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# Temperature factor for magnetic instability conditions of type – II superconductors

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ABSTRACT

The macroscopic development of interrelated electrodynamics and thermal states taking place both before and after instability onset in type-II superconductors are studied using the critical state and the flux creep concepts. The physical mechanisms of the non-isothermal formation of the critical state are discussed solving the set of unsteady thermo-electrodynamics equations taking into consideration the unknown moving penetration boundary of the magnetic flux. To make it, the numerical method, which allows to study diffusion phenomena with unknown moving phase-two boundary, is developed. The corresponding non-isothermal flux jump criteria are written. It is proved for the first time that, first, the diffusion phenomena in superconductors have the fission-chain-reaction nature, second, the stability conditions, losses in superconductor and its stable overheating before instability onset are mutually dependent. The results are compared with those following from the existing magnetic instability theory, which does not take into consideration the stable temperature increase of superconductor before the instability onset. It is shown that errors of isothermal approximation are significant for modes closed to adiabatic ones. Therefore, the well-known adiabatic flux jump criterion limits the range of possible stable superconducting states since a correct determination of their stability states must take into account the thermal prehistory of the stable magnetic flux penetration. As a result, the calculation errors in the isothermal approximation will rise when the sweep rate of an external magnetic field or the size of the superconductor's cross-sectional area increase. The basic conclusions formulated in the framework of the critical state model are verified comparing the experimental results and the numerical analysis of the stability conditions and the temperature dynamics of the helicoid-type superconducting current-carrying element having real voltage–current characteristic.

On the whole, the non-isothermal stability conditions expand the existence of allowable stable superconducting states. The non-isothermal approximation permits also to link the theories of the losses, the magnetic instability and the thermal stabilization of superconductors, which are independently developed.



## 1. Introduction

The macroscopic electrodynamics of superconductors is one of the most important issues in their investigations. The results obtained are not only valuable to understanding the fundamental properties of superconducting materials but also permit the limits of their practical applications to be determined [1–4]. Therefore, the investigations of macroscopic formation mechanisms of superconducting states are important in basic studies of superconductors. They allow one to determine the margin of stable operating regimes of superconducting devices subjected to many external perturbations of different nature. As a result, they have made it possible to formulate the main principles lying on the basis of the theories of magnetic, current and thermal instabilities, which, in turn, lead to the conditions of retaining the superconductivity against electrodynamics and thermal perturbations.

In the past years many investigations have focused on experimental and theoretical studies of the magnetic instability problem [5–18]. Recently, the interest connected with this phenomenon increases again [19–26] since high-temperature superconductors have been widely used. However, the main conclusions of the existing magnetic instability theory were, as a rule, formulated basing on studies of the initial stage of thermal and electrodynamics states occurring inside the superconductor and resulting from the effect of infinitely small perturbation. Moreover, the finite

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temperature change in the superconductor before onset of instability depending on its operating mode was not taken into account in the framework of this approximation. At the same time, one of the main features of type-II superconductors is the dissipative processes occurring in them, which are due to thermal activated motion of vortices. Therefore, the thermal state of superconductor before instability may influence on its stability conditions. However, the existing investigations of the forming regimes of the superconducting states in the non-isothermal approximation [13–18,26] do not permit the general regularities determining the effect of stable variations in their temperature on the instability conditions to be formulated. To recognize this peculiarity, let us discuss early formulated conclusions, which deal with the magnetic instability conditions of type-II superconductors.

## 2. Existing flux jump conditions of type-II superconductors

As it follows from the critical state model, the magnetic instability does not exist [1–4], if the superconductor satisfies the so-called adiabatic stability condition

$$\beta = \mu_0 a^2 J_c^2(T_0, B_a)/[C(T_0)(T_{cB} - T_0)] < 3 \tag{1}$$

which has been obtained for a thermally isolated superconducting slab in the approximation $\Lambda = \mu_0 \lambda(T_0)/C(T_0)\rho_f \to 0$. Here, $C$ is the specific heat capacity of the superconductor, $\lambda$ is the coefficient of its heat conductivity, $a$ is the half-thickness of a slab, $T_0$ is the coolant temperature, $B_a$ is the external magnetic field, $J_c(T,B)$ is the critical current density of superconductor, $T_{cB}$ is the critical temperature of the superconductor at $B_a$, $\rho_f$ is the superconductor resistance in the resistive regime, $\mu_0$ is the magnetic permeability of vacuum.

According to [1,3], the allowance for the final value of $\Lambda$ leads to the corresponding correction in the right side of the criterion (1). In this case, it can be written as

$$\beta < \pi^2(1 + 2\sqrt{\Lambda})/4 \tag{2}$$

Criteria (1) and (2) obviously show that isothermal approximation was used to formulate the magnetic instability conditions. Indeed, the critical current density, specific heat capacity and heat conductivity coefficient are determined at coolant temperature $T_0$. In other words, these criteria describe the stability conditions of thermally isolated superconductor whose temperature is equal to the coolant temperature upon adiabatic penetration of magnetic flux in it.

1D-analysis of the magnetic instability conditions in the type-II superconductors taking into account the flux creep states described by the exponential equation of the voltage–current characteristic has been performed in [14–17,26] both for the low- and high-temperature superconductors. In particular, it was proposed a model in whose framework the instability criterion is written as follows [14]

$$\frac{1}{S}\int_S E ds > E_m = \frac{hp}{S}\frac{J_\delta}{J_c|\partial J_c/\partial T|} \tag{3}$$

using the $E$–$J$ relation, which may generally written in the form $E = E_c \exp[J/J_\delta + (T_0 - T_{cB})/T_\delta]$. Here, $h$ is the heat transfer coefficient, $p$ is the cooled perimeter of the conductor, $S$ is its cross-sectional area, $T_\delta$ and $J_\delta$ are the temperature and current creep parameters, $E_c$ is the electric field criterion in $J_c$ definition. Accordingly, the temperature of superconductor $T_i$ before instability is small and equals

$$T_i = T_0 + J_\delta/|\partial J_c/\partial T| \tag{4}$$

when the current density $J$ over the cross-sectional area of superconductor is constant.

The physical sense of condition (3) is obvious: the superconducting state is stable when the overage value of the electric field over its cross-sectional area is lower than the characteristic value $E_m$.

In [15] the low perturbation method was used for 1D-investigation of the superconducting state stability with allowance for variation in the back ground temperature of the superconductor prior to the magnetic instability onset. The corresponding flux jump field $B_m$ was obtained. It satisfies the solution of equation

$$B_m^2 = \left(2\frac{h\mu_0^2 J_\delta}{\dot{B}} + 6\frac{h\mu_0^2 J_\delta}{\dot{B}}\sqrt{\frac{C\dot{B}B_p}{h\mu_0 J_\delta B_m}}\right)(T_{cB} - T_0) \tag{5}$$

Here, $\dot{B}$ is sweep rate of an external magnetic field and $B_p = \mu_0 a J_c|_{T=T_0}$ is the fully penetrated field.

According to [16], an allowable overheating of superconductor before the instability onset is low and equal to the temperature creep parameter $T_\delta$, which satisfies the condition $T_\delta/(T_{cB} - T_0) \ll 1$. As a result of this feature, the following stability criterion

$$\int_S EJds \leqslant hpT_\delta \tag{6}$$

was formulated in [16] for a superconducting composite. Condition (6) has the following physical meaning: superconductivity is retained if the heat release in composite does not exceed the heat flux to the coolant at a constant permissible overheating, which is equal to $T_\delta$ regardless of the sweep rate, cooling conditions and the transverse dimension of a composite.

It is easy to find that criteria (3) and (5), which determine the first flux jump field in superconductors during flux creep, do not satisfy the limiting transition to criterion (1) at $h \to 0$ or $J_\delta \to 0$ because the flux jump field will be equal to zero in these transitions. As it follows from (6), superconductivity of composite conductor will be also inevitably destroyed by any infinitesimal perturbation at $h \to 0$ or $T_\delta \to 0$, i.e., the superconducting state of an adiabatically cooled superconductor with very steep transition $E$–$J$ characteristic must be in principle unstable. In other words, the limiting transition to the critical state model ($J_\delta \to 0$ or $T_\delta \to 0$) is not possible in the framework of such approximations. As a result, the existing theory does not interconnect magnetic instability conditions obtained in the framework of the critical state model with ones taken place for superconductors with real $E$–$J$ characteristics. Besides, theoretical approximations used do not give exact answer to the question on the dependence of an allowable overheating of superconductor before the onset of instability on the operating conditions and its influences on the stability conditions. For example, the conclusions made in [26] for Bi-based superconducting slab with real $E$–$J$ characteristic contend that the temperature of slab jumps from the coolant one to a peak one during flux jump and then back down the coolant one. This statement is not consistent with criteria (4) and (6).

Note that the stability analysis is the most favorable, if analytical expressions may be formulated. However, in practical cases, numerical methods are needed to find magnetic instability boundary since low- and high-temperature superconductors have essentially non-linear shape of $E$–$J$ characteristics. Therefore, the numerical investigations of the macroscopic thermo-electrodynamics phenomena taking place in technical superconductors are used [24–26]. Nevertheless, such investigations are not numerous because they not only have very large computation time of the corresponding system of unsteady thermo-electrodynamics equations but require the development of appropriate methods of numerical calculations of such system of differential equations. In particular, it should be stressed that there is still no the modeling method, which allows one to make simulation of the unsteady electrodynamics states of type-II superconductor in the framework of the non-isothermal critical state model.

Thus, the main results of the existing magnetic instability theory are as follow:

– The magnetic instability theories based on the critical state model and the real $E$–$J$ characteristics of superconductors are developed independently of each other.
– The change in the thermal state of superconductor occurring before the instability onset are not taken into account in terms of the critical state model.
– The analysis of the magnetic stability boundary of the superconductors with the real $E$–$J$ characteristics is based on fact that the instability occurs under the conditions of small overheating of superconductor independently of the conditions of its cooling, cross-sectional area and sweep rate.
– Some stability criteria proposed for the superconductors with the real $E$–$J$ characteristics do not lead to the limiting transition to ones described by the critical state approximation.

In the present investigation, the results of the numerical simulations and analytical analysis of the thermo-electrodynamics instability mechanisms of flux penetration in type-II superconductors with both extremely steep and real $E$–$J$ characteristics are discussed in detailed. They were investigated taking into account the thermal prehistory of superconductor before instability onset. To do this study, the numerical method for simulation of the thermo-electrodynamics phenomena with an unknown moving phase boundary was developed. A method, which allows one to determine the stability boundary of superconductors with different types of non-linearity of its $E$–$J$ characteristics, is used. It permitted to formulate specific features of the formation of mutually connected thermal and electrodynamics states of superconductors, which are observed both prior to the onset of instabilities and after their development. Besides, the performed analysis enabled the non-isothermal magnetic instability criteria to be formulated. It has been shown that when the superconductor's temperature, whose change can occur even in the stable state formation, is set a priory, the existing stability criteria may lead to uncorrected flux jump fields. It is proved for the first time that the stability conditions may be formulated on the basis of general theoretical concept which is independent of the perturbation nature. In this case, the stability conditions will satisfy to the limiting transition to known stability criteria.

## 3. Non-isothermal critical state model

Let us consider a cooled superconducting slab ($-a < x < a$) placed in an external magnetic field $B_a(t)$, which is parallel to its surface and rises with the constant sweep rate $\dot{B} = dB/dt$. To understand the boundary of the existing magnetic instability theory ignoring an acceptable overheating of the superconductor before the instability onset, let us consider interrelated variations of the temperature $T(x,t)$, the electric field $E(x,t)$ and the current density $J(x,t)$ inside a type-II superconductor in terms of the extended critical state model (so-called viscous flow model, which is based on the finite value of differential resistance of superconductor $\rho_f$ in the resistivity state [3]). This approximation may be described by the system of equations

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + \begin{cases} 0, & 0 < x < x_p \\ EJ, & x_p \leqslant x \leqslant a \end{cases}, \tag{7}$$

$$\mu_0 \frac{\partial J}{\partial t} = \frac{\partial^2 E}{\partial x^2}, \tag{8}$$

$$J = J_c(T, B_a) + E/\rho_f \tag{9}$$

with the following initial and boundary conditions

$$T(x,0) = T_0, \quad E(x,0) = 0, \tag{10}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad \lambda\frac{\partial T}{\partial x} + h(T - T_0)|_{x=a} = 0, \tag{11}$$

$$E(x_p, t) = 0, \quad \left.\frac{\partial E}{\partial x}\right|_{x=a} = \begin{cases} \frac{dB}{dt}, & 0 < t < t_s \\ 0, & t \geq t_s \end{cases} \tag{12}$$

Here, $t_s$ is the time of an external magnetic field rise after which $dB/dt = 0$, $J_c(T, B)$ is the critical current density and $x_p$ is the time-varying depth of magnetic flux penetration determined by equation

$$\mu_0 \int_{x_p}^{a} J(x,t)dx = B_a(t) = \frac{dB}{dt}t, \tag{13}$$

The problem defined by Eqs. (7)–(13) describes the dissipative process of magnetic field diffusion inside the cooled type-II superconductor whose dynamics depends not only on the variations of the external magnetic field but also on the corresponding temperature change. The implicit form of the law (13) defining the unknown moving flux boundary $x_p(t)$ complicates significantly the use of the known methods for solving the sets of equations of parabolic type describing the diffusion phenomena in the conductors with moving phase boundary. Therefore, the numerical method, which enables to study diffusion phenomena with a second-type moving phase boundary, was developed. Accordingly, to determine $x_p(t)$, the iteration procedure is based on the determination of the root of Eq. (13), in which the value $x_p(t)$ is the desired quantity. In the framework of this method, the calculations to be carried out involve the separation of the root of Eq. (13), which then is defined more exactly. Consequently, the sign of the expression $r^{(s)} = \mu_0 \int_{x_p^{(s)}}^{a} J(x,t)dx - \frac{dB}{dt}t$ is found at each successive iteration step $s = 1, 2\ldots$ during the separating of the root. Based on the simple physical sense of $r^{(s)}$, it is easy to see that the sign of $r^{(s)}$ will be negative when ${x_p}^{(s)}$ is larger than the real value $x_p(t)$. On the contrary, if ${x_p}^{(s)} < x_p$, then $r^{(s)} > 0$. That is why the stage of root separation breaks as soon as the sign of $r^{(s)}$ for two successive iterations changes for the opposite sign. After this it is easy to perform the root refinement with a given accuracy. Without any serious difficulties, this algorithm is reproduced in the analysis of non-isothermal dynamics of the critical state in the cylindrical superconductors as well as in the superconducting composites.

To verify the proposed method, the results of the numerical analysis in the non-isothermal approximation were compared with the analytical formulas obtained in terms of the well-known critical state model [1,3]. In these calculations, the linear temperature dependence of the critical current density is assumed

$$J_c(T, B) = J_{c0}(T_{cB} - T)/(T_{cB} - T_0) \tag{14}$$

Here, $J_{c0}$ and $T_{cB}$ are the known critical current density and critical temperature of a superconductor, respectively.

Then the isothermal time variations of the magnetic flux penetration depth $x_p$ and volume density of the losses $G = \frac{1}{a}\int_0^t \int_0^a EJdxdt$ (the heat being released during magnetic flux diffusion in a slab) can be described in the form [1,3] $x_p(t) = a - \dot{B}t/(\mu_0 J_{c0})$, $G = \left(\dot{B}t\right)^3/(6\mu_0^2 a J_{c0})$. Calculation results of the penetration depth and the volume density of losses obtained both in the non-isothermal (dashed lines) and in the isothermal (solid lines) approximations are shown in Fig. 1. They were made under the adiabatic conditions ($h = 0$). The parameters of the calculations were set as follows $T_0 = 4.2$ K, $T_{cB} = 8.5$ K, $C = 30T^3$ J/(m$^3$ × K), $\lambda = 0.0075T^{1.8}$ W/(m × K), $\rho_f = 5\times10^{-7}$ Ω m and $J_{c0} = 4\times10^9$ A/m$^2$.

It is easy to see that isothermal and non-isothermal approximations lead to the similar results in the initial formation stage. At the same time, the non-isothermal dynamics of the critical state is

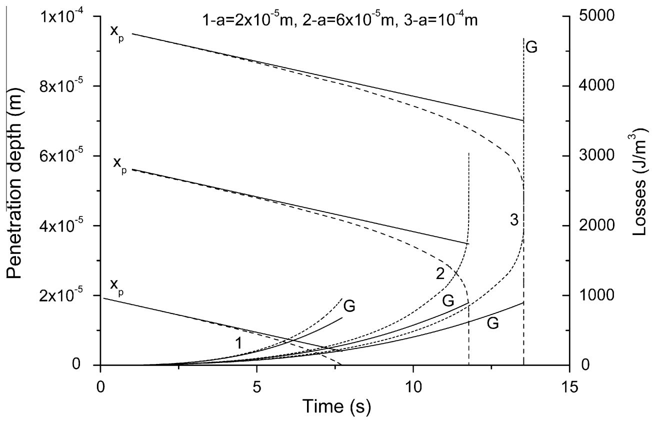


**Fig. 1.** Time dependence of the flux penetration depth and hysteresis losses during the magnetic flux diffusion in an uncooled superconductor at $dB/dt = 0.01$ $T/s$: $1 - a = 2 \times 10^{-5}$ m, $2 - a = 6 \times 10^{-5}$ m, $3 - a = 10^{-4}$ m.

characterized by a faster penetration of the magnetic flux into the type-II superconductor compared with the isothermal approximation. Therefore, the losses calculated in the non-isothermal approximation may differ from those calculated in the isothermal one before the instability onset. Consequently, the thermal prehistory of a superconductor may influence on the conditions of the magnetic instability onset and losses. This feature is discussed below in detail.

## 4. Basic mechanisms underlying the non-isothermal formation of the critical state

Let us discuss the non-isothermal formation mechanisms of the critical state making the numerical analysis of the magnetic flux penetration into a niobium–titanium superconductor ($a = 10^{-4}$ m) under different heat transfer conditions. The results discussed below were obtained by solving the system of Eqs. (7)–(13). The thermal and electrical properties of the superconductor were set as follows

$$C = 0.812 \cdot 10^3 T \frac{B}{B_{c0}} + 42.73T^3 \quad \mathrm{J/(m^3 \cdot K)} - \textit{according to } [27],$$

$$\lambda = 0.0075T^{1.8} \ \mathrm{W/(m \cdot K)} - \textit{according to } [28],$$

$$\rho_f = \rho_n \frac{B}{B_{c2}(T)}, \quad \rho_n = 10^{-6} \ \Omega \cdot \mathrm{m} - \textit{according to } [3]$$

The critical current density $J_c(T, B)$ was defined according to the Kim–Anderson model [3]

$$J_c(T,B) = \frac{\alpha_0}{B+B_0}\left(1 - \frac{T}{T_{cB}(B)}\right), \quad T_{cB}(B) = T_{c0}\sqrt{1 - B/B_{c2}(T)},$$
$$B_{c2}(T) = B_{c0}\left[1 - (T/T_{c0})^2\right] \tag{15}$$

where $\alpha_0 = 1.5 \cdot 10^{10} A \cdot T/\mathrm{m}^2$, $T_{c0} = 9K$, $B_{c0} = 14T$, $B_0 = 1.5T$.

Field of the flux jump $B_m$ that defines the magnetic instability boundary was found by the calculations of a series of the magnetic fields $B_1 \to B_2 \to B_3 \to \ldots B_s \to B_{s+1}$. Accordingly, the boundary values $B_s$ and $B_{s+1}$ are determined under the condition that the following inequality $|B_{s+1}–B_s| < \varepsilon$ takes place for a given accuracy of the calculations $\varepsilon$. Hence, the temperature of the superconductor and induced electric field are stabilized at $B = B_s$ (magnetic instability is absent) or increase spontaneously at $B = B_{s+1}$ (the penetrated magnetic flux is unstable). It should be noted that this method, which can be called as the method of final perturbation of the inner equilibrium state, has been established for the first time in [29] to solve the ramp-rate limitation problem for low-$T_c$ superconducting composites.

Curves demonstrating the dynamics of the stable and unstable thermo-electrodynamics states of the superconductor, which occur near the stability boundary, are plotted in Fig. 2. The states in which the external magnetic field $B_a$ remained constant are shown by dashed and dotted lines. As it follows from Fig. 2, the temperature of the superconductor varies during stable magnetic flux penetration in a manner that is typical under the action of subcritical or overcritical pulsed thermal disturbances [30]. In fact, the superconductor may both save the superconducting properties despite its appreciable allowable overheating and go to the normal state after a slight increase in the temperature even when an external magnetic field becomes constant. In particular, when the magnetic field rapidly rises or when the superconductor is thermally insulated (time of stable magnetic flux penetration $t_s$ is short in both cases), an increase in the temperature before the magnetic instability onset is appreciable. Note that in the given numerical experiment, the allowable overheating reaches 10% of the superconductor's critical temperature. Simultaneously, the time of stable magnetic field penetration grows when $dB/dt$ decreases or the heat transfer coefficient increases. In these cases, the allowable overheating decreases.

To analyze the allowable overheating of the superconducting slab, let us use the formula

$$\Delta T \sim \frac{1}{aC_v(1 + t_s/t_h)} \int_0^{t_s} \int_0^a EJdxdt, \quad t_s = B_a/\dot{B}, \quad t_h = aC_v/h \tag{16}$$

which may be obtained after transition from the heat conduction equation to thermal balance equation. Here, $C_v$ is the averaged heat capacity of a superconductor; $t_h$ is the characteristic overheating time. Then, if the instability occurs within a short time, namely, at $t_s \ll t_h$ (for example, when the magnetic flux penetrates into a bulk or non-intensive cooled superconductor), the allowable overheating depends on the heat released $G_m = \frac{1}{a}\int_0^{t_s}\int_0^a EJdxdt$. In this case, the heat transfer conditions will weakly influence on the value of $\Delta T$ and the instability conditions will weakly depend on the time behavior of an external magnetic field. When the time of the magnetic flux diffusion before the flux jump grows, the allowable overheating of the superconductor will decrease, but the heat release will increase. They will have limiting value at $t_s \gg t_h$ (for example, if the sweep rate is very low or the cooling conditions are intensive). In this case, the time variation of the magnetic field will influence on the stable formation of the superconducting states. These conclusions allow to explain the unexpected feature of the magnetic flux penetration, which follows from Fig. 3. Namely, if the heat transfer coefficient increases (the time of the magnetic flux diffusion before the flux jump increases), then allowable overheating decreases before the magnetic instability onset. At the same time, if the heat transfer conditions are improved then the allowable heat release that does not cause the instability increases. As a result, they show that the relations (3) and (4) will take place at intensive cooling conditions.

Thus, the magnetic instability onset start at a finite allowable overheating, which depends on the sweep rate of an external magnetic field, cooling conditions or the cross-sectional area of the superconductor. Essential overheating of superconductor will be observed in the absence of its cooling or when the sweep rate is high. This feature, first of all, should be taken into account when the stability conditions of superconducting magnet with tight winding subjected to rapidly varying magnetic fields.

Note that the above-discussed results obtained for the $E–J$ characteristic of a superconductor described by the viscous flow model are very important. As it was discussed above, small constant overheating of the superconductor is allowable in the framework of the existing theory but only in terms of models assuming continuous rise of the superconductor's $E–J$ characteristic. However, when the critical state stability is analyzed under the assumption of

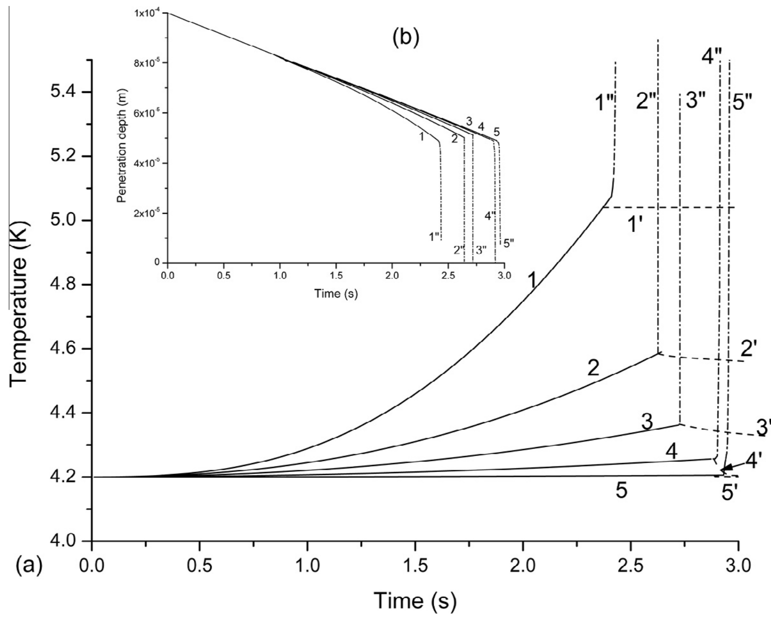


**Fig. 2.** Time dependence of the superconductor's surface temperature (a) and penetration depth of magnetic flux (b) before and after instability onset at different cooling conditions and $dB/dt = 0.1$ $T/s$: 1 – continuous external magnetic field increase at $h = 0$, 1′ – $B_a = 0.255$ $T$, 1″ – $B_a = 0.256$ $T$; 2 – continuous external magnetic field increase at $h = 1$ W/(m$^2$ K), 2′ – $B_a = 0.279$ $T$, 2″ – $B_a = 0.280$ $T$; 3 – continuous external magnetic field increase at $h = 3$ W/(m$^2$ K), 3′ – $B_a = 0.287$ $T$, 3″ – $B_a = 0.288$ $T$; 4 – continuous external magnetic field increase at $h = 10$ W/(m$^2$ K), 4′ – $B_a = 0.303$ $T$, 4″ – $B_a = 0.304$ $T$; 5 – continuous external magnetic field increase at $h = 100$ W/(m$^2$ K), 5′ – $B_a = 0.304$ $T$, 5″ – $B_a = 0.305$ $T$.

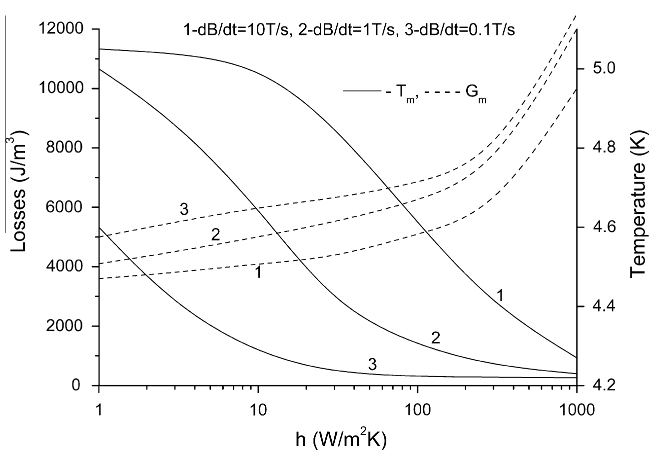


**Fig. 3.** Relationship between the allowable rise in the temperature of the superconductor $T_m$ and heat release $G_m$ before the magnetic instability onset as a function of the heat transfer coefficient at different sweep rates of the external magnetic field.

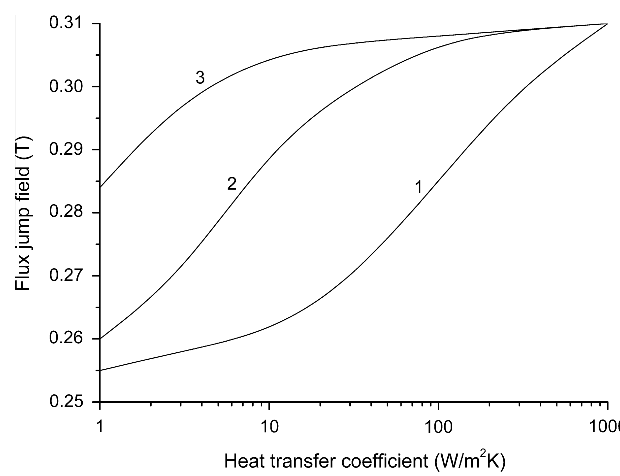


**Fig. 4.** Influence of the heat transfer coefficient and the sweep rate of an external magnetic field on the flux jump field: 1 – $dB/dt = 10$ $T/s$, 2 – $dB/dt = 1$ $T/s$, 3 – $dB/dt = 0.1$ $T/s$.

the collective formation of the thermal and electrodynamics states, which occurs throughout the magnetic flux diffusion, a type-II superconductor will exhibit both a finite allowable overheating and its dependence on the peculiarity of the electromagnetic field penetration before the flux jump (Fig. 3). Therefore, according to the variation of allowable superconductor's overheating, which depends on the sweep rate of an external magnetic field, cooling conditions or the cross-sectional area of the superconductor, the flux jump field correspondingly varies. The latter is illustrated by Fig. 4 where the field of the flux jump in a type-II superconductor is plotted as a function of the heat transfer coefficient at different sweep rates. It proves that thermal features of the critical state formation inherently influences on the sweep rate dependence of the flux jump. Consequently, the artificial limitation of the allowable overheating before the instability onset may distort the flux jump conditions.

The above-discussed results lead to the basic physical conclusion obtained for the first time under the viscous flow model. Namely, there exists nontrivial relation between allowable losses in a superconductor and a stable rise in its temperature before the instability onset. In particular, in the case of intensive cooling conditions, the losses result in the instability onset at a small but finite allowable overheating. As a result, the allowable stable temperature rise of the superconductor before the instability onset will be lower than the critical temperature of superconductor under the ideal cooling conditions ($h \to \infty$). Therefore, the generally accepted opinion according to which the losses in a superconductor under the intensive heat transfer conditions will lead to the transition of a superconductor to the normal state only when its temperature is higher than the critical temperature is erroneous.

Thus, a nontrivial relation between allowable losses and the overheating of a superconductor, which exists at arbitrary values of the heat transfer coefficients, is observed even in the framework of the critical state model. This conclusion leads to the new formulation of the of critical state stability conditions: the magnetic instability occurs when the heat release exceeds the characteristic (critical) value, which depends, for example, on the cooling conditions or the intensity of the electromagnetic disturbances. These features link the theory of the magnetic instability, the theory of the losses and the theory of the thermal stabilization, which are being now developed independently. Indeed, the thermal stabilization theory admits the existence of a considerable allowable overheating depending on the type of disturbances and the cooling conditions [30]. At the same time, the existing magnetic instability theory does not allow the existence of the finite stable overheating of a superconductor, which depends on a character of an external magnetic field variation. Meanwhile, a general thermal mechanism of the magnetic and/or thermal instabilities exits. This concept allows one to formulate the stability conditions under the action of different disturbances, which have various nature, taking into account the interrelated changes in the macroscopic thermal and electrodynamics states of superconductor before instability onset.

## 5. Interconnection of the critical state adiabatic stability condition with losses and allowable overheating

Stable finite rise in the temperature of a superconductor before magnetic instability onset, which is maximum under the adiabatic cooling condition, must change the known adiabatic stability condition of the critical state. Let us formulate the corresponding criterion in the non-isothermal approximation for a superconducting slab with a linear temperature dependence of the critical current density (4) in the case of the partially penetrated magnetic flux into the superconductor under the assumption that its specific heat of a superconductor does not depend on temperature. Let us introduce the dimensionless variables

$$X = x/a,\ \tau = t/t_x\ (t_x = \mu_0 a^2/\rho_f),\ e = E/\left(J_{c0}\rho_f\right),\ \theta = (T - T_0)/(T_{cB} - T_0)$$

Excluding the current density from system of Eqs. (7)–(9), it easy to obtain the following system of equations

$$\frac{\partial \theta}{\partial \tau} = \Lambda \frac{\partial^2 \theta}{\partial X^2} + \begin{cases} 0, & 0 < X < X_p \\ \gamma e, & X_p \le X \le 1 \end{cases} \tag{17}$$

$$\frac{\partial e}{\partial \tau} = \Lambda \frac{\partial^2 e}{\partial X^2} + \gamma e + \Lambda \frac{\partial^2 \theta}{\partial X^2}, \quad \gamma = \beta(1 - \theta) \tag{18}$$

where

$$\Lambda = \frac{\lambda \mu_0}{C \rho_f}, \quad \beta = \frac{\mu_0 J_{c0} a^2}{C} \left| \frac{dJ_c}{dT} \right|$$

are the characteristic dimensionless parameters of the critical state formation influencing on its stability conditions [1,3].

Eq. (18) describing the electric field evolution inside a superconductor belongs to the class of the diffusion equations with volume multiplication [31]. A typical process of this type is the diffusion of neutrons in an active medium with multiplication rate $\gamma = \gamma_1 - \gamma_2$, where $\gamma_1 = \beta$ is the neutron generation rate and $\gamma_2 = \beta\theta$ is the neutron absorption rate. As known, the volume generation of neutrons prevails over the neutron absorption at $\gamma > 0$. Therefore, the nuclear chain reaction may be initiated under certain conditions depending on $\gamma$. Following that analogy, it may take place a spontaneous growth of the electric field and the temperature in the superconductor leading to the destruction of the superconducting state at the critical value of the $\gamma = \beta(1-\theta)$. Note that the temperature rises before the magnetic instability onset that is equivalent to an increase in the absorption coefficient will extend the range of stable states in accordance with the value and sign of the term $\Lambda \partial^2\theta/\partial X^2$, which also influences on the electric field distribution inside the superconductor. In other words, the thermal stabilization of the critical state will take place when the possible overheating of the superconductor takes into account.

In general case, the stability analysis based even on the simplified system of Eqs. (7) and (8) needs the numerical methods to be used. To formulate the magnetic stability criteria in the non-isothermal approximation, let us study the initial stage of the electric field evolution when the temperature $\theta_m$ of the superconductor, to which it is heated before instability, and the boundary of the magnetization area $X_p$ are insignificantly changed during instability onset. This assumption is valid for type-II superconductors because the variation of them temperature is determined by the magnetic field diffusion at $\Lambda \ll 1$ [3]. Besides, let us also take into account that the temperature distribution inside a type-II superconductor under the adiabatic cooling conditions is almost uniform [3]. Therefore, the initial stage of the electric field redistribution inside a superconductor on the surface of which the magnetic field is constant can be analyzed by investigating the eigenvalues of the reduced equation

$$\frac{\partial e}{\partial \tau} = \frac{\partial^2 e}{\partial X^2} + \gamma e, \quad \gamma = \beta(1 - \theta_m) \sim const \tag{19}$$

with the boundary conditions

$$e(X_p, \tau) = 0, \quad \partial e/\partial X(1, \tau) = 0 \tag{20}$$

Solution of the problem described by Eqs. (19) and (20) may be written in the form

$$e(X, \tau) = \sum_{k=1}^{\infty} A_k \exp\left[(\gamma - \nu_k)\tau\right] Q_k(X) \tag{21}$$

where $A_k$ are the constants of integration, $Q_k(X)$ are the eigenfunctions and $\nu_k$ are the eigenvalues obtained by solving the following Sturm–Liouville problem

$$\frac{d^2 Q_k}{dX^2} + \nu_k Q_k = 0, \quad Q_k(X_p) = 0, \quad dQ_k/dX(1) = 0$$

In this spectral problem, the eigenvalues satisfy the equality

$$\sqrt{\nu_k}(1 - X_p) = (2k - 1)\pi/2, \quad 0 < \nu_1 < \nu_2 < \ldots, k = 1, 2, 3\ldots$$

Therefore, an electric field induced by varying external magnetic field will increase spontaneously (Fig. 2) under the condition $\gamma - \nu_1 > 0$ even when the external magnetic field is fixed. In this case, the first term ($k = 1$) will exponentially grow with time in (21). Therefore, the superconducting state is stable if the non-isothermal adiabatic stability condition

$$\beta < \frac{\pi^2}{4(1 - \theta_m)(1 - X_p)^2} \tag{22}$$

is satisfied. In terms of the dimensional variables, this inequality can be written as

$$\frac{\mu_0 a^2 J_{c0}}{C} \left| \frac{dJ_c}{dT} \right| \left(1 - \frac{x_p}{a}\right)^2 \frac{T_{cB} - T_m}{T_{cB} - T_0} < \frac{\pi^2}{4} \tag{23}$$

It is easy to see that criteria (22) and (23) lead to the corresponding isothermal adiabatic stability conditions in the limiting transition $T_m \to T_0$ and $X_p \to 0$ [1,3]. Criteria (22) and (23) also clearly demonstrate how an allowable stable rise in the temperature of the superconductor $T_m$ influences on the stability conditions of the superconducting state, in particular, how it influences on an allowable increase of the magnetization area when the screening current incompletely penetrates into the superconductor. According to (23), the critical state is stable if the moving coordinate $x_p$ of the magnetic flux satisfies the condition

$$x_p < a - \frac{\pi}{2}\sqrt{\frac{C(T_{cB}-T_0)^2}{\mu_0 J_{c0}^2 (T_{cB}-T_m)}} \tag{24}$$

or an external magnetic field is not higher than

$$B_a < B_m = \frac{\pi}{2}\sqrt{\mu_0 C(T_{cB}-T_m)} \tag{25}$$

Thus, the conditions of the magnetic instability may depend on the superconductor's thermal state even in the cases when the cross section of the superconductor is stably partially filled by the screening currents. In particular, the more stable increase in the temperature of the superconductor $T_m$ before the magnetic instability onset, the larger $x_p$. As a result, the temperature of the superconductor will influence on the adiabatic stability conditions most significantly when the screening currents completely penetrate into the superconductor.

The above simplified analysis of the temperature influence on the adiabatic stability conditions of the critical state was carried out only for the initial stage of the electric field redistribution at constant value of an external magnetic field. Consider now another aspect of the magnetic instability problem. Namely, let us investigate the temperature dynamics of a type-II superconductor under the assumption that the electric field distribution inside the magnetization area satisfies the Bean approximation [1,3] and, in terms of the above-introduced dimensionless variables, can be expressed as follows

$$e(X,\tau) = \frac{a\dot{B}}{J_{c0}\rho_f}(X - X_p) \tag{26}$$

Then, let us go from the heat conduction Eq. (17) to equation of thermal balance integrating it with respect to $X$ from 0 to 1, taking into account the adiabatic thermal boundary conditions ($\partial\theta/\partial X(0,\tau) = 0$ and $\partial\theta/\partial X(1,\tau) = 0$) and the uniform temperature distribution, which is observed under the adiabatic conditions [3]. After simple algebra, one can obtain

$$\frac{d\theta}{d\tau} = \beta(1-\theta)\int_{X_p}^{1} e(X,\tau)dX$$

Substituting formula (26) into this expression, it is easy to get a relationship between the rate of the temperature rise and the magnetic flux penetration depth. Namely, they satisfy the equality

$$(1 - X_p)^2 = \frac{2J_{co}\rho_f}{\beta(1-\theta)\dot{B}a}\frac{d\theta}{d\tau}$$

Then, as it follows from criterion (22), the critical state is stable, if the rate of the temperature rise is no higher than some characteristic value, i.e., if it satisfies the thermal stabilization condition

$$\frac{d\theta}{d\tau} < \frac{\pi^2}{8}\frac{\dot{B}a}{J_{c0}\rho_f}$$

In the dimensional form, this condition is written as

$$\frac{dT}{dt} < \frac{\pi^2}{8}\frac{\dot{B}(T_{cB}-T_0)}{\mu_0 J_{c0} a}$$

From this estimation and equation of thermal balance written in the form

$$C\frac{dT}{dt} = \frac{1}{a}\int_{x_p}^{a} EJdx$$

one easily finds a restriction on the heat losses. Namely, at

$$\int_{x_p}^{a} EJdx < \frac{\pi^2}{8}\frac{C(T_{cB}-T_0)\dot{B}}{\mu_0 J_{c0}}$$

and, hence, at

$$\int_0^t \int_{x_p}^{a} EJdxdt < \frac{\pi^2}{8}\frac{C(T_{cB}-T_0)B_a(t)}{\mu_0 J_{c0}}$$

the critical state is stable under the adiabatic cooling conditions. The physical meaning of this inequality is obvious. There exists an upper limit of the volume density of the losses above which the magnetic instability occurs at the adiabatic critical state formation.

Thus, the above-formulated criteria conclusively prove what thermal features of the stable magnetic flux penetration into superconductor exist. They are characterized by the existence of

- The critical rate of the superconductor's temperature rise.
- The critical value of the heat release, which is linked with the value of a rising external magnetic field and, thus, leads to the existence of the flux jump.

It should be emphasized that these features cannot be correctly formulated using models ignoring the thermal prehistory of the superconducting state formation. Besides, they will also take place when the stability conditions of practical low- and high-temperature superconductors are investigated.

## 6. Non-isothermal adiabatic stability of the critical state of composite superconductor (multilayer model "superconductor + normal metal")

Let us formulate the non-isothermal criterion of the magnetic instability onset in the composite superconductor consisting of alternating superconducting and non-superconducting (normal metal) layers (Fig. 5) in the adiabatic approximation taking into account the inevitable difference of the composite temperature $T_i$ before instability from the coolant one ($T_i > T_0$). Let us consider an infinitely long thermally insulated slab placed into the external magnetic field that is parallel to its surface. Suppose that the magnetic field has completely penetrated into the composite. Let the critical current density depends only on the temperature and changes according to the relation (14). Then the temperature of composite, which is equal to $T_i$ at the initial moment, should change to $T_k$ as a result of the energy release stored in the screening currents during instability development. After that heat release, a new distribution of the magnetic field should be established inside the composite. Accordingly, the heat release generated by source creating the external magnetic field should change the enthalpy of the composite. Let us consider two typical cases supposing that the distribution of the temperature inside the composite is practically uniform due to fulfilling the condition $ha/\lambda_k \ll 1$.

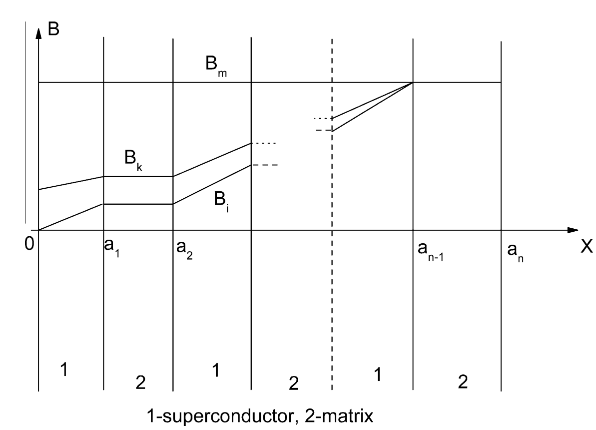


**Fig. 5.** Distribution of the magnetic field inside the composite with the complete penetration of the magnetic current: $B_i$ – at the initial moment of time, $B_m$ – after the transition of the superconductor to the normal state, $B_k$ – in the partial heat release of the magnetic energy.

*6.1. Complete heat release of stored energy ($T_k > T_{cB}$)*

In this case, the superconducting composite will be in the normal state. Then, according to the energy conservation law the change in its enthalpy satisfies the following equality

$$\int_0^{a_n}\int_{T_i}^{T_k} C_k(T)dTdx = \int_0^{a_n}\frac{B_i^2}{2\mu_0}dx$$

because its change is only connected with a part of the magnetic energy, which changes due to the heat variations in the screening current inside the superconductor (Fig. 5). The above equality can be easily rewritten as

$$\int_{T_i}^{T_k}(\Delta_s C_s(T) + \Delta_m C_m(T))dT = \frac{B_m^3}{6\mu_0^2 J_c(T_i)}$$

taking into account two components of the specific heat capacity of composite and considering that the distribution of the magnetic field in each superconducting layer is described by equation $dB_i/dx = \mu_0 J_c(T_i)$ in accordance with the critical state model. Here, $\Delta_s$ and $C_s$ are the common thickness of the superconductor and its specific heat capacity, respectively; $\Delta_m$ and $C_m$ are the common thickness of the normal metal and its specific heat capacity, respectively; $B_m$ is the magnetic field on the composite surface before the instability onset, which is equal to $B_m = \mu_0 J_c(T_i)\Delta_s$. Substituting $B_m$ one shall obtain the following equation

$$\int_{T_i}^{T_k}[\eta C_s(T) + (1-\eta)C_m(T)]dT = \frac{\mu_0\eta^3 J_{c0}^2}{6}\left(\frac{T_{cB}-T_i}{T_{cB}-T_0}\right)^2 a_n^2 \qquad (27)$$

Here, $\eta$ is the filling coefficient of the composite ($\eta = \Delta_s/a_n$). This relation permits to determine the final temperature of composite after the transition of the superconducting layers to the normal state. Besides, it is easy to find from (27) the non-isothermal criterion for irreversible transition of the superconducting composite to the normal state. Its final temperature will exceed the critical temperature of the superconductor due to the spontaneous development of the magnetic instability when the following inequality

$$\int_{T_i}^{T_{cB}}[\eta C_s + (1-\eta)C_m]dT < \frac{\mu_0\eta^3 J_{c0}^2}{6}\left(\frac{T_{cB}-T_i}{T_{cB}-T_0}\right)^2 a_n^2 \qquad (28)$$

takes place or when the external magnetic field will exceed the value

$$B_m > \sqrt{6\frac{\mu_0}{\eta}\int_{T_i}^{T_{cB}}[\eta C_s + (1-\eta)C_m]dT} \qquad (29)$$

Criterion (28) makes also possible to determine characteristic value of the composite thickness or its initial temperature $T_i$ when the energy stored by the screening currents is not sufficient for total destruction of the superconducting state of composite. Let us consider the stability conditions of such states.

*6.2. Partial release of the magnetic energy stored by the screening currents ($T_k < T_{cB}$)*

For these states, the energy conservation law is written as follows

$$\int_0^{a_n}\int_{T_i}^{T_k} C(T)dTdx = \int_0^{a_n}\frac{B_i^2}{2\mu_0}dx - \int_0^{a_n}\frac{B_k^2}{2\mu_0}dx$$

which may be transformed into the equality

$$\int_{T_i}^{T_k}[\eta C_s + (1-\eta)C_m]dT = \frac{\mu_0\eta^3 J_{c0}^2 a_n^2}{6(T_{cB}-T_0)^2}(T_k - T_i)(2T_{cB} - T_i - T_k) \qquad (30)$$

in the framework of the critical state model. As the right side of this equality decreases with increasing $T_i$, the corresponding value of the final temperature of the superconducting composite as a result of the developing magnetic instability also decreases with increasing $T_i$. This is explained physically by that the energy of the screening currents, which results in spontaneous increase in the composite temperature, decreases with the increase in $T_i$. Therefore, the temperature of composite will not change under some conditions, i.e., $T_k = T_i$, and, thus, the magnetic instability does not occur. Fulfilling the limiting transition $T_k \to T_i$ and taking into account that the enthalpy of the composite does not change, we shall find the criterion

$$\frac{\mu_0 a^2\eta^2 J_{c0}^2}{[\eta C_s(T_i) + (1-\eta)C_m(T_i)](T_{cB}-T_0)} < \frac{3}{\eta}\frac{T_{cB}-T_0}{T_{cB}-T_i} \qquad (31)$$

at which the critical state is stable in spite of difference between the composite temperature $T_i$ from that of the coolant $T_0$ at the initial moment. In this case, the external magnetic field should satisfy the condition

$$B_m < \sqrt{\frac{3\mu_0}{\eta}[\eta C_s(T_i) + (1-\eta)C_m(T_i)](T_{cB}-T_i)}$$

Let us discuss the above written criteria and compare them with the known stability condition of the critical state of composite. First of all, note that these criteria fulfill the limited transition to the non-isothermal conditions at $\eta \to 1$ obtained above which describe the destruction of the critical state of type-II superconductor having no stabilizing matrix. At the same time, they demonstrate the effect of the composite structure on the stability conditions which cannot be formulated in terms of the model with averaged physical properties. In order to estimate the effect of non-uniformity of the physical properties of the composite on the conditions of the magnetic instability onset, let us use the known isothermal criterion obtained under the assumption $C_s$, $C_m \sim const$ and $T_i = T_0$. Then, according to [1,3], the critical state of the composite is stable, if the magnetic instability parameter of composite $\beta_k$ satisfies the condition

$$\beta_k = \frac{\mu_0 a^2\eta^2 J_{c0}^2}{[\eta C_s + (1-\eta)C_m](T_{cB}-T_0)} < 3 \qquad (32)$$

However, criterion (31) gives the condition $\beta_k < 3/\eta$. Since $\eta < 1$, the stability condition (32) leads to harder restrictions imposed on the composite parameters. It is easy to understand that this singularity due to averaging the screening current over the full composite cross-sectional area, which was used in the model of anisotropic continuum. At the same time, this assumption was not used in deriving condition (28) and (31) and they were obtained for the case when the screening currents flow only in the superconducting part of the composite. Thus, the model of anisotropic continuum underestimates the range of the critical state stability accurate to factor $1/\eta > 1$.

## 7. Non-isothermal magnetic instability conditions of superconducting composite in the flux creep state

Discussed above features of the critical state formation of superconductors and superconducting composites show that the non-isothermal analysis of the magnetic instability conditions extends the boundaries of the existing theory even when it is made in terms of the simplified model of viscous flow accounting for the thermal prehistory of the occurring processes. However, in the real cases, the voltage–current characteristics of the superconductors rise continuously with the increase in the current. As it was shown in [32], the superconductors with the real $E$–$J$ characteristics have the differential resistance, which is non-uniformly distributed over the superconductor's cross-section area and goes to zero in the moving front of the magnetization region in the incomplete penetration of the magnetic flux. This regularity should affect the dynamics of the magnetic flux penetration into technical superconductors with the real $E$–$J$ characteristics contrary to the electrodynamics states of the superconductors with idealized $E$–$J$ characteristic. Taking into account the thermal prehistory of the formation of electrodynamics states in the composite, let us discuss the features of the magnetic instability onset in the superconducting composite whose real $E$–$J$ characteristic is described by the exponential equation [15–17]. For this purpose, let us investigate the conditions of the instability onset in the superconducting helicoid [33].

The superconducting helicoid represents a number of separate superconducting strands connected as single whole over the helical plane. It has some advantages comparing with the traditional current-carrying elements, namely, an increased engineering current density and high mechanical strength. The first is due to the natural optimization of the current in its distribution over the helicoid cross section area. The second advantage is that the Lorentz forces and bending mechanical moments are received by the whole stranded turn. However, their operability is limited by some circumstances. Since the helicoid represents a massive composite structure, it is sensitive to the electromagnetic perturbations. So, let us study the stability conditions of its superconducting state depending on the sweep rate.

To investigate the diffusion of the temperature, electric and magnetic fields inside the cooled helicoid, approximate it as a hollow infinitely long cylinder. Let us consider the regime when the external magnetic field changes on its inner surface. Then this approximation is described by the set of equations

$$C_k(T)\frac{\partial T}{\partial t} = div(\lambda_k(T) gradT) + EJ, \qquad (33)$$
$$\mu_0\frac{\partial J}{\partial t} = \Delta E, \quad B = \frac{dB}{dt}t - \mu_0\int_{r_1}^{r} J(r,t)rdr$$

with initial conditions

$$T(r,0) = T_0, \quad E(r,0) = 0, \quad B(r,0) = 0 \qquad (34)$$

and with the conditions on the inner and external surfaces

$$-\lambda_k\partial T/\partial r + h(T-T_0)|_{r=r_1} = 0, \quad rotE|_{r=r_1} = -dB/dt, \qquad (35)$$
$$\lambda_k\partial T/\partial r + h(T-T_0)|_{r=r_2} = 0, \quad rotE|_{r=r_2} = 0$$

For the description of the connection between the electric field and the currents inside the superconductor and the matrix, one shall use equation of voltage–current characteristic and the Kirchhoff's equations in the form

$$E = J_s\rho_n \exp\left(\frac{J_s}{J_\delta} + \frac{T-T_{cB}}{T_\delta}\right) = J_m\rho_m, \quad J = \eta J_s + (1-\eta)J_m \qquad (36)$$

In performing the numerical calculations, we set the initial parameters as

$$r_1 = 1.2\cdot 10^{-2}\mathrm{m}, \quad r_2 = 2.7\cdot 10^{-2}\ \mathrm{m}, \quad \eta = 0.2, \quad T_0 = 4.2\mathrm{K},$$
$$h = 2500\mathrm{W}/(\mathrm{m}^2\cdot\mathrm{K}),$$
$$J_\delta = 8\cdot 10^7\ \mathrm{A/m^2}, \quad T_\delta = 0.096K, \quad T_{cB} = 9.5 - 0.643B,$$
$$\rho_n = 6\cdot 10^{-7}\Omega\cdot\mathrm{m}, \quad \rho_m = (2.13 + 0.605B)10^{-10}\Omega\cdot\mathrm{m},$$
$$\lambda_k = 2.45\cdot 10^{-8}\frac{T}{\rho_m}\frac{1-\eta}{1+\eta}\frac{W}{\mathrm{m}\cdot\mathrm{K}}, \quad C_s = 13T^3\frac{J}{\mathrm{m^3K}},$$
$$C_m = 8T^3\frac{J}{\mathrm{m^3K}}$$

They describe the thermal and electrical properties of the helicoid containing 71 plane turns. Each turn consisted of 15 alternating niobium-titanium strands 0.5 mm in diameter contained in the copper matrix and 15 copper wires of the same diameter which were soldered together. The heat transfer coefficient was chosen so that the theoretical and experimental values of the flux jump field would insignificantly differ. Such approximation of the effective value of the heat transfer coefficient, which is often used in the comparison of the experimental data with the simulation results, was made because of a significant quantity of experimental factors affecting the true value of the heat transfer coefficient, whose analysis requires performance of special experiments. At the same time, our chosen effective value of the heat transfer coefficient remains within the range of the values commonly used in the estimation of the conditions of the magnetic instability onset in the composite superconductors [3].

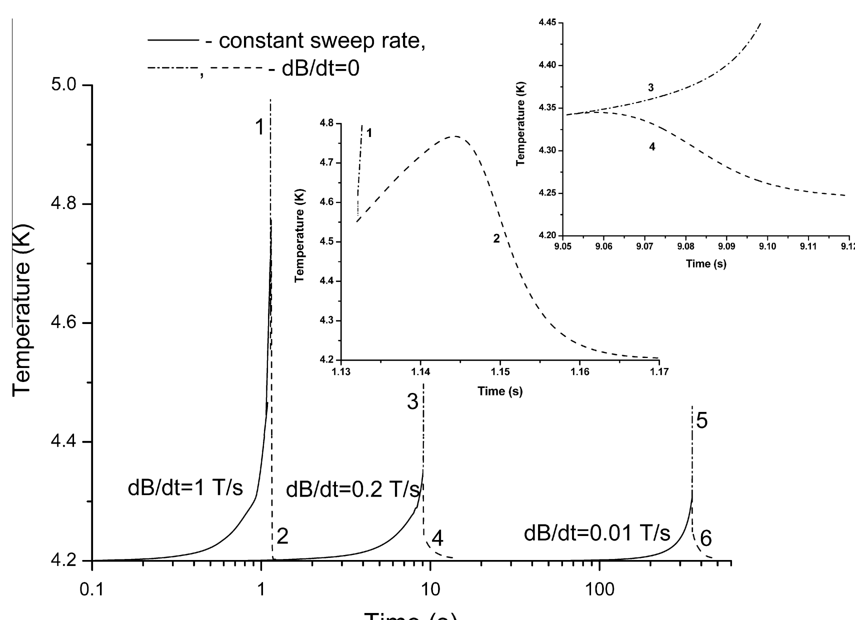


**Fig. 6.** Change in the temperature on the inner surface of the helicoid when the external magnetic field reaches the values corresponding to the stable and non-stable states: 1 – $B_0$ = 1.14 $T$, 2 – $B_0$ = 1.13 $T$, 3 – $B_0$ = 1.811 $T$, 4 – $B_0$ = 1.81 $T$, 5 – $B_0$ = 3.528 $T$, 6 – $B_0$ = 3.526 $T$.

Fig. 6 presents the results of the numerical experiment for the determination of the stability conditions of the superconducting state of the helicoid. It was made in accordance with the above proposed method of final perturbation of the inner equilibrium state. Accordingly, Fig. 6 shows the dynamics of its temperature during the magnetic flux diffusion both in the continuous rise in the external magnetic field (solid line) and at its fixed value close to the flux jump field. In these cases, the dashed lines show the stable change in the helicoid temperature and the dashed-pointed lines demonstrate the change in the temperature in the non-stable states. Insets present the stable and unstable variations in the temperature near the boundary of the stable states during relaxation states ($dB/dt = 0$) in more detail. The results presented demonstrate the existence of same thermal stability mechanisms as discussed above and should be the basis of the determination in the boundary of the magnetic instability onset in the composite superconductor with the arbitrary $E$–$J$ characteristic. In particular, the initial stage of the helicoid temperature dynamics may be characterized by the rise in its temperature in the stable relaxation of the electrodynamics field. Accordingly, depending on the $dB/dt$ value, the stable variation in the electric field may occur, first, in its monotonous rise and then in its subsequent decrease. As a whole, stable overheating of the helicoid depends on the sweep rate according to the estimate (16). Namely, it decreases monotonically with an increase in the time of stable magnetic flux diffusion.

It may be proved that potential modes of the thermal and electromagnetic states of superconductors with real $E$–$J$ characteristics also have the fission-chain-reaction nature. It may be also shown that the stable and unstable development of thermal and electrodynamics states of both low- and high-temperature superconductors depends not only on the joint temperature variation of the $C(T)$, $\lambda(T)$, $J_c(T)$ but $\partial J_c/\partial T$ [34]. The importance of this conclusion should be emphasized. To understand the stability mechanism of superconductors, one usually allows for only the heat capacity of a superconductor. In the meantime, it appears that the instability in technical superconductors may be absent due to the temperature influence of all above-mentioned properties of superconductors. As a result of this temperature effect, the fission-chain-reaction character of the electrodynamics state evolution in high-temperature superconductors may be absent in the high operating temperature range where its swept development may become stable.

Fig. 7 shows the corresponding values of the magnetic flux jump field as a function of the sweep rate. The results of direct measurement of $B_m(dB/dt)$ are also presented. The reasonable agreement of the theoretical and experimental results verifies the non-isothermal approximation used. As a result, they prove the dependence of the permissible overheating of composite superconductor on the rate of the external magnetic field variations and demonstrate the workability of the models proposed. Therefore, they can be used in solving practical problems arising in the development of new superconducting devices and, first of all, those suffering from the influence of the rapidly varying magnetic fields when the thermal prehistory of the current-carrying element cannot be neglected.

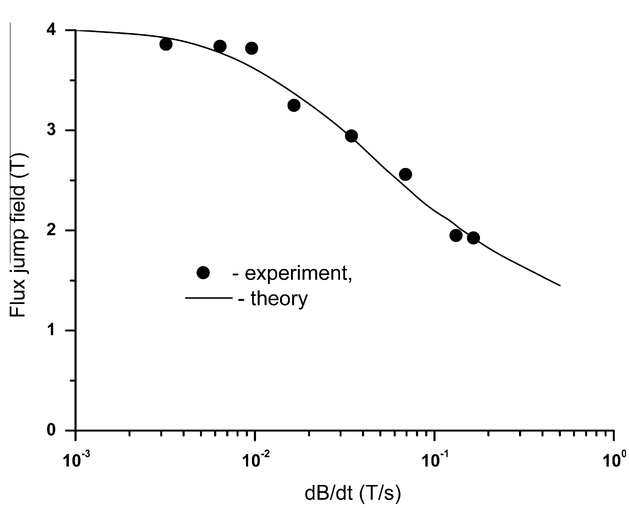


**Fig. 7.** Effect of the rate of rise in the external magnetic field on the stability of the superconducting state of the helicoid.

## 8. Conclusions

The analysis made shows for the first time that the interrelated electrodynamics and thermal phenomena in superconductors have the fission-chain-reaction nature. They are characterized by non-trivial relations between stability conditions, allowable losses and the corresponding overheating of a superconductor before magnetic instability onset. Therefore, not only the electrodynamics conditions of the superconducting state stability but also the temperature factor are responsible for the onset of the magnetic instability in superconductors when the heat release exceeds the corresponding critical value during flux penetration. In this case, a method of the final perturbation of the initial equilibrium state is advised, which permits the boundary of the stable states to be determined taking into account the processes passing before the instability onset in the superconductors with different types of non-linearity of $E$–$J$ characteristics. It validates the use of a unified method allowing to find the conditions of the instability onset of different nature both in low- and high-temperature superconductors. This concept allows to make the limiting transitions to the known stability criteria of superconducting states, which were obtained using different models of the $E$–$J$ characteristic of the superconductor. Eventually, the stability conditions will be direct consequence of the state when the corresponding critical energy depending on the type of an external disturbance (electromagnetic, mechanical, thermal or a combination of them) is exceeded.

The determination of the conditions of the magnetic instability onset in type-II superconductors, whose $E$–$J$ characteristics are described even by the idealized viscous flow model, have correctly demonstrated for the first time that the conditions of the critical state stability depend on the sweep rate of the external magnetic field due to the corresponding dependence of the stable overheating of the superconductor before the instability onset. The corresponding non-isothermal magnetic instability criteria of type-II superconductors and superconducting composites based on them are formulated. They prove that the generally accepted opinion according to which the thermal losses in a superconductor under an intensive heat transfer conditions will lead to the transition of a superconductor to the normal state only when its temperature is higher than the critical temperature is erroneous. Therefore, the analysis of the violation of the stability of the superconducting state made without accounting for the increase in temperature before instability may give underestimated values of the flux jump field. It is also shown that the anisotropic continuum model leads to more strict requirements imposed on the parameters of composite responsible for conservation of its superconducting properties.

The non-isothermal behavior of superconductor before instability was verified comparing the experimental results and the numerical analysis of the stability conditions and temperature dynamics of the helicoid-type conductor.

Since the macroscopic electrodynamics of superconductors is investigated, as a rule, in the isothermal approximation, these results will be very important to describe the phenomena in non-intensive cooled superconducting devices with bulk current-carrying elements placed in rapidly varying magnetic fields or alternating current.

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