## LO: Dividing polynomials.

4	14/ I		•	· ·
1.	Work	out the	inverse	function

$$f(x) = 2x - 3$$

2. Work out the value of the following by first finding an expression for the inverse function.

$$f^{-1}(4)$$
 when  $f(x) = 5x - 1$ 

Prerequisite
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#### Prerequisite

Retrieval

**Problem Solving** 

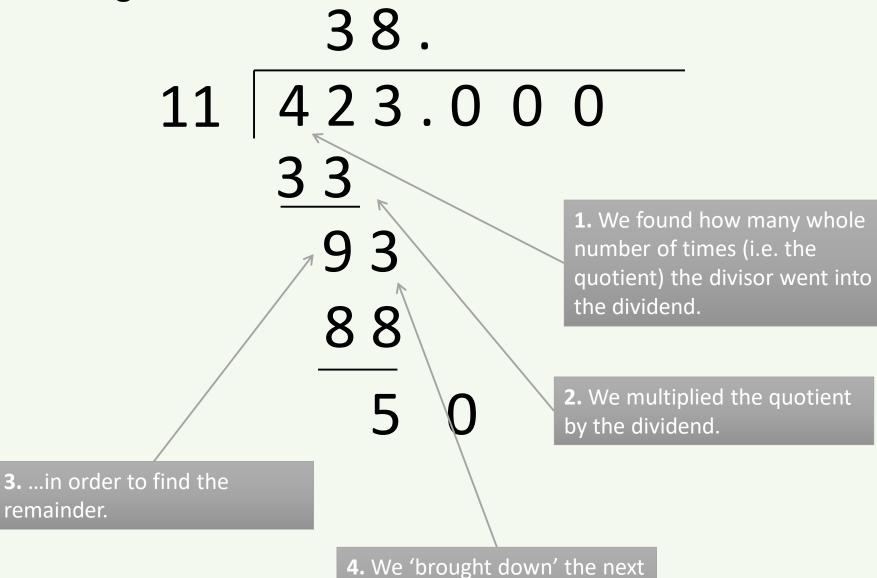
3. Work out the value of the following by first finding an expression for the inverse function.

$$f^{-1}(3)$$
 when  $f(x) = 2 + \frac{1}{x}$  for  $x > 0$ 

4.

### Fluency and understanding





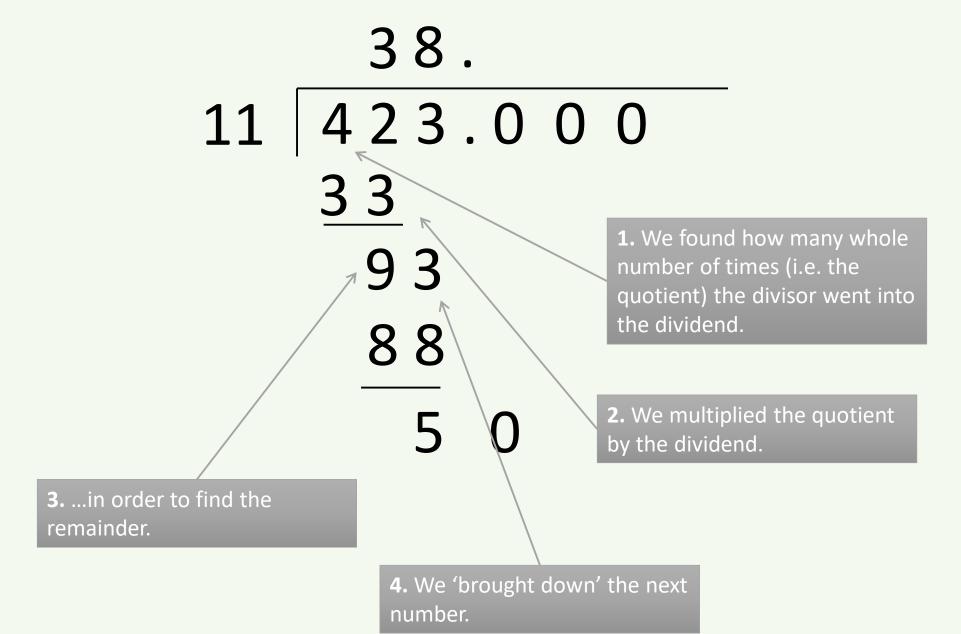
number.

The Big Picture

remainder.

#### Simplify algebraic fractions and introduction to dividing polynomials.

#### Recap on long division



Evaluate 1026 ÷ 3

Evaluate 7182 ÷ 42

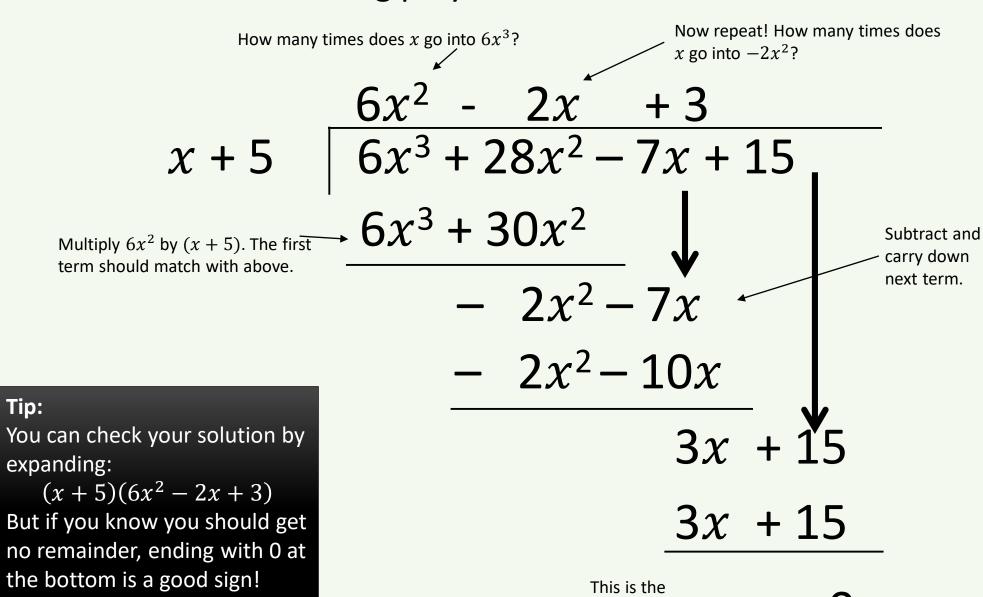
1) Use long division to evaluate 197041 ÷ 23

2) Use long division to evaluate 56168 ÷ 34

## Fluency and understanding

<del>remainder</del>

Introduction to diving polynomials



THE DISTIBLITIES

Find the remainder when  $3x^3 - 2x + 4$  is divided by x - 1.

$$\begin{array}{r}
3x^{2} + 3x + 1 \\
x - 1 \overline{\smash)3x^{3} + 0x^{2} - 2x + 4} \\
\underline{3x^{3} - 3x^{2}} \overline{\smash)3x^{2} - 2x} \\
\underline{3x^{2} - 2x} \\
\underline{3x^{2} - 3x} \\
x + 4 \\
\underline{x - 1}
\end{array}$$
The remainder is 5.

Find the remainder when  $2x^3 - 5x^2 - 16x + 10$  is divided by x - 4.

$$\begin{array}{r}
2x^{2} + 3x - 4 \\
x - 4 \overline{\smash)2x^{3} - 5x^{2} - 16x + 10} \\
\underline{2x^{3} - 8x^{2}} \\
3x^{2} - 16x \\
\underline{3x^{2} - 12x} \\
-4x + 10 \\
\underline{-4x + 16} \\
\end{array}$$
The remainder is -6.

Divide 
$$8x^3 - 1$$
 by  $2x - 1$ .

$$\begin{array}{r}
4x^{2} + 2x + 1 \\
2x - 1 \overline{\smash)8x^{3} + 0x^{2} + 0x - 1} \\
\underline{8x^{3} - 4x^{2}} \\
4x^{2} + 0x \\
\underline{4x^{2} + 0x} \\
4x^{2} - 2x
\end{array}$$

### Fluency and understanding

#### The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

We can see that (x - 2) is a factor of  $x^3 + x^2 - 4x - 4$ . What would happen if x is 2?

2-2=0 so the RHS, and hence LHS would be 0.

The converse is also true: if we could find a value a such that the LHS is 0 when we substitute in a for x, then (x-a) would be a factor.

The Factor Theorem states that if f(x) is a polynomial then:

- If f(p) = 0, then (x p) is a factor of f(x).
- Conversely, if (x p) is a factor of f(x), then f(p) = 0.

Show that (x-2) is a factor of  $x^3 + x^2 - 4x - 4$  by:

a) algebraic division

b) the factor theorem

Let 
$$f(x) = x^3 + x^2 - 4x - 4$$
  
 $f(2) = 2^3 + 2^2 - 4(2) - 4 = 0$   
 $\therefore (x - 2)$  is a factor of  $x^3 + x^2 - 4x - 4$ .

Fully factorise  $2x^3 + x^2 - 18x - 9$  hence sketch the graph of  $y = 2x^3 + x^2 - 18x - 9$ 

Keep on trying values until you find one where f(p) = 0

Let 
$$f(x) = 2x^3 + x^2 - 18x - 9$$
  
 $f(1) = 2(1)^3 + 1^2 - 18(1) - 9 = -24$   
 $f(-1) = 2(-1)^3 + (-1)^2 + 18(-1) - 9 = 24$   
...

 $f(3) = 0 \implies (x - 3)$  is a factor.

Using algebraic division we find that:
$$2x^3 + x^2 - 18x - 9 = (x - 3)(2x^2 + 7x + 3)$$

$$= (x - 3)(2x + 1)(x + 3)$$

**Using Factor Theorem to find unknown coefficients** 

Given that 2x + 1 is a factor of  $6x^3 + ax^2 + 1$ , determine the value of a.

$$f(x) = 6x^{3} + ax^{2} + 1$$

$$f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^{3} + a\left(-\frac{1}{2}\right)^{2} + 1$$

$$= -\frac{3}{4} + \frac{1}{4}a + 1 = 0$$

$$\frac{1}{4}a = -\frac{1}{4}$$

$$a = -1$$

#### Edexcel C2 May 2016 Q2

$$f(x) = 6x^3 + 13x^2 - 4$$

No long in spec.

- (a) Use the remainder theorem to find the remainder when f(x) is divided by (2x + 3). (2)
- (b) Use the factor theorem to show that (x + 2) is a factor of f(x).

(2)

(c) Factorise f(x) completely.

**(4)** 

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Given that 3x - 1 is a factor of  $3x^3 + 11x^2 + ax + 1$ , determine the value of a.

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# Plenary

# **Textbook**

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Exercise 4c

Q1. Odd number

Q2. Odd numbers

Q3. Odd numbers

Q4.

Q5.

Q6.

Q8.

Challenge: Q7 (not compulsory)