Numerical Examples of the Surrogate Model for the Monodomain Equations with the Minimal Reaction Model

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1 Mesh for the Spatial Discretization

The mesh for the exact problem is composed by triangles of characteristic size 0.1 [mm], and is refined nearby the collagen inclusions, in order to correctly consider the mesoscale into the model. The refined elements have a characteristic size of 0.02 [mm]. On the other hand, the mesh used to solve the homogenized problem have a characteristic length of 0.5 [mm]. This difference is a consequence of the fact that the homogenized model does not need explicitily the mircoscale in the model, because the microscopic propiertes are considered in the effective macroscopic tensor.
Note that the mesh size for the homogenized problem is selected in order to have at least (ussualy more) one node per each inclusion, which is useful for randomly generated fibrotic mesh, that will be used in the numerical experiment # 2. For this case, a even more coarse mesh can be used, given the constant value of θ_c and θ_f.

2 Common parameters for the Experiments

Table 1: common parameters for all the experiments.

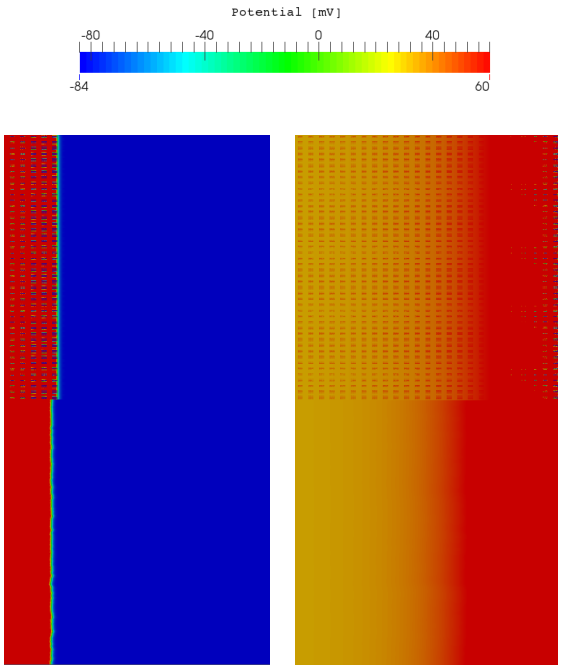
Paramter	Meaning	Value
d_1	main fiber diffusivity	0.1 $[mm/s]$
γ	cross fiber difusivity	4 $[mm/s]$
$\hat{\beta}$	fraction of d_1 taken as the collagen diffusivity	10^{-5}
$\hat{f}$	main fiber direction	$(1,\ 0)$
$\hat{c}$	cross fiber direction	$(0,\ 1)$
I_{est}	estimulus intensity	1.587 $[mV/mm]$
I_{dur}	estimulus duration	3 $[ms]$
a	radius of the myocites array with collagen	0.1 [mm]
b	distance between the start of two contiguous collagen inclusions (in direction e_2)	1 [mm]
$\Delta\ t$	time-step	0.1 $[ms]$

Boundary Conditions $\partial_x \phi = 1.587\delta, \forall \vec{x} \in \partial\Omega$
Initial Conditions $\phi = 0$, $r = 1$, $w = 1$ and $s = 0$, $\forall \vec{x} \in \Omega$

bc afuera de la tabla sigma ettc

3 Experiment # 1: Tissue with High and Constant Fibrosis

The goal is poner cuuanto vale theta c y teta f con mono



(b) 3 ms (c) 17 ms

Figure 1: Exact (up) and Homogenized (down) solutions.

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Experiment # 2: Tissue with Randomly Generated Fibrosis

The collagen laminations are usually randomly distributed. In this example we try to emulate that by setting $\theta_c \sim \mathcal{N}(\mu_c, \sigma_c)$ and $\theta_f \sim \mathcal{N}(\mu_f, \sigma_f)$, where σ_c and μ_c are the mean and the standard deviation of a normal distribution. In particular, we will use the paramters of the table 2:

Table 2: parameters use to generate the random fibrotic tissue mesh.

Parameter	Value
μ_c	0.3
μ_f	0.5
σ_c	0.2
σ_f	0.3

Note that the $\mathbb{E}(\theta_c)$ and $\mathbb{E}(\theta_f)$ represent a tissue with a moderated level of fibrosis. In the figure 2 a portion of a domain generated with the parameters of the table 2 parameters can be observed.

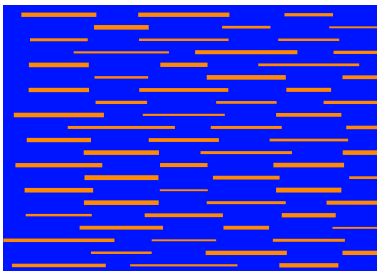

Figure 2: a zoom of a randomly generated mesh.

If the random values are not within a physiolocal range are recalculated. For θ_c we admit values from 0.15 (healthy tissue) to 0.45 (highly fibrotic tissue), and for $theta_f$ we admit values from 0.35 to 0.9. The choise of θ_f is done in order to get a tissue that emulates diffuse fibrosis, because values over 0.9 tends to generate collagen block-walls, i.e., stringy fibrosis.

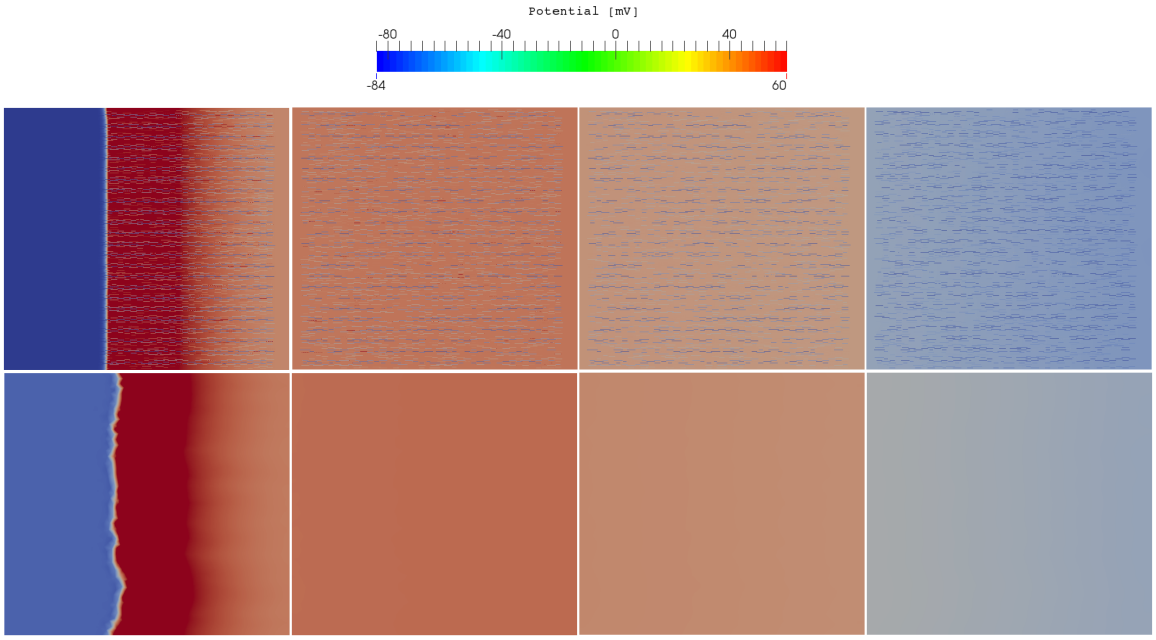

Figure 3: results of the experiment for $t = 30ms$, $t = 50ms$, $t = 150ms$ and $t = 250ms$ from left to right. The exact solution is shown at top, and the homogenized at the bottom.

The evolution of the error with time can be appreciated in table 4.

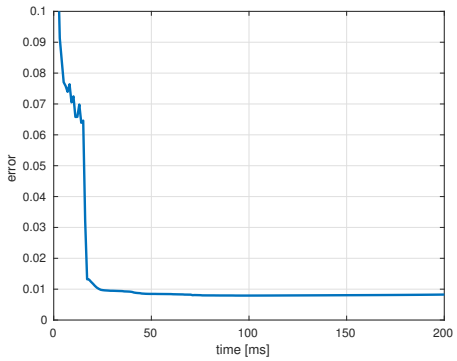

Figure 4: temporal evolution of the error.

The results are pretty similar to the non-random case.