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THE EFFECT OF CLEARANCE ON SHROUDED AND UNSHROUDED TURBINES AT TWO LEVELS OF REACTION

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ABSTRACT

In this paper, the effect of seal clearance on the efficiency of a turbine with a shrouded rotor is compared with the effect of the tip clearance when the same turbine has an unshrouded rotor. The shrouded versus unshrouded comparison was undertaken for two turbine stage designs one having 50% reaction the other having 24% reaction. Measurements for a range of clearances, including very small clearances, showed three important phenomena.

Firstly, as the clearance is reduced, there is a "break-even clearance" at which both the shrouded turbine and the unshrouded turbine have the same efficiency. If the clearance is reduced further, the unshrouded turbine performs better than the shrouded turbine, with the difference at zero clearance termed the "offset loss". This is contrary to the traditional assumption that both shrouded and unshrouded turbines have the same efficiency at zero clearance. The physics of the break-even clearance and the offset loss are discussed.

Secondly, the use of a lower reaction had the effect of reducing the tip leakage efficiency penalty for both the shrouded and the unshrouded turbines. In order to understand the effect of reaction on the tip leakage, an analytical model was used and it was found that the tip leakage efficiency penalty should be understood as the dissipated kinetic energy rather than either the tip leakage mass flow rate or the tip leakage loss coefficient.

Thirdly, it was also observed that, at a fixed flow coefficient, the fractional change in the output power with clearance was approximately twice the fractional change in efficiency with clearance. This was explained by using an analytical model.

NOMENCLATURE

b, h	blade pitch, blade span
c, c_m	chord, meridional chord
c_p	specific heat capacity at constant pressure
C_D	tip leakage discharge coefficient
g/h	clearance-to-span ratio
Δh_{0stage}	stage stagnation enthalpy drop
Δh_{0rotor}	stagnation enthalpy drop through the rotor
$\dot{m}$, $\dot{m}_L$	mass flow rate, leakage mass flow rate
p, T, ρ	pressure, temperature, density
p_0, T_0	stagnation pressure, stagnation temperature
Δs	change in specific entropy
U, V_m	blade speed at mid-span, meridional velocity
V, W	absolute and relative velocity
$\frac{1}{2}W_3^2$	specific relative kinetic energy
Z_w	Zweifel coefficient = $(b/c_m)\times V_m \Delta V_\theta / \frac{1}{2}V_{exit}^2$
α, β	absolute and relative flow angle
Λ	reaction = $\Delta h_{rotor}/\Delta h_{stage}$
ζ	loss coefficient = $T\Delta s/\frac{1}{2}W_3^2$
ψ	stage loading coefficient = $\Delta h_{0stage}/U^2$
Ψ	output power = torque$\times$angular velocity
η	total-to-total efficiency
$\Delta\eta$	tip leakage efficiency penalty = $\eta_{zero\ clearance} - \eta$
η_{offset}	$\eta_{unshrouded} - \eta_{shrouded}$ evaluated at zero clearance
ϕ	flow coefficient = V_m/U

Subscripts:

1, 2, 3	NGV inlet, rotor inlet, rotor exit
p, s	pressure side, suction side at the tip.
m, θ	meridional, tangential directions

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INTRODUCTION

Two of the key decisions in the design of a turbine are the choice of the tip architecture (i.e. shrouded or unshrouded) and the level of reaction. Aerodynamically these are interlinked and one of the purposes of this piece of research was to undertake a detailed study where the clearance[2] was varied (including very small clearances) in both shrouded and unshrouded turbines and at two levels of reaction.

Shrouded versus unshrouded

In a turbine, there is an inevitable clearance between a moving rotor and the stationary casing. This allows the tip leakage flow that causes the tip leakage efficiency penalty. In an unshrouded turbine, the leakage flow is driven by the pressure difference between the pressure side and the suction side, whereas, in a shrouded turbine, the driving pressure for the leakage flow is the pressure drop across the rotor shroud (from the upstream to the downstream of the rotor).

Both unshrouded and shrouded turbines are commonly used in the gas turbine industry. The main benefit of employing an unshrouded turbine is that it reduces the stage weight and the stress in the rotor blade and disc. Hence the turbine can be run faster which could increase the power output or allow the aerodynamic loading to be reduced. Shrouded turbines are known to be aerodynamically more efficient than unshrouded turbines, at the same flow conditions, because the tip leakage flows can be minimized by the use of multiple seals, Denton [1] and Harvey [2].

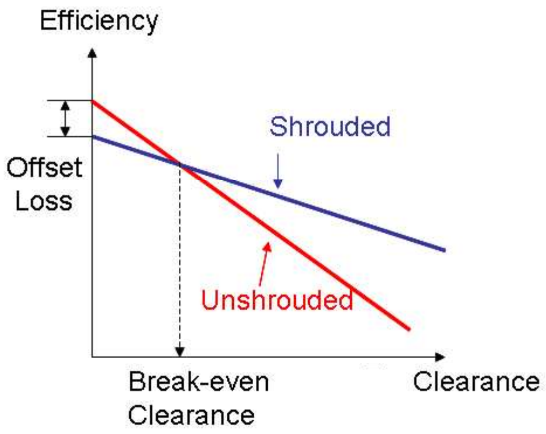

Fig. 1. The break-even clearance and the offset loss, reproduced from Harvey [3].

Traditionally, it has usually been assumed that the aerodynamic advantage of a shrouded turbine over an unshrouded turbine is reduced as the clearance is decreased, displaying the same efficiency at zero clearance. However, Harvey [3] suggested that there should be a break-even clearance at which the two turbines have the same efficiency. If the clearance is reduced further, the unshrouded turbine should perform better than the shrouded turbine, with the difference at zero clearance termed the "offset loss" as shown in Fig. 1. This offset loss is caused by entropy generation in the cavities upstream and downstream of the shroud, by change in the secondary flow loss within the rotor blade row and the windage drag due to skin friction on the upper surface of the shroud. The existence of the offset loss has been supported by the CFD studies of Denton [4] but it has not previously been experimentally demonstrated.

Level of reaction

It can be shown that for repeating stages, the stage loading coefficient (ψ) can be expressed as a function of the flow coefficient (ϕ), reaction (Λ) and the absolute swirl angle at the stage exit (α_3):

$$\psi = 2(1 - \Lambda - \phi \tan \alpha_3) \qquad (1)$$

It should be noted that in the above equation α_3 is usually negative, opposite to the direction of rotation. A reduction in the level of reaction at constant flow coefficient has the effect of either increasing the stage loading coefficient or reducing the magnitude of the exit swirl angle.

Reaction clearly influences the pressure drop across the rotor shroud and hence the tip leakage in shrouded turbines. However, even in an unshrouded turbine the level of reaction can influence the tip leakage by changing the pressure surface to suction surface pressure difference across the rotor tip.

The effect of reaction on the tip leakage efficiency penalty has not been investigated fully despite its importance. The authors are not aware of a systematic study examining the effect of reaction on shrouded turbines although there are a few studies regarding the effect of reaction on unshrouded turbines.

Abianc [5] compared[3] an unshrouded impulse turbine with an unshrouded 50% reaction turbine and showed that the impulse turbine reduced the tip leakage efficiency penalty compared to the 50% reaction turbine at a given clearance, see Fig. 2. Stechkin et al. [6] and Cordes [7] referred to these results and Cordes interpolated the results in order to correlate the level of reaction. These interpolated results have been widely circulated within the turbomachinery community through the literature of Hong and Groh [8], Glassman [9] and Harvey [2]. Booth [10] presented data for unshrouded turbines which showed that reducing the tip reaction decreased the tip leakage efficiency penalty. Haas and Kofskey [11] examined the effect of clearance on various turbines and showed that reducing the tip reaction had a slight benefit in both shrouded and unshrouded turbines. However, it has not yet been fully explained why a lower reaction should reduce the tip leakage efficiency penalty in an unshrouded turbine.

[2] Clearance refers to both the shrouded and unshrouded configurations.

[3] Assumed to be measured but not explicitly stated in the source text.

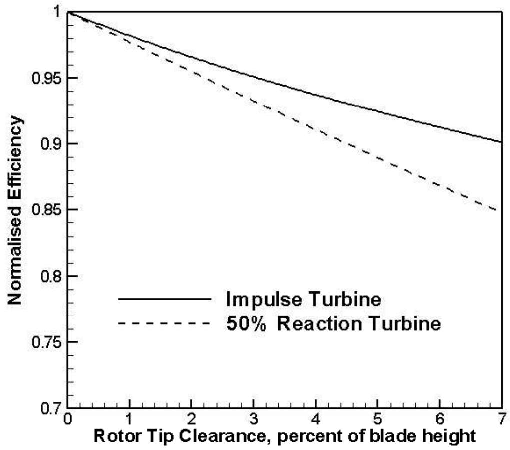

Fig. 2: The effect of reaction on the efficiency of unshrouded turbines, Abianc [5].

Denton [1] noted that for a 50% reaction turbine, the pressure drop between the pressure side and the suction side is similar to that between the upstream and the downstream of the rotor. For such a turbine, the tip leakage efficiency penalty would therefore be similar for both the unshrouded rotor and the shrouded rotor with a single seal. Denton also argued that as the reaction is reduced, the pressure drop across the shroud becomes smaller which would reduce the tip leakage efficiency penalty[4] in shrouded turbines. However, in unshrouded turbines as the reaction is reduced the pressure difference between the pressure side and the suction side would tend to increase which would increase the tip leakage mass flow rate.

This argument suggests that using a lower reaction would be beneficial in shrouded turbines but not in unshrouded turbines. However, the results shown in Fig. 2 suggest that, as the reaction is reduced, the tip leakage efficiency penalty is reduced even in unshrouded turbines.

Paper outline

In this paper the effects of clearance, turbine architecture (i.e. shrouded or unshrouded) and level of reaction are investigated experimentally. The efficiency of four different turbine builds (shrouded versus unshrouded and 50% versus 24% reaction) were measured each operating with seven different sizes of clearance. Three important phenomena associated with the tip leakage are discussed using the experimental results along with analytical/empirical models.

Firstly, the aerodynamic performance of the two shrouded turbines (50% and 24% reaction) are compared with their performance when the rotor shroud had been removed (unshrouded) over a range of clearances which included very small clearances. The break-even clearance and the offset loss were determined for both the 50% and 24% reaction turbines. It is believed that this is the first time that this has been done.

Secondly, the results are processed to examine the effect of reaction on the fractional change of efficiency with clearance. As will be described later, the experimental results showed that the 24% reaction turbine had a smaller fractional change of efficiency with clearance than the 50% reaction turbine in both the shrouded and unshrouded rotor configurations. The physical mechanism behind this reduced tip leakage efficiency penalty is explained using analytic and empirical models.

Finally, this paper also addresses the observation that at fixed flow coefficient, contrary to the common assumption, the fractional change in the output power with clearance is almost twice that of the fractional change of efficiency with clearance. This is explained using an analytic model.

Reaction (Λ)	50%	24%
Flow coefficient (ϕ)	0.5	
Rotational speed	1100 rpm	
Rotor tip radius	0.52 m	
NGV: number of blades	24	
Rotor: number of blades	126	
Stage loading coefficient (ψ)	1.45	1.67
Stage exit flow angle (α_3)	-30.3°	-16.4°
NGV: pitch/merid-chord (b/c_m)	1.20	1.20
NGV: aspect ratio (h/c_m)	0.78	0.78
NGV: Zweifel coefficient (Z_w)	0.66	0.58
NGV: Reynolds number	4.8×10^5	6.4×10^5
Rotor: pitch/merid-chord (b/c_m)	1.00	0.68
Rotor: aspect ratio (h/c_m)	3.42	2.48
Rotor: Zweifel coefficient (Z_w)	0.74	0.77
Rotor Reynolds number	1.2×10^5	1.8×10^5

Table 1: Design parameters for the 50% and 24% reaction turbines tested in the IP turbine facility (each in shrouded and unshrouded architecture).

EXPERIMENTAL FACILITY

The experimental work presented here was undertaken using a single-stage, low-speed, axial-flow turbine facility at the Whittle laboratory. The basic aerodynamic parameters are listed in Table 1 and a schematic of the working section of the facility is shown in Fig. 3. A separate 200 kW electrically driven fan (not shown) supplies the air to the facility. The turbine shaft speed is controlled by an eddy current dynamometer to within ±0.5 rpm. Further details of the facility are given in Pullan et al [13]. Efficiency measurements within the facility have been shown to be repeatable to 0.1%.

The mainstream flow path of the turbine is characterized by a large change in radius through the stage. Inlet Guide Vanes (IGVs) were used to generate an inlet yaw angle

[4] Early steam turbine designers often used impulse turbines in order to minimise the tip leakage flow over the shroud, Harris [12].

distribution representative of engine conditions. An engine representative inlet stagnation pressure profile was generated using a metal gauze upstream of the IGVs and by selectively obstructing the flow at both the hub and casing at that location.

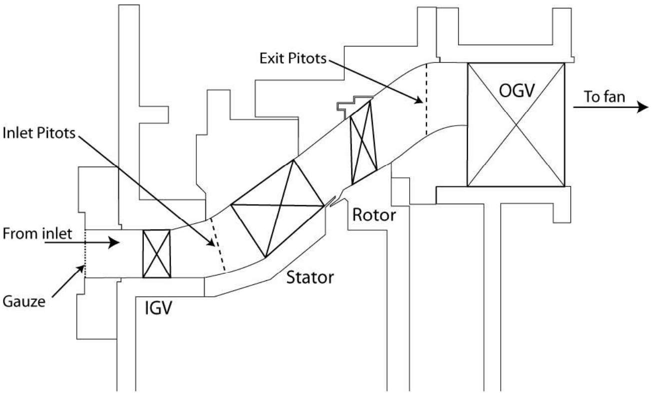

Fig. 3: Intermediate Pressure (IP) turbine rig.

For all of the turbines investigated the total-to-total efficiency was calculated according to:

$$\eta = \frac{(\text{brake torque}) \times (\text{angular velocity})}{\dot{m}_{inlet} c_p T_{0\,inlet}\left[1 - \left(\frac{p_{0exit}}{p_{0inlet}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{2}$$

Throughout all of the experimental tests no significant non-axisymmetric flow features were encountered.

The 24% reaction turbine was designed to increase the work output and to reduce the stage exit swirl angle compared to the 50% reaction turbine at the same flow coefficient.

ASSESSMENT OF OPERATING CLEARANCE

In this paper, the clearance was varied systematically, in both the 50% and 24% reaction turbines, with both shrouded and unshrouded tip configurations. For the shrouded turbines the casings were machined to produce a 3 mm gap between the casing and the entire shroud. A balsa seal was then used to control the clearance in the shrouded turbines, Fig. 4. Seven clearances were investigated with the smallest clearance being when the balsa seal rubbed against the shroud. The clearance was gradually increased by reducing the balsa seal. The maximum possible clearance was 3 mm (the size of the casing-shroud gap). In this experimental study an axial clearance was used, instead of a radial clearance, because this allowed the seal clearance to be easily changed.

After the measurements with the shrouded turbines were completed, the shroud and the balsa seal were removed. Synthetic material was then used to produce a flat casing for the unshrouded turbines, Fig. 4. Machining away the tips of the rotors gradually increased the clearance in the unshrouded turbines. Seven clearances, of similar size to those in the shrouded turbines, were used to investigate the effect of clearance on the efficiency of the unshrouded turbines.

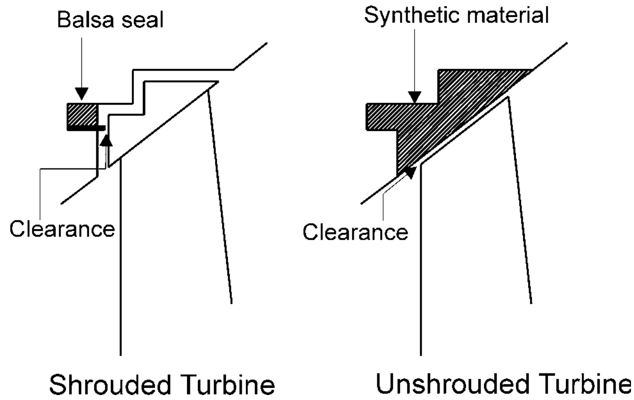

Fig. 4: Shrouded and unshrouded rotor configuration.

When the IP rig was in operation at the design condition, the running clearance was larger than the static clearance due to the movement of the rotor, Fig. 5. In the shrouded turbines, the rotor moved axially due to the pressure drop across the rotor and there was also a small radial movement due to the centrifugal stress. However, the balsa seal was against the front face of the shroud and so the small radial movement did not affect the clearance. In the unshrouded turbines the clearance was measured perpendicular to the casing (the rotor moved both axially and radially).

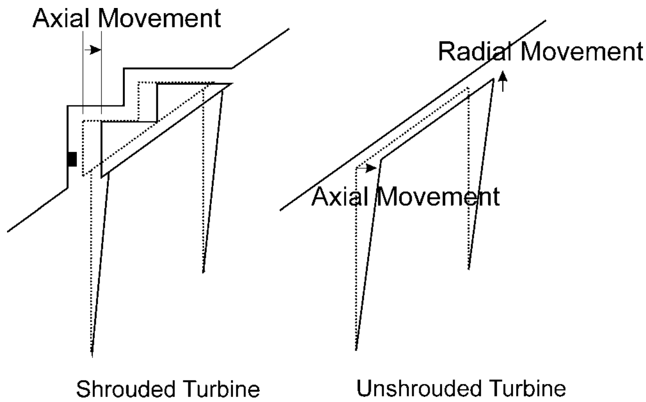

Fig. 5: Axial-radial movement of the rotor blade between static and running conditions.

In all four turbines the opening clearance was measured[5] whilst the turbine was running by inserting an electrical contact probe through the casing, orthogonal to the casing. The measured opening clearances are summarized in Table 2 for all four turbines. It can be seen that the opening clearance

[5] At the smallest clearance.

of the shrouded 50% reaction turbine was larger than that of shrouded 24% reaction turbine. This is due to the two factors. Firstly, the lower reaction reduced the pressure drop across the rotor and thus reduced the axial movement. Secondly, the 24% reaction rotor has a longer meridional chord (a lower aspect ratio, Table 1) and a higher rotor camber angle so it is stiffer to the mechanical bending force.

Reaction (Λ)		50%	24%
Opening clearance	Shrouded	0.41%	0.18%
	Unshrouded	0.21%	0.16%

Table 2: Measured opening clearance (difference between running and static clearance) for the four turbines tested (expressed as percentage of span).

RESULTS: EFFICIENCY VERSUS CLEARANCE

The efficiencies of the 50% and 24% reaction turbines were measured at the fixed design flow coefficient[6], over a range of clearances, in both the shrouded and the unshrouded configurations. The results are shown in Figs 6 and 7. The running clearance is expressed as a fraction of span and was determined by taking the static clearance (measured when the rotor was stationary) and adding the opening clearance.

For the 50% reaction turbines, the effect of clearance on the measured efficiency is shown in Fig. 6. As the clearance increased, the efficiency decreased in both the unshrouded and shrouded turbines. This is due to the increased leakage mass flow rate and, therefore, the increased tip leakage efficiency penalty. The shrouded turbine had a higher efficiency than the unshrouded turbine when the clearance was large. At a clearance of 3.5% of span, the efficiency of the shrouded turbine was greater than that of the unshrouded turbine by approximately 3.5%. However, as the clearance was reduced, the difference between the two efficiencies became smaller. At the smallest clearances measured (0.54% and 0.49% of span for the shrouded and the unshrouded turbines respectively), the unshrouded turbine had a higher efficiency than the shrouded turbine by 0.1%. Considering that the accuracy of the efficiency measurement is within 0.1%, this difference may not be significant, but the trend of the measured efficiencies suggests that at zero clearance the efficiency of the unshrouded turbine would be higher than the shrouded turbine.

Figure 7 shows the measured efficiencies of the 24% reaction turbines. These turbines show the same trend as the 50% reaction turbines. However, in the 24% reaction turbines, the measured efficiency of the unshrouded turbine was always less than the shrouded turbine. Even at the smallest clearances measured (0.37% and 0.44% of span for the shrouded and unshrouded turbines respectively), the unshrouded turbine was less efficient by approximately 0.1%. However, the trend in the measured data suggests that at zero clearance the efficiency of the unshrouded turbine would be higher than the shrouded turbine.

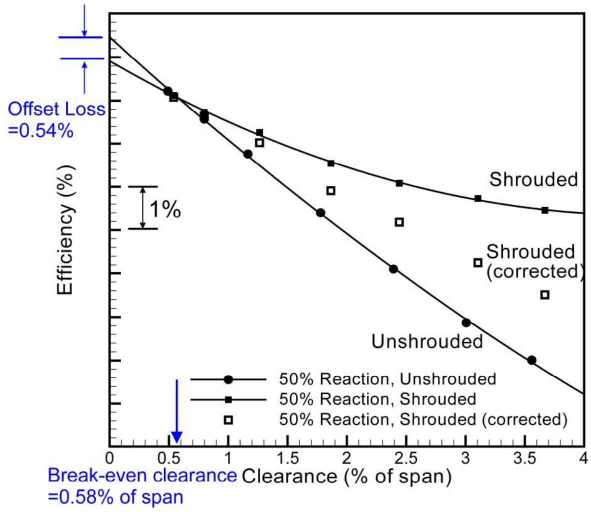

Fig. 6: Effect of clearance on the 50% reaction shrouded and unshrouded turbines.

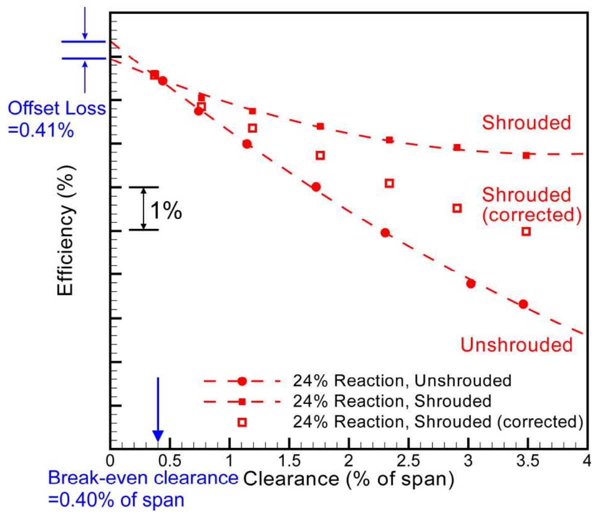

Fig. 7: Effect of clearance on the 24% reaction shrouded and unshrouded turbines.

Shrouded versus Unshrouded Turbine

It should be noted that the efficiency decreased almost linearly with clearance in the unshrouded turbines. However, the shrouded turbines were less sensitive to the effect of increasing the clearance when the clearance was large, resulting in the parabolic curves, Figs 6 and 7. This was

[6] The performance was also measured at the fixed design work coefficient. Since the results and the conclusions drawn from the fixed design work coefficient are very similar to those from the fixed flow coefficient, they are not included.

surprising because most published studies regarding the effect of clearance on turbine efficiency show a linear relationship.

The reason for the non-linear relationship between the clearance and the efficiency in the shrouded turbines can be understood by considering the geometry of the leakage path between the casing and the shroud. The casing-shroud gap was 3 mm except at the balsa seal. Therefore, when the balsa seal clearance was large, the pressure drop across the rotor shroud was determined not only by the balsa seal but also by the casing-shroud gap. In order to estimate the pressure drop across the balsa seal, the pressure was measured at three locations around the shroud, Fig. 8. At each location, four static tappings, distributed around the circumference, were used to measure the static pressure. The fraction of pressure drop across the balsa seal is defined by $(p_1-p_2)/(p_1-p_3)$.

Figure 8 shows the fraction of the shroud pressure drop that occurred across the balsa seal for all the clearances used in the shrouded turbines. When the clearance was small, most of the pressure drop occurred across the balsa seal. However, when the seal clearance approached the size of the casing-shroud gap less of the pressure drop occurred across the balsa seal. That is to say, the leakage mass flow rate was mainly determined by the balsa seal when the clearance was small. When the clearance approached the size of the casing-shroud gap the gap itself behaved like an extra seal.

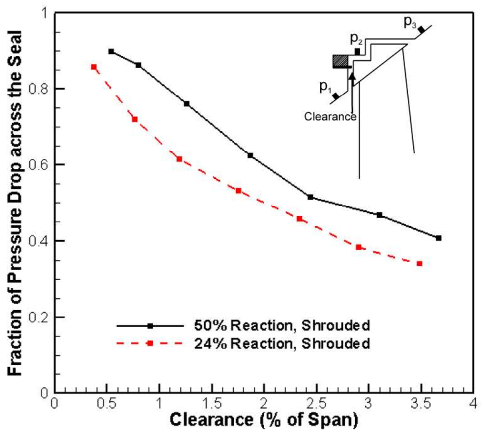

Fig. 8: Fraction of the measured shroud pressure drop that occurred across the balsa seal for the shrouded turbines.

The fraction of the shroud pressure drop across the balsa seal was measured at each seal clearance in the shrouded turbines. Therefore, it was possible to estimate the corrected efficiency if all of the shroud pressure drop occurred across the balsa seal. Firstly, the efficiency at zero clearance was estimated by extrapolating the measured data using a second order best-fit curve. Secondly, it was assumed that the leakage mass flow rate was proportional to the square root of the pressure drop across the balsa seal ($\dot{m}_L \propto \Delta p^{0.5}$). The overall pressure drop (p_1-p_3) was then used to estimate the leakage mass flow that would have occurred if the entire shroud pressure drop occurred across the balsa seal. Finally, the tip leakage efficiency penalty was assumed, for a fixed turbine stage, to vary linearly with the tip leakage mass flow rate ($\Delta\eta_{leak} \propto \dot{m}_L$). The corrected efficiencies for the shrouded turbines are indicated in Figs 6 and 7. They show a linear variation with seal clearance in agreement with the literature.

DISCUSSION:
BREAK-EVEN CLEARANCE AND OFFSET LOSS

Figures 6 and 7 show that there is a "break-even clearance" at which the efficiencies of both the shrouded and the unshrouded turbines are the same. If the clearance is less than the break-even clearance, the unshrouded turbine is expected to be better than the shrouded turbine. If there were no clearance, then the unshrouded turbine would be more efficient than the shrouded turbine by the "offset loss" (η_{offset}).

In order to estimate the offset loss and the break-even clearance, the experimental data were curve fitted using second order polynomials. The offset loss and break-even clearance for the 50% and 24% reaction turbines are listed in Table 3. The offset loss measured for the 24% reaction turbine was 0.13% smaller than that measured for the 50% reaction turbine. This difference is slightly larger than the efficiency uncertainty (0.1%). Both turbines have similar shroud geometries and cavity sizes and so would be expected to produce similar levels of entropy generation within them when operating with zero clearance[7]. However, the 24% reaction turbine has a 15% higher stage loading than the 50% reaction turbine and so, by Eqn. 6, the same level of entropy generation would be expected to produce an offset loss which was 0.07% smaller. This is consistent with the measured 0.13% reduction in the offset loss when the efficiency uncertainty is included.

Reaction (Λ)	50%	24%
Break-even clearance (% span)	0.58%	0.40%
Offset loss (η_{offset}/η)	0.54%	0.41%

Table 3: Measured break-even clearance and offset loss for the 50% and 24% reaction turbines.

From Fig. 1, the relationship between the offset loss and the break-even clearance is given by:

$$\frac{\eta_{offset}}{\eta} = \left(\left.\frac{\Delta\eta/\eta}{g/h}\right|_{unshrouded} - \left.\frac{\Delta\eta/\eta}{g/h}\right|_{shrouded} \right) \left.\frac{g}{h}\right|_{break-even} \qquad (3)$$

It will be shown later (Table 6) that adequate estimates of the efficiency exchange rate, $(\Delta\eta/\eta)/(g/h)$, can be obtained using Denton's analytic expressions for the tip leakage loss coefficient combined with velocity triangles. Therefore, Eqn. 3

[7] The 24% reaction turbine has a higher flow velocity across the upstream cavity than the 50% reaction turbine but it has a lower flow velocity across the downstream cavity.

can be used early in the design process to determine the break-even clearance if the offset loss can be estimated based on the sizes of the cavities and the geometry of the rotor shroud.

The design implication of the offset loss and the break-even clearance is that, if the turbine manufacturer can control the clearance to be smaller than the break-even clearance, the unshrouded turbine could be employed because this will enhance the efficiency, reduce the weight of the turbine and reduce the stress in the rotor blade and disc.

It should be noted that in the present investigation the cavities were quite small in the shrouded turbines and so the offset loss and the break-even clearance may be larger in practice. Further, it should be emphasized that the results presented above apply to turbines with a single shroud seal. For a shrouded turbine with more seals the efficiency penalty and the break-even clearance will both be smaller, roughly in inverse proportion to the square root of the number of seals.

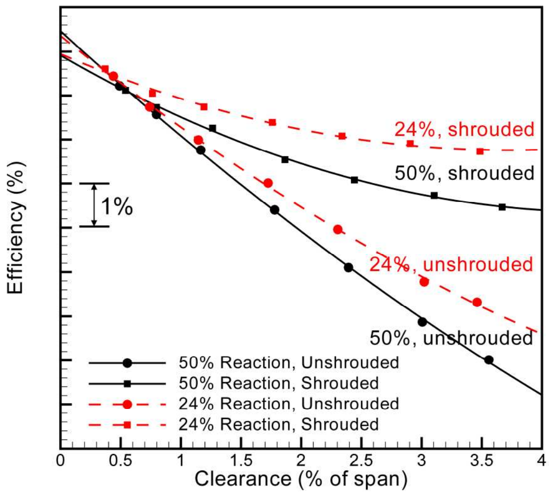

Fig. 9: Measured efficiency versus clearance for the four turbines tested (with best-fit curves).

DISCUSSION: EFFICIENCY VERSUS CLEARANCE

The measured variation of efficiency with clearance for all the four turbines tested is shown in Fig. 9. Based on the best-fit curves, both the 50% and 24% reaction shrouded turbines are expected to have the same efficiency at zero clearance. The unshrouded turbines are expected to have efficiencies within 0.2% at zero clearance. For each turbine the tip leakage efficiency penalty is calculated using:

$$\Delta\eta = \eta_{\text{zero clearance}} - \eta \qquad (4)$$

where $\eta_{\text{zero clearance}}$ is the expected efficiency (based on the curve-fit) for the turbine in question if it were operated with zero clearance. The efficiency penalty is the sensitivity of each turbine to the effects of clearance and is not compromised by the rotor blades in the 50% and 24% reaction turbines having different Reynolds numbers (Table 1).

In order to quantify the effect of clearance on the tip leakage efficiency penalty, the efficiency exchange rate is estimated. This is the ratio[8] of the fractional change of efficiency ($\Delta\eta/\eta$) to the clearance-to-span value (g/h), Table 4.

Reaction (Λ)		50%	24%
Efficiency exchange rate $(\Delta\eta/\eta)/(g/h)$	Shrouded	1.64	1.26
	Unshrouded	2.31	1.91
Work exchange rate $(\Delta\Psi/\Psi)/(g/h)$	Shrouded	-	-
	Unshrouded	4.50	3.57

Table 4: Measured efficiency and output power exchange rates. (The corrected efficiency data were used for the shrouded turbines.)

In both the shrouded and the unshrouded configurations, the 24% reaction turbine has a smaller efficiency exchange rate than the 50% reaction turbine, Table 4. This is also shown in Fig. 9. In the shrouded turbines, the smaller efficiency exchange rate of the 24% reaction turbine was expected because a reduced reaction would cause a lower pressure drop across the rotor shroud. In the unshrouded turbines, however, the slightly increased rotor Zweifel coefficient (Table 1) of the 24% reaction turbine would be expected to drive an increased tip leakage flow and cause a larger tip leakage loss coefficient than in the 50% reaction turbine. Therefore, the unshrouded 24% reaction turbine having a smaller efficiency exchange rate than the 50% reaction turbine requires explanation.

Analytic/empirical model

An analytical model was used to understand why the 24% reaction turbines reduced the tip leakage efficiency penalty in both the shrouded and unshrouded tip configurations compared to the 50% reaction turbines. Firstly, the Denton loss model [1] was applied to estimate the tip leakage loss coefficient in both the shrouded and unshrouded configurations. According to the Denton model, the loss coefficient (ζ) can be expressed as a function of the relative angles at the rotor inlet (β_2) and the rotor exit (β_3), the clearance-to-span ratio (g/h), the pitch-to-chord ratio (b/c) and the discharge coefficient (C_D):

$$\varsigma = \frac{T\Delta s}{\frac{1}{2}W_3^2} = \text{fnc}\left(\beta_2, \beta_3, \frac{g}{h}, \frac{b}{c}, C_D\right) \qquad (5)$$

Different analytical expressions for this relationship are given by Denton for shrouded and unshrouded turbines (see Appendix). The loss coefficient is based on the mixing loss which is related to the tip leakage mass flow rate. The above tip leakage loss coefficient can be converted to the efficiency penalty, in a turbine stage, by the following equation:

$$\Delta\eta = \frac{T\Delta s}{\Delta h_{0\,\text{stage}}} = \frac{1}{2}\frac{\varsigma}{\psi}\left(\frac{W_3}{U}\right)^2 \qquad (6)$$

[8] Estimated using the best-fit curves through the data.

The difference between the loss coefficient (ζ) and the tip leakage efficiency penalty (Δη) needs to be clarified. The loss coefficient is the amount of entropy generation (TΔs) normalized by the specific relative kinetic energy at the rotor exit ($\frac{1}{2}W_3^2$). The efficiency penalty is the amount of the entropy generation normalized by the stagnation enthalpy change (Δh_{0stage}) through the turbine stage. What this implies is that the efficiency penalty can be significantly different from the tip leakage loss coefficient depending on the relationship between the specific relative kinetic energy at the rotor exit ($\frac{1}{2}W_3^2$) and the stagnation enthalpy change (ψU^2).

Traditionally, the tip leakage efficiency penalty was understood to be primarily determined by the tip leakage mass flow rate. However, Eqn. 6 suggests that the tip leakage efficiency penalty should be thought of in terms of the dissipated kinetic energy which is determined not only by the leakage mass flow rate but also by the specific relative kinetic energy at the rotor exit and the stagnation enthalpy change.

The 24% reaction turbine was designed to reduce the stage absolute exit flow angle by 14º, Table 1. This in turn substantially decreases the specific relative kinetic energy at the stage exit. Moreover, the 24% reaction turbine increased the work coefficient by 15%. The reduction of the relative kinetic energy and the increase of the work coefficient have the effect of reducing the efficiency penalty even though the loss coefficient and the leakage mass flow rate may have increased.

To illustrate the importance of the above effect on the unshrouded turbines the values for the tip leakage fraction, loss coefficient and efficiency penalty were estimated using Denton's model [1]. These are shown in Table 5 for the case where the clearance-to-span is 1%. The 24% reaction turbine is estimated to have both an increased leakage flow and an increased loss coefficient because the rotor Zweifel coefficient and the leakage area are both larger than in the 50% reaction turbine. However, because of the different velocity triangles, the 24% reaction turbine has a lower efficiency penalty.

Reaction (Λ)	50%	24%
Leakage fraction ($\dot{m}_L / \dot{m}$)	0.024	0.026
Leakage loss coefficient (ζ)	0.040	0.056
Efficiency penalty (Δη)	2.08%	1.95%

Table 5: Estimated tip leakage parameters for the unshrouded turbines using Denton's model [1] (at clearance-to-span g/h=0.01).

If Denton's equations for the tip leakage loss coefficients (see Appendix) and Eqn. 6 are applied to the turbines examined, then the efficiency exchange rates can be estimated[9], Table 6. There is a fairly good agreement between the measured and the estimated efficiency exchange rates. In particular, the estimated values correctly show the smaller efficiency exchange rate for the 24% reaction turbine in both the shrouded and unshrouded configurations. This agreement emphasizes that the tip leakage efficiency penalty should be interpreted as the dissipation of (relative) kinetic energy rather than either the tip leakage loss coefficient or the tip leakage mass flow rate.

Reaction (Λ)		50%	24%
Measured $(\Delta\eta/\eta)/(g/h)$	Shrouded	1.64	1.26
	Unshrouded	2.31	1.91
Estimated (Denton) $(\Delta\eta/\eta)/(g/h)$	Shrouded	1.87	1.24
	Unshrouded	2.08	1.95

Table 6: Comparison of the measured efficiency exchange rate with that estimated using Denton's analytic model and velocity triangles.

DISCUSSION: OUTPUT POWER

Figure 10 shows the effect of clearance on the stage loading coefficient, at the fixed design flow coefficient, for the four turbines tested. It should be noted that, by design, the 24% reaction turbine has a higher stage loading coefficient than the 50% reaction turbine by approximately 15%. Similar to the efficiency measurements, the unshrouded turbines show a linear relationship between the stage loading coefficient and the clearance. However, the shrouded turbines show that the effect of increasing the clearance is reduced when the clearance is comparable to the size of the casing-shroud gap. This is again due to the fact that, at large clearances, the gap between the casing and the shroud behaved as an extra seal.

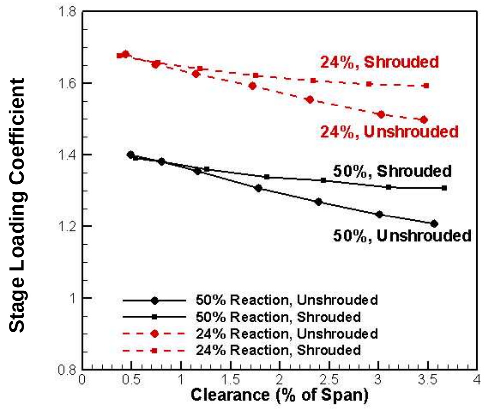

Fig. 10: Measured stage loading coefficient versus clearance (at the design flow coefficient).

To quantify the effect of clearance on the turbine output power the exchange rate was calculated. This is the ratio of

[9] The equations can be directly applied with the flow angles at the rotor inlet and the exit which can be obtained from traverse measurements. Alternatively, the flow angles can be expressed as a function of the flow coefficient, the work coefficient and the level of reaction as demonstrated in Yoon [14]. Then the data in Table 1 is sufficient to predict the efficiency penalty.

($\Delta\Psi_{leak} / \Psi_{no\ leak}$) to the clearance-to-span value (g/h). It was estimated for the unshrouded turbine using the best-fit curves through the measured data, Table 4. The output power exchange rate for the shrouded turbine could not be estimated because the measurement did not show a linear relation between the stage loading coefficient and the clearance. Contrary to the efficiency data, there was no straightforward way to correct the stage loading coefficient.

A striking feature from Table 4 is that the output power exchange rate is approximately twice the efficiency exchange rate. Traditionally, it has been assumed that the tip leakage efficiency penalty was simply due to the reduced output power (for example, Farokhi [15]). However, the experimental results suggest that this is not true when the turbine is operated at a fixed flow coefficient and an explanation is needed.

Analytic model

In order to examine the relationship between the efficiency exchange rate and the output power exchange rate, an analytic model is developed. If the flow coefficient (ϕ) is held constant and all the mass flow passes through the NGV then the tangential velocity at the NGV exit ($V_{\theta 2}$) does not change. Between the NGV exit and the rotor inlet the leakage flow ($\dot{m}_L$) leaves the mainstream. However, the angular momentum ($rV_{\theta 2}$) is conserved and so the tangential velocity entering the rotor is independent of the amount of leakage flow. The relative flow angle at the rotor exit (β_3) is not affected by the tip leakage flow because β_3 is primarily determined by the rotor exit metal angle. By applying Euler's equation to just the flow that passes through the rotor blade the change in the stagnation enthalpy is:

$$\Delta h_{0\,rotor} = U(V_{\theta 2} - V_{\theta 3}) \tag{7a}$$

$$\Delta h_{0\,rotor} = U\left[V_{m2} \tan\alpha_2 - (U + V_{m3} \tan\beta_3)\right] \tag{7b}$$

Because the tip leakage reduces the mass flow ($\dot{m} - \dot{m}_L$) passing through the rotor blade, the meridional velocity at rotor exit is related to the meridional velocity at the NGV exit (prior to any leakage) by:

$$V_{m3} = V_{m2}\left(1 - \frac{\dot{m}_L}{\dot{m}}\right) \tag{8}$$

By combining these two expressions, the stagnation enthalpy change of the flow that passes through the rotor is given by:

$$\Delta h_{0\,rotor} = U^2\left[\phi(\tan\alpha_2 - \tan\beta_3) - 1 + \phi\frac{\dot{m}_L}{\dot{m}}\tan\beta_3\right] \tag{9}$$

where the flow coefficient is defined by:

$$\phi = V_{m2} / U \tag{10}$$

The turbine output power is given by:

$$\Psi = (\dot{m} - \dot{m}_L)\Delta h_{0\,rotor} \tag{11}$$

Combining Eqns 9 and 11 yields:

$$\Psi = \dot{m}\, U^2\left[\phi\,(\tan\alpha_2 - \tan\beta_3) - 1\right] + \dot{m}_L U^2 \phi \tan\beta_3 - \dot{m}_L\, U^2\left[\phi\,(\tan\alpha_2 - \tan\beta_3) - 1\right] \tag{12}$$

In the above, terms of order $\dot{m}_L^2$ have been ignored.

It is convenient to rewrite Eqn. 12 in the form:

$$\Psi = \Psi_{no\ leak} - \Delta\Psi_{leak} \tag{13}$$

where:

$$\Psi_{no\ leak} = \dot{m}\, U^2\left[\phi\,(\tan\alpha_2 - \tan\beta_3) - 1\right] \tag{14}$$

$$\Delta\Psi_{leak} = \dot{m}_L\, U^2\left[\phi\,(\tan\alpha_2 - 2\tan\beta_3) - 1\right] \tag{15}$$

The two factor in front of the $\tan\beta_3$ in the expression for $\Delta\Psi_{leak}$ is due to leakage flow both reducing the mass flow through the rotor and also reducing the meridional velocity at the rotor exit which in turn affects the tangential velocity.

If the efficiency penalty is approximated to vary with the tip leakage mass flow rate for a fixed design point then:

$$\frac{\Delta\eta_{leak}}{\eta_{no\ leak}} \approx \frac{\dot{m}_L}{\dot{m}} \tag{16}$$

Combining Eqns 14, 15 and 16 yields:

$$\frac{\Delta\Psi_{leak}}{\Psi_{no\ leak}} \bigg/ \frac{\Delta\eta_{leak}}{\eta_{no\ leak}} \approx \frac{\phi\,(\tan\alpha_2 - 2\tan\beta_3) - 1}{\phi\,(\tan\alpha_2 - \tan\beta_3) - 1} \tag{17}$$

The ratio of the fractional change in the output power ($\Delta\Psi_{leak} / \Psi_{no\ leak}$) to the fractional change in efficiency ($\Delta\eta_{leak} / \eta_{no\ leak}$) of the 50% and 24% reaction turbines can be estimated using Eqn. 17. These are compared with the measured values in Table 7. There is good agreement between the measured and estimated values for the ratio of the fractional change in the output power to the fractional change in efficiency for the unshrouded turbines. Thus Eqn. 17 explains why the work exchange rates shown in Table 4 are approximately twice the efficiency exchange rates.

Reaction (Λ)	50%	24%
Measured $(\Delta\Psi/\Psi)/(\Delta\eta/\eta)$	1.95	1.87
Estimated $(\Delta\Psi/\Psi)/(\Delta\eta/\eta)$	1.84	1.67

Table 7: Comparison of measured and estimated ratio of the fractional change in the output power to the fractional change in efficiency for both 50% and 24% reaction unshrouded turbines.

For the turbine designs investigated here $\phi\tan\alpha_2 \approx 1$ and so the fractional change in output power is approximately twice the fractional change in efficiency (Eqn. 17). This ratio could be different in other turbine designs (e.g. when β_3 is small this ratio could be nearer to unity).

It is noteworthy that little attention has been paid to the change in the output power due to the tip leakage although many studies in the past examined the tip leakage efficiency penalty. It appears that most researchers assumed that the change in the output power would be the same as the efficiency penalty. However, both the experimental and analytical studies show that, at fixed flow coefficient, the fractional change of the output power is approximately twice the fractional change in efficiency for the turbines examined.

This result is specific to a turbine that is operated at a fixed flow coefficient[10]. If, however, the turbine is operated at a fixed pressure drop, the mass flow rate will change and the turbine will operate at a slightly off-design condition. This will give a smaller reduction in power but possibly a greater reduction in the tip leakage efficiency penalty.

CONCLUSIONS

In this paper, the effect of tip leakage in shrouded and unshrouded turbines was examined at two levels of reaction by systematically varying the clearances in four turbines. The experimental results were complemented and generalized by analytic studies and the following conclusions can be drawn.

Firstly, the experimental data shows clearly that a 'break-even clearance' exists. This is because the presence of the shroud and associated cavities increases the loss at very small clearances. The significance of this finding is that, if the clearance can be controlled to be less than the break-even clearance, an unshrouded turbine could be used instead of a shrouded turbine. This will not only increase the aerodynamic efficiency but also reduce the weight and stress of the turbine blade and disc. In the current investigation the cavities were quite small so the break-even clearance and the offset loss may be larger in typical applications.

Secondly, measurements show that the 24% reaction turbines have a smaller tip leakage efficiency penalty than the 50% reaction turbines in both the shrouded and unshrouded configurations. From analytic studies, it was found that the tip leakage efficiency penalty should be understood in terms of the dissipated relative kinetic energy. A reduction of reaction can decrease the relative kinetic energy at the stage exit although it may increase the tip leakage mass flow rate and loss coefficient in unshrouded turbines.

Finally, the fractional change in the output power at a fixed flow coefficient was measured to be approximately twice the fractional change in efficiency. The relationship between the efficiency exchange rate and the output power exchange rate was derived analytically and agrees well with the measurements.

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[10] The reduced mass flow through the rotor will reduce the rotor pressure drop and hence the turbine exit pressure will increase.

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APPENDIX

Denton [1] presented analytic models for tip leakage flow in turbines. For shrouded turbines the shroud leakage mass flow fraction can be approximated by:

$$\frac{\dot{m}_L}{\dot{m}} = C_D \frac{g}{h} \frac{1}{\cos\beta_3} \sqrt{\sec^2\beta_3 - \tan^2\beta_2} \qquad \text{(A1)}$$

and the associated shroud leakage loss coefficient by:

$$\varsigma = \frac{T\Delta s}{\frac{1}{2}W_3^2} = 2\frac{\dot{m}_L}{\dot{m}}\left(1 - \frac{\tan\beta_2}{\tan\beta_3}\sin^2\beta_3\right) \qquad \text{(A2)}$$

For unshrouded turbines the tip leakage mass flow fraction can be approximated by:

$$\frac{\dot{m}_L}{\dot{m}} = C_D \frac{g}{h}\frac{c}{p}\frac{1}{\cos\beta_3}\int_0^c \left(\frac{W_s}{W_2}\right)\sqrt{1-\left(\frac{W_p}{W_s}\right)^2}\,\frac{dc}{c} \qquad (A3)$$

and the associated tip leakage loss coefficient (ζ) by:

$$\varsigma = 2\,C_D \frac{g}{h}\frac{c}{b}\frac{1}{\cos\beta_3}\int_0^c \left(\frac{W_s}{W_2}\right)^3\left(1-\frac{W_p}{W_s}\right)\sqrt{1-\left(\frac{W_p}{W_s}\right)^2}\,\frac{dc}{c} \quad (A4)$$

For both shrouded and unshrouded turbines the loss coefficient depends on the leakage fraction multiplied by a term that represents the fraction of relative kinetic energy that is dissipated.

It is worth noting that for both shrouded and unshrouded turbines the leakage mass flow fraction depends on the clearance-to-span (g/h) ratio but in unshrouded turbines there is also a chord-to-pitch (c/b) factor.