LO: Sequences and limiting values.

22/10/2024

1.

The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.

(i) Find the remainder when $f(x)$ is divided by $(x - 2)$.

(ii) Show that $(2x + 1)$ is a factor of $f(x)$ and hence factorise $f(x)$ completely.

2.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

Prerequisite Retrieval

Prerequisite Problem Solving

3.

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(b) Factorise $f(x)$ completely.

4.

Ready to Learn

Problem solving with linear sequences

Examples – I do

Find the n^{th} term and the 60^{th} term of the following sequence:

-5, -9, -13, -17, -21

Independent Practice- You do

1) Here are the first five terms of an arithmetic sequence 6, 11, 16, 21 and 26

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 100th term of the sequence.

2) Here are the first five terms of an arithmetic sequence 1, 7, 13, 19 and 25

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 50th term of the sequence.

3) Here are the first five terms of an arithmetic sequence -4, -1, 2, 5 and 8

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 1000th term of the sequence.

Answers

1) Here are the first five terms of an arithmetic sequence 6, 11, 16, 21 and 26

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 100th term of the sequence.

a) 5n + 1
b) 501

2) Here are the first five terms of an arithmetic sequence 1, 7, 13, 19 and 25

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 50th term of the sequence.

a) 6n - 5
b) 295

3) Here are the first five terms of an arithmetic sequence -4, -1, 2, 5 and 8

a Write down, in terms of n, an expression for the nth term of this sequence.

b Find the 1000th term of the sequence.

a) 3n - 7
b) 2993

Worked Examples – I do

1) The nth term of the sequence is given by the rule 2n +1.
Is 36 a term in this sequence?

Worked Examples – We do

2) Is 52 a term in the following sequence?
7, 10, 13, 16,….
Justify your answer.

Build it Together

Independent Practice- You do

1) A sequence is given by the rule 3n – 5.
Is 55 a term in this sequence? Explain your answer.

2) Find the nth term of the following sequence:
2, 7, 12, 17, ….
Is 75 a term in this sequence? Explain your answer.

3) Is 67 a term in the sequence describe by the nth term 4n – 1?
Explain your answer.

4) Find the nth term of the following sequence:
1,-5, -11, -17, ….
Is -48 a term in this sequence? Explain your answer.

5) A sequence is given by the rule 3n + 2.
Is 38 a term in this sequence? Explain your answer.

Answers

1) A sequence is given by the rule 3n – 5.
Is 55 a term in this sequence? Explain your answer.

2) Find the nth term of the following sequence:
2, 7, 12, 17, ….
Is 75 a term in this sequence? Explain your answer.

3) Is 67 a term in the sequence describe by the nth term 4n – 1? Explain your answer.

4) Find the nth term of the following sequence:
1,-5, -11, -17, ….
Is -48 a term in this sequence? Explain your answer.

5) A sequence is given by the rule 3n + 2.
Is 38 a term in this sequence? Explain your answer.

1) Yes, 20th term.
2) No, term has to be an integer.
3) Yes, 17th term
4) No, not an integer.
5) Yes, 12th term.

Quadratic sequences

Find the n^{th} term of a quadratic sequence.

Quadratic Sequences (Example 1)

n =	1st	2nd	3rd	4th	5th
Sequence =	-1	5	15	29	47
1st difference =		+6	+10	+14	+18
2nd difference =			+4	+4	+4

Find the n^{th} term of a quadratic sequence.

The nth term of a quadratic sequence is always in the form

$$an^2 + bn + c$$

Is found using the second differences and dividing by 2

Deliberate practice

Find the 'second differences'.
Which value do we use for 'a'?

a) 1, 4, 9, 16, 25

b) 4, 10, 20, 34, 52

c) 4, 13, 28, 49, 76

Deliberate practice

Find the 'second differences'.
Which value do we use for 'a'?

a) 1, 4, 9, 16, 25, 36, 49

2^{nd} dif. is 2 so a = 1; n^2.....

b) 4, 10, 20, 34, 52, 74, 100

2^{nd} dif. is 4 so a = 2; $2n^2$.....

c) 4, 13, 28, 49, 76, 109, 148

2^{nd} dif. is 6 so a = 3; $3n^2$.....

Find the n^{th} term of a quadratic sequence.

The nth term of a quadratic sequence is always in the form

$$an^2 + bn + c$$

How do we find this part of the formula?

Find the n^{th} term of a quadratic sequence.

Quadratic Sequences (Example 1)

Find the difference between these sequences.
In other words how close did we come?

	-1	5	15	29	47
$2n^2$ =	2	8	18	32	50
	-3	-3	-3	-3	-3

Our sequence was off by -3.

Since we were off by -3 the nth term is: $2n^2 - 3$

Find the n^{th} term of a quadratic sequence.

Quadratic Sequences (Example 2)

n =	1	2	3	4	5
Sequence =	5	14	29	50	77
1st difference =	+ 9	+15	+21	+27	
2nd difference =	+6	+6	+6		

$6 \div 2 = 3$
Sequence has $3n^2$

	5	14	29	50	77
$3n^2$ =	3	12	27	48	75
	2	2	2	2	2

Therefore the n^{th} term = $3n^2 + 2$

Find the n^{th} term of a quadratic sequence.

Deliberate practice

Find the n^{th} term of these sequences

CORE

1) 5, 11, 21, 35, 53

EXT

2) 1, 10, 25, 46, 73

3) 6, 12, 22, 36, 54

KILLER

4) 0, 3, 8, 15, 24

5) 3, 9, 17, 27, 39

Answers

1) $2n^2 + 3$

2) $3n^2 - 2$

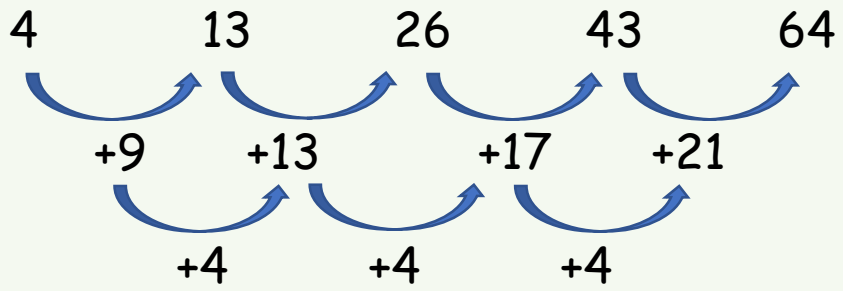
3) $2n^2 + 4$

4) $n^2 - 1$

5) $n^2 - 3n - 1$

Find the n^{th} term of a quadratic sequence.

Quadratic Sequences (Example 3)



term	4	13	26	43
$2n^2$	2	8	18	32
subtract	2	5	8	11

This sequence is $3n - 1$

Sequence is $2n^2 + 3n - 1$

$4 \div 2 = 2$
Sequence has $2n^2$

Find the n^{th} term of a quadratic sequence.

Deliberate practice

Find the n^{th} term of these sequences

1)	6,	17,	34,	57,	86
2)	3,	18,	41,	72,	111
3)	7,	14,	23,	34,	47
4)	8,	19,	34,	53,	76
5)	2,	12,	30,	56,	90

Answers

1) $3n^2 + 2n + 1$
2) $4n^2 + 3n - 4$
3) $n^2 + 4n + 2$
4) $2n^2 + 5n + 1$
5) $4n^2 - 2n$

Limiting value of a sequence

Worked Examples – I do

The nth terms of sequences are shown.

Work out the limiting value of each sequence as $n \to \infty$.

(ii) $\dfrac{n+2}{n+3}$

(v) $\dfrac{3n}{4n+1}$

(viii) $\dfrac{2-6n}{5-2n}$

Worked Examples – We do

The nth terms of sequences are shown.

Work out the limiting value of each sequence as $n \to \infty$.

(i) $\dfrac{2n}{n+1}$

(iv) $\dfrac{2n-1}{4n+1}$

(vii) $\dfrac{2n}{3-4n}$

Worked Examples – I do

The nth term of a sequence is $\frac{2n-1}{3n+2}$.

Prove that the limiting value of the sequence as $n \to \infty$ is $\frac{2}{3}$.

Worked Examples – We do

④ The nth term of a sequence is $\frac{5n+1}{2n+1}$.

Prove that the limiting value of the sequence as $n \to \infty$ is $\frac{5}{2}$.

Exam Style/Problem Solving

PS ⑦ The nth term of a sequence is $\frac{an+3}{cn-1}$.

The 1st term of the sequence is 11, and its limiting value as $n \to \infty$ is 4

Work out the value of the 2nd term of the sequence.

Homework

Textbook

Page 114
Exercise 4J

Q2
Q3. iii) and vi)
Q5
Q6

Page 110
Exercise 4H

Q2
Q5
Q6

Page 112
Exercise 4I

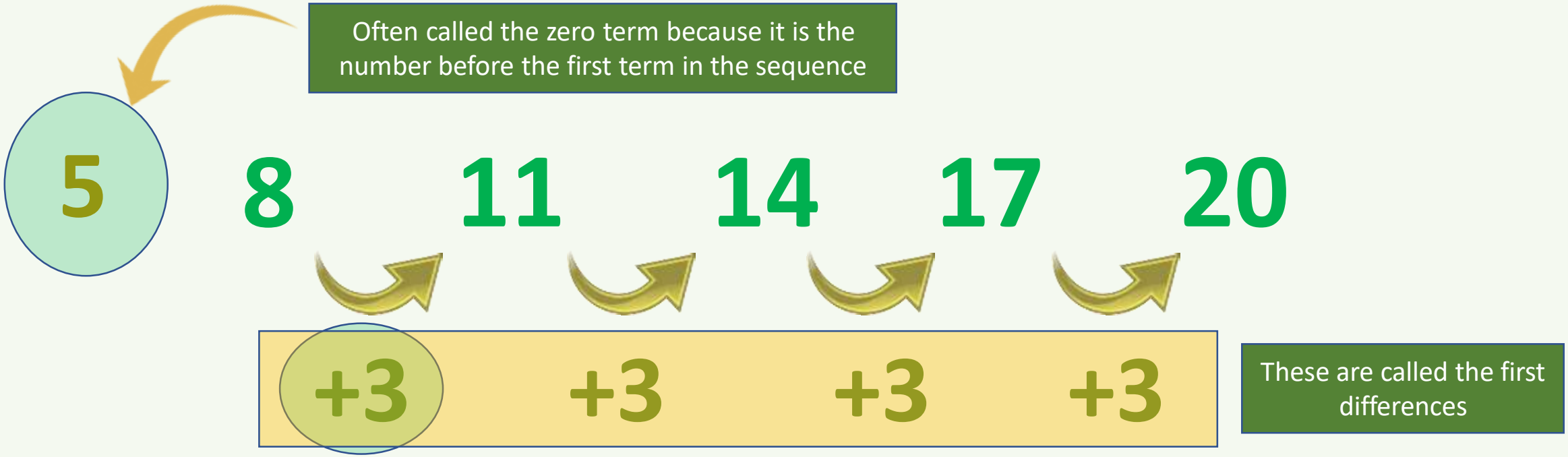
Q1. Even
Q4
Q7

Find the n^{th} term and substitute into quadratic expression.

Prior Knowledge

Linear Sequences Recap – Find the nth term rule

Often called the zero term because it is the number before the first term in the sequence



These are called the first differences

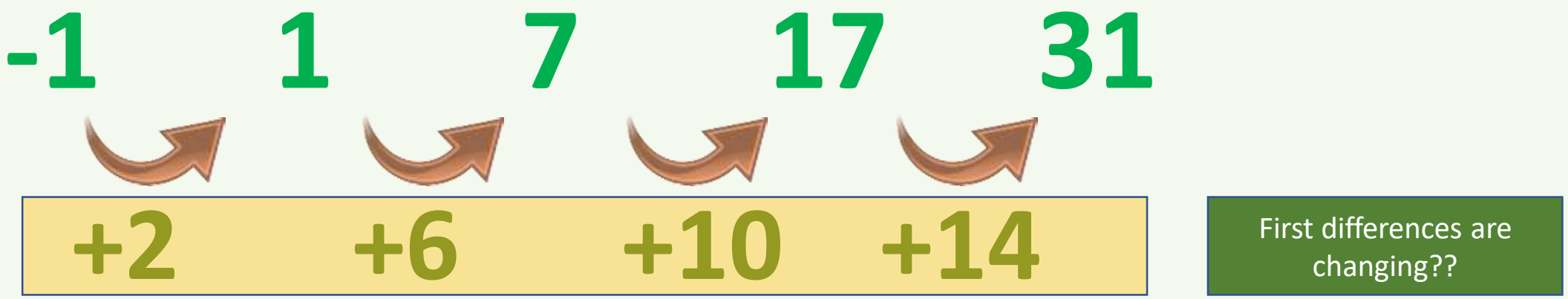
Nth term rule

$3n + 5$

Find the n^{th} term and substitute into quadratic expression.

Worked examples

Sequences Recap



First differences are changing??

Find the n^{th} term and substitute into quadratic expression.

Worked examples

Sequences Recap

-1 1 7 17 31

+2 +6 +10 +14

First differences are changing??

+4 +4 +4

Let's have a look at the second differences.

The second differences are constant.

This tells us that it is a QUADRATIC SEQUENCE.

Find the n^{th} term and substitute into quadratic expression.

Worked examples

Finding an expression for quadratic sequences

The general formula for a quadratic sequence is:

$$an^2 + bn + c$$

where a, b and c are numbers we need to calculate

$$2n^2 + bn + c$$

-1 1 7 17 31

+2 +6 +10 +14

+4 +4 +4

Divide the second the difference by 2.

This is the value for a

Find the n^{th} term and substitute into quadratic expression.

Worked examples

Finding an expression for quadratic sequences

$$2n^2 + bn + c$$

For the next part, you will need to create a small table

Substitute these values in to **$2n^2$**

n	1	2	3	4	5
Term	-1	1	7	17	31
$2n^2$	2	8	18	32	50

Term $2n^2$	-3	-7	-11	-15	-19

This is linear sequence because the first differences are always the same

Find the n^{th} term and substitute into quadratic expression.

Worked examples

$2n^2 + bn + c$

n	1	2	3	4	5
Term	-1	1	7	17	31
$2n^2$	2	8	18	32	50

Calculate nth term of this linear sequence

-3 -7 -11 -15 -19

-4 -4 -4 -4

Nth term of linear sequence =

-4n +1

Expression for quadratic sequence

$2n^2 - 4n + 1$

Find the n^{th} term and substitute into quadratic expression.

Deliberate practice

Find the n^{th} term of these sequences

1)	6,	17,	34,	57,	86
2)	3,	18,	41,	72,	111
3)	7,	14,	23,	34,	47
4)	8,	19,	34,	53,	76
5)	2,	12,	30,	56,	90

Answers

1) $3n^2 + 2n + 1$
2) $4n^2 + 3n - 4$
3) $n^2 + 4n + 2$
4) $2n^2 + 5n + 1$
5) $4n^2 - 2n$

Find the n^{th} term and substitute into quadratic expression.

Deliberate practice

Finding an expression for the following quadratic sequences:

1)

n	1	2	3	4	5
Term	6	9	14	21	30

2)

n	1	2	3	4	5
Term	11	20	35	56	83

3)

n	1	2	3	4	5
Term	5	14	31	56	89

Extension: Calculate the 10^{th} term in each sequence

Find the n^{th} term and substitute into quadratic expression.

Answers

n	1	2	3	4	5
Term	6	9	14	21	30

$1n^2 + 5$

10th term = 105

n	1	2	3	4	5
Term	11	20	35	56	83

$3n^2 + 8$

10th term = 308

n	1	2	3	4	5
Term	5	14	31	56	89

$4n^2 - 3n + 4$

10th term = 374