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# Improvements to the Ainley-Mathieson Method of Turbine Performance Prediction

*In 1951 Ainley and Mathieson published a method of predicting the design and off-design performance of an axial turbine (British ARC, R & M 2974). The flow and hence the losses were calculated at a single "reference diameter" for each blade row. This method has been widely used ever since. A critical review of the method has been made, based on detailed comparisons between the measured and predicted performance of a wide range of modern turbines. As a result, improvements have been made in the formulas for secondary loss and tip clearance loss prediction. The accuracy of the improved method has been assessed. Despite its relatively simple approach, it is believed that it will remain of great value in project work and preliminary design work.*

## Introduction

If a turbine designer is to select the best design for his purpose, he must be able to predict the design point performance of a wide range of possible turbines. In addition, he would like to be able to predict the performance of his designs over a wide range of operating conditions. The Ainley-Mathieson performance prediction method [1][1] is widely used, sometimes with local amendments. Nearly 20 years old, it is based on several broad assumptions and a correlation of the data then available on blade losses. This paper assesses its accuracy applied to a range of modern turbine stages and presents detailed improvements to some of the correlations derived from more recent data. The value of the revised method to the modern designer is discussed.

[1] Numbers in brackets designate references at end of paper.

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## The Ainley-Mathieson Method

The flow through the turbine stage is calculated only at a single "reference diameter." The gas temperature, pressure, and velocity vector are calculated at this diameter at one axial station between each blade row by assuming that the mass flow passing through the turbine at that station is equal to the mass flow per unit height at the reference diameter multiplied by the annulus height. The gas outlet angle relative to each row there is expressed as an empirical function of the blade throat, pitch, and outlet relative Mach number. The profile losses are expressed as an empirical function of blade inlet angle, maximum thickness, trailing edge thickness, pitch, chord, incidence, and gas relative outlet angle, but not Mach number. To these profile losses at the reference diameter are added secondary loss terms and tip clearance terms, also empirical functions of the reference diameter conditions to represent the effect of these losses on the turbine flow (although they are actually incurred near the ends of the blade, not at the reference diameter). Full details appear in reference [1].

This model obviously simplifies the flow greatly. It implies that changes to the detailed profile shape have little effect, provided the maximum thickness and trailing edge thickness are

## Nomenclature

$A_1$ = flow area at inlet to blade row
$A_2$ = flow area at exit from blade row
$c$ = true chord
$C_L$ = lift coefficient
$h$ = mean blade height
$ID$ = hub dia
$k$ = tip clearance
$M_n$ = relative Mach number at exit from blade row
$OD$ = casing dia
Re = Reynolds number based on blade chord and blade row outlet conditions
$s$ = blade pitch
$Y_p$ = profile loss coefficient
$Y_s$ = secondary loss coefficient
$Y_k$ = tip clearance loss coefficient
$Z$ = Ainley's loading parameter
$\alpha_1$ = relative gas angle at blade row inlet
$\alpha_2$ = relative gas angle at blade row exit
$\alpha_m$ = vector mean gas angle = $\tan^{-1} [(\tan \alpha_1 + \tan \alpha_2)/2]$
$\beta_1$ = blade inlet angle
$\eta$ = isentropic efficiency

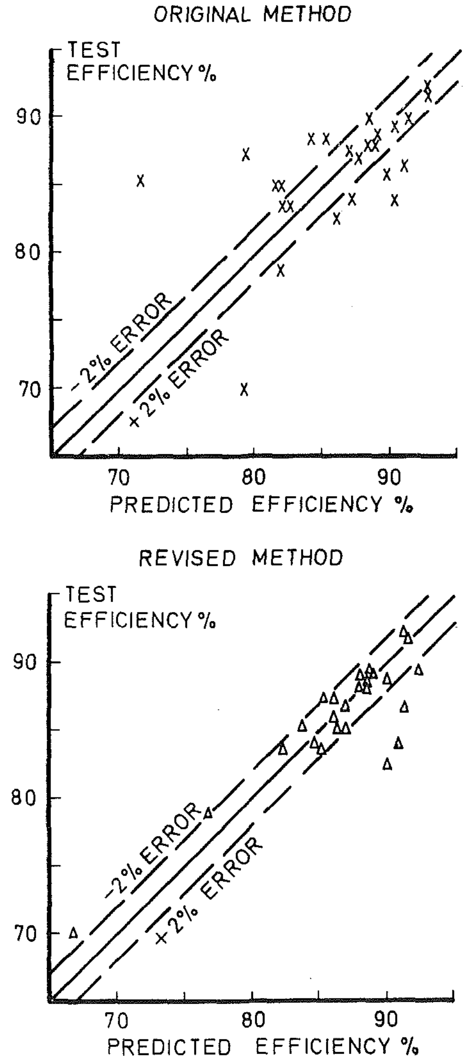


**Fig. 1 Comparison of design point efficiencies of 25 turbines**

unaltered. It also implies that the rule chosen for radial variation of blade design (for example, free vortex, or constant section) has little effect. The implicit assumption is that the design is competent, and conforms to the well known rules that recompression must be avoided at all radii, and that blade passages must converge steadily from inlet to outlet at all radii. These assumptions are crucial to the method, whereas the particular empirical formulas for losses can be altered without affecting the framework of the calculation.

## Improvements

As a result of detailed comparisons between the predicted and measured performance of 16 turbines, changes seemed desirable in the empirical formulas for profile losses, secondary losses, and tip clearance losses. Individual analyses were made of all available data on each type of loss, which will be published separately. Here a brief outline of the analyses is given, and the formulas finally adopted are quoted.

The profile loss depends primarily on the blade inlet angle, gas inlet and outlet angles, and blade pitch/chord ratio which will be collectively referred to as "blade loading," and also on the blade maximum thickness/chord and trailing edge thickness/chord. In reference [1] the profile loss coefficient $Y_p$ was expressed as a complicated function of these factors alone. The additional losses incurred when a blade row chokes, due to the development of shock waves, have now been accounted for by the introduction of an arbitrary correction factor incorporating a numerical constant chosen to fit available overall efficiency data. Insufficient experimental results could be found to support a more detailed study. The expression is

$$Y_p = (Y_p \text{ from reference [1]}) \times [1 + 60(\mathbf{M}_n - 1)^2] \quad (1)$$

the [ ] correction being applied only for $\mathbf{M}_n > 1$.

In reference [1] a Reynolds number correction to overall efficiency was recommended for $\mathrm{Re} < 2 \times 10^5$

$$(1 - \eta_{corrected}) = (1 - \eta)\left(\frac{\mathrm{Re}_{\mathrm{mean}}}{2 \times 10^5}\right)^{-0.2} \quad (2)$$

In the revised method an optional correction has been applied directly to the profile and secondary losses using the Reynolds number appropriate to the particular blade row

$$(Y_p + Y_s)_{\mathrm{corrected}} = (Y_p + Y_s)\left(\frac{\mathrm{Re}}{2 \times 10^5}\right)^{-0.2} \quad (3)$$

In all the examples but one, the Reynolds numbers were near enough to $2 \times 10^5$ to make little difference and were not applied.

The secondary losses arise from the interaction between the flow through the blade passage and the end wall boundary layer. They must therefore depend on the blade loading, the blade shape, and the end wall boundary layer. Several empirical formulas have been suggested for the effect of blade loading, but it was found in reference [2] that cascade data are best fitted by the new blade loading parameter

$$Z \cos \alpha_2 / \cos \beta_1$$

where $Z$ is the Ainley loading parameter [1]

$$\left(\frac{C_L}{s/c}\right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m}$$

No experiments have been conducted on the effect of blade shape, and few on the influence of the upstream wall boundary layer, but there is no doubt that a thicker wall boundary layer gives rise to higher secondary losses. For more advanced methods of performance prediction it will be necessary to calculate the wall boundary layer development, but here this complication is avoided by the adoption of a single numerical constant in place of a function of wall boundary layer thickness and blade shape. The numerical constant was chosen from comparisons with overall efficiency data, and it also compensates for the use of reference diameter blade loading values instead of values at the ends of the blade.

The final expression is

$$Y_s = 0.0334\left(\frac{c}{h}\right)\left(\frac{\cos \alpha_2}{\cos \beta_1}\right) Z \quad (4)$$

in place of Ainley's

$$Y_s = \lambda Z \quad (5)$$

($\lambda$ being an empirical function, Fig. 8 of reference [1]).

The tip clearance loss is defined here as the increase in end loss due to tip clearance. It is due to several factors, discussed for example in reference [3], arising from the clearances between the ends of a blade and the adjacent wall. For convenience, the secondary loss coefficient $Y_s$ is taken to be unaltered although the secondary flow is of course affected by the clearance. The tip clearance loss coefficient $Y_k$ depends on the blade loading and the size and nature of the clearance. Examination of both cascade and turbine data confirmed that Ainley's blade loading parameter $Z$ represents the effect of blade loading satisfactorily.

Hubert's cascade data [4] for plain clearances suggested that the usual linear dependence of loss on tip clearance should be replaced by the power law

$$Y_k \propto k^{0.78} \quad (6)$$

and this proved better for turbine results too.

The final expression is

$$Y_k = B \frac{c}{h} \left(\frac{k}{c}\right)^{0.78} Z \quad (7)$$

($B$ = 0.47 for plain tip clearance, 0.37 for shrouded) replacing Ainley's

$$Y_k = BkZ/h \quad (8)$$

($B$ = 0.5 for plain tip clearance, 0.25 for shrouded).

Some shrouded blades have multiple tip seals which are more effective than single seals. This is represented by using an effective value

$$k = (\text{geometrical } k) \times (\text{number of seals})^{-0.42} \quad (9)$$

The extent to which the geometrical design of the seals influences their effective clearance is not yet known.

The method had already been revised to take account of the aerodynamic losses due to blade cooling. This revision has already been published [5] and will not be referred to again.

## Comparison With Test Results

Sixteen single-stage gas turbines were initially chosen as examples, and later nine more were added. They covered a wide range of designs and sizes, but in every case the performance had been carefully measured at some time on a cold air rig.

The original method predicted the choking flow of most example turbines to within ±3 percent. This flow depends primarily on the nozzle throat area, and the extent to which flow prediction errors arose simply from inaccurate throat areas is impossible to say. The variation of flow with pressure ratio and speed was usually nearly correct.

Fig. 1 compares the design point efficiency of all 25 examples

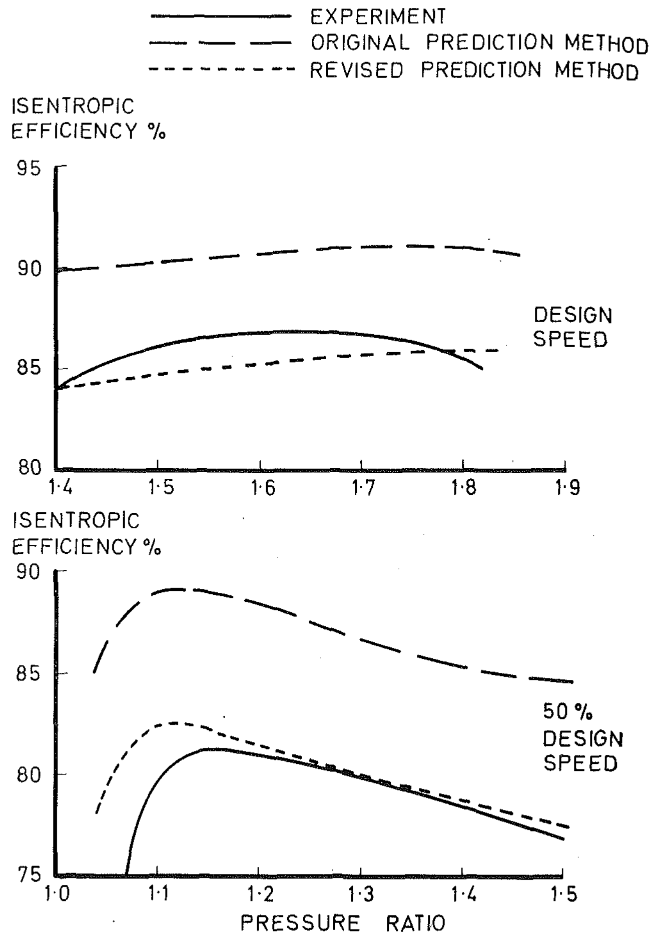


**Fig. 3 Low aspect ratio turbine**

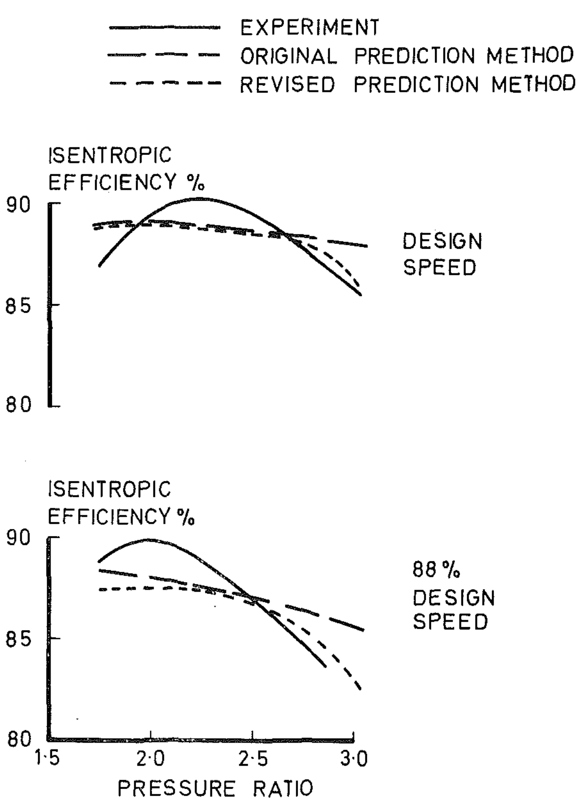


**Fig. 2 Effect of choking on losses**

with predictions. Typical aircraft engine turbines usually fell within ±3 percent of the original predictions, but much larger errors occurred with certain unusual designs and with small turbines. The revised method brought most of the examples within ±2 percent irrespective of their size or design. The three turbines which fell seriously short of their predicted efficiency were three versions of the same stage with different radial variations of blade design, and the authors strongly suspect negative reaction at the rotor blade roots. This condition is well known to lose efficiency. The examples were included to emphasize the "competent design" criterion already introduced as essential if the method is to work.

Individual examples will now be used to illustrate off design results and the nature of the improvements. In all cases, "efficiency" refers to total-to-total isentropic efficiency, and "speed" refers to corrected speed, i.e.,

rev/min $\sqrt{\dfrac{288}{\text{inlet temperature, } ^\circ K}}$, "pressure ratio" refers to stagnation pressure ratio.

Fig. 2 shows some characteristics for a typical aircraft engine turbine. The efficiency level was predicted satisfactorily by the original method, and the revised method shows little change except when choking occurred. Here the measured efficiency fell away, a trend indicated satisfactorily by the revised method but not by the original method.

The main improvement is in the prediction of secondary losses for both low aspect ratio turbines and low reaction turbines. Small turbines inevitably have low aspect ratio blading. Fig. 3 shows an example with a mean aspect ratio of about unity. The revised efficiency prediction is much better at both speeds. A speed as low as 50 percent is a severe test of any method. Fig.

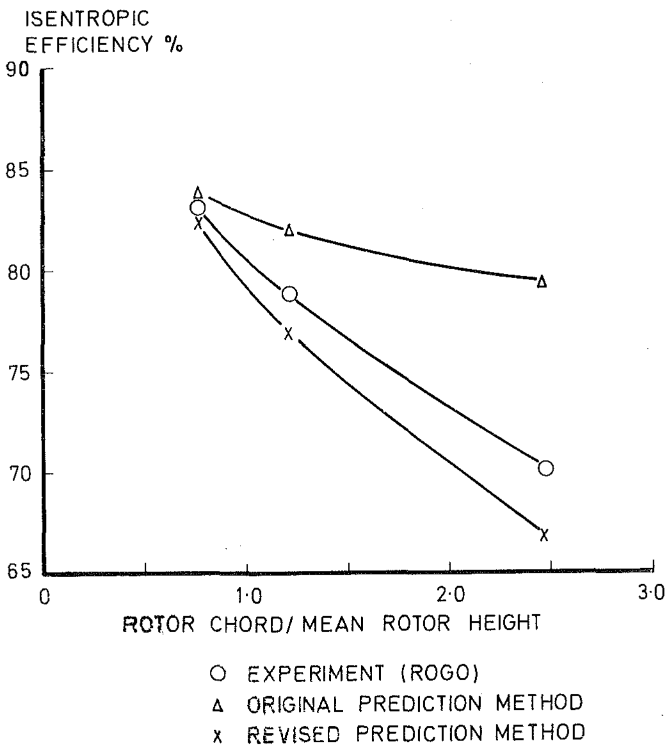


**Fig. 4 Effect of aspect ratio**

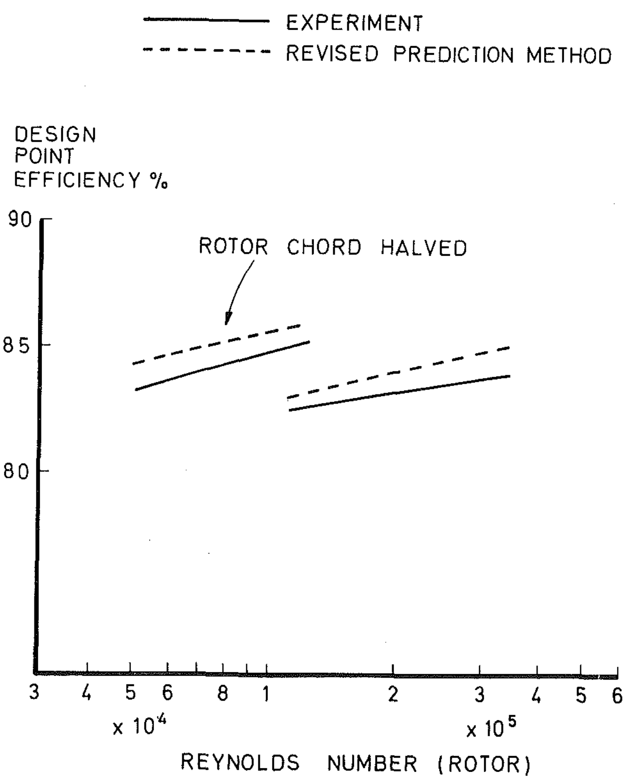


**Fig. 5 Effect of halving rotor chord**

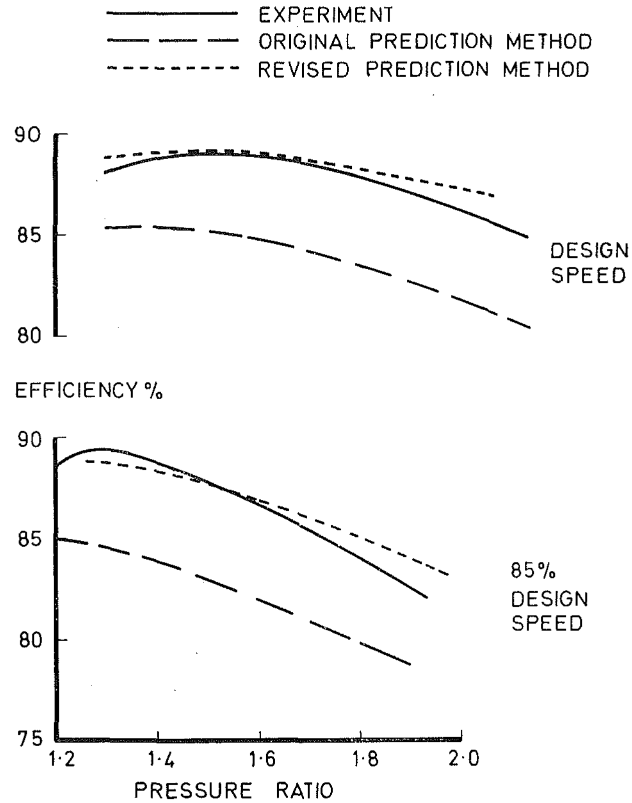


**Fig. 6 Low reaction turbine**

4 shows the design speed and pressure ratio efficiency comparisons for Rogo's turbines [6]. Again the revised method demonstrates its superiority. The secondary losses were overestimated, perhaps because nozzle wall profiling was employed to reduce them. Rogo's results were not used in choosing the constant in equation (2).

Rogo varied the aspect ratio by changing the blade height but keeping the chord constant. On the other hand, Fig. 5 shows the effect of a change in aspect ratio by halving the rotor blade chord (doubling the number of blades to maintain aerodynamic similarity) but keeping the height constant. This halved the Reynolds number for given inlet conditions, so the gain in efficiency due to high aspect ratio was offset by the loss in efficiency due to reduced Reynolds number. The net change was accurately predicted.

Fig. 6 shows the predicted and measured efficiency of a high aspect ratio turbine of about 25 percent reaction. In the original predictions, the value of Ainley's parameter $\left[\frac{(A_2/A_1)^2}{1 + ID/OD}\right]$ exceeded the range of his correlation (Fig. 8 of reference [1]) with the result that the secondary losses were overestimated. The revised method which does not use that correlation proved much more accurate.

Figs. 7 and 8 show the effect of tip clearance on a rotor and a nozzle row, respectively. Fig. 7 shows that both the original and revised methods agree on the efficiency changes of the Rogo turbines [6]; as already noted the revised method predicts the efficiency level better. Fig. 8 refers to the same small turbine as Fig. 3, which was tested with small clearances at the ends of the nozzle blades, and again with those clearances sealed. The revised method predicted the efficiency change more accurately.

## Discussion

The original profile loss correlation appears reasonably good, and remains unchanged except at supersonic Mach numbers. The very simple correction suggested in equation (1) penalizes supersonic outlet velocities heavily, probably because a high outlet Mach number at the reference diameter normally implies an even higher one at the hub. The correction would not apply to blading specially designed for high Mach numbers as in low pressure stages of steam turbines. The subsonic profile loss correlation is not wholly satisfactory because it sometimes suggests different optimum blade numbers from more recent data; see for example reference [7].

The revised method shows advantage over the original one chiefly when applied to small turbines (with which reference [1] was not primarily concerned). The change in secondary loss correlation is evidently an improvement, though it too cannot be wholly satisfactory as it takes no account of the effect of inlet annulus wall boundary layer thickness. Fig. 5 is particularly striking because it concerns the designer's dilemma in choosing blade chord. The revised method can evidently offer useful guidance.

It is reasonable to inquire as to the value of this type of approximate method when modern computers enable much more detailed calculations to be attempted. Various methods [8, 9] are now in use in which the flow is calculated at several radial stations instead of only one, and the advance of boundary layer prediction methods can replace the purely empirical loss formulas, first for unstalled profile losses and later perhaps for all losses. Experience of advanced methods has drawn attention to certain disadvantages, however. The major difficulty is data preparation, since they need perhaps 10 times as much information as the simple Ainley-Mathieson method. Most of the extra information has not been decided at the initial design stage, and usually has only a minor effect on efficiency. It would be a nuisance to have to choose precise blade and annulus shapes for every tentative design. Furthermore, computers are still not fast enough for the designer to afford to try dozens of cases at a time by an advanced method. It is preferable to select a few final contenders for the design by using a simple method, and only then to check them by a more detailed method. The authors therefore believe that the simple Ainley-Mathieson method will continue to find everyday use in small and large firms alike.

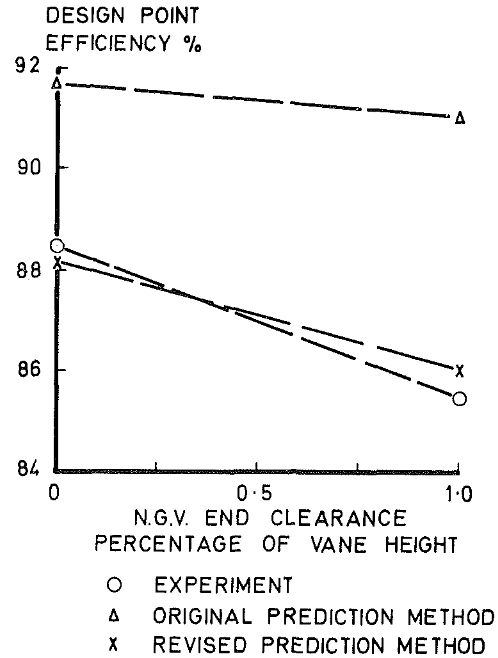


Fig. 8 Effect of nozzle guide vane end clearance

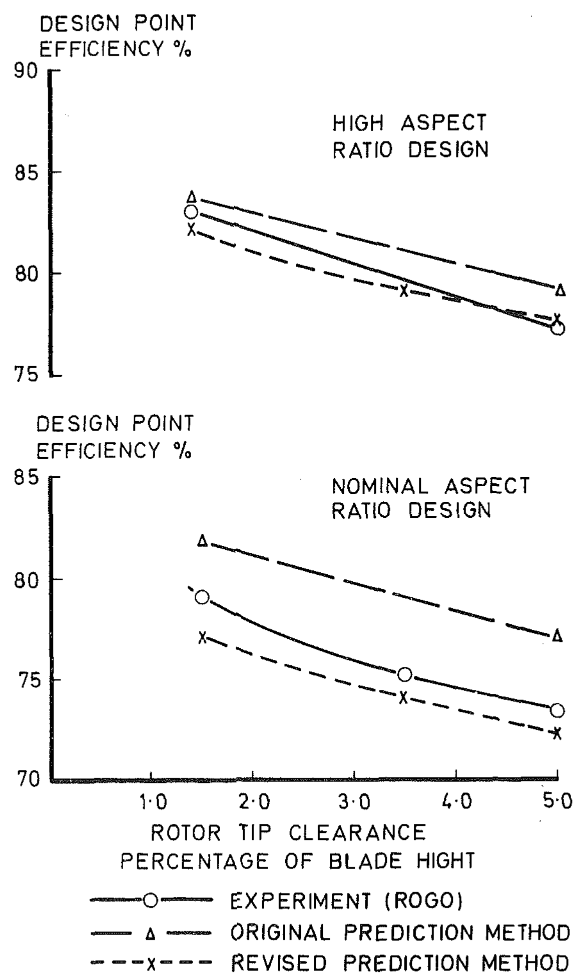


Fig. 7 Effect of rotor tip clearance

## Conclusions

The Ainley-Mathieson performance prediction method [1] has been tested against experimental data from 25 turbines. Although it was satisfactory for typical aircraft turbines, it proved to be misleading for small turbines. This has been corrected by changing some of the loss correlations, especially the secondary loss correlation. Provided the blading is competently designed, the revised method appears reliable over the range of designs and conditions tried to within $\pm 3$ percent flow (depending on the accuracy of the throat area) and to within $\pm 2$ percent on efficiency. It is a useful aid to design.

## Acknowledgments

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