LO: Dividing polynomials.

22/10/2024

1. Work out the inverse function

$f(x) = 2x - 3$

2. Work out the value of the following by first finding an expression for the inverse function.

$f^{-1}(4)$ when $f(x) = 5x - 1$

Prerequisite	Prerequisite
Retrieval	Problem Solving

3. Work out the value of the following by first finding an expression for the inverse function.

$f^{-1}(3)$ when $f(x) = 2 + \frac{1}{x}$ for $x > 0$

4.

Ready to Learn					

Fluency and understanding

Recap on long division

```
       3 8 .
    ___________________
11 | 4 2 3 . 0  0  0
     3 3
     ---
       9 3
       8 8
       ---
         5   0
```

1. We found how many whole number of times (i.e. the quotient) the divisor went into the dividend.

2. We multiplied the quotient by the dividend.

3. …in order to find the remainder.

4. We 'brought down' the next number.

Simplify algebraic fractions and introduction to dividing polynomials.

Recap on long division

$$\begin{array}{rl} & 38. \\ 11 & \overline{\big)\,423.0\ 0\ 0} \\ & \underline{33} \\ & 93 \\ & \underline{88} \\ & 5\ \ 0 \end{array}$$

1. We found how many whole number of times (i.e. the quotient) the divisor went into the dividend.

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3. …in order to find the remainder.

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Worked Examples - I do

Evaluate $1026 \div 3$

Worked Examples - We do

Evaluate $7182 \div 42$

Build it

Independent Practice- You do

1) Use long division to evaluate
197041 ÷ 23

2) Use long division to evaluate
56168 ÷ 34

Show Me

Fluency and understanding

- Introduction to diving polynomials

How many times does x go into $6x^3$?

Now repeat! How many times does x go into $-2x^2$?

$$
\begin{array}{r|l}
 & 6x^2 \;-\; 2x \;+3 \\ \hline
x+5 & 6x^3 + 28x^2 - 7x + 15 \\
 & \underline{6x^3 + 30x^2} \\
 & \quad -2x^2 - 7x \\
 & \quad \underline{-2x^2 - 10x} \\
 & \qquad\qquad 3x + 15 \\
 & \qquad\qquad \underline{3x + 15} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Multiply $6x^2$ by $(x+5)$. The first term should match with above.

Subtract and carry down next term.

This is the remainder.

Tip:
You can check your solution by expanding:
$(x+5)(6x^2-2x+3)$
But if you know you should get no remainder, ending with 0 at the bottom is a good sign!

Examples - I do

Find the remainder when $3x^3 - 2x + 4$ is divided by $x - 1$.

$$
\begin{array}{r|l}
 & 3x^2 + 3x + 1 \\
\hline
x - 1 & 3x^3 + 0x^2 - 2x + 4 \\
 & 3x^3 - 3x^2 \\
\cline{2-2}
 & \qquad 3x^2 - 2x \\
 & \qquad 3x^2 - 3x \\
\cline{2-2}
 & \qquad\qquad x + 4 \\
 & \qquad\qquad x - 1 \\
\cline{2-2}
 & \qquad\qquad\quad 5
\end{array}
$$

The remainder is 5.

Independent Practice- You do

Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $x - 4$.

$$
\begin{array}{r|l}
 & 2x^2 \quad +3x \quad -4 \\
\hline
x - 4 & 2x^3 - 5x^2 - 16x + 10 \\
 & \underline{2x^3 - 8x^2} \downarrow \qquad \downarrow \\
 & \qquad 3x^2 - 16x \\
 & \qquad \underline{3x^2 - 12x} \\
 & \qquad\qquad -4x + 10 \\
 & \qquad\qquad \underline{-4x + 16} \\
 & \qquad\qquad\qquad -6
\end{array}
$$

The remainder is -6.

Show Me

Independent Practice- You do

Divide $8x^3 - 1$ by $2x - 1$.

$$
\begin{array}{r|l}
 & 4x^2 + 2x \quad + 1 \\
\hline
2x - 1 & 8x^3 + 0x^2 + 0x - 1 \\
 & \underline{8x^3 - 4x^2} \quad \downarrow \qquad \downarrow \\
 & \qquad 4x^2 + 0x \\
 & \qquad \underline{4x^2 - 2x} \qquad \downarrow \\
 & \qquad\qquad 2x - 1 \\
 & \qquad\qquad \underline{2x - 1} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Show Me

Fluency and understanding

The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

We can see that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.
What would happen if x is 2?

$2 - 2 = 0$ so the RHS, and hence LHS would be 0.
The converse is also true: if we could find a value a such that the LHS is 0 when we substitute in a for x, then $(x - a)$ would be a factor.

The Factor Theorem states that if $f(x)$ is a polynomial then:

- If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$.
- Conversely, if $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$.

Examples - I do

Show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$ by:

a) algebraic division

b) the factor theorem

Let $f(x) = x^3 + x^2 - 4x - 4$

$f(2) = 2^3 + 2^2 - 4(2) - 4 = 0$

$\therefore$ $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.

Examples - I do

Fully factorise $2x^3 + x^2 - 18x - 9$ hence sketch the graph of y = $2x^3 + x^2 - 18x - 9$

Keep on trying values until you find one where $f(p) = 0$

Let $f(x) = 2x^3 + x^2 - 18x - 9$

$f(1) = 2(1)^3 + 1^2 - 18(1) - 9 = -24$

$f(-1) = 2(-1)^3 + (-1)^2 + 18(-1) - 9 = 24$

...

$f(3) = 0 \quad \therefore \quad (x - 3)$ is a factor.

Using algebraic division we find that:

$$2x^3 + x^2 - 18x - 9 = (x - 3)(2x^2 + 7x + 3)$$
$$= (x - 3)(2x + 1)(x + 3)$$

Examples - I do

Using Factor Theorem to find unknown coefficients

Given that $2x + 1$ is a factor of $6x^3 + ax^2 + 1$, determine the value of a.

$$f(x) = 6x^3 + ax^2 + 1$$

$$f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 + 1$$

$$= -\frac{3}{4} + \frac{1}{4}a + 1 = 0$$

$$\frac{1}{4}a = -\frac{1}{4}$$

$$a = -1$$

Independent Practice- You do

Edexcel C2 May 2016 Q2

$$f(x) = 6x^3 + 13x^2 - 4$$

No long in spec.

~~(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$. (2)~~

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. **(2)**

(c) Factorise $f(x)$ completely. **(4)**

?

Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, determine the value of a.

?

Show Me

Plenary

Textbook

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Exercise 4c

Q1. Odd number

Q2. Odd numbers

Q3. Odd numbers

Q4.

Q5.

Q6.

Q8.

Challenge: Q7 (not compulsory)

Evaluate