Stochastic Lanchester Simulation

Lanchester Equations have been used to model attrition in combat for nearly a century. The original models proposed by Lanchester used differential equations to approximate attrition via per-capita kill rates—the per-capita kill rate times the number of survivors determined the rate at which enemy forces were attrited. This model, despite its shortcomings (or perhaps because of them!), could be solved mathematically to yield insights into the course of a battle. Virtually any improvement in the realism quickly destroyed mathematicians' abilities to solve the models in the days before computers.

With modern computing we can introduce more realistic behaviors, such as randomness. It is also more realistic to model attrition as an integer process, rather than approximating attrition as a continuous process (which yields fractional soldiers) in order to get a mathematically tractable model.

We will build a simple version of a stochastic Lanchester model, based on kill rates which are exponentially distributed. As with the original differential equation model, we will assume that each of the two forces is homogeneous in terms of abilities, i.e., all red forces share a common per-capita kill rate λ_R and all blue forces share a common per-capita kill rate λ_B. These represent the rate at which one red soldier kills blue forces, and one blue soldier kills red forces, respectively. We will further assume that the times between kills follow an exponential distribution. We will denote the number of red forces as R, and the number of blue forces as B at any particular point in time.

A battle proceeds as follows:

1. Red and Blue forces start with initial force levels R_0 and B_0, respectively.

2. They commence firing, with effective rates of $(R \cdot \lambda_R)$ and $(B \cdot \lambda_B)$. With the exponential distribution we can pool the rates, and say that deaths on the battlefield occur at a combined rate of $(R \cdot \lambda_R + B \cdot \lambda_B)$.

3. When a death occurs, we need to determine which side suffered attrition. Again, we can use the properties of exponential distributions to answer the question:

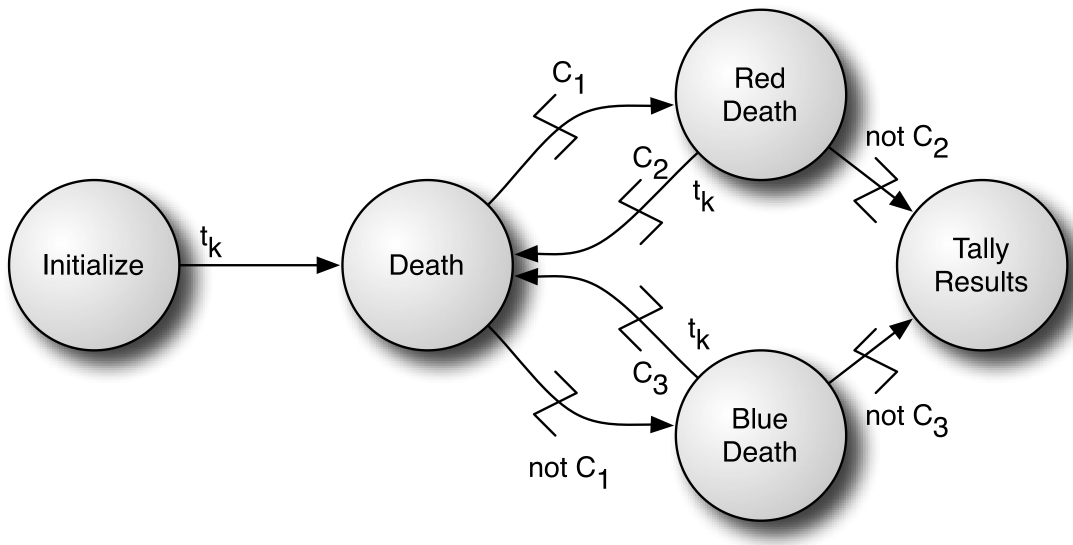
$$P\{\text{Blue attrition}\} = \frac{R \cdot \lambda_R}{R \cdot \lambda_R + B \cdot \lambda_B}$$

$$P\{\text{Red attrition}\} = \frac{B \cdot \lambda_B}{R \cdot \lambda_R + B \cdot \lambda_B}$$

4. Use a random number to decide which side suffered attrition, and decrement their force level.

5. If both sides have forces remaining, repeat the process from step 2. Note that the loss of a soldier on one side or the other reduces their effective kill rate, so even though the logic remains unchanged the combined kill rate and probabilities which determine red vs. blue attrition will change as the battle progresses.

The model described above can be represented by the following event graph.



State Transitions:

Initialize: $R = R_0$; $B = B_0$

Death: $\text{pRedKill} = B \cdot \lambda_B / (R \cdot \lambda_R + B \cdot \lambda_B)$

Red Death: decrement R

Blue Death: decrement B

Tally Results: gather any statistics needed.

Conditions:

C_1**:** $U \leq \text{pRedKill}$, where $U \sim \text{Uniform}(0,1)$.

C_2**:** $R > 0$

C_3**:** $B > 0$

Delays:

$t_k \sim \text{exponential}(R \cdot \lambda_R + B \cdot \lambda_B)$.