Real Analysis - A Long Form Mathematics Textbook Chapter 2: Cardinality

Yen-Ting Chen

May 2025

2.1 Bijections and Cardinality

Principle 2.1 (The bijection principle). Two sets have the same size if and only if there is a bijection between them.

2.2 Counting Infinities

Theorem 2.8 ($|Z| = |Q|$). There are the same number of integers as rational numbers.

Theorem 2.9 ($|R| > |N|$). There are more real numbers than natural numbers.

Theorem 2.11 (Sizes of infinity). There are different sizes of infinity, with countable infinity being the smallest. Moreover, N, Z, and Q are countable while R is uncountable.

Theorem 2.13 (—A— ¡ —P(A)—). If A is a set and P(A) is the power set of A, then

$$|A| < |P(A)| \tag{1}$$

Corollary 2.14 (There exist infinitely many infinities). There exist infinitely many distinct infinite cardinalities.