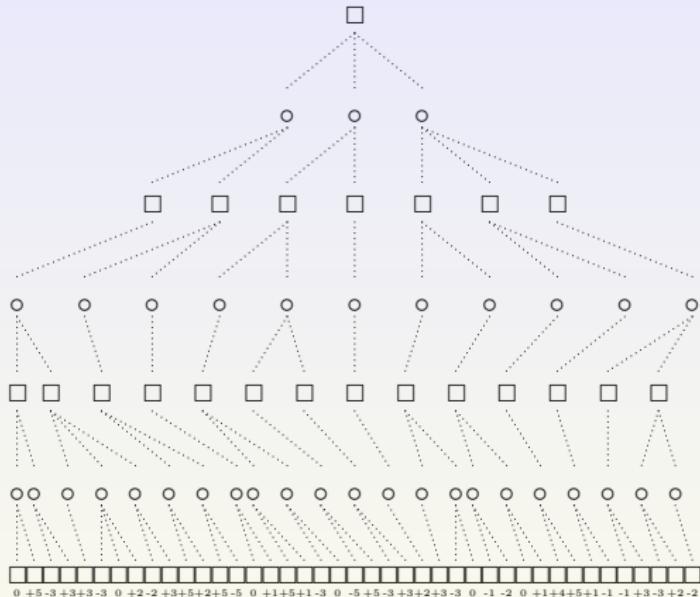


```
→function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 
```

$$\alpha = 0 \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
→ v ← MAX-VALUE(state, $-\infty$ , $+\infty$ )
   return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ←  $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha$  ← MAX( $\alpha$ , v)
  return v

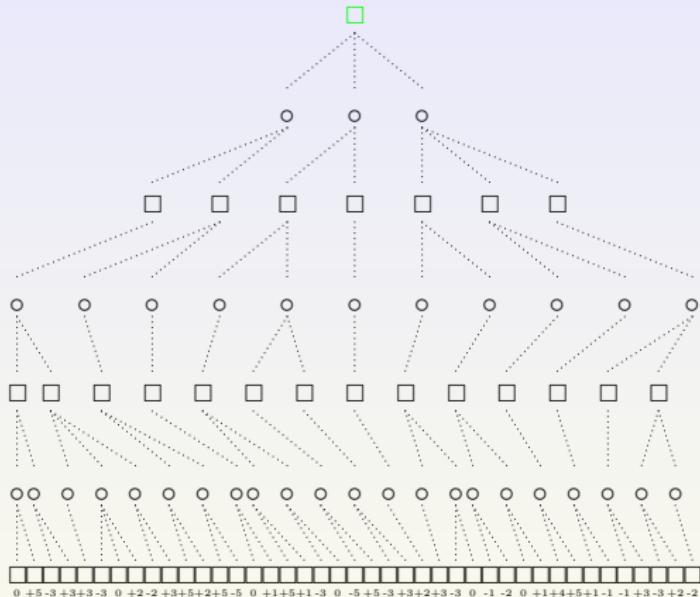
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ←  $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$  ← MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

→ **function** MAX-VALUE(*state*, α , β) **returns** a utility value

if TERMINAL-TEST(*state*) **then**
return UTILITY(*state*)

v \leftarrow $-\infty$

for *a, s* in SUCCESSORS(*state*) **do**
v \leftarrow MAX(*v*, MIN-VALUE(*s*, α , β))
 if *v* $\geq \beta$ **then**
return *v*
a \leftarrow MAX(α , *v*)

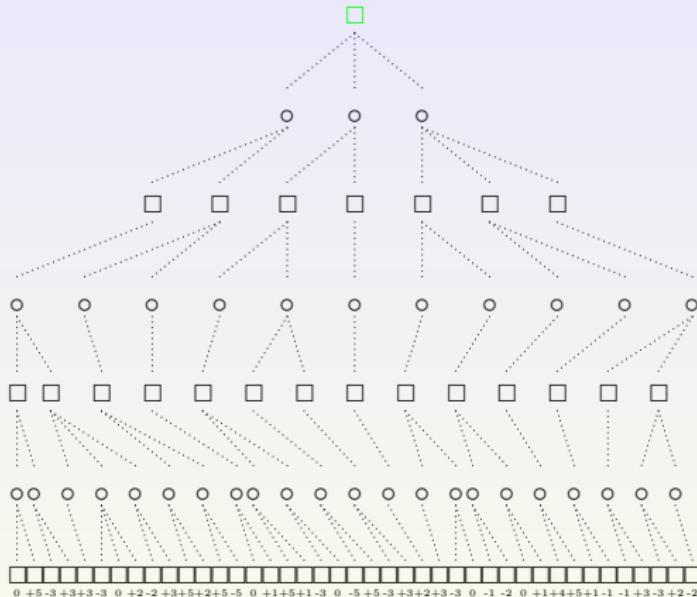
return *v*

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
    
v  $\leftarrow$   $+\infty$ 
    
for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
    b  $\leftarrow$  MIN( $\beta$ , v)
    
return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

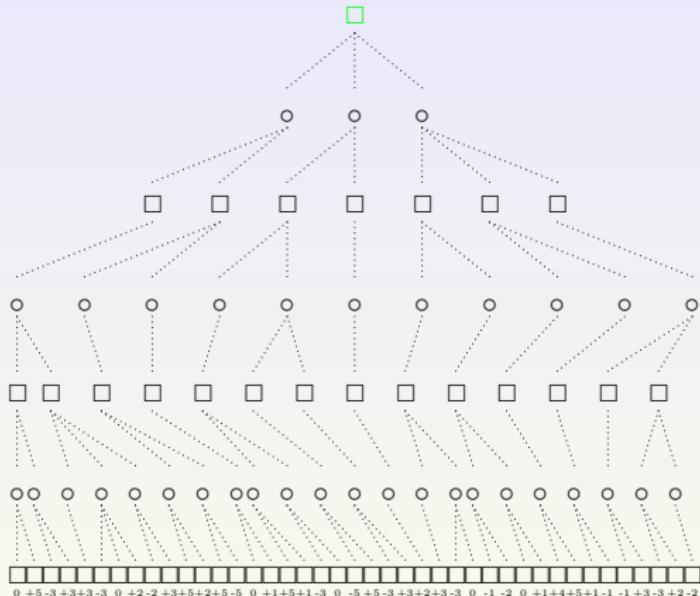
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

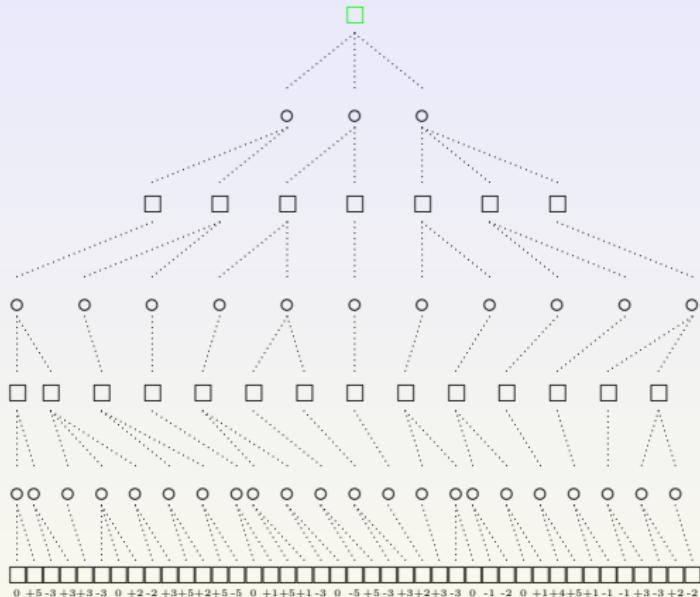
```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v$ 
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v$ 
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

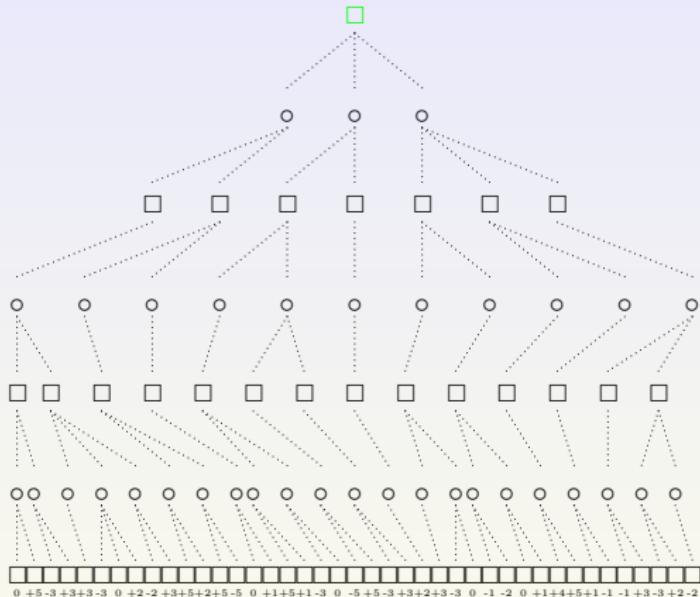
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

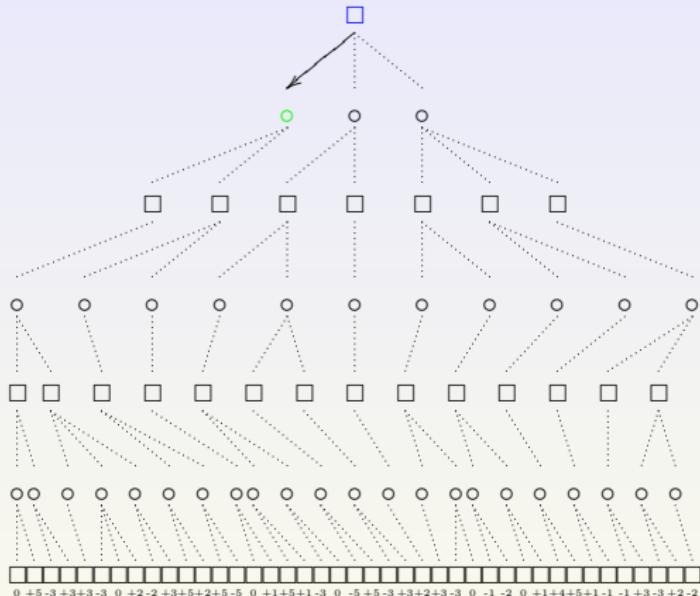
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

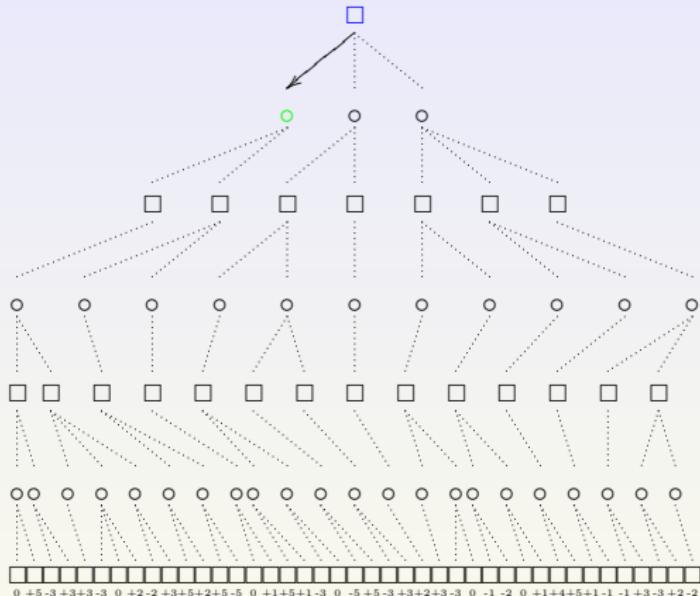
→

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

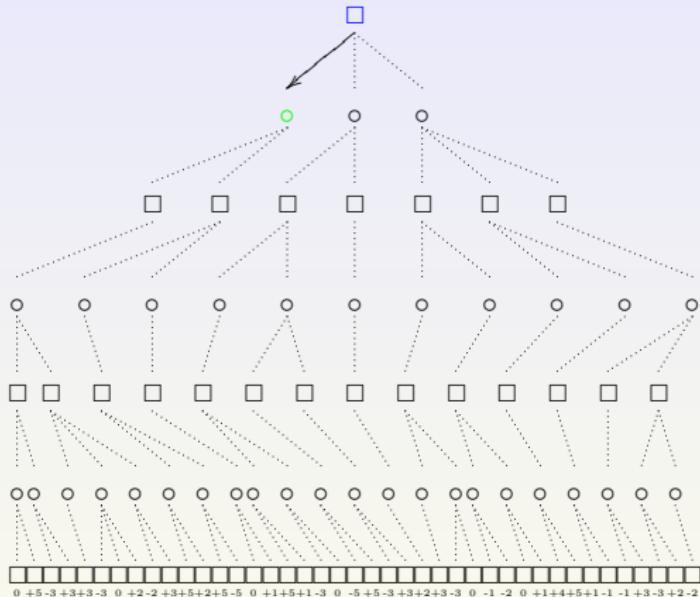


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

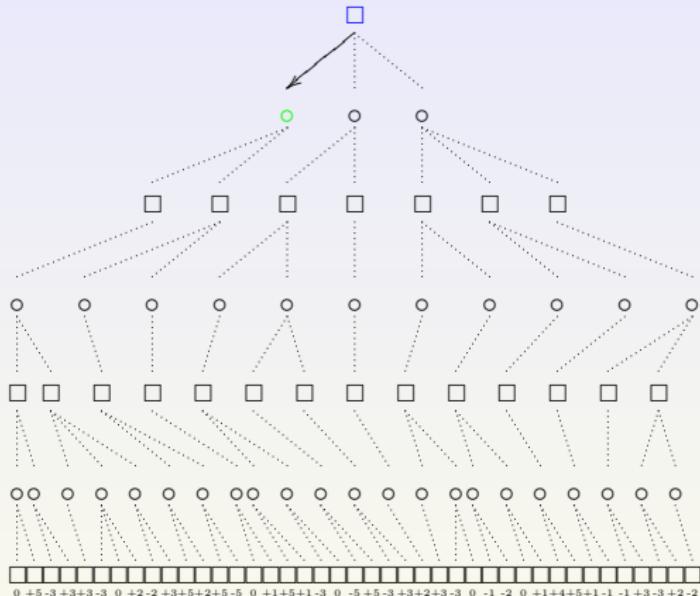


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function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

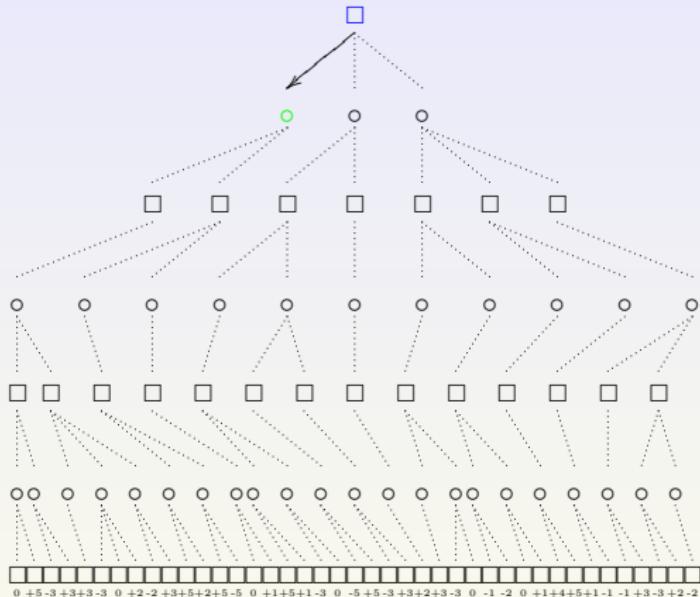
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

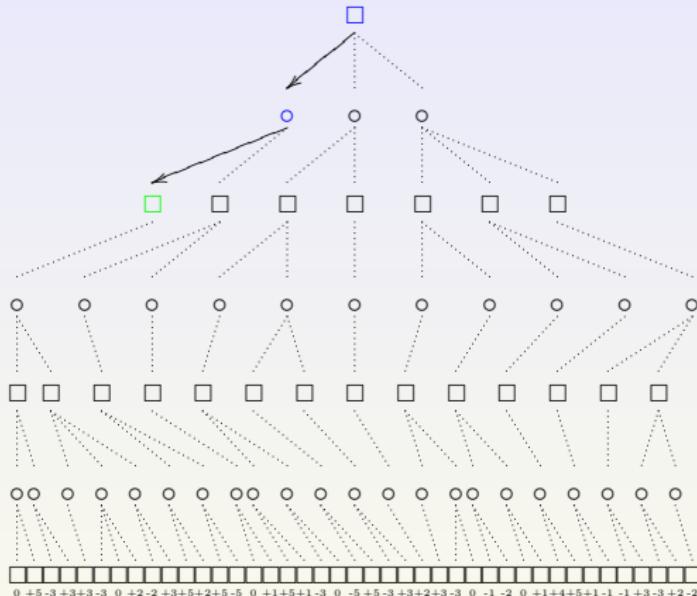
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

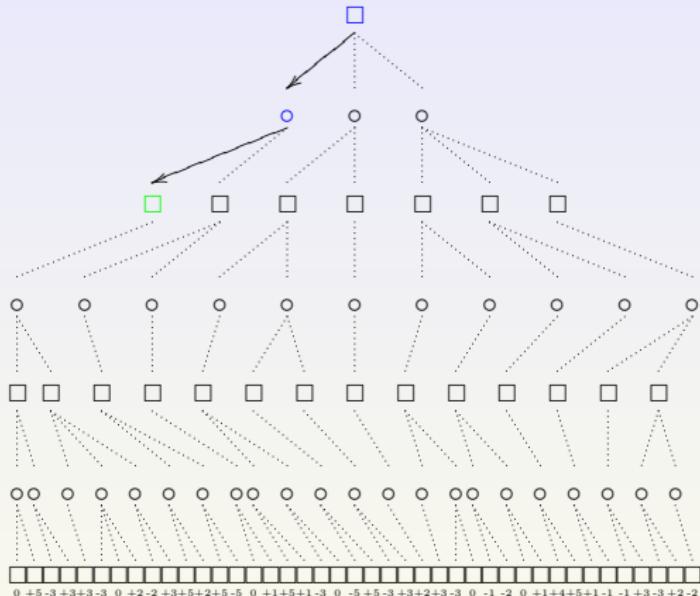
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
    β  $\leftarrow$  MIN(β, v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

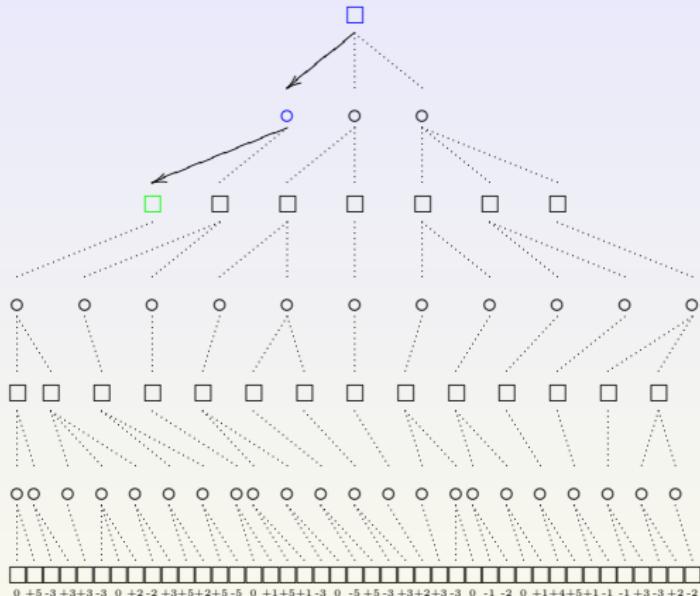
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

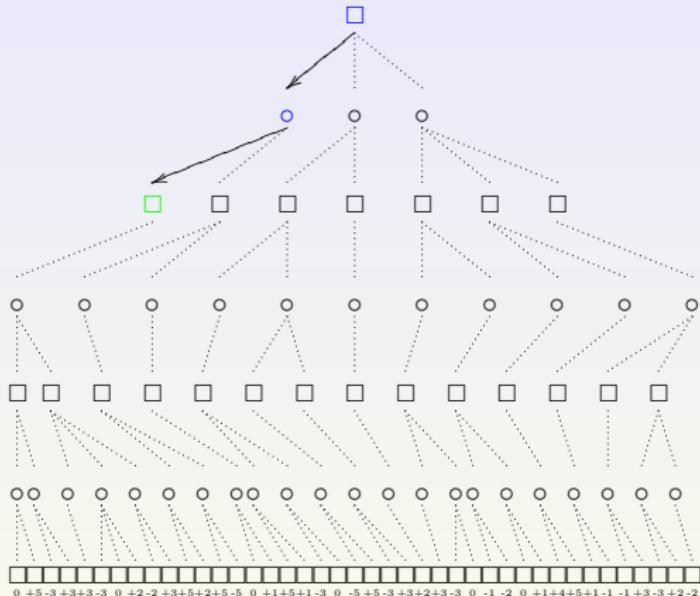
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

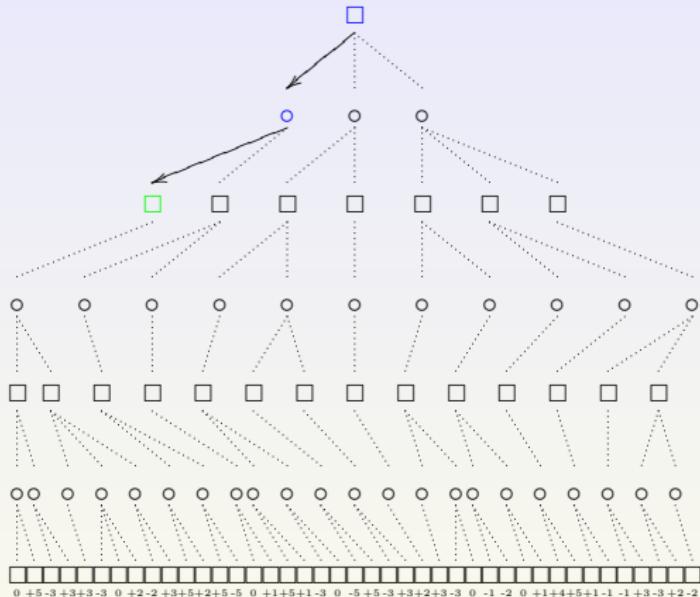
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

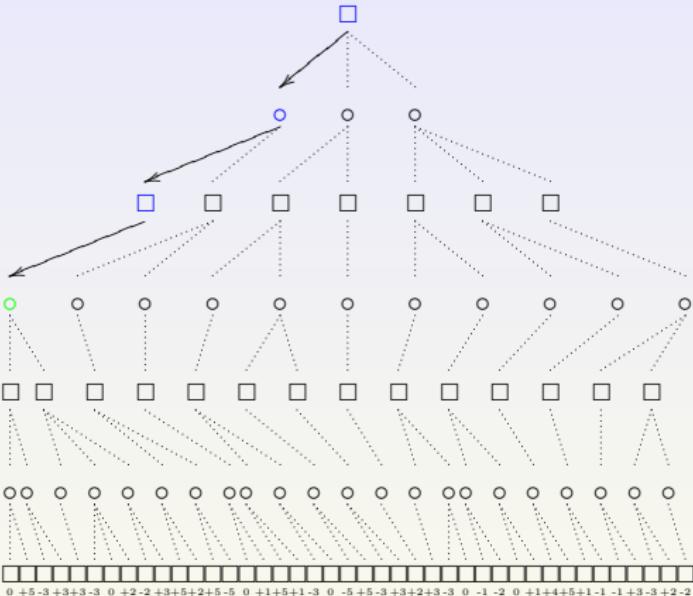


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
        return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

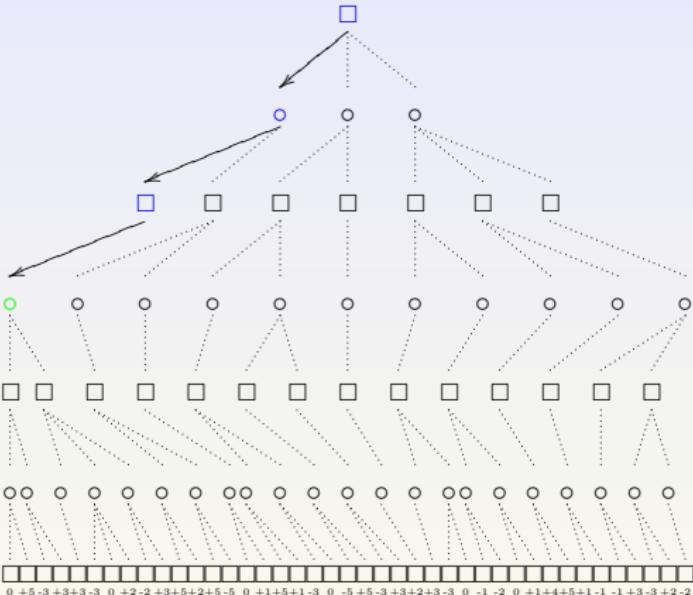


```
function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v
```

→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value
 if TERMINAL-TEST(*state*) **then**
return UTILITY(*state*)
 v \leftarrow $+\infty$
for *a, s* in SUCCESSORS(*state*) **do**
v \leftarrow MIN(*v*, MAX-VALUE(*s*, α , β))
 if *v* $\leq \alpha$ **then**
return *v*
 β \leftarrow MIN(β , *v*)
 return *v*

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

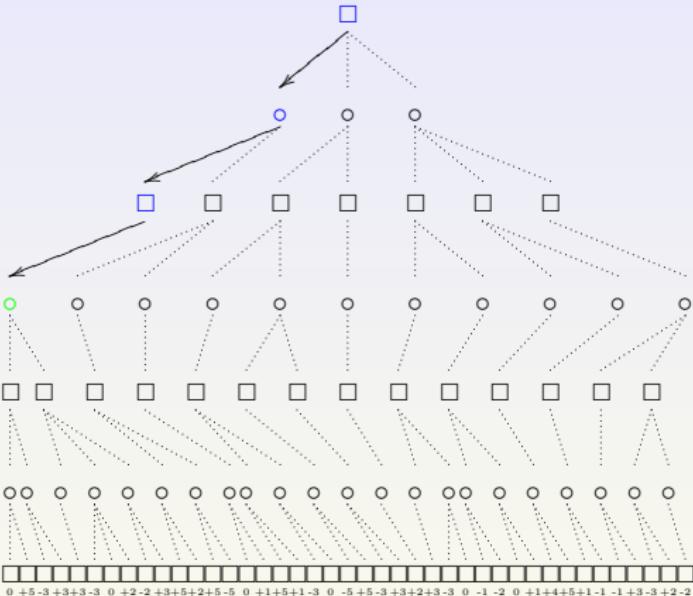
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

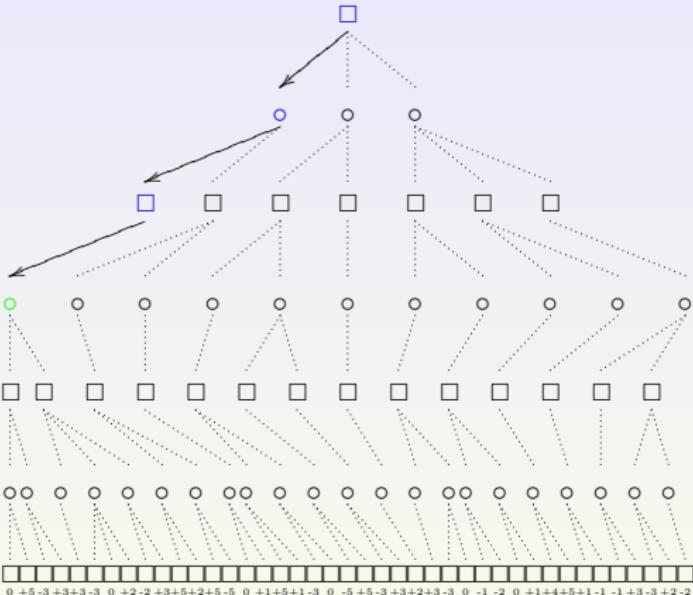


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

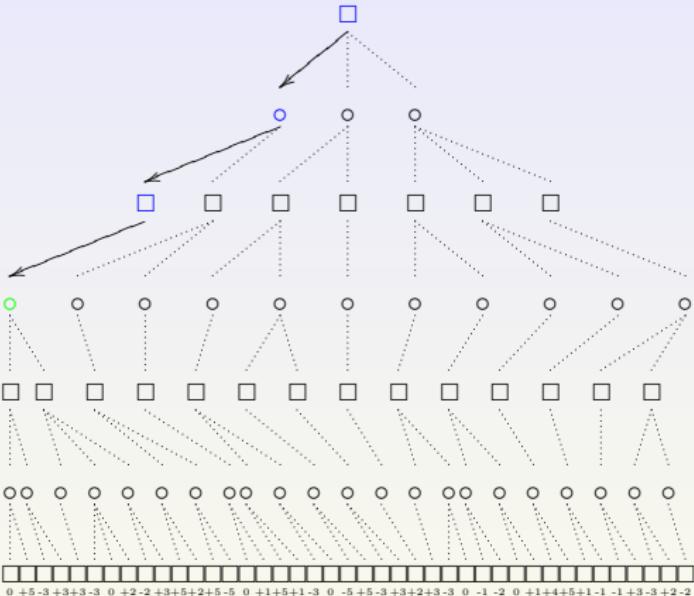


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  → for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

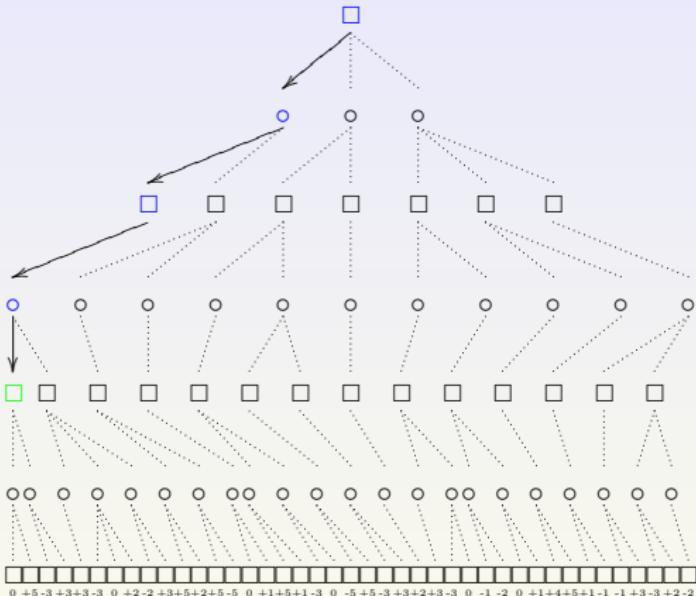
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

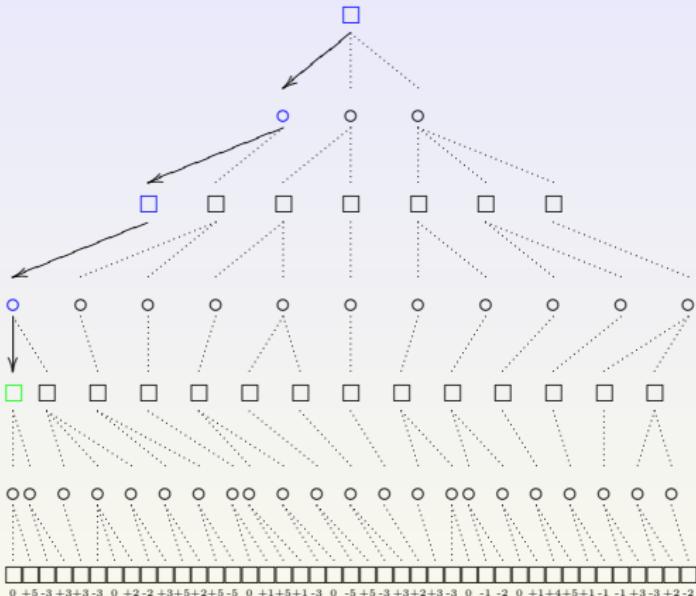
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

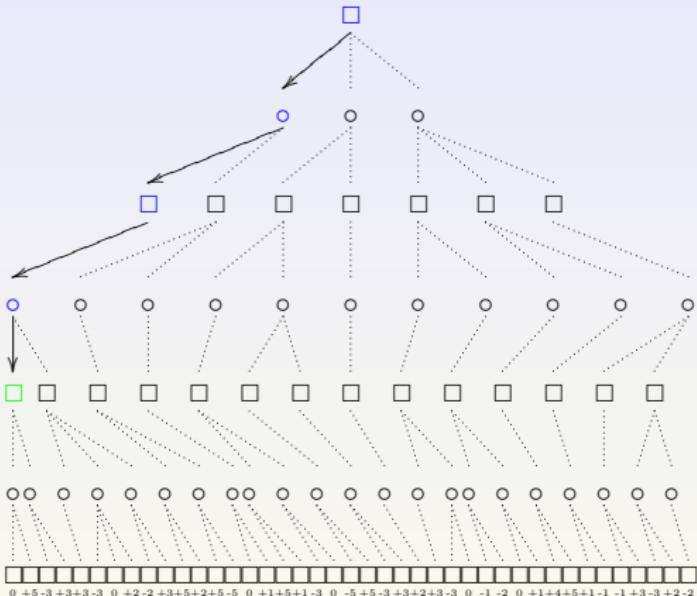


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

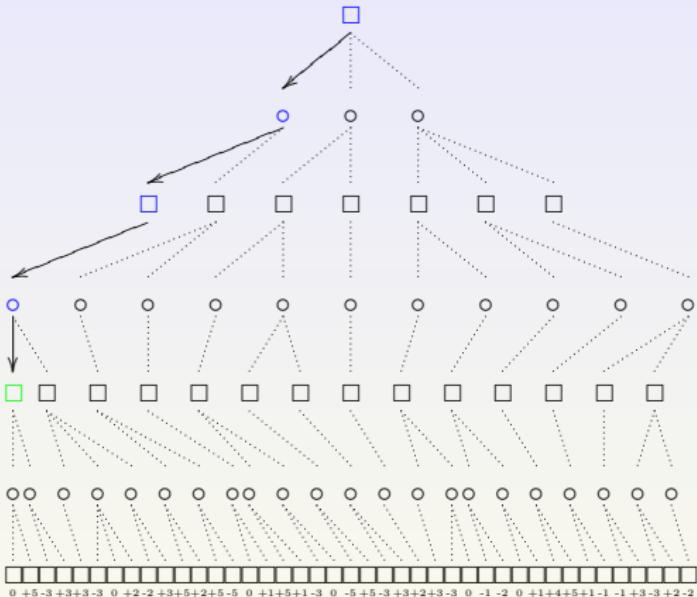
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

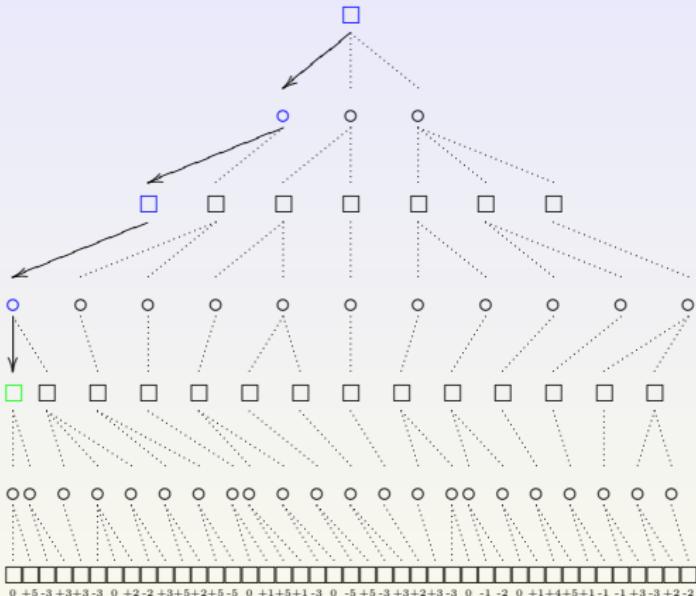


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

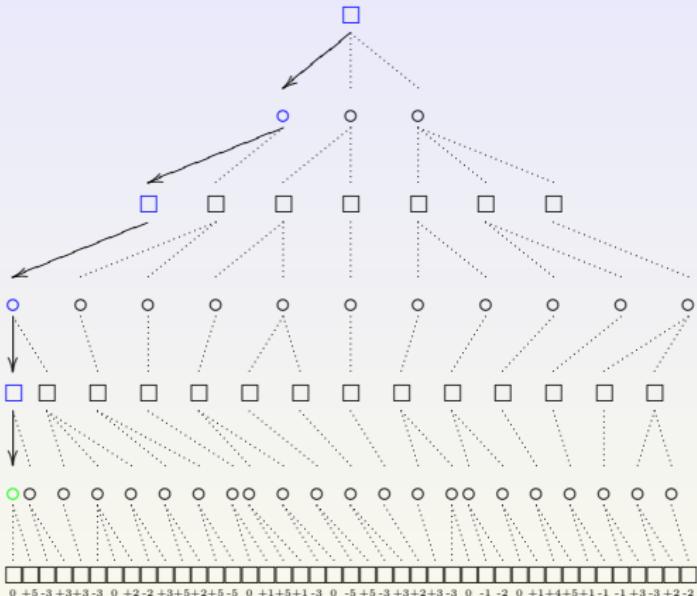
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

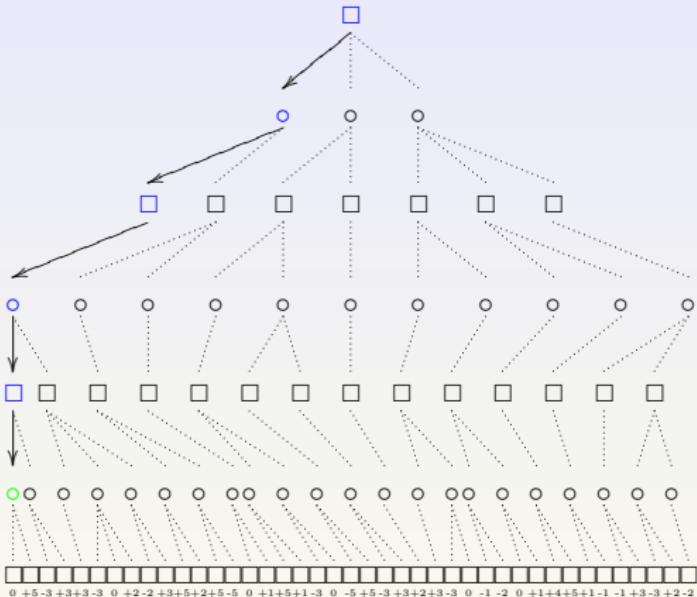
→

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v
  
```

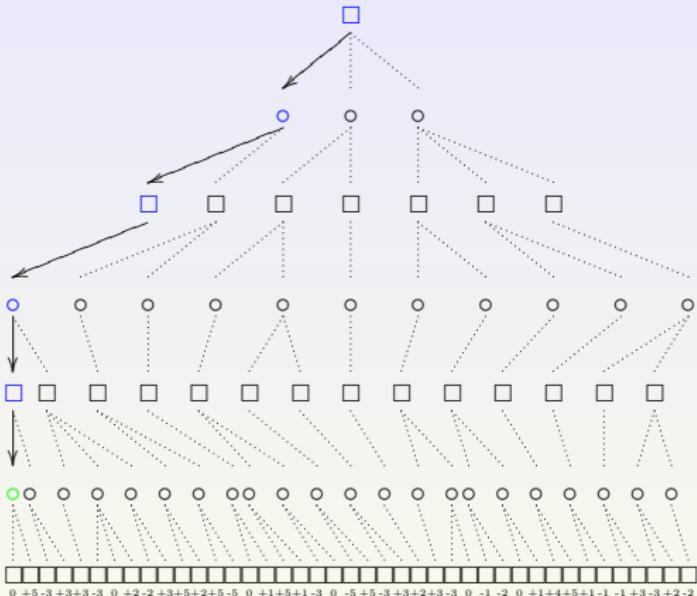
```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v
  
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v
  
```

$$\alpha = -\infty \quad \beta = +\infty$$

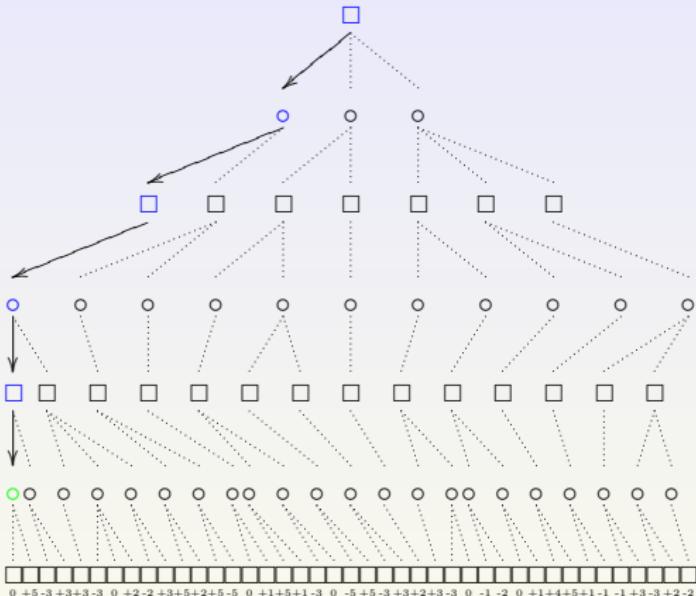


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

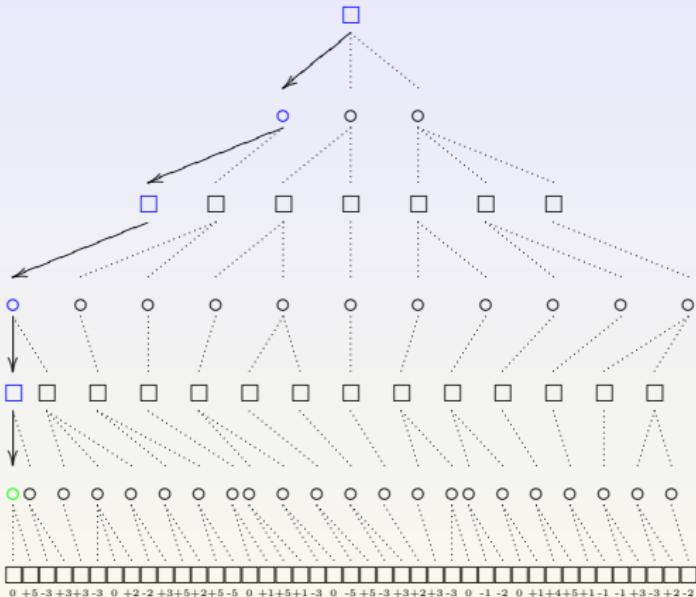
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
    β  $\leftarrow$  MIN(β, v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

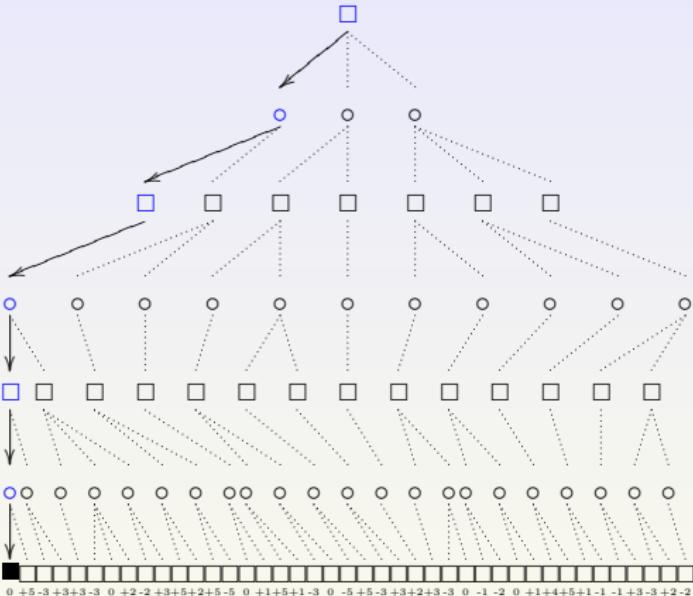
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
    β  $\leftarrow$  MIN(β, v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

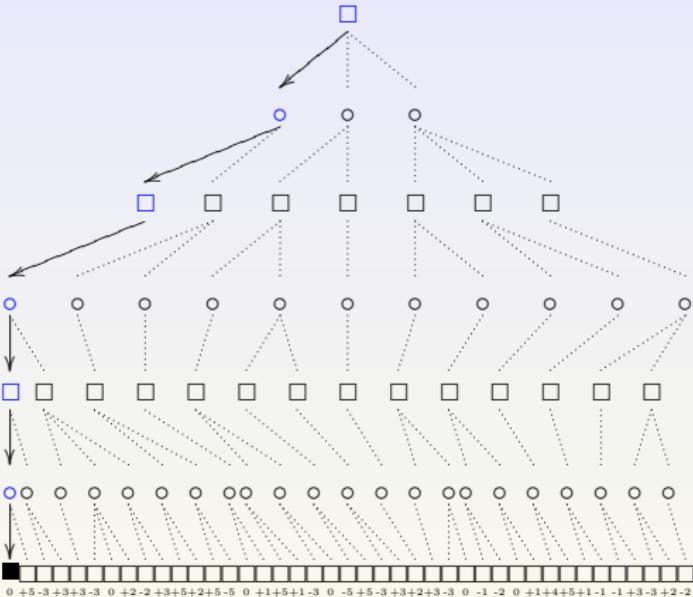
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```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

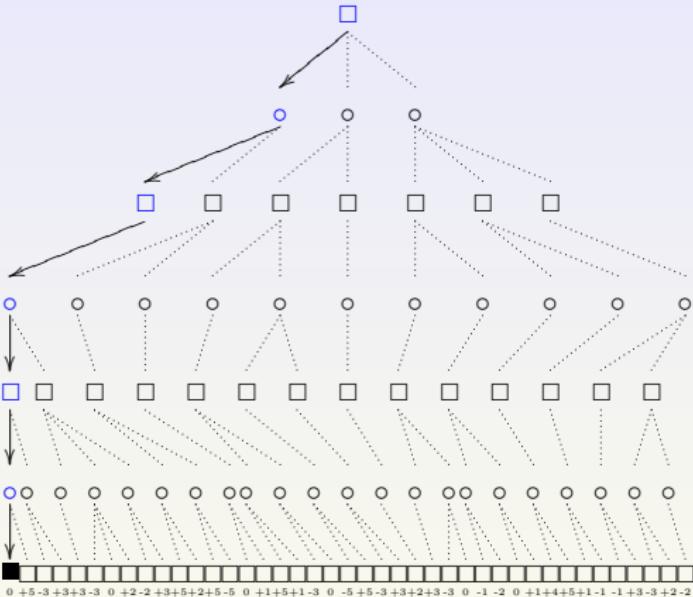


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$ 
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

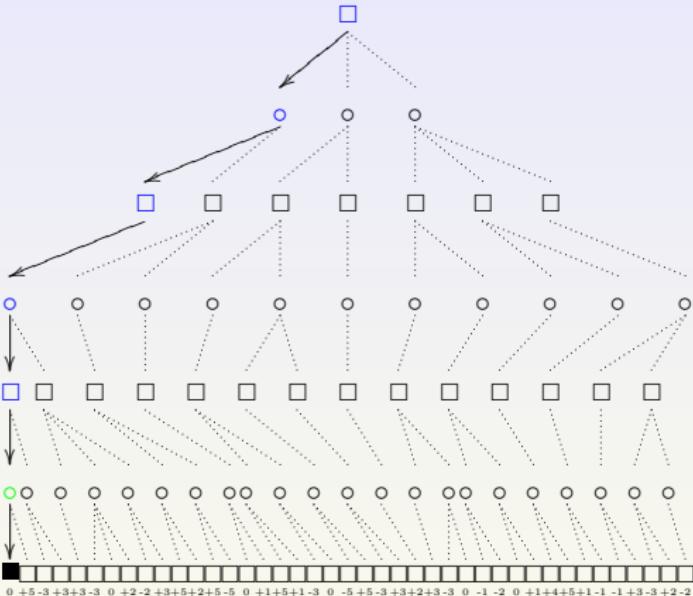
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

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  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
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    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

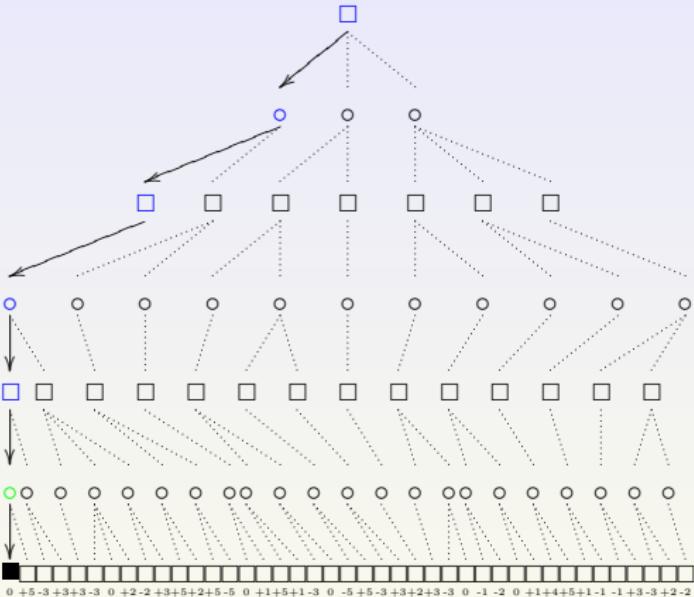
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```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

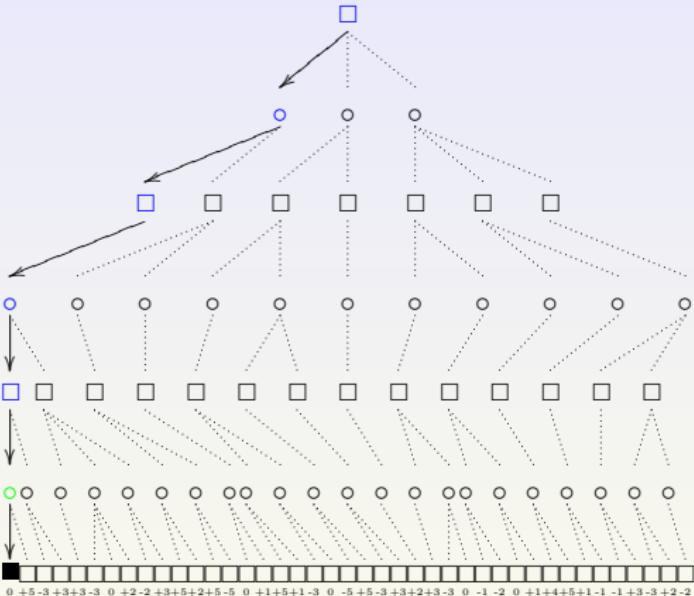


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

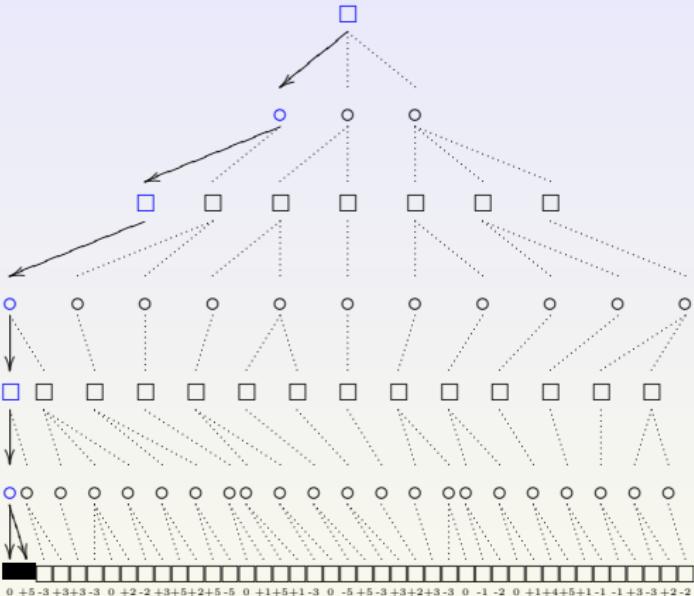
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

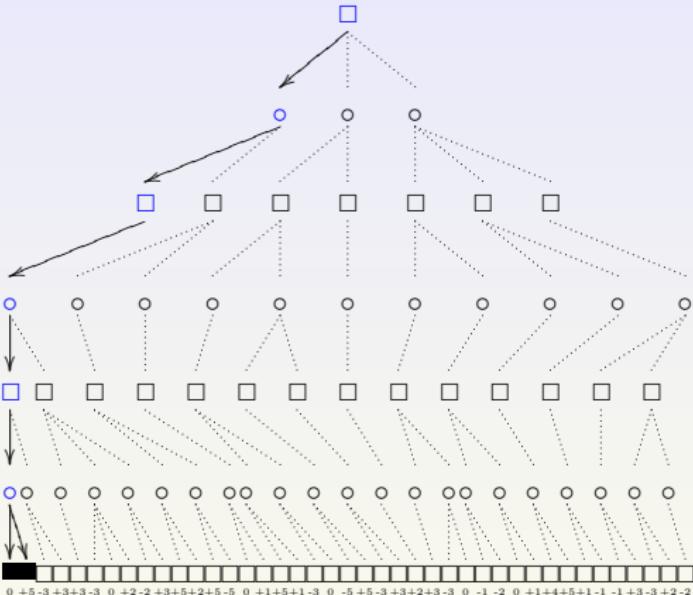


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

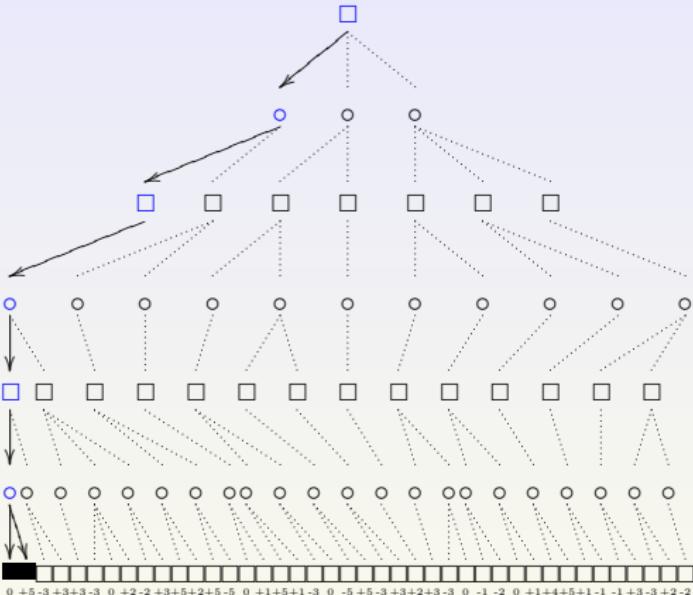


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

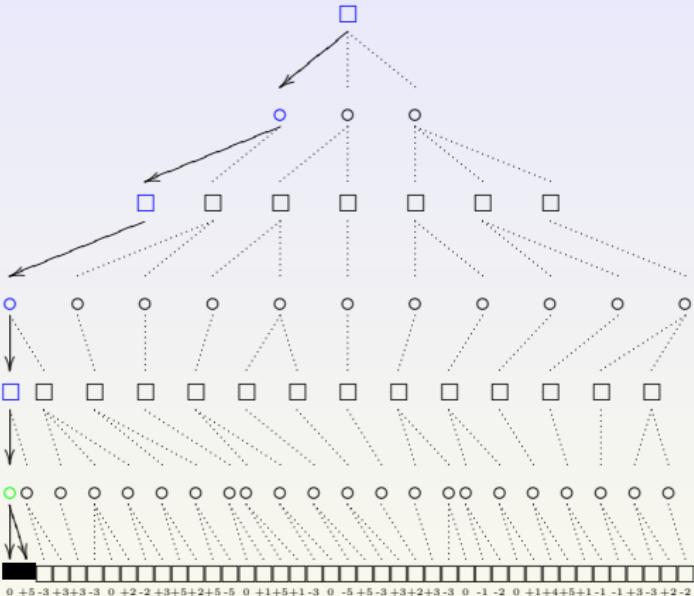


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

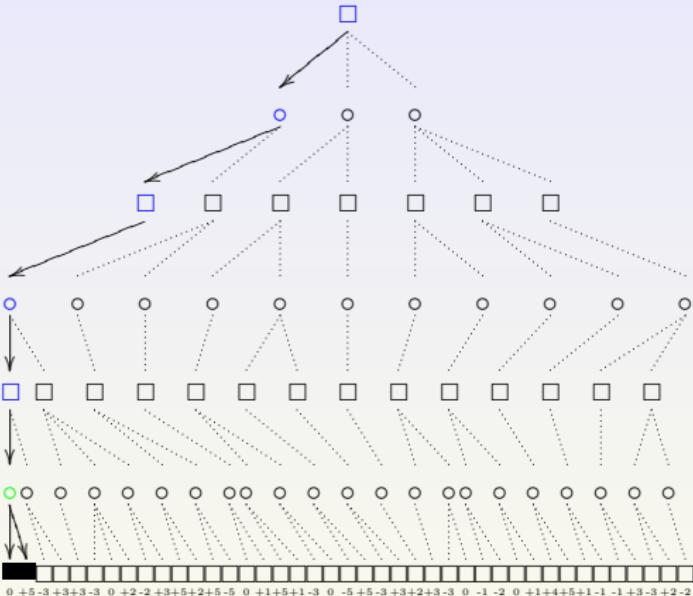


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

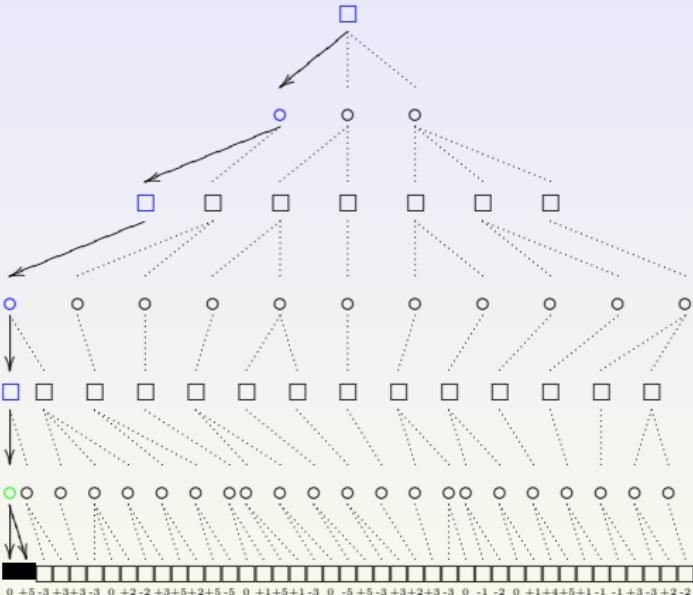


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

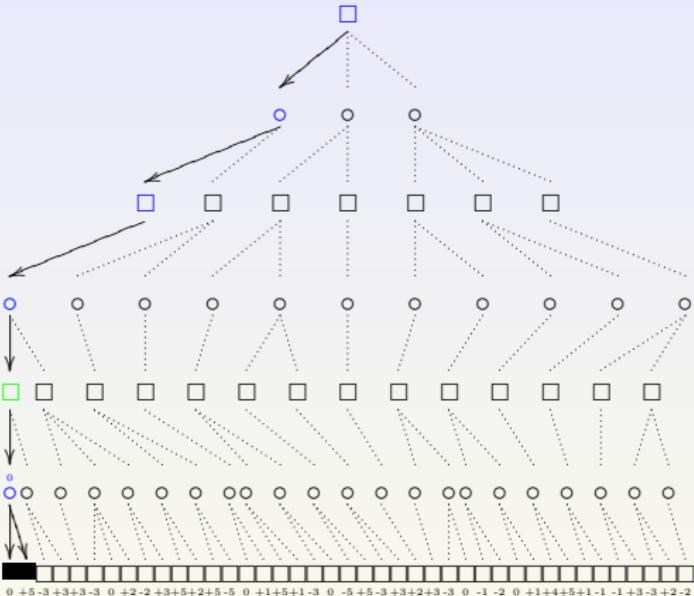


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  → return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

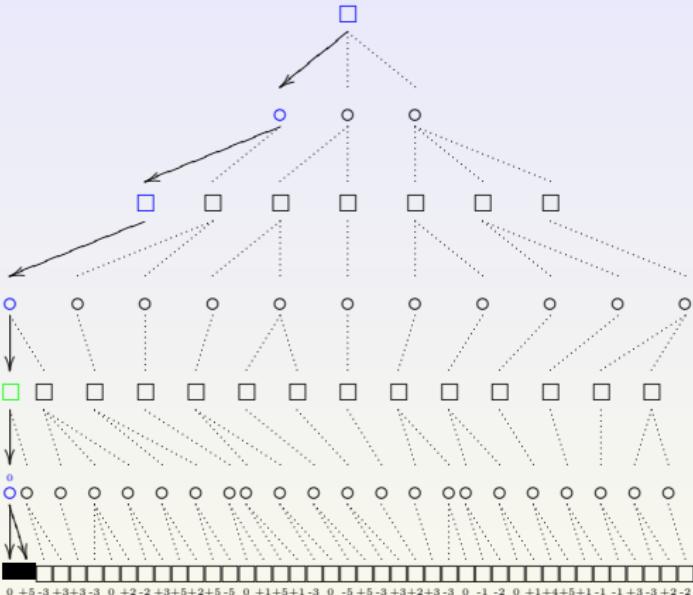
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  return  $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

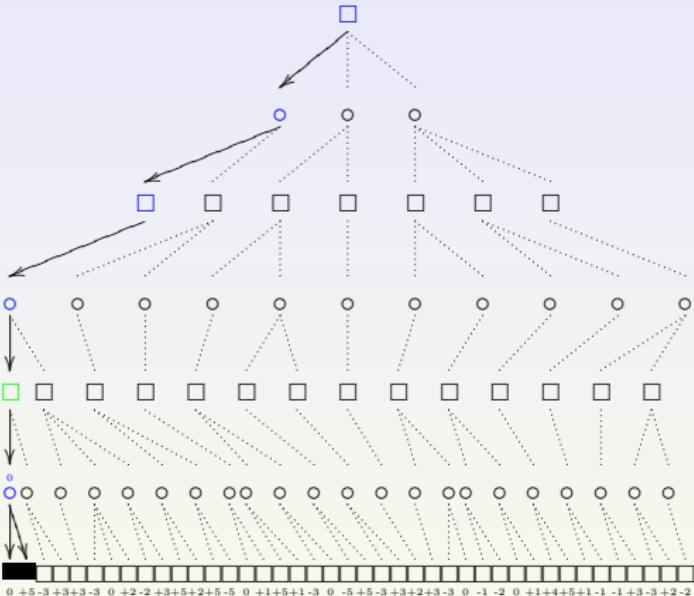
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

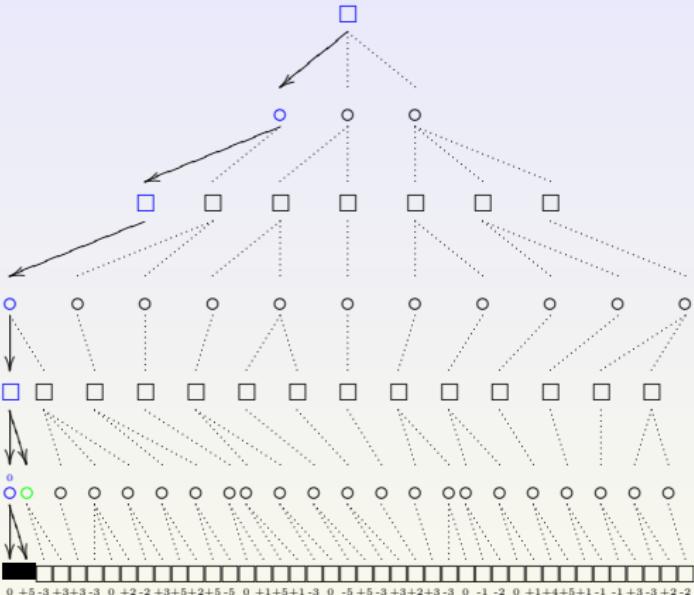


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

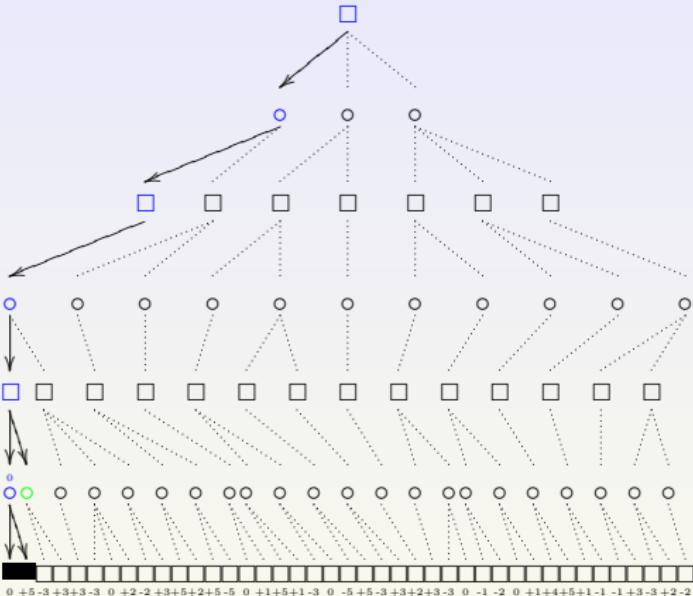


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

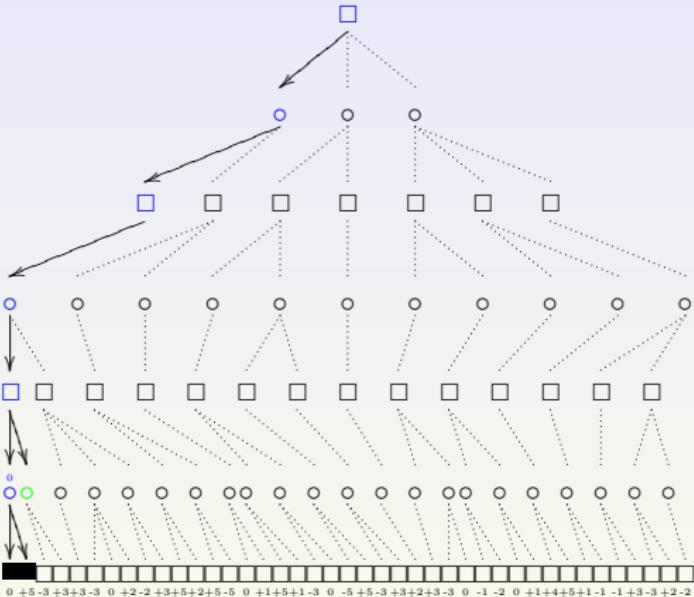


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

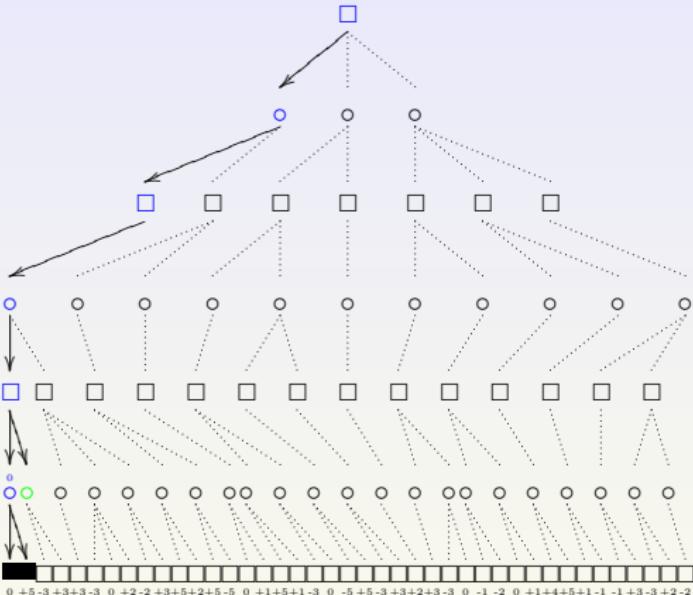


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

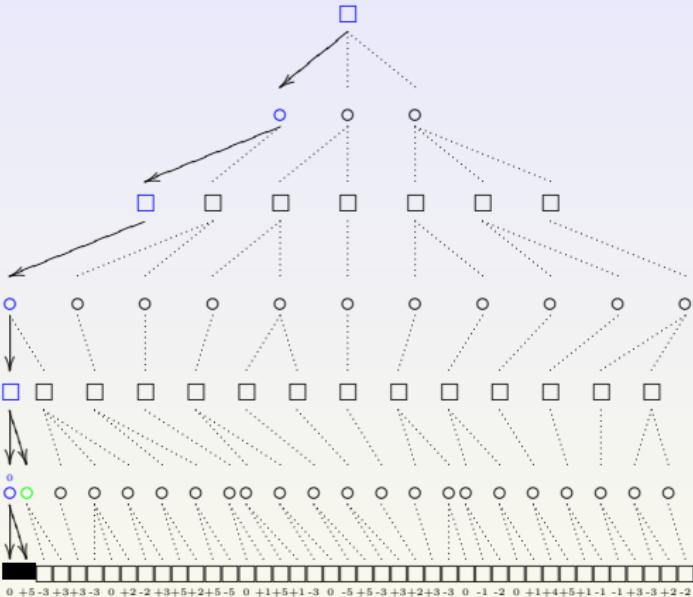
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

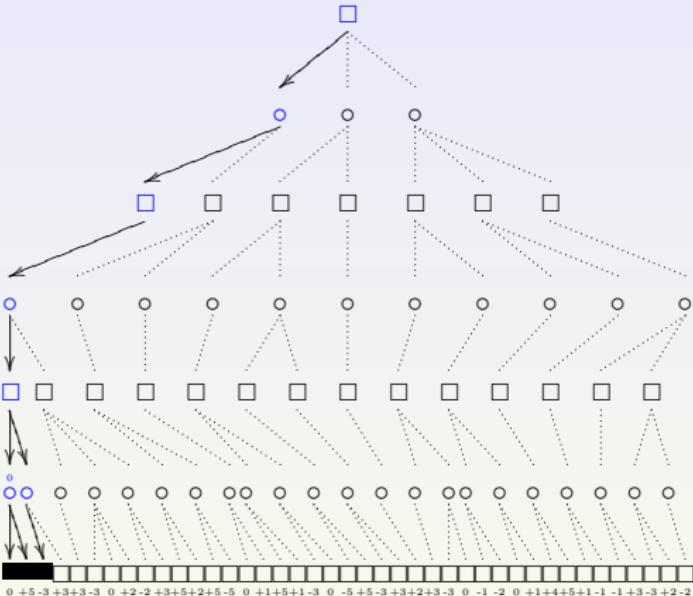
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

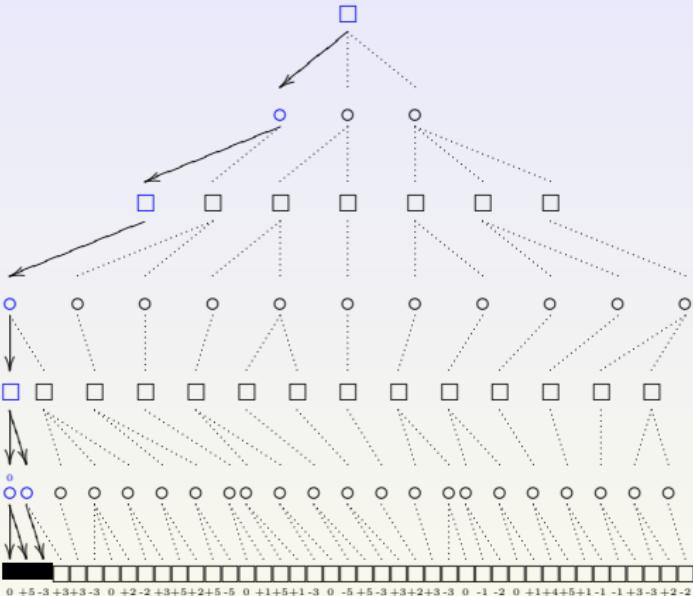


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

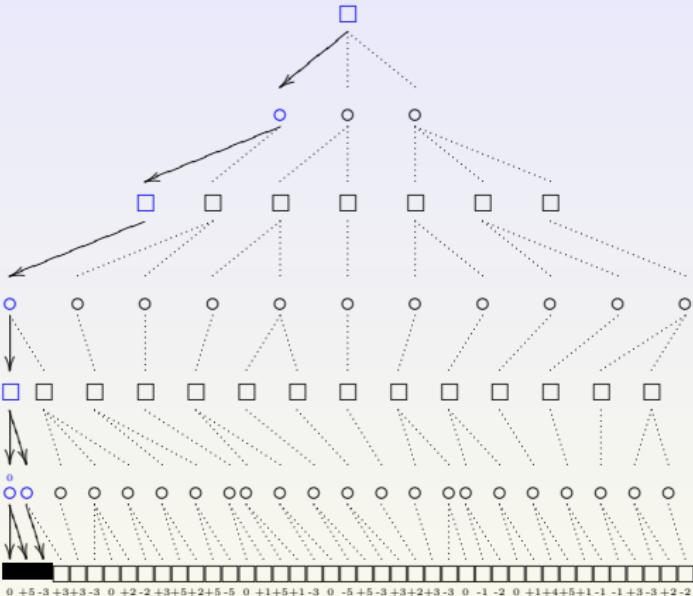
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

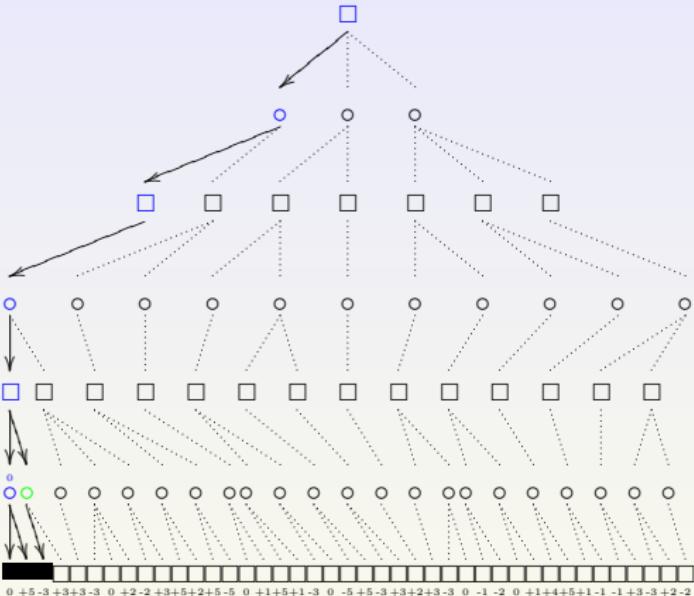


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  →
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

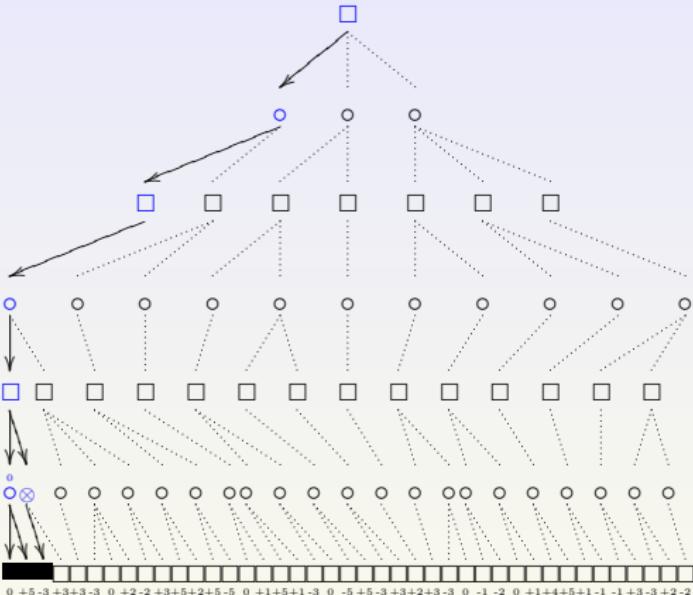


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

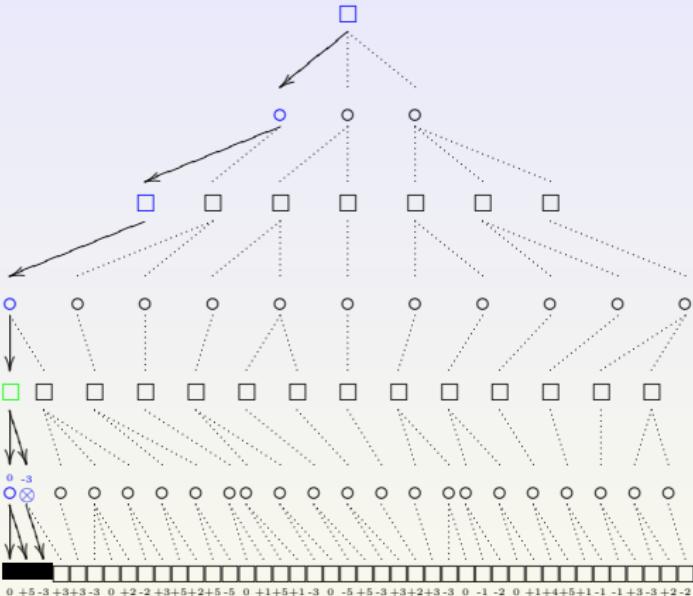
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

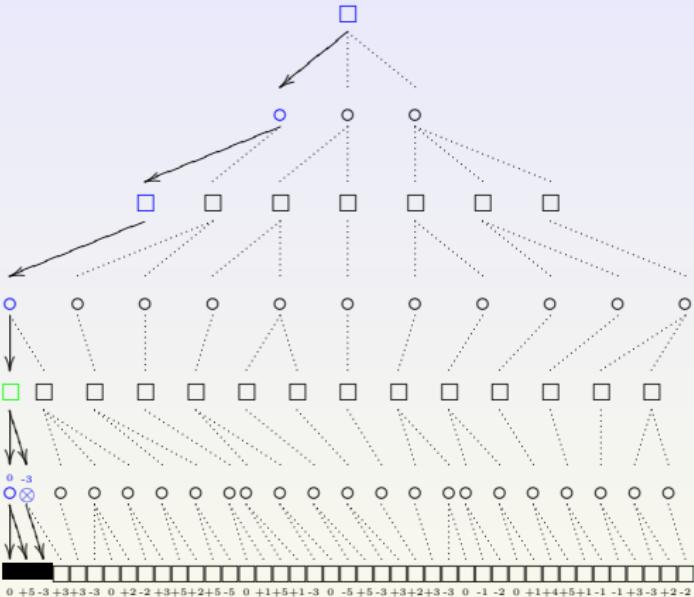


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
  →   if v ≥ β then
    return v
  α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
  if v ≤ α then
    return v
  β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  return  $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

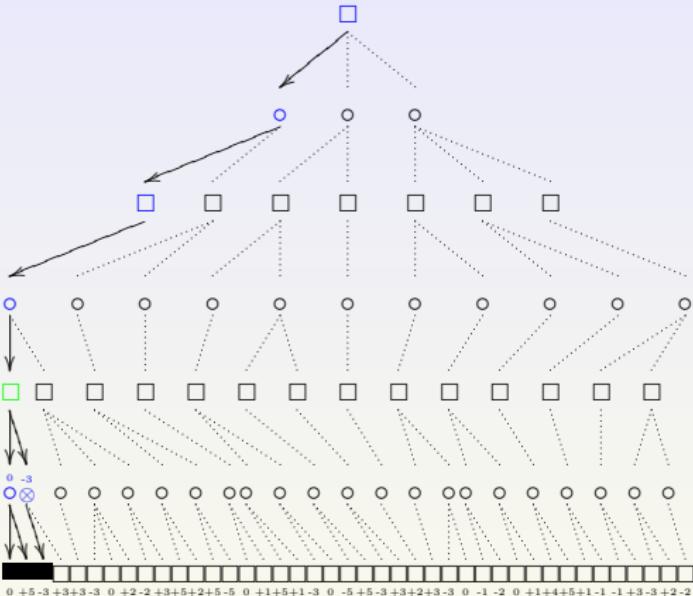
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

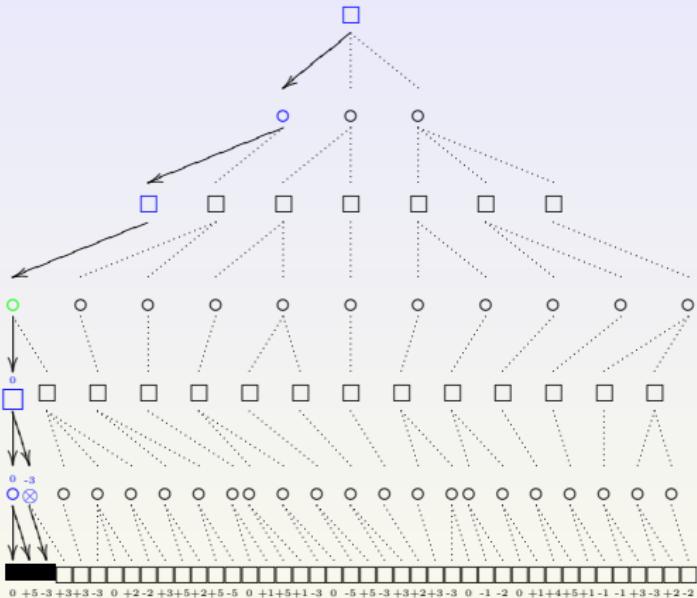


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  → return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

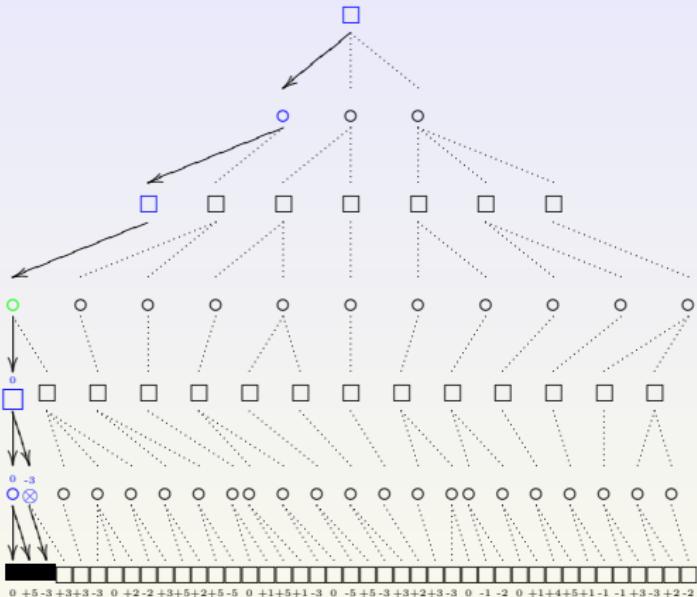


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

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function MIN-VALUE(state, α, β) returns a utility value
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

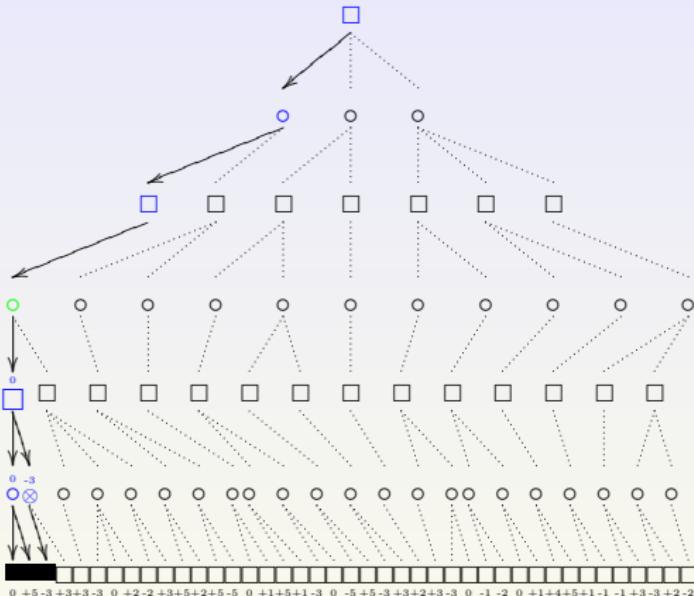


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

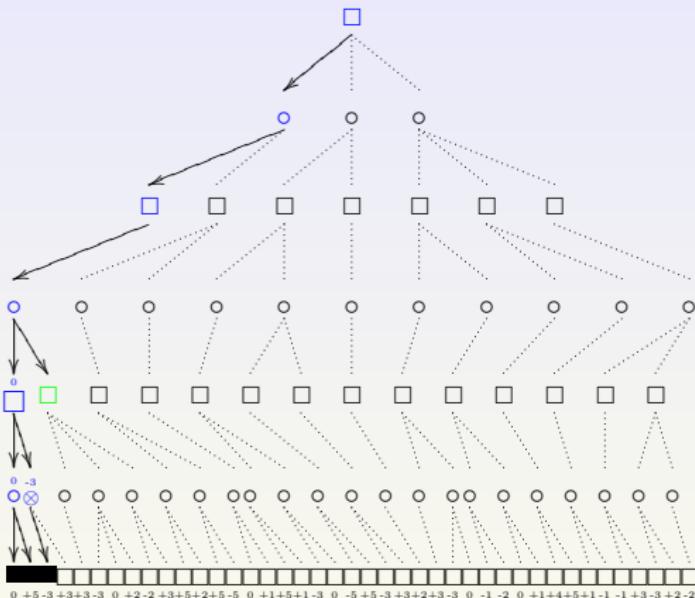
```

function MIN-VALUE(state, $\alpha,\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha,\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state, $\alpha,\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

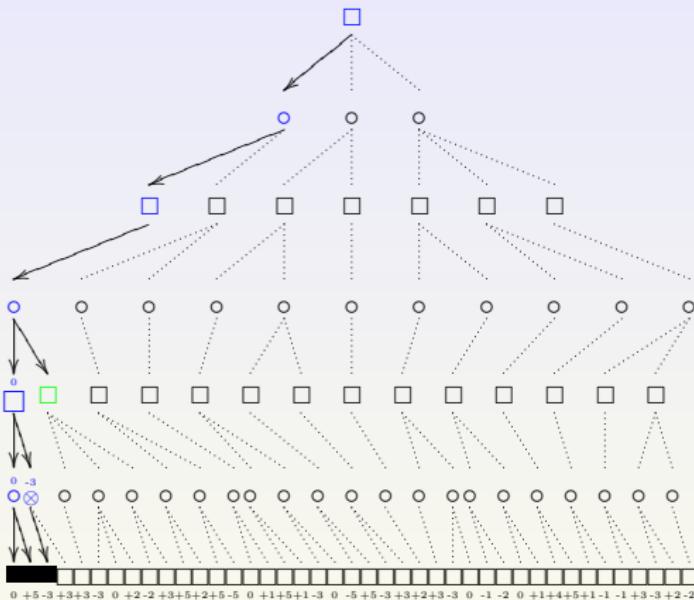
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

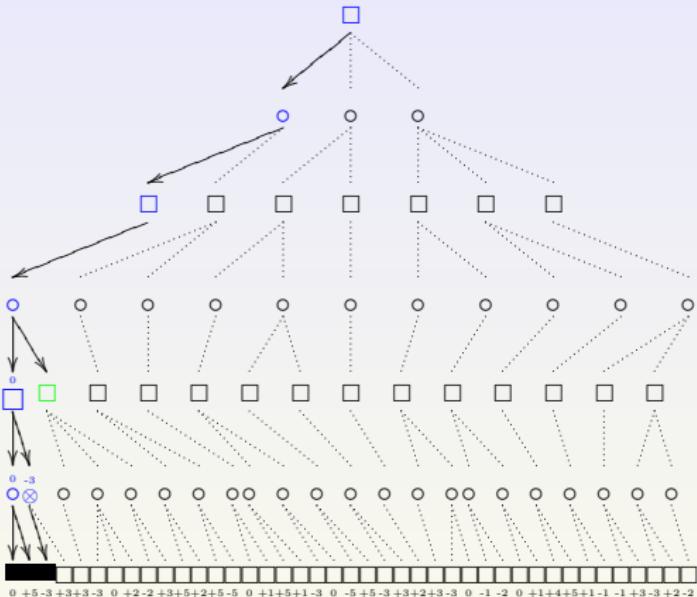
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = -\infty \quad \beta = 0$$

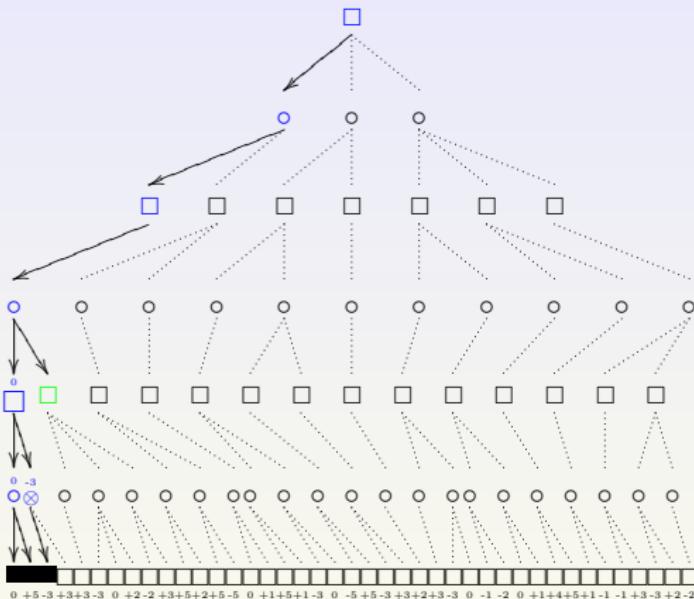


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

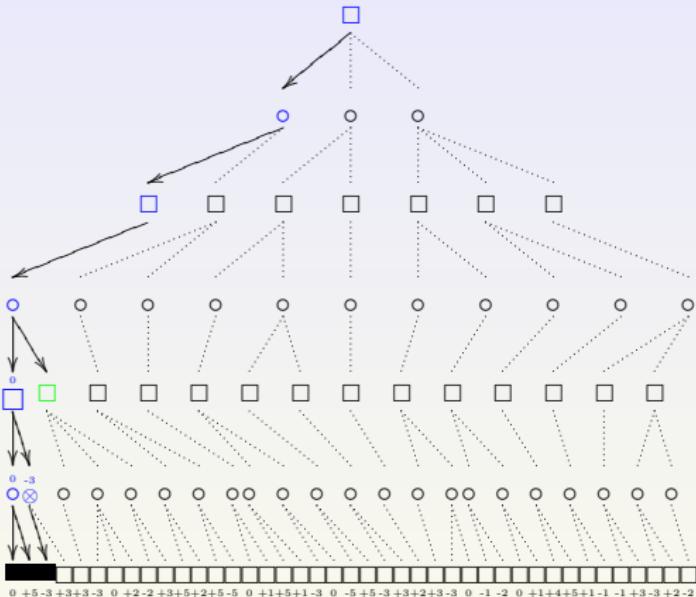
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = -\infty \quad \beta = 0$$

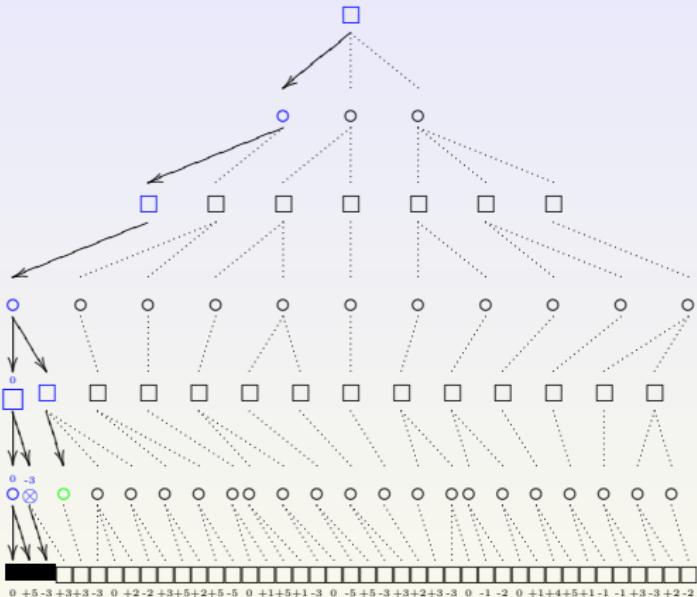


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    → v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

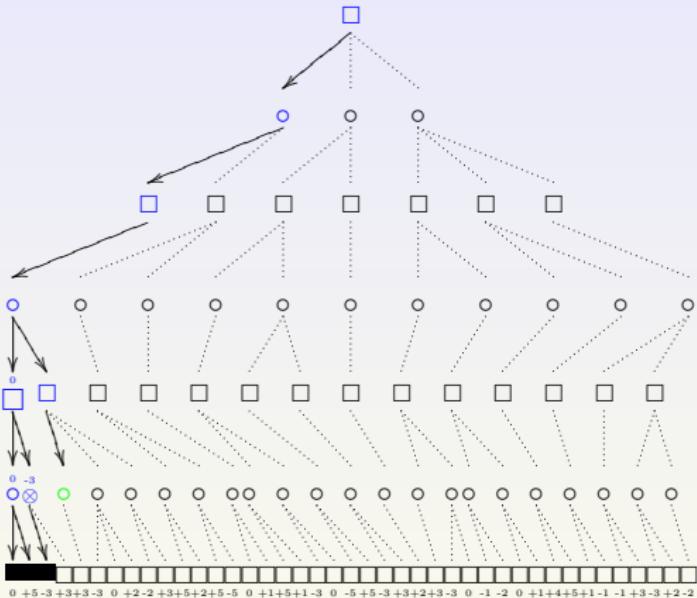


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

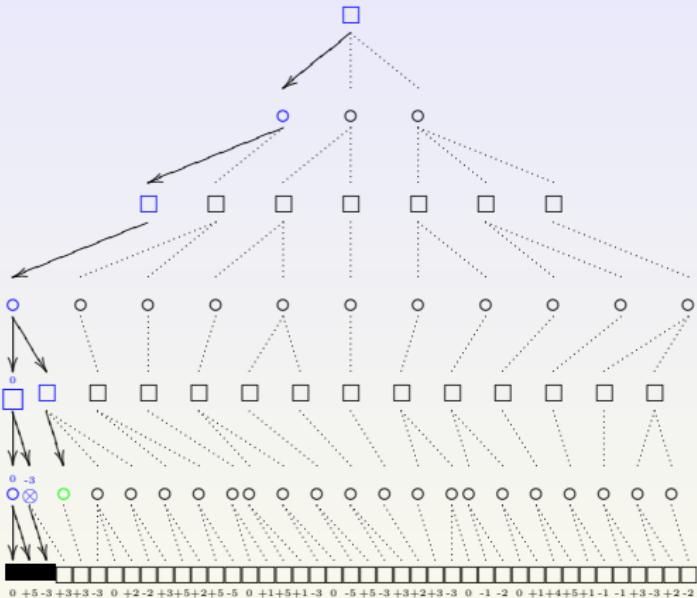


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

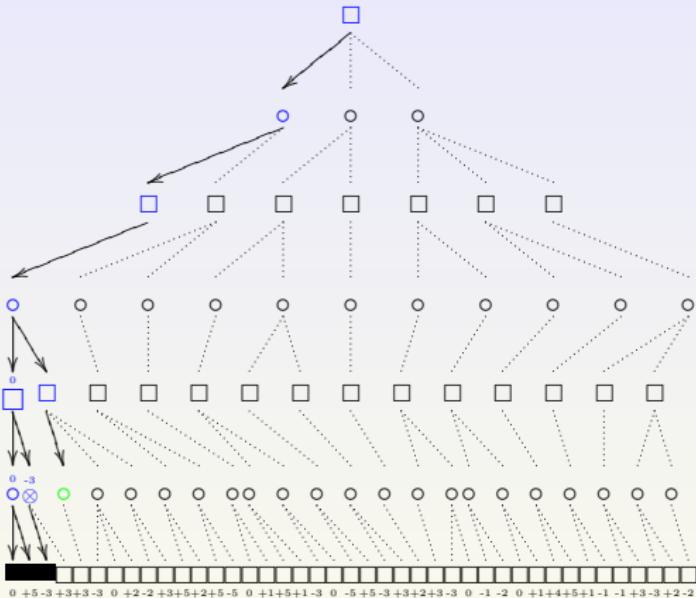
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

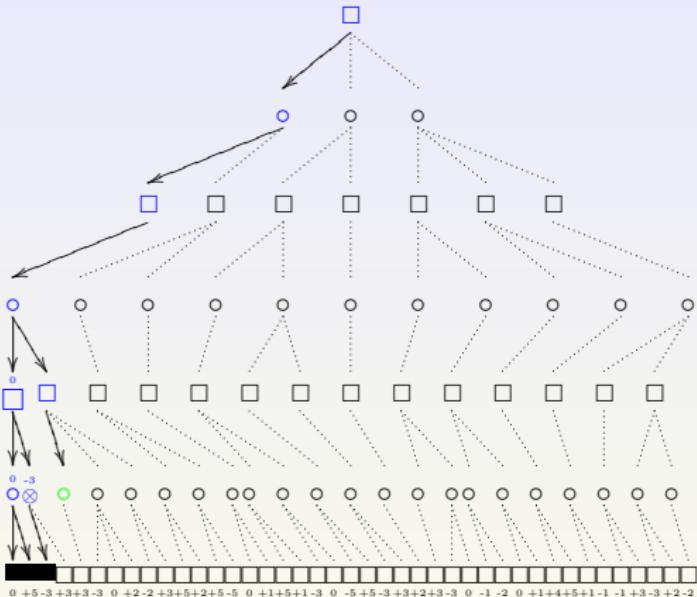


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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
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    if  $v \leq \alpha$  then
      return v
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$$\alpha = -\infty \quad \beta = 0$$

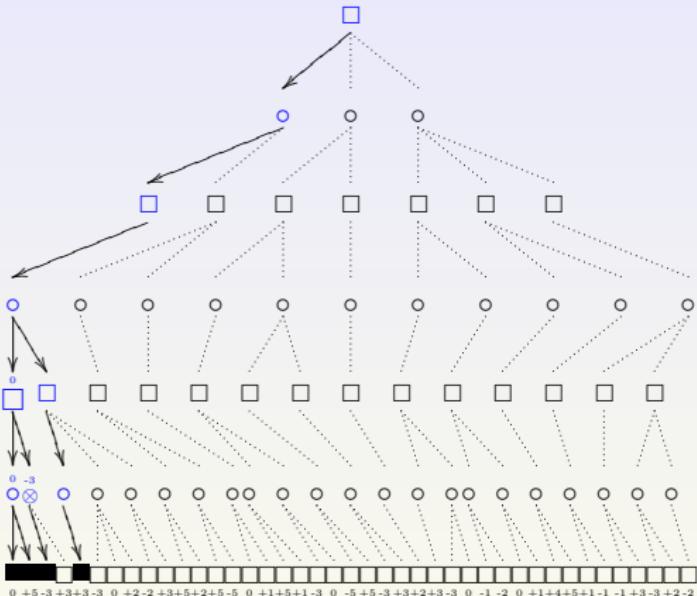


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  return the action in SUCCESSORS(state) with value v
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

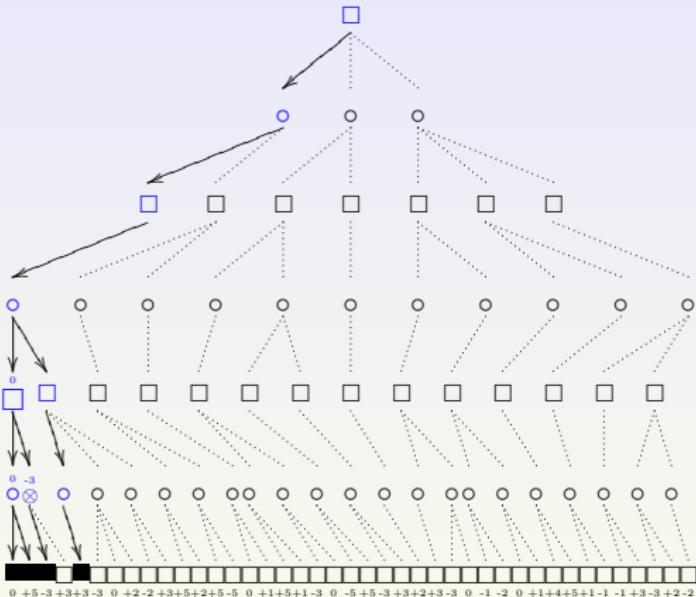
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

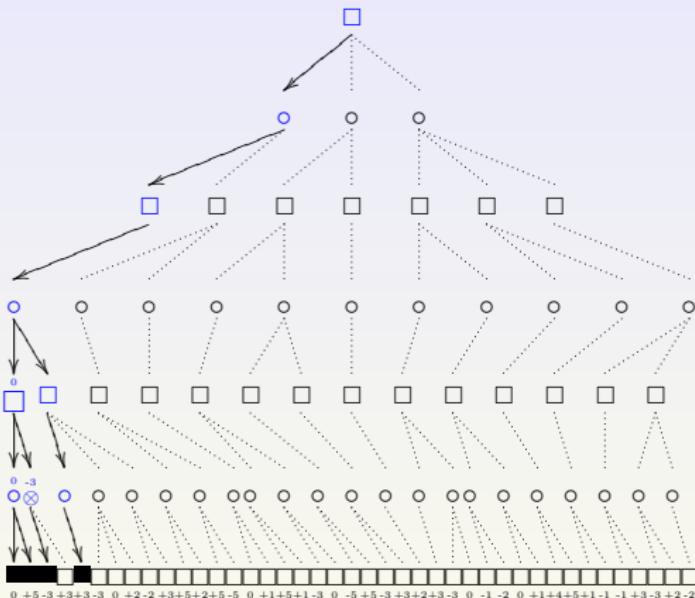


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

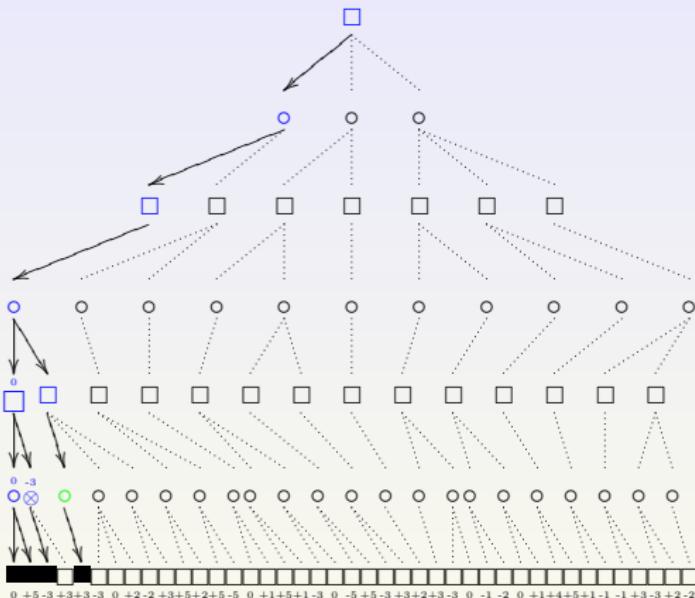
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = -\infty \quad \beta = 0$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

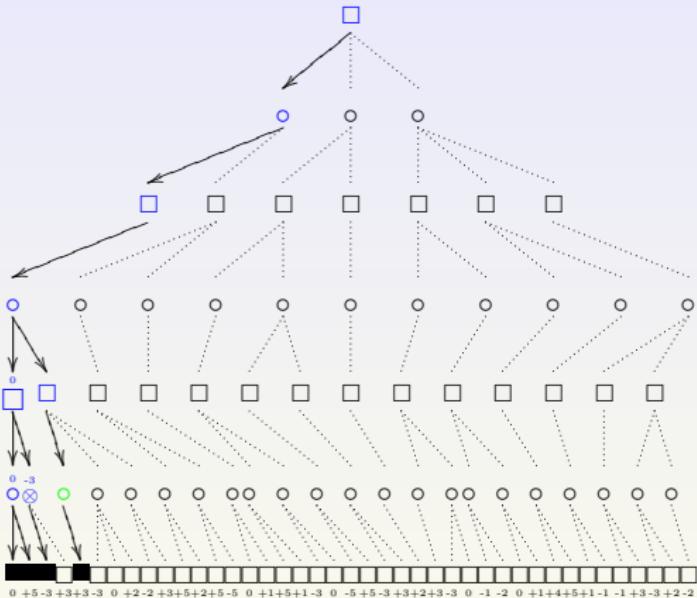
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
  if  $v \leq \alpha$  then
    return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

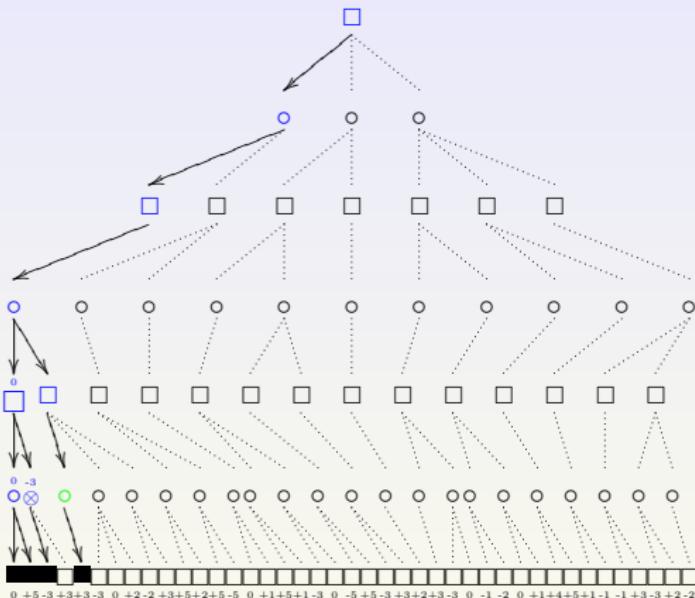
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

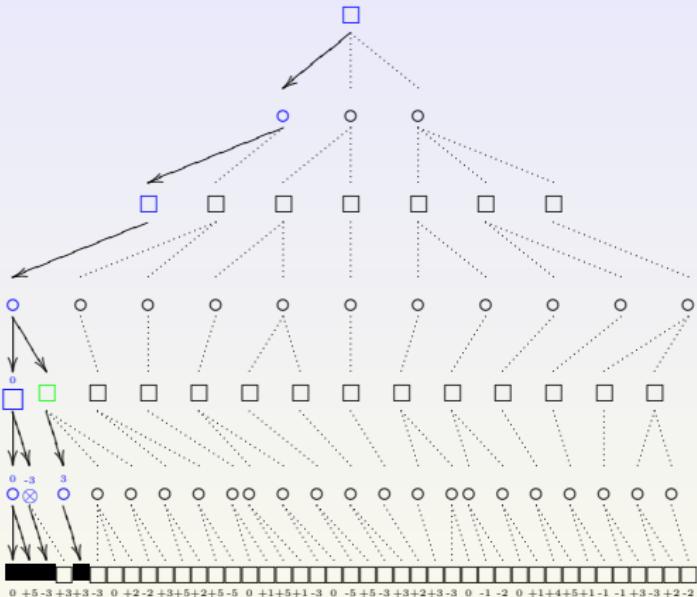
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = -\infty \quad \beta = 0$$



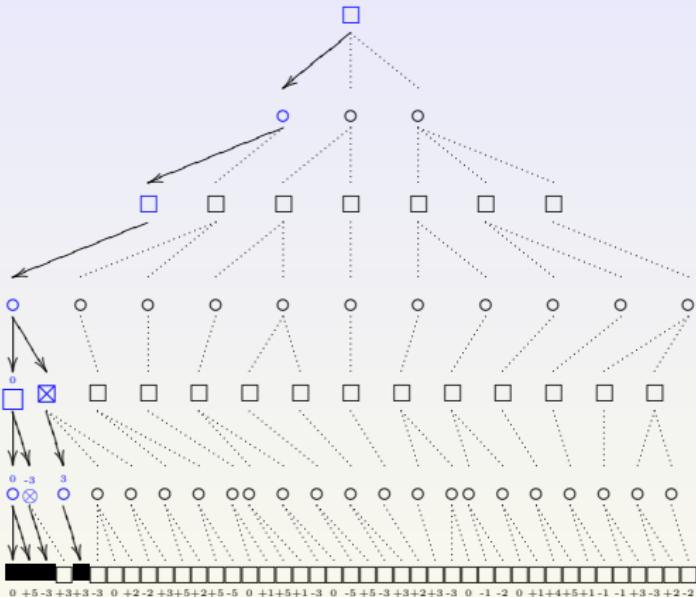


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
  →   if v ≥ β then
    return v
  α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
  if v ≤ α then
    return v
  β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

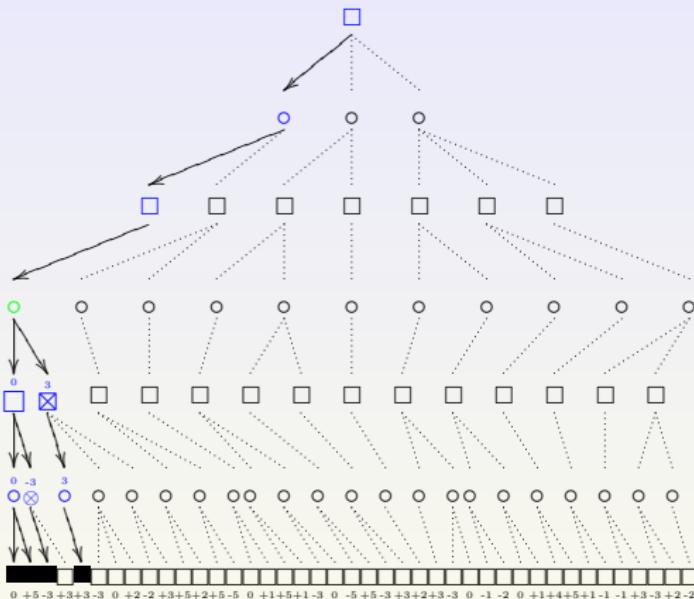


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      → return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

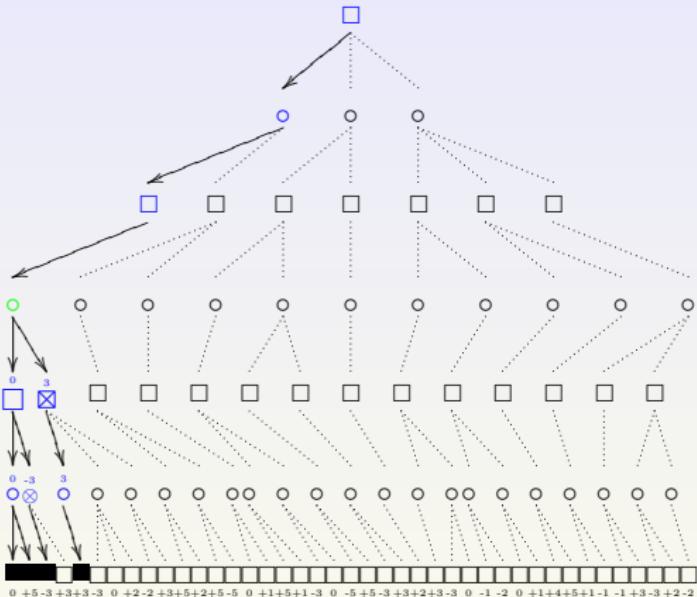
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow \text{MINI}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

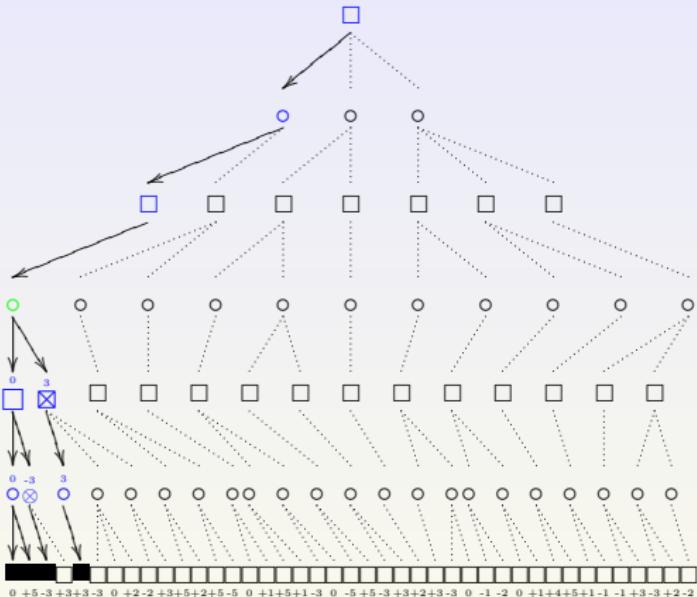
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

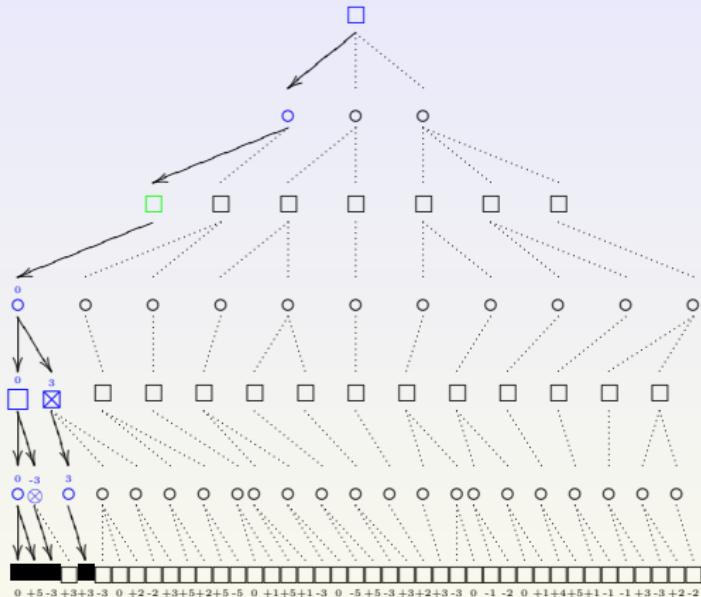


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  → return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

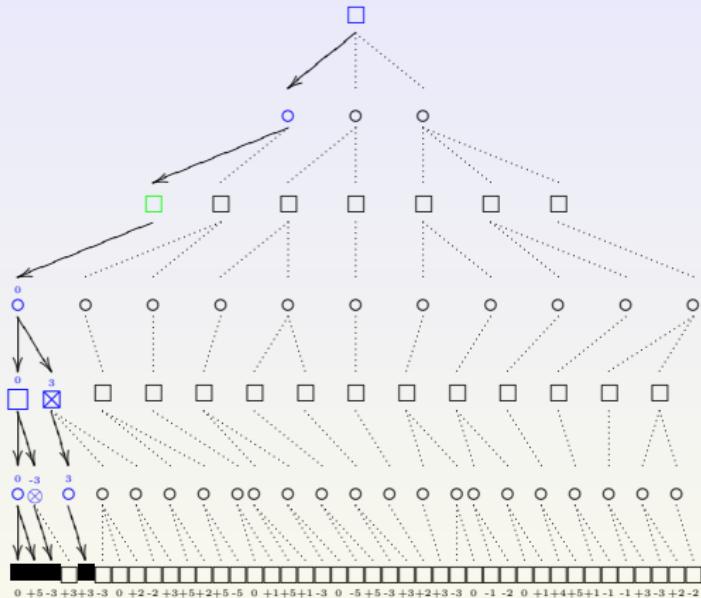
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$

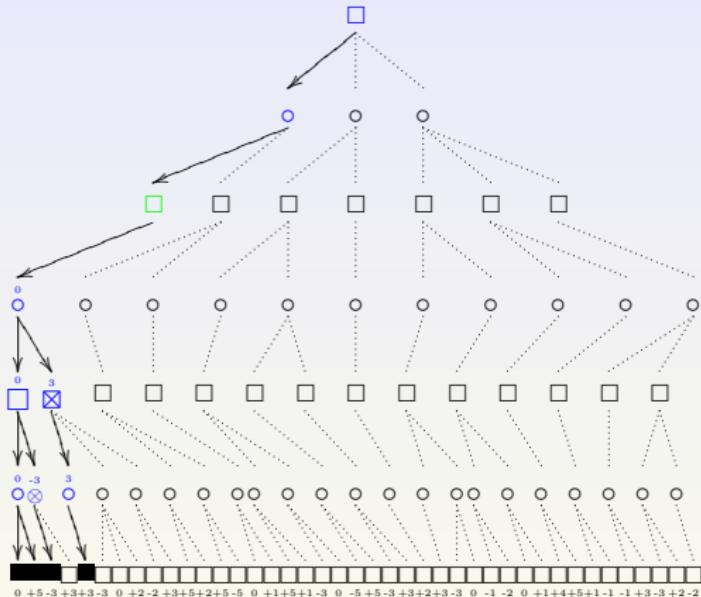


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
  →  $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

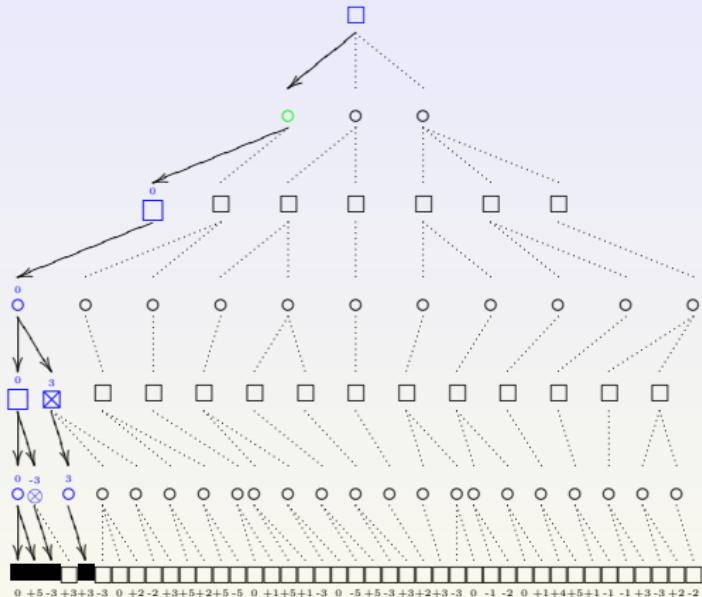
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

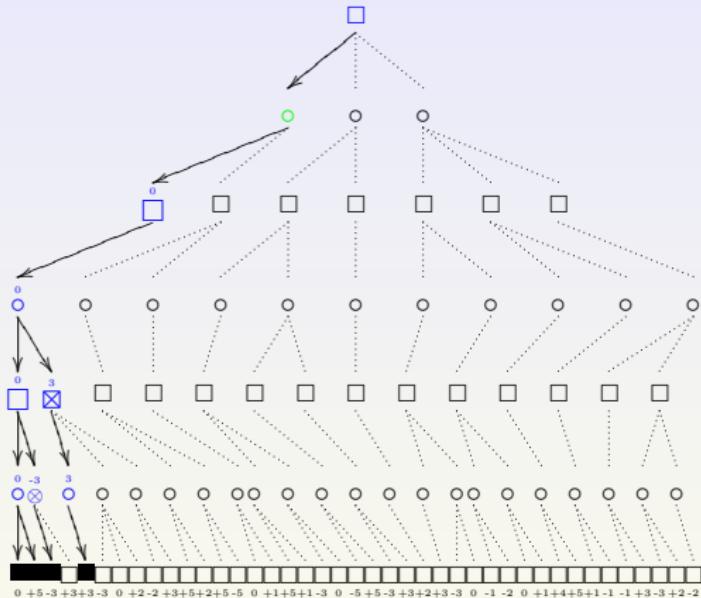


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

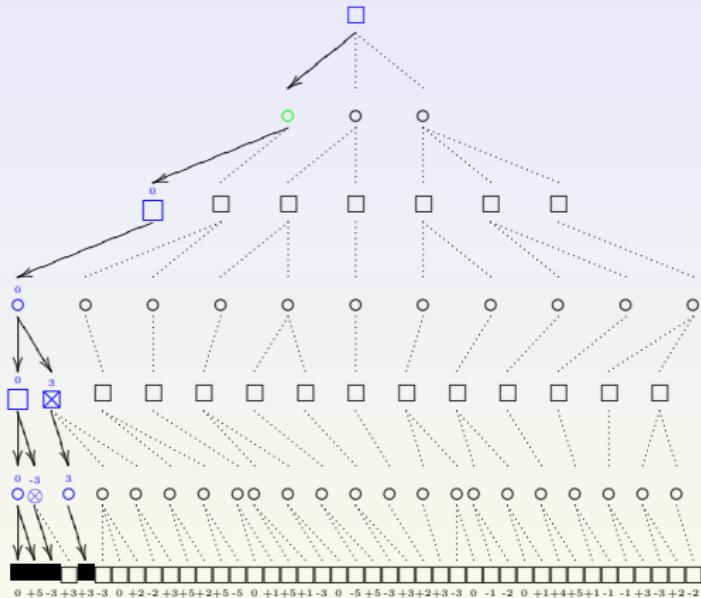


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function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
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function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$

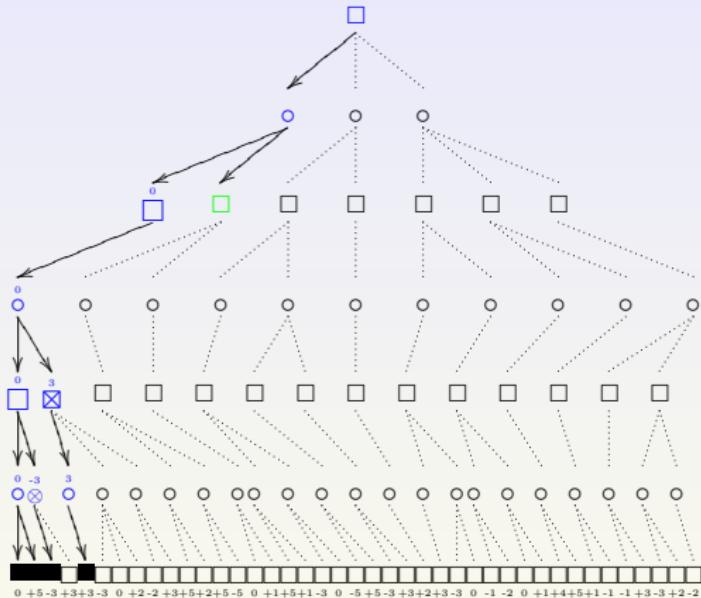


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  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

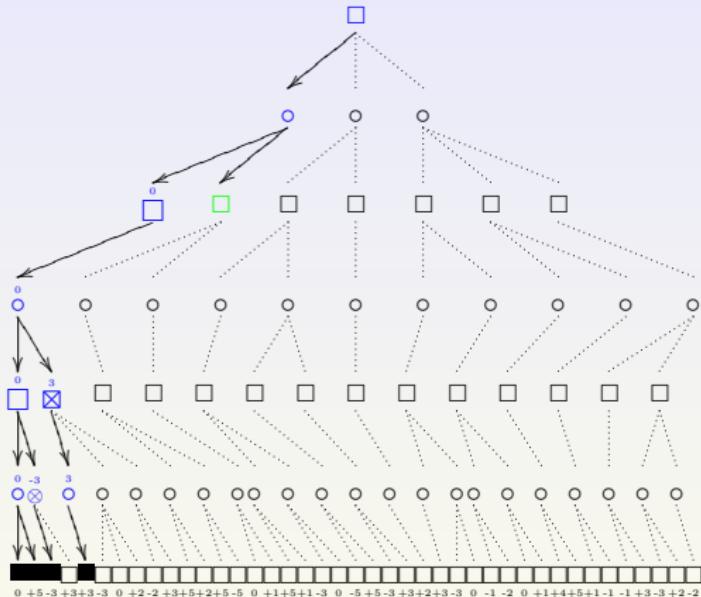
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

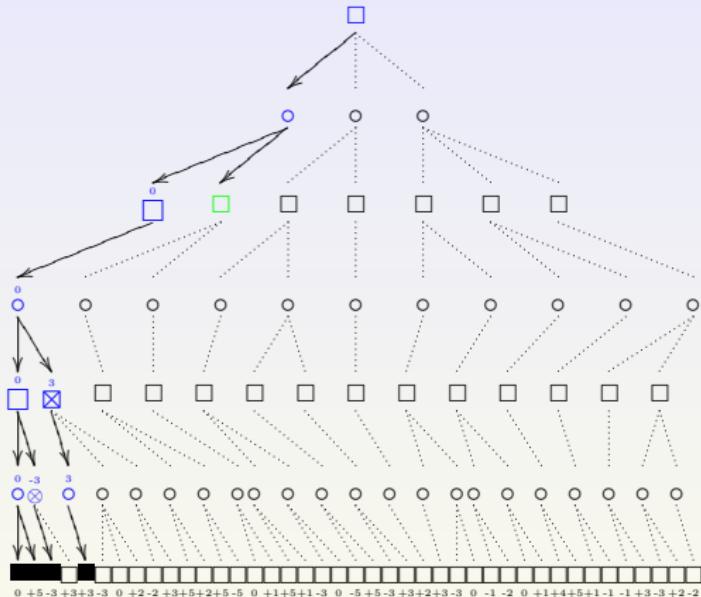
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

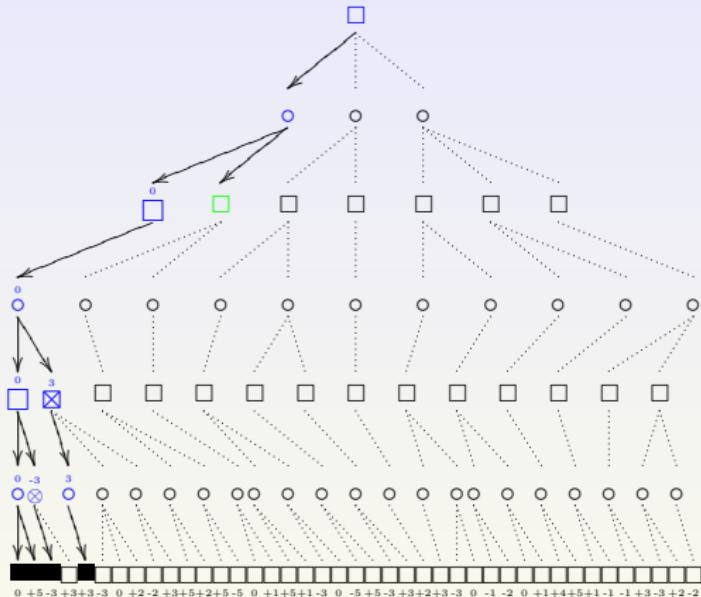


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

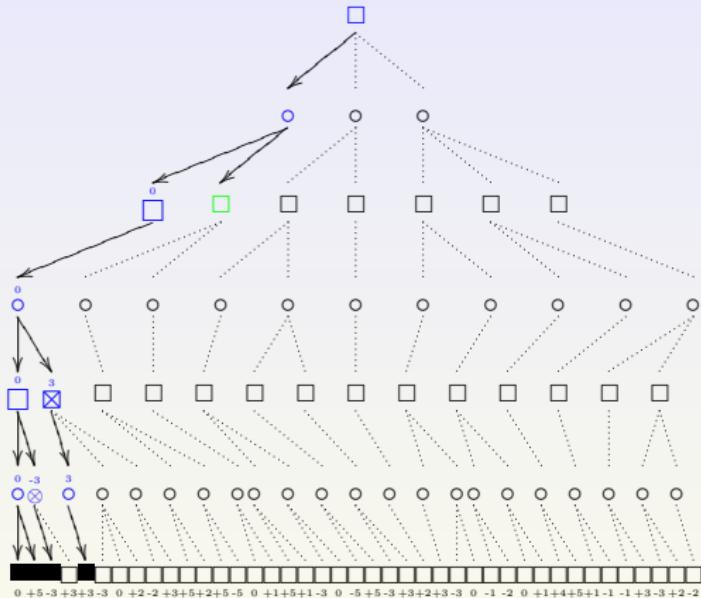


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

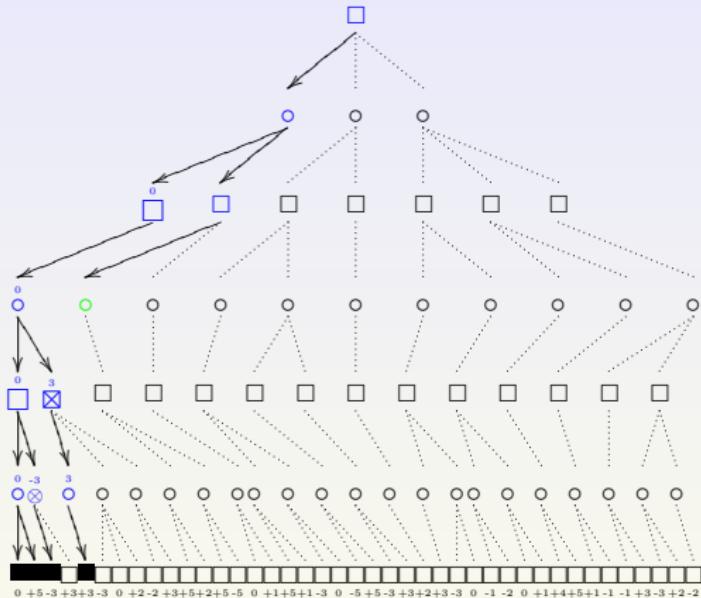


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

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function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

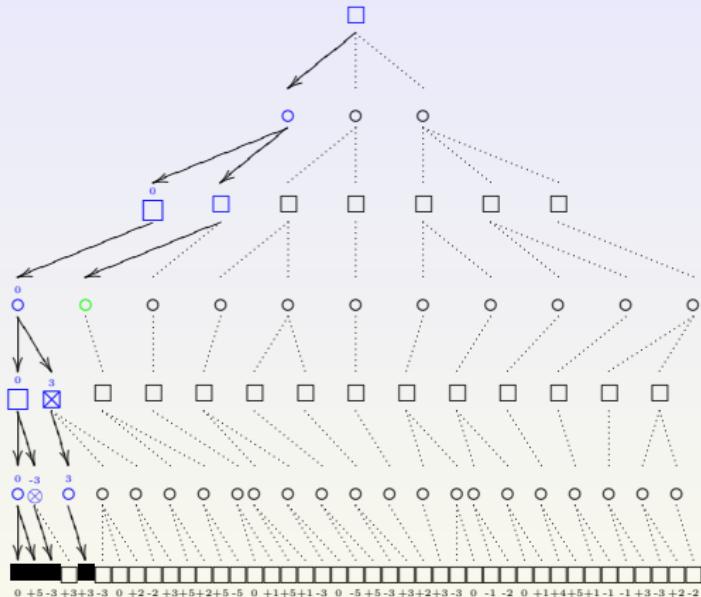
→

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    return UTILITY(state)
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

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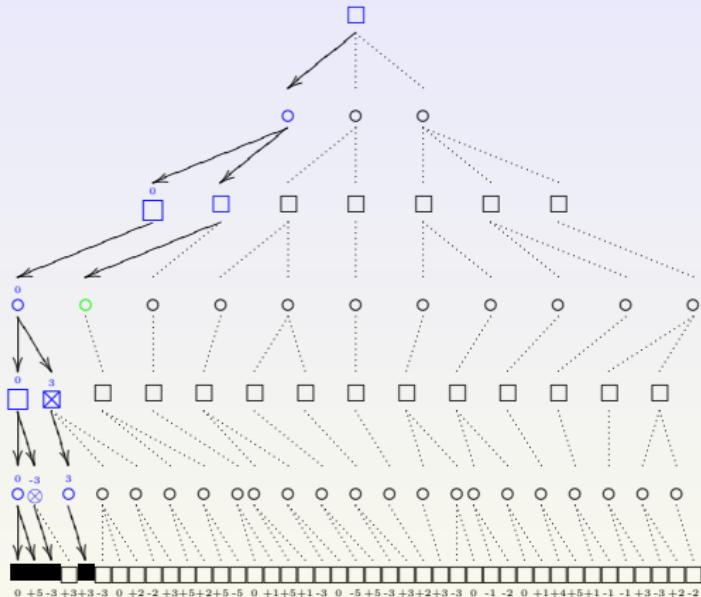


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    if v ≤ α then
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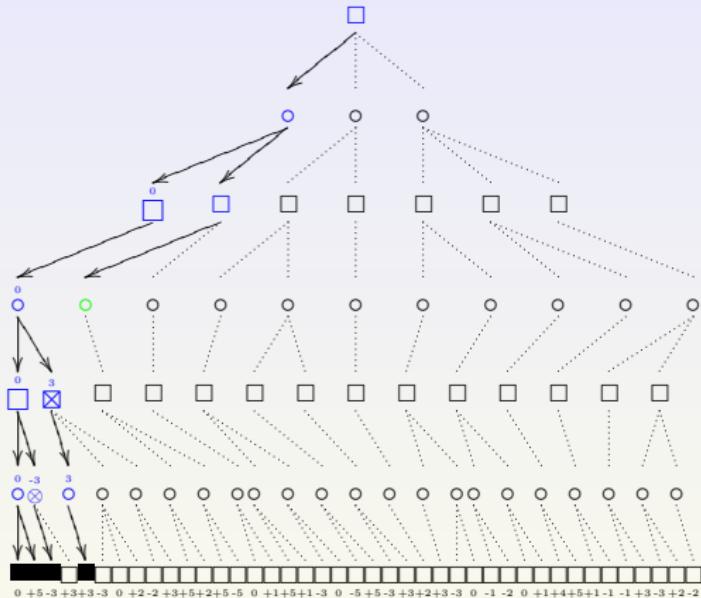


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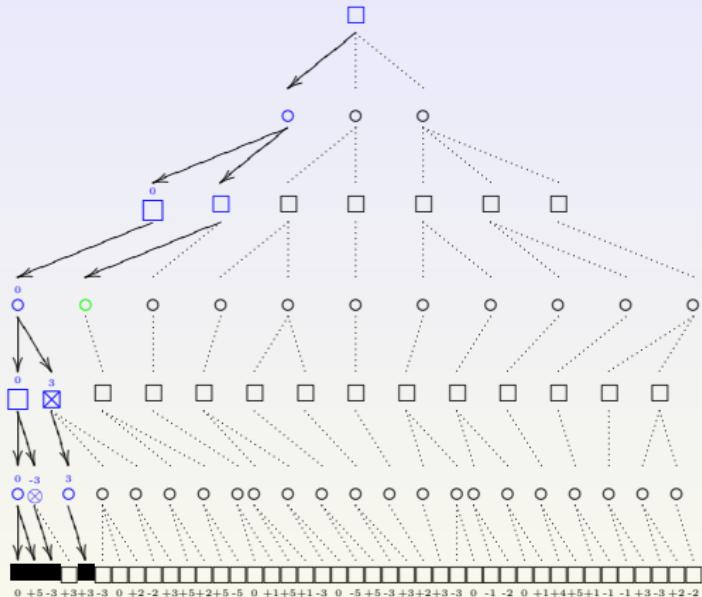


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$$\alpha = -\infty \quad \beta = 0$$

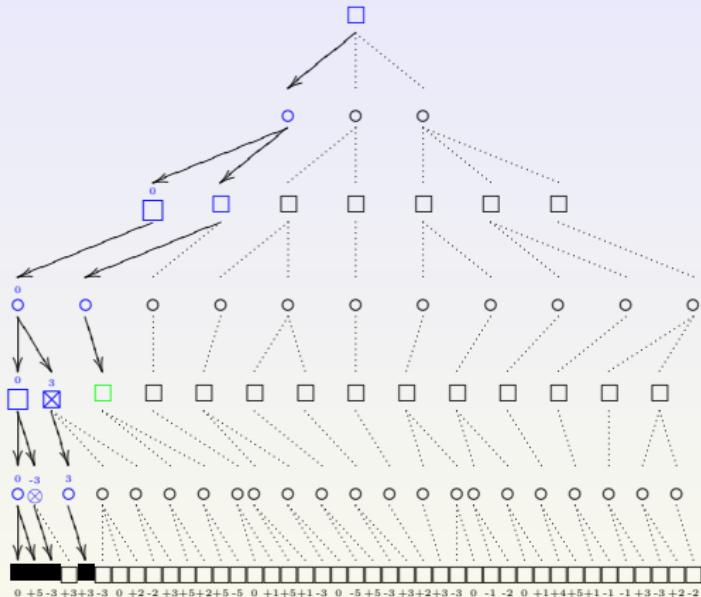


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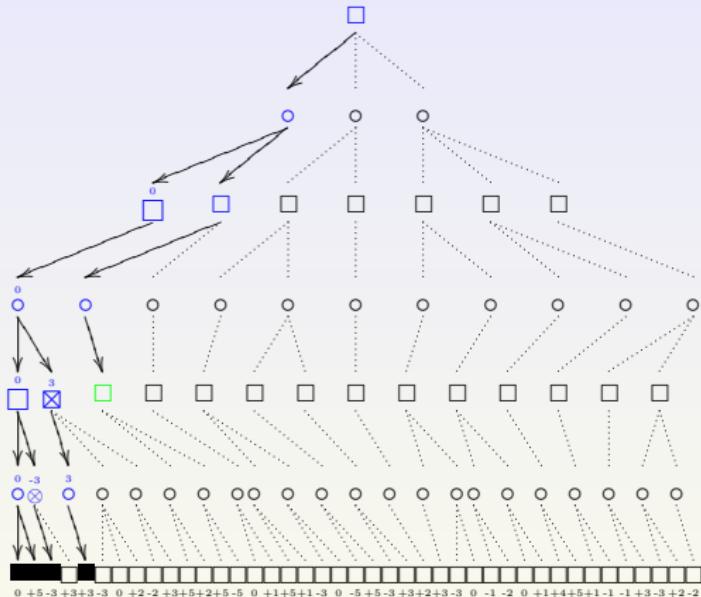


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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

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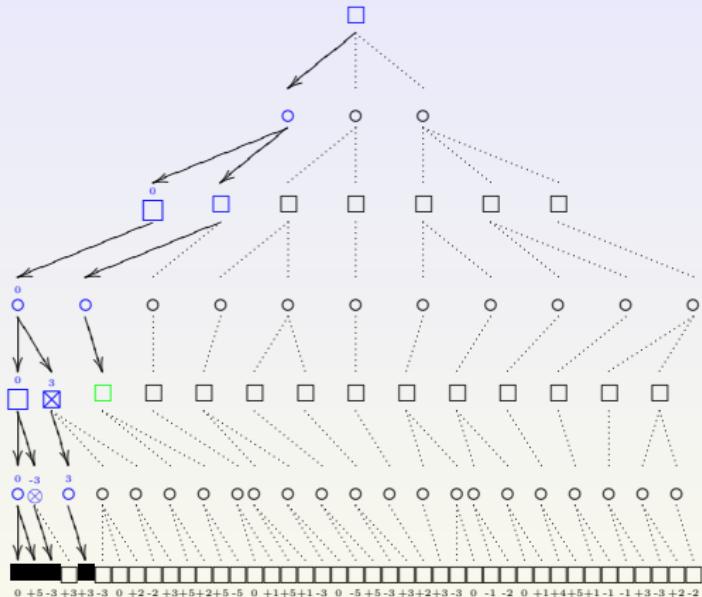


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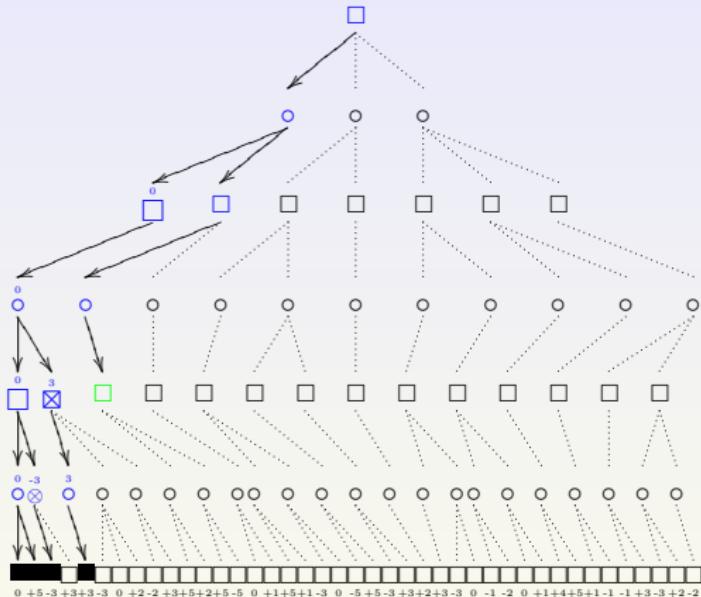


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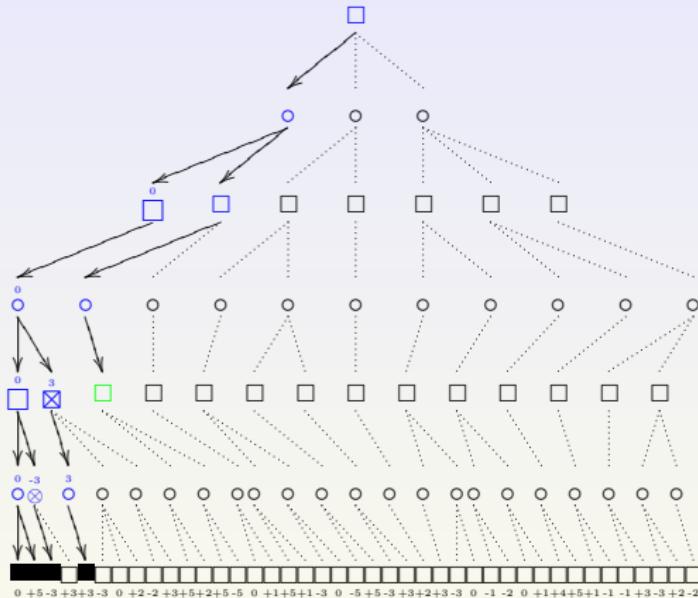


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      return v
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    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

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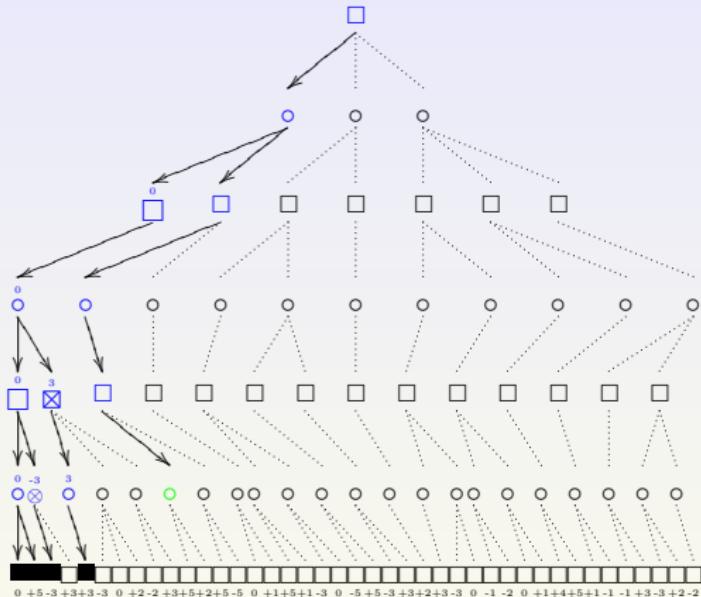


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    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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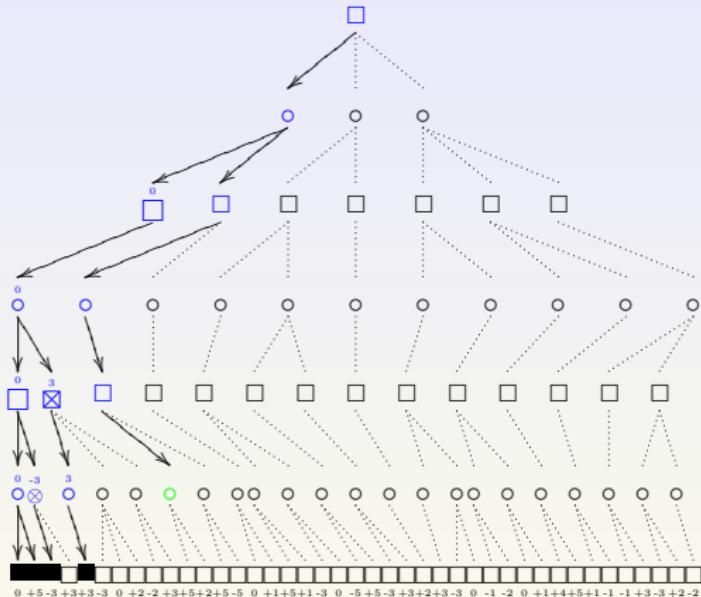
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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v
  
```

```

→function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v
  
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$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

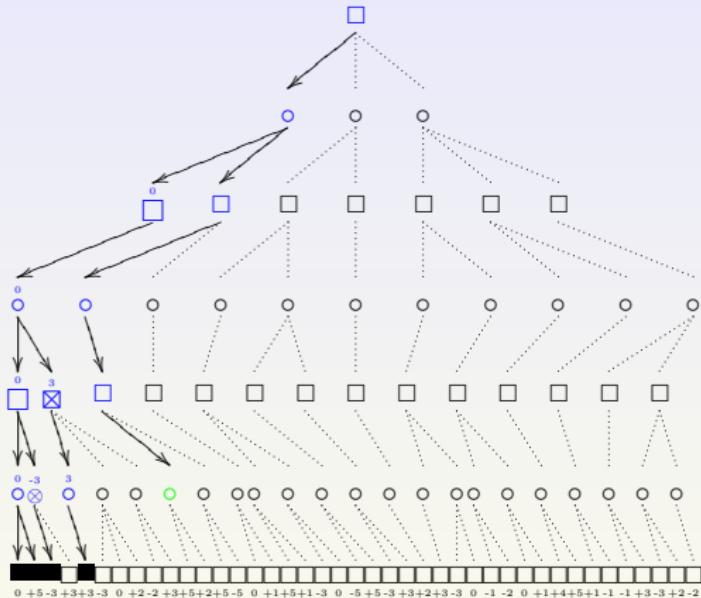
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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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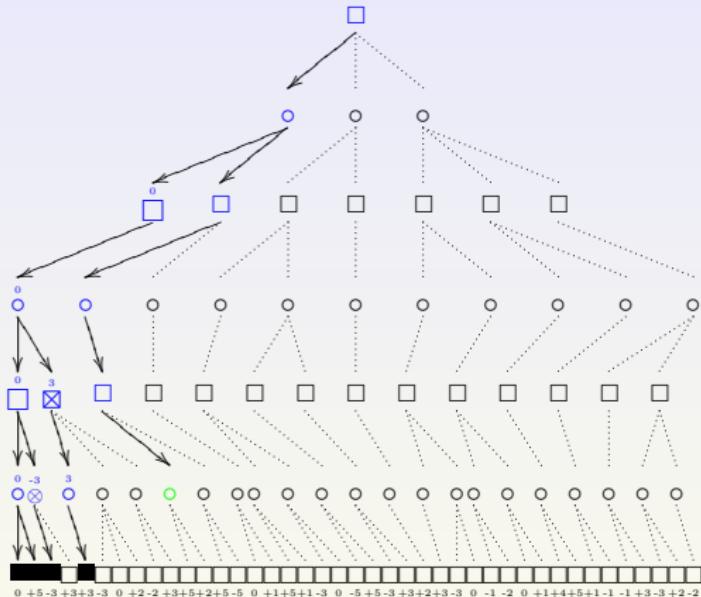
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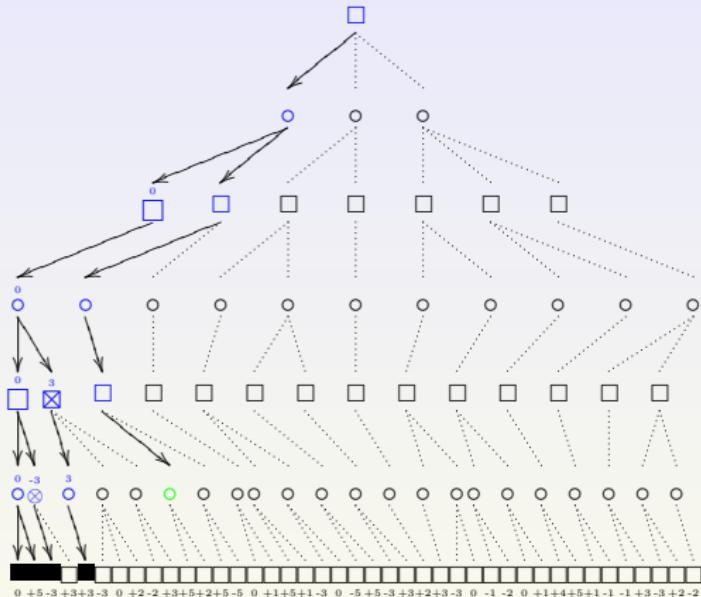
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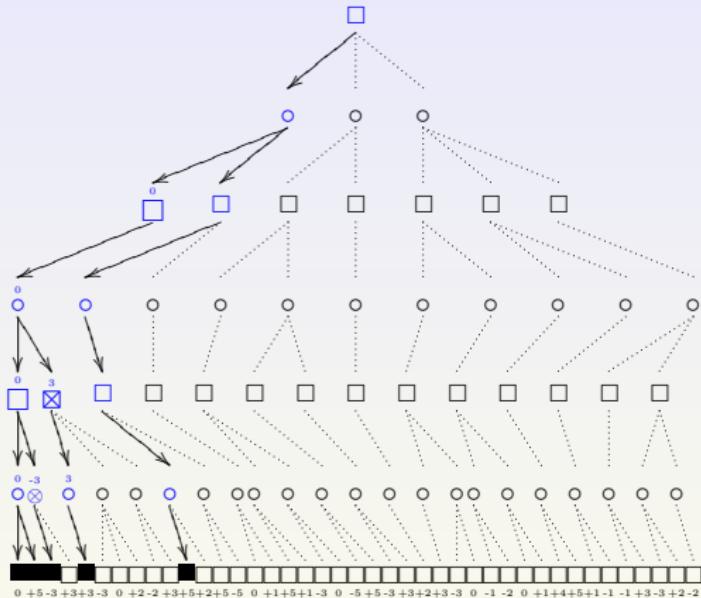
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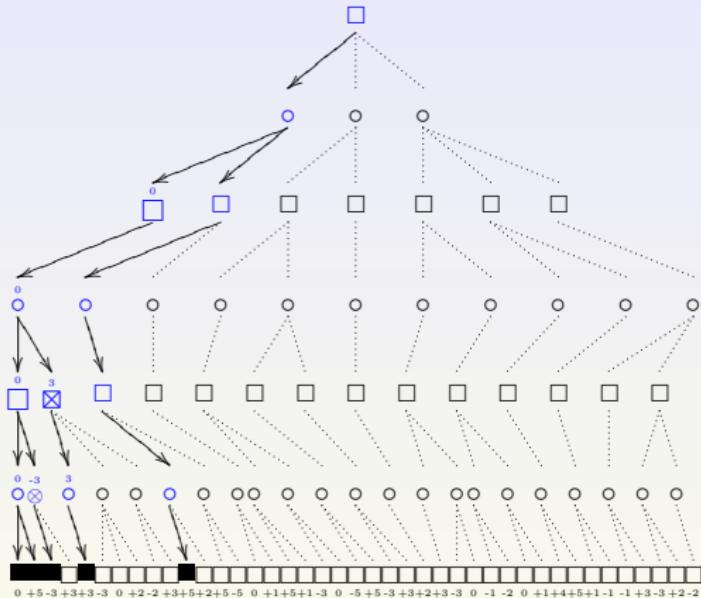
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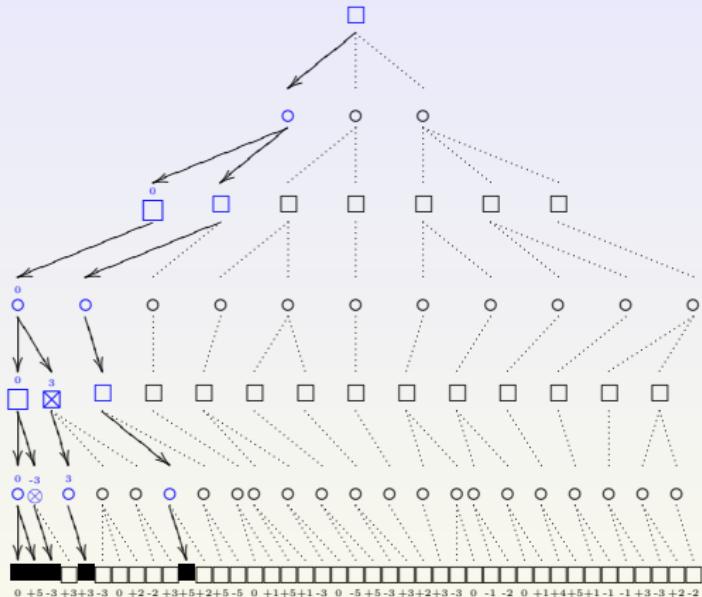
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      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

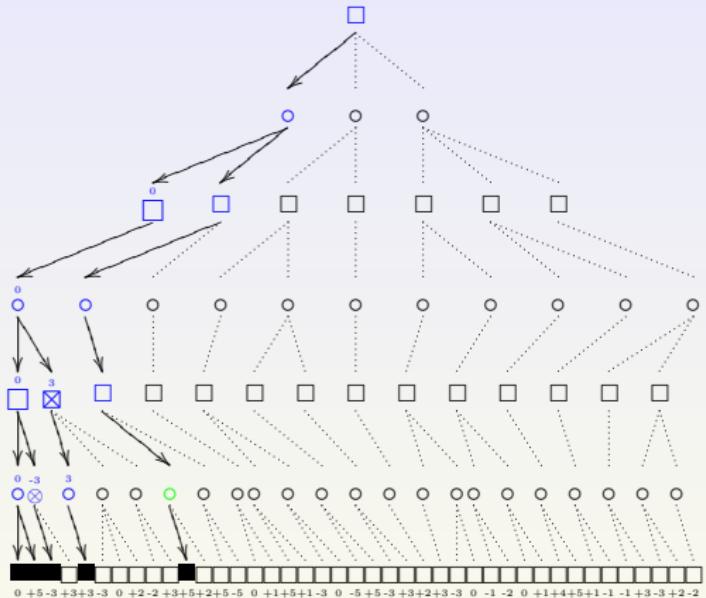
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

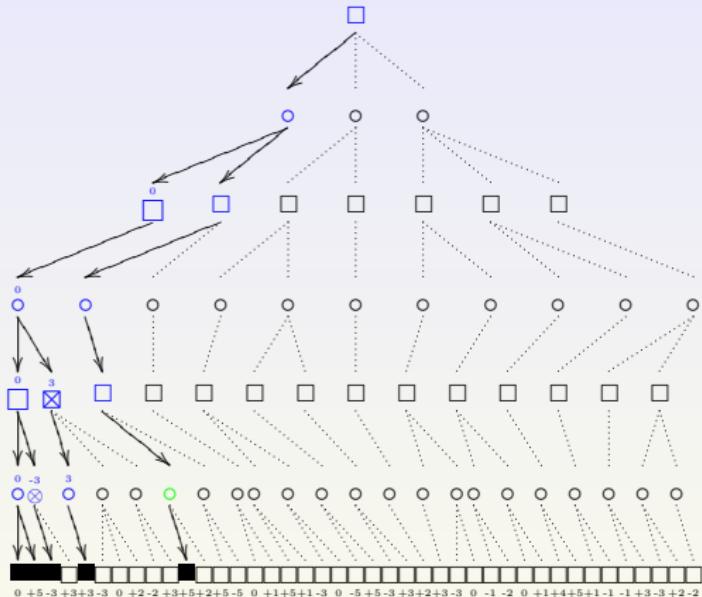
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

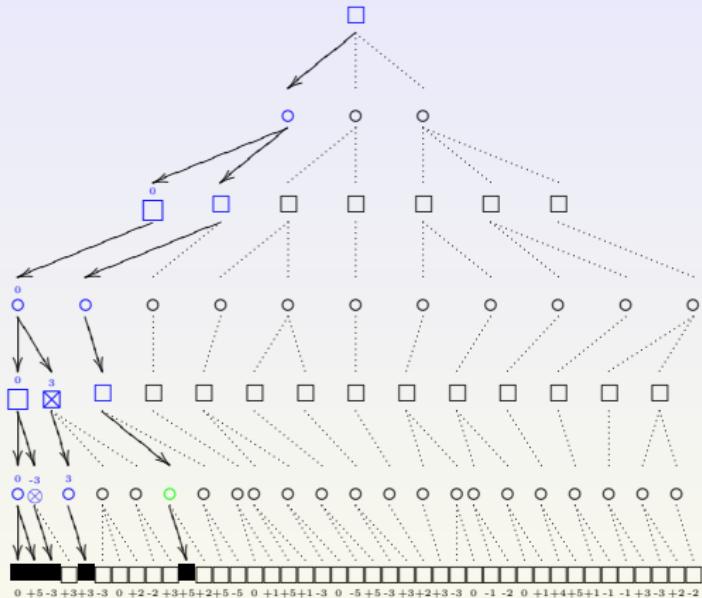
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

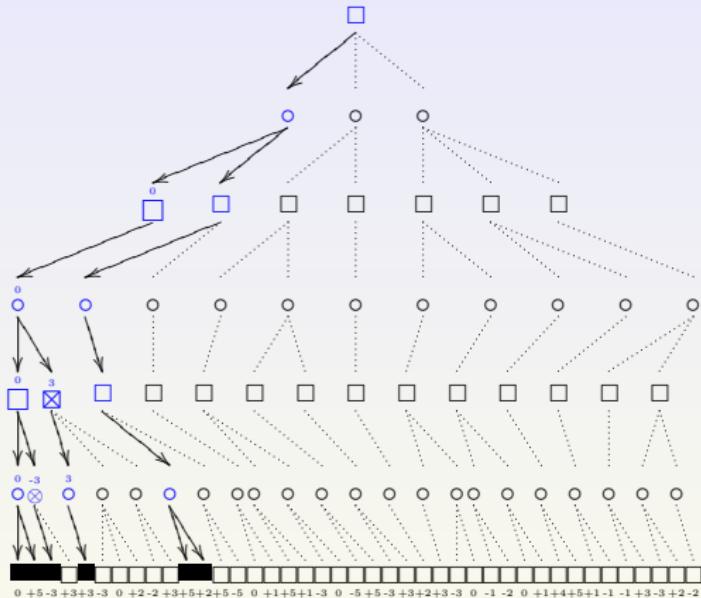
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

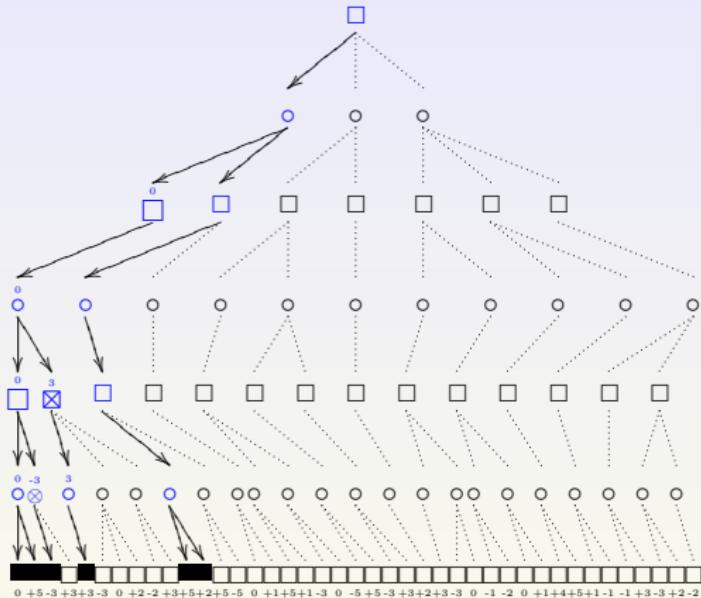
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
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    if v  $\leq \alpha$  then
      return v
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  return v

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$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

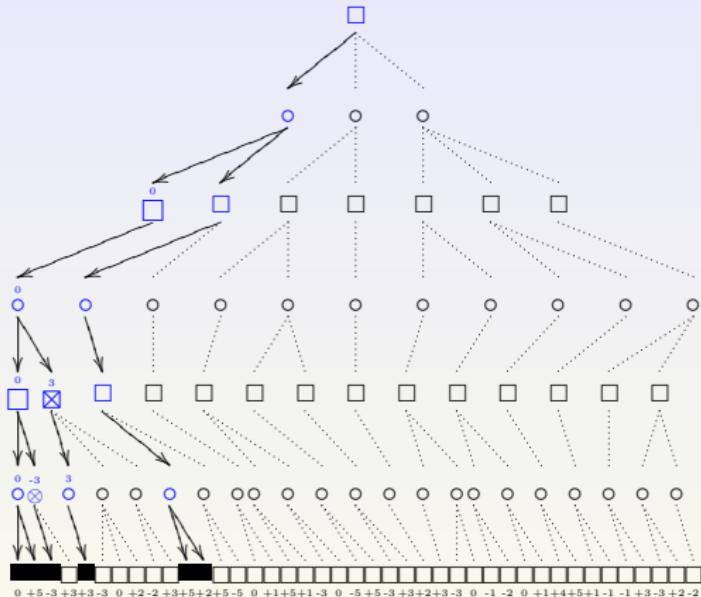
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$ 
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

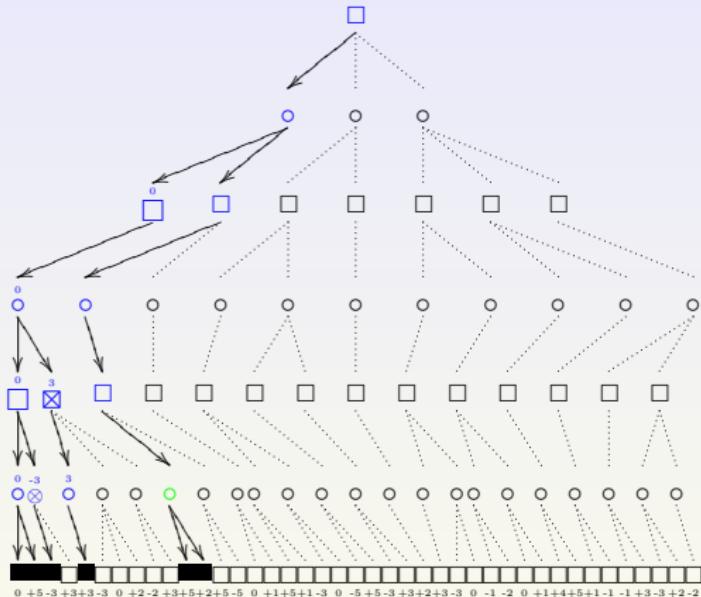
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

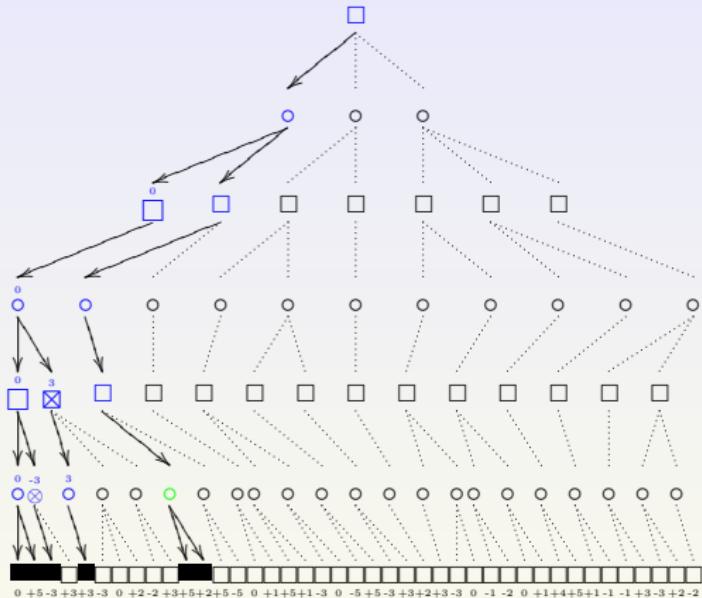


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

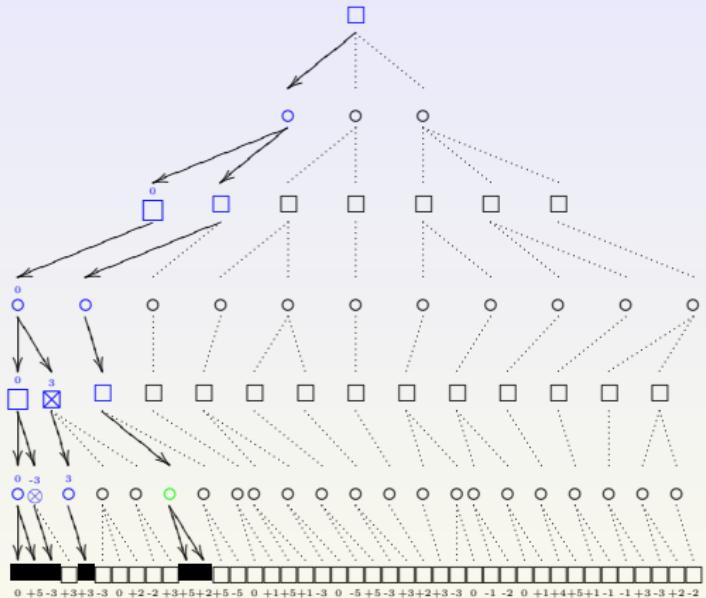
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

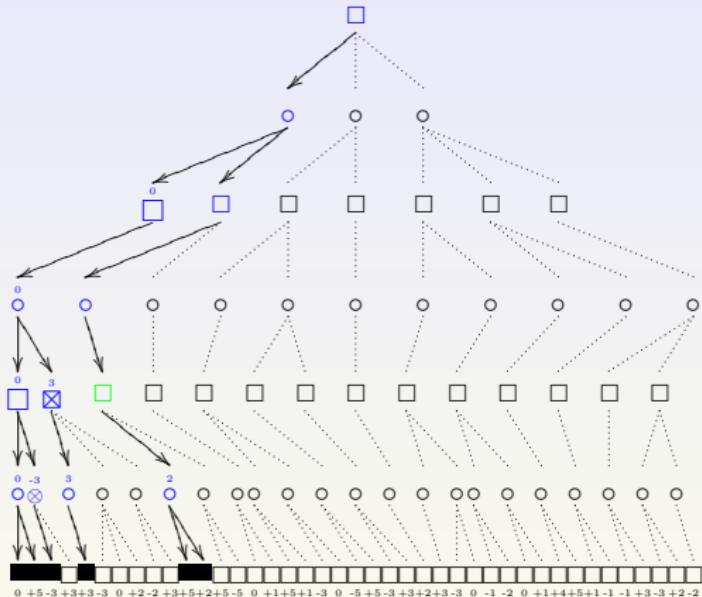
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

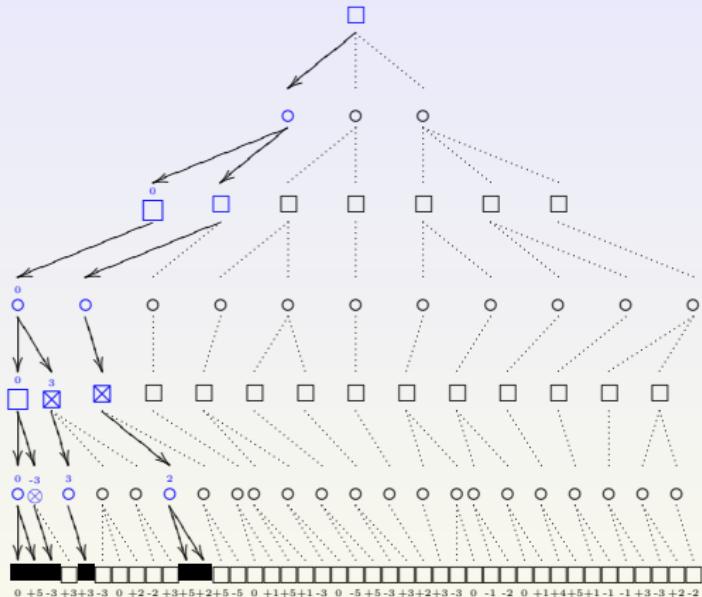


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
  →   if v ≥ β then
    return v
  α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
  if v ≤ α then
    return v
  β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

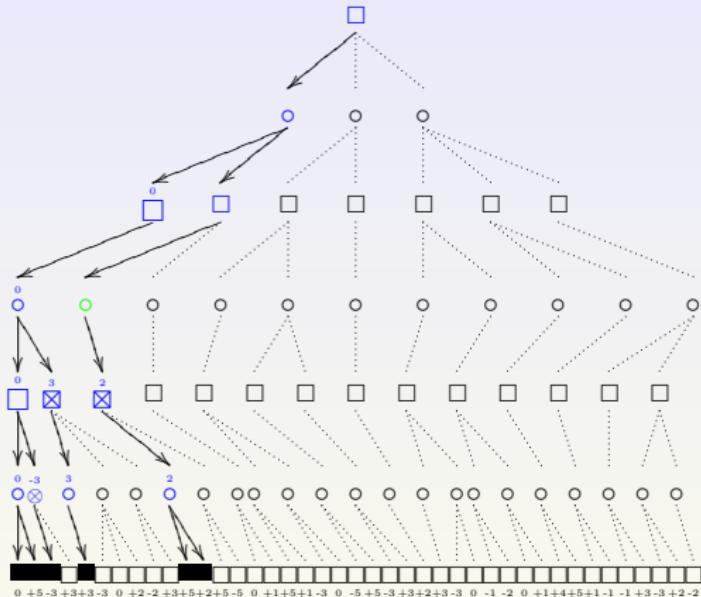
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```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

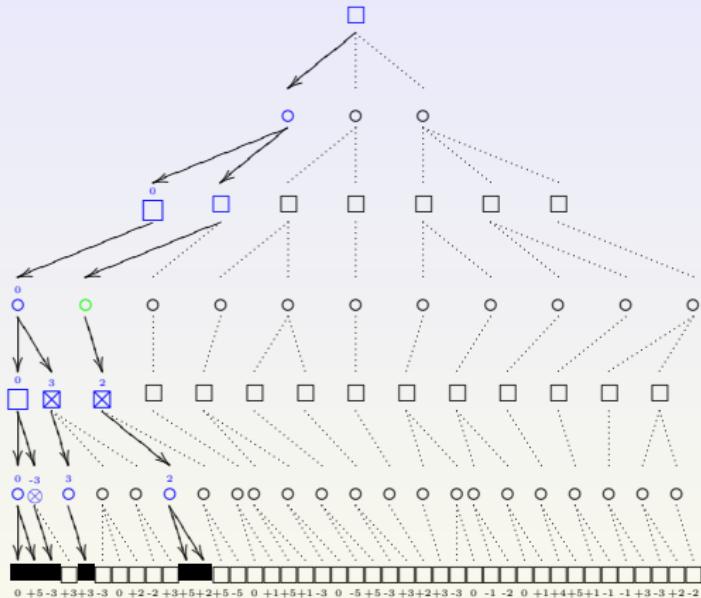


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = 0$$

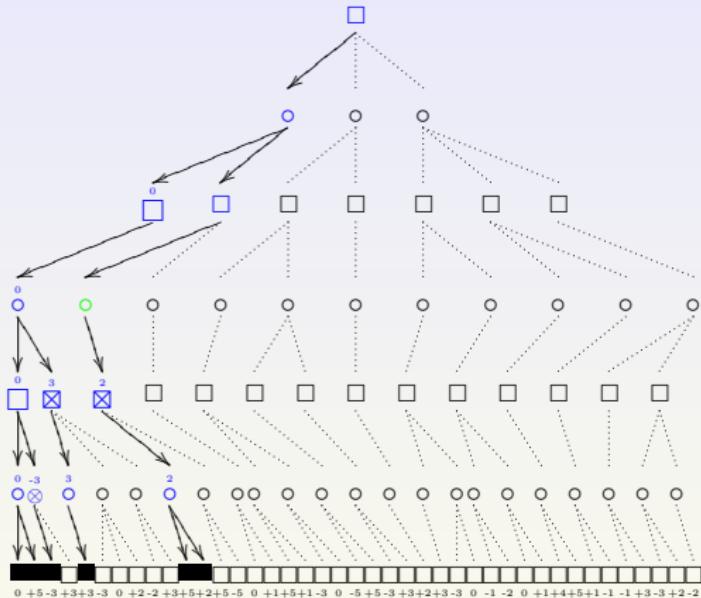


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function ALPHA-BETA-SEARCH(state) returns an action
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  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

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function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    → β ← MIN(β, v)
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$$\alpha = -\infty \quad \beta = 0$$



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function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

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  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

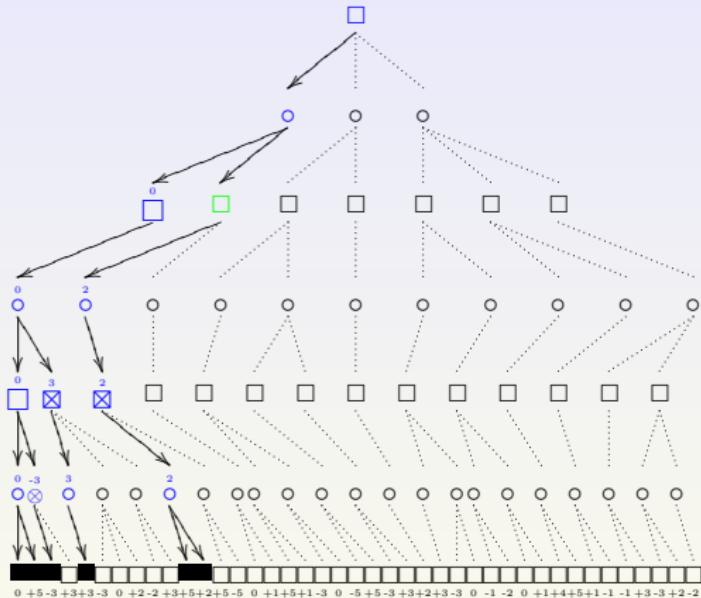
```

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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
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  if v  $\geq \beta$  then
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   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

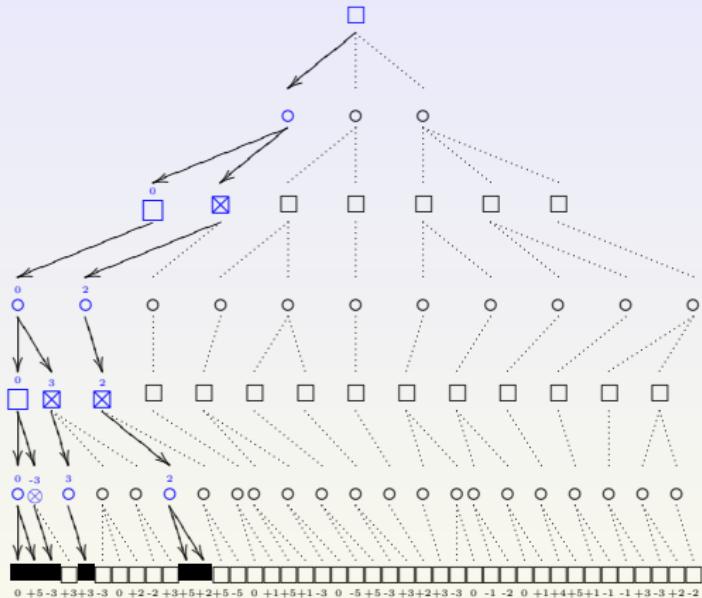
```

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```

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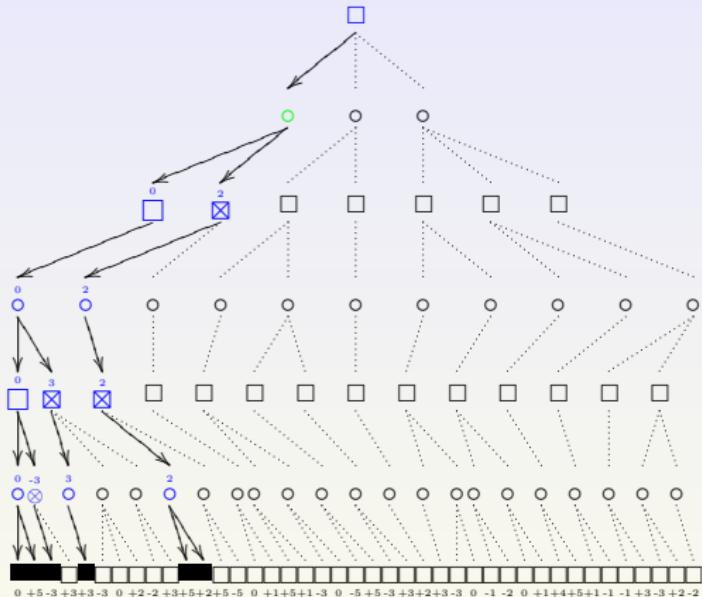


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function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      → return v
    α ← MAX(α, v)
  return v
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```
function MIN-VALUE(state, α, β) returns a utility value
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    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
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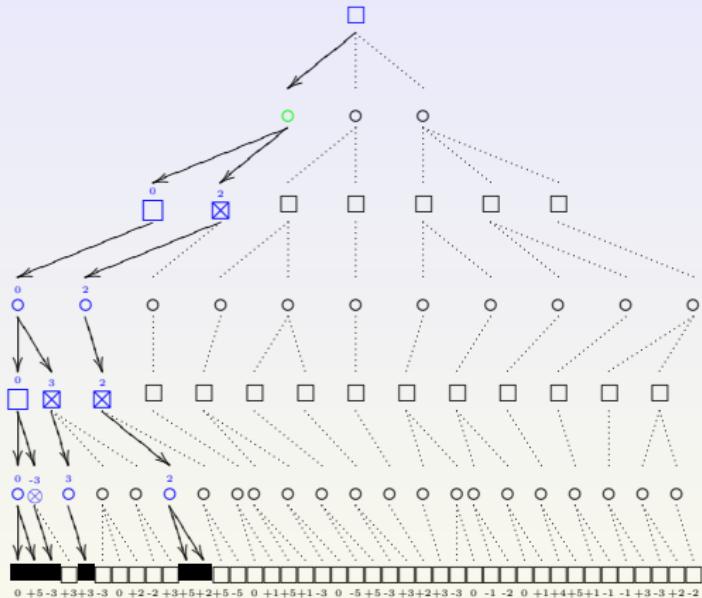


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  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
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```
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      return v
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  return v
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function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

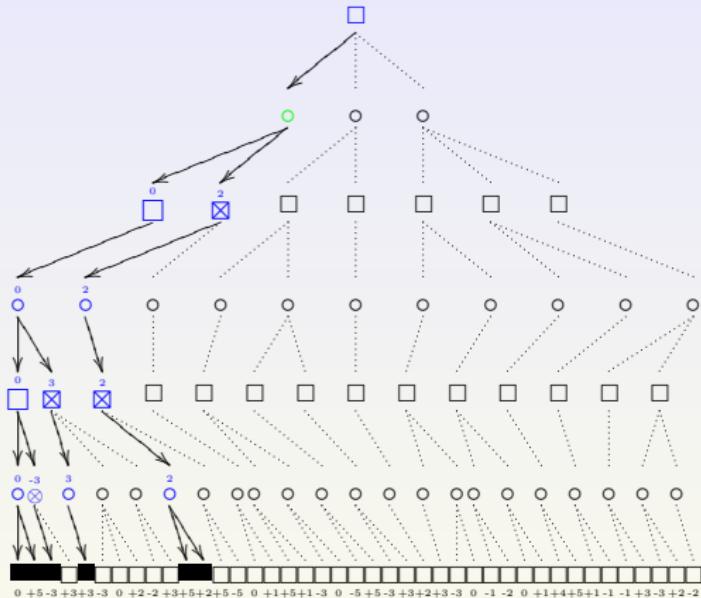
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = -\infty \quad \beta = 0$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

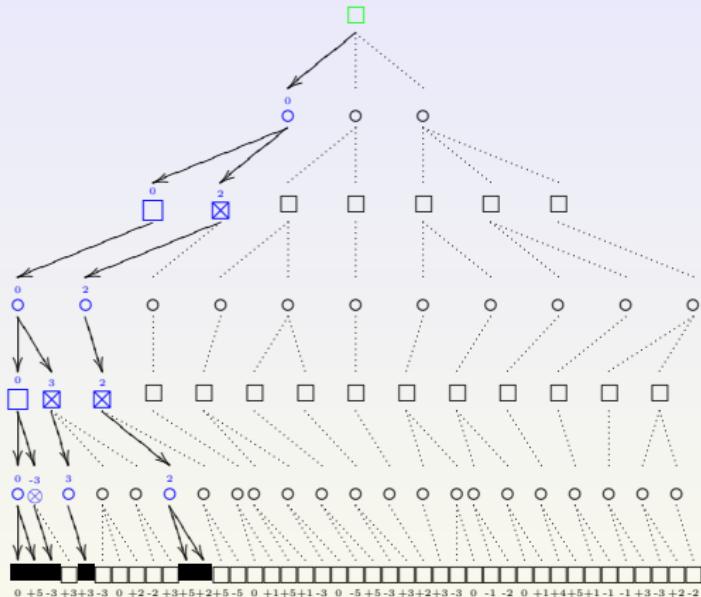
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = 0$$

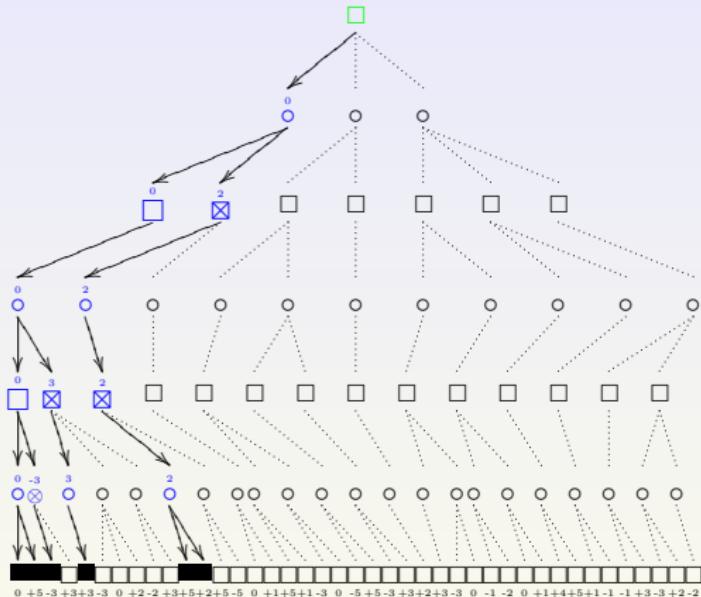


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
  →   if v ≥ β then
    return v
  α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
  if v ≤ α then
    return v
  β ← MIN(β, v)
  return v
```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  return  $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v
  →

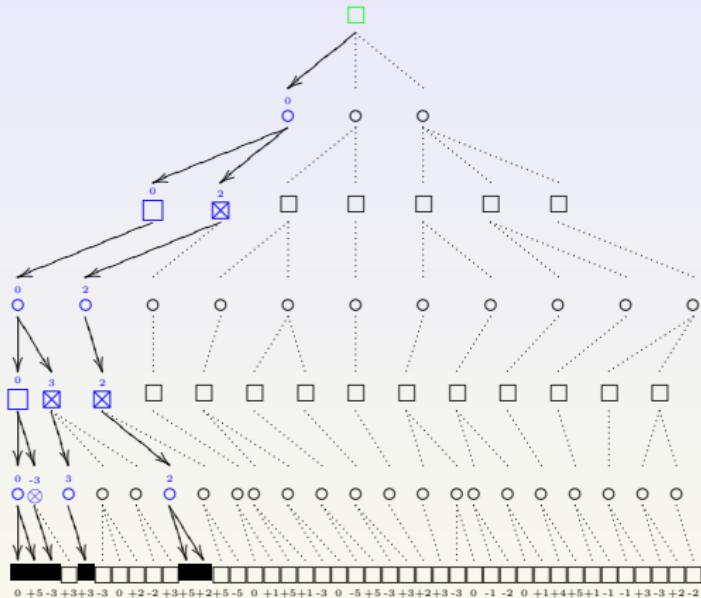
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = -\infty \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

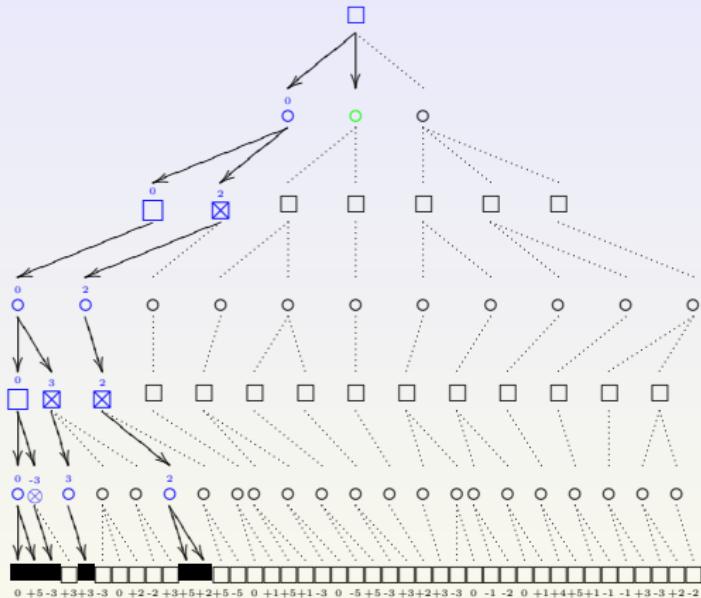
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

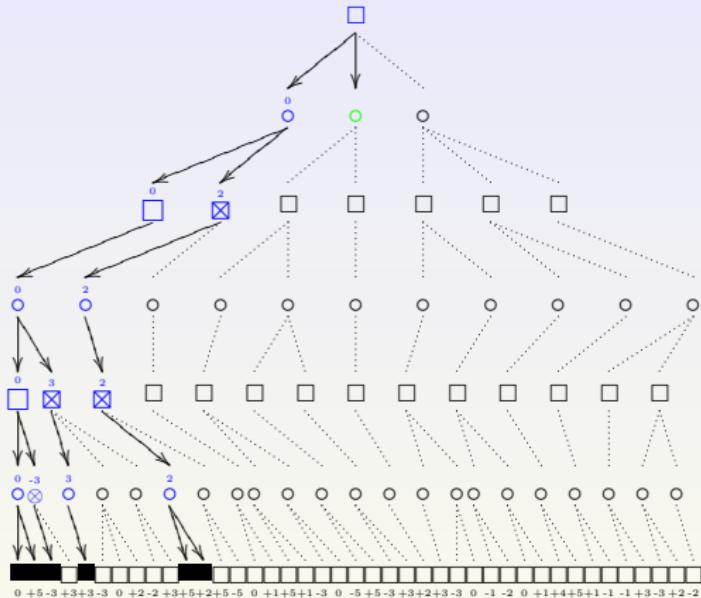
→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value

```

  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

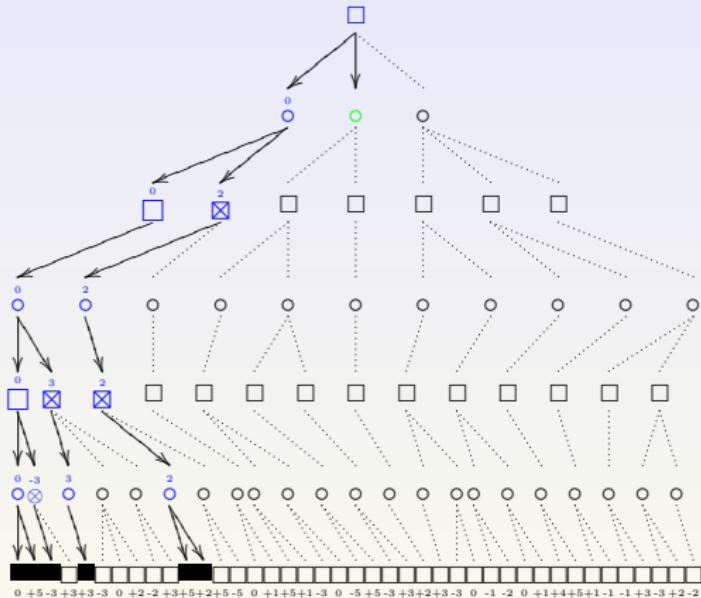
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

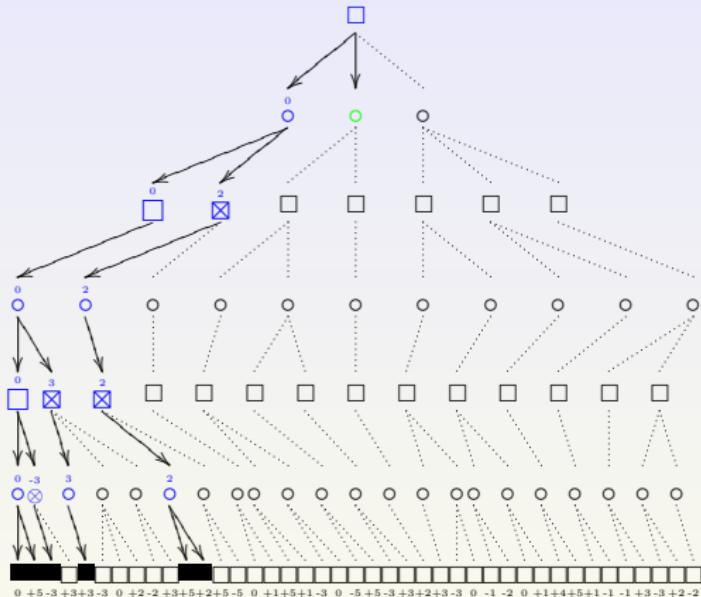
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

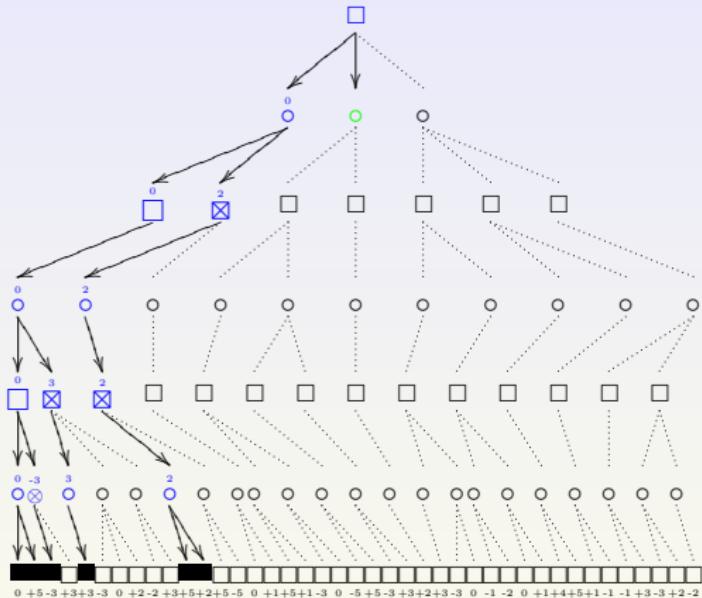
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

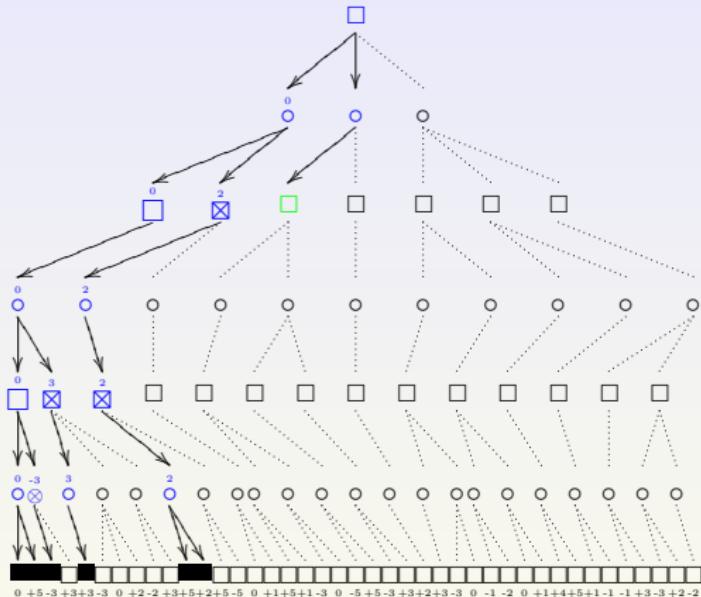
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v
  
```

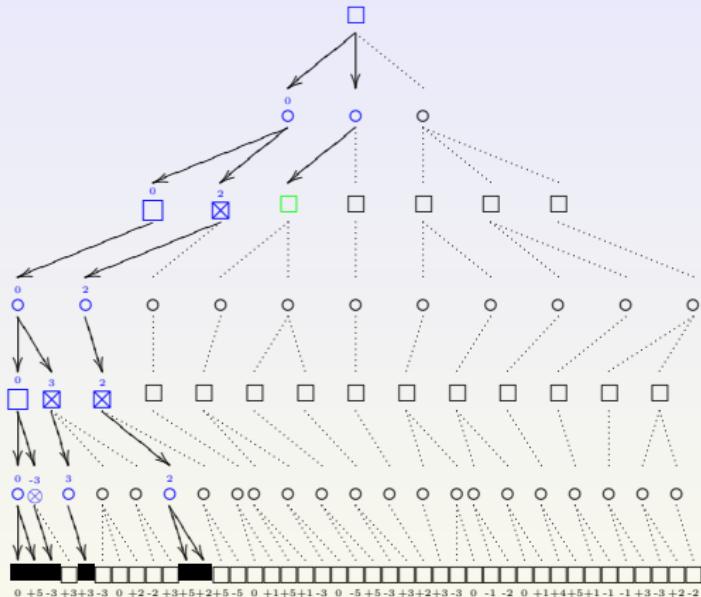
```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v
  
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v
  
```

$$\alpha = 0 \quad \beta = +\infty$$

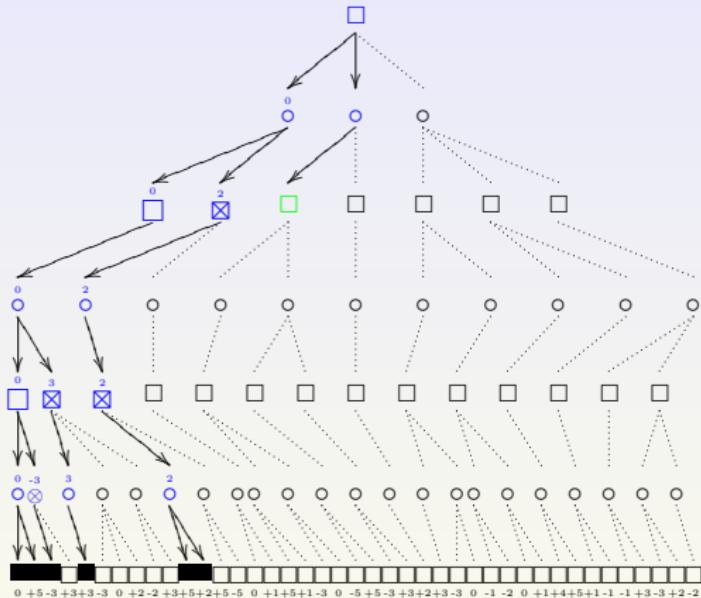


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

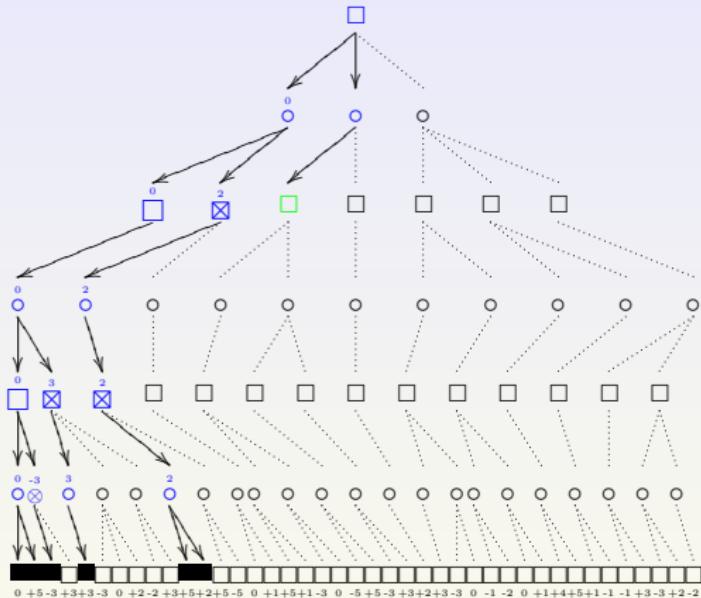


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

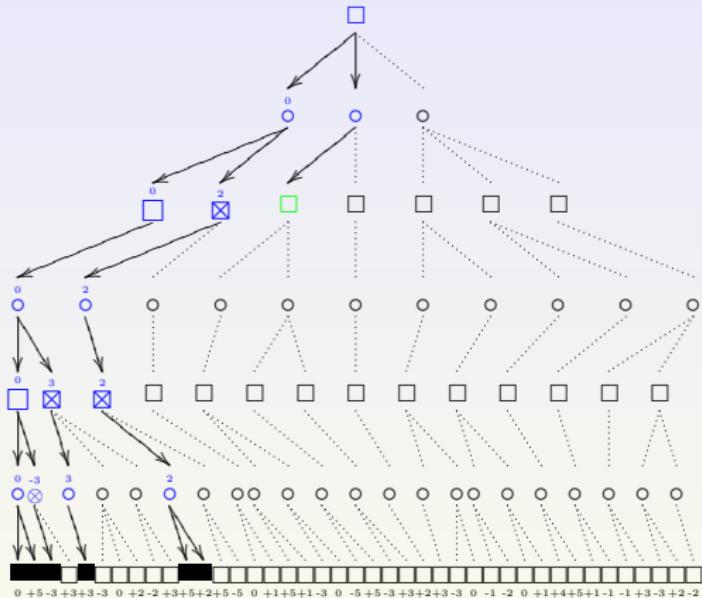


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

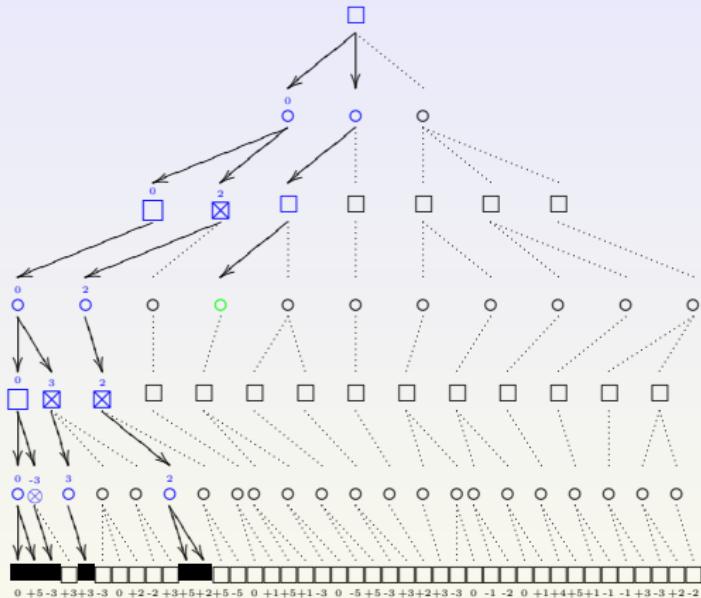


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    → v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

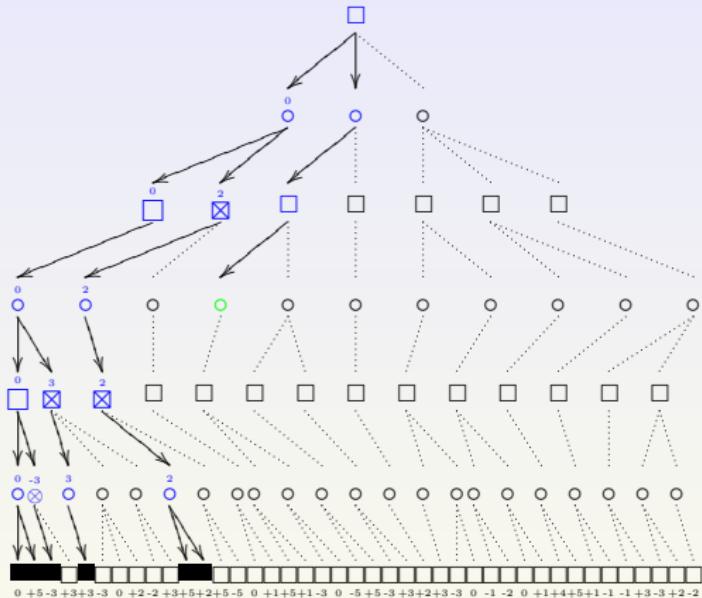
→

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

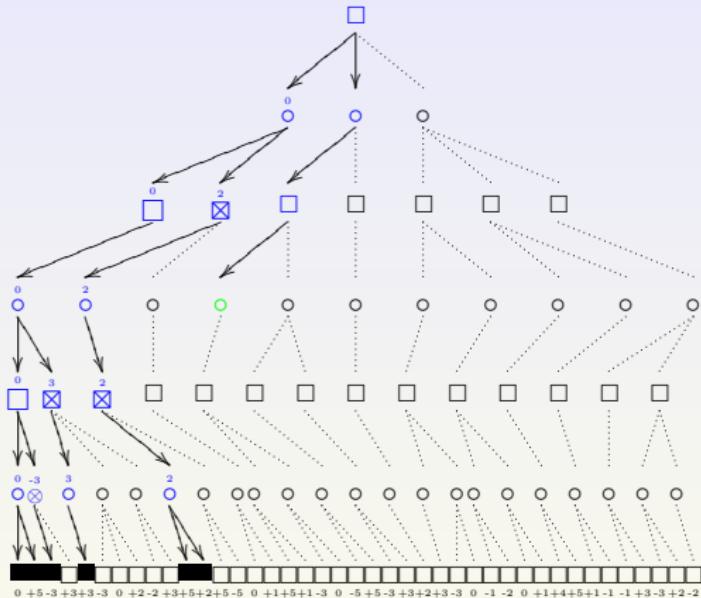


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

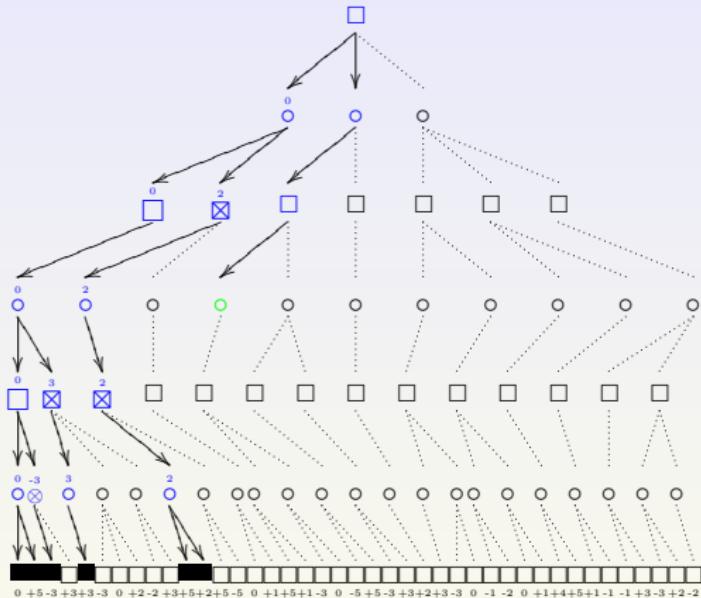
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

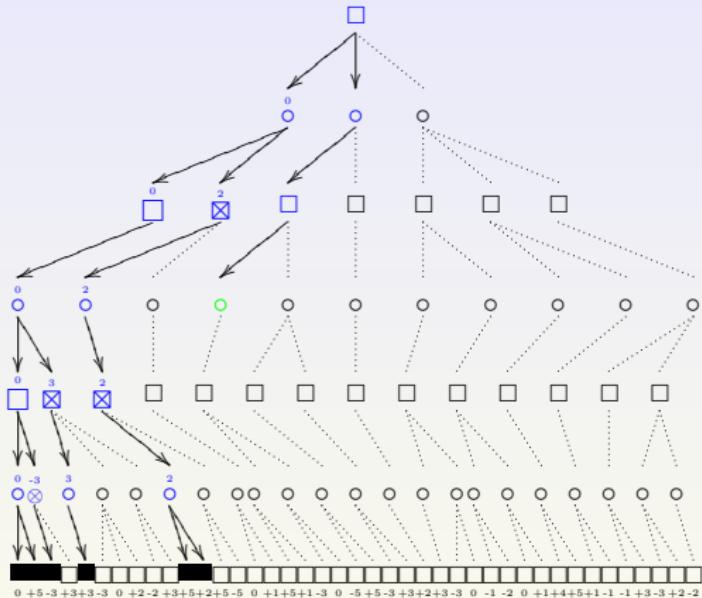
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
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    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



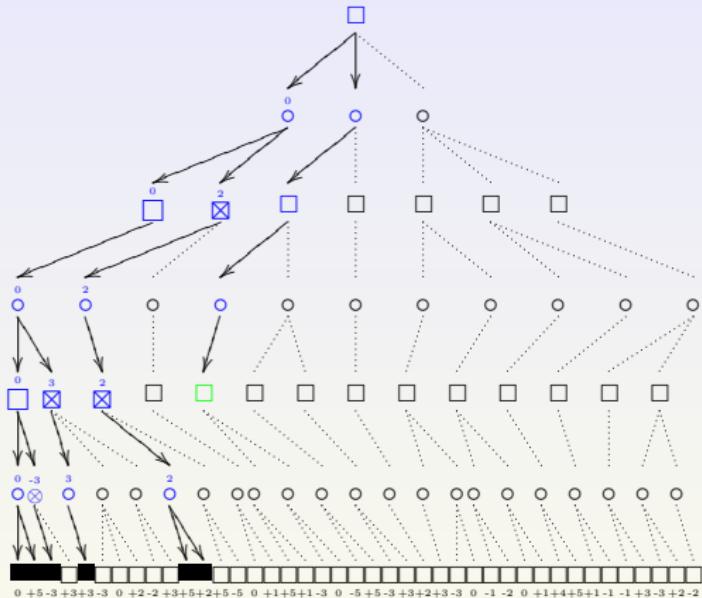


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

```

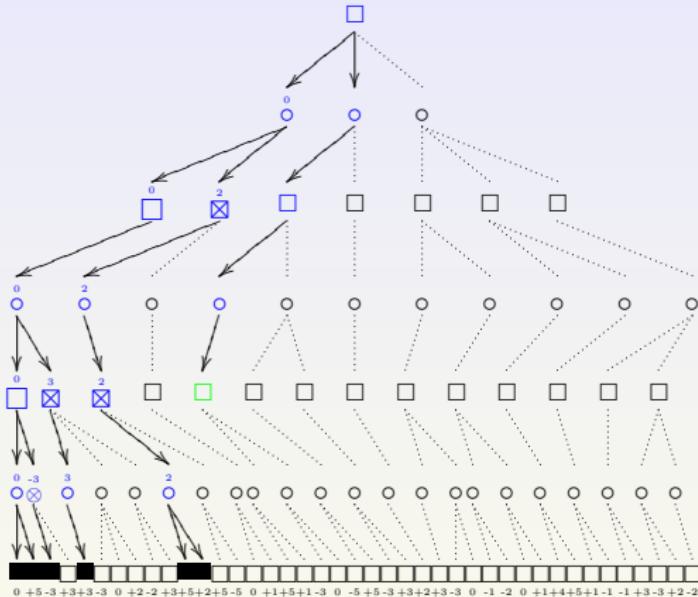
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

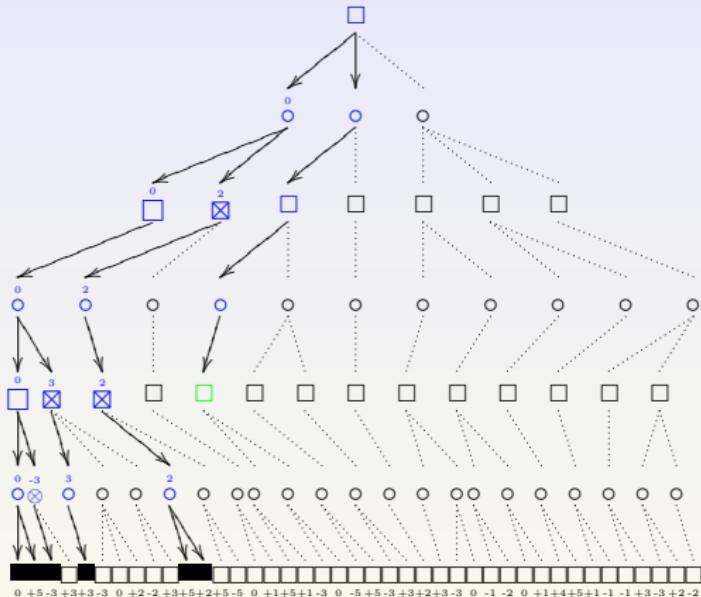
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\text{v}, \text{MAX-VALUE}(\iota, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, \iota)$ 
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

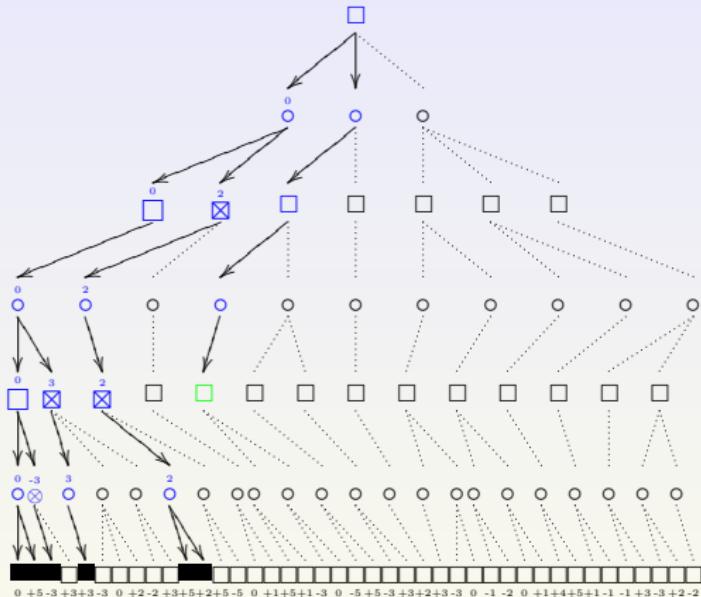
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v$ 
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

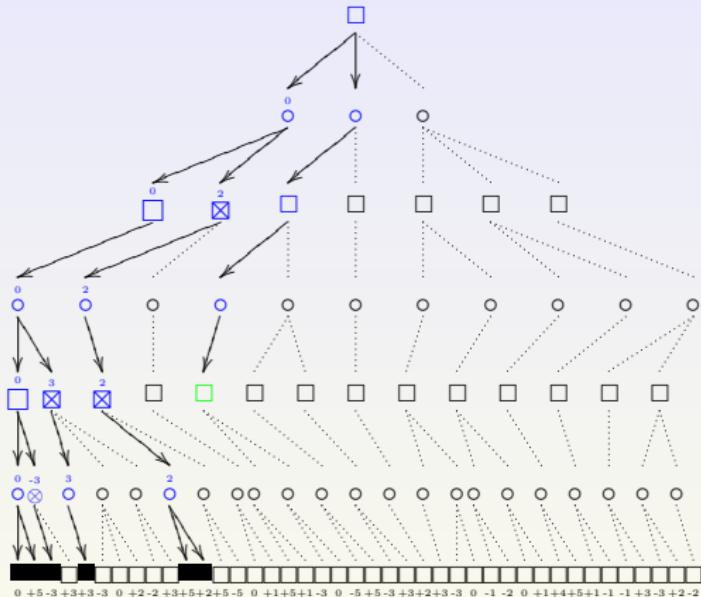


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

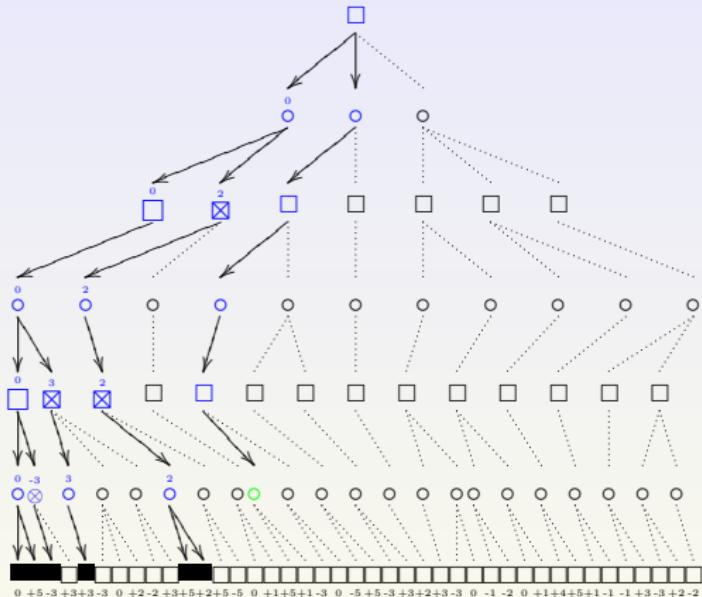


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    → v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

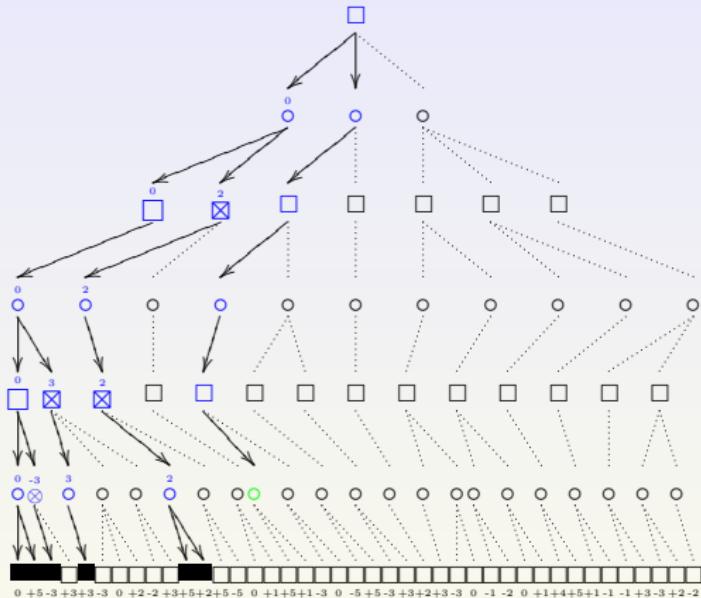


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
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  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



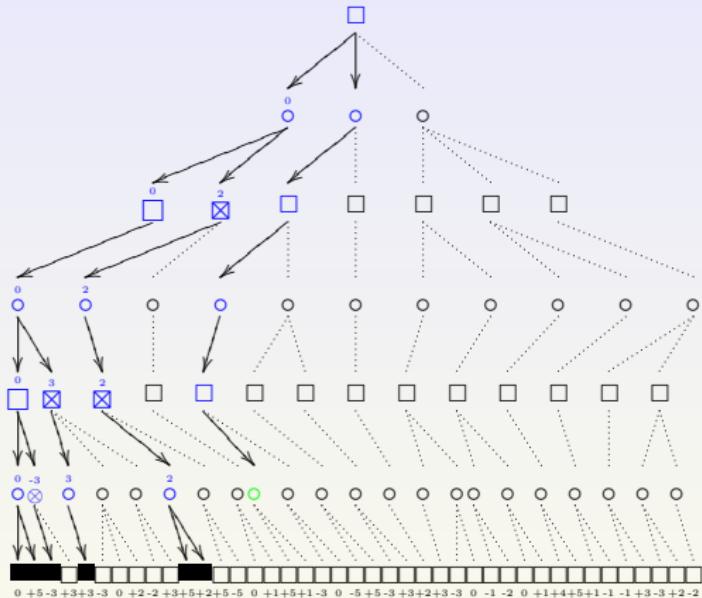
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```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

→

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

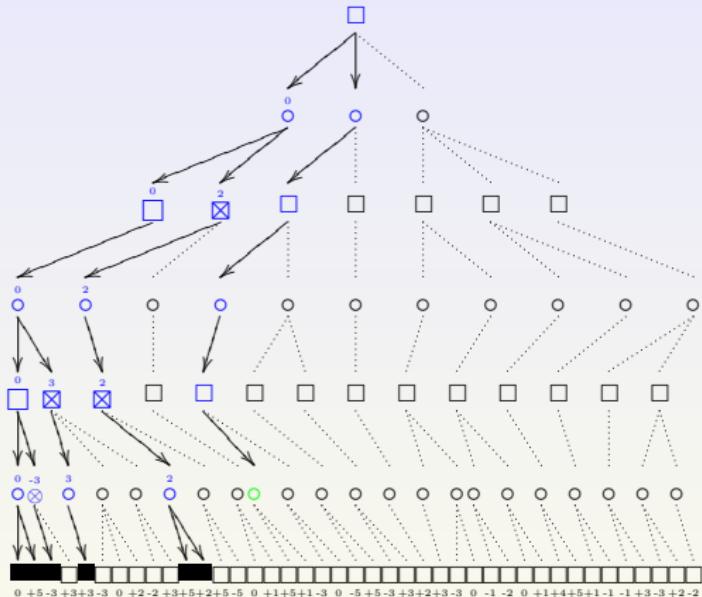


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  v ← -∞
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    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
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    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

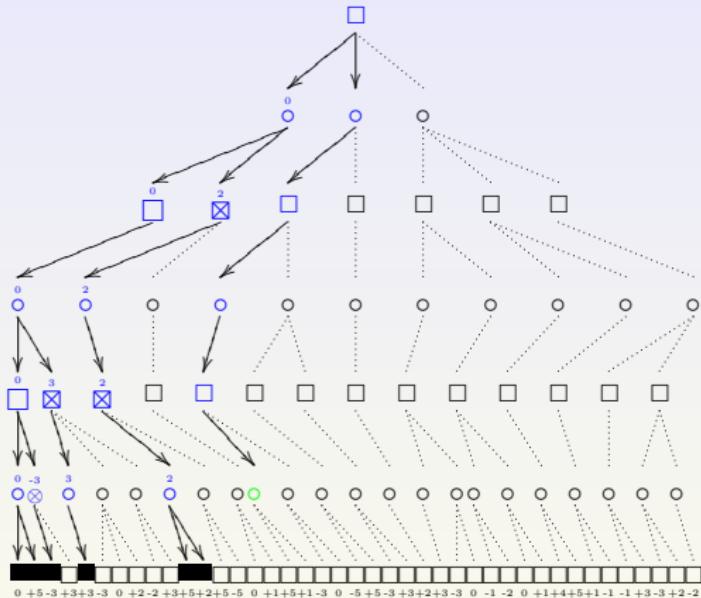


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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

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  v ← +∞
  → for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

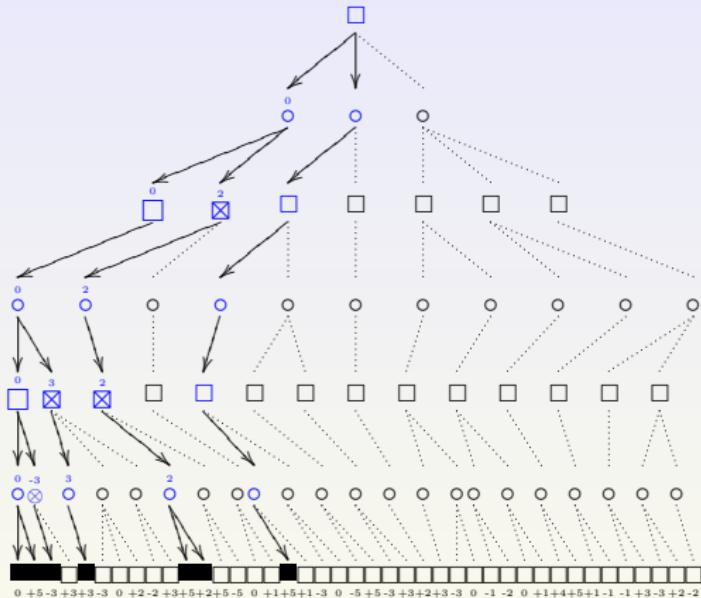


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    if v ≥ β then
      return v
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  return v
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

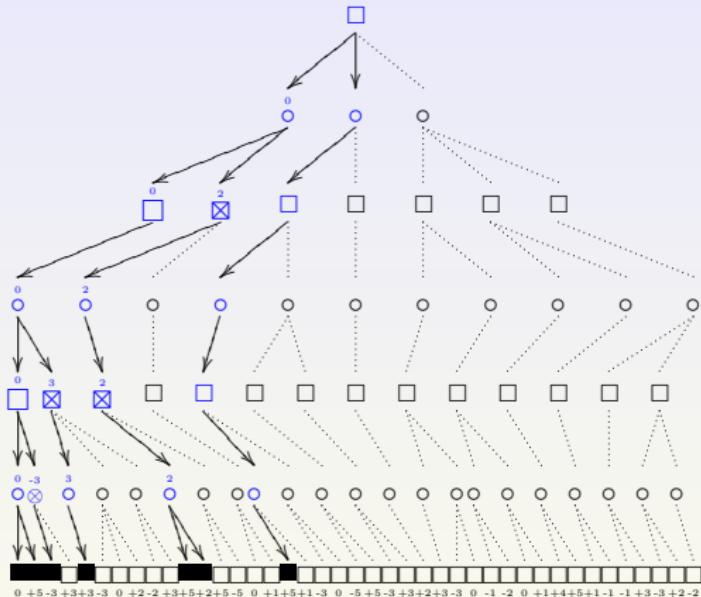
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

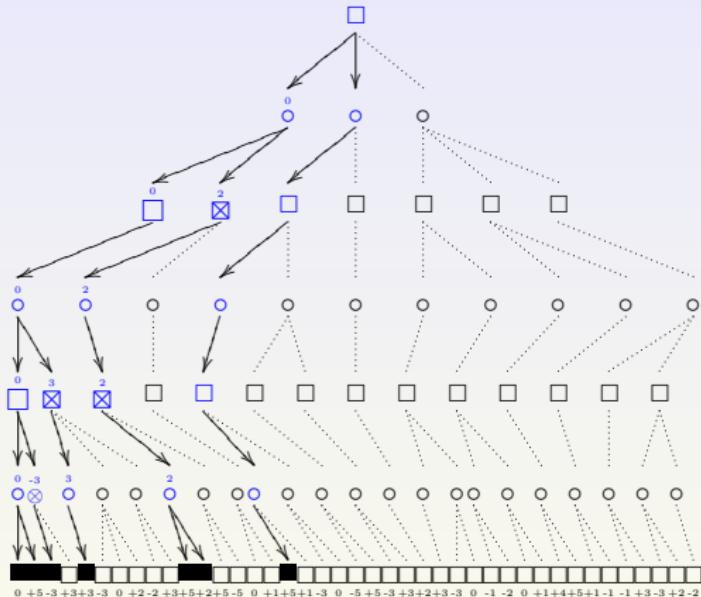


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function ALPHA-BETA-SEARCH(state) returns an action
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```
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  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

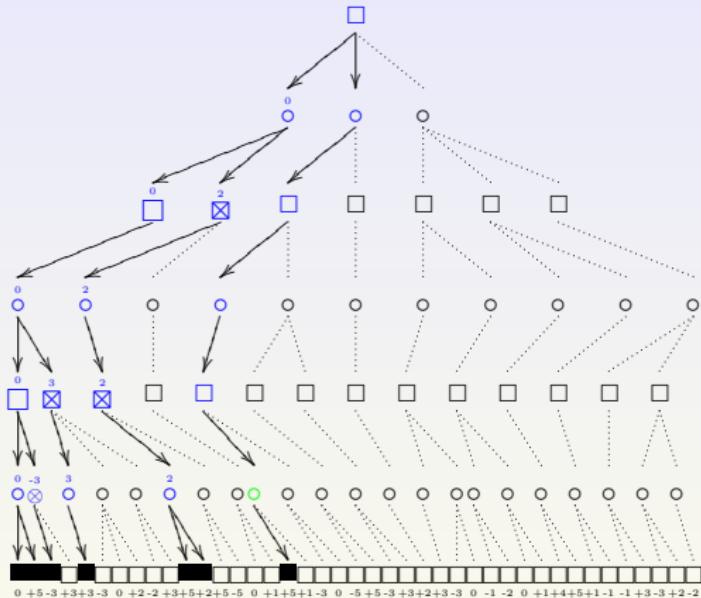


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  →
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

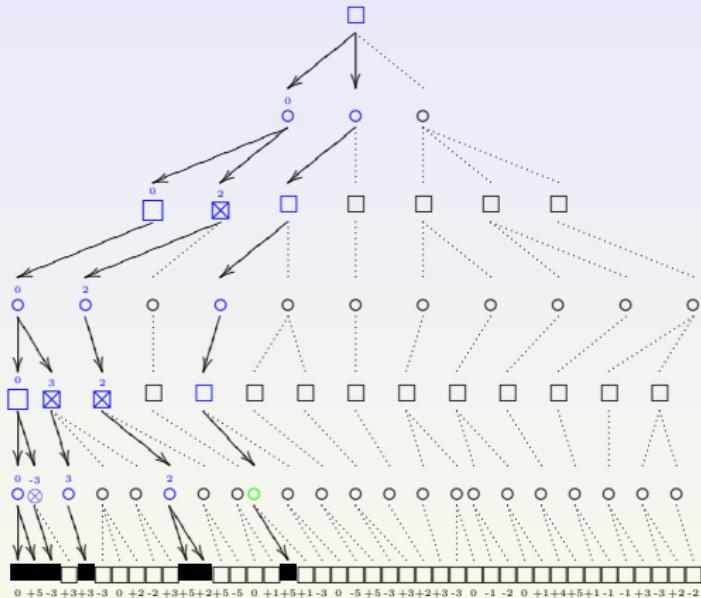


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

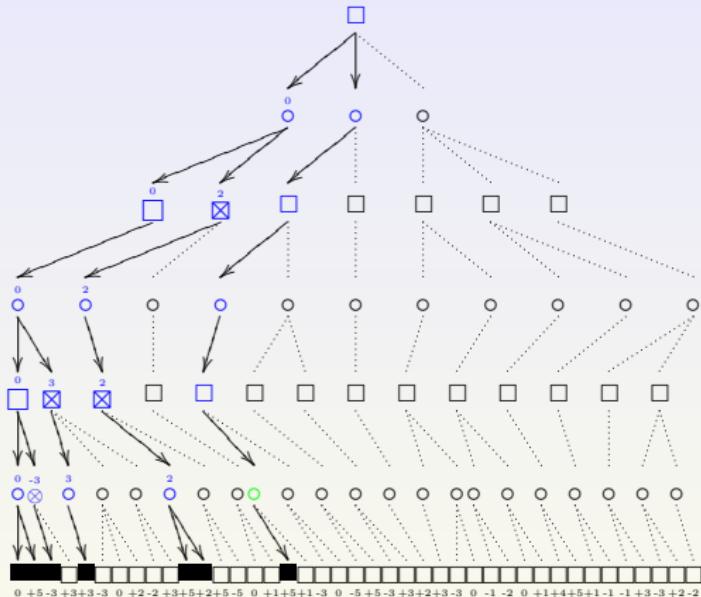


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    → β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

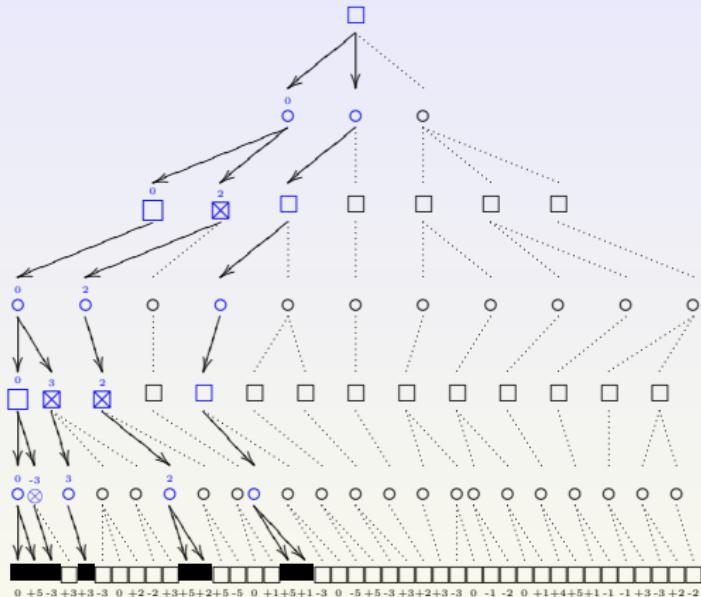


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    → v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 5$$

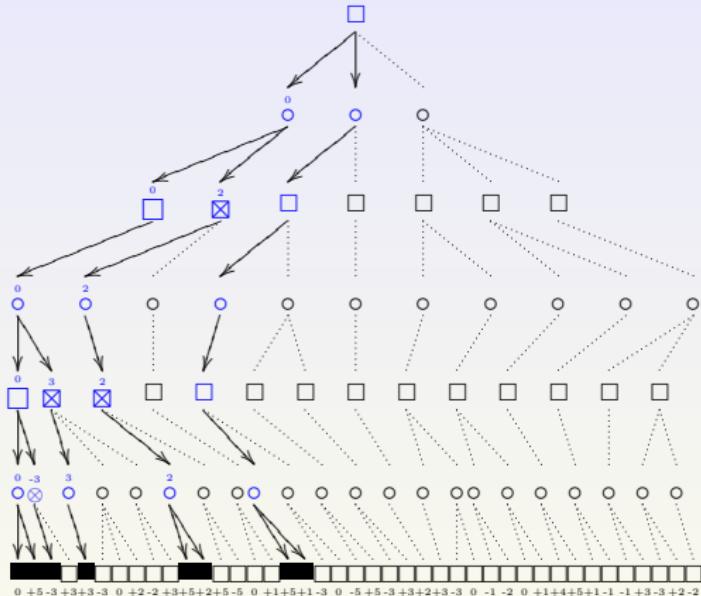


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = 5$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

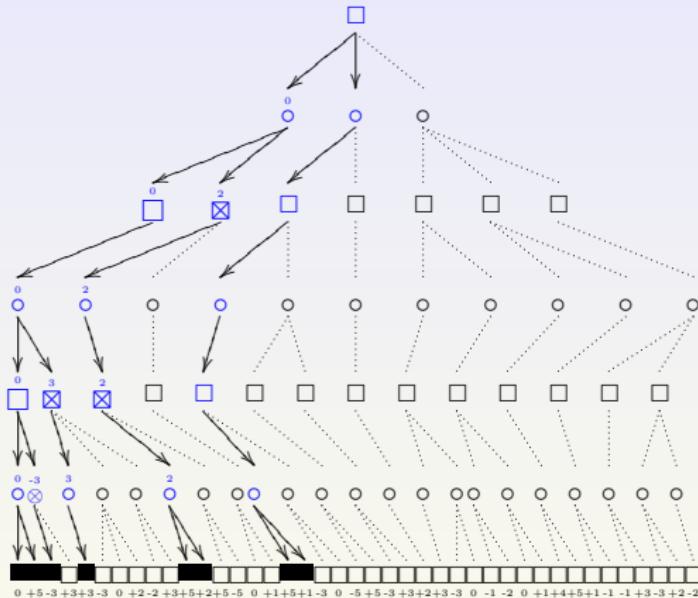
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 5$$

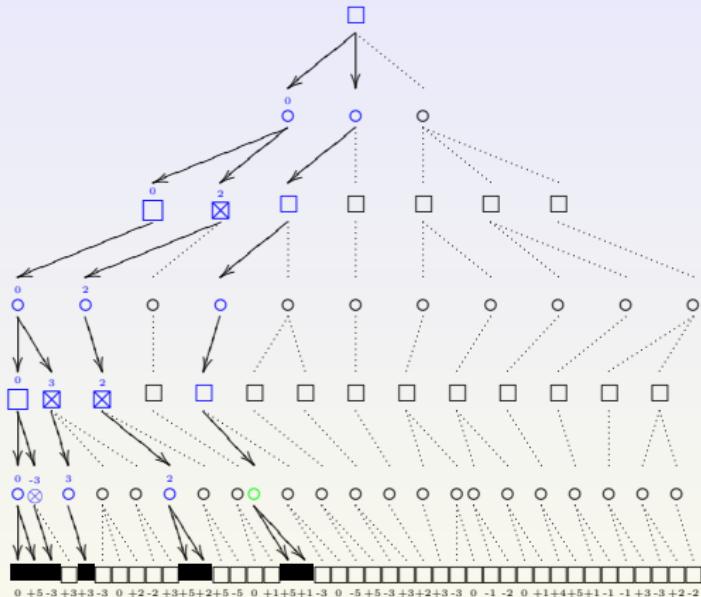


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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  →
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 5$$



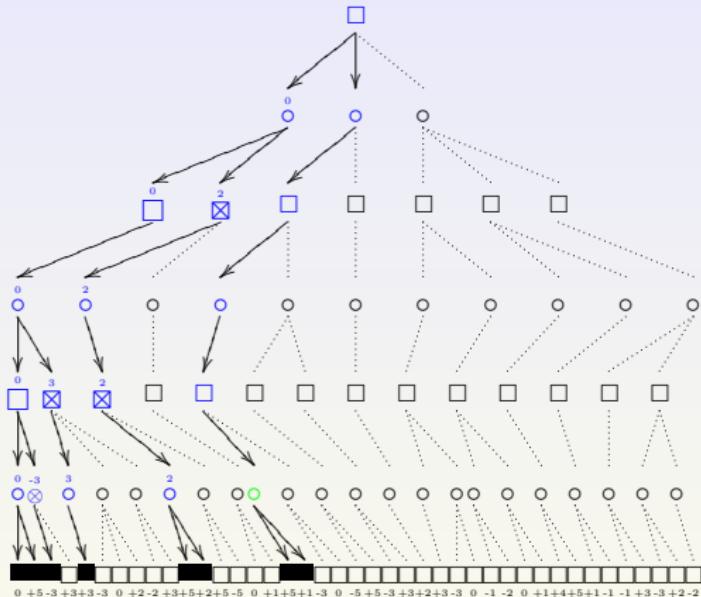
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 5$$



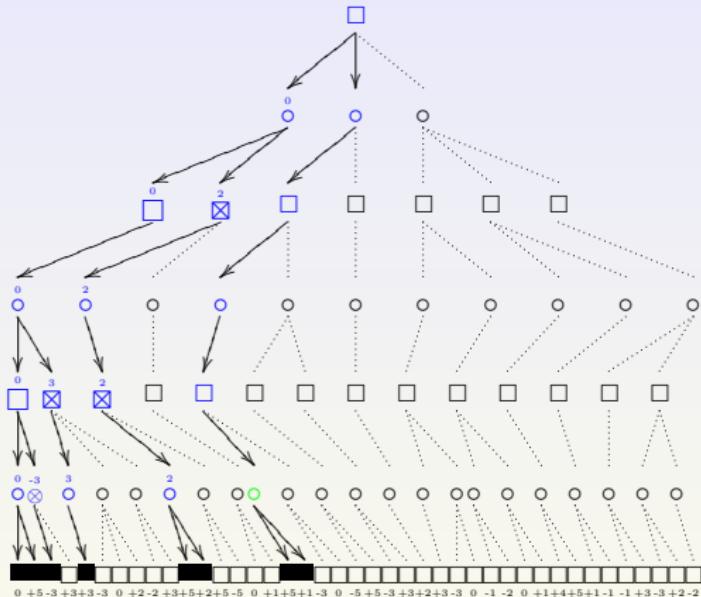


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 5$$

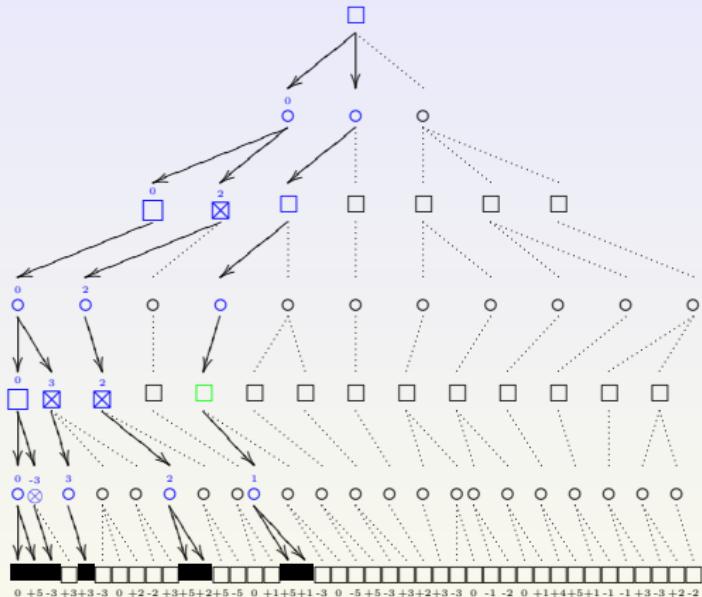


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  → return v
```

$$\alpha = 0 \quad \beta = 1$$

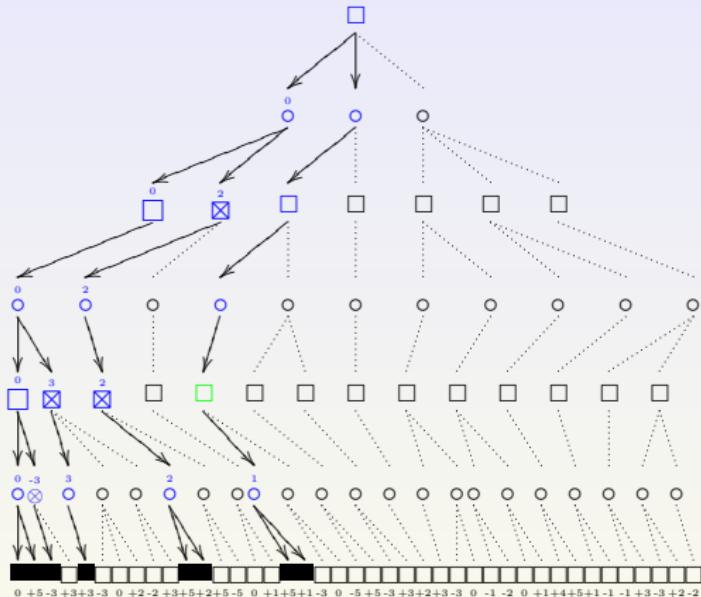


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
        return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

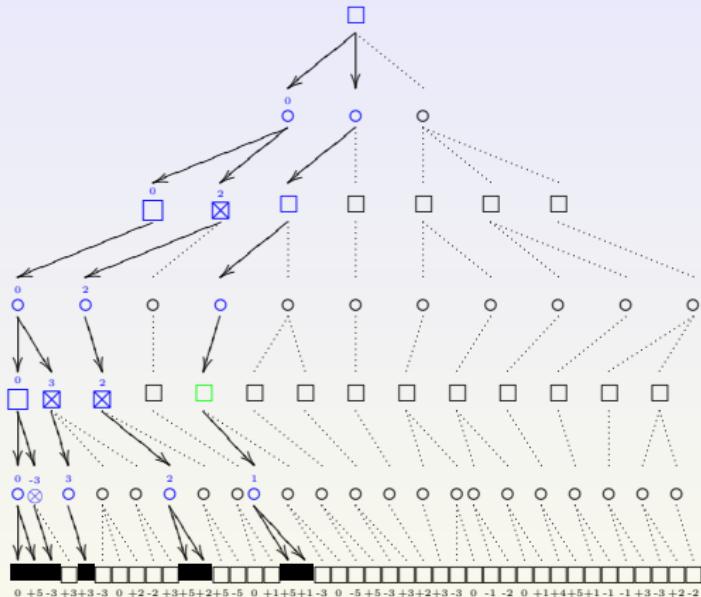


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
  →  $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

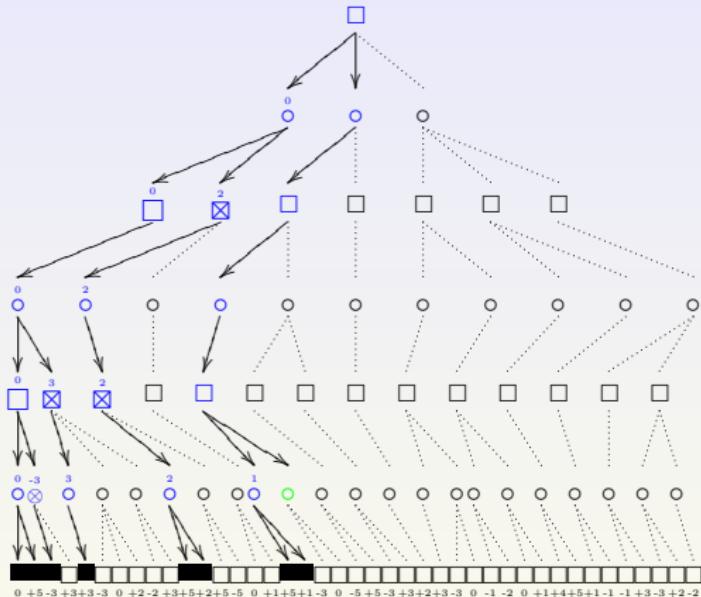


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    → v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

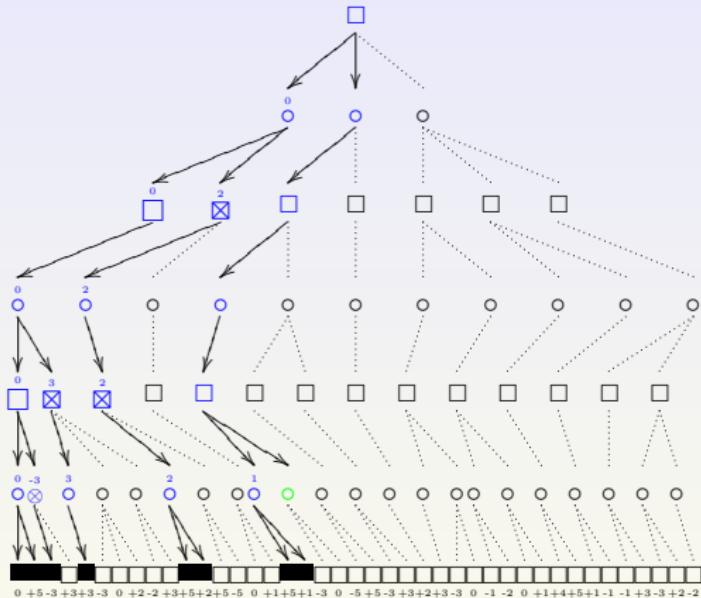


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
→ function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

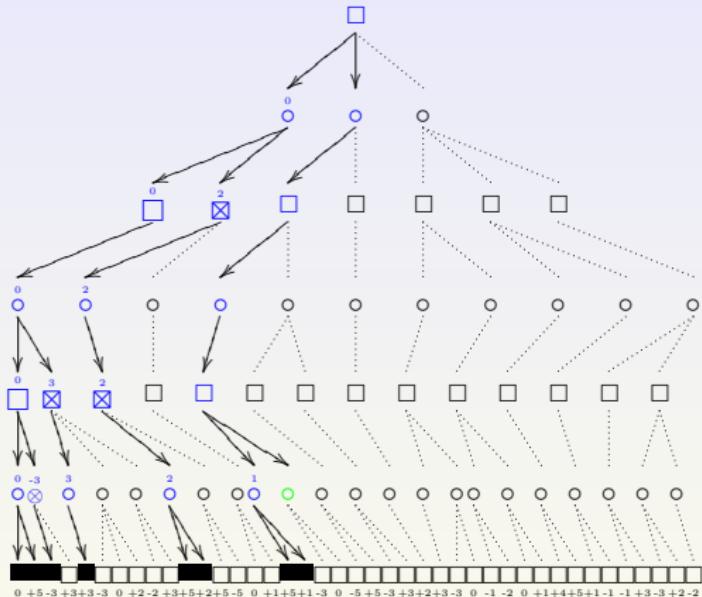


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function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
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  v ← -∞
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  return v
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

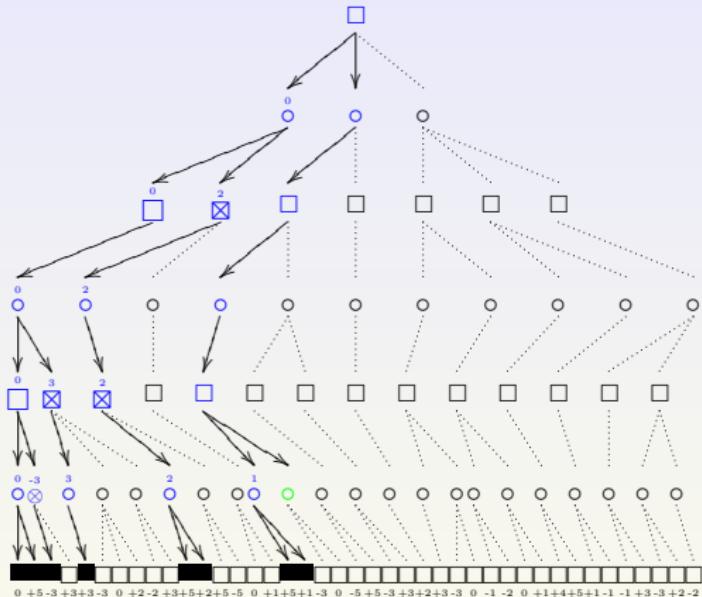


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  return the action in SUCCESSORS(state) with value v
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      return v
    α ← MAX(α, v)
  return v
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

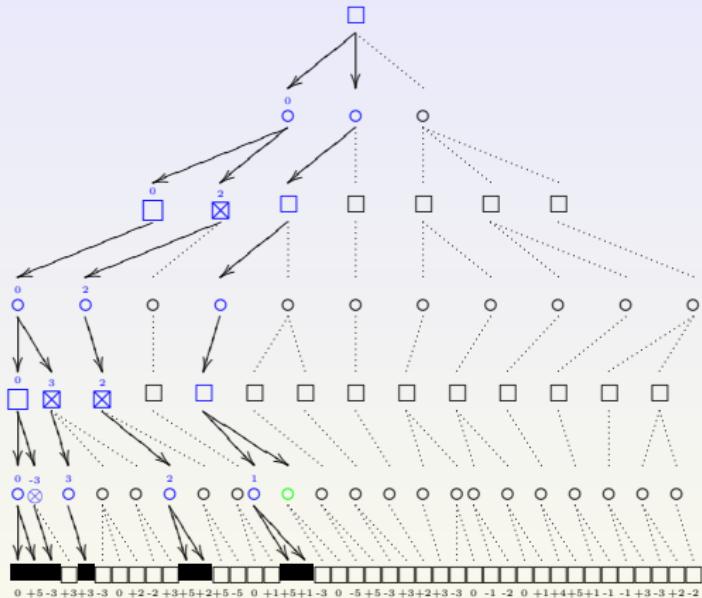


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  return the action in SUCCESSORS(state) with value v
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  → for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

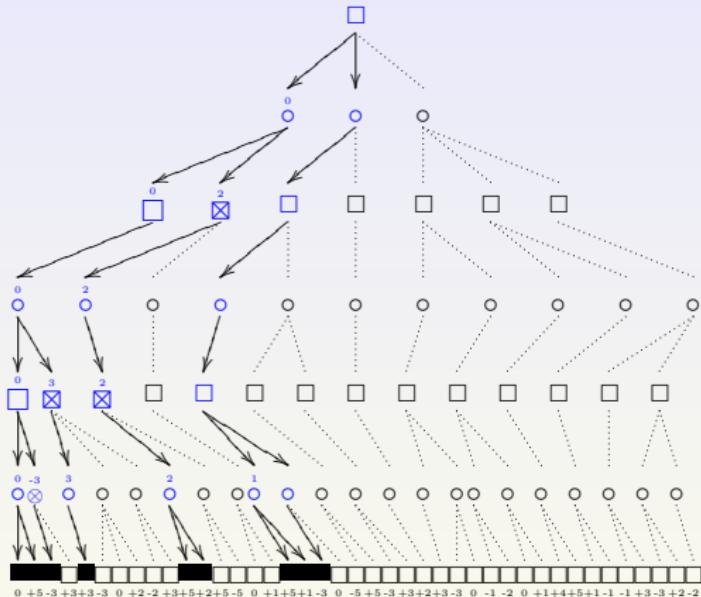


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
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function MIN-VALUE(state, α, β) returns a utility value
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

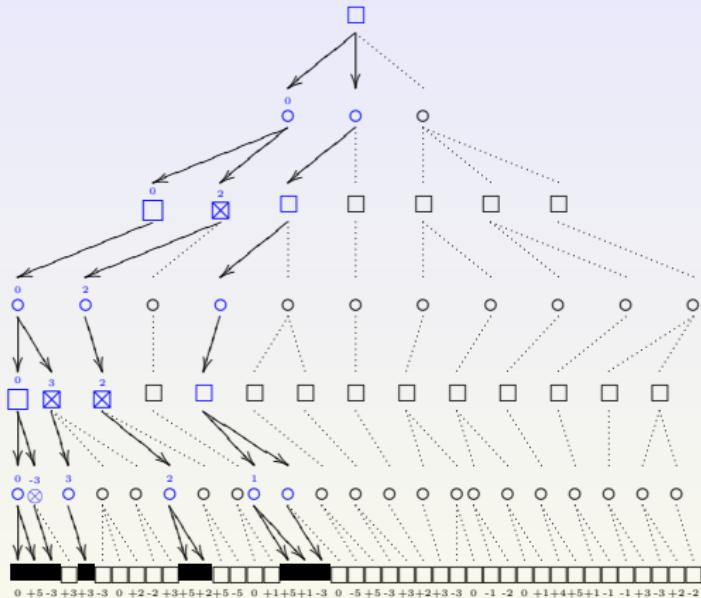


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

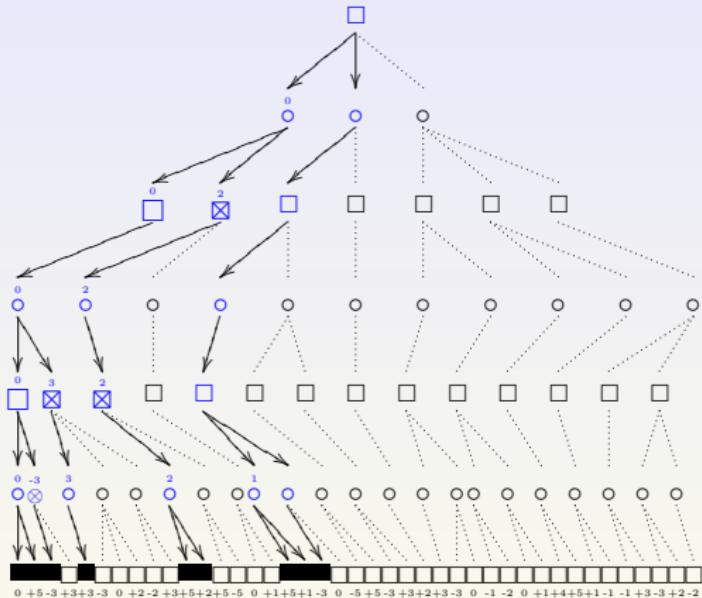


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

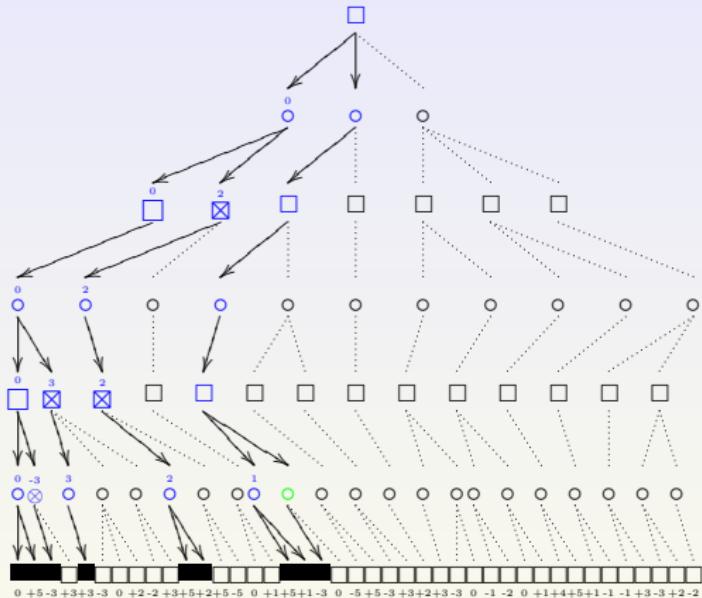


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  →
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

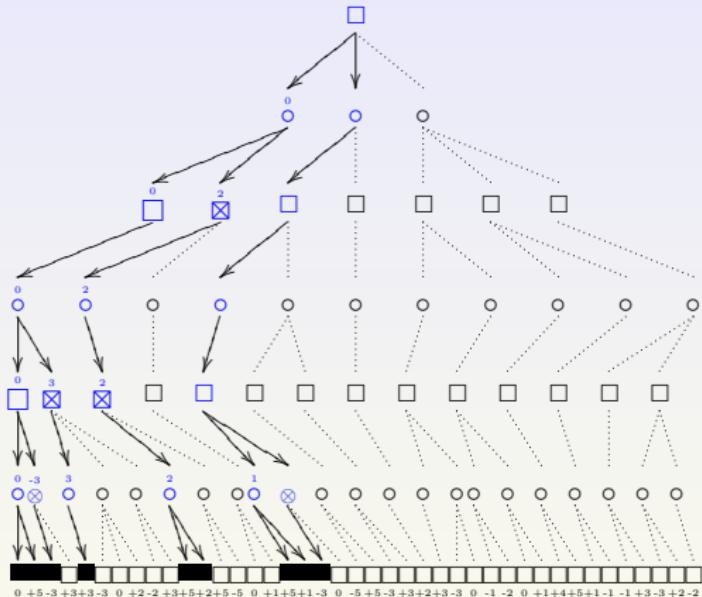


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

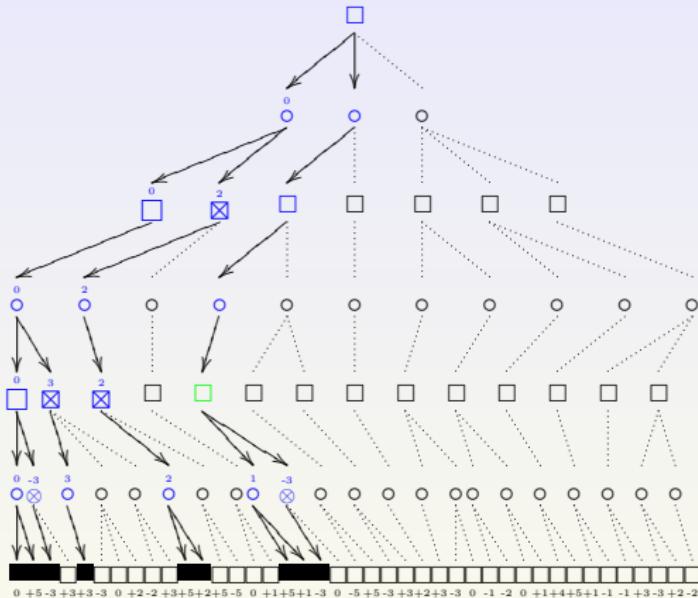
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

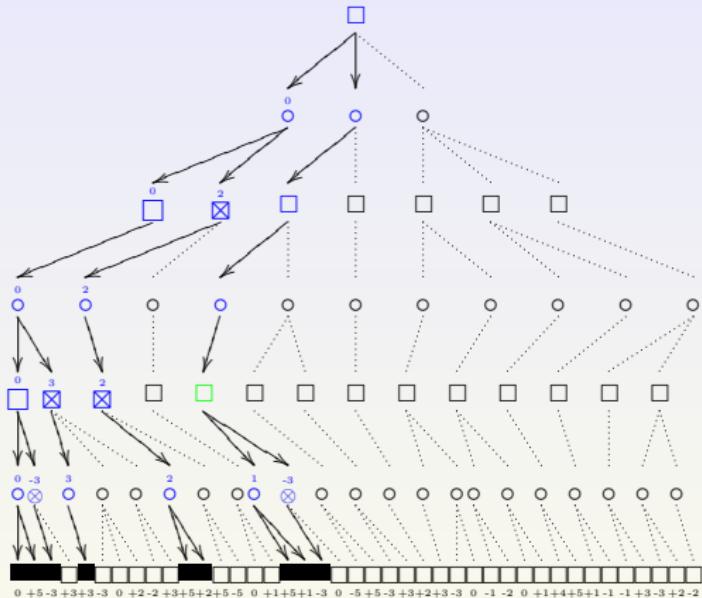
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

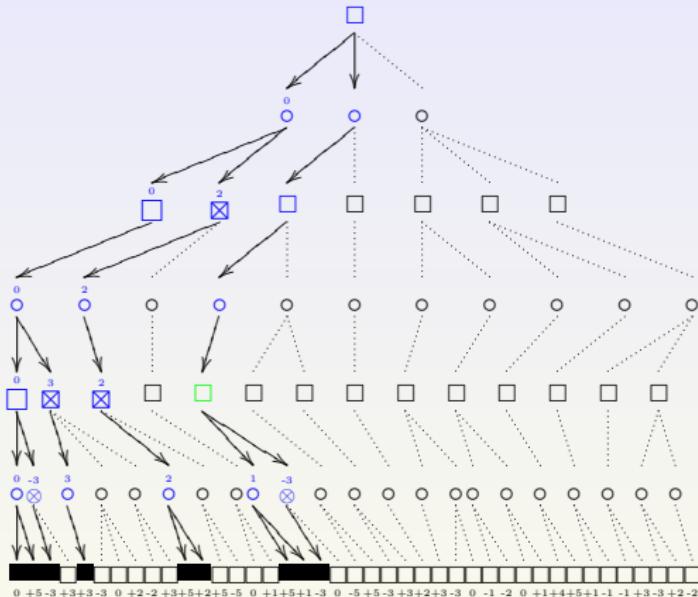


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
  →  $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

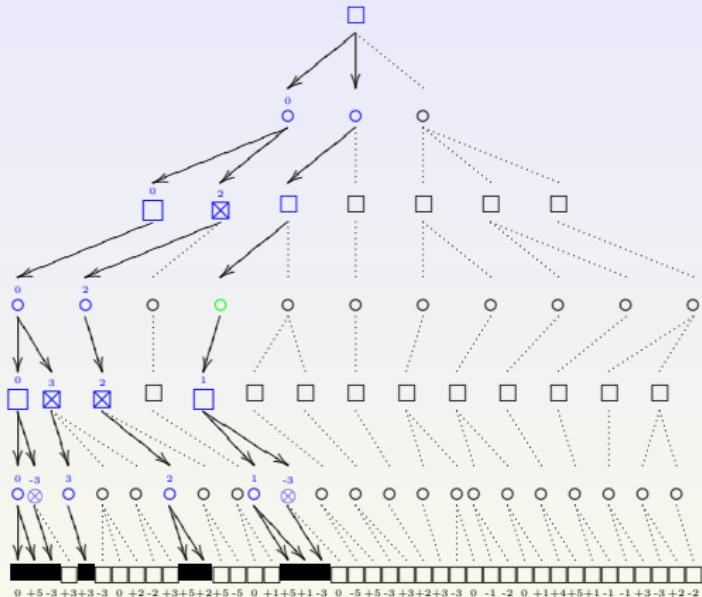
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

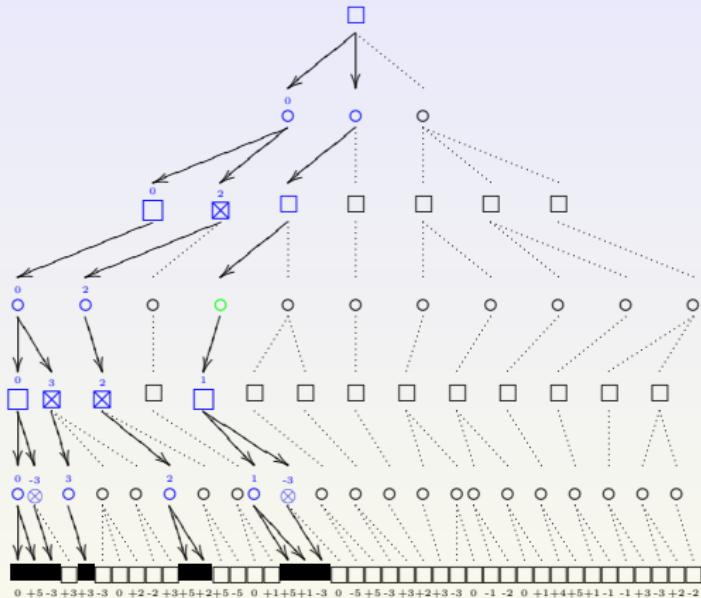


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

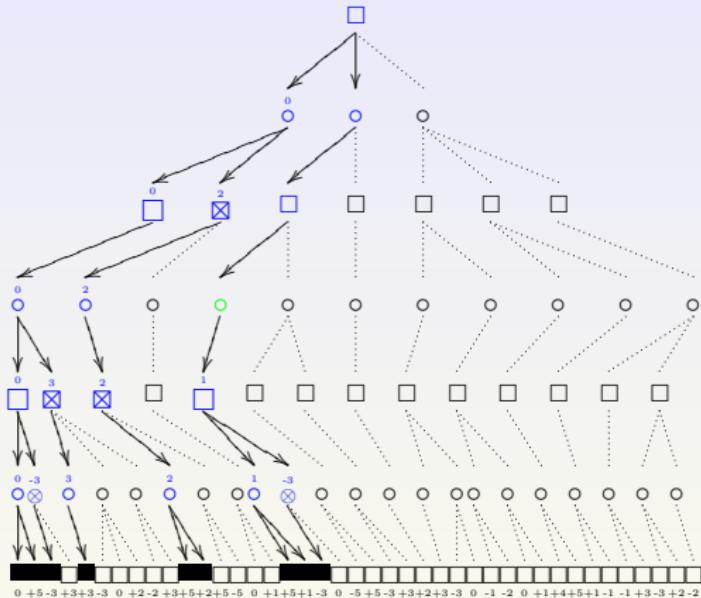


```
function ALPHA-BETA-SEARCH(state) returns an action
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

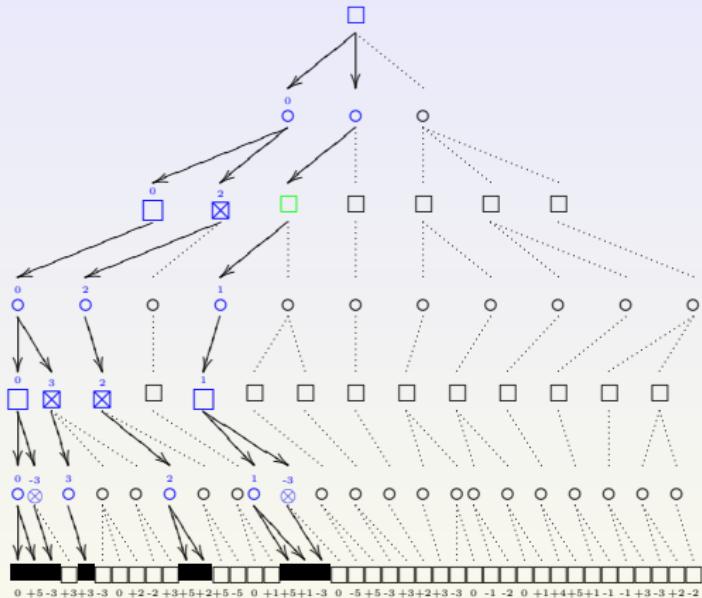
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$

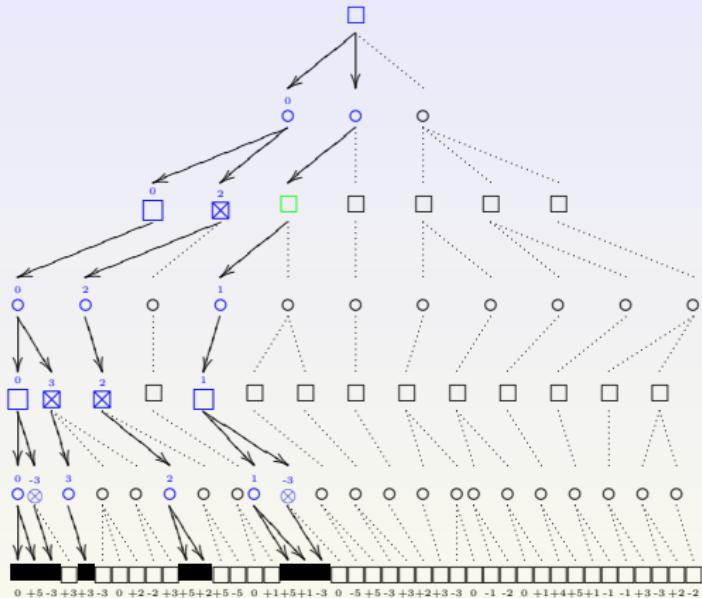


```
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```
function MAX-VALUE(state, α, β) returns a utility value
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    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
        return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$

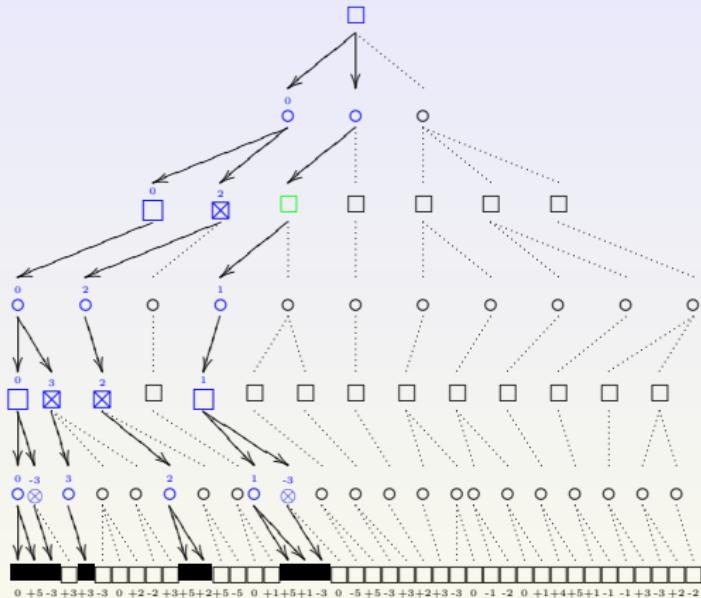


```
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  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
  → α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
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    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

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```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

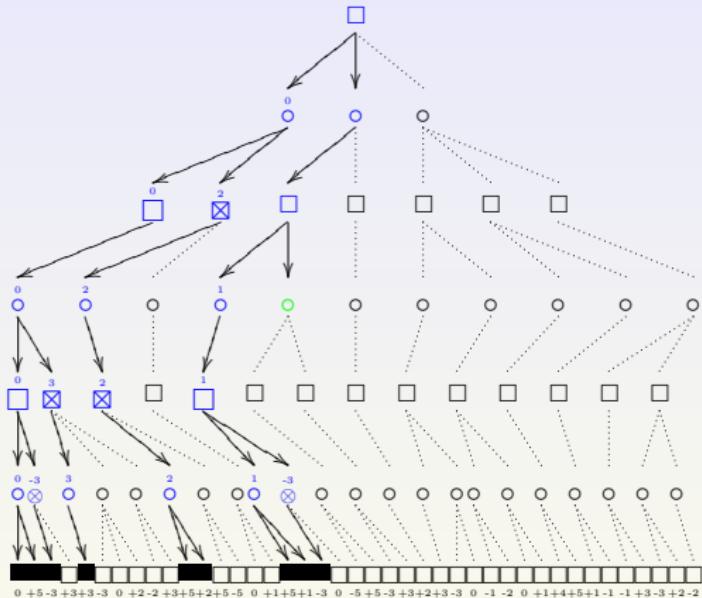
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
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    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

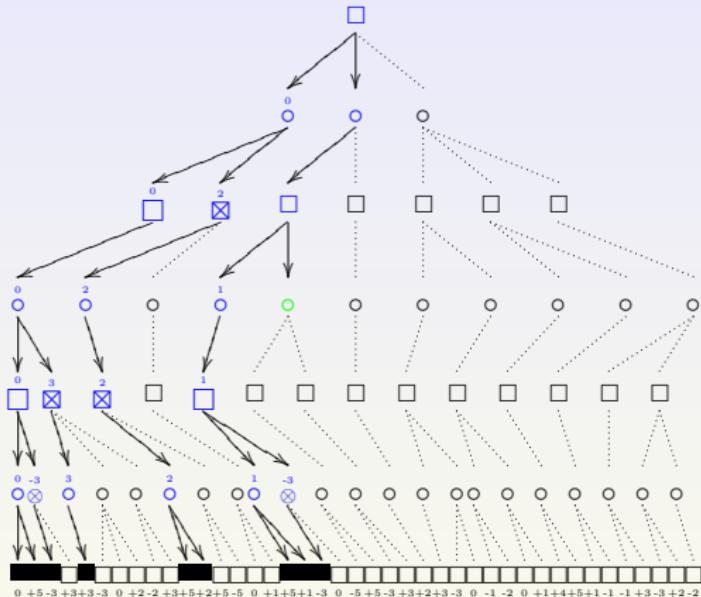
→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value

```

  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

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```

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    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

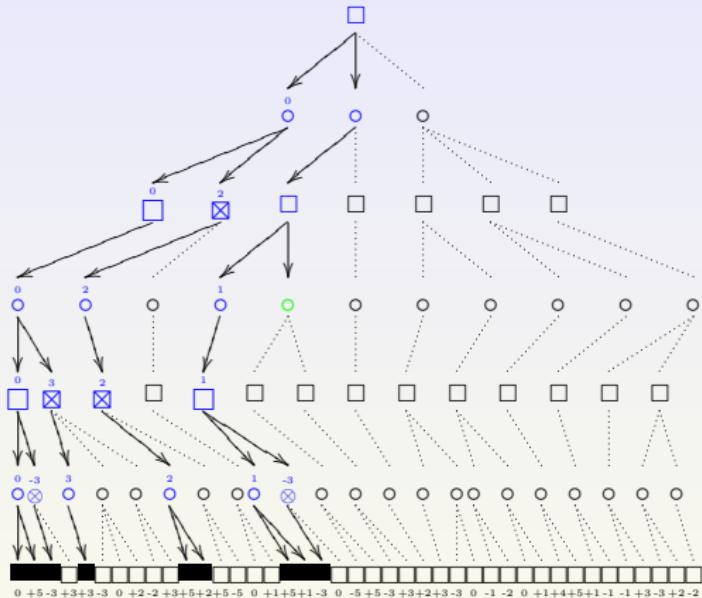
```

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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

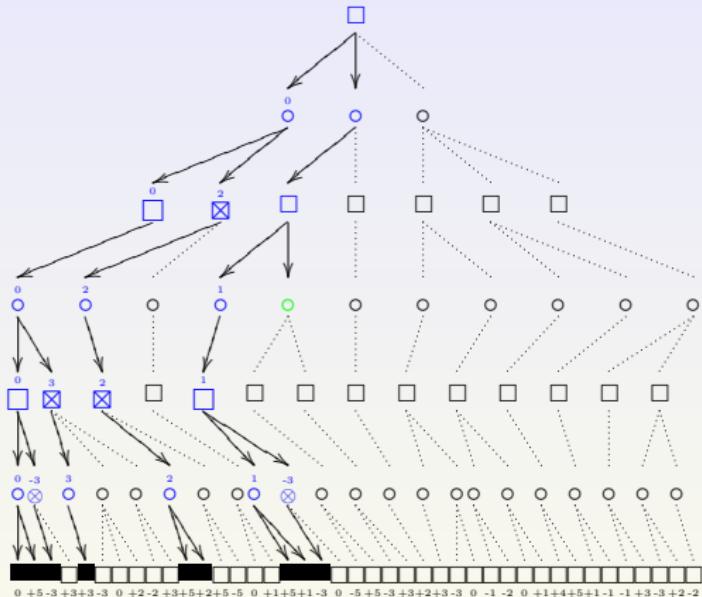
```

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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    if v  $\leq \alpha$  then
      return v
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  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



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  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

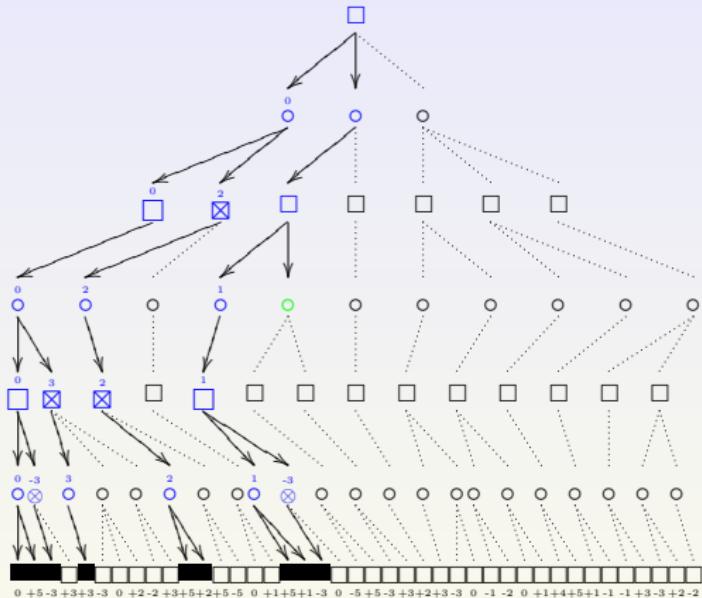
```

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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    if v  $\leq \alpha$  then
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    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

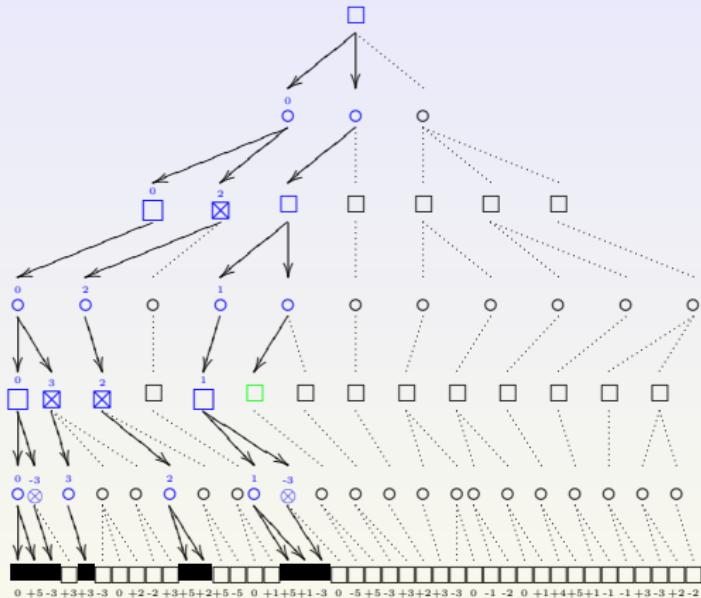
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
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```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

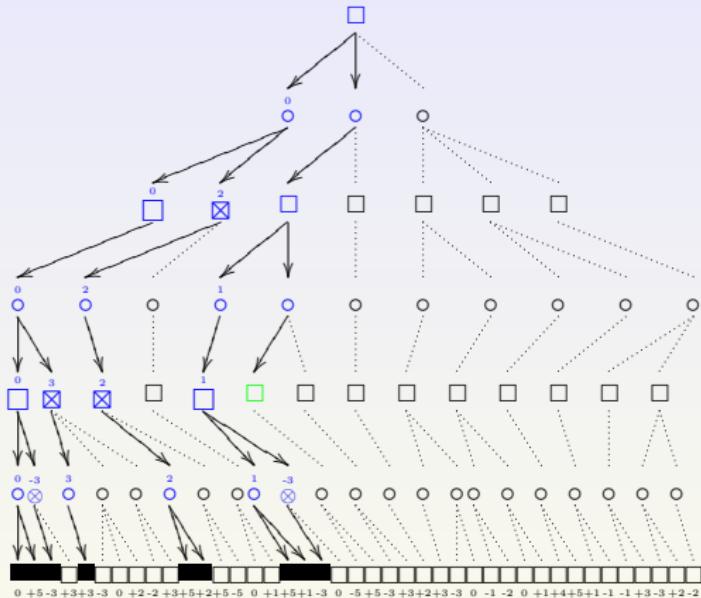
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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

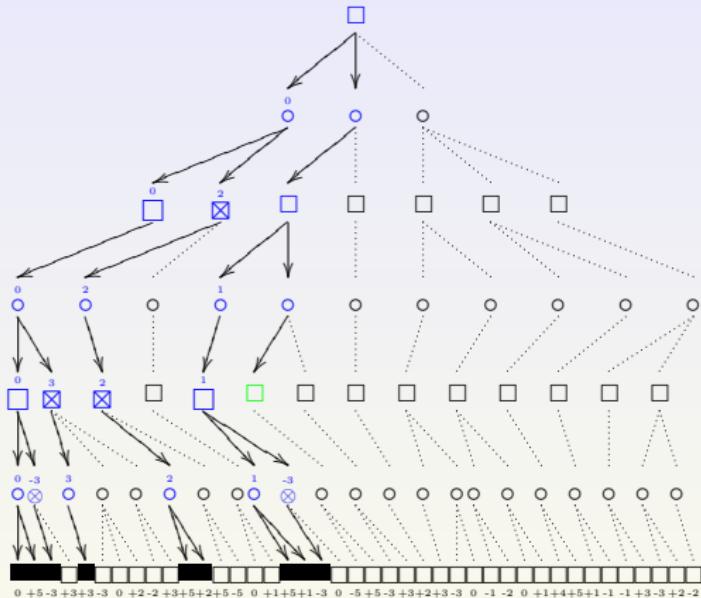
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
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  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)

```

→ $v \leftarrow -\infty$

```

  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)

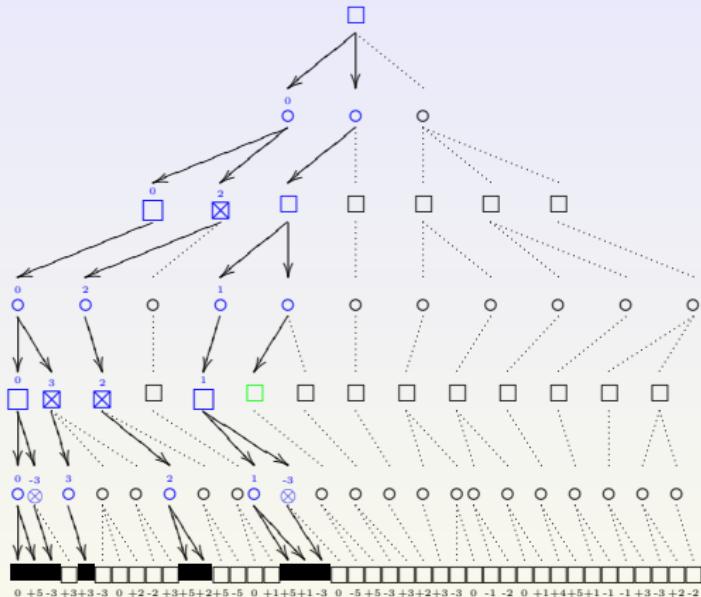
```

```

   $v \leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



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function ALPHA-BETA-SEARCH(state) returns an action
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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

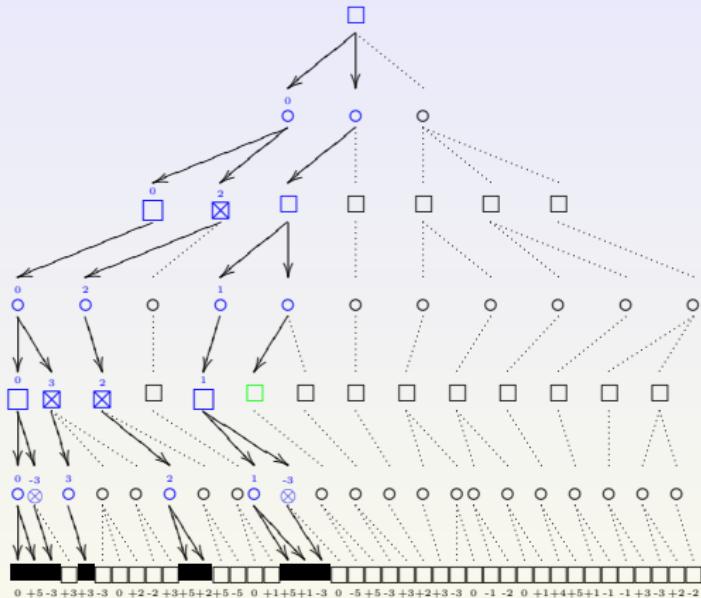
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

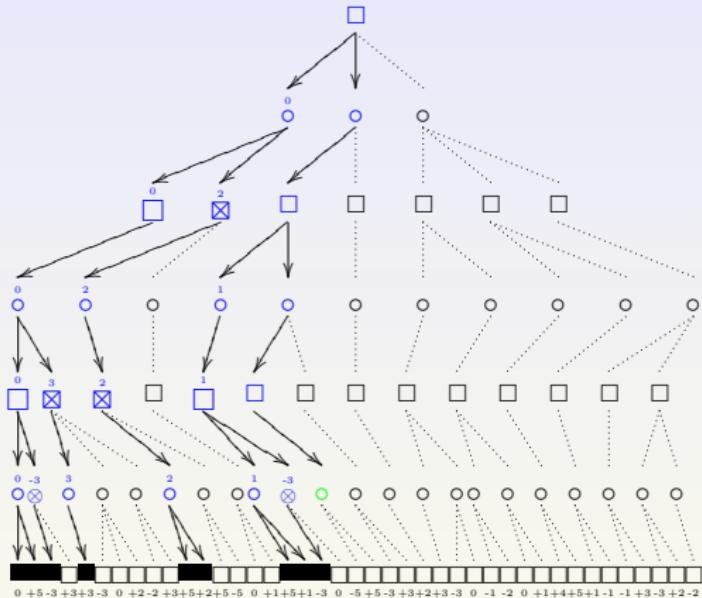
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

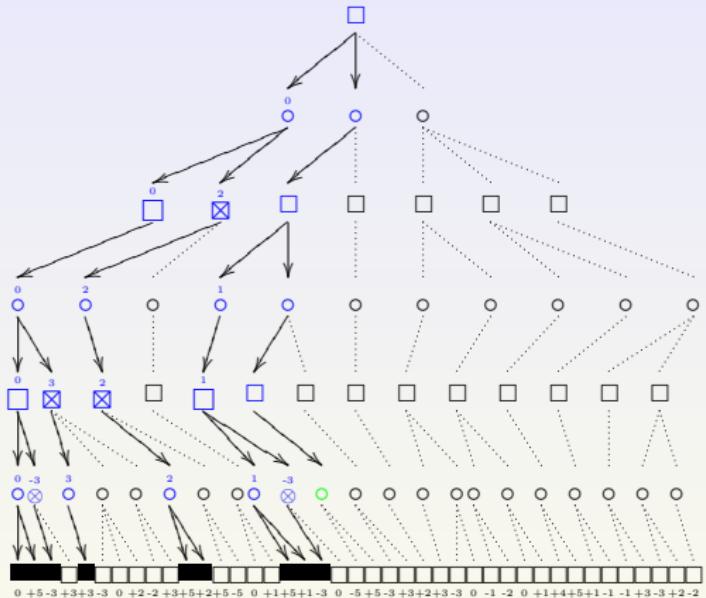
→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value

```

  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

function MIN-VALUE(*state*, α , β) **returns** a utility value

→ **if** TERMINAL-TEST(*state*) **then**

return UTILITY(*state*)

v \leftarrow $+\infty$

for *a, s* in SUCCESSORS(*state*) **do**

v \leftarrow MIN(*v*, MAX-VALUE(*s*, α , β))

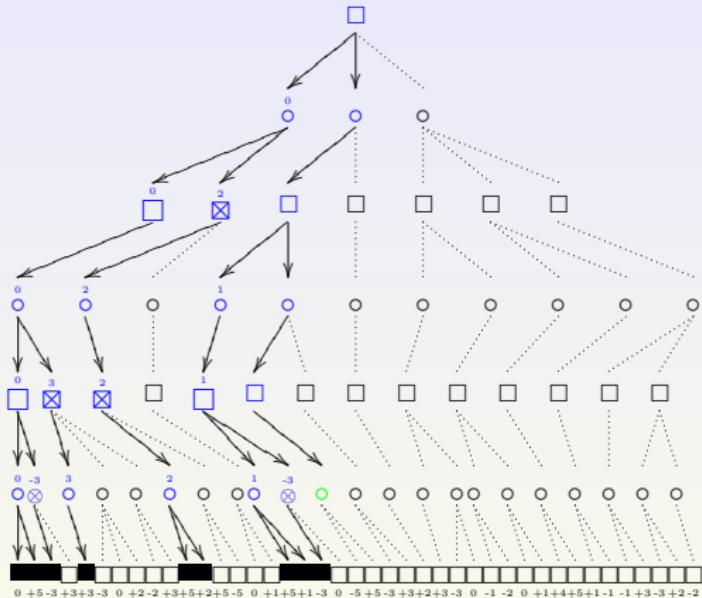
if *v* $\leq \alpha$ **then**

return *v*

$\beta \leftarrow$ MIN(β , *v*)

return *v*

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

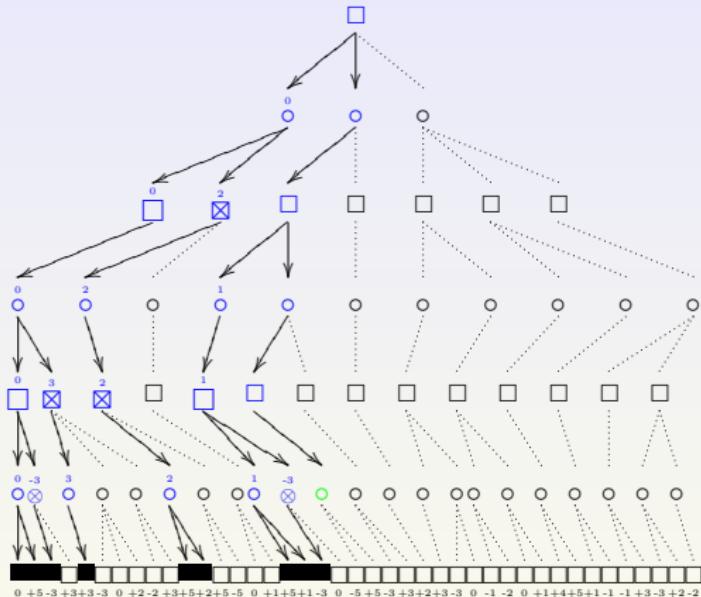
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

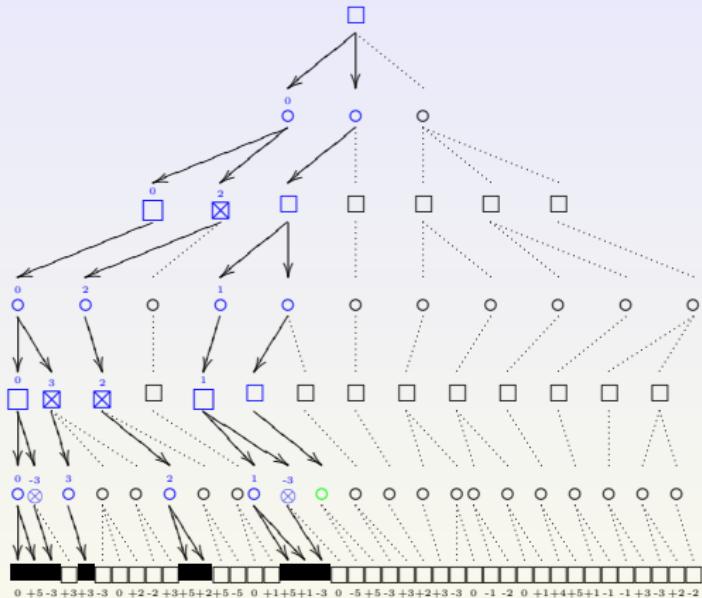


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

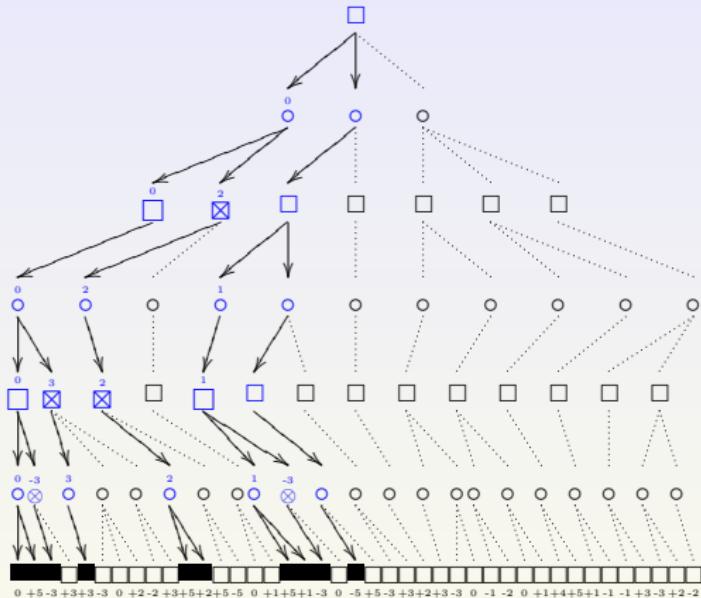


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

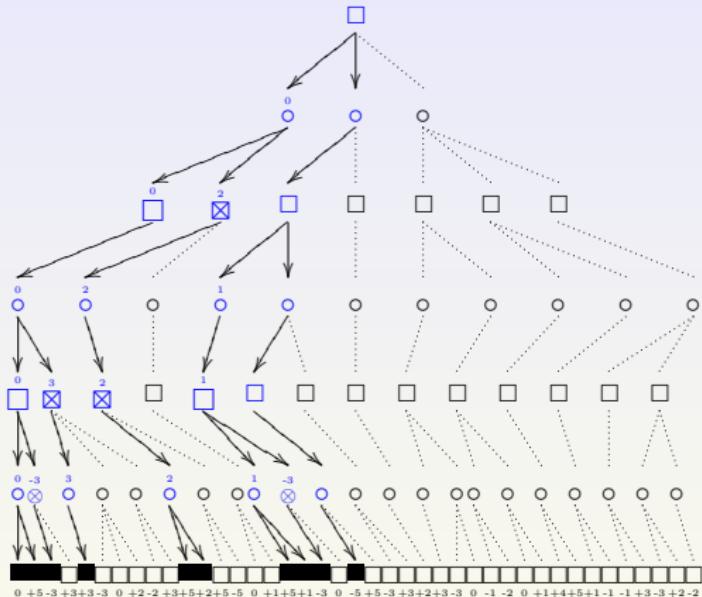
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

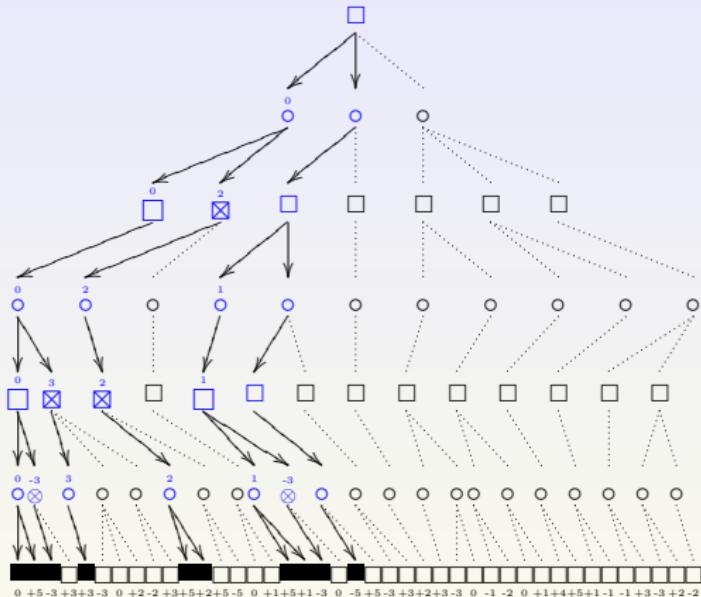
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$ 
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

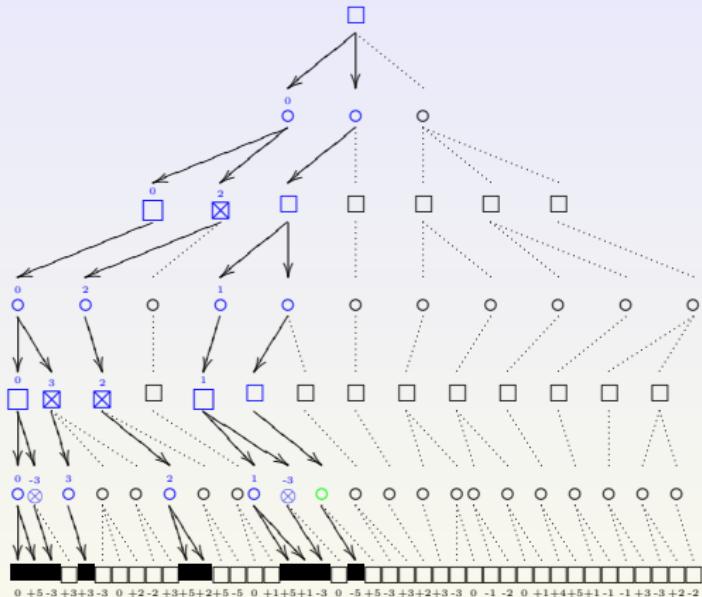
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

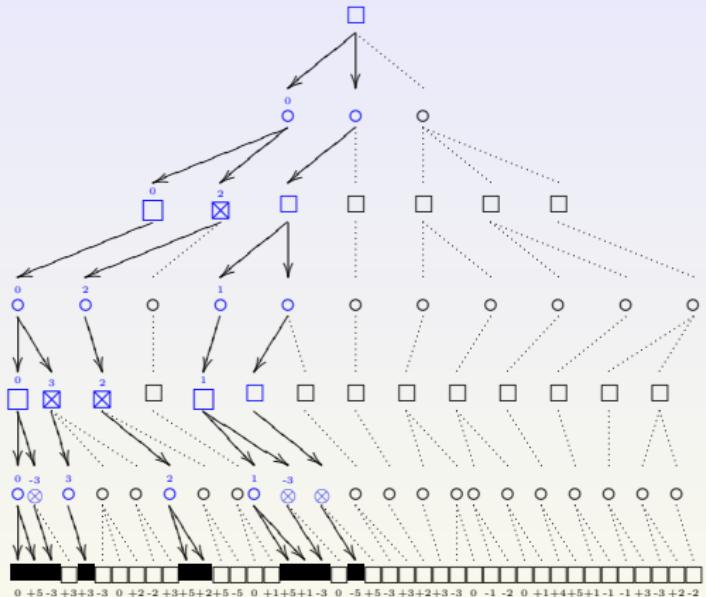


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

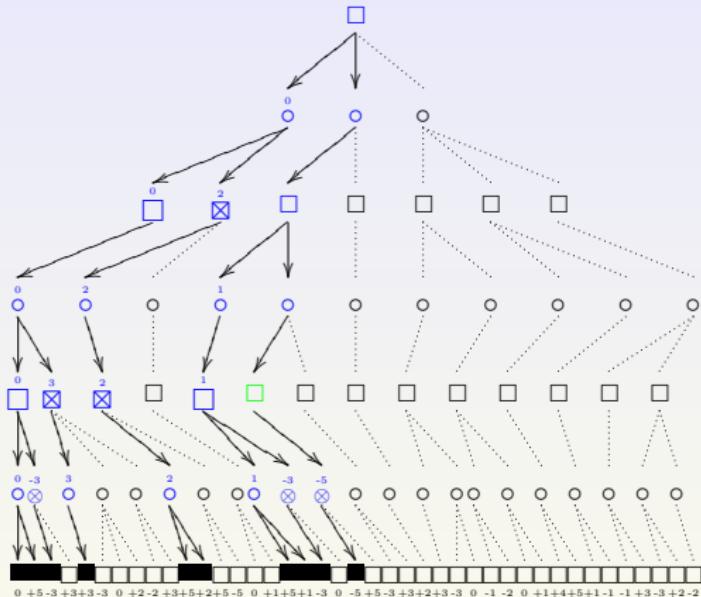
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

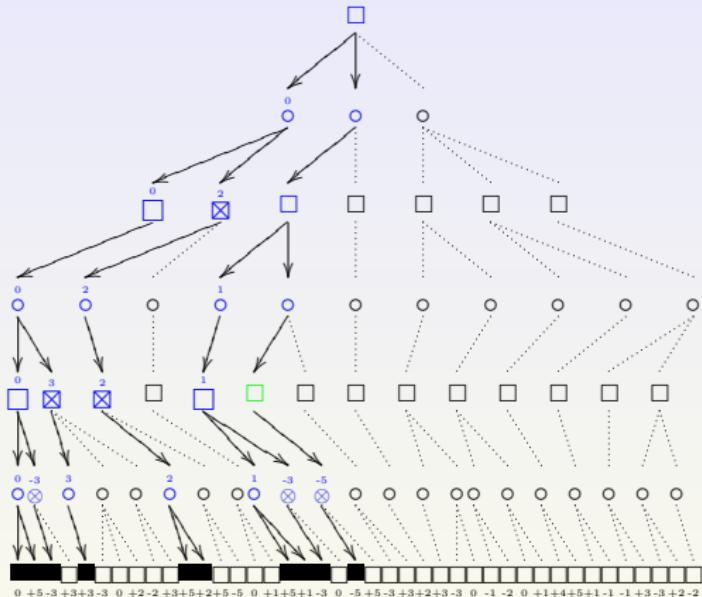
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  →
   $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

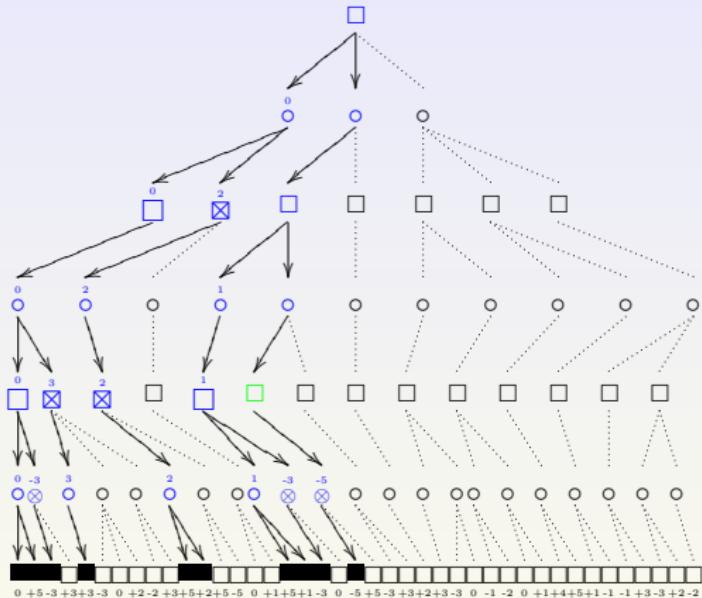
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

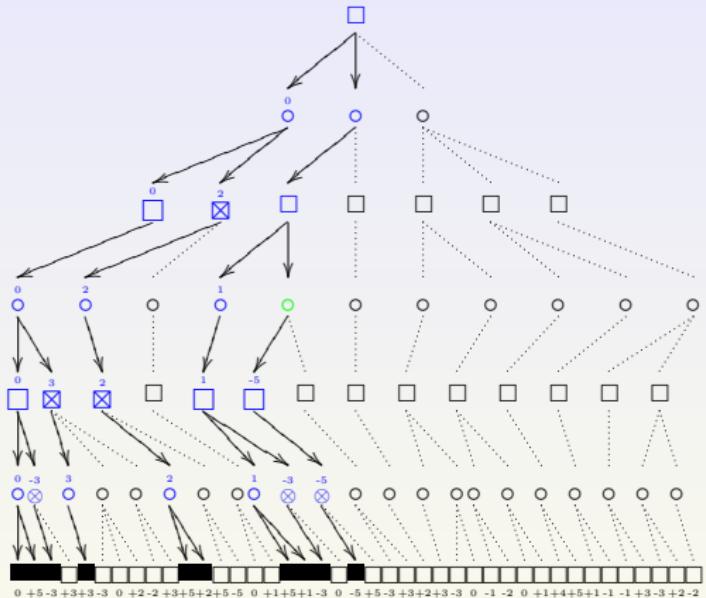
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

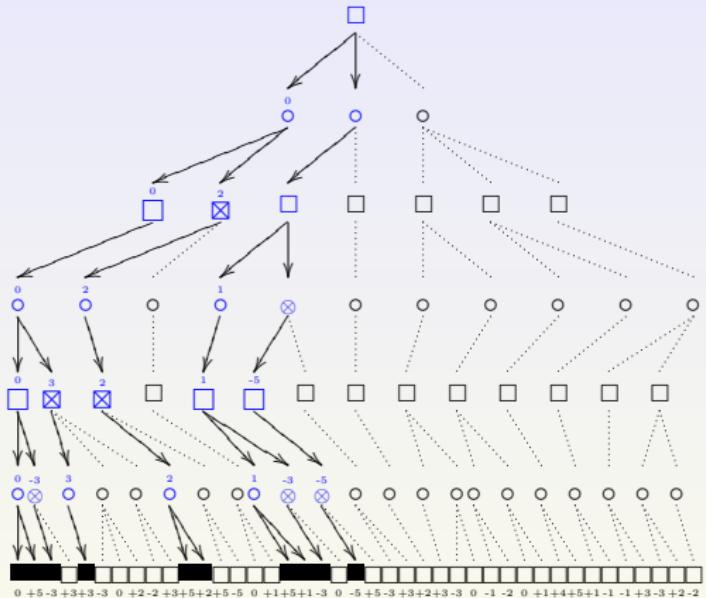
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

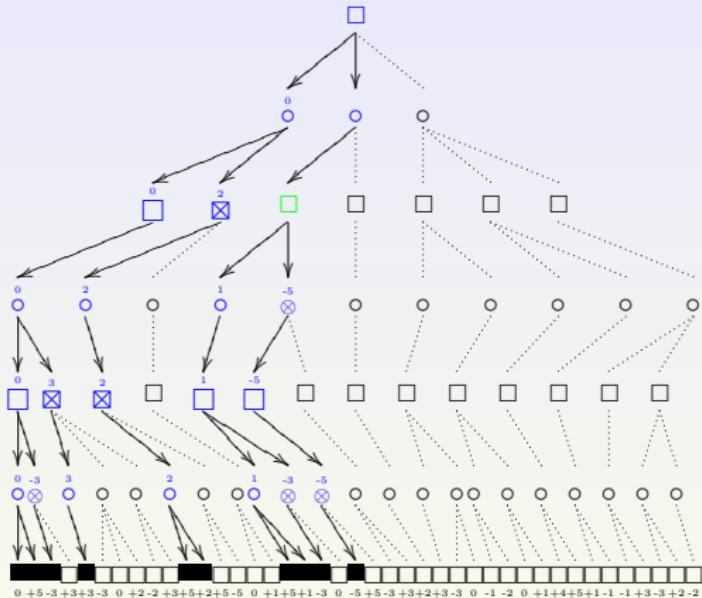
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

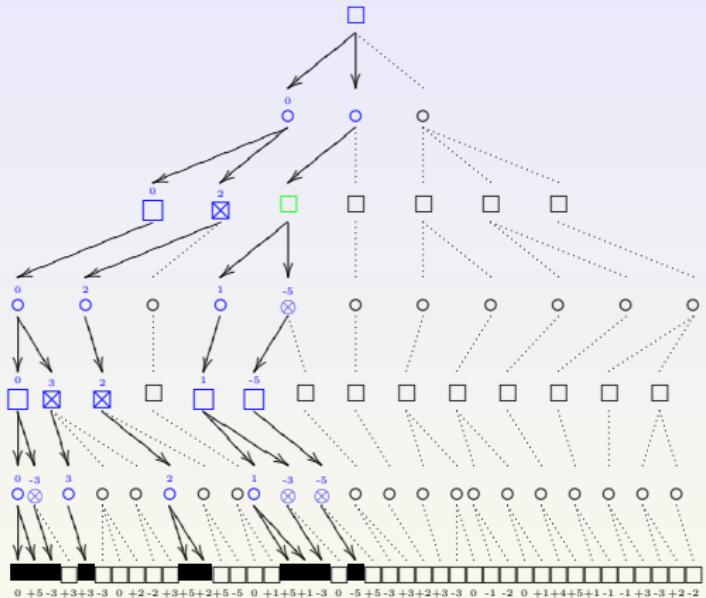
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

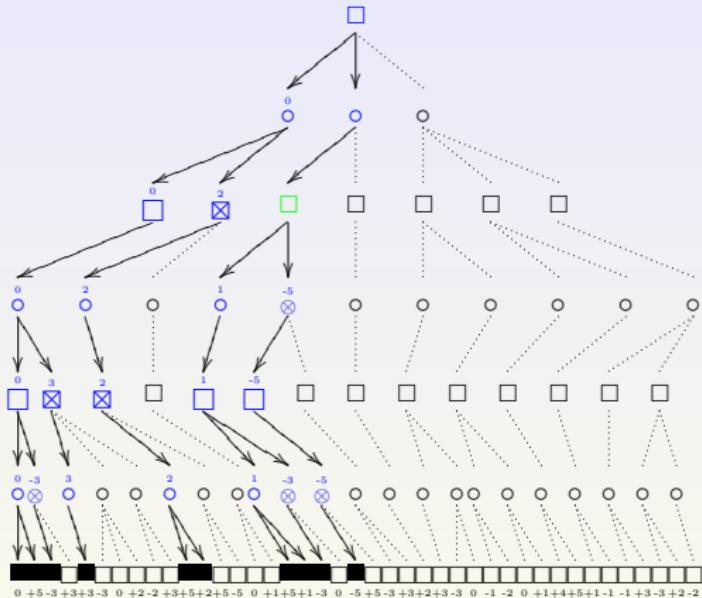


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
  → α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

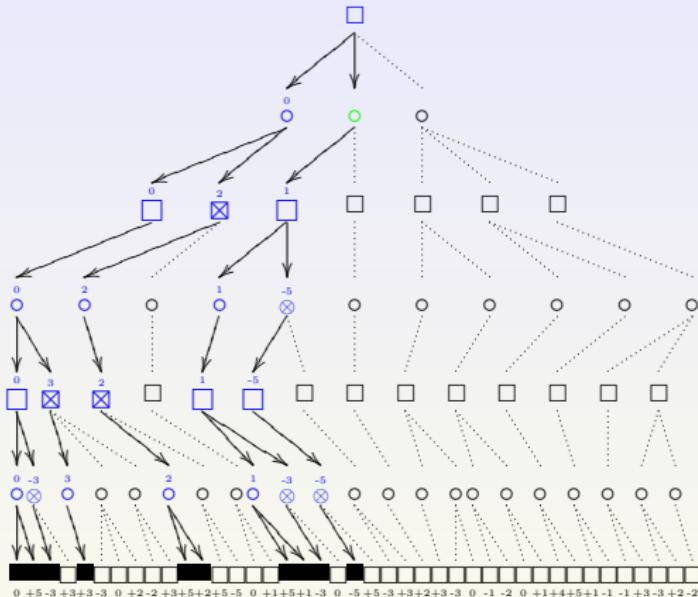


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  → return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

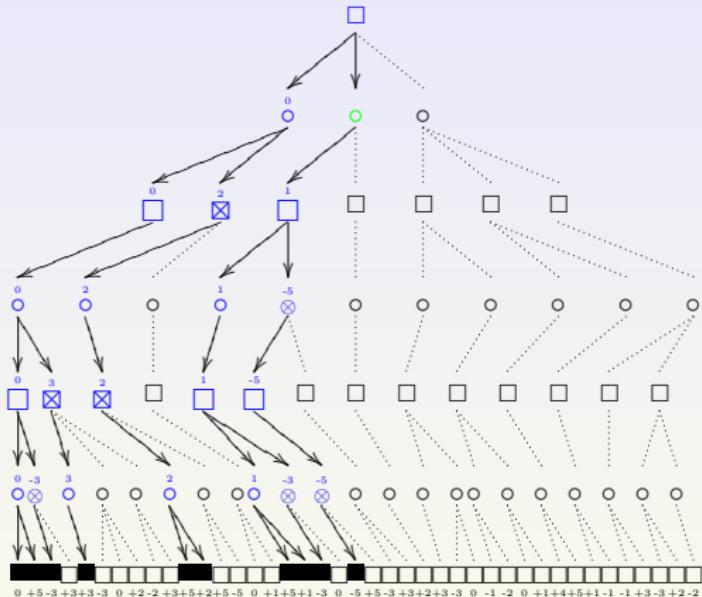
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow \text{MINI}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

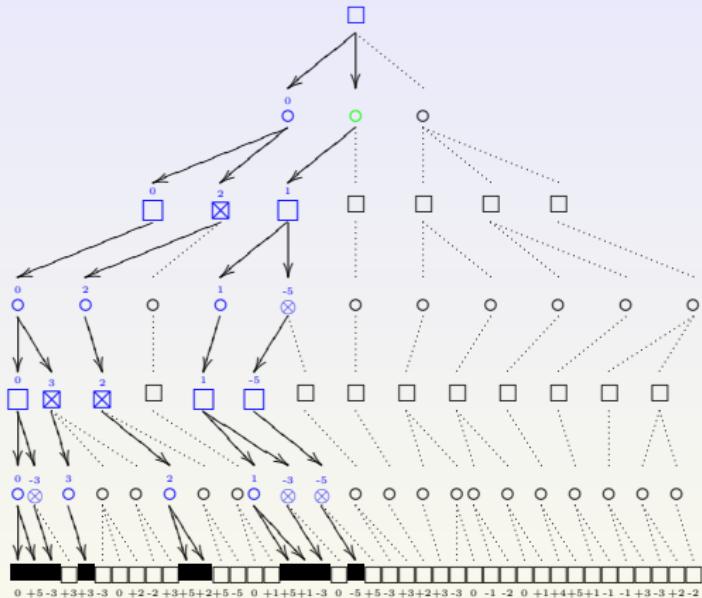
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

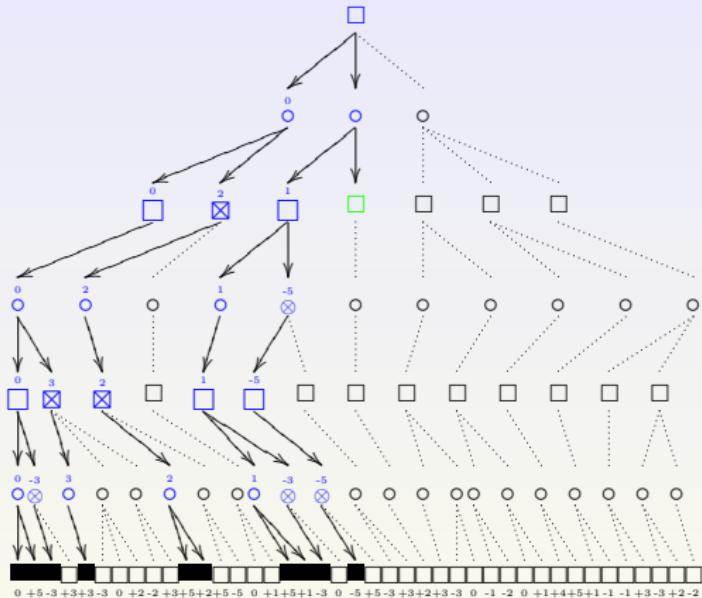


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    → v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

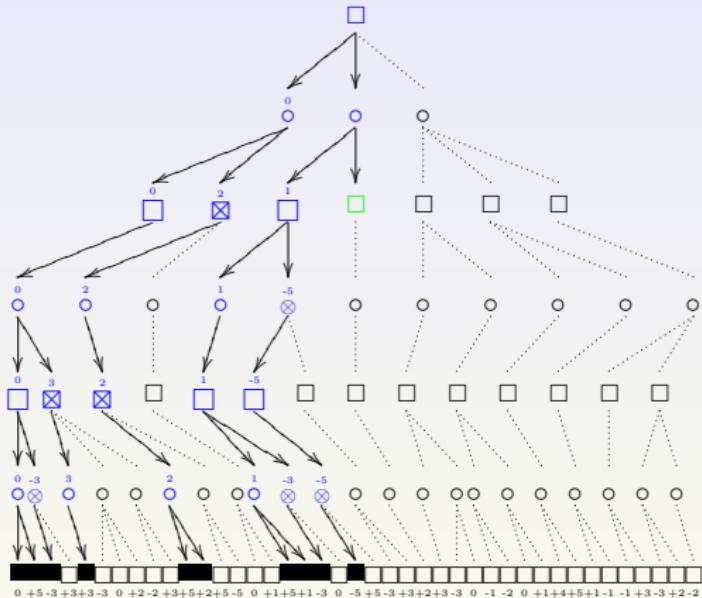


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = 1$$

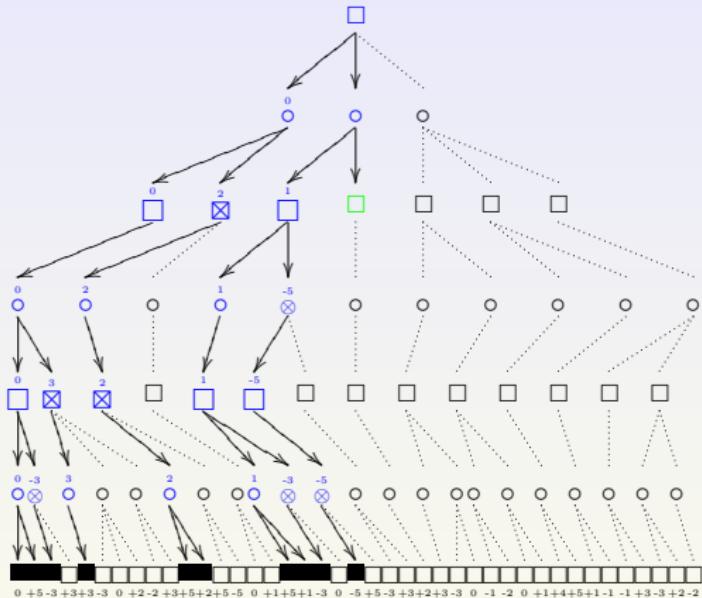


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



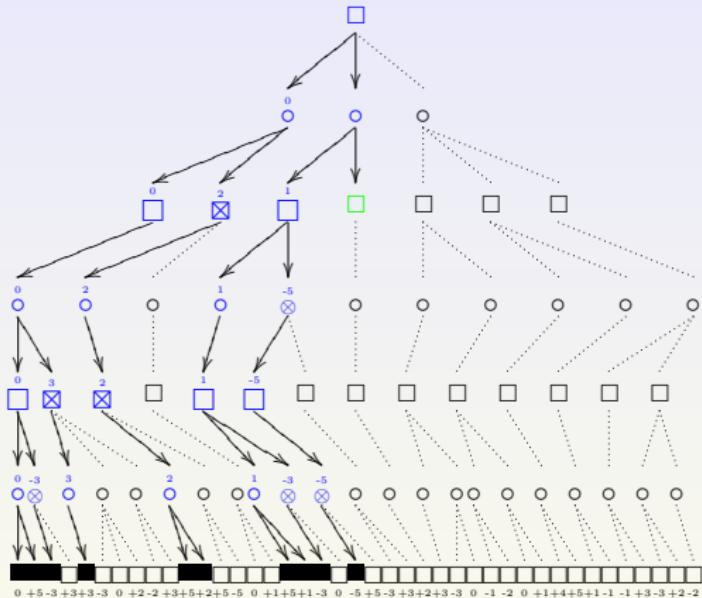
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
```

```
→   v ← -∞
    for a, s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s, α, β))
      if  $v \geq \beta$  then
        return v
      α ← MAX(α, v)
    return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

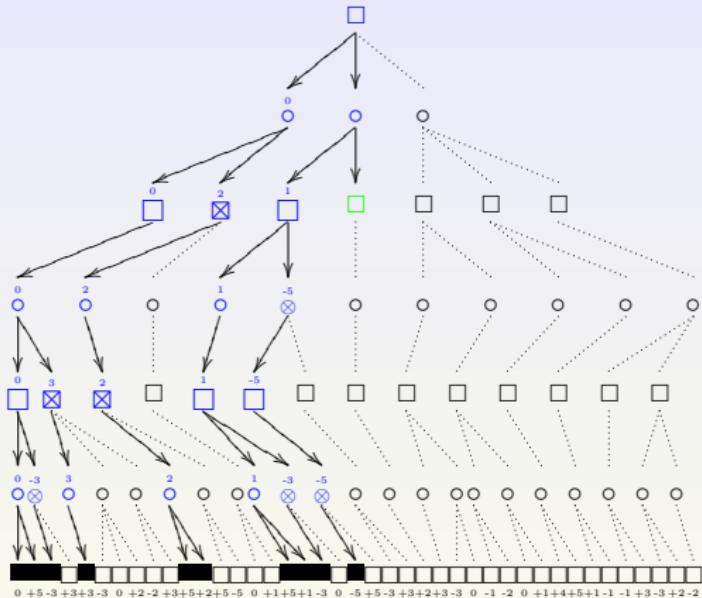


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
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  v ← -∞
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    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
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function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

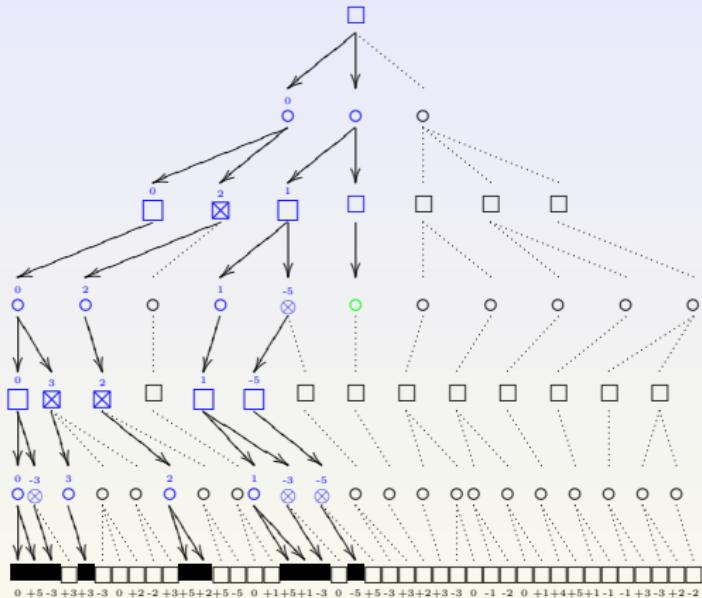
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$

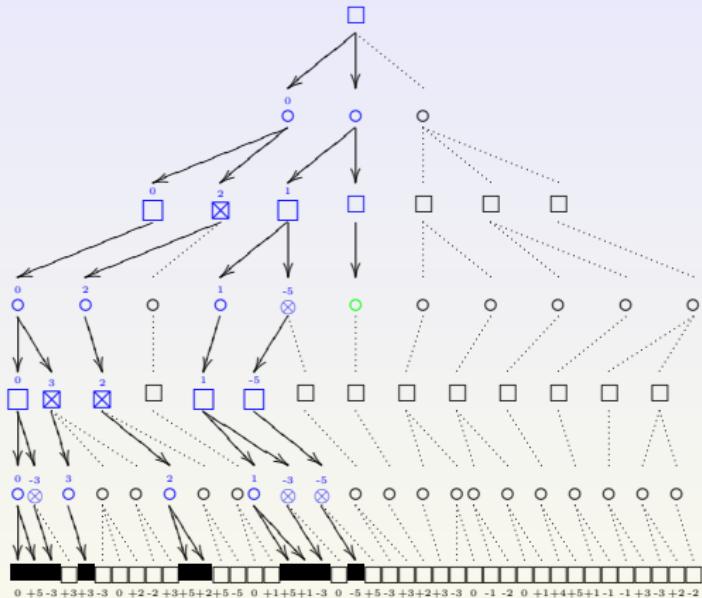


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    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
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```

$$\alpha = 0 \quad \beta = 1$$

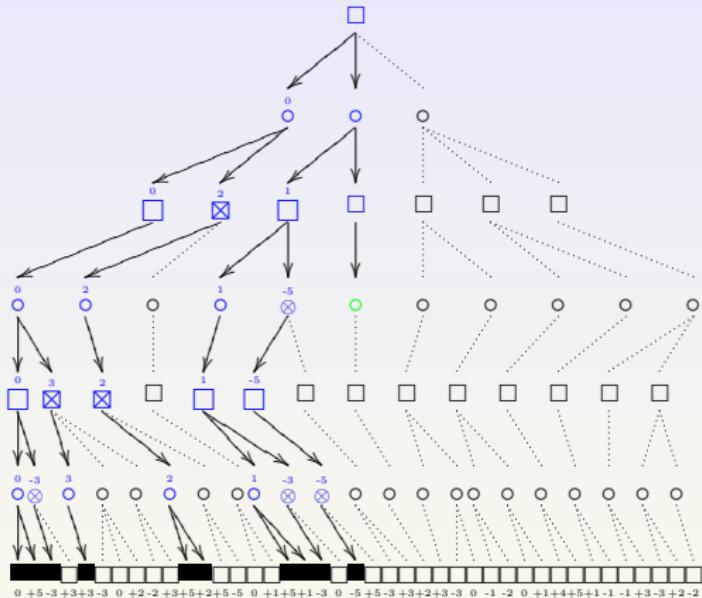


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    if v ≥ β then
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    α ← MAX(α, v)
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    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
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$$\alpha = 0 \quad \beta = 1$$

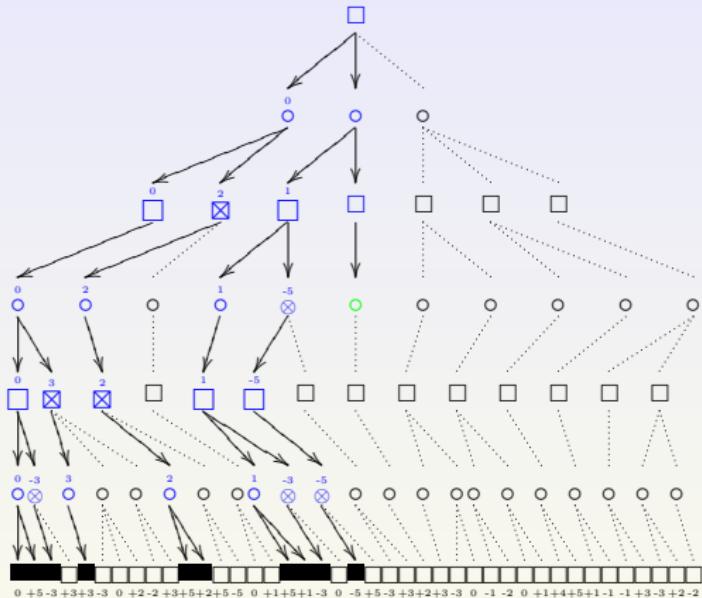


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    if v ≥ β then
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

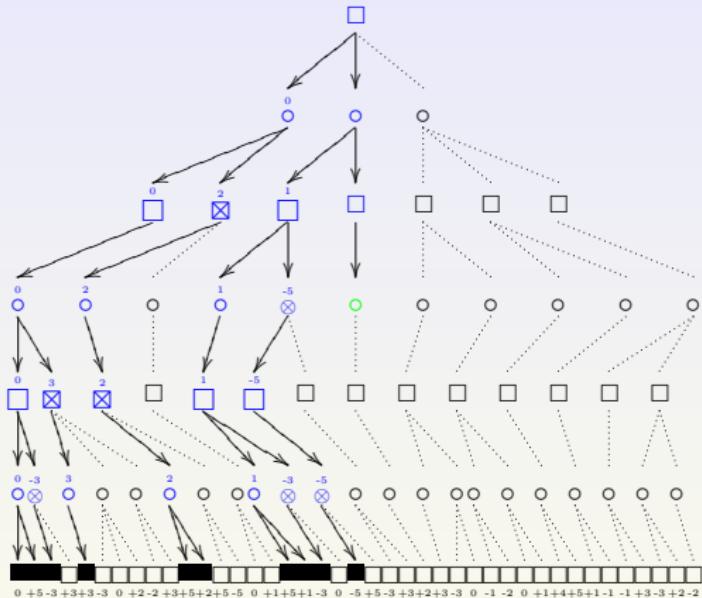


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    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

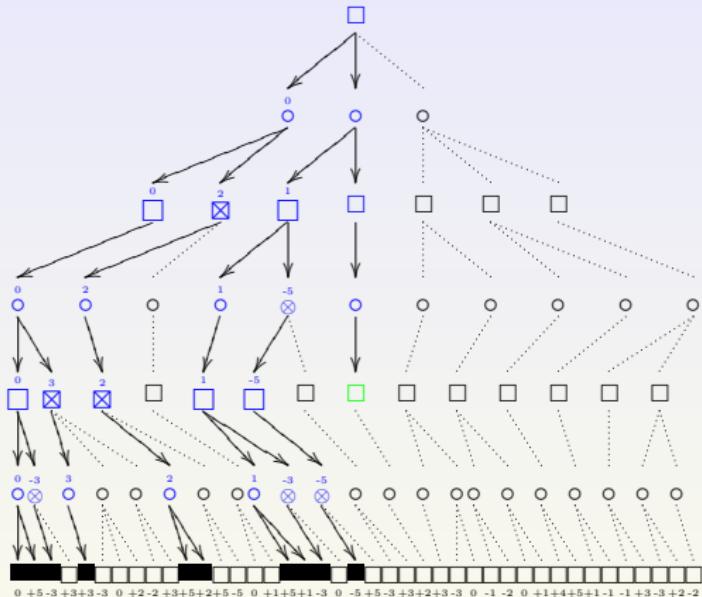


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  v ← +∞
  for a, s in SUCCESSORS(state) do
    → v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

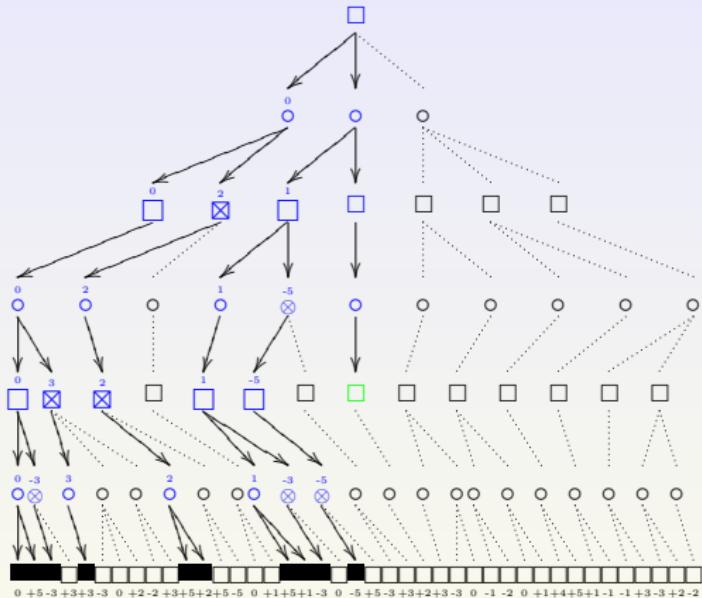


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

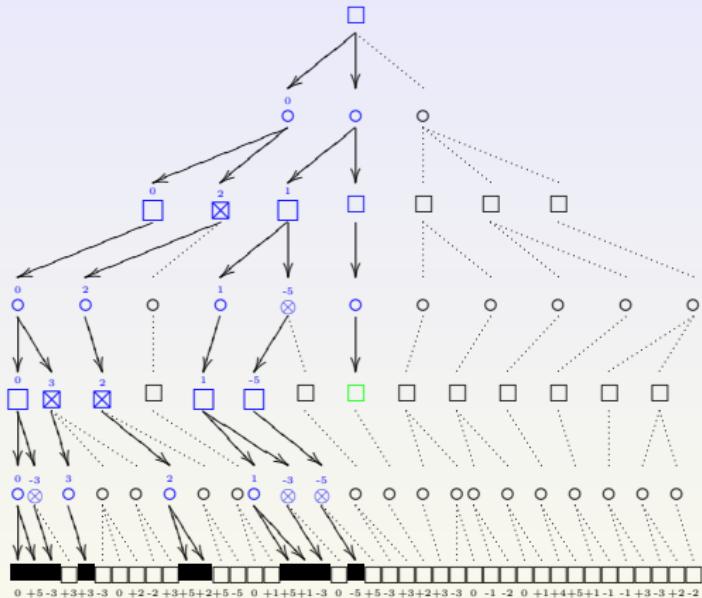


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

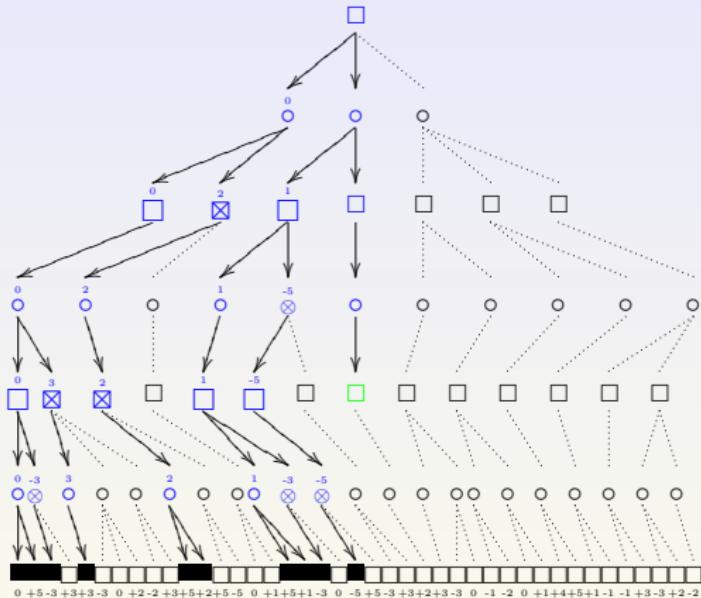
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$

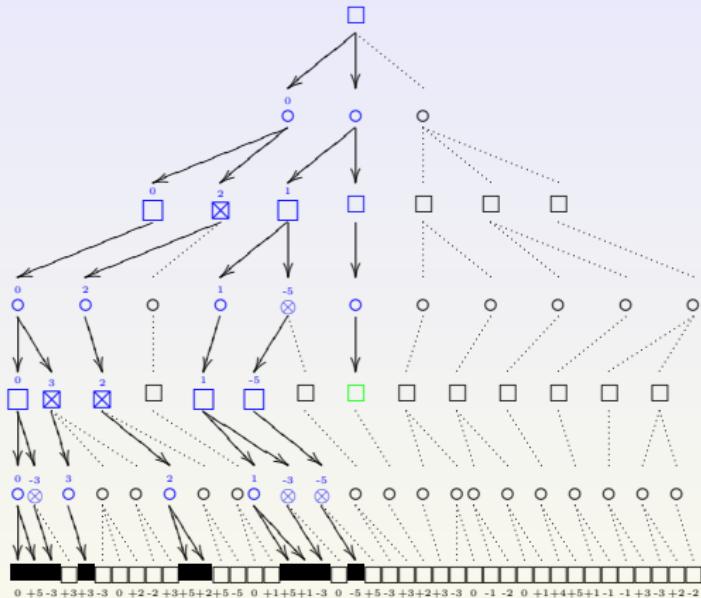


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function ALPHA-BETA-SEARCH(state) returns an action
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```
function MAX-VALUE(state, α, β) returns a utility value
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    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

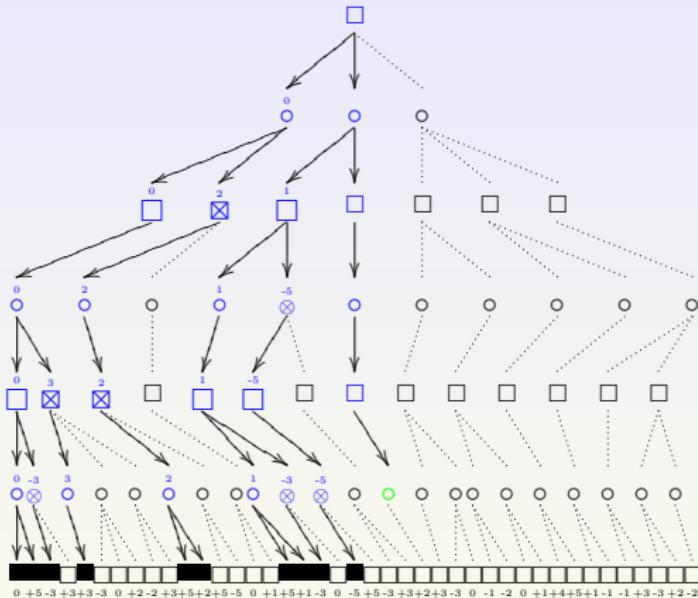


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function ALPHA-BETA-SEARCH(state) returns an action
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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

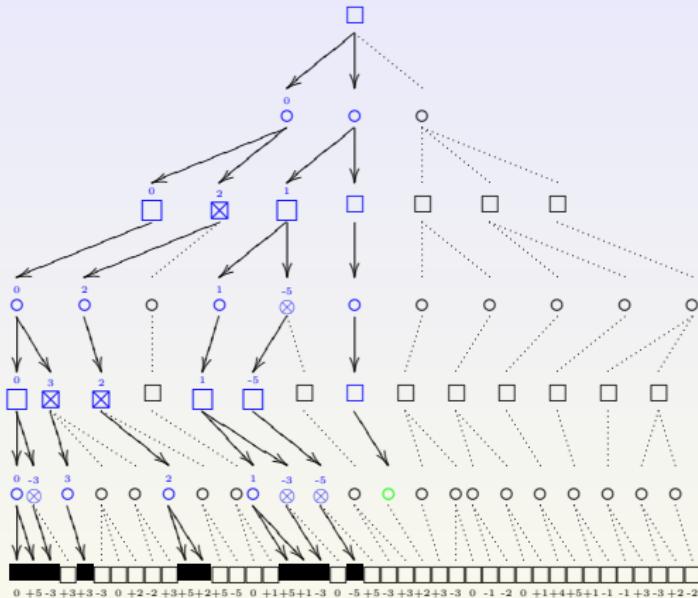
```

```

→function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

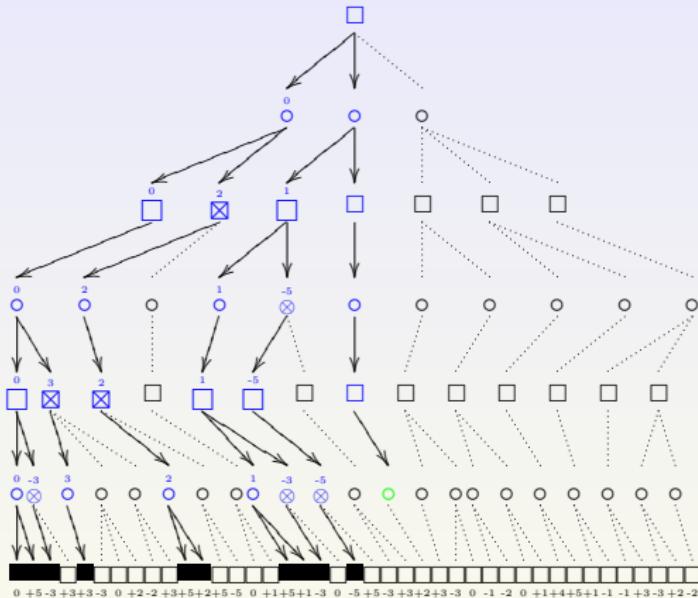
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function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
→ if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
         $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
        if  $v \leq \alpha$  then
            return  $v$ 
         $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 

```

$$\alpha = 0 \quad \beta = 1$$





```

function ALPHA-BETA-SEARCH(state) returns an action
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```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

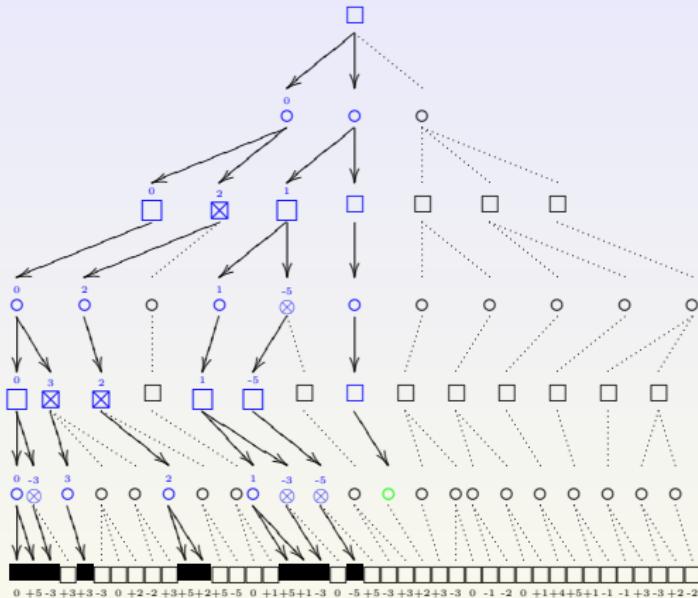
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 0 \quad \beta = 1$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
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function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
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```

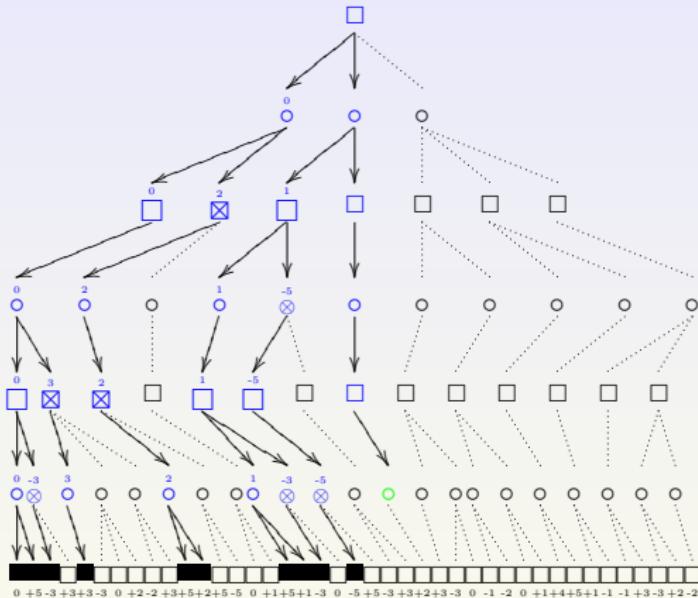
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    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = 1$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

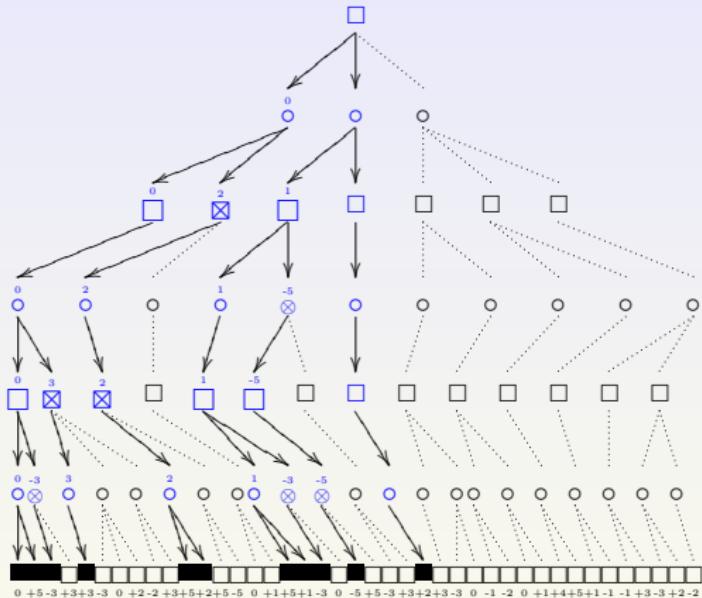
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 0 \quad \beta = 1$$



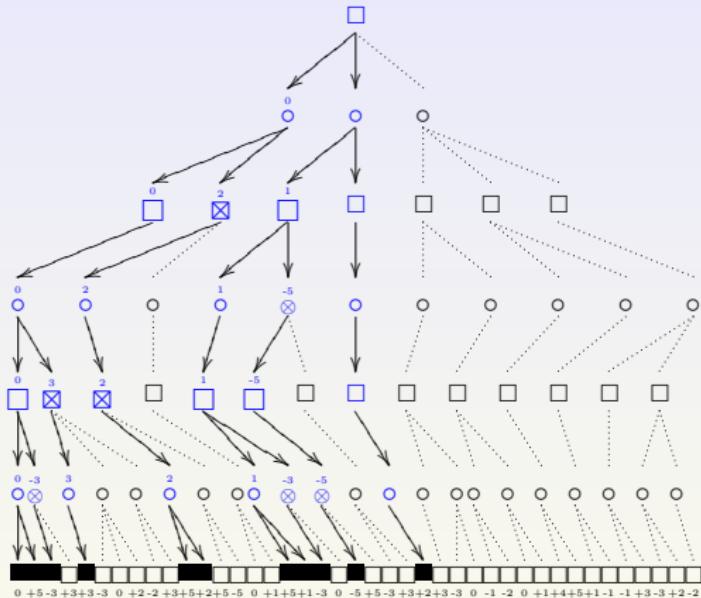


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

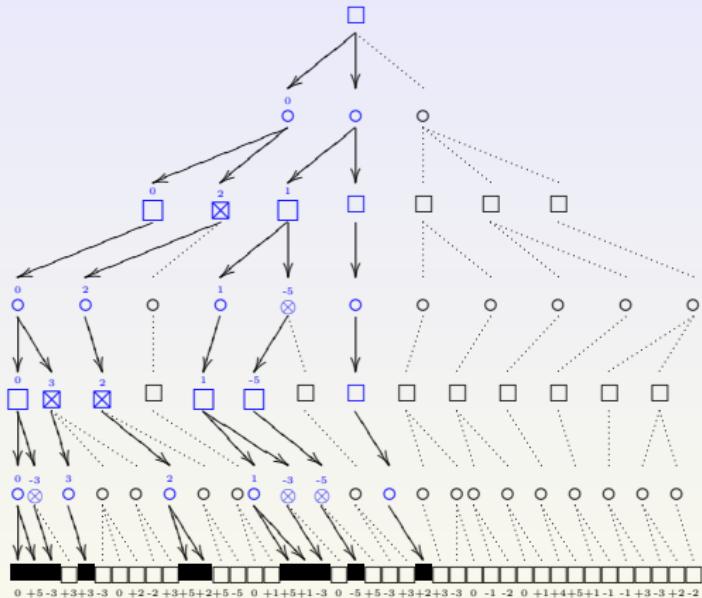


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$ 
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

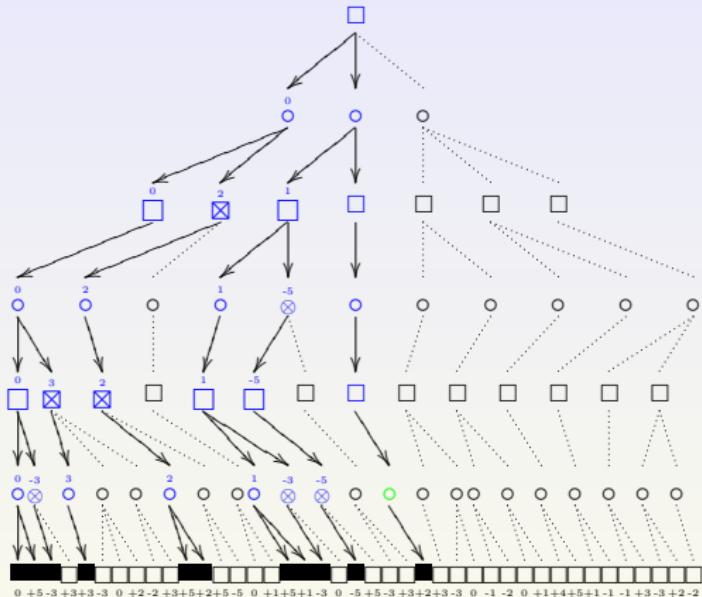
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
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    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = 1$$

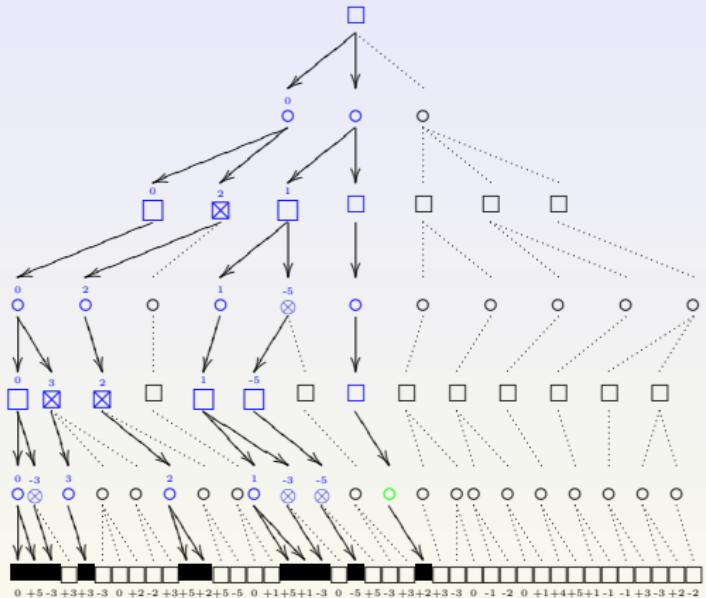


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function ALPHA-BETA-SEARCH(state) returns an action
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

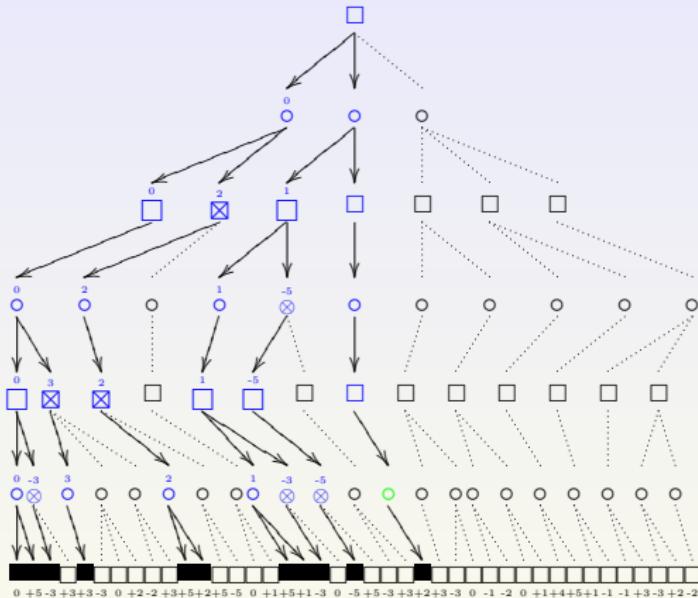


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
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function MAX-VALUE(state, α, β) returns a utility value
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```

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function MIN-VALUE(state, α, β) returns a utility value
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

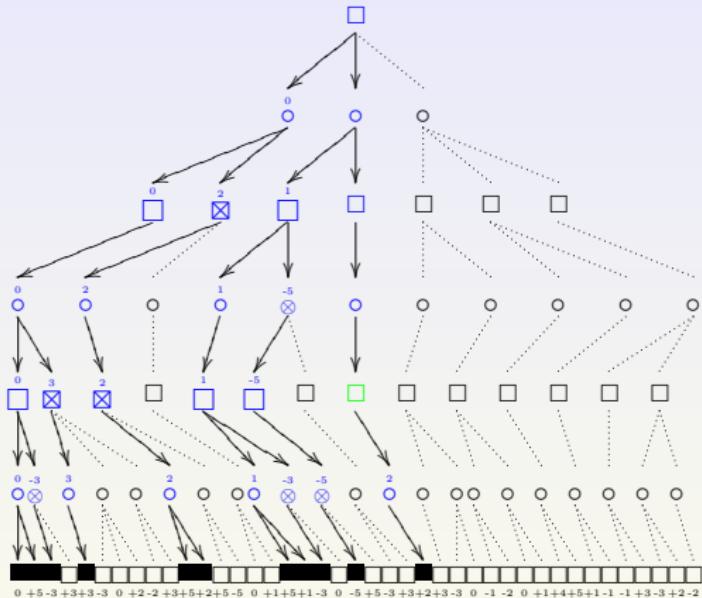
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 0 \quad \beta = 1$$



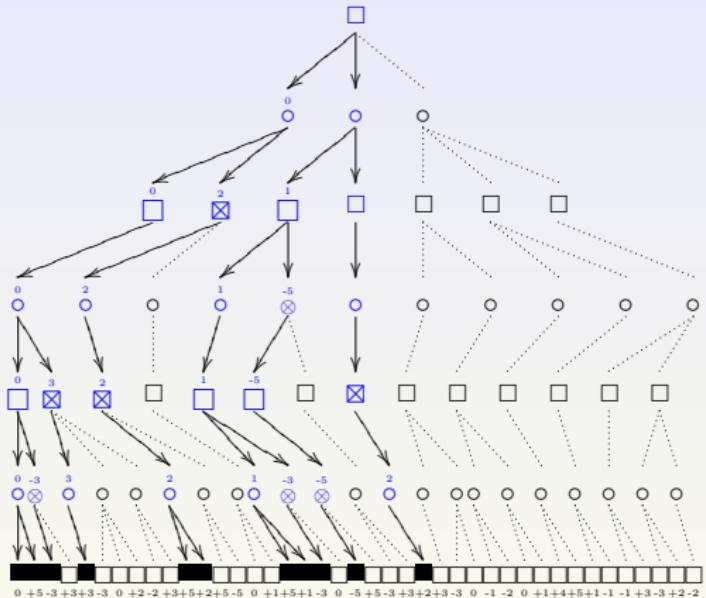


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function ALPHA-BETA-SEARCH(state) returns an action
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  return the action in SUCCESSORS(state) with value v
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```
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  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
        return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

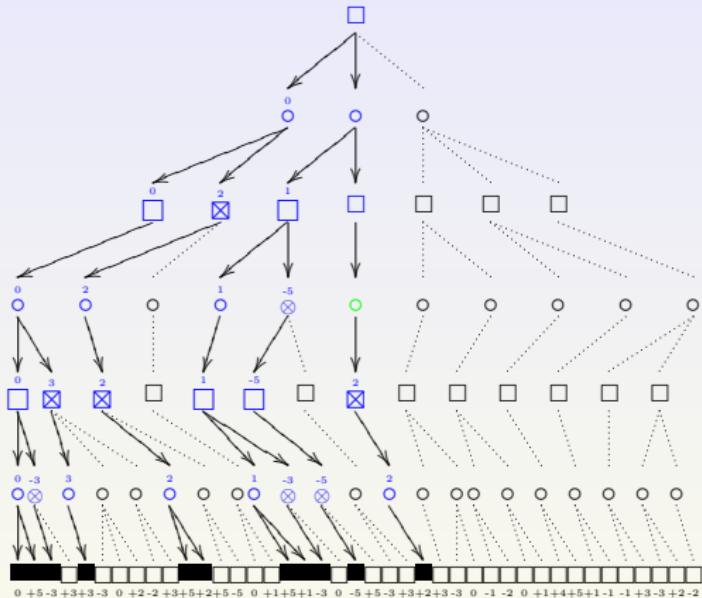


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function ALPHA-BETA-SEARCH(state) returns an action
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    if v ≥ β then
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function MIN-VALUE(state, α, β) returns a utility value
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    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
      β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

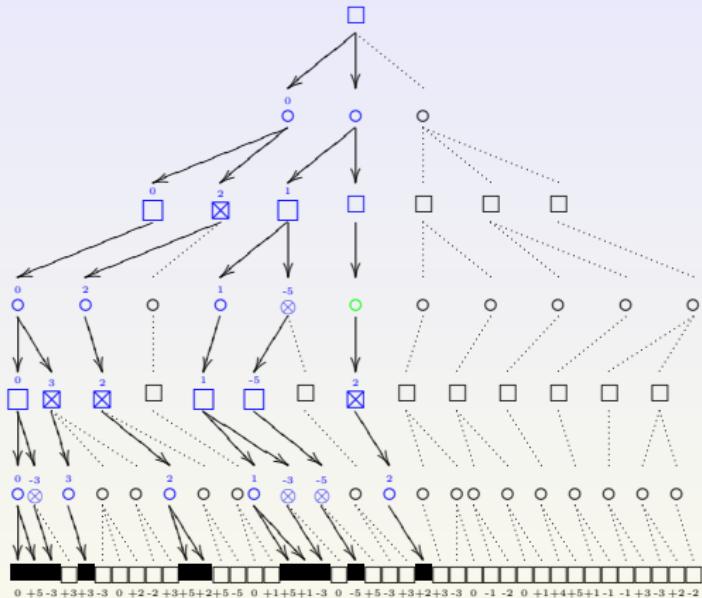


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    if v ≥ β then
      return v
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  return v
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    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$

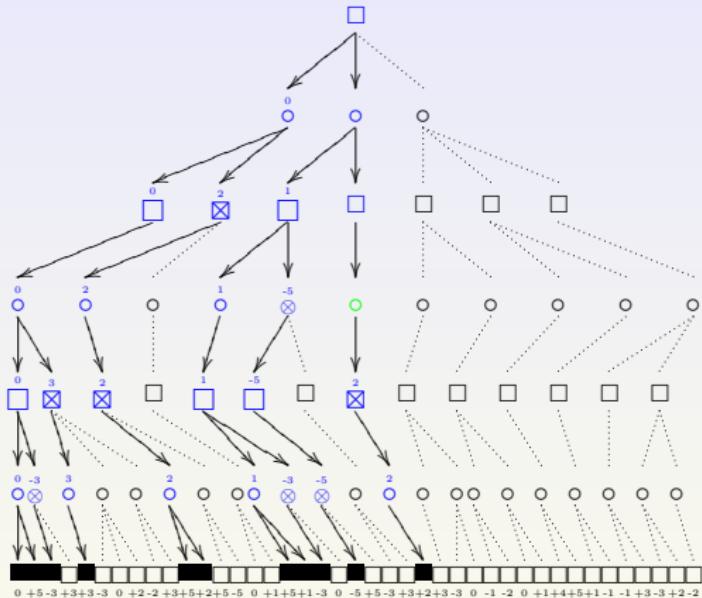


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  for a, s in SUCCESSORS(state) do
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    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
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function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

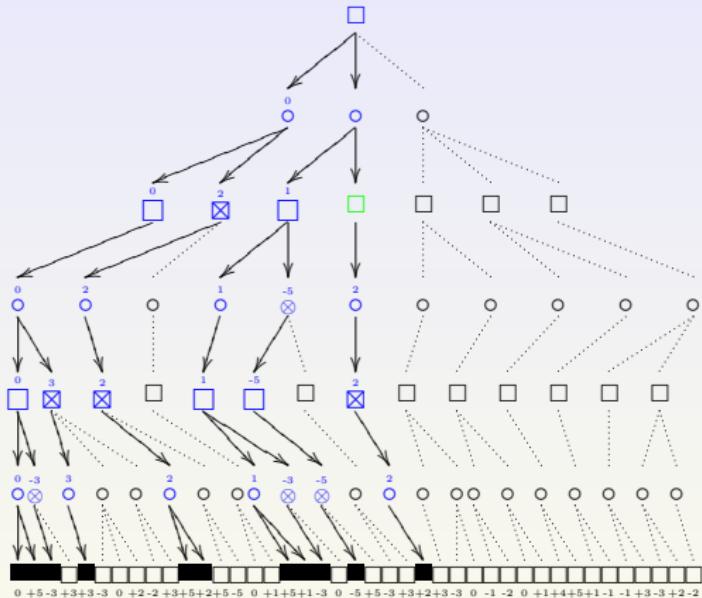
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$

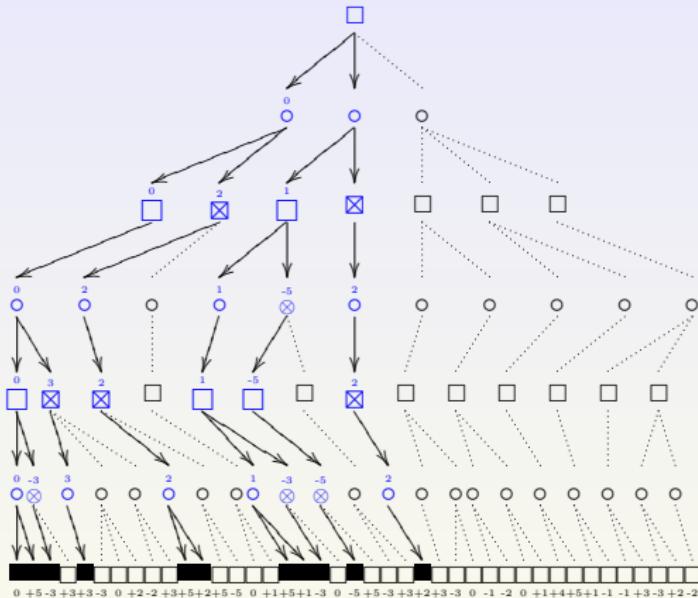


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
        return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

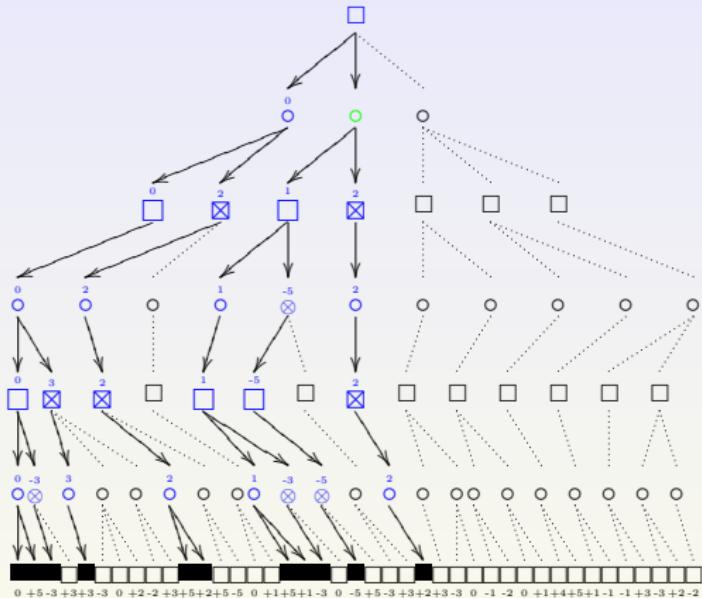
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 0 \quad \beta = 1$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

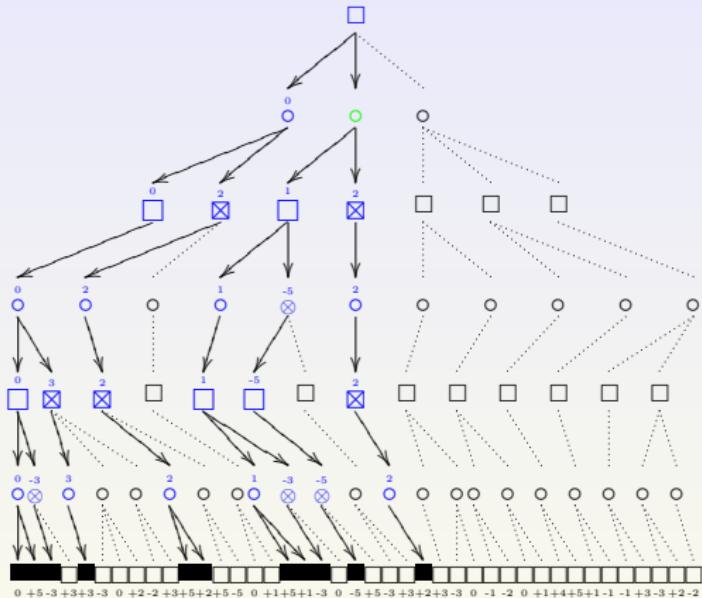
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$

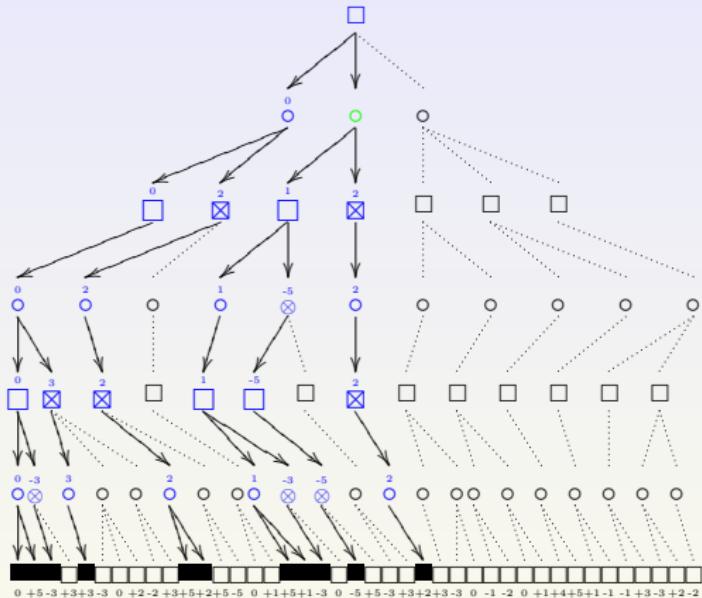


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    → β ← MIN(β, v)
  return v
```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

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```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

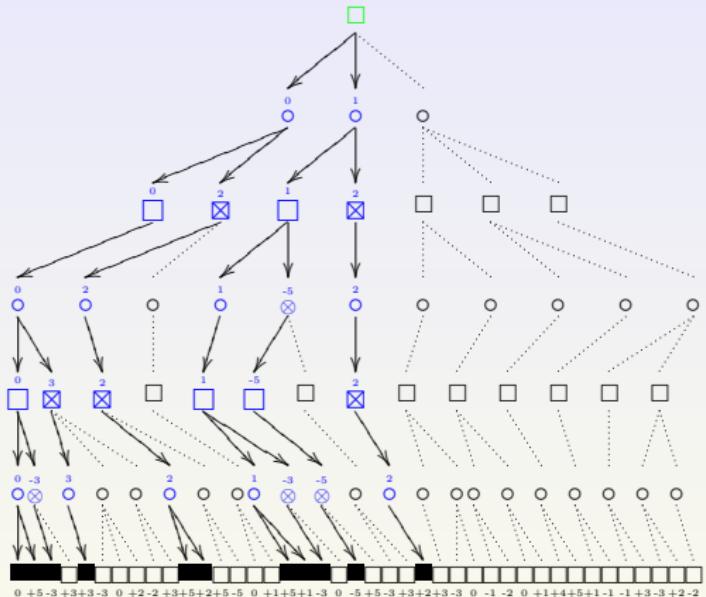
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = 1$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

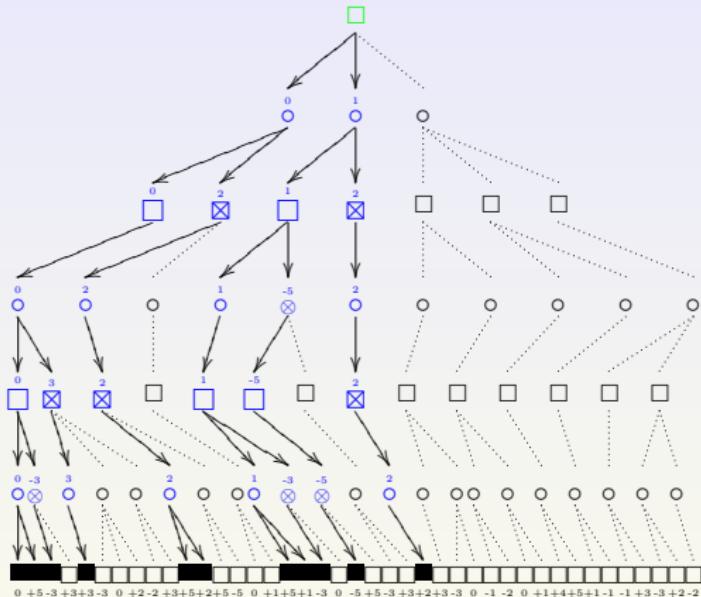
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 0 \quad \beta = +\infty$$

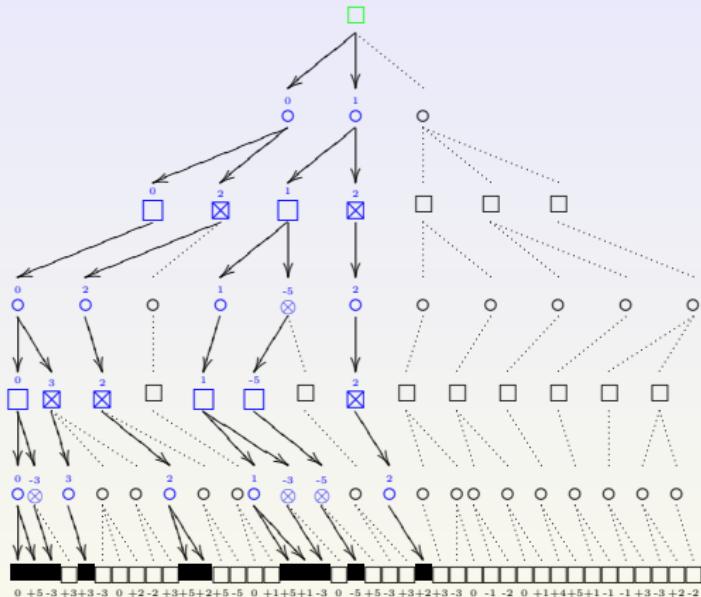


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
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```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
  →  $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 0 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

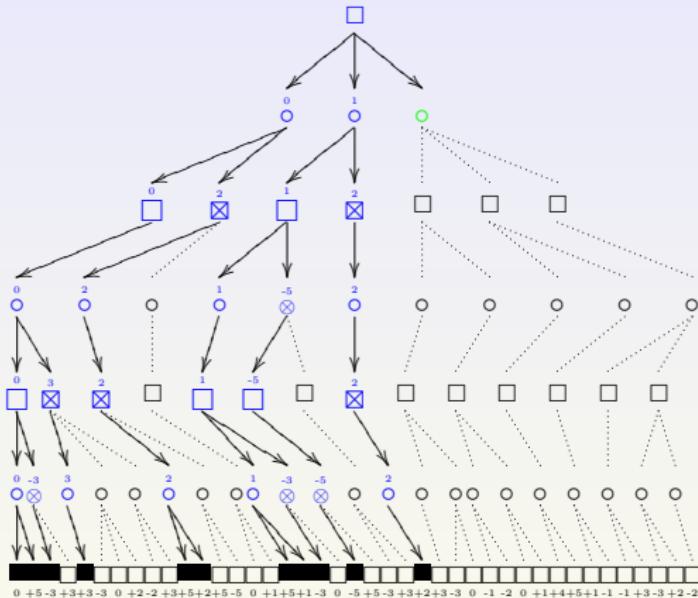
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

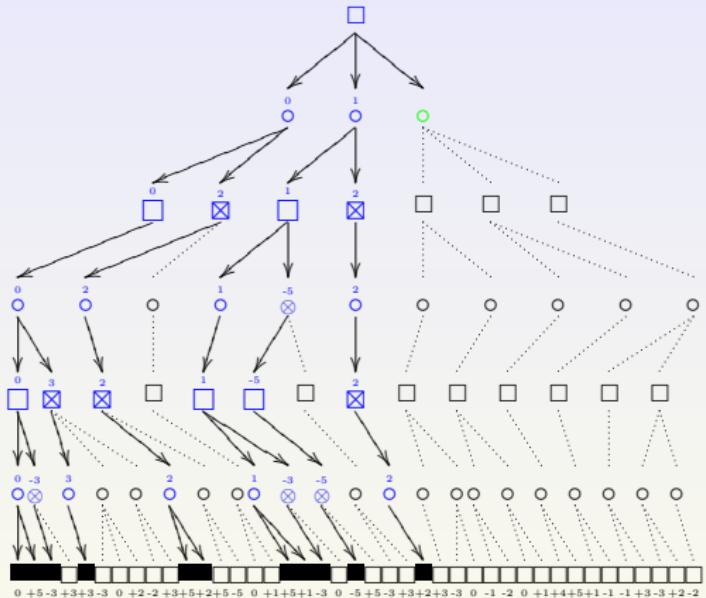
```

```

→function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = +\infty$$

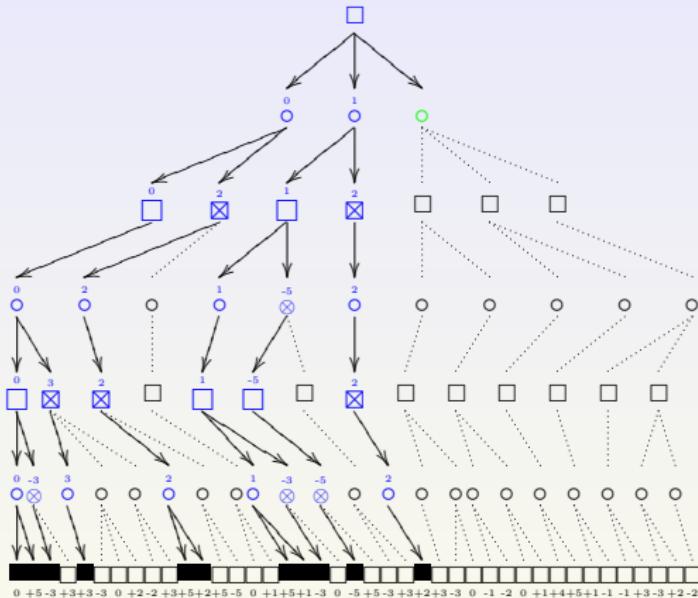


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

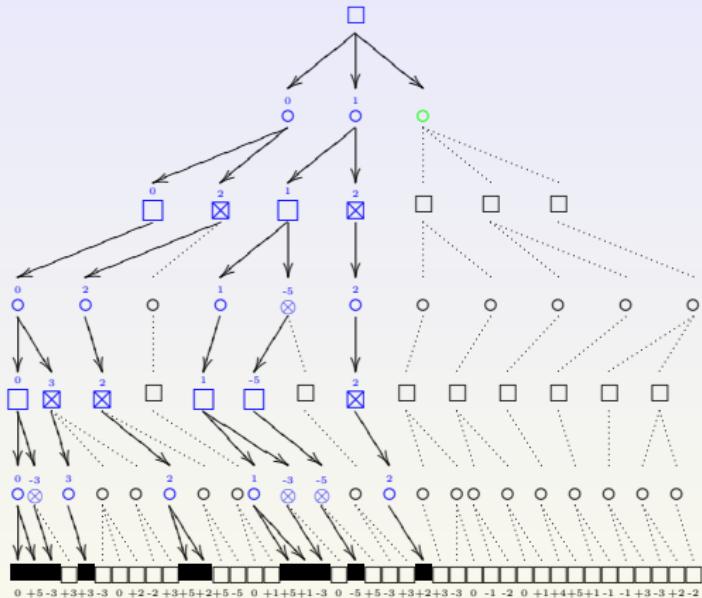
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

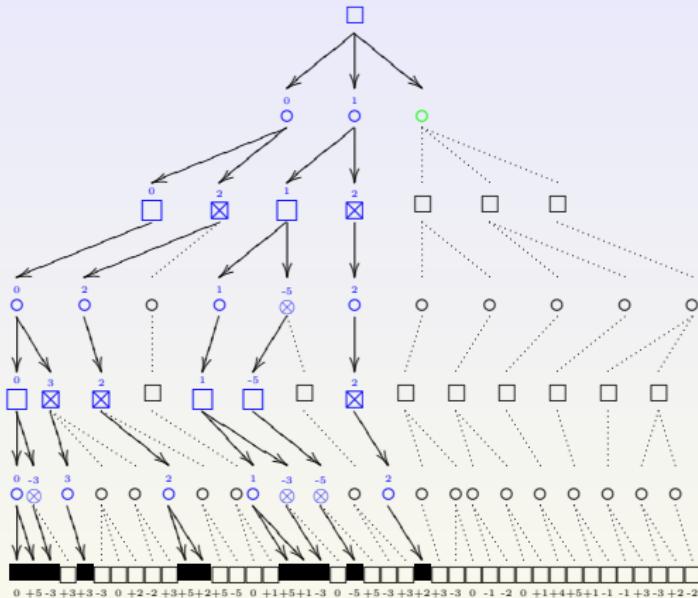
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

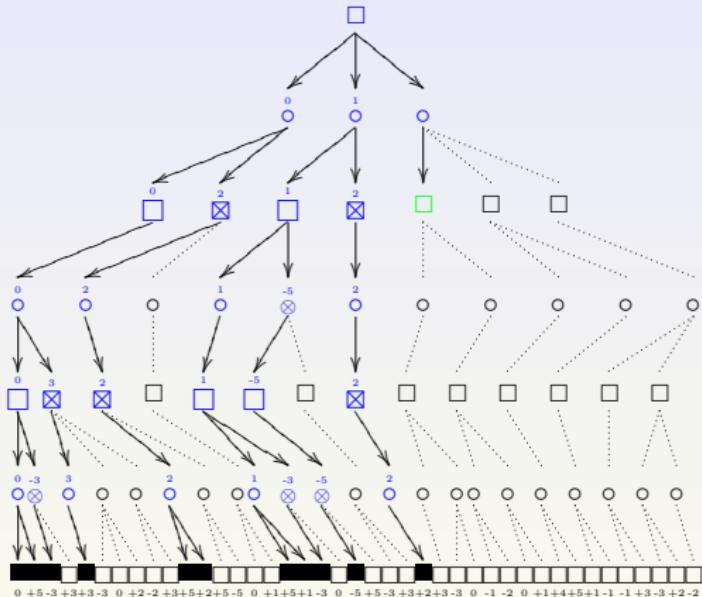
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

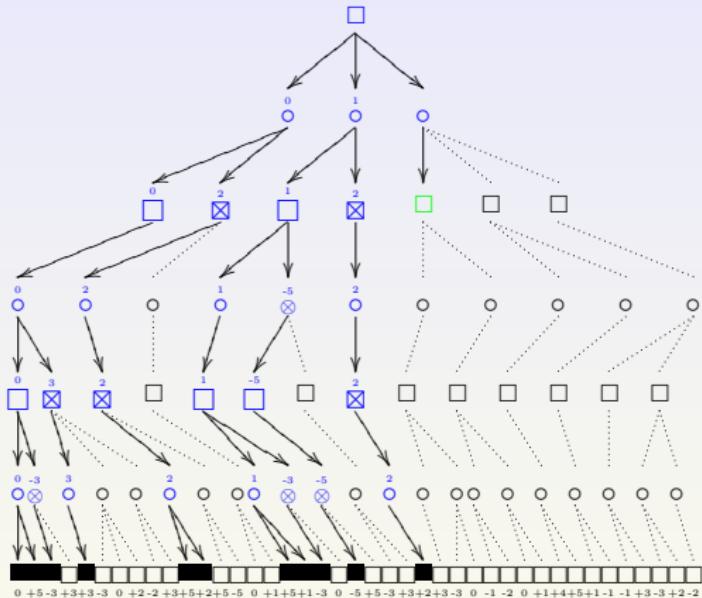
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

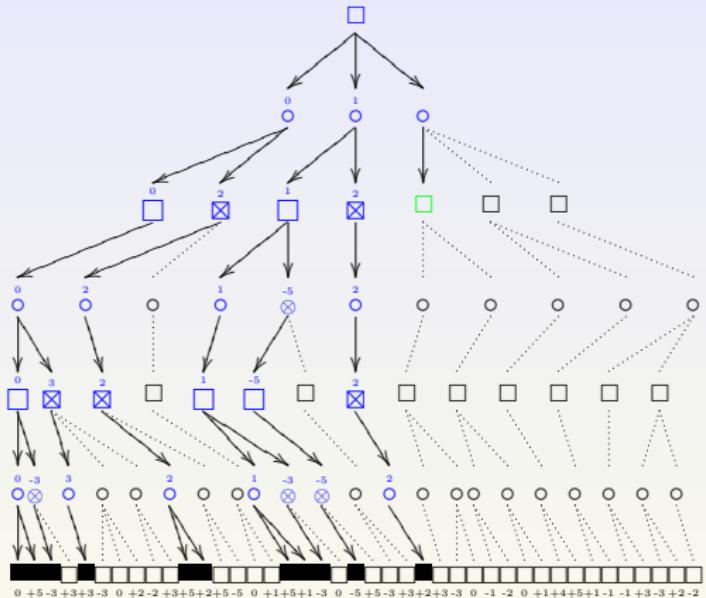
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

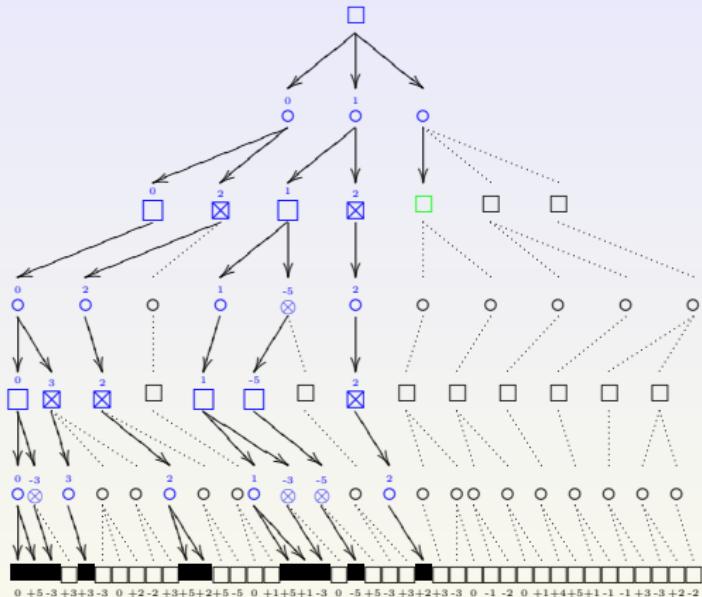
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

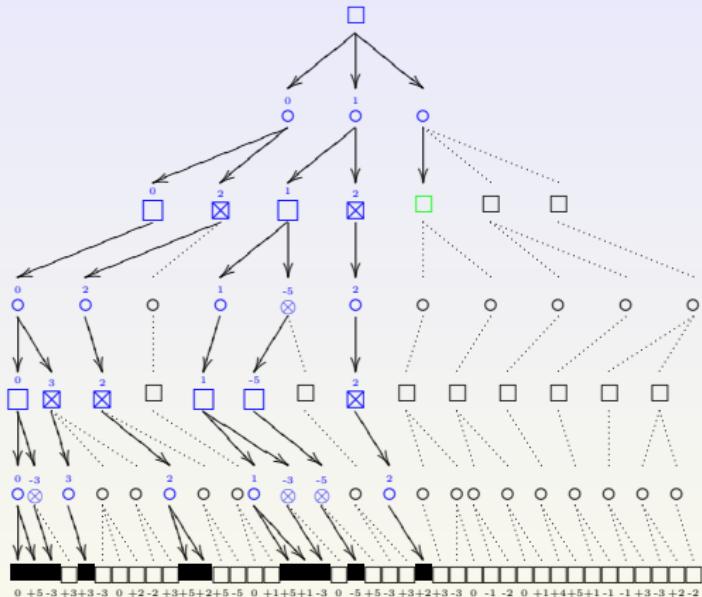
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

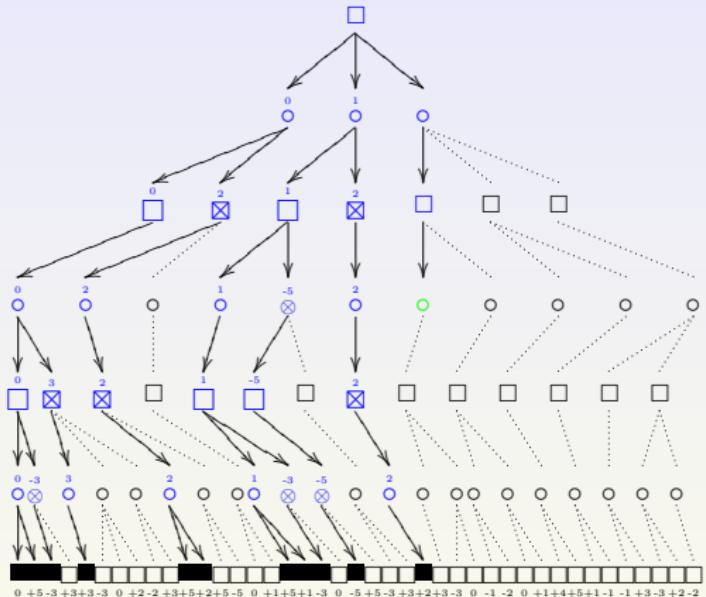
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

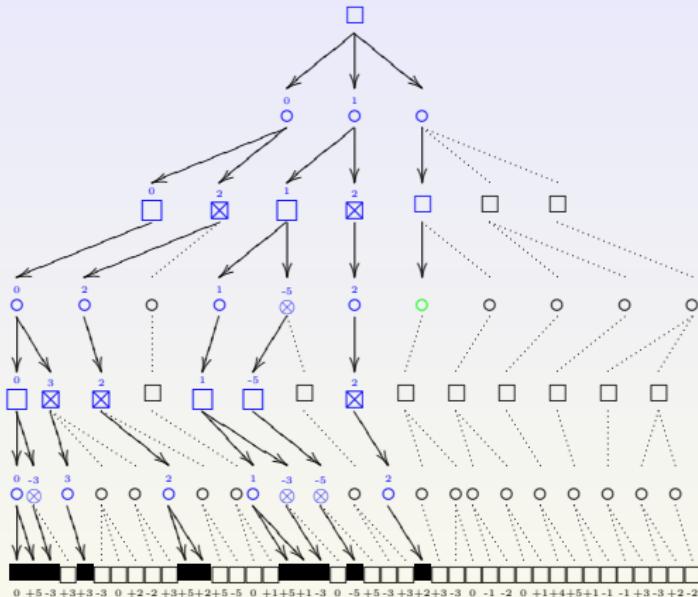


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

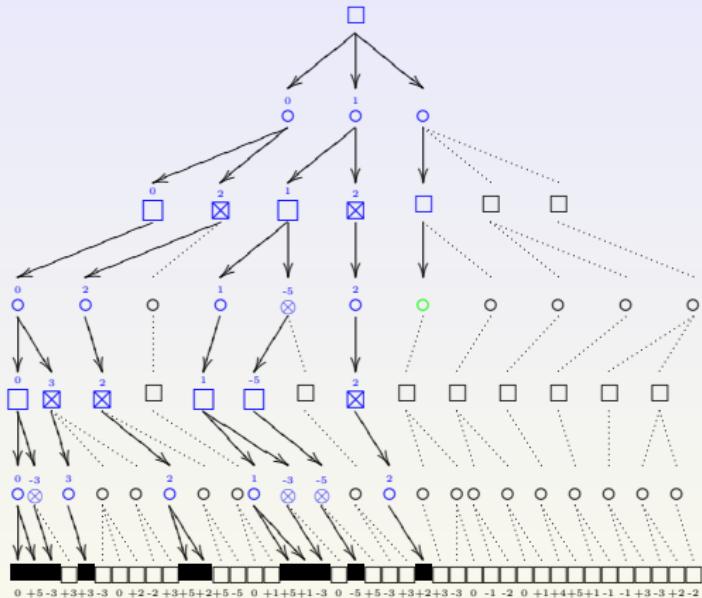
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  → if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
       $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
      if  $v \leq \alpha$  then
        return  $v$ 
       $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 
  
```

$$\alpha = 1 \quad \beta = +\infty$$



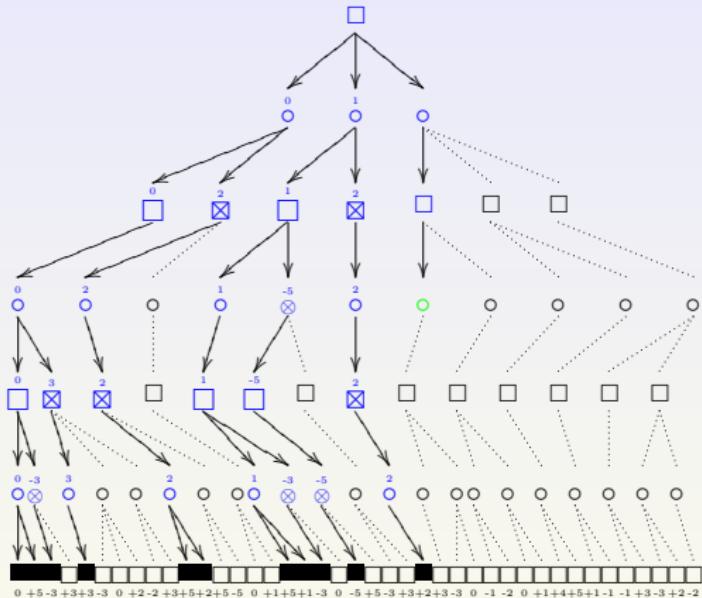


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

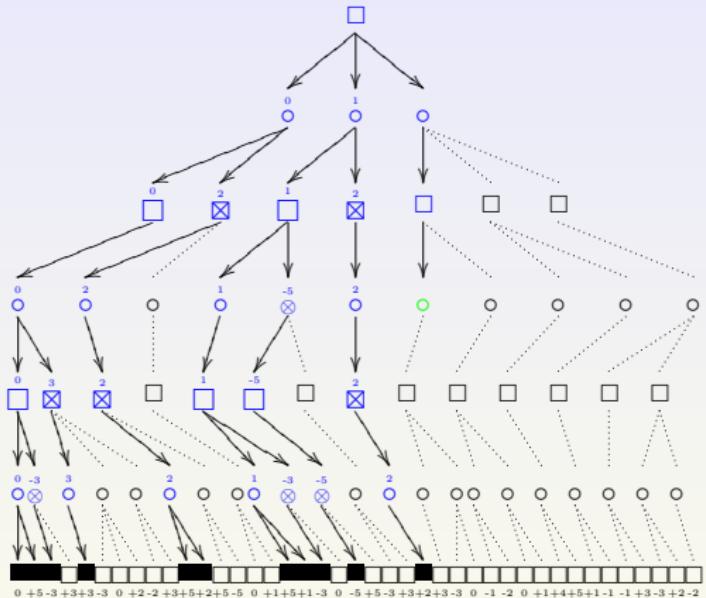


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  → for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

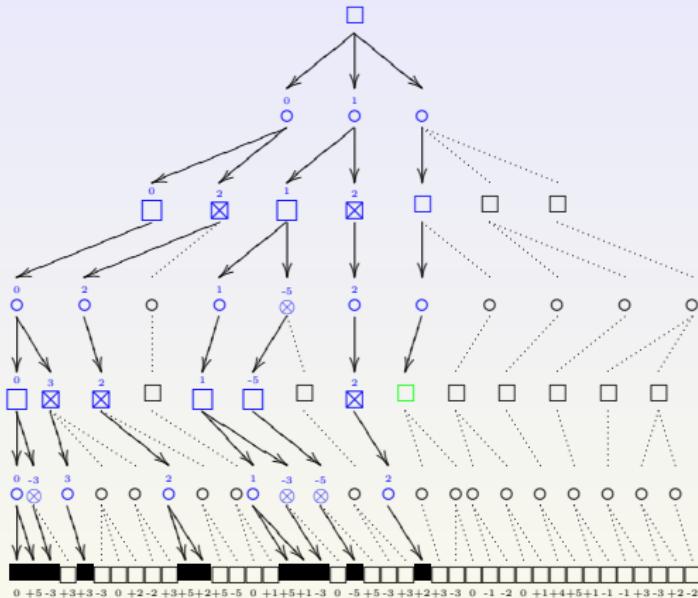


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

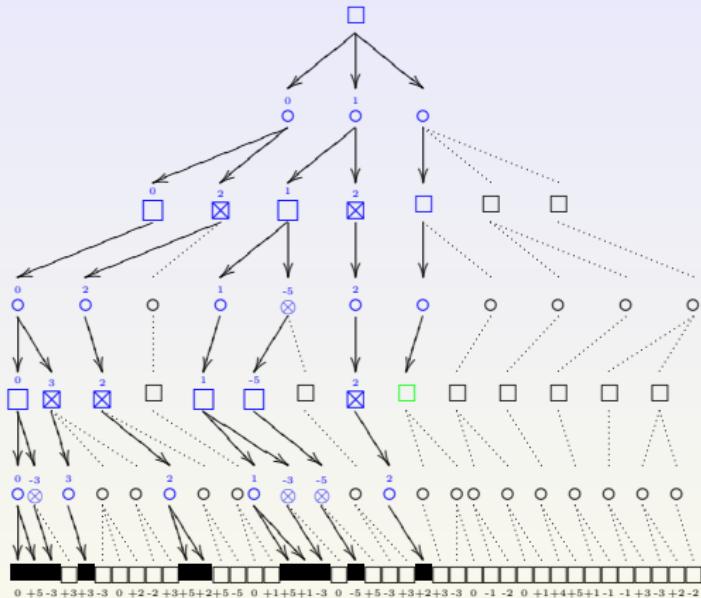
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

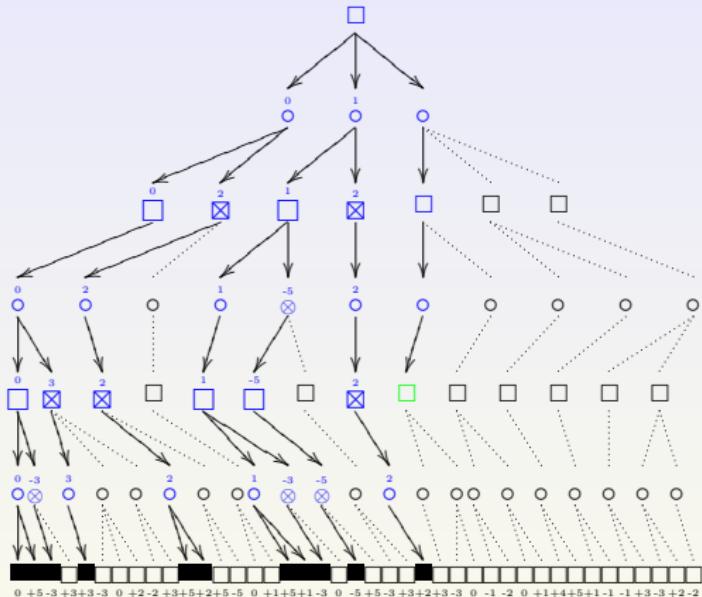
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

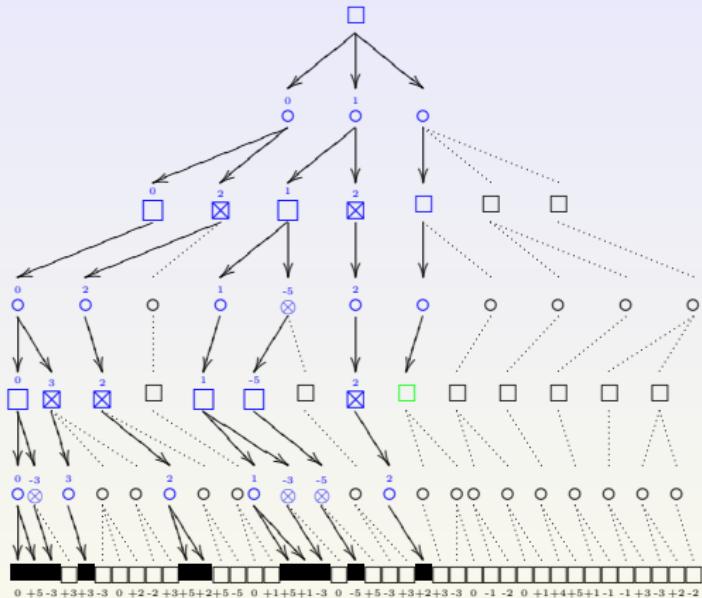


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

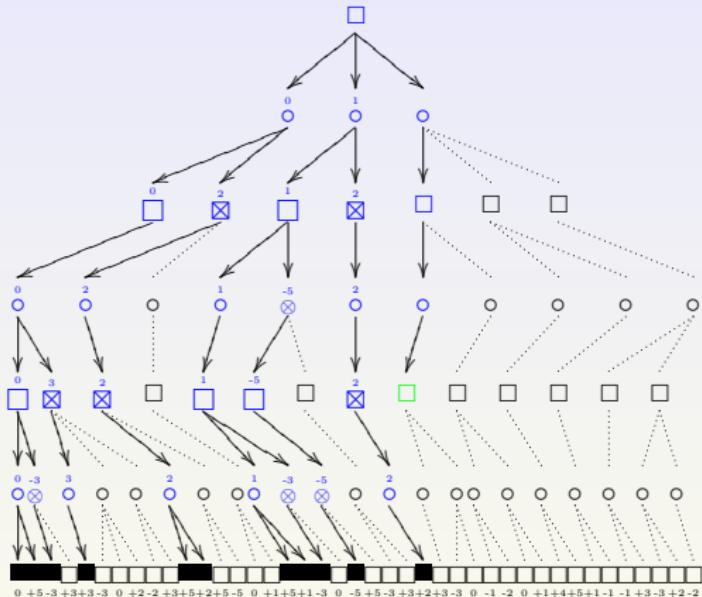


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

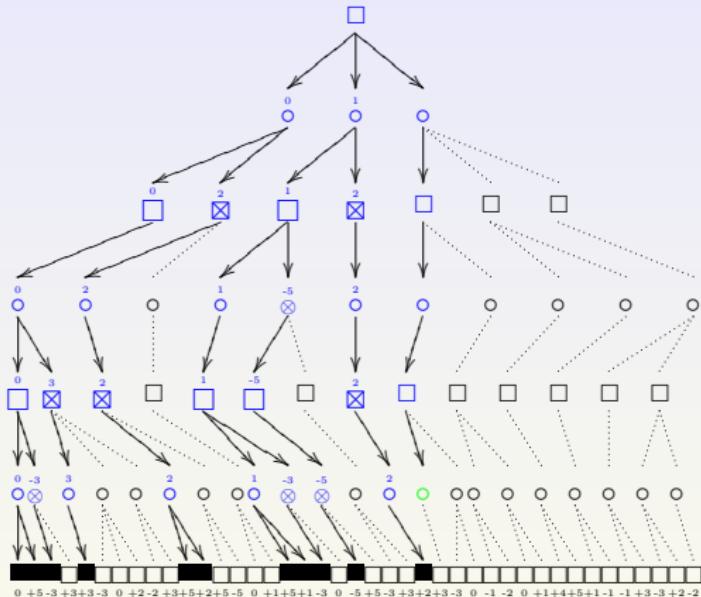


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
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  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

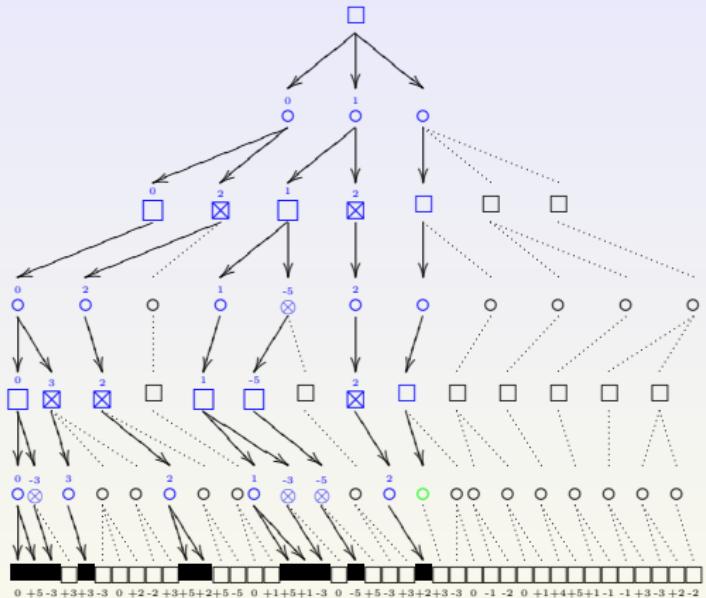


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



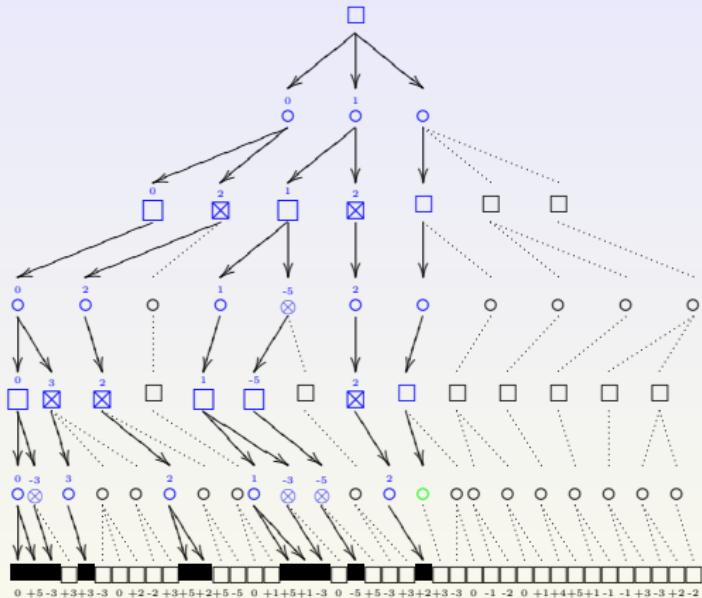
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

→

```
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    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

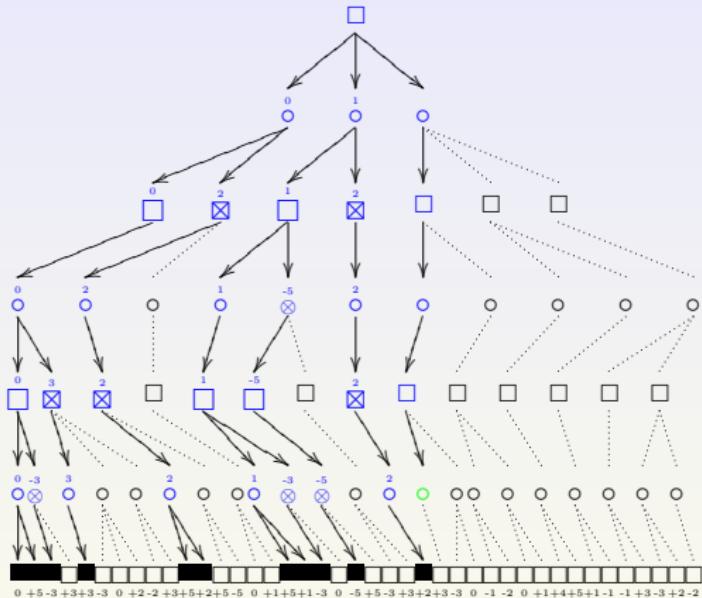


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  if TERMINAL-TEST(state) then
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  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
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  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

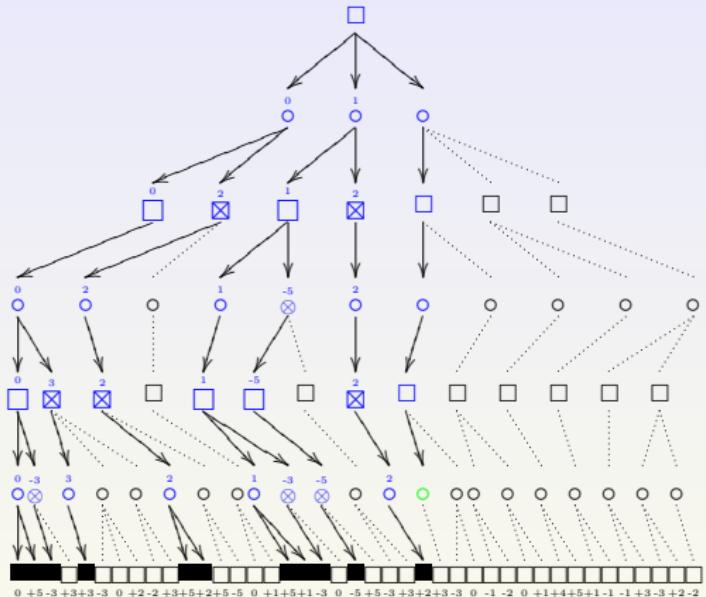


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
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  v ← +∞
  → for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

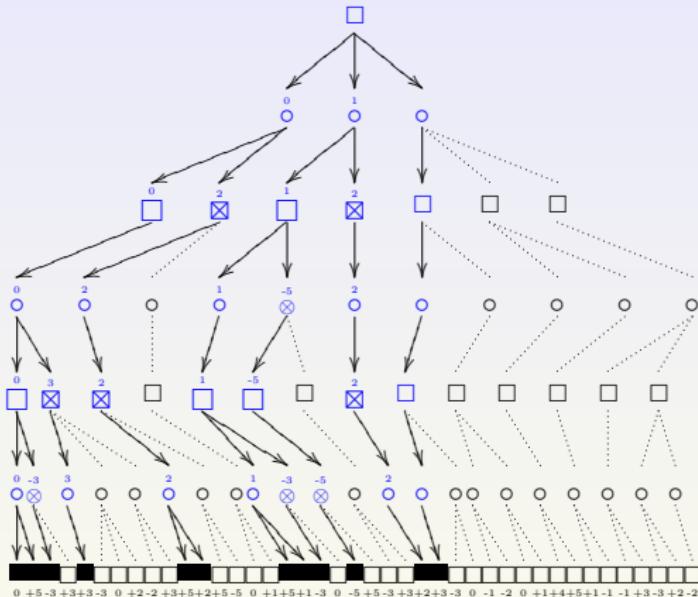


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $s, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $s$ ) then
    return UTILITY( $s$ )
   $v \leftarrow -\infty$ 
  for  $a, s'$  in SUCCESSORS( $s$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s', \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

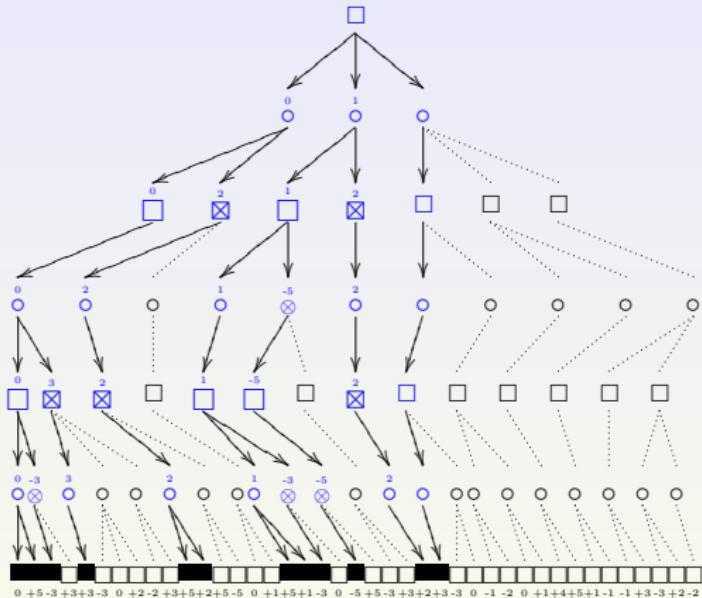
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\text{v}, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

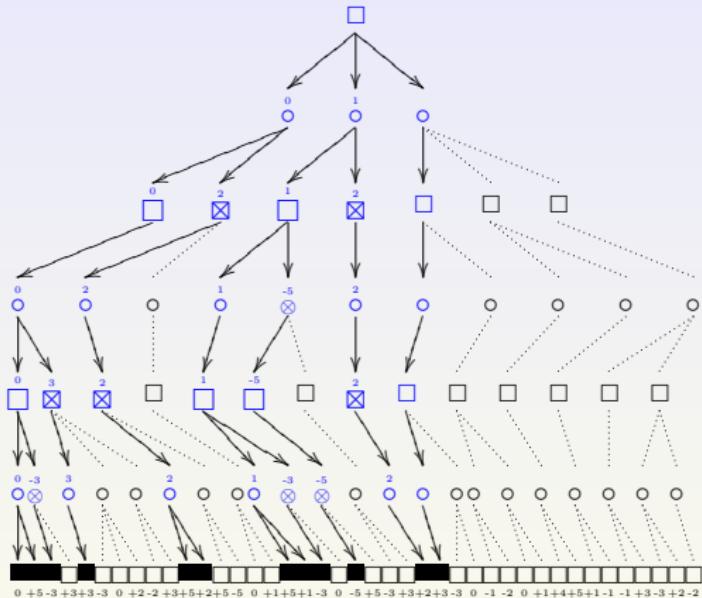
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$

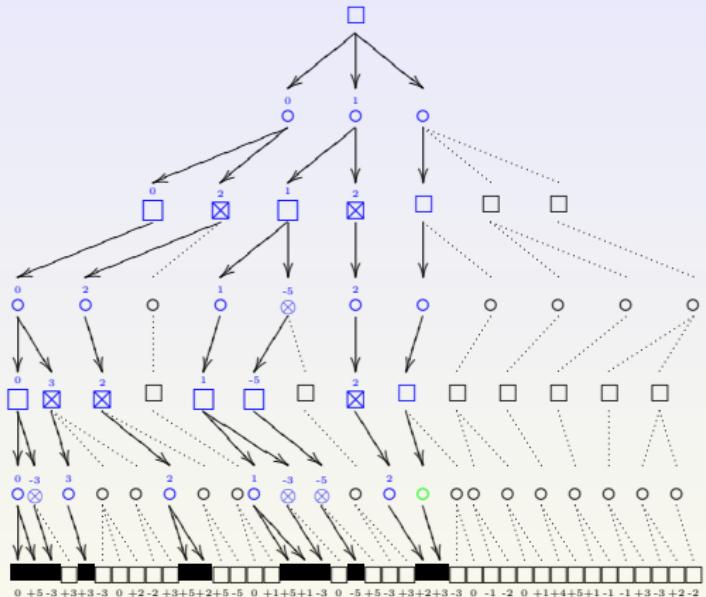


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    → return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

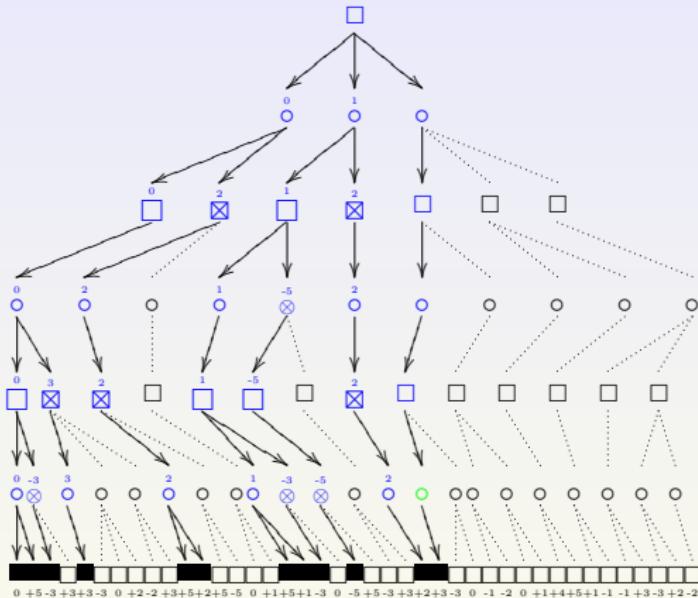
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)
  return v$ 
```

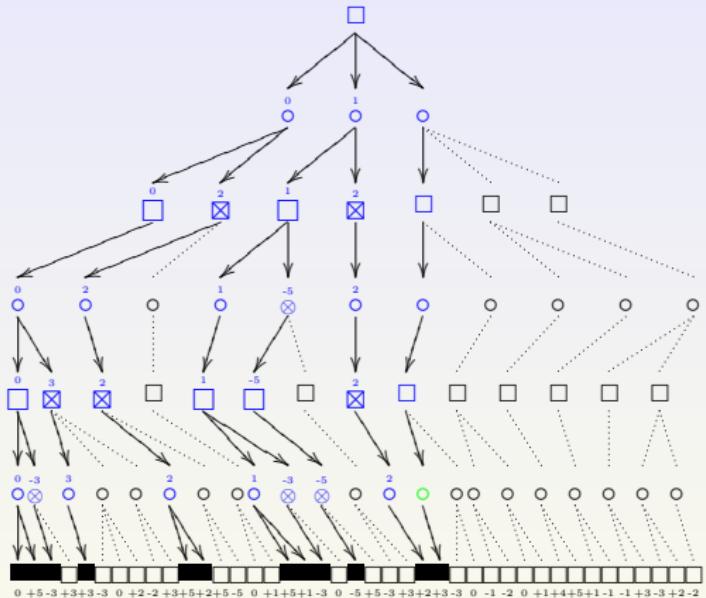
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

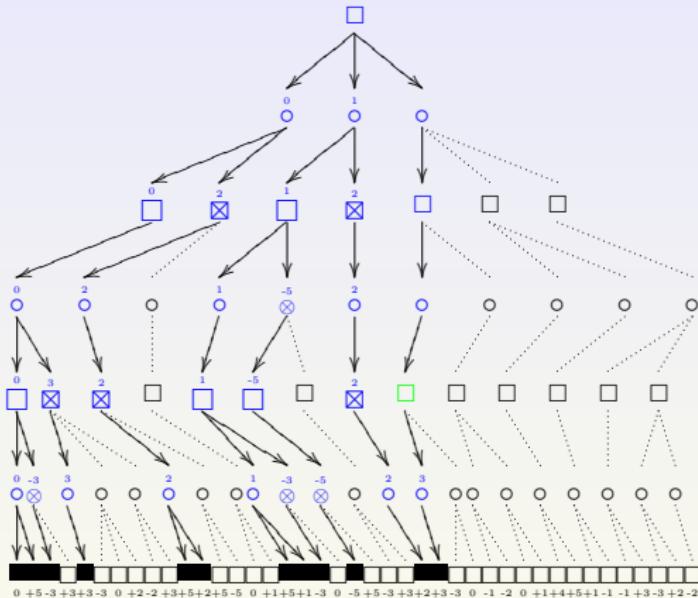
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

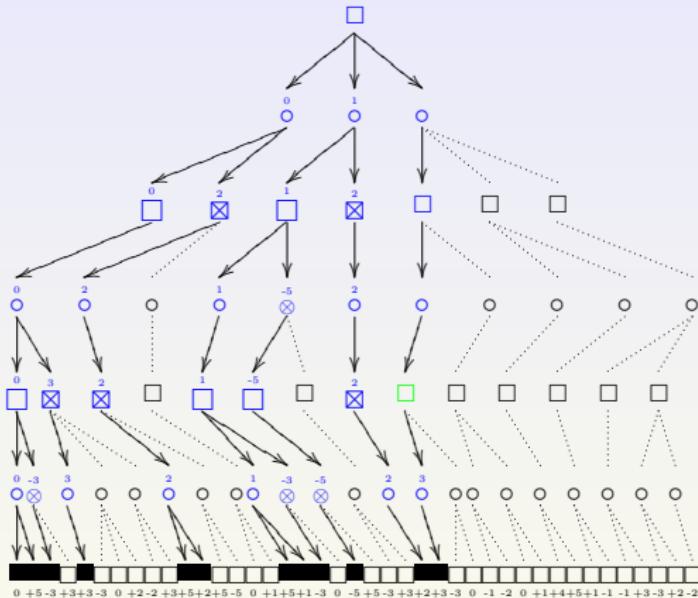
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
  v  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

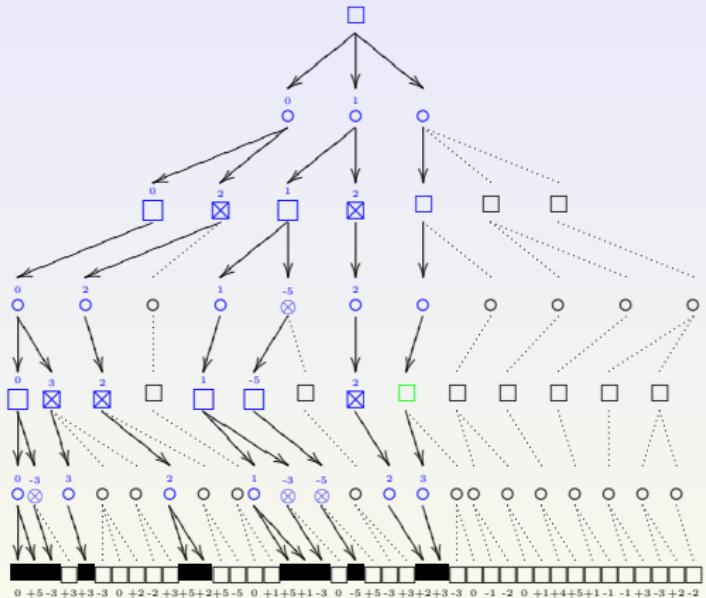
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

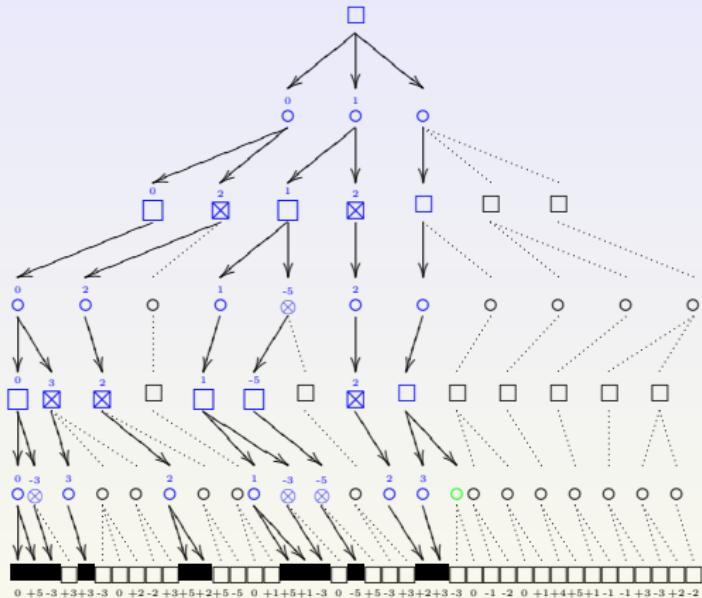
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
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    return UTILITY(state)
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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

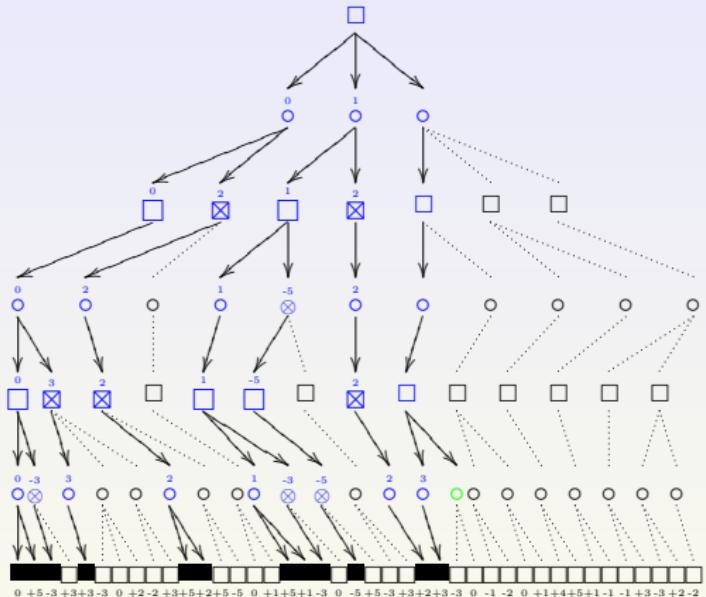
→

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
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```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

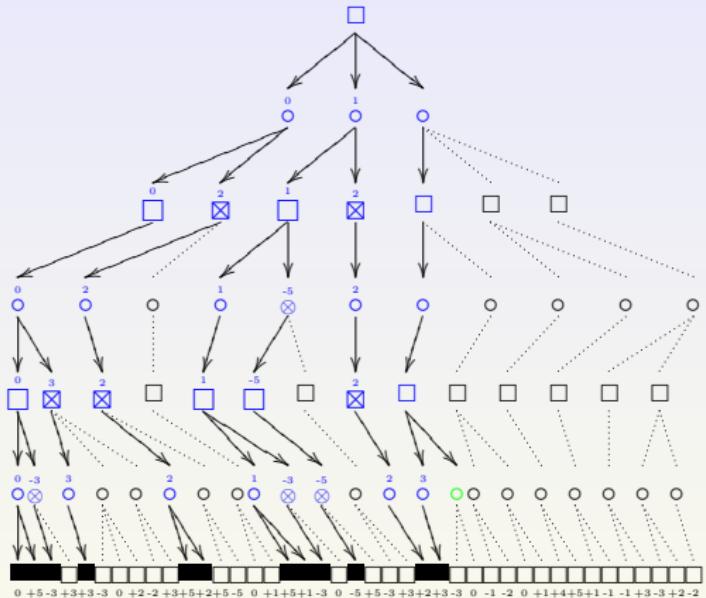
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

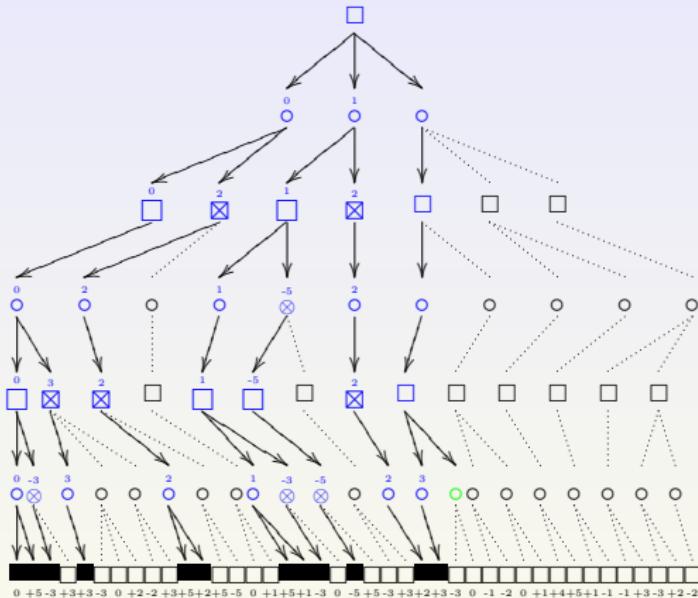
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

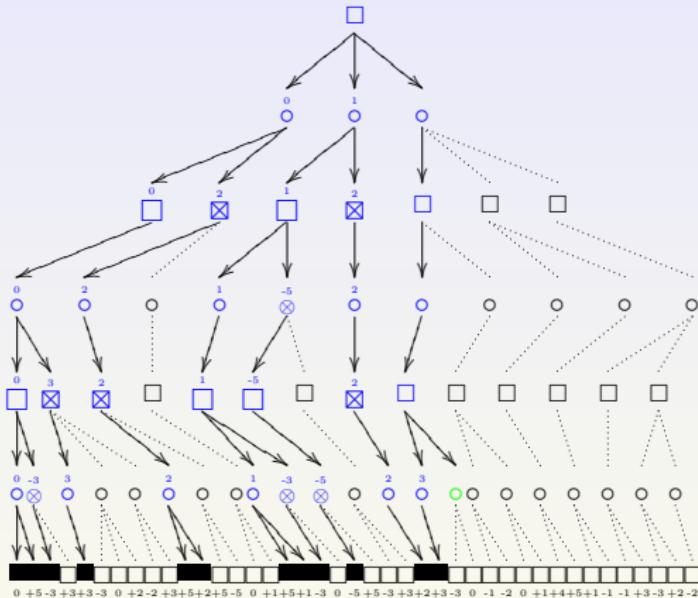
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

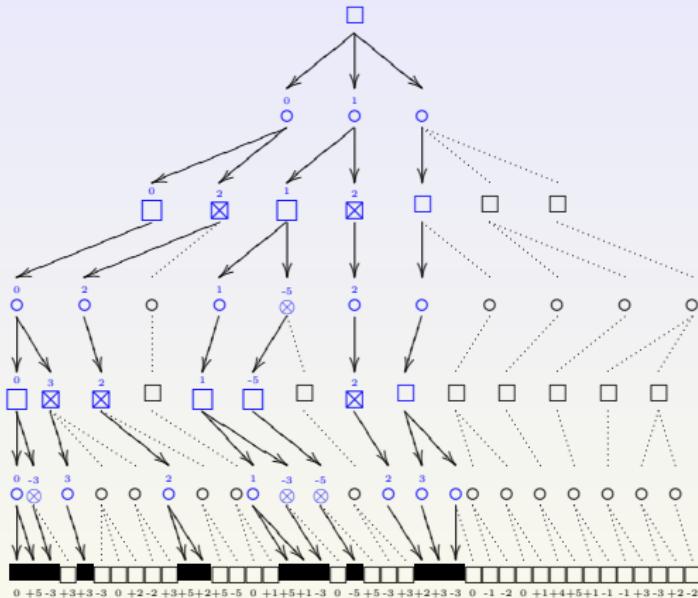
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $s, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $s$ ) then
    return UTILITY( $s$ )
   $v \leftarrow -\infty$ 
  for  $a, s'$  in SUCCESSORS( $s$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s', \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

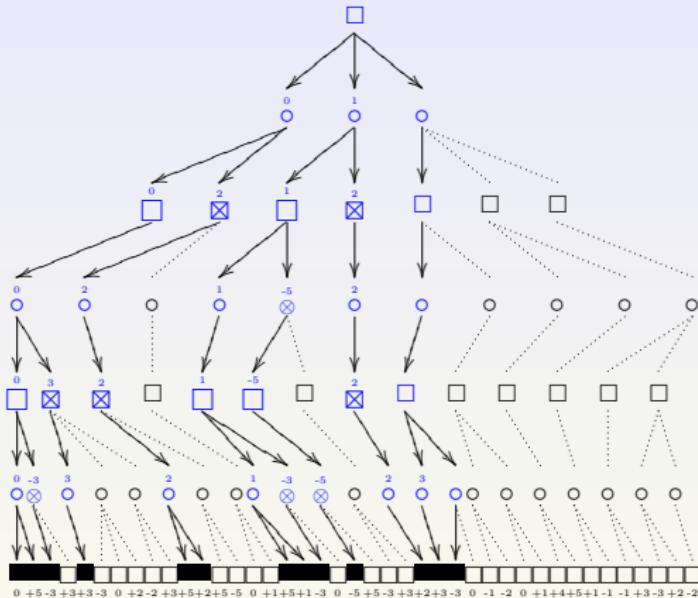
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq$   $\alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

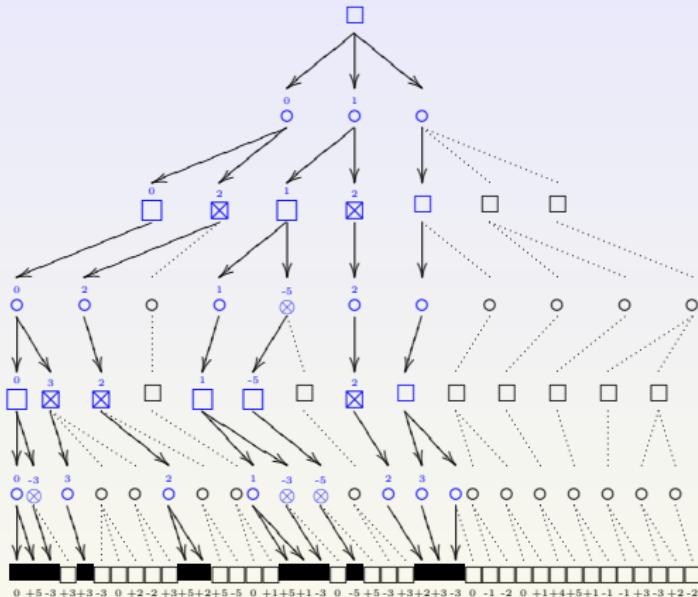
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\text{v}, \text{MAX-VALUE}(\iota, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

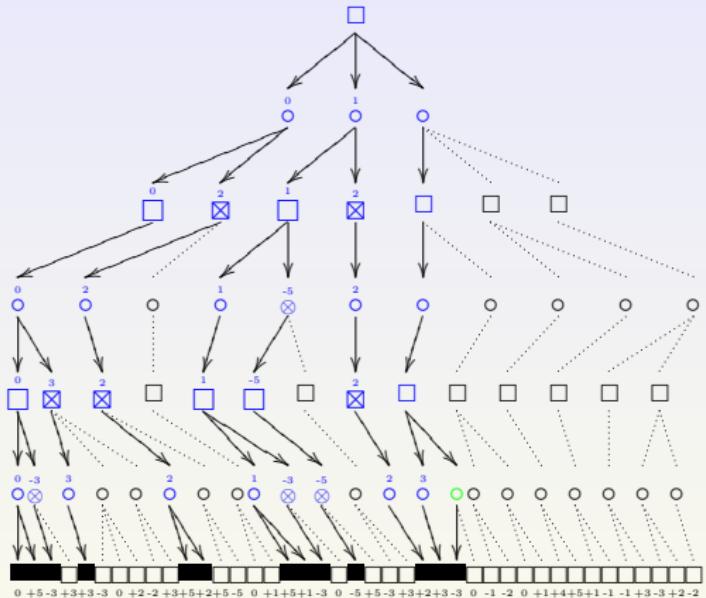
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





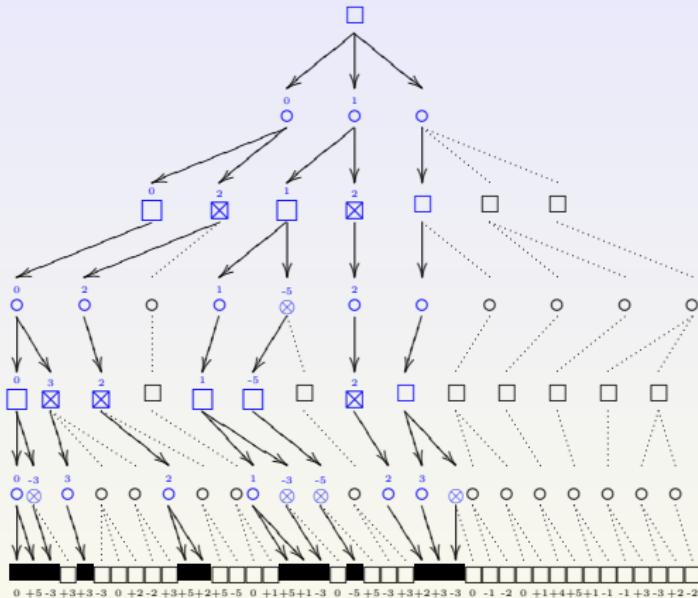
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

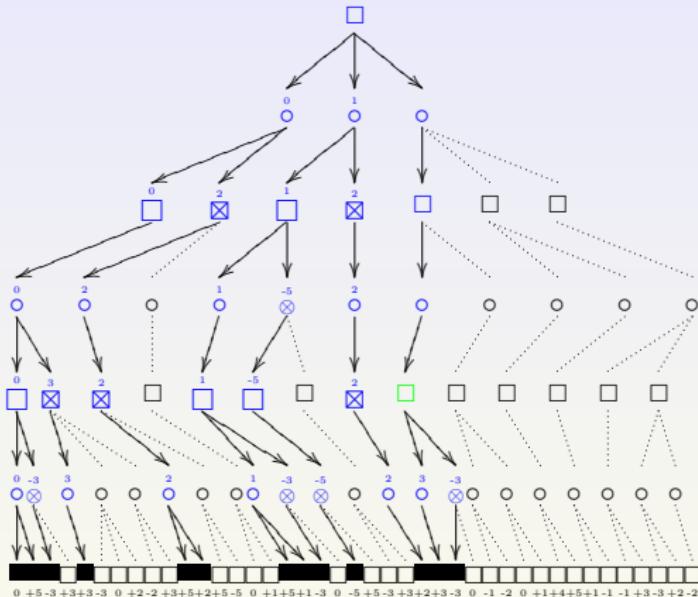
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

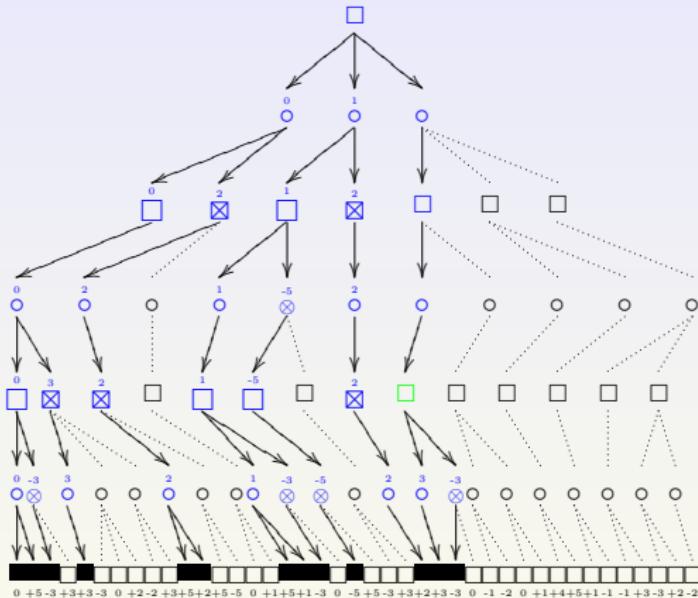
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
  v  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

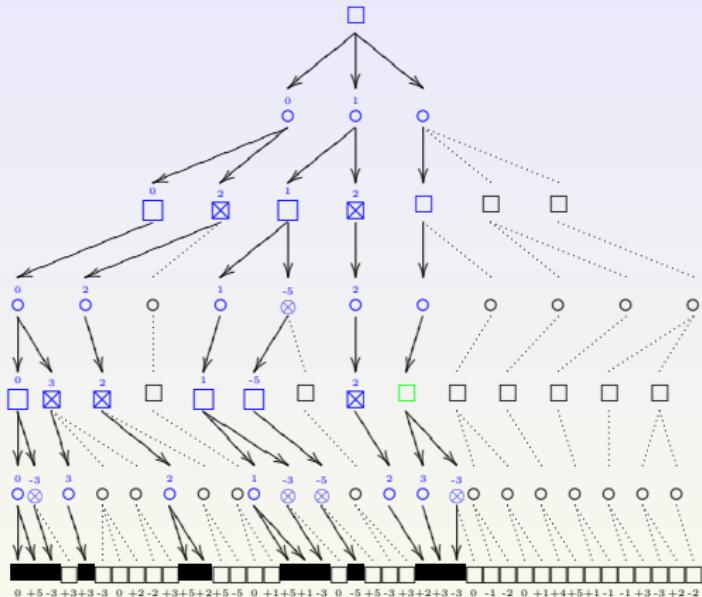
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

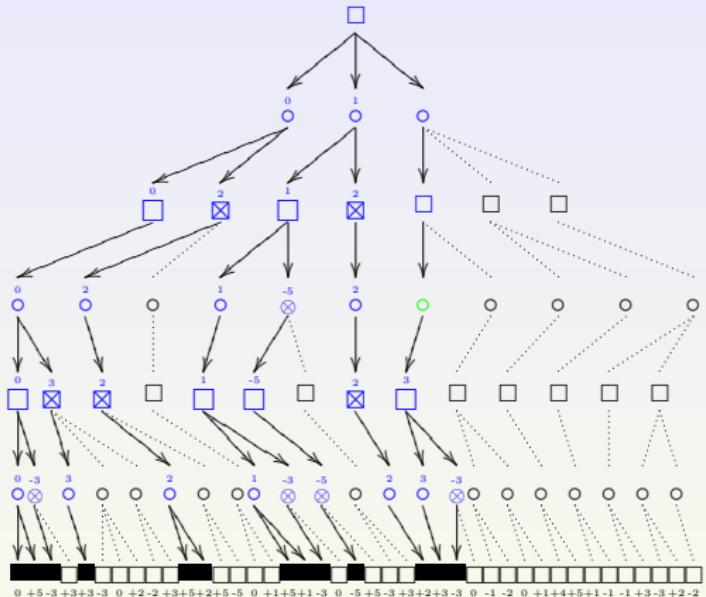
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
    β  $\leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



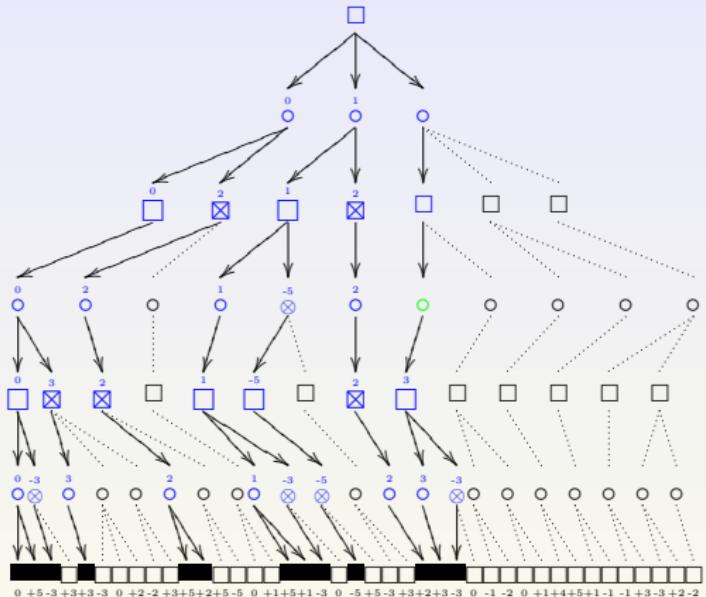
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



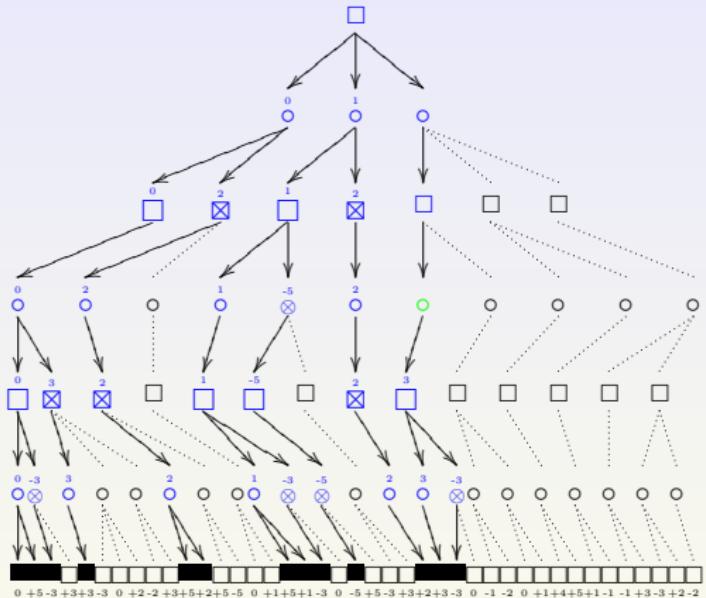


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    → β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow$  MAX(α, v)
  return v

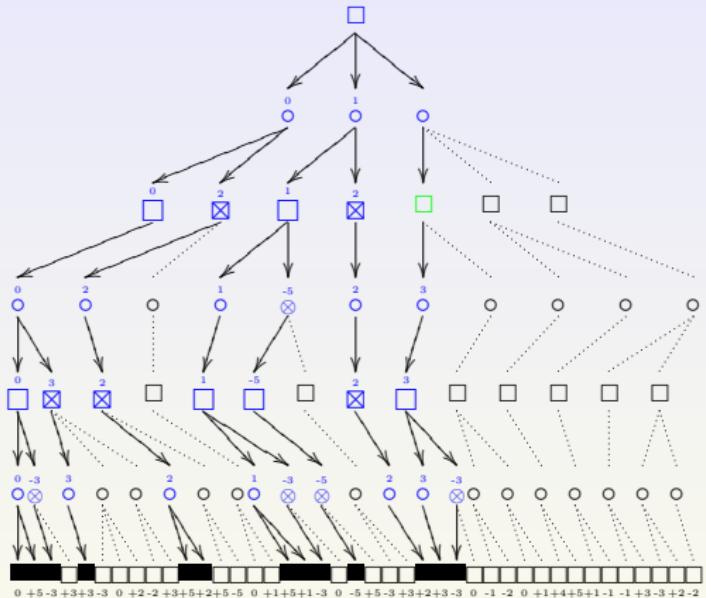
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
    β  $\leftarrow$  MIN(β, v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$

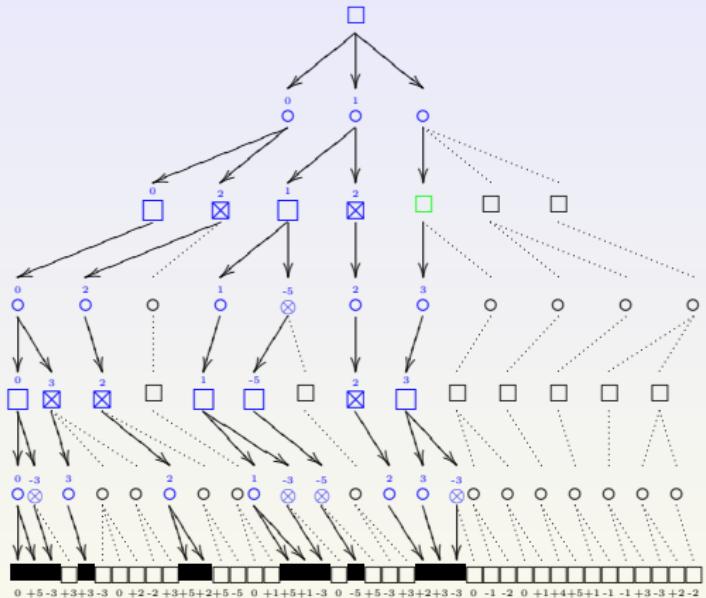


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    → if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

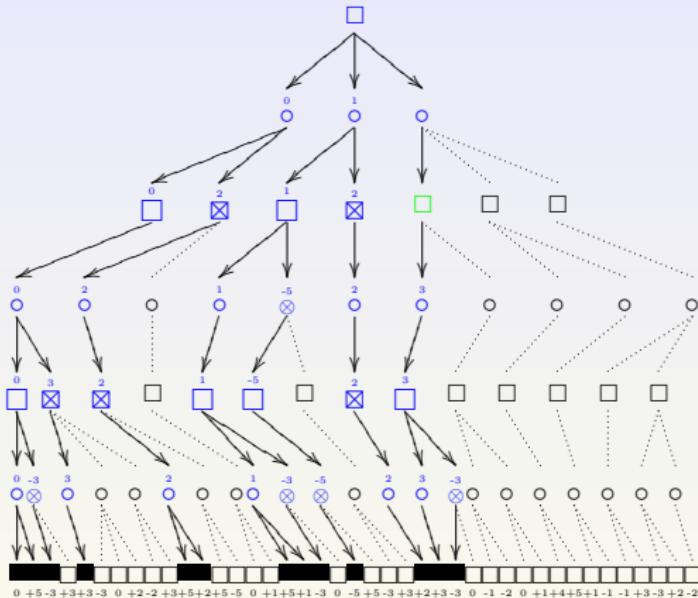


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
  → α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)
  return v$ 
```

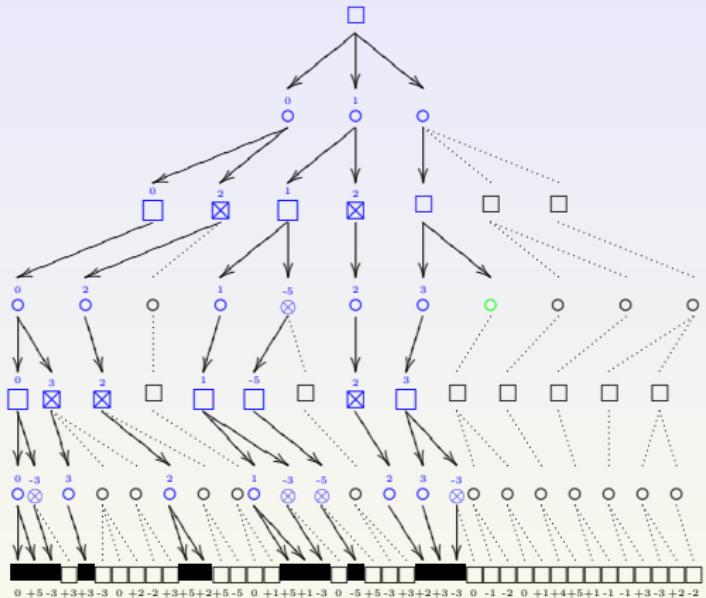
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



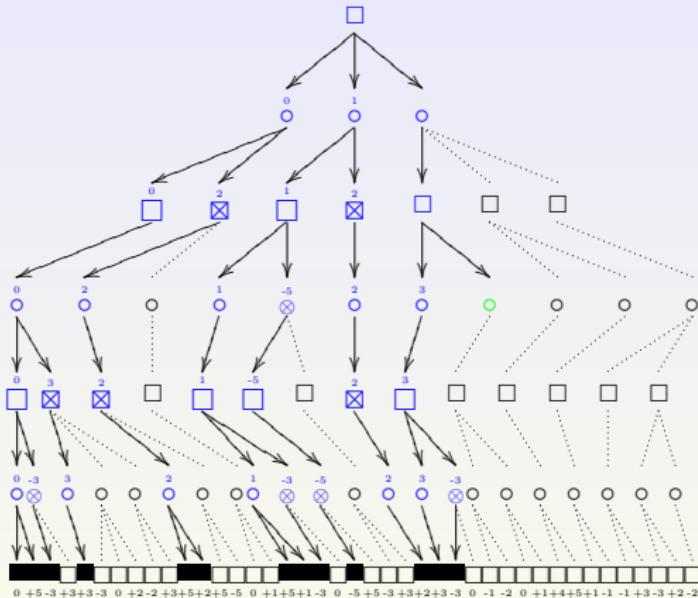


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

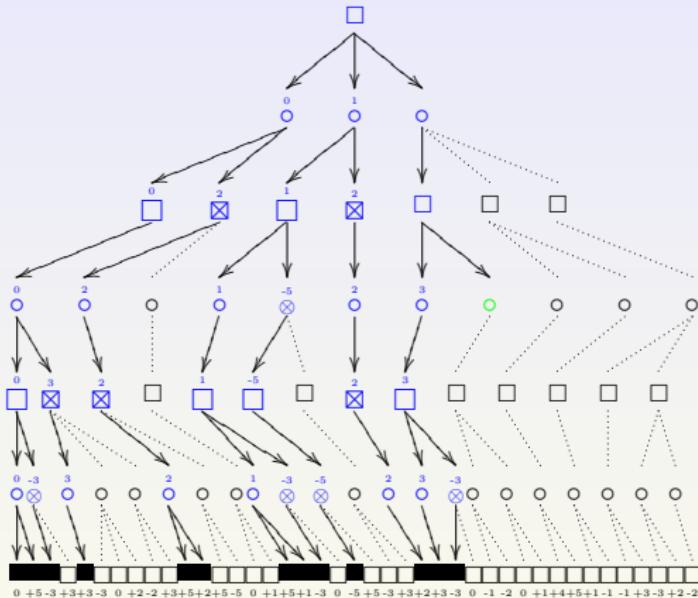
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  → if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
       $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
      if  $v \leq \alpha$  then
        return  $v$ 
       $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 
  
```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

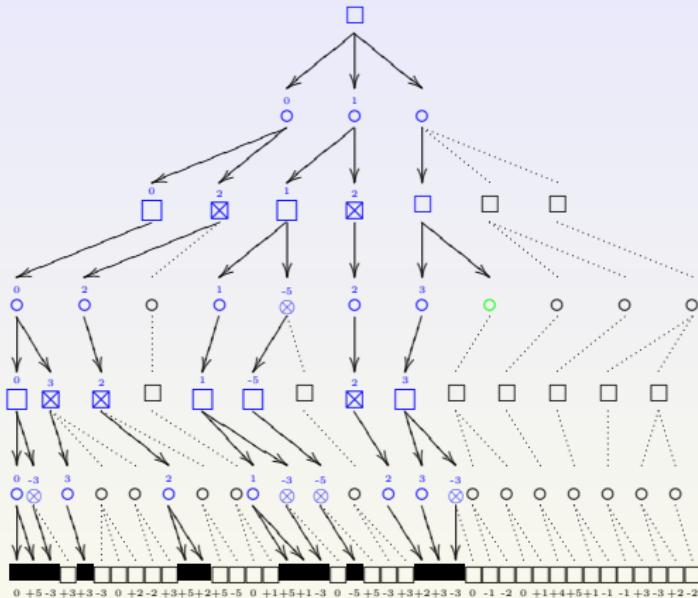
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

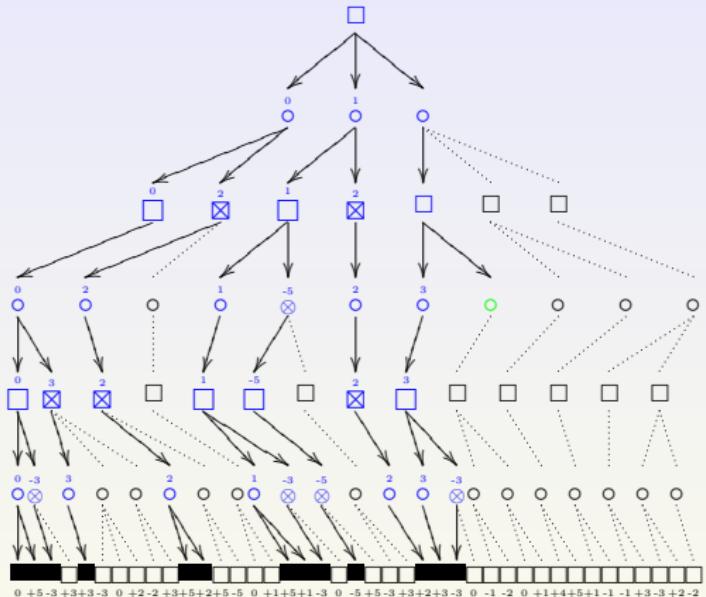
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



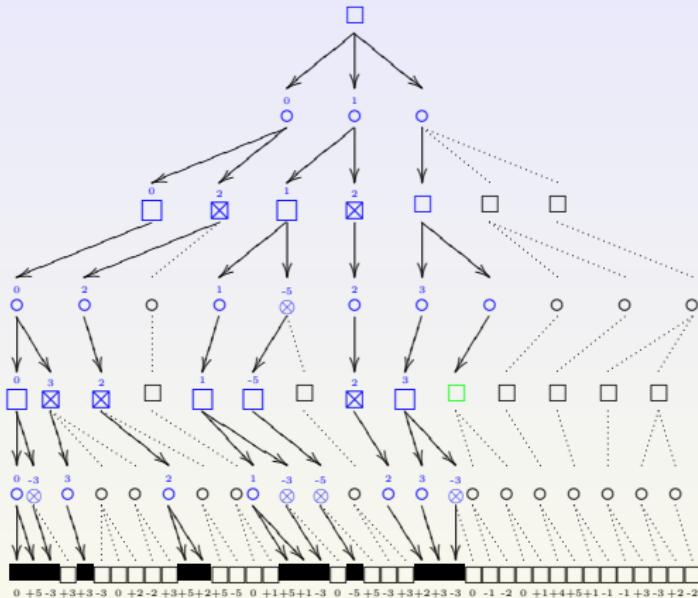


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

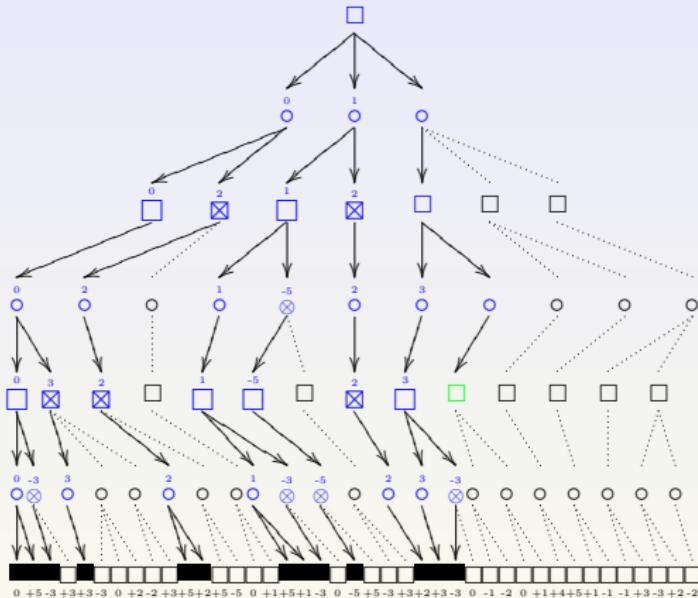
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq$   $\alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

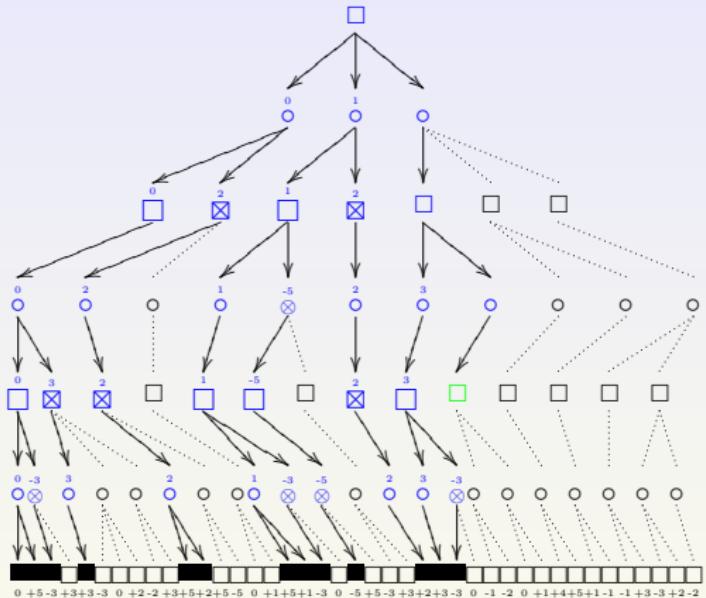
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

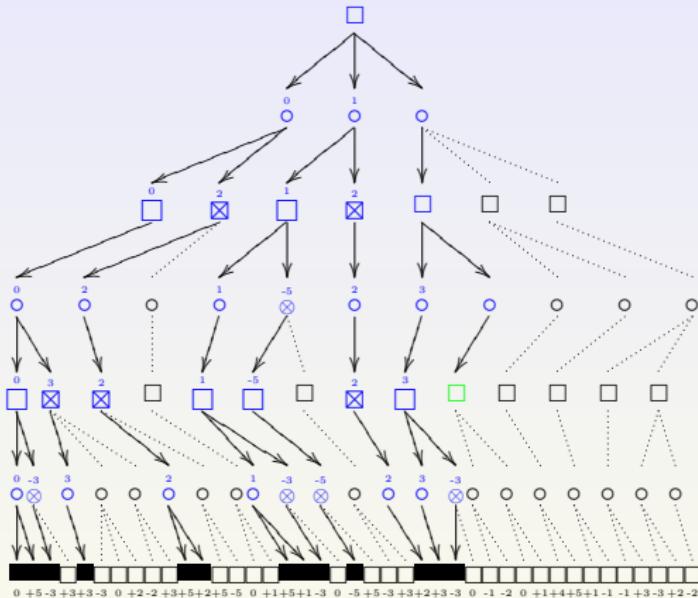
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

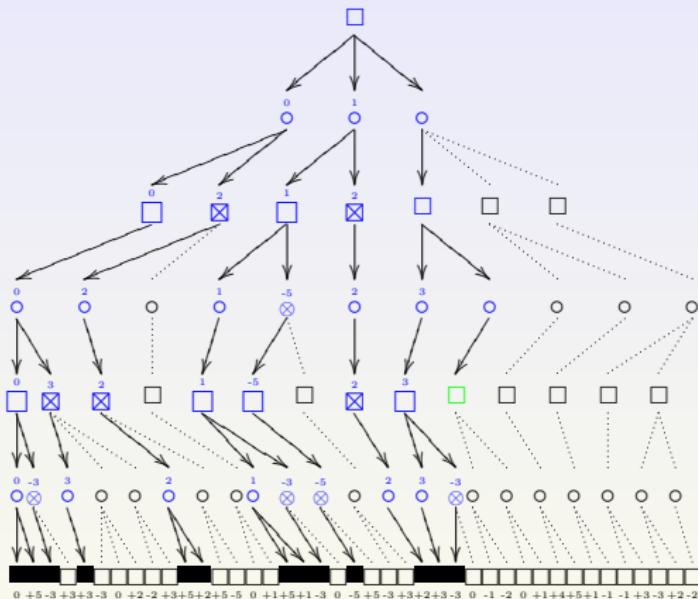
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

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```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow$  MAX(alpha, v)
  return v

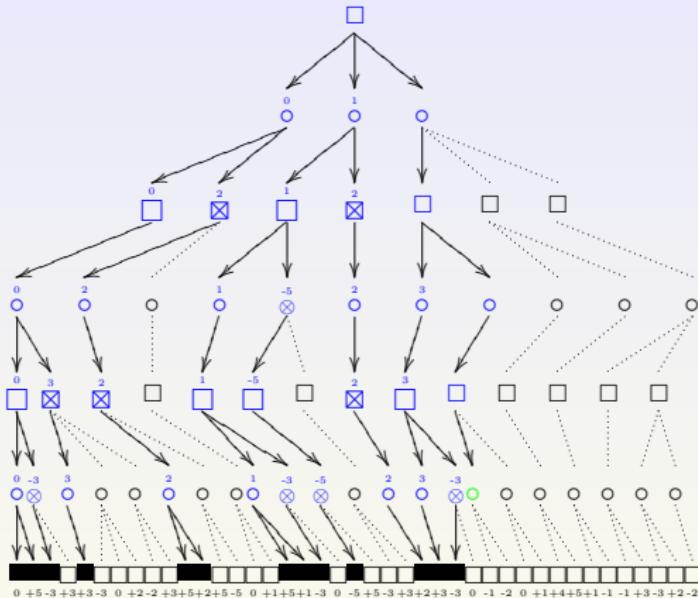
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

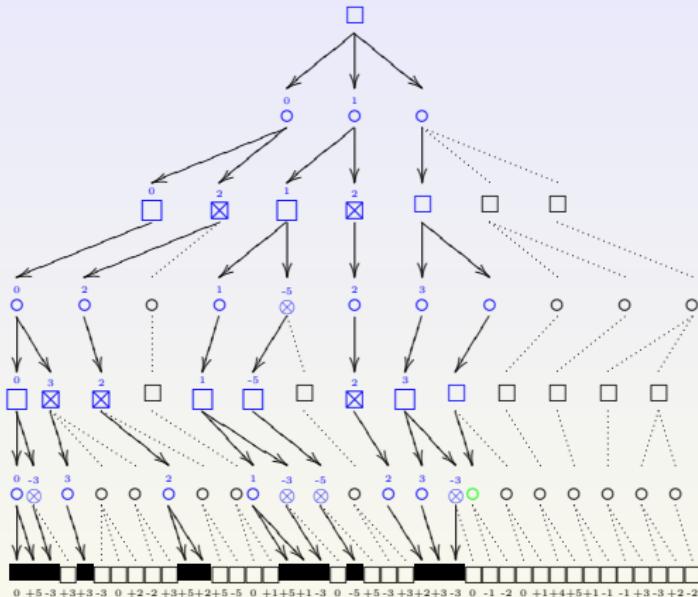
```

→function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

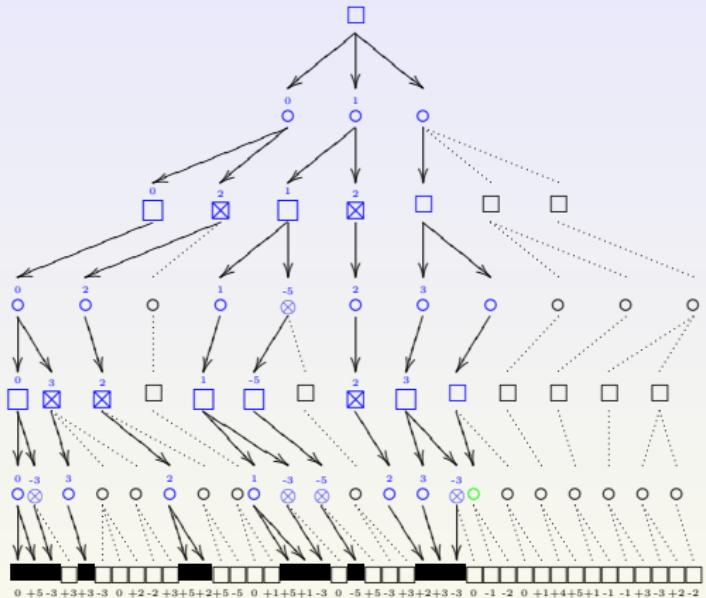
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  → if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
       $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
      if  $v \leq \alpha$  then
        return  $v$ 
       $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 
  
```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

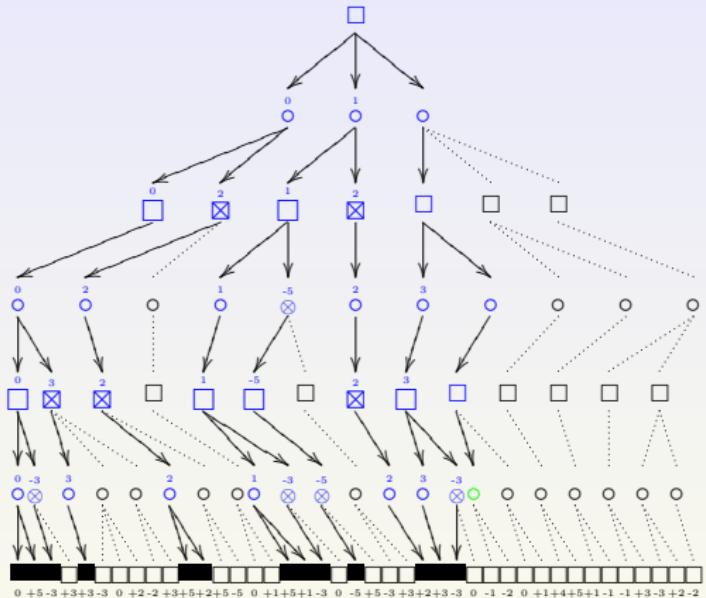
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

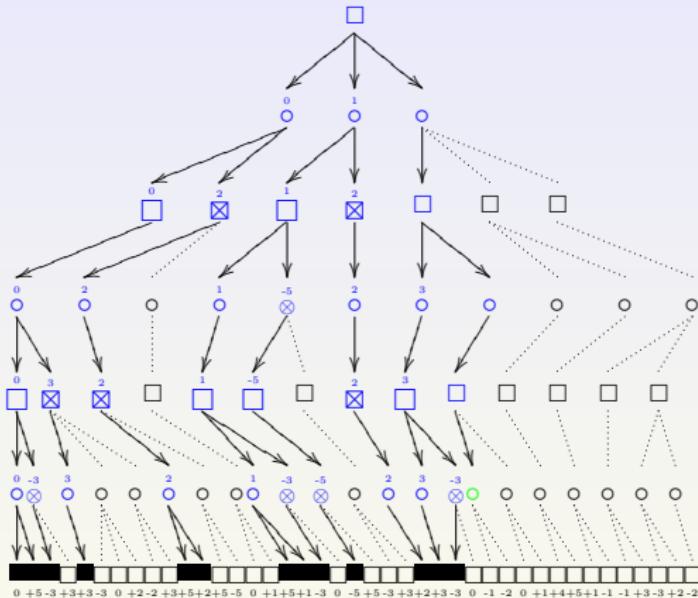
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

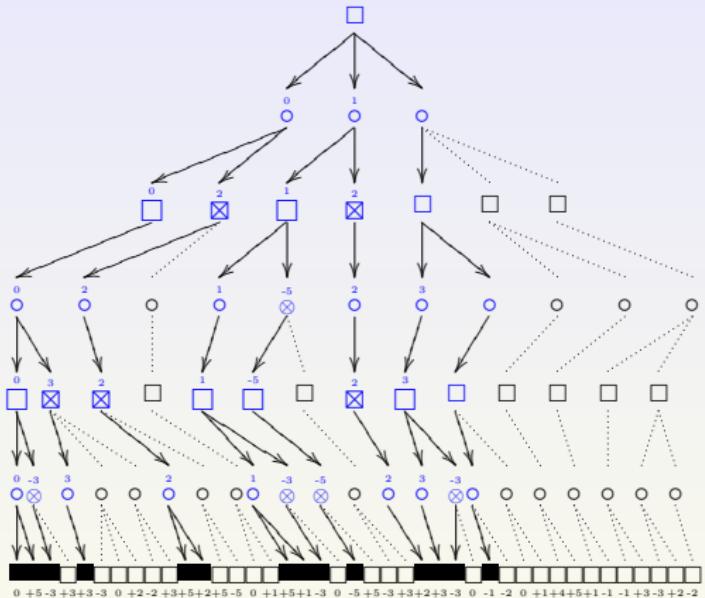
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

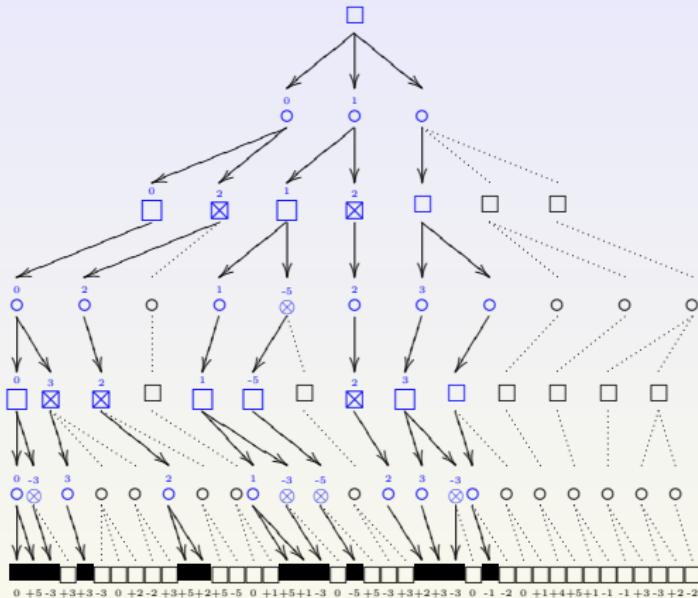
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

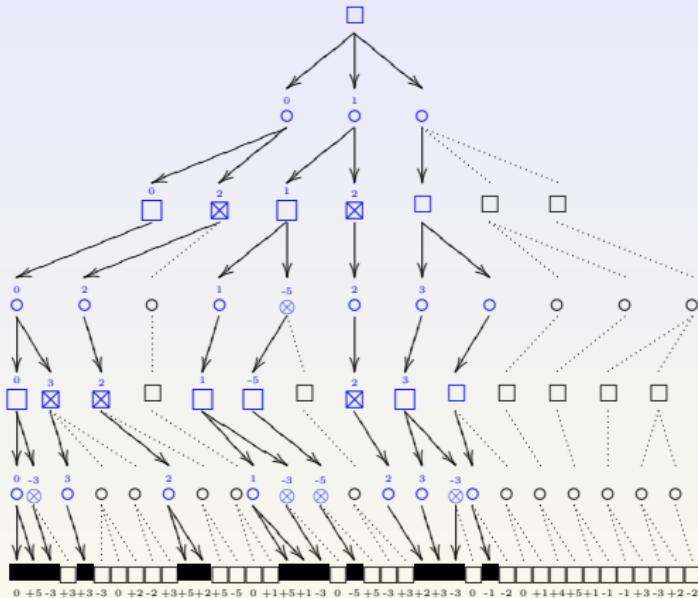
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

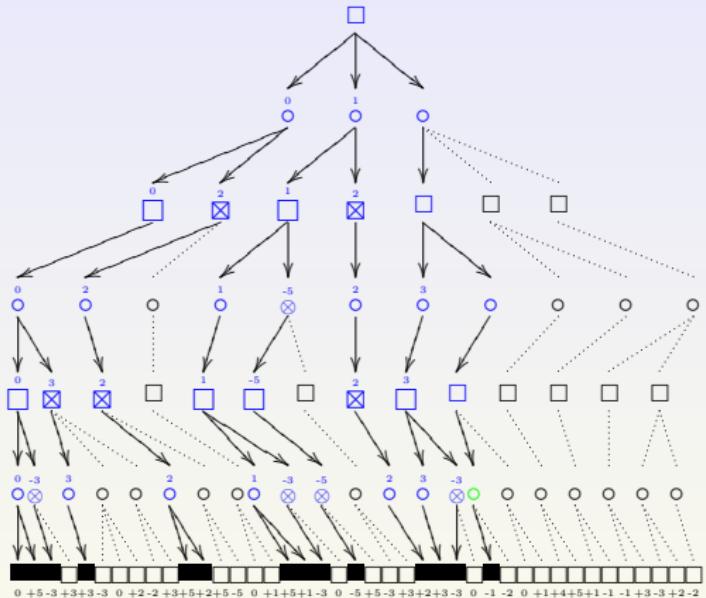
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



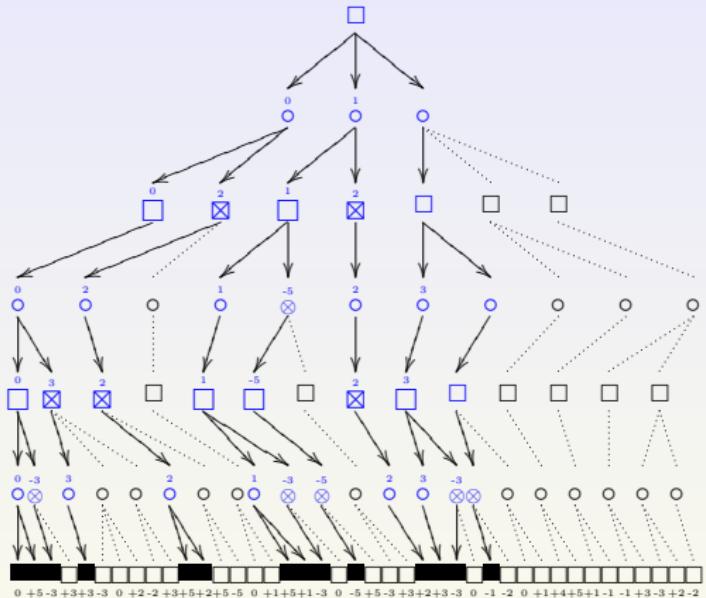


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

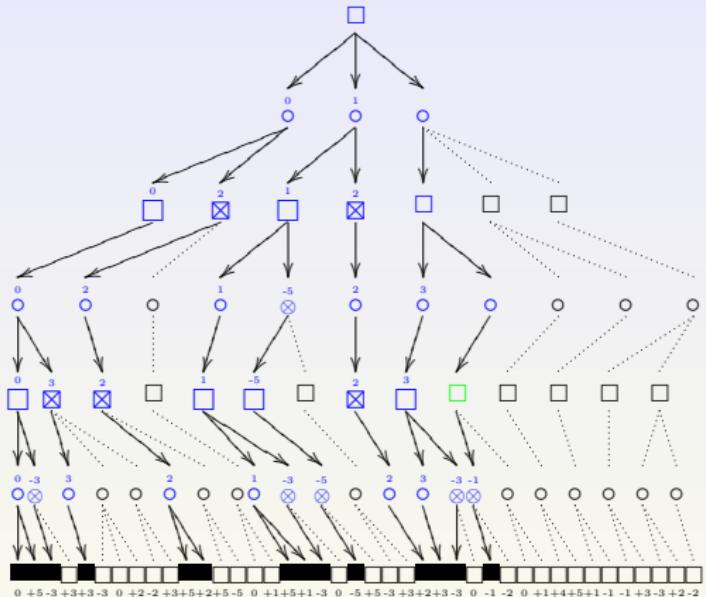
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

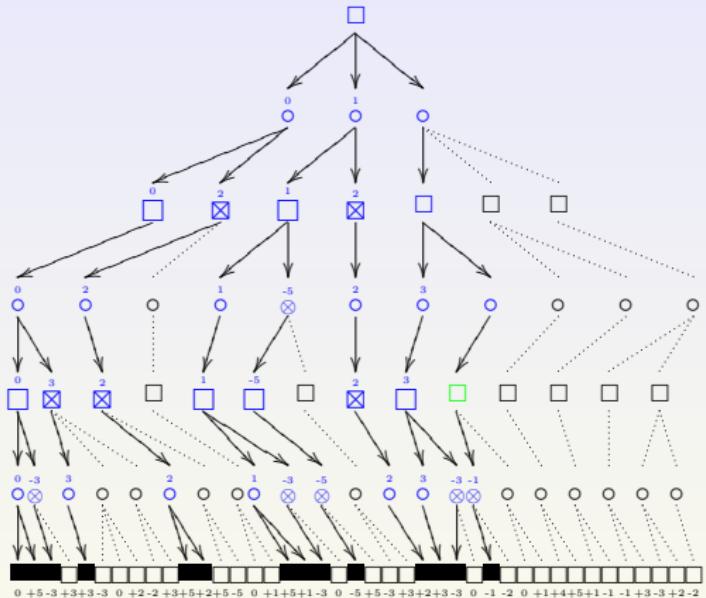
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$

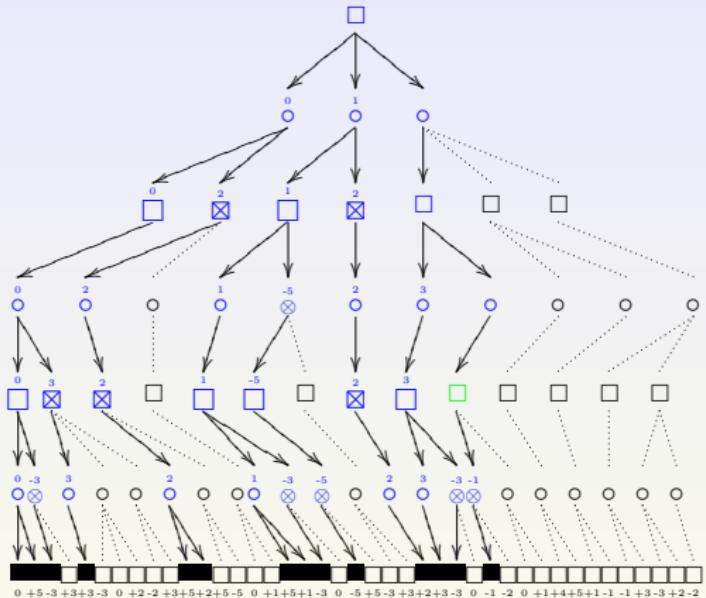


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
  →  $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$

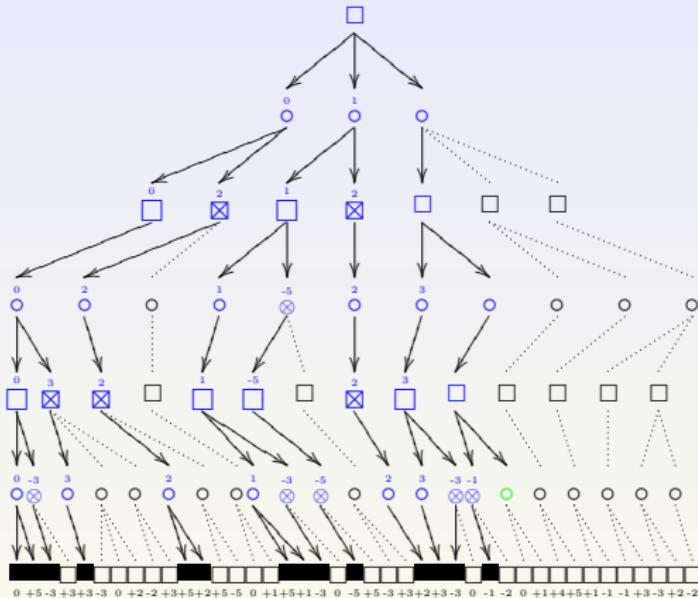


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

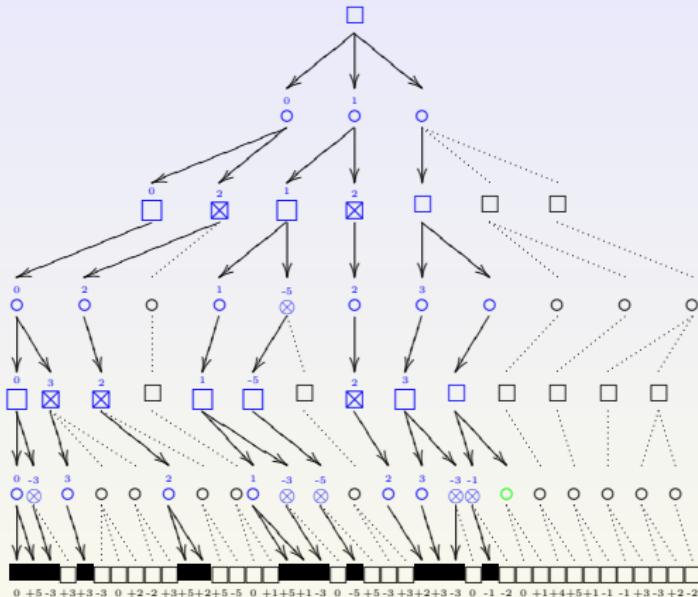
```

→function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

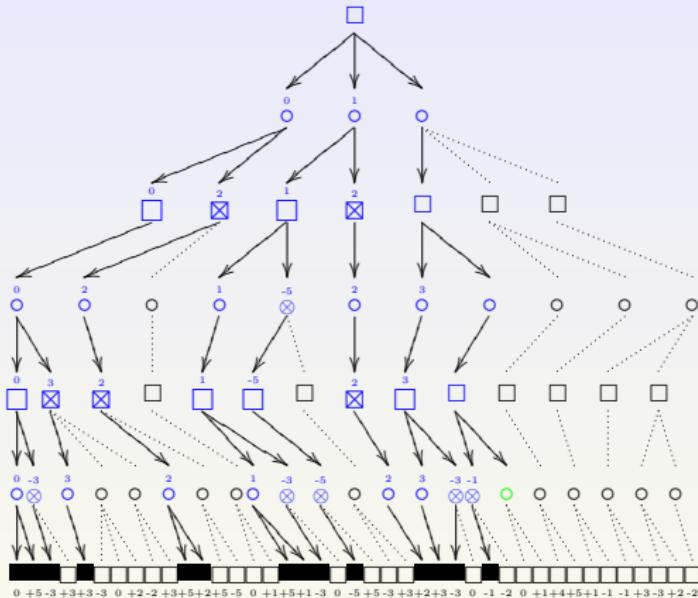
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
→ if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
         $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
        if  $v \leq \alpha$  then
            return  $v$ 
         $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

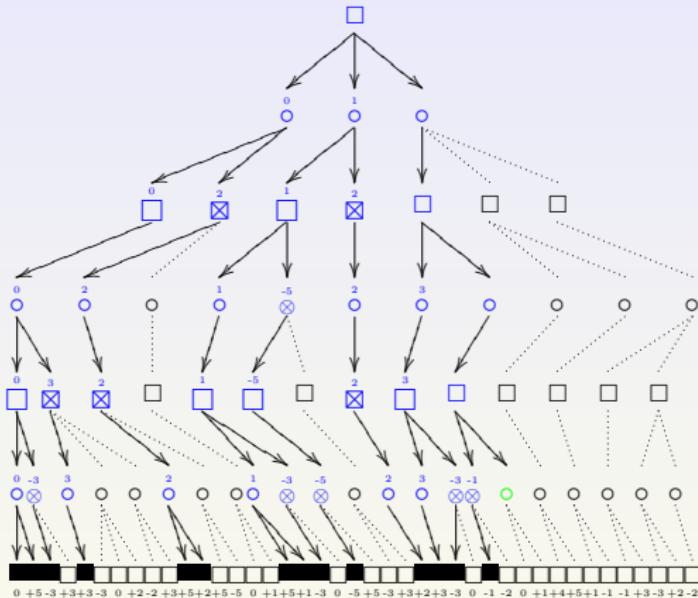
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

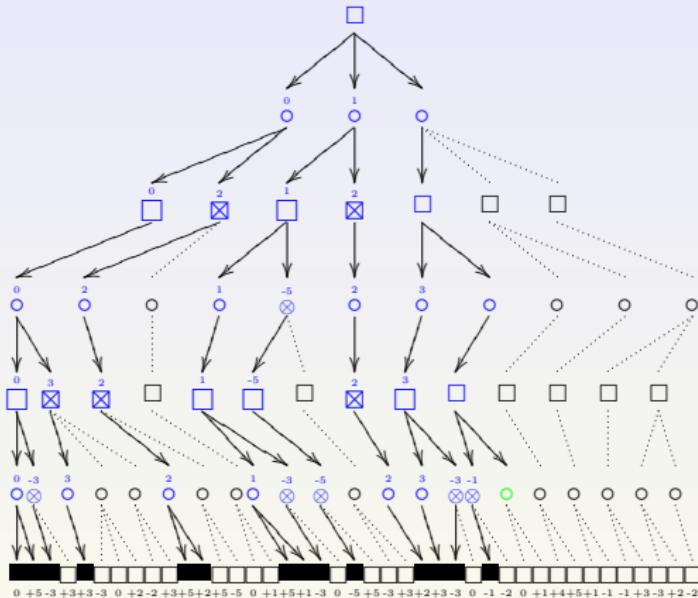
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

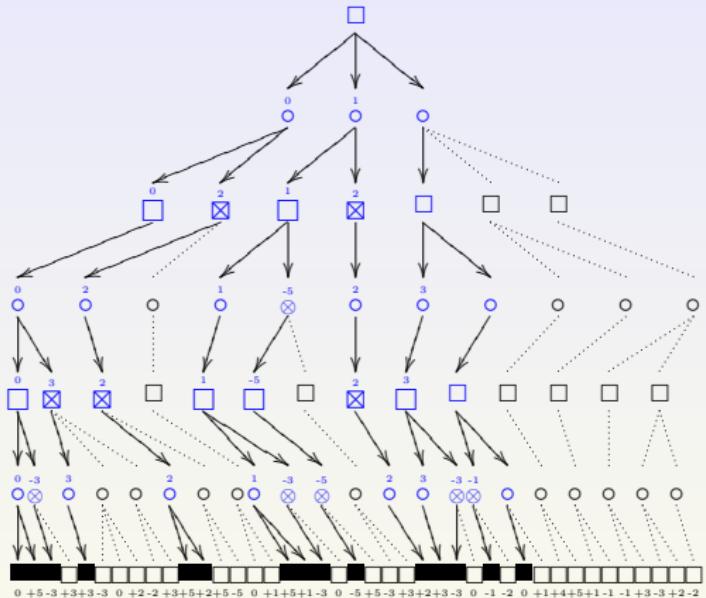
```

function MIN-VALUE(state, $\alpha,\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha,\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



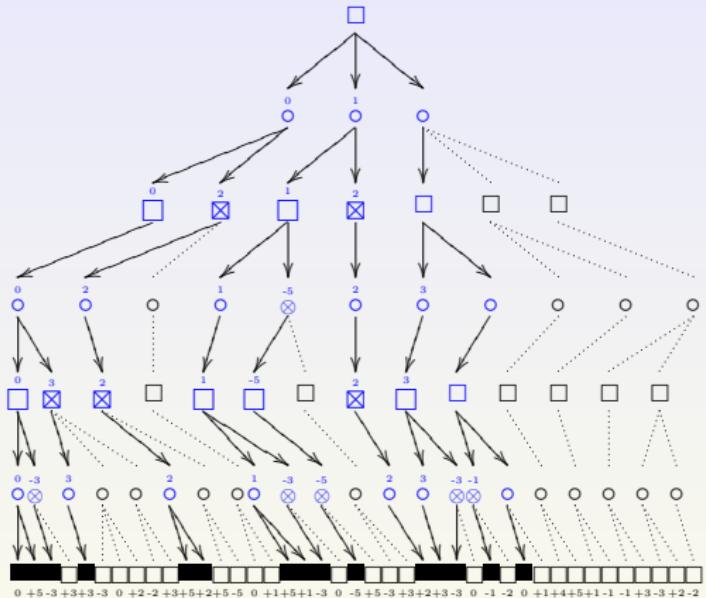


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

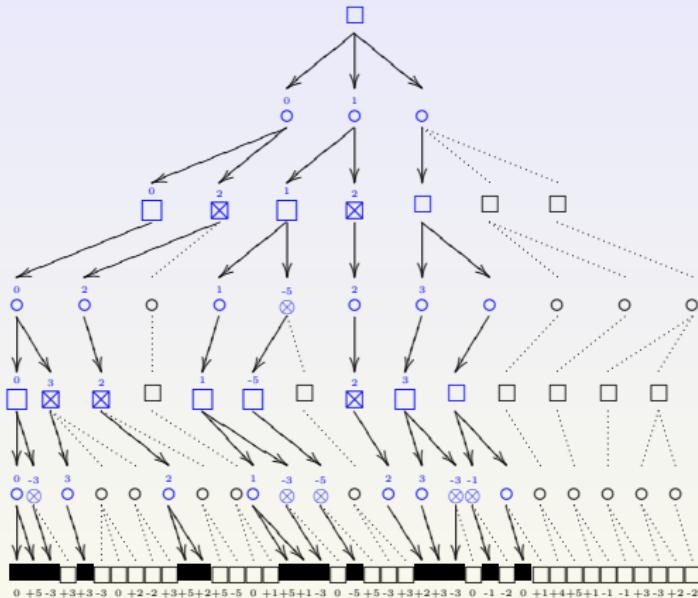
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

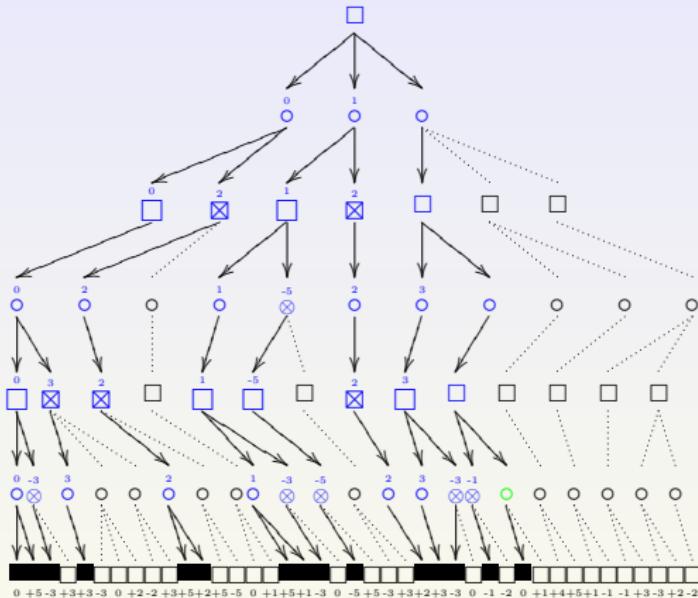
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

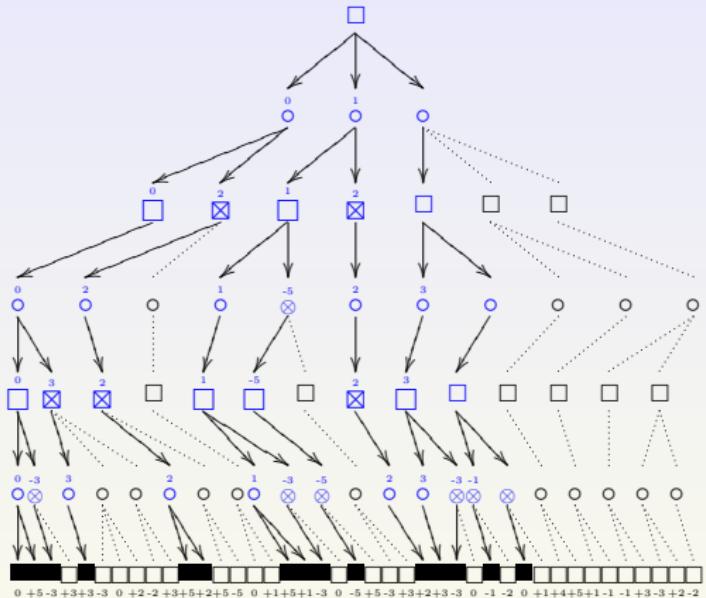
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow \text{MINI}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



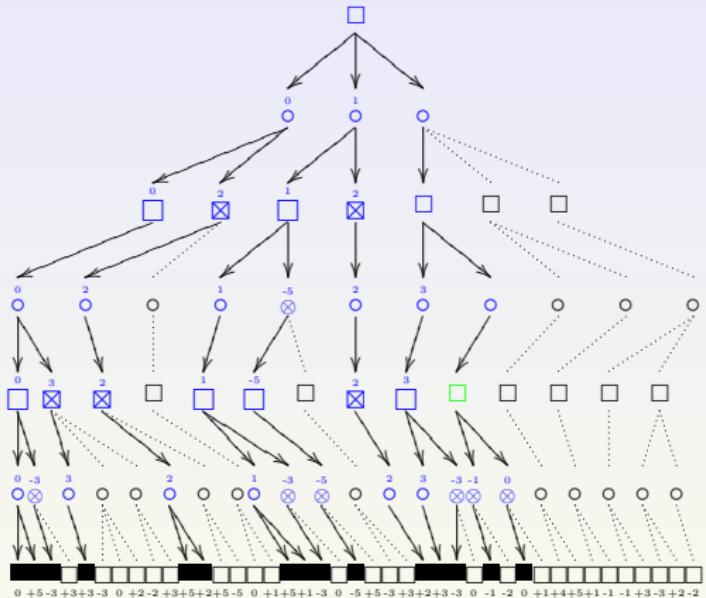


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  → if v  $\geq \beta$  then
    return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

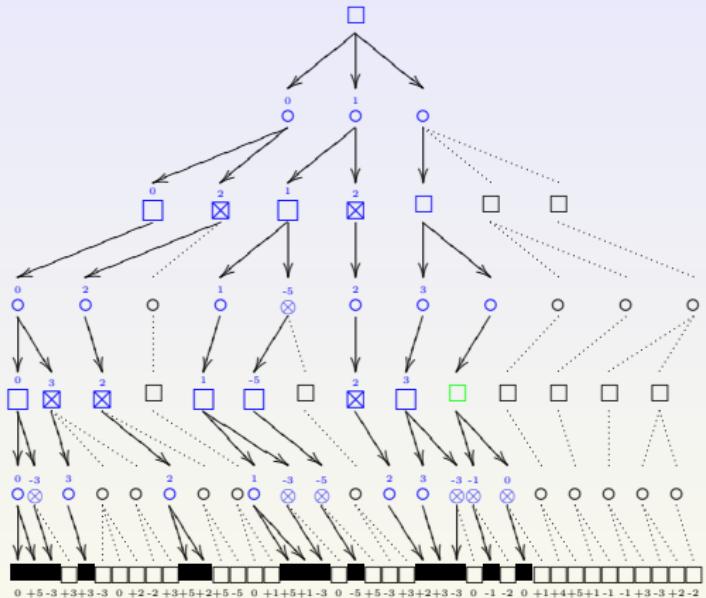
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  →
   $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

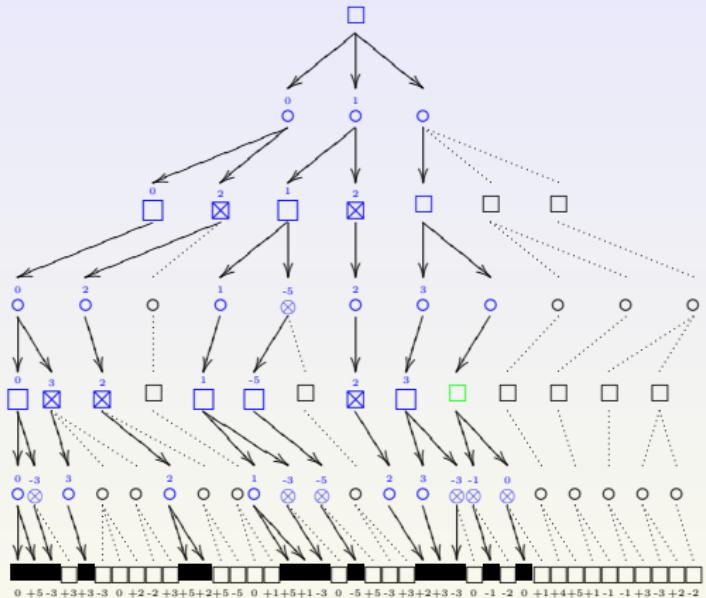
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$

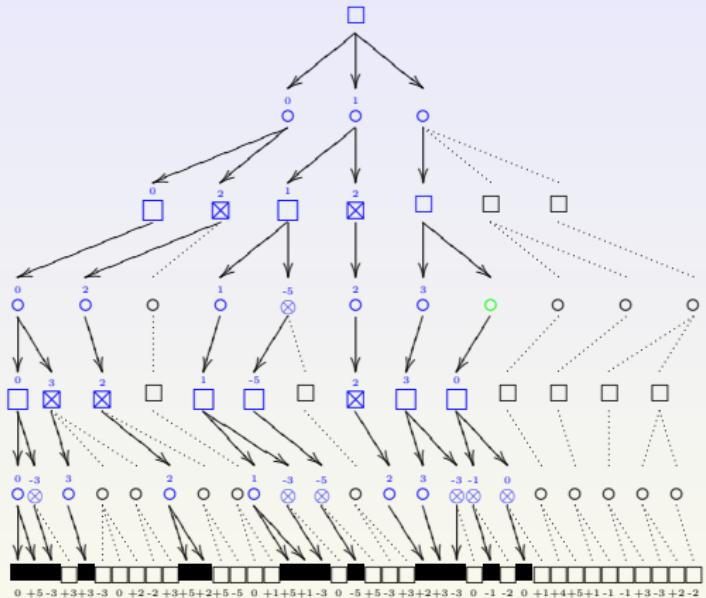


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  → return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$

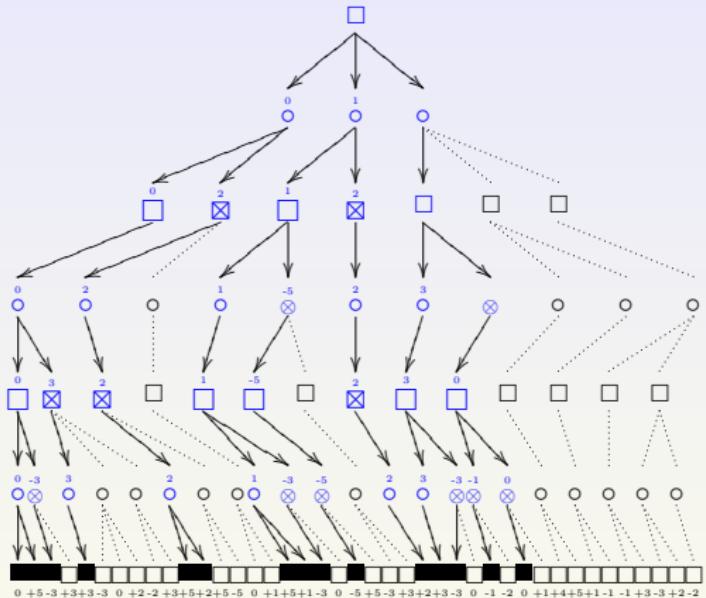


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

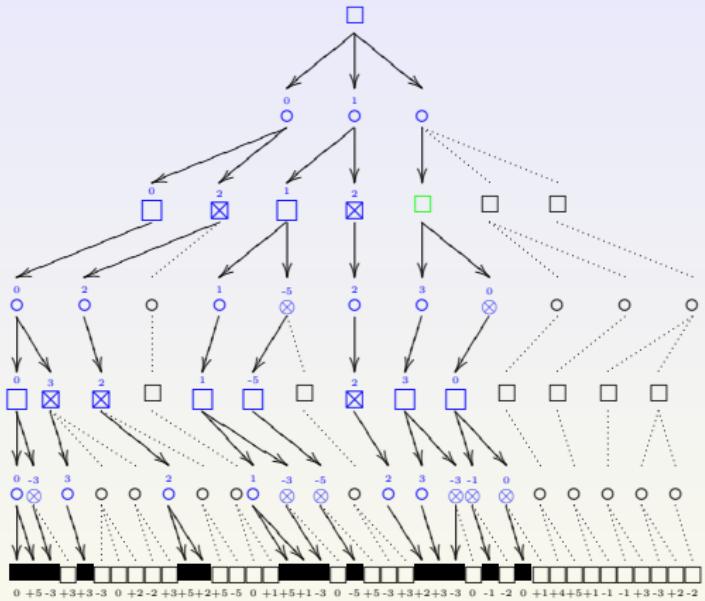
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

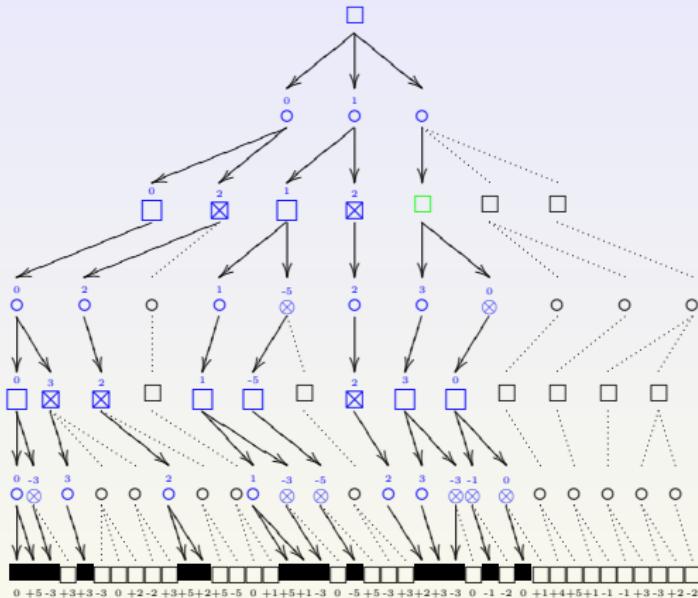
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
  v  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

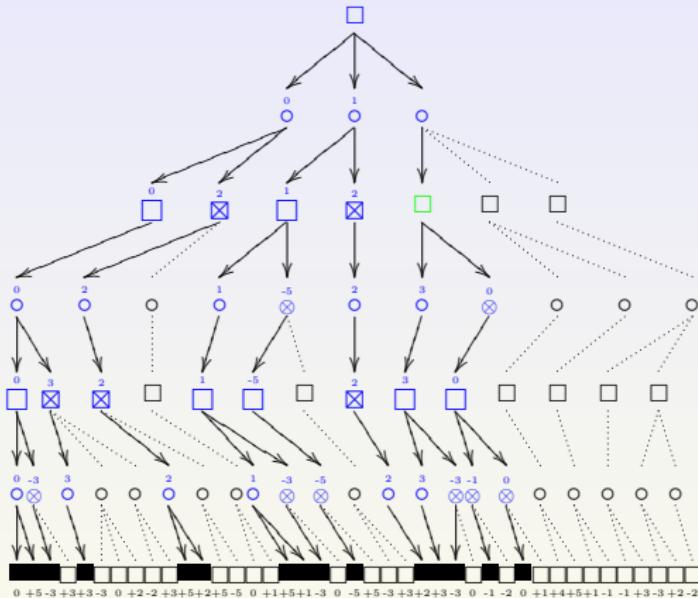
```

```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

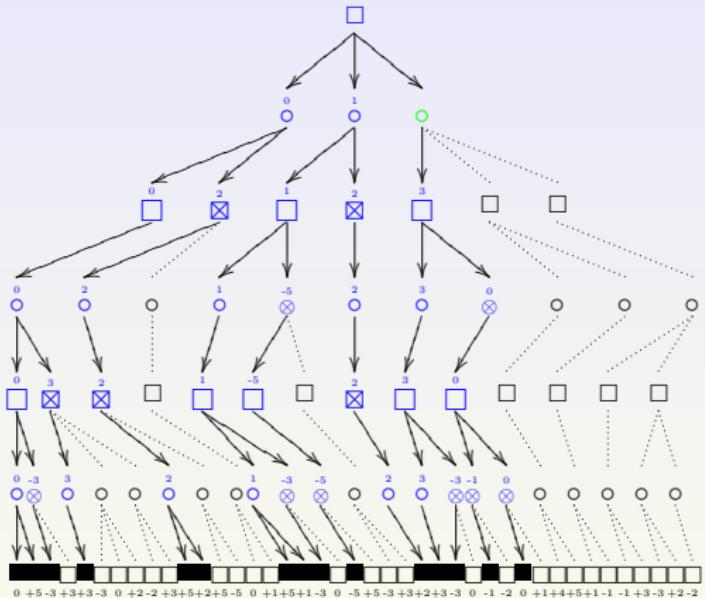
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 3 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

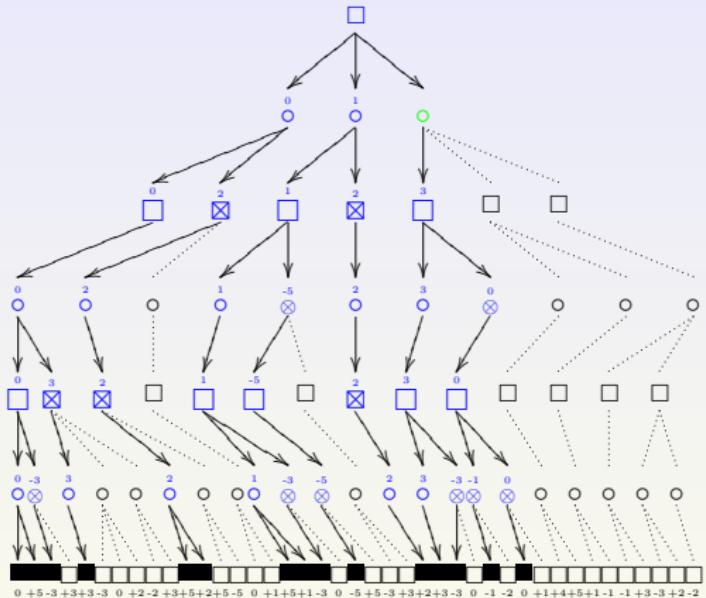
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

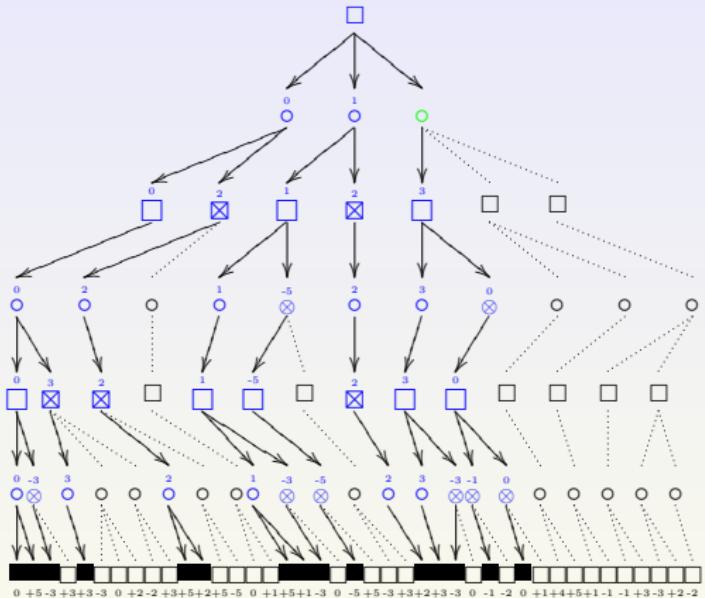
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

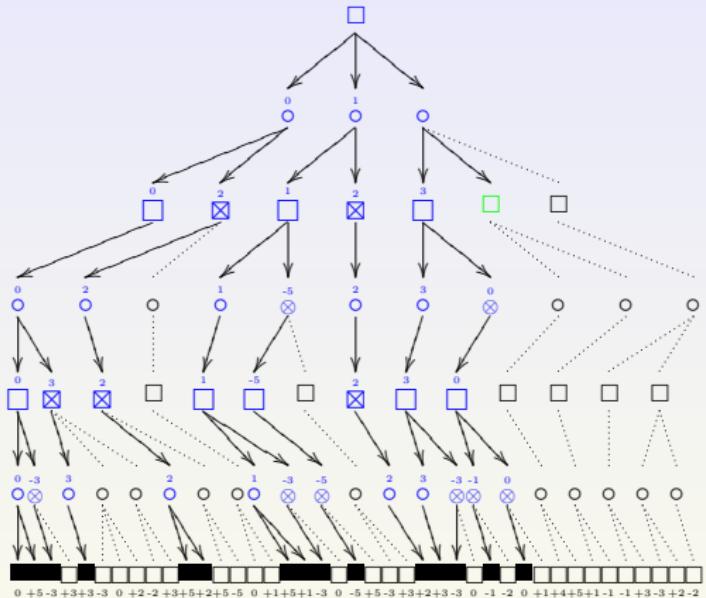
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



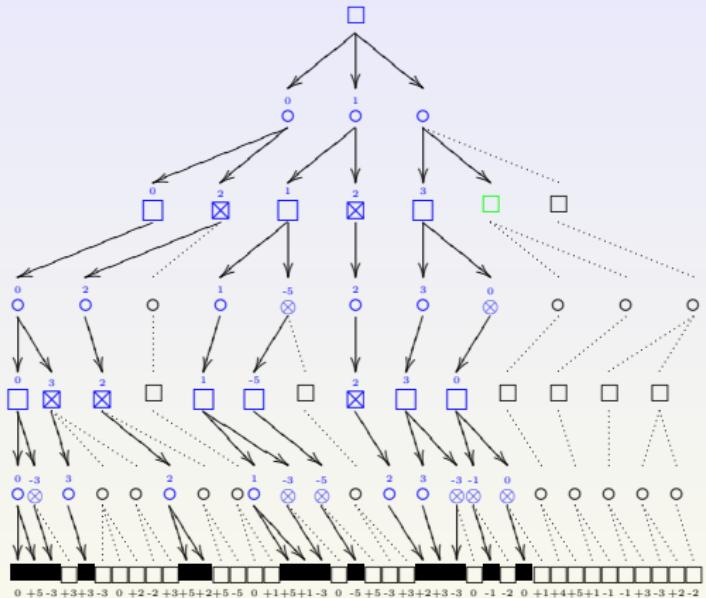


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
→ function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

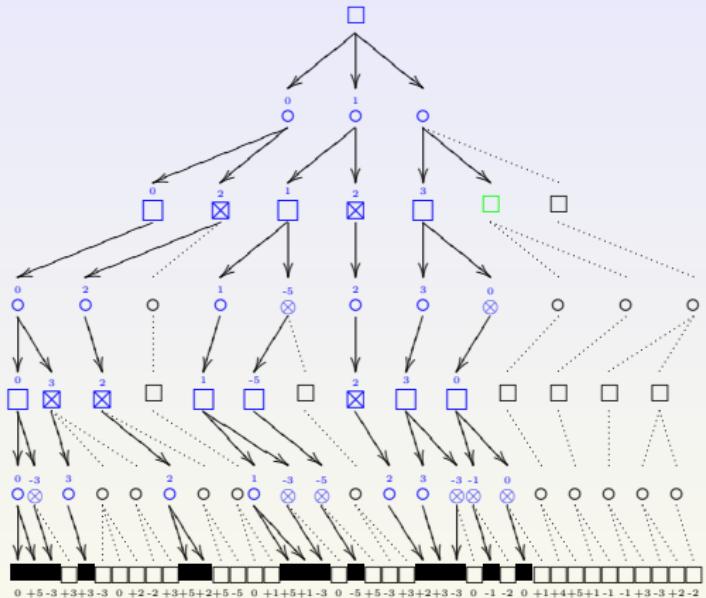
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$

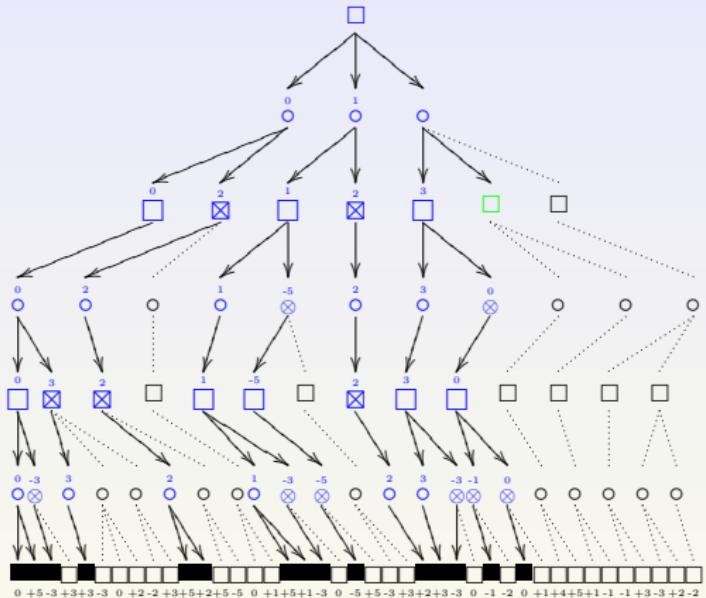


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

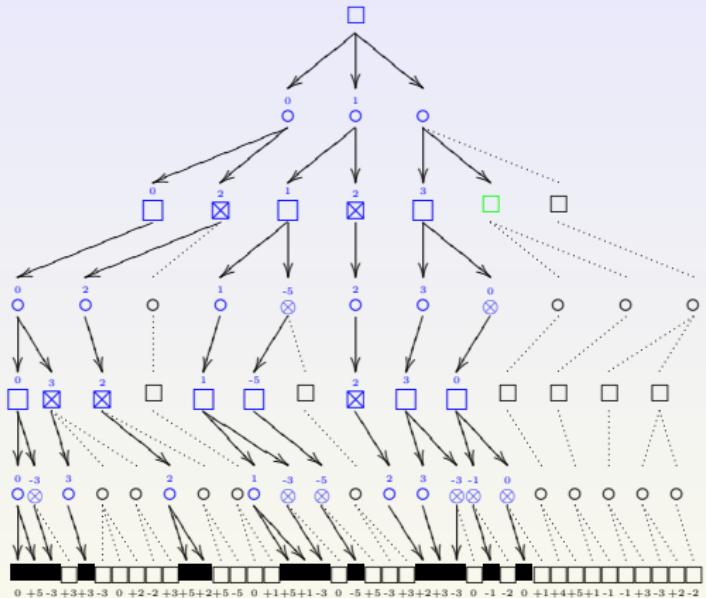
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$

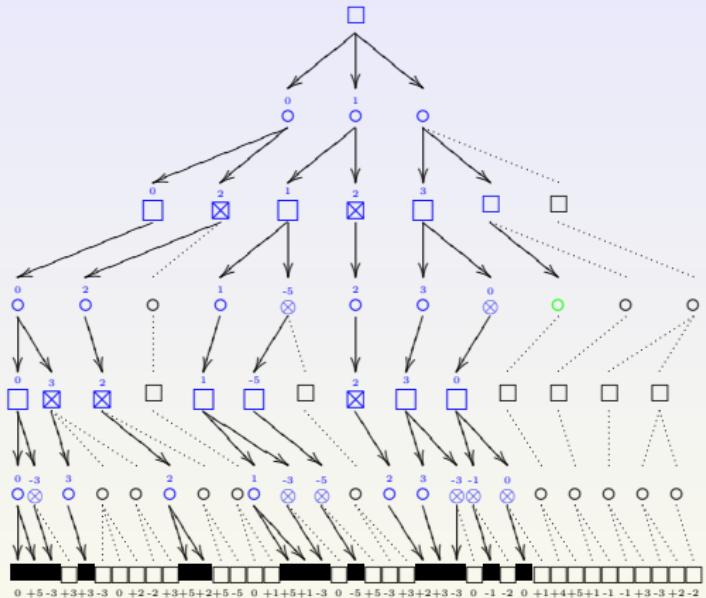


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

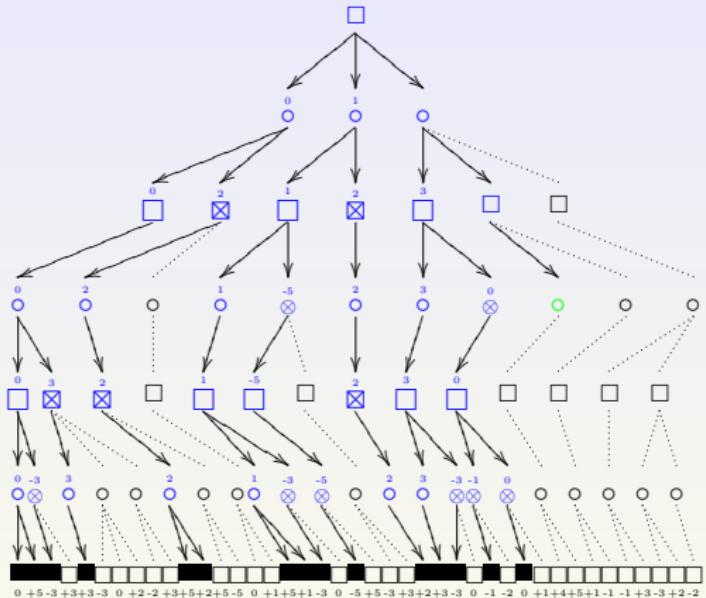
→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value

```

  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$

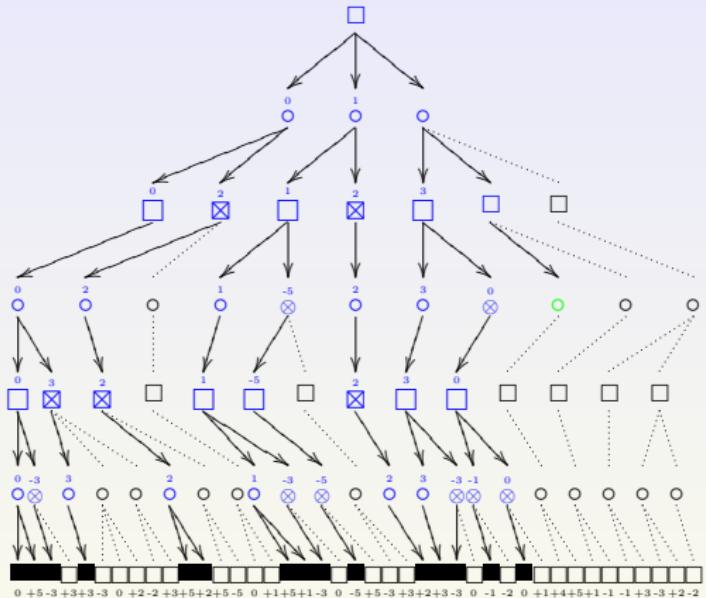


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

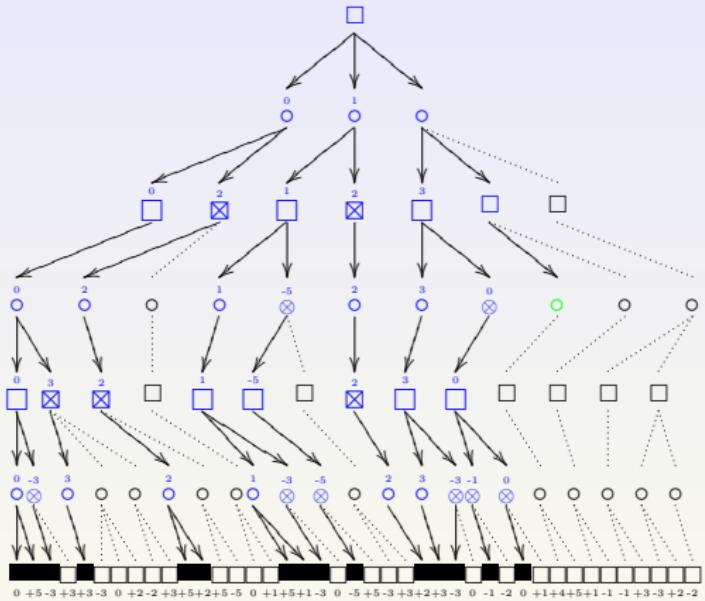
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

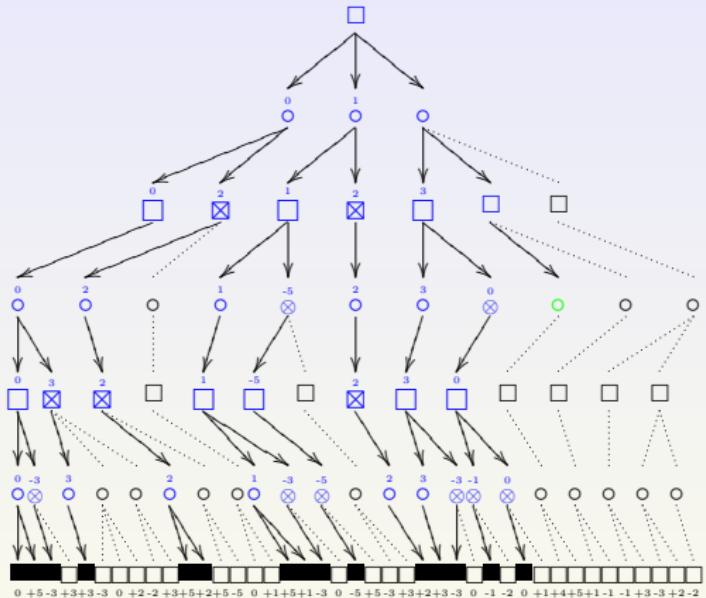
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

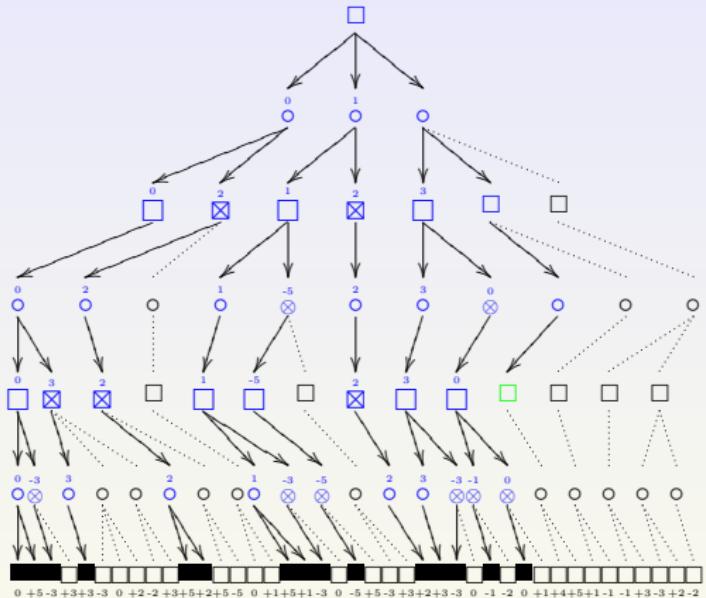
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

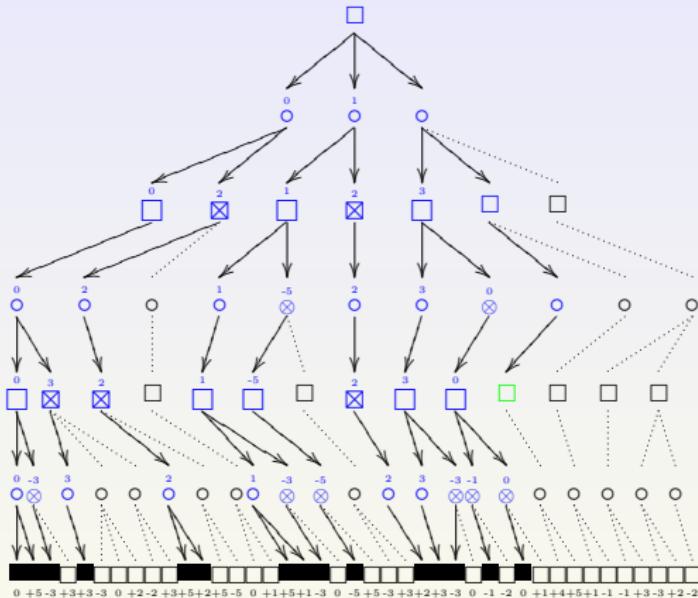
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

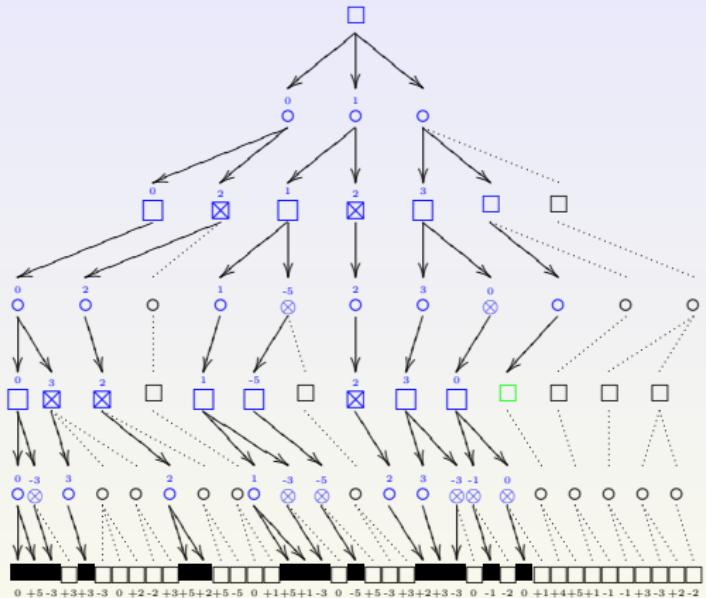
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$

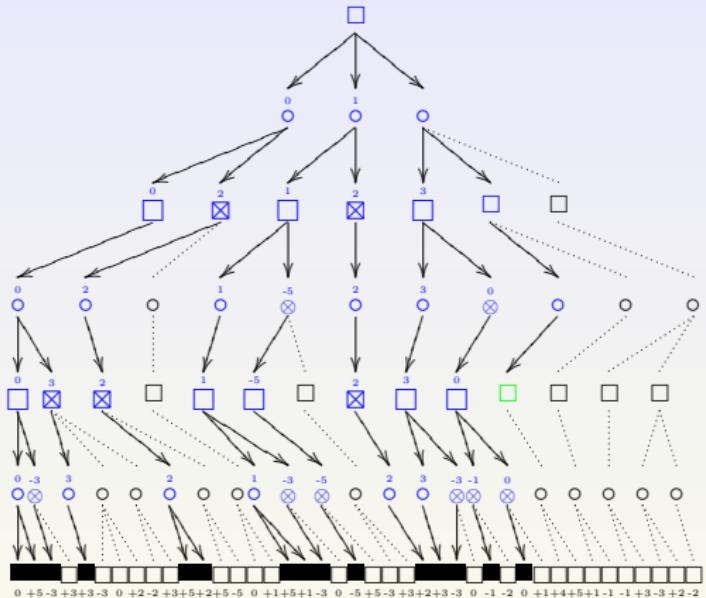


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  → v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

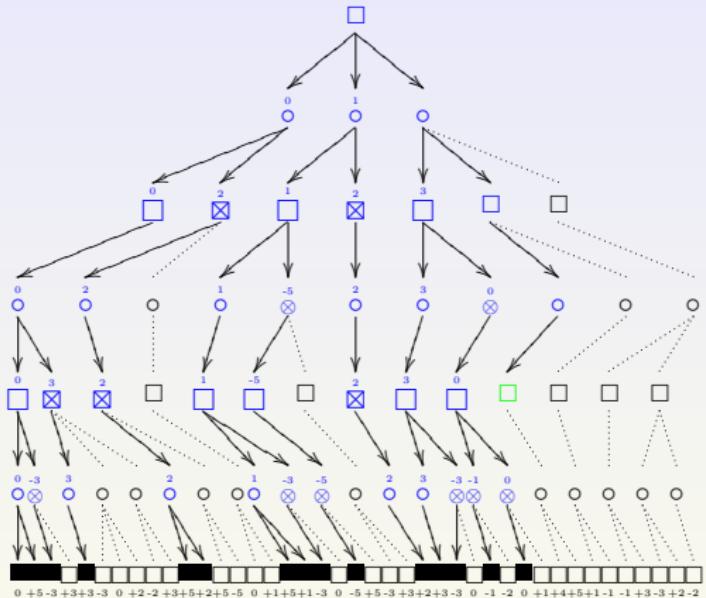
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

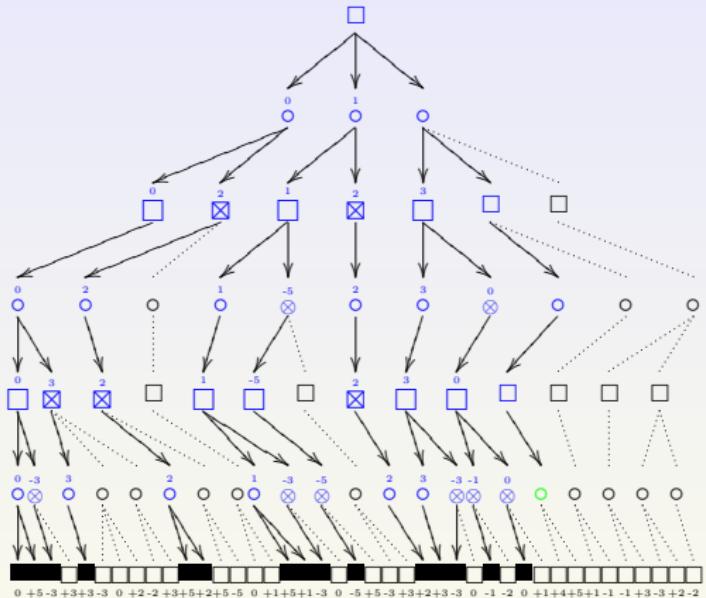
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

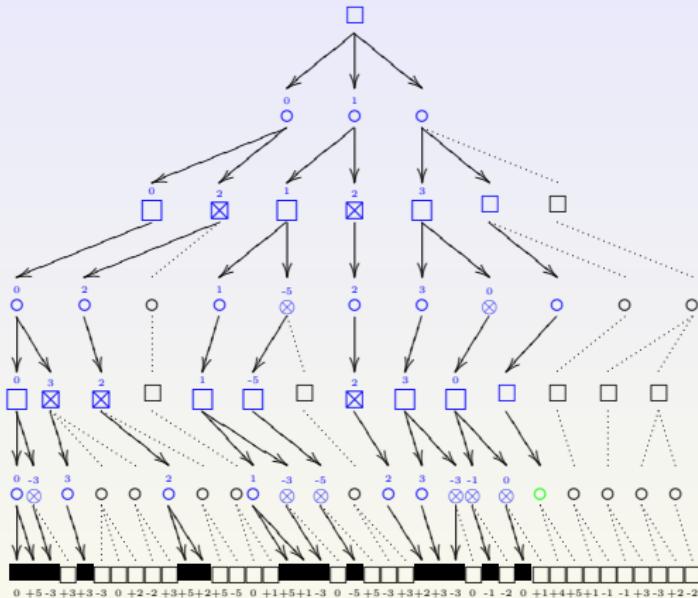
→ **function** MIN-VALUE(*state*, α , β) **returns** a utility value

```

  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

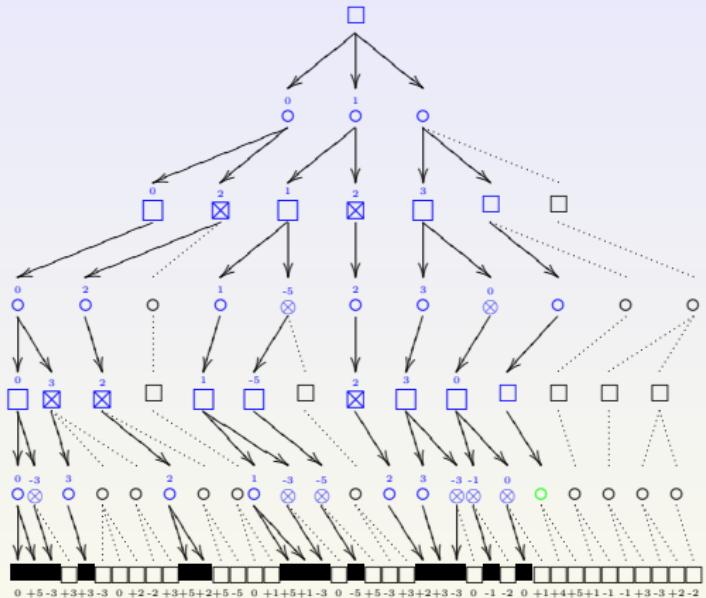
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
→ if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
     $v \leftarrow +\infty$ 
    for  $a, s$  in SUCCESSORS( $state$ ) do
         $v \leftarrow \text{Min}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
        if  $v \leq \alpha$  then
            return  $v$ 
         $\beta \leftarrow \text{MIN}(\beta, v)$ 
    return  $v$ 

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

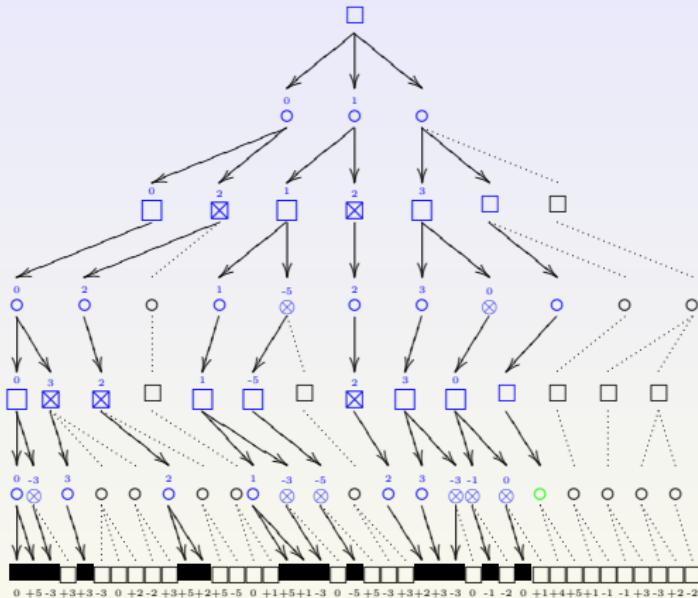
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

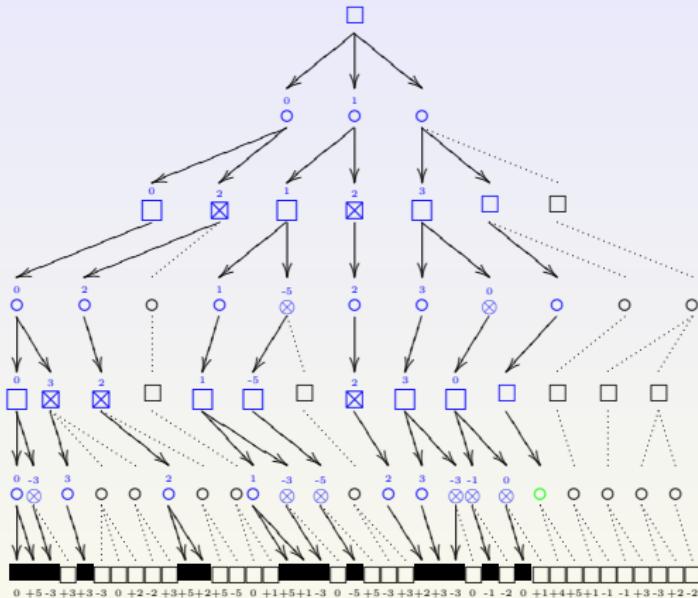
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  → for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

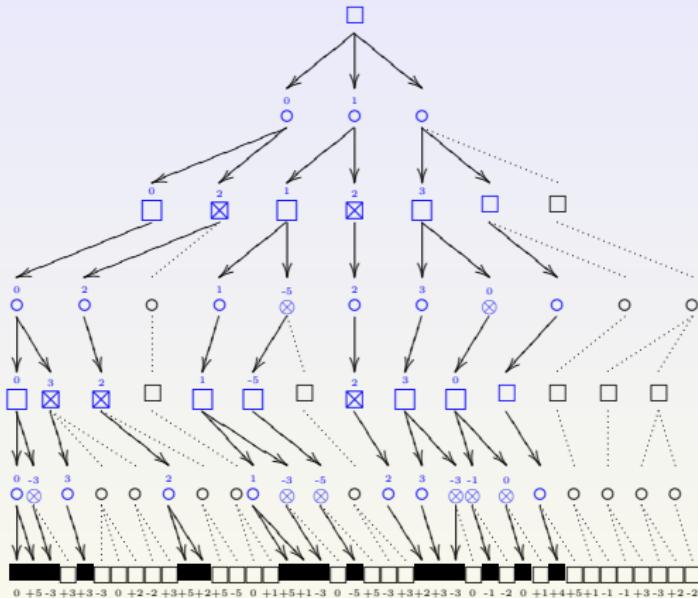
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $s, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $s$ ) then
    return UTILITY( $s$ )
   $v \leftarrow -\infty$ 
  for  $a, s'$  in SUCCESSORS( $s$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s', \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

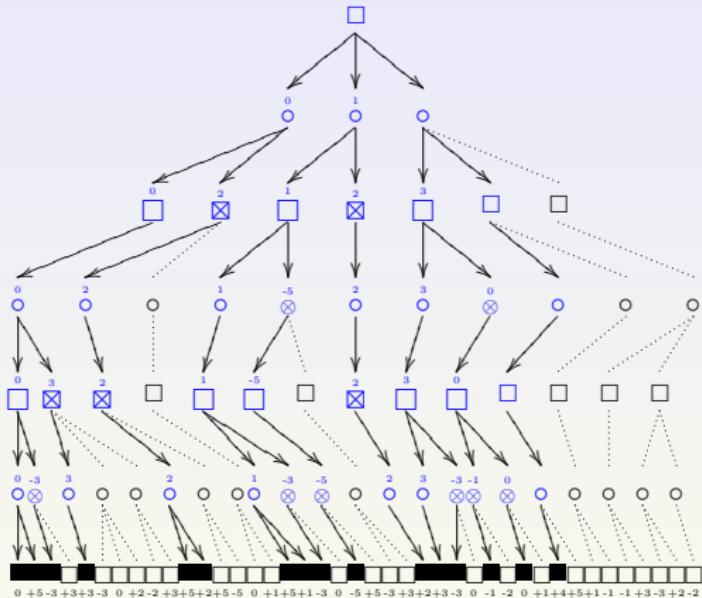
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq$   $\alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

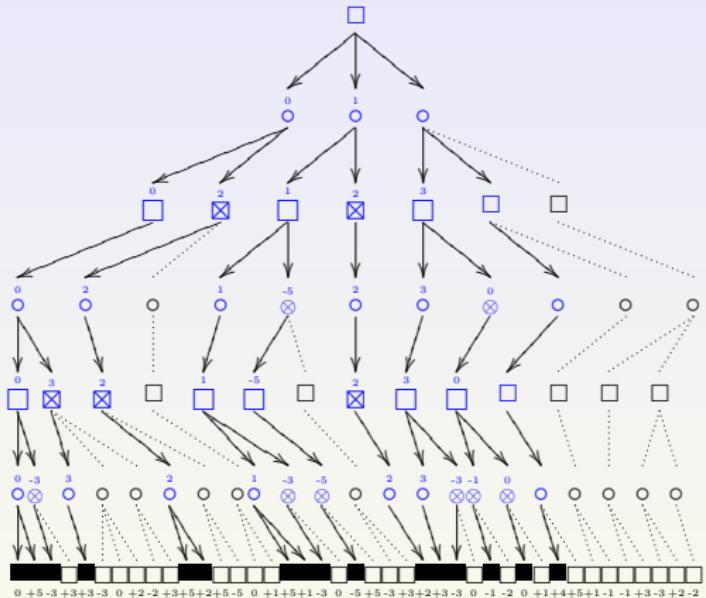
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$ 
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

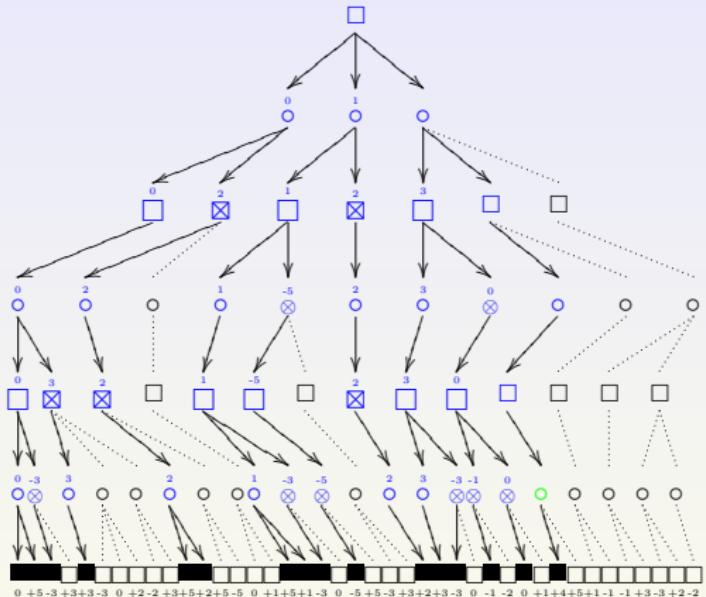
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

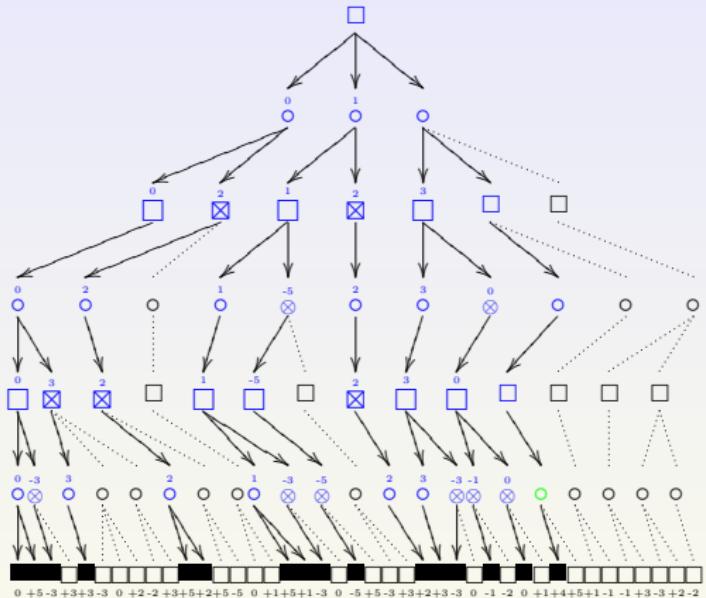
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



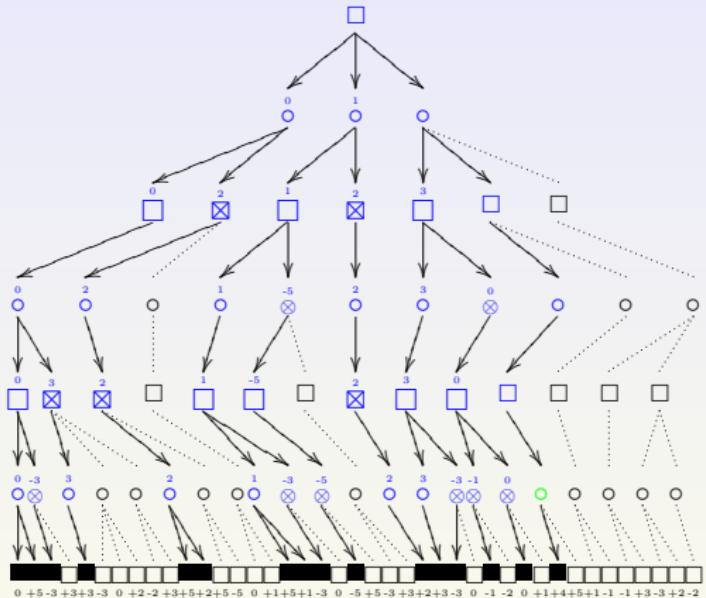


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

```

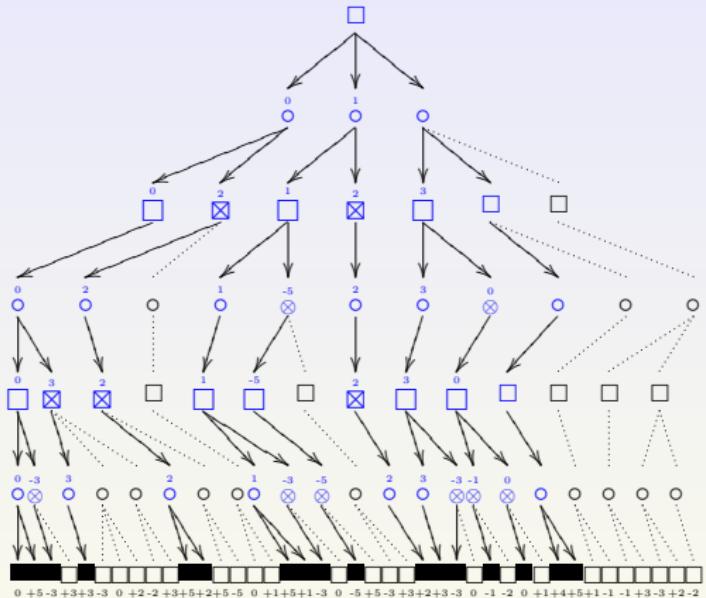
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

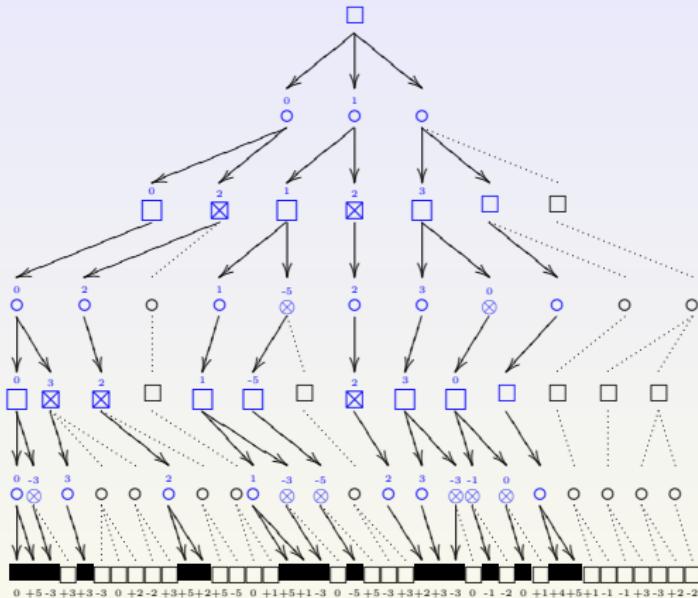
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

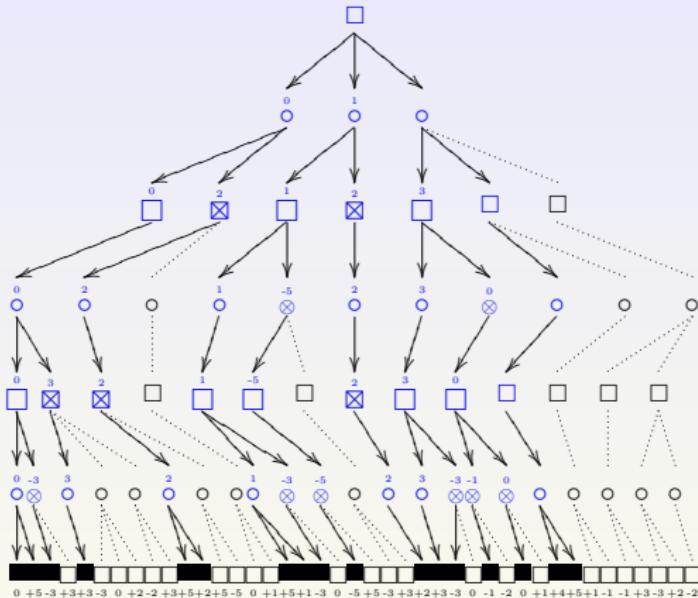
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\text{v}, \text{MAX-VALUE}(\iota, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

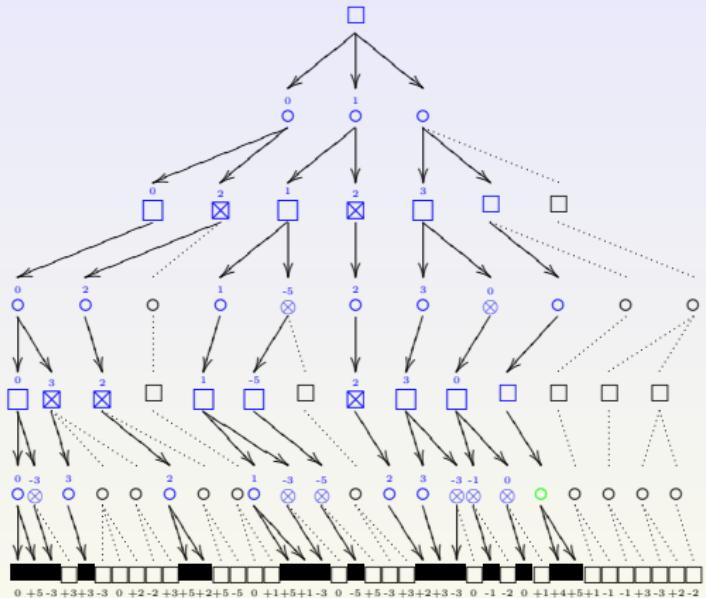
```

function MIN-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

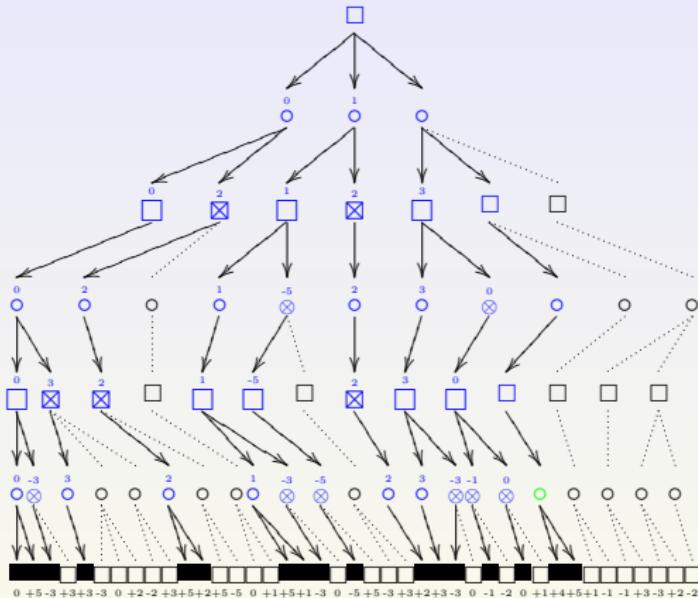
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

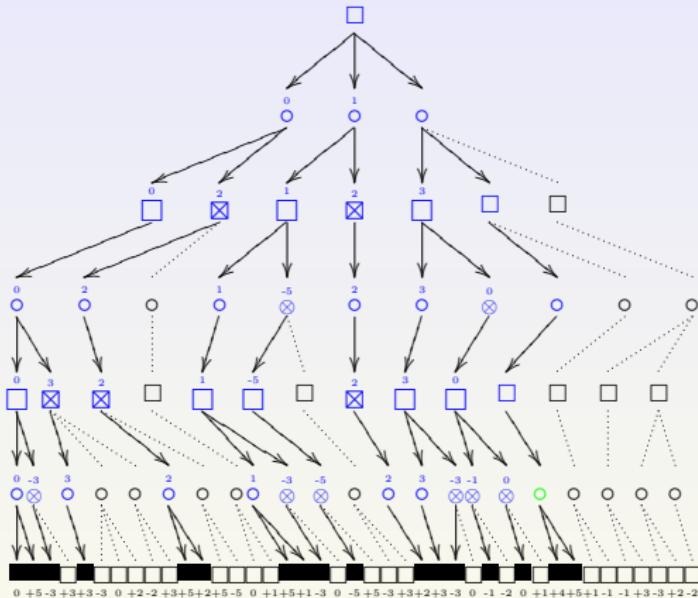
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

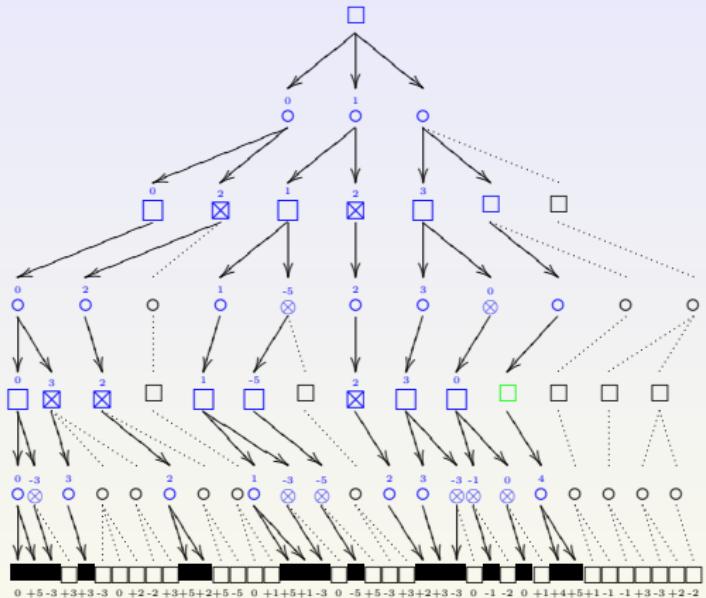
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

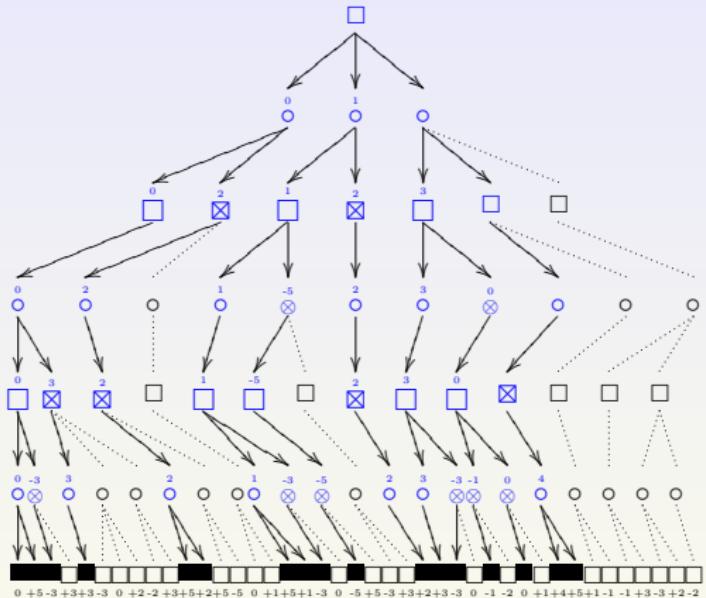
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

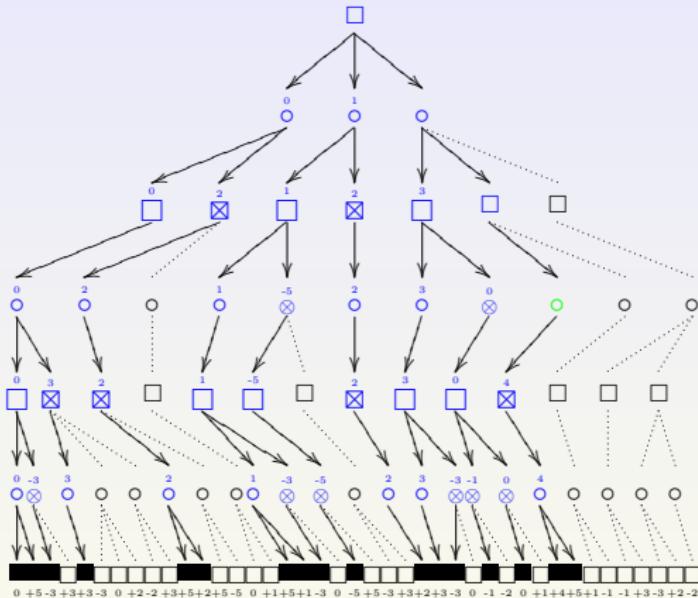
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

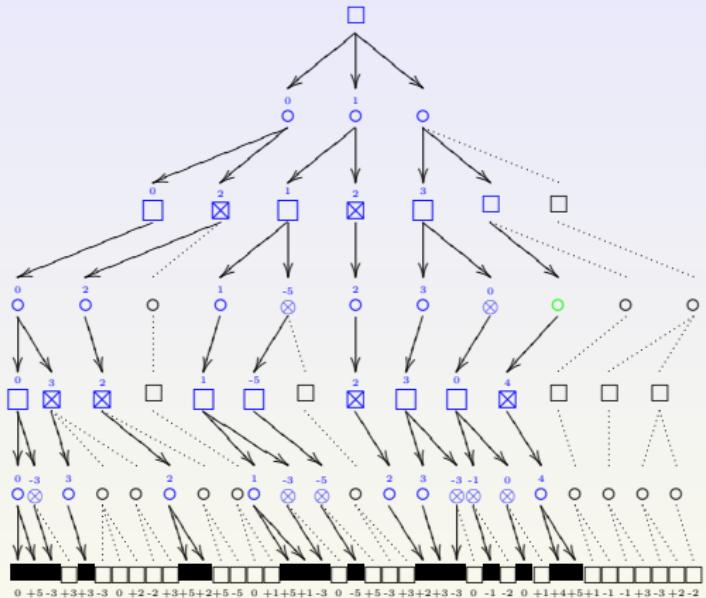
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow \text{MINI}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    α  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

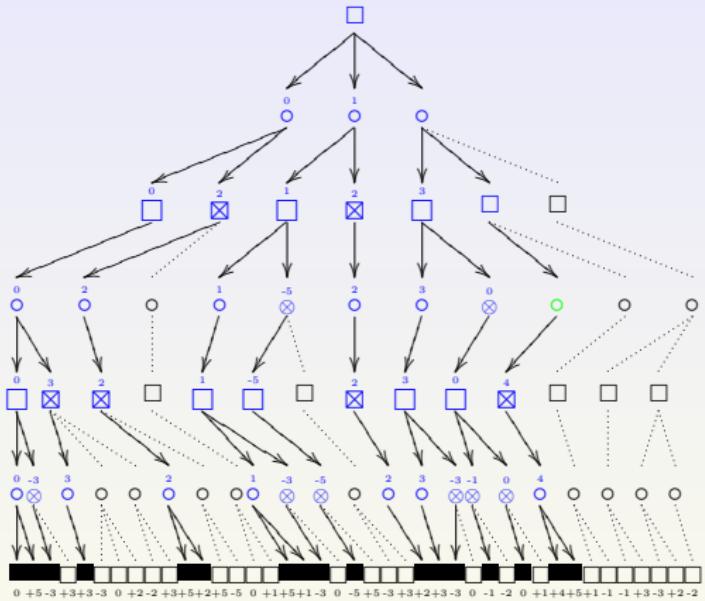
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
   $\rightarrow$   $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha,\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

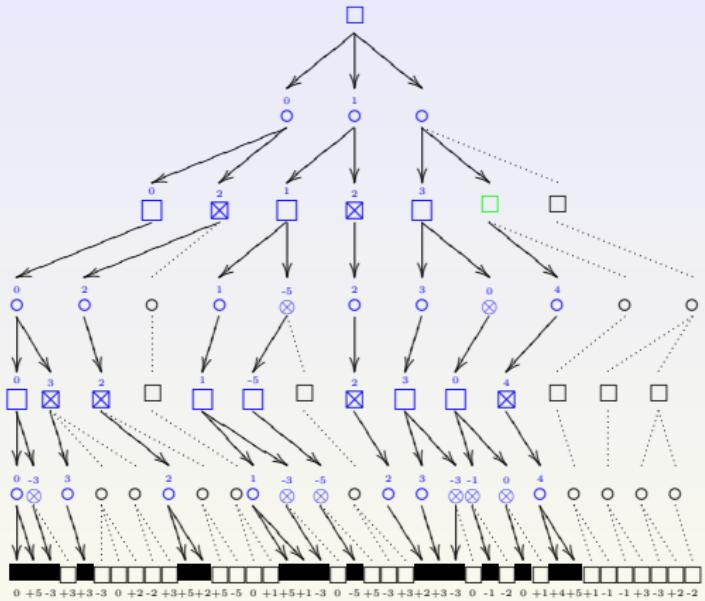
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s, $\alpha$ , $\beta$ ))
    if v  $\leq$   $\alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

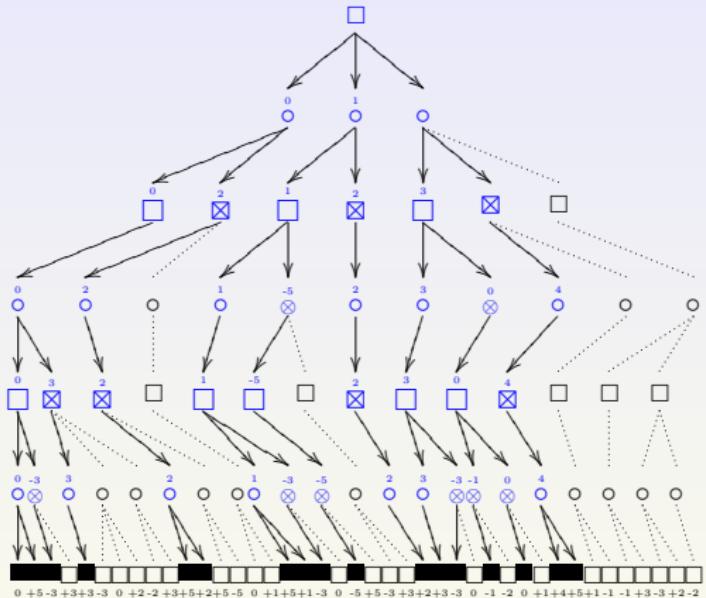
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



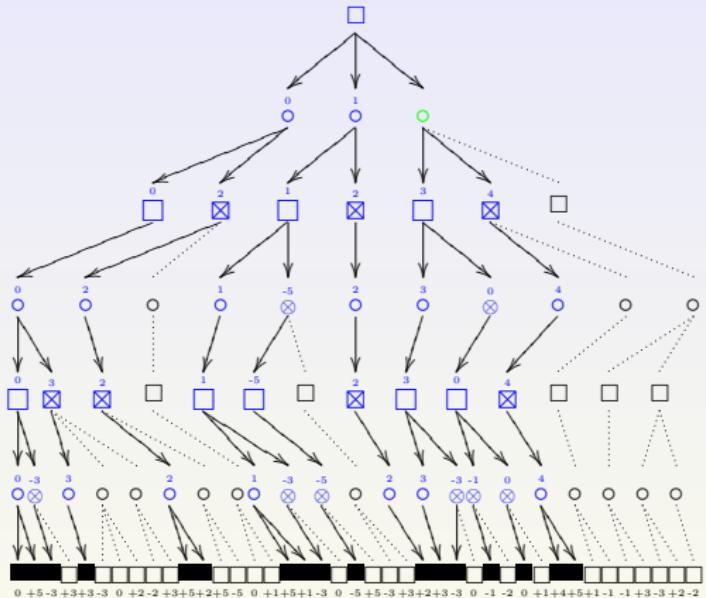


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      → return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



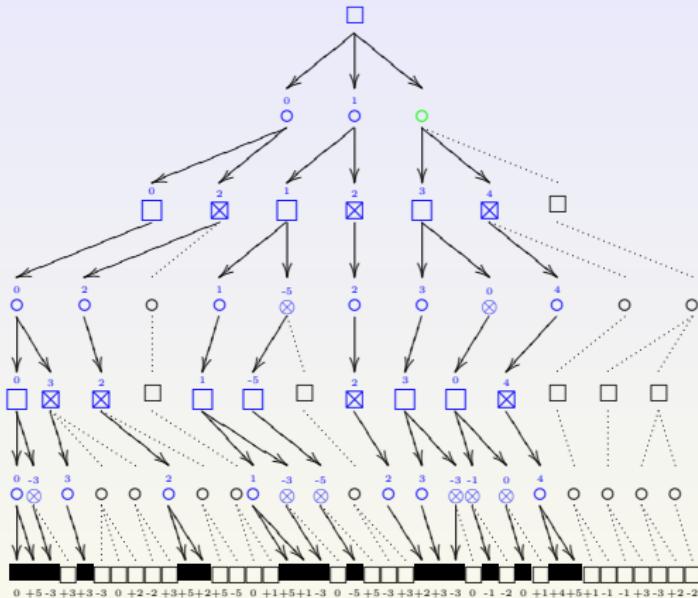
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

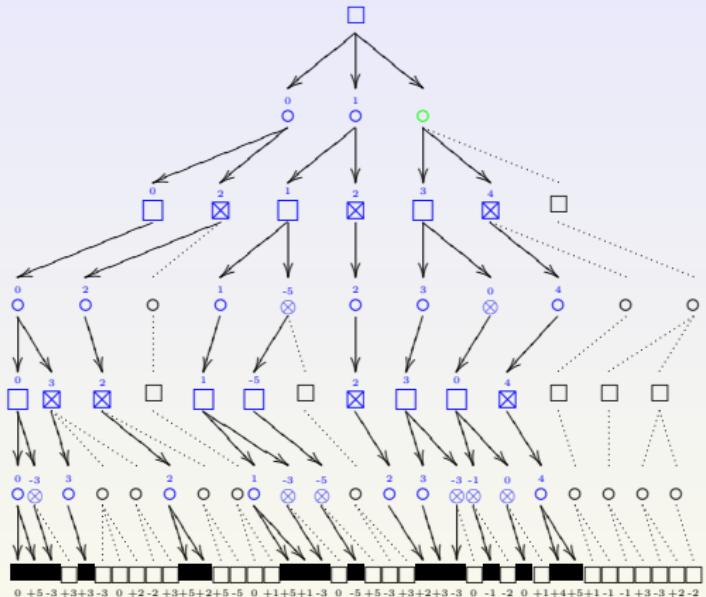
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



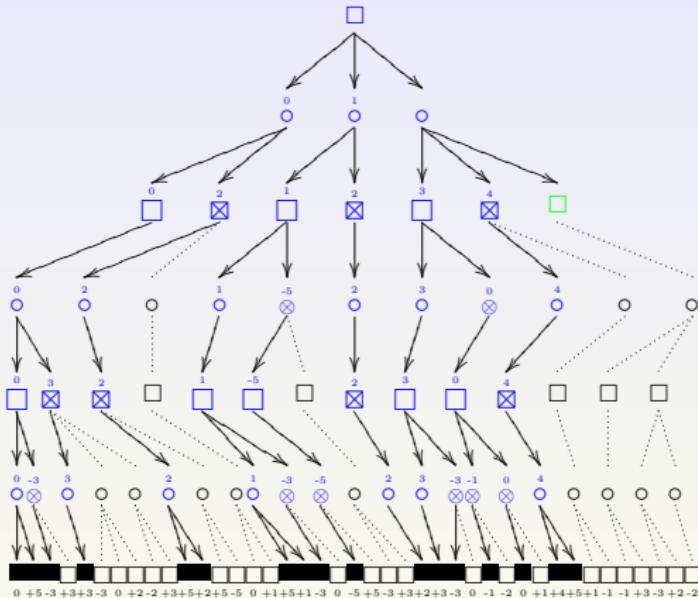


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    → v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $s, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $s$ ) then
    return UTILITY( $s$ )
   $v \leftarrow -\infty$ 
  for  $a, s'$  in SUCCESSORS( $s$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s', \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

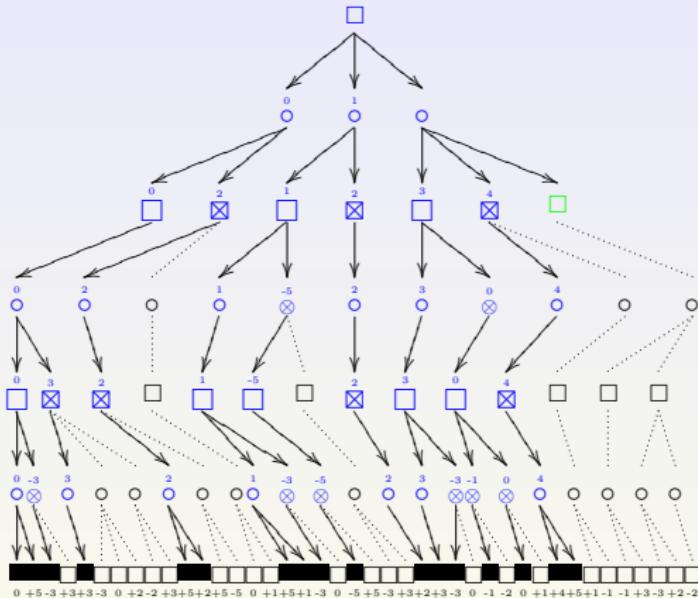
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s, $\alpha$ , $\beta$ ))
    if v  $\leq$   $\alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

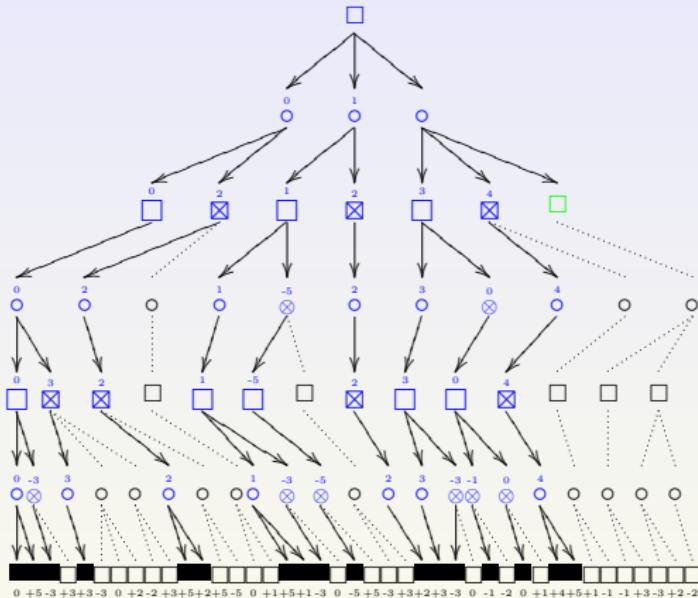
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

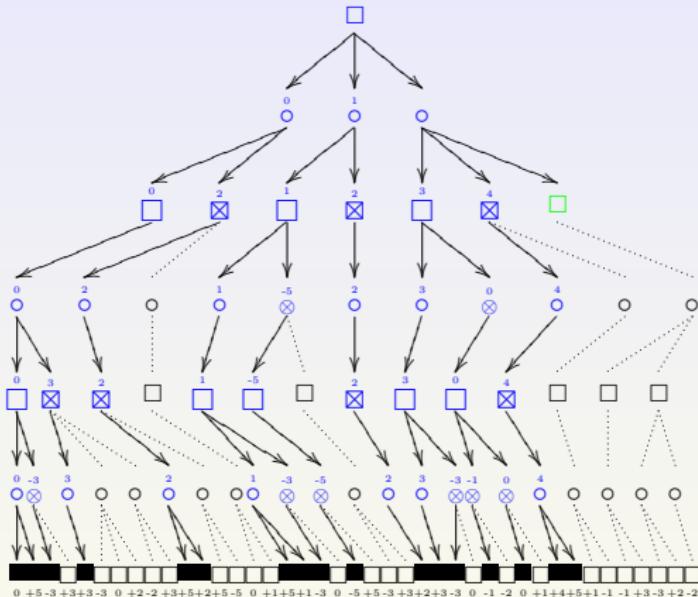
```

```

function MIN-VALUE(state,  $\alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a,s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

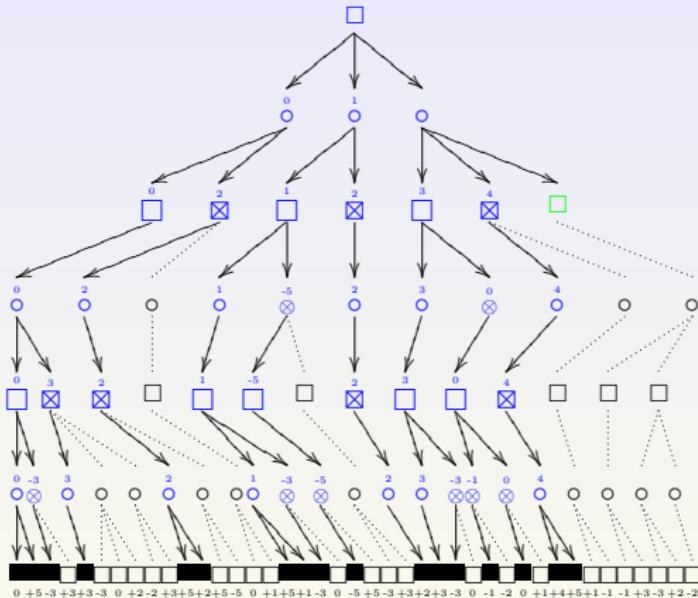
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

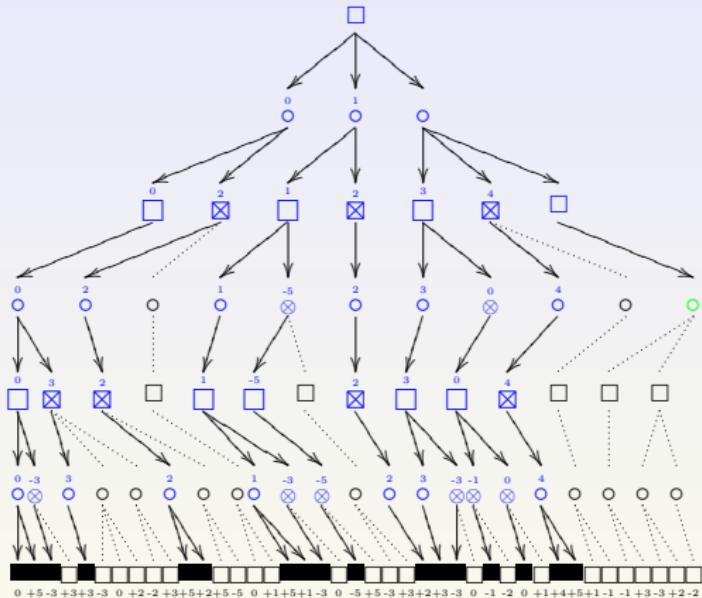
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



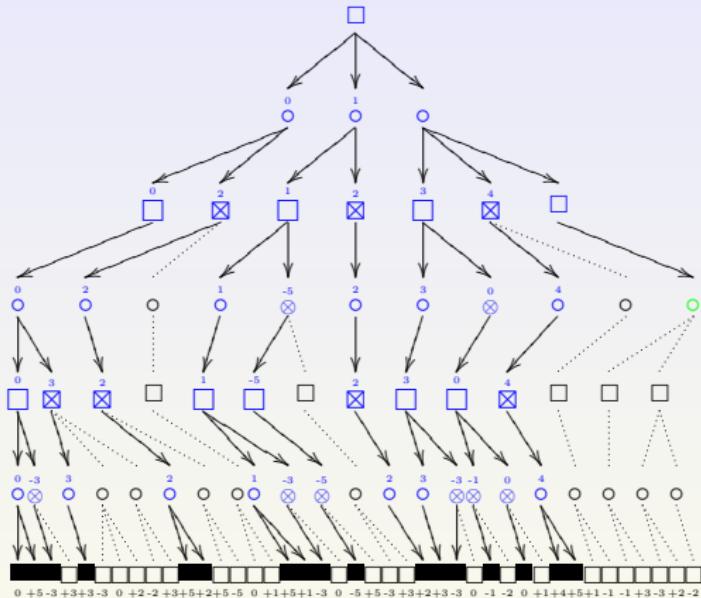


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
→function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$

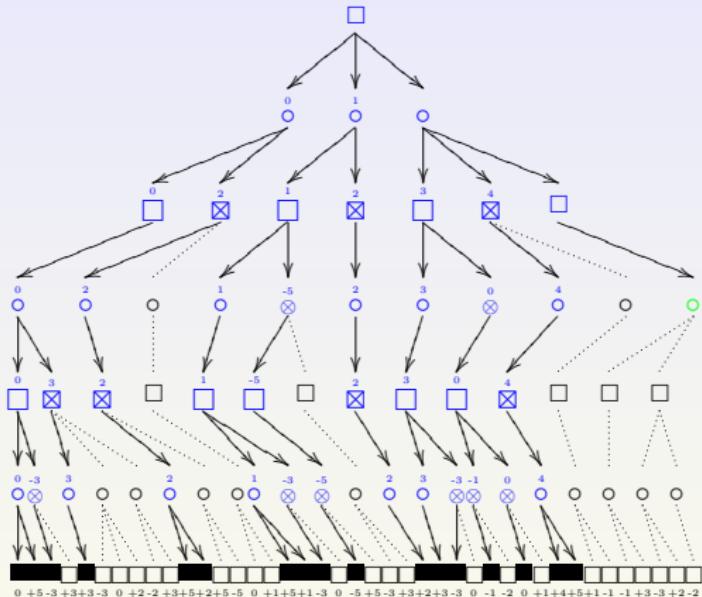


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$

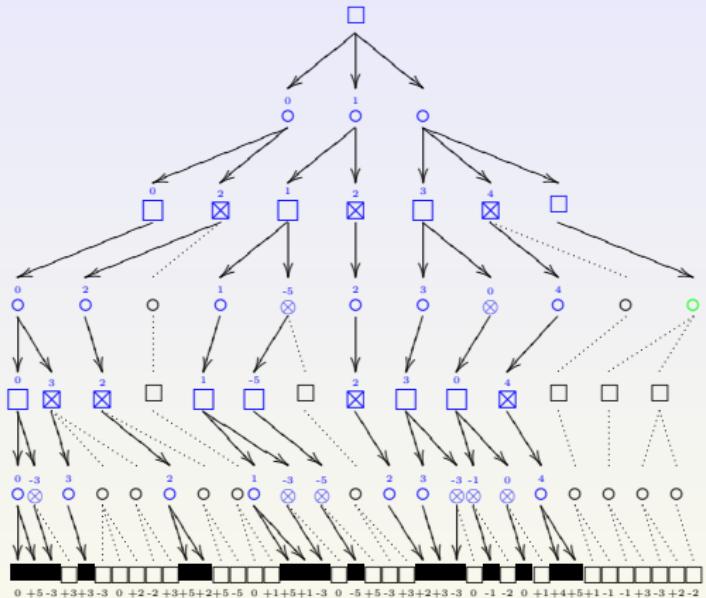


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

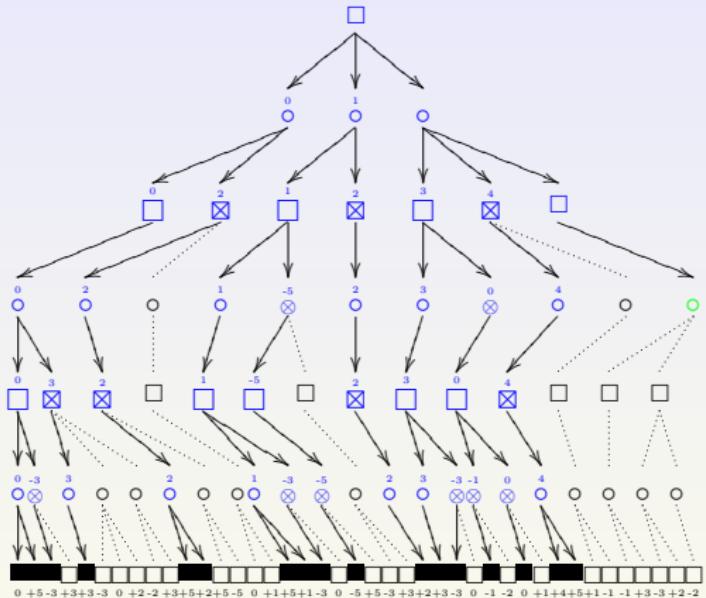
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

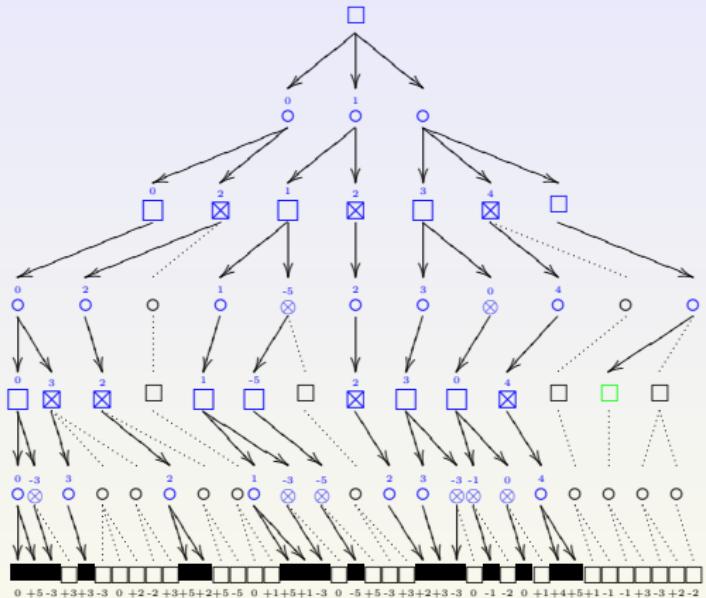
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

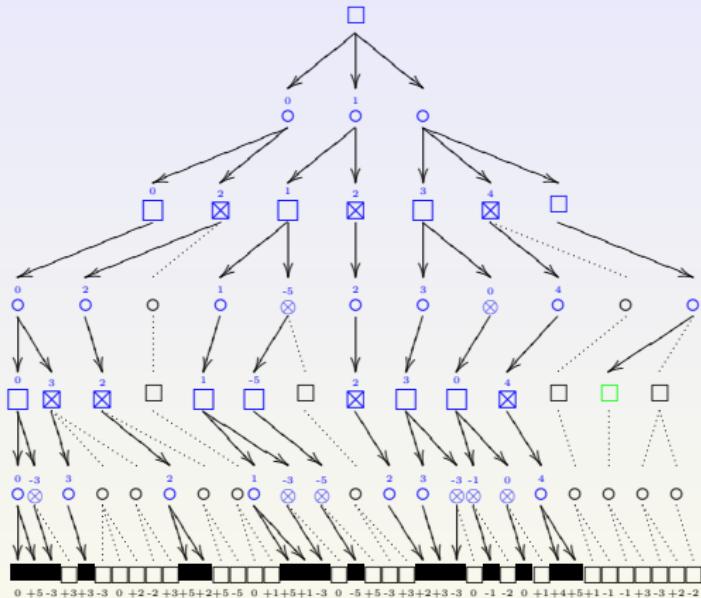
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
   $\rightarrow$  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

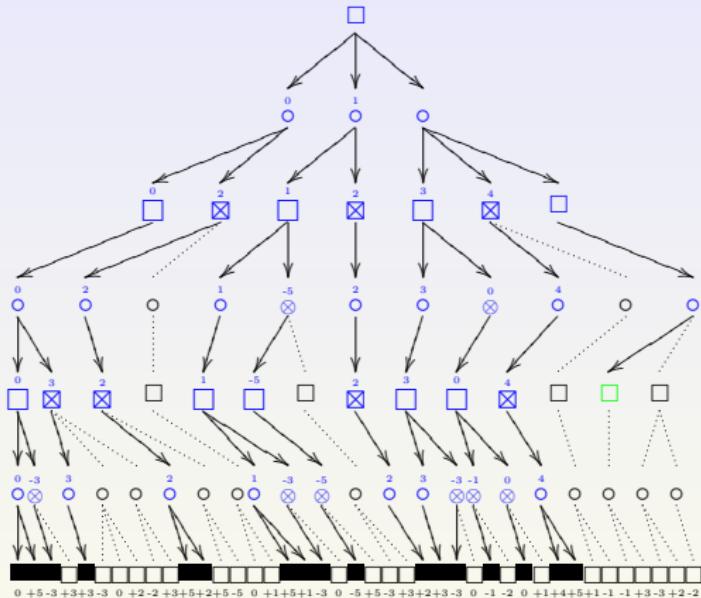
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

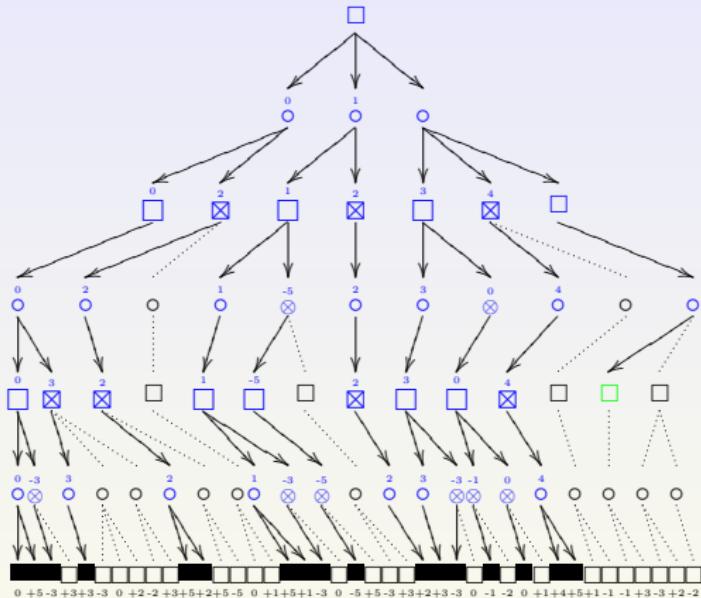
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$

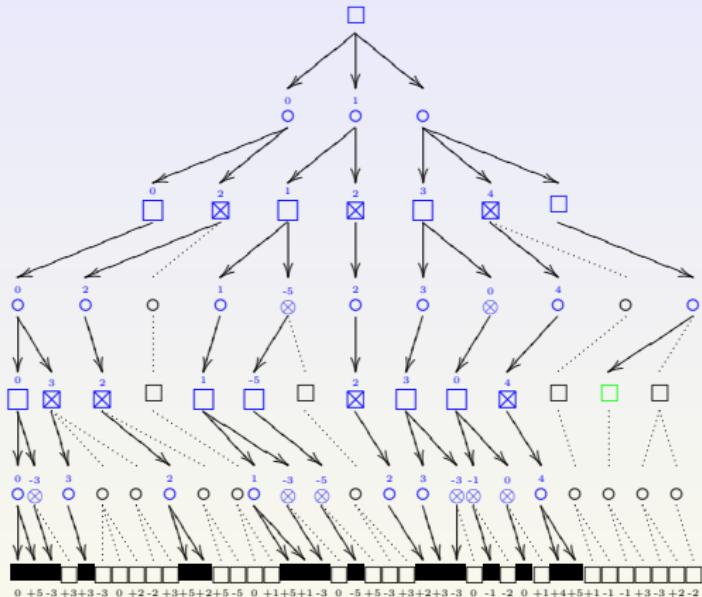


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  → for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

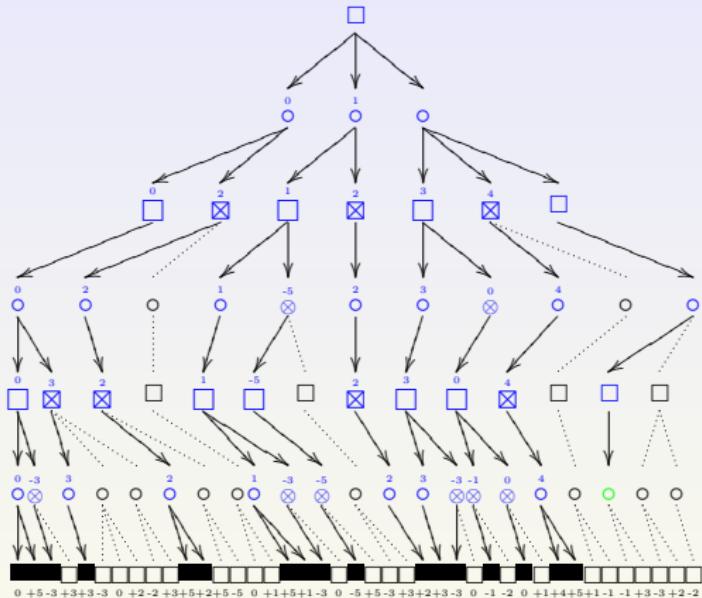
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

→

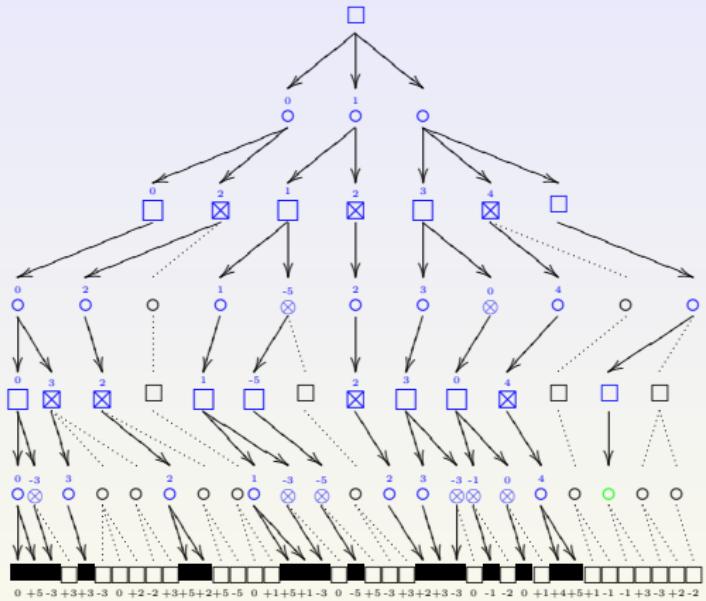
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



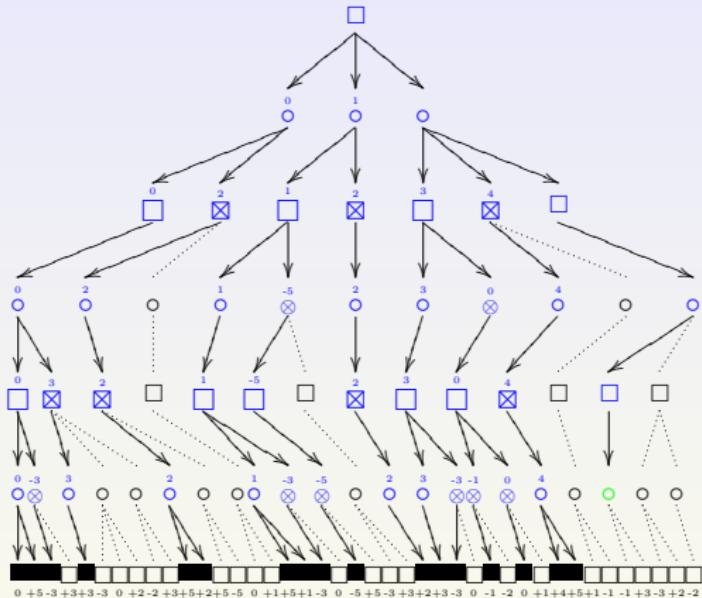


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

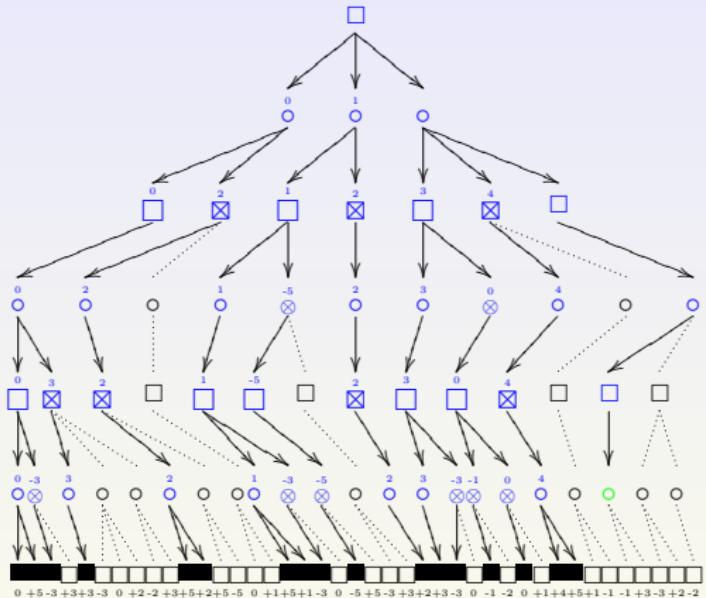
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $\rightarrow$  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

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function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
    a  $\leftarrow$  MAX( $\alpha$ , v)
  return v

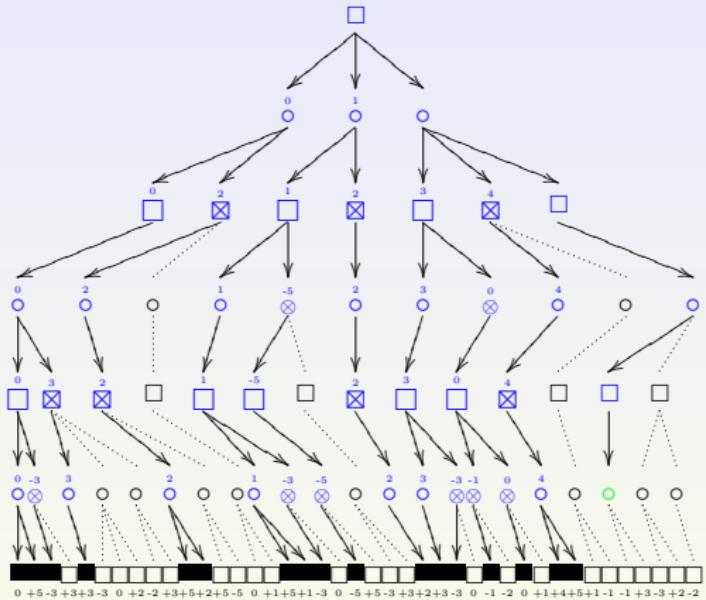
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

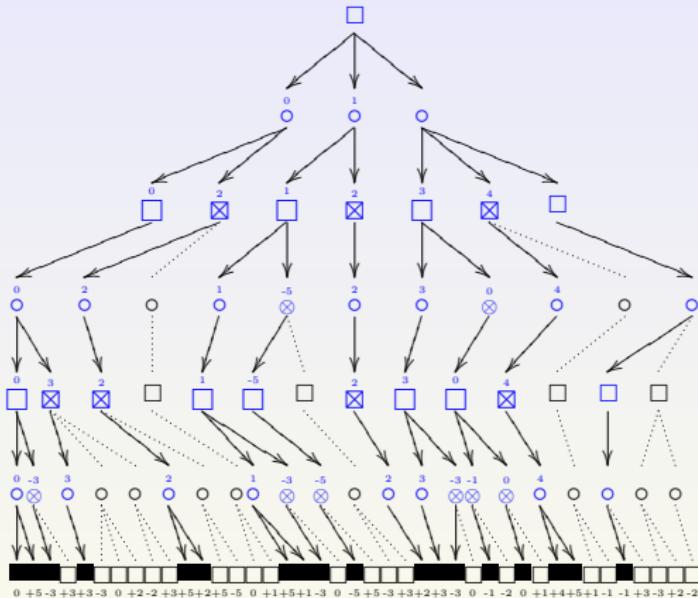
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

→function MAX-VALUE( $state, \alpha, \beta$ ) returns a utility value
  if TERMINAL-TEST( $state$ ) then
    return UTILITY( $state$ )
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS( $state$ ) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then
      return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 

```

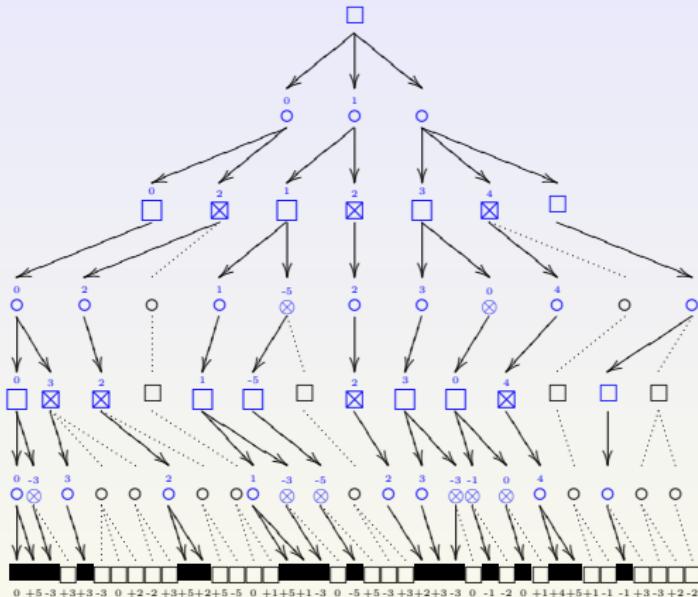
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  → if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\text{v}, \text{MIN-VALUE}(s,  $\alpha$ ,  $\beta$ ))
    \text{if } v \geq \beta \text{ then}
      \text{return } v
    \mathbf{\alpha} \leftarrow \text{MAX}(\mathbf{\alpha}, v)
  \text{return } v$ 
```

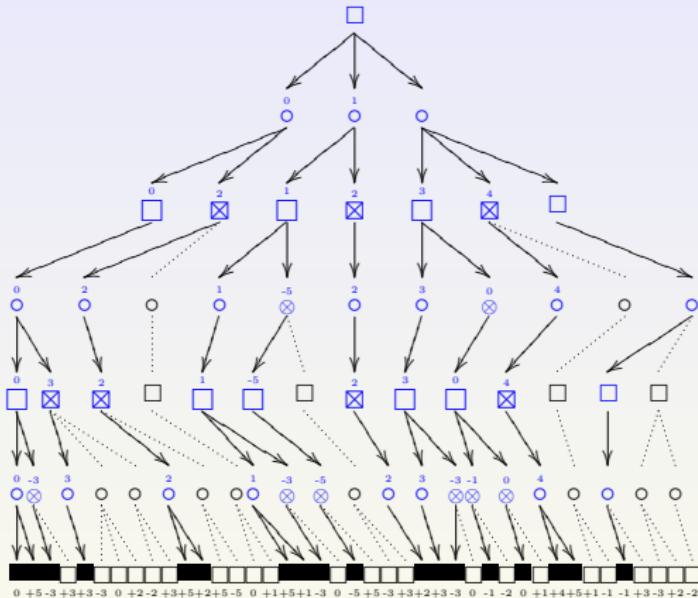
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  if TERMINAL-TEST(state) then
    return UTILITY(state)
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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

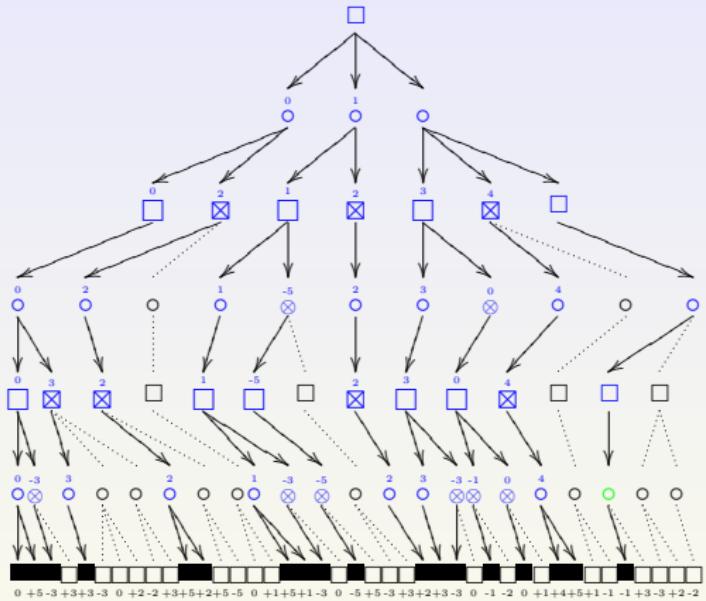
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta$   $\leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

```

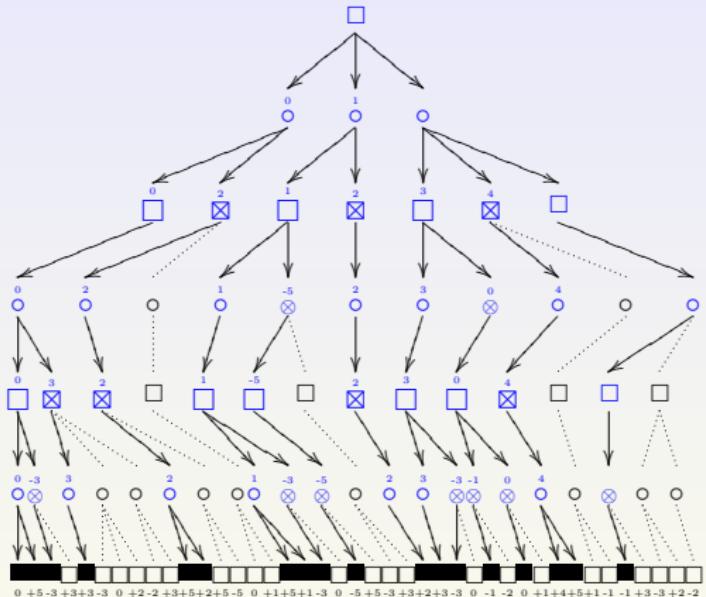
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function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





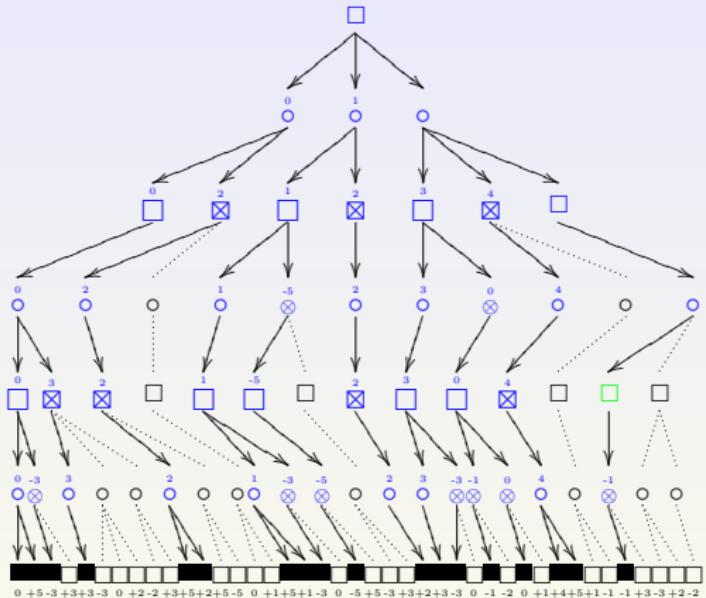
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\geq \beta$  then
    return v
   $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

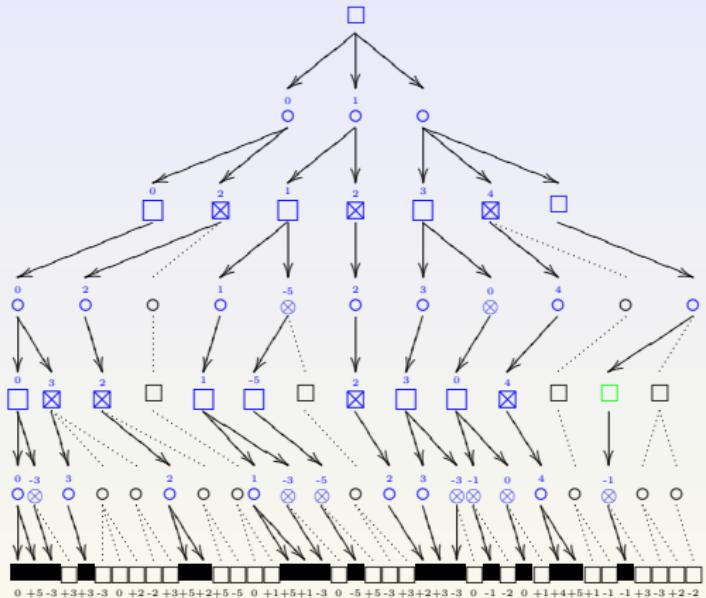
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```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
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  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
  →
   $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

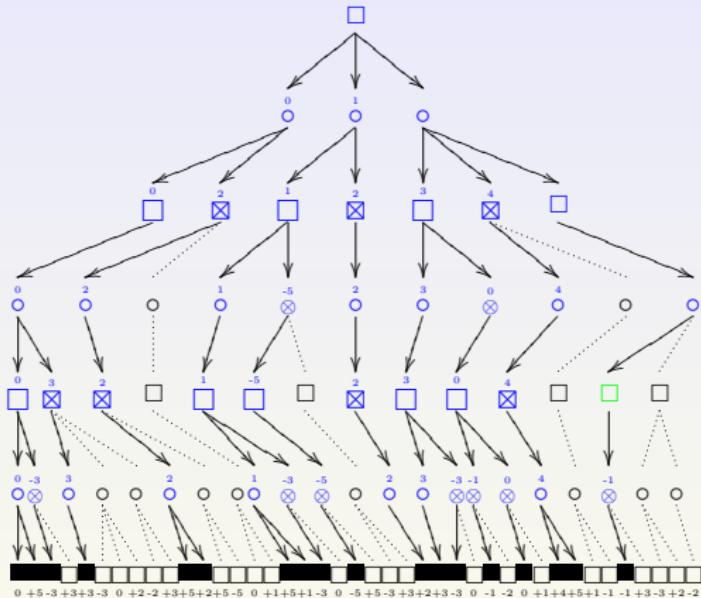
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $-\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha$ , v)
  return v

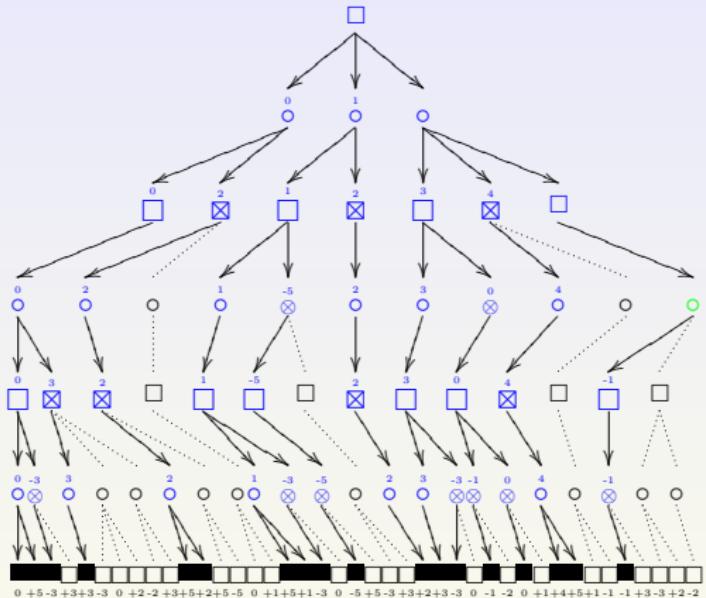
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow$   $+\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



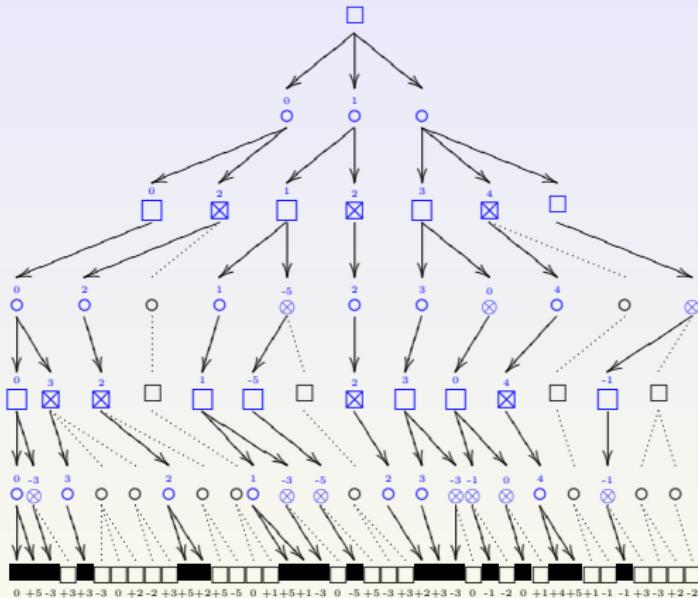
```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
    alpha  $\leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

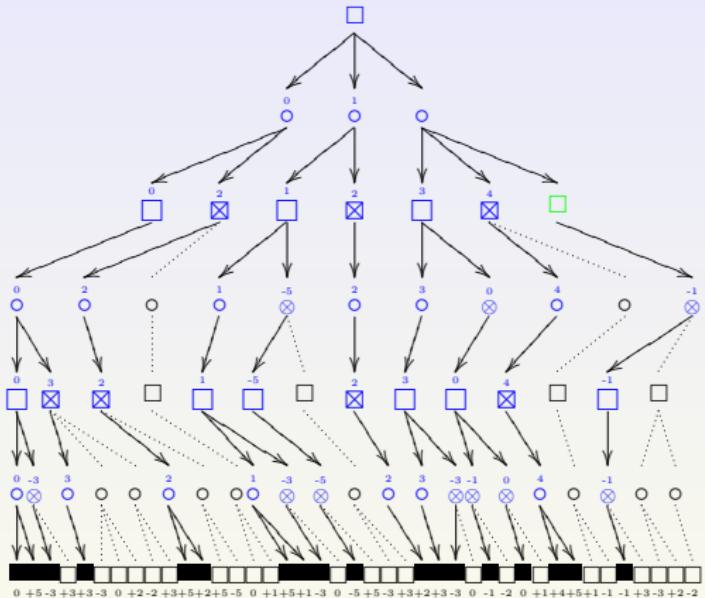
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s, a,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow$  MAX( $\alpha, v$ )
  return v

```

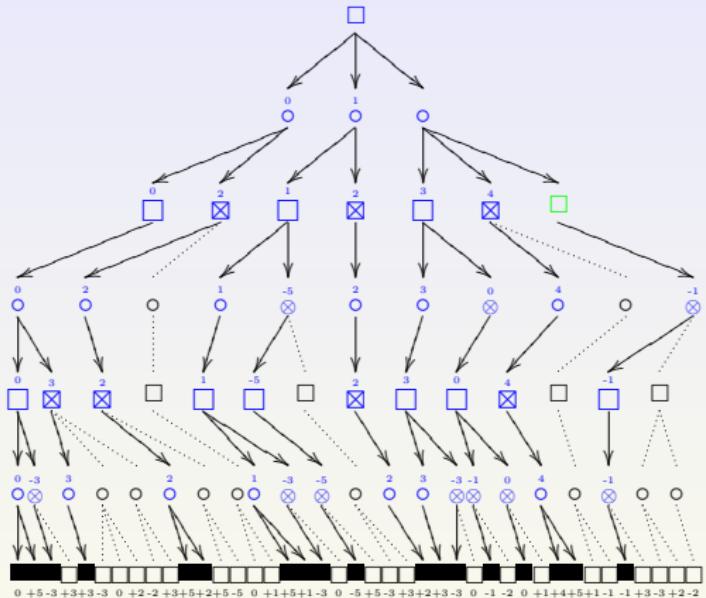
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
  v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
  return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MAX(v, MIN-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

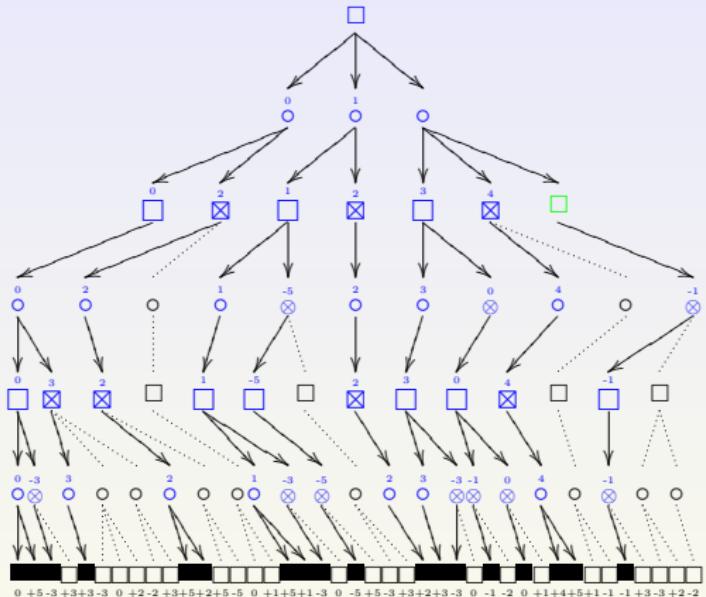
```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$

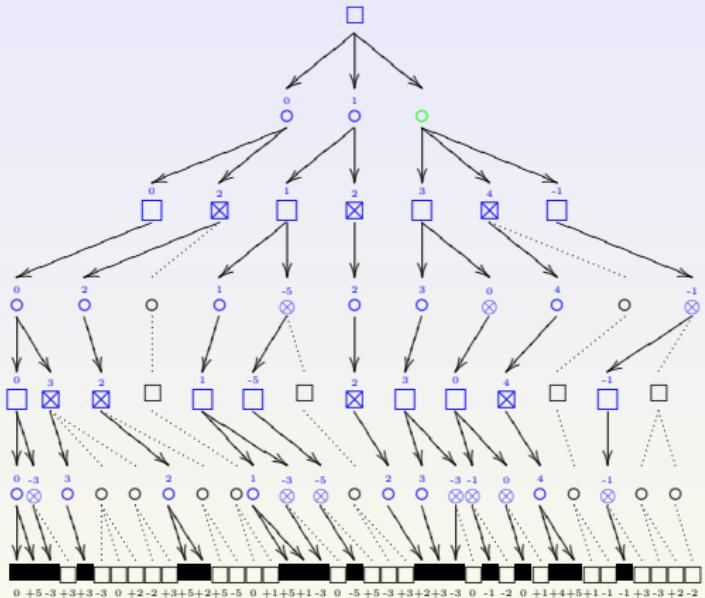


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if  $v \geq \beta$  then
      return v
    α ← MAX(α, v)
  → return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if  $v \leq \alpha$  then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

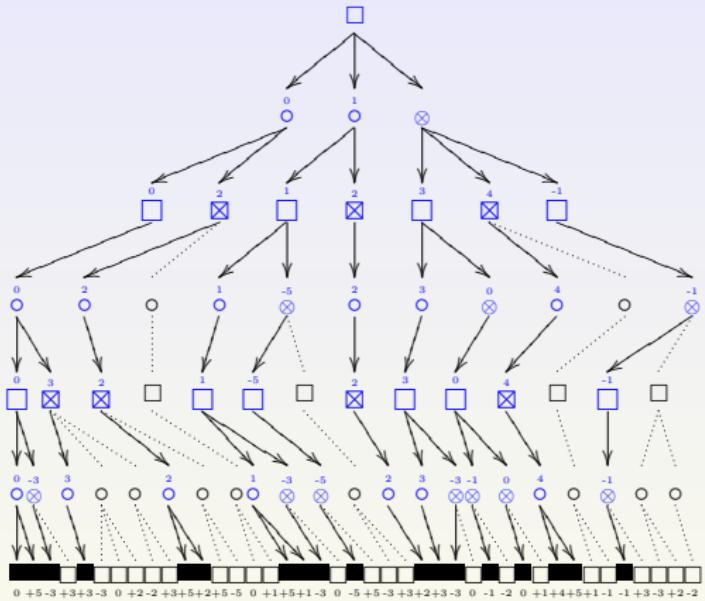
```

```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow$  MIN(v, MAX-VALUE(s,  $\alpha$ ,  $\beta$ ))
  if v  $\leq \alpha$  then
    return v
   $\beta \leftarrow$  MIN( $\beta$ , v)
  return v

```

$$\alpha = 1 \quad \beta = 3$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty, +\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

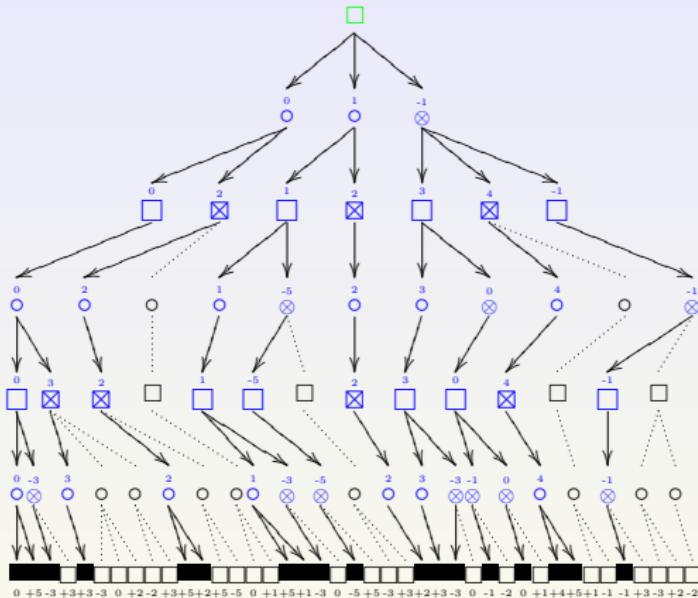
```

function MIN-VALUE(state, $\alpha,\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{Min}(\text{v}, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{Min}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = 3$$





```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

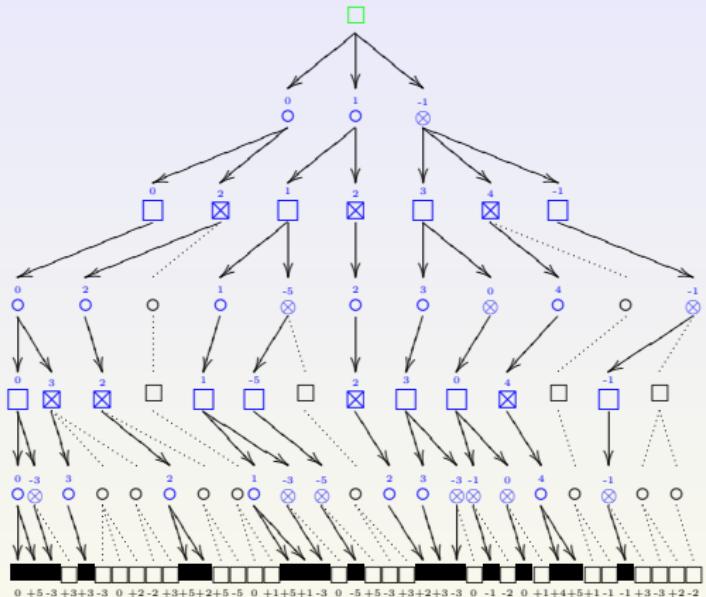
```

function MIN-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MIN}(\iota, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if v  $\leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = 1 \quad \beta = +\infty$$



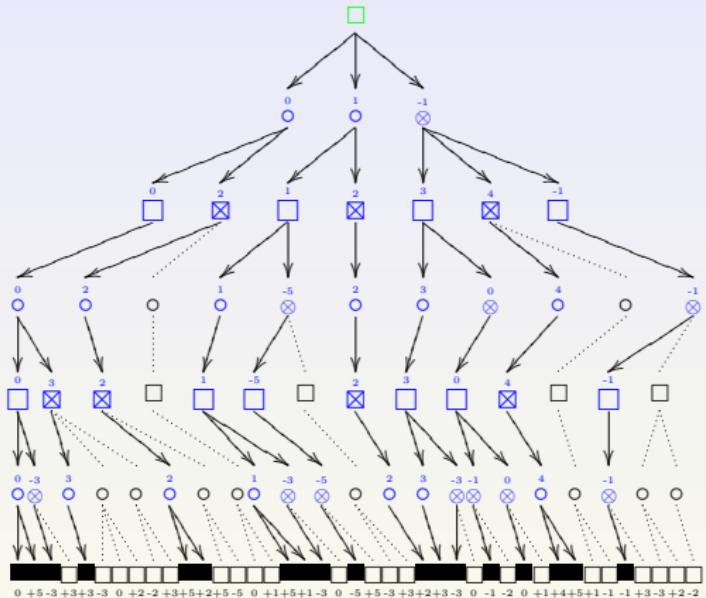


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$

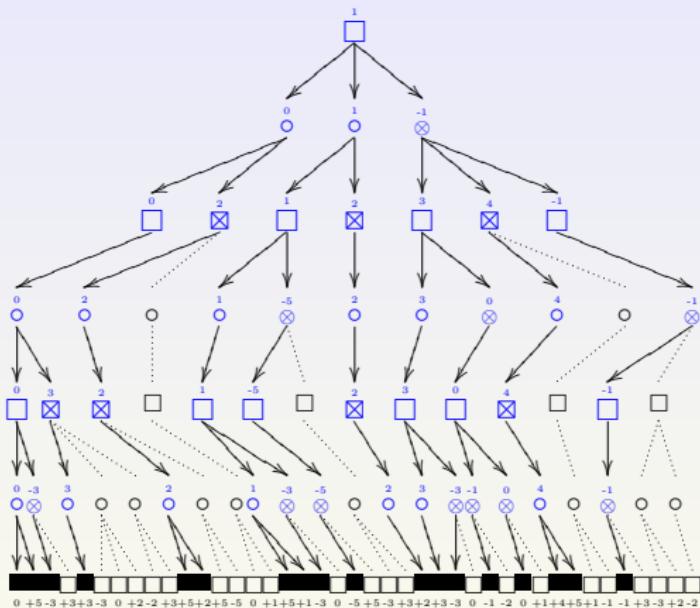


```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← -∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then
      return v
    α ← MAX(α, v)
  → return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then
      return v
    β ← MIN(β, v)
  return v
```

$$\alpha = 1 \quad \beta = +\infty$$



```

function ALPHA-BETA-SEARCH(state) returns an action
    v  $\leftarrow$  MAX-VALUE(state,  $-\infty$ ,  $+\infty$ )
    return the action in SUCCESSORS(state) with value v

```

```

function MAX-VALUE(state, $\alpha$ , $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
  v  $\leftarrow -\infty$ 
  for a, s in SUCCESSORS(state) do
    v  $\leftarrow \text{MAX}(\iota, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if v  $\geq \beta$  then
      return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then
    return UTILITY(state)
   $v \leftarrow +\infty$ 
  for a, s in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then
      return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

$$\alpha = - \quad \beta = -$$

