European Centre for Soft Computing

Genetic Fuzzy Systems

Fuzzy Knowledge Extraction by Evolutionary Algorithms

Oscar Cordón

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Outline

1. Brief introduction to genetic fuzzy systems
2. The birth of GFSs: 1991. GFSs roadmap and milestones
3. Evolutionary tuning of fuzzy rule-based systems
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What’s next?

OUTLINE

1. **Introduction to GFSs**
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- The use of genetic/evolutionary algorithms (GAs) to design fuzzy systems constitutes one of the branches of the Soft Computing paradigm: genetic fuzzy systems (GFSs)
- The most known approach is that of genetic fuzzy rule-based systems, where some components of a fuzzy rule-based system (FRBS) are derived (adapted or learnt) using a GA
- Some other approaches include genetic fuzzy neural networks and genetic fuzzy clustering, among others

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

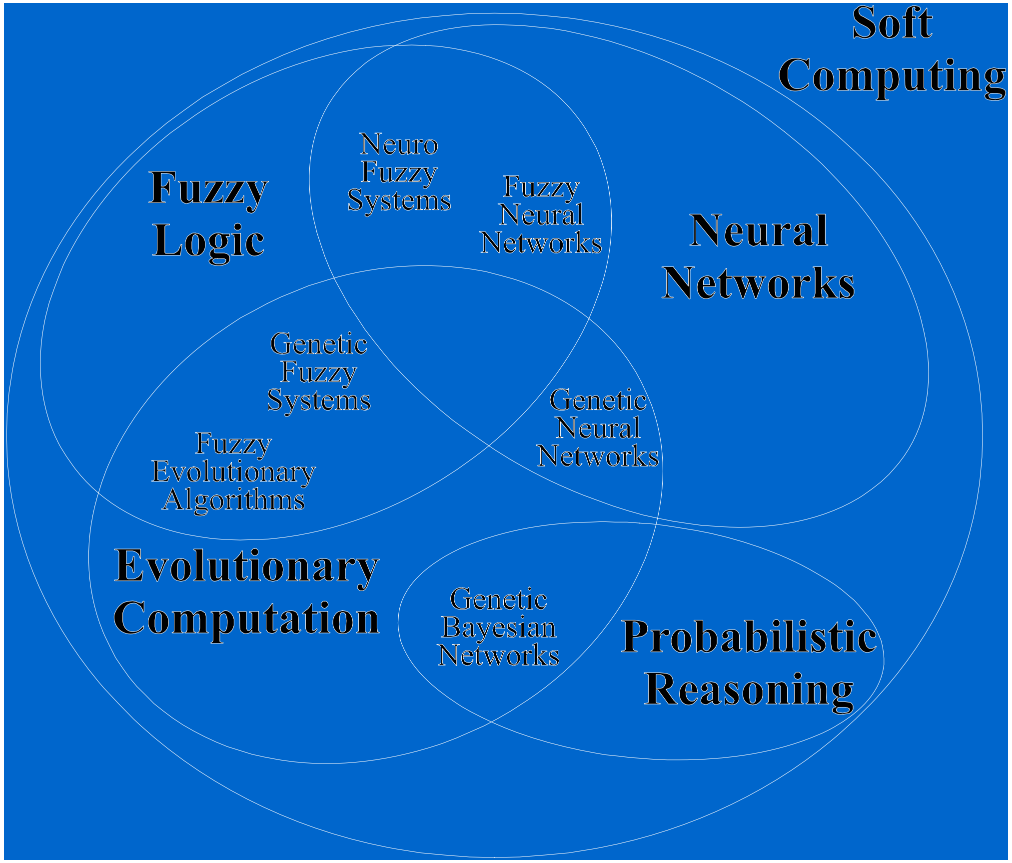
4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

GFSs and Soft Computing:



OUTLINE

1. Introduction to GFSs
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3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
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Evolutionary algorithms and machine learning:

- Evolutionary algorithms were not specifically designed as machine learning techniques, like other approaches like neural networks
- However, it is well known that a learning task can be modelled as an optimization problem, and thus solved through evolution
- Their powerful search in complex, ill-defined problem spaces has permitted applying evolutionary algorithms successfully to a huge variety of machine learning and knowledge discovery tasks
- Their flexibility and capability to incorporate existing knowledge are also very interesting characteristics for the problem solving.

OUTLINE

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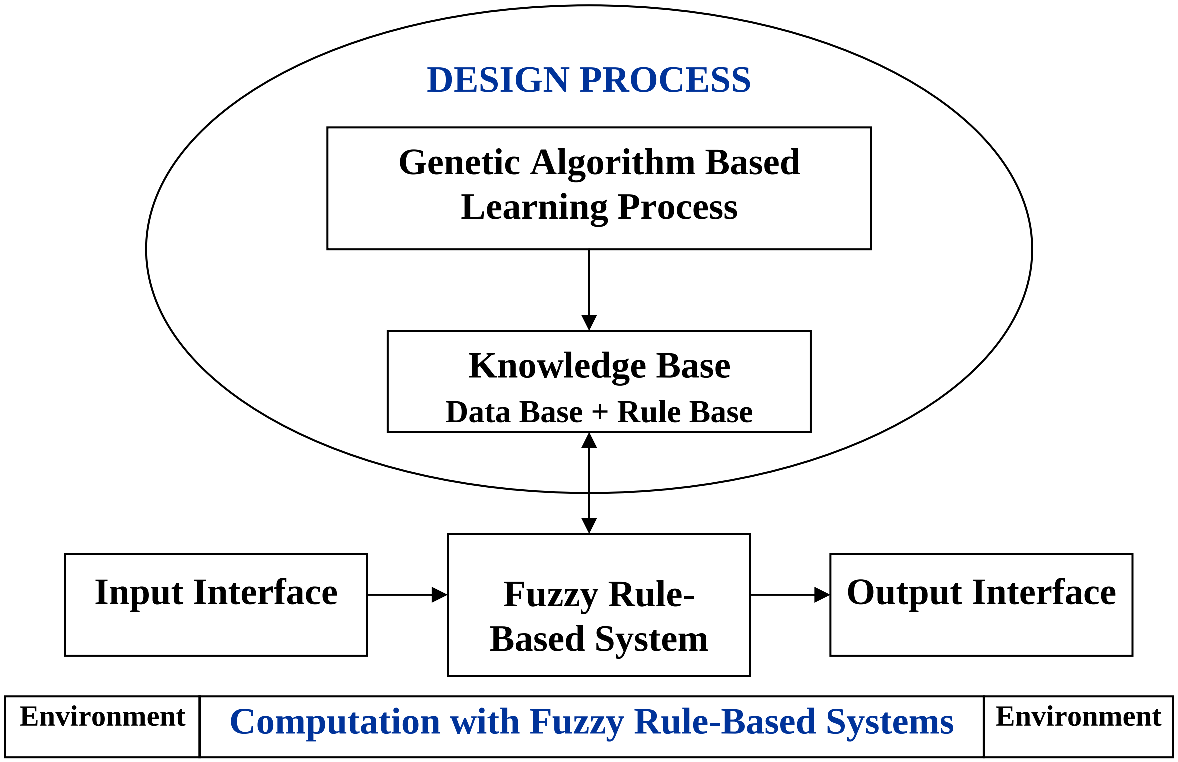
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Genetic Fuzzy Rule-Based Systems:



OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

Design of fuzzy rule-based systems:

- An FRBS (regardless it is a fuzzy model, a fuzzy logic controller or a fuzzy classifier), is comprised by two main components:
  - The Knowledge Base (KB), storing the available problem knowledge in the form of fuzzy rules
  - The Inference System, applying a fuzzy reasoning method on the inputs and the KB rules to give a system output

- Both must be designed to build an FRBS for a specific application:
  - The KB is obtained from expert knowledge or by machine learning methods
  - The Inference System is set up by choosing the fuzzy operator for each component (conjunction, implication, defuzzifier, etc.)

    Sometimes, the latter operators are also parametric and can be tuned using automatic methods

OUTLINE

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The KB design involves two subproblems, related to its two subcomponents:

- Definition of the Data Base (DB):
  - Variable universes of discourse
  - Scaling factors or functions
  - Granularity (number of linguistic terms/labels) per variable
  - Membership functions associated to the labels
- Derivation of the Rule Base (RB): fuzzy rule composition

As said, there are two different ways to design the KB:

- From human expert information
- By means of machine learning methods guided by the existing numerical information (fuzzy modeling and classification) or by a model of the system being controlled

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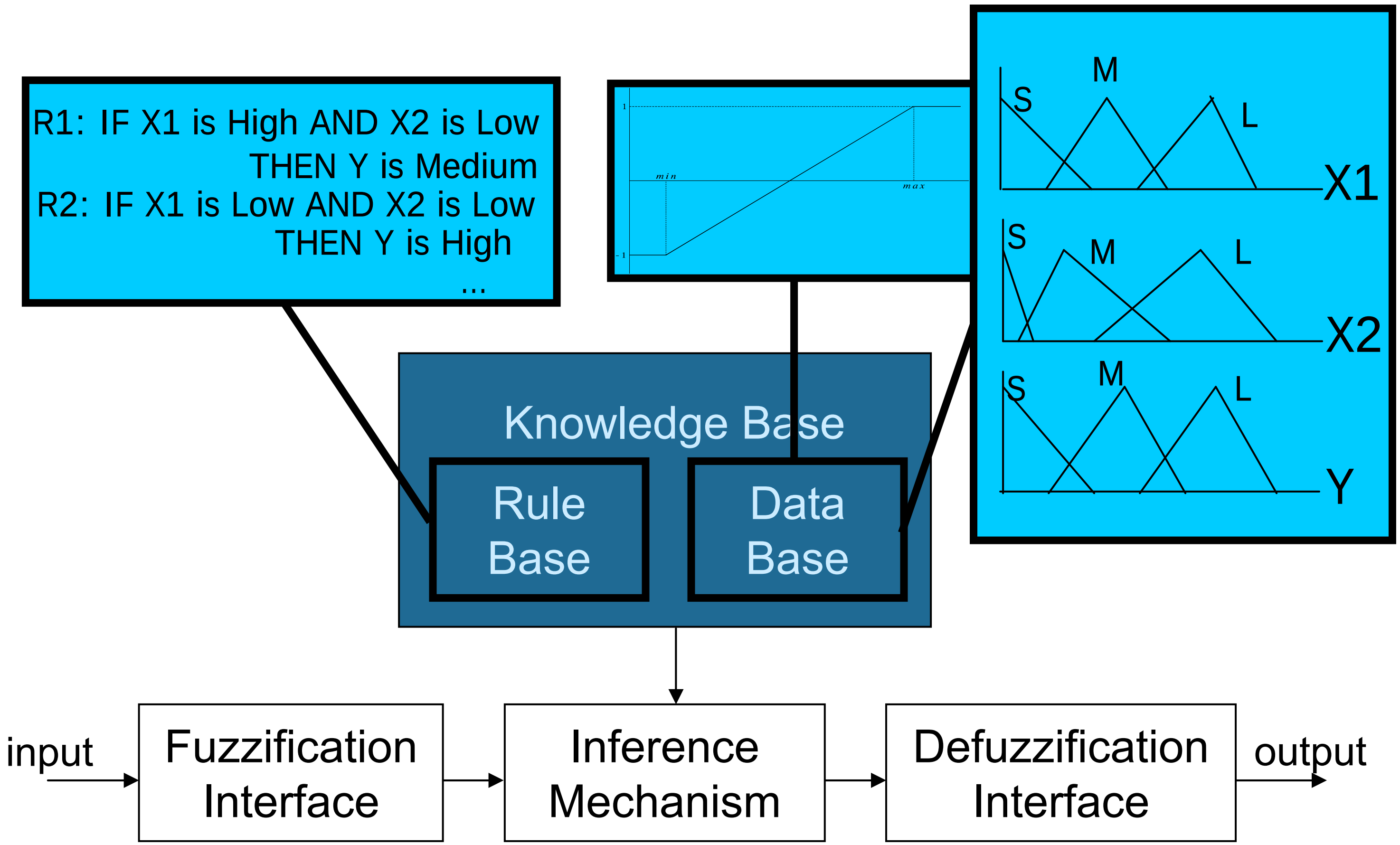
7. Conclusions. What's next?



Fuzzy rule-based system

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
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Classical Taxonomy of GFRBSs:

There are there different groups of GFRBSs according to the KB components, DB and/or RB, included in the learning process:

- Genetic definition of the FRBS Data Base
- Genetic derivation of the FRBS Rule Base
- Genetic derivation of the FRBS Knowledge Base

Additionally:

- Genetic design of the Inference Mechanism (less usual)

OUTLINE

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2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

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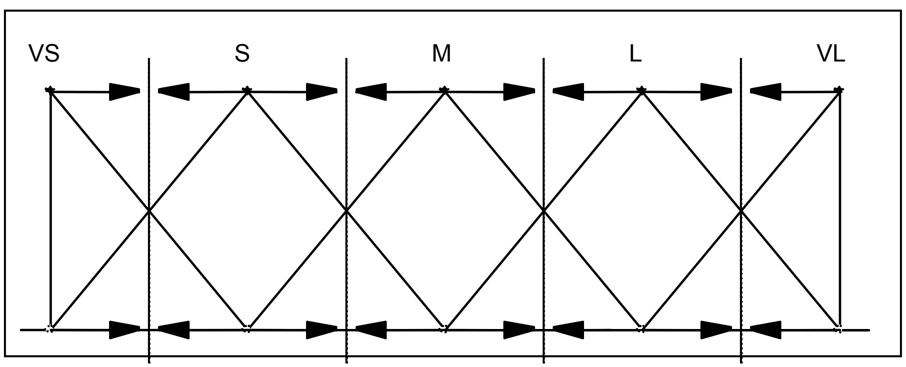
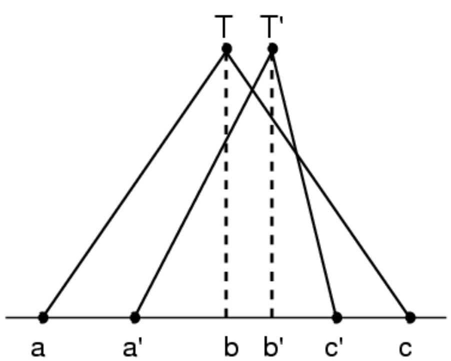
1. Genetic definition of the Data Base:

Classically:

- performed on a predefined DB definition
- tuning of the membership function shapes by a GA





OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
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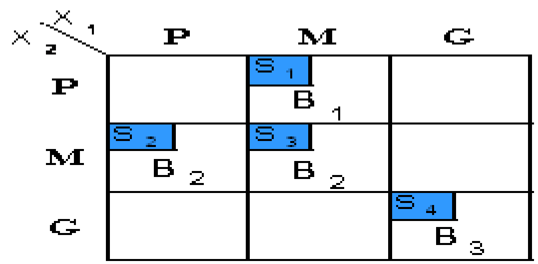
2. Genetic Derivation of the Rule Base:

- A predefined Data Base definition is assumed
- The fuzzy rules (usually Mamdani-type) are derived by a GA



Rule Base

R_1 = IF X_1 is **M** and X_2 is **P** THEN Y is $\mathbf{B}_1$

R_2 = IF X_1 is **P** and X_2 is **M** THEN Y is $\mathbf{B}_2$

R_3 = IF X_1 is **M** and X_2 is **M** THEN Y is $\mathbf{B}_2$

R_4 = IF X_1 is **G** and X_2 is **G** THEN Y is $\mathbf{B}_3$

OUTLINE

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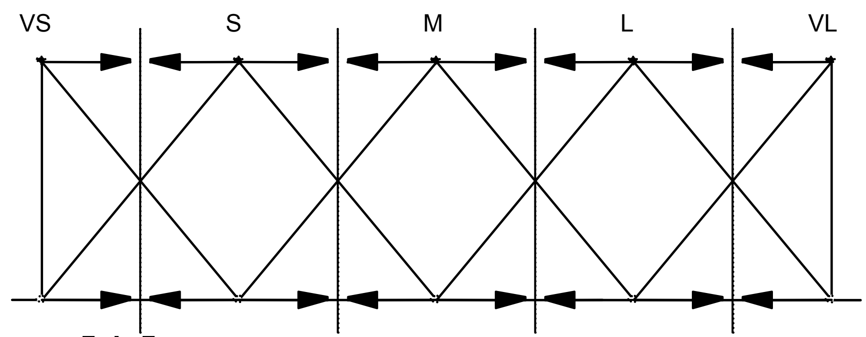
3. Genetic Derivation of the Knowledge Base:

- The simultaneous derivation properly addresses the strong dependency existing between the RB and the DB



Rule Base

R_1 = IF X_1 is **M** and X_2 is **P** THEN Y is $\mathbf{B_1}$

R_2 = IF X_1 is **P** and X_2 is **M** THEN Y is $\mathbf{B_2}$

R_3 = IF X_1 is **M** and X_2 is **M** THEN Y is $\mathbf{B_2}$

R_4 = IF X_1 is **G** and X_2 is **G** THEN Y is $\mathbf{B_3}$

1. Brief introduction to genetic fuzzy systems

OUTLINE

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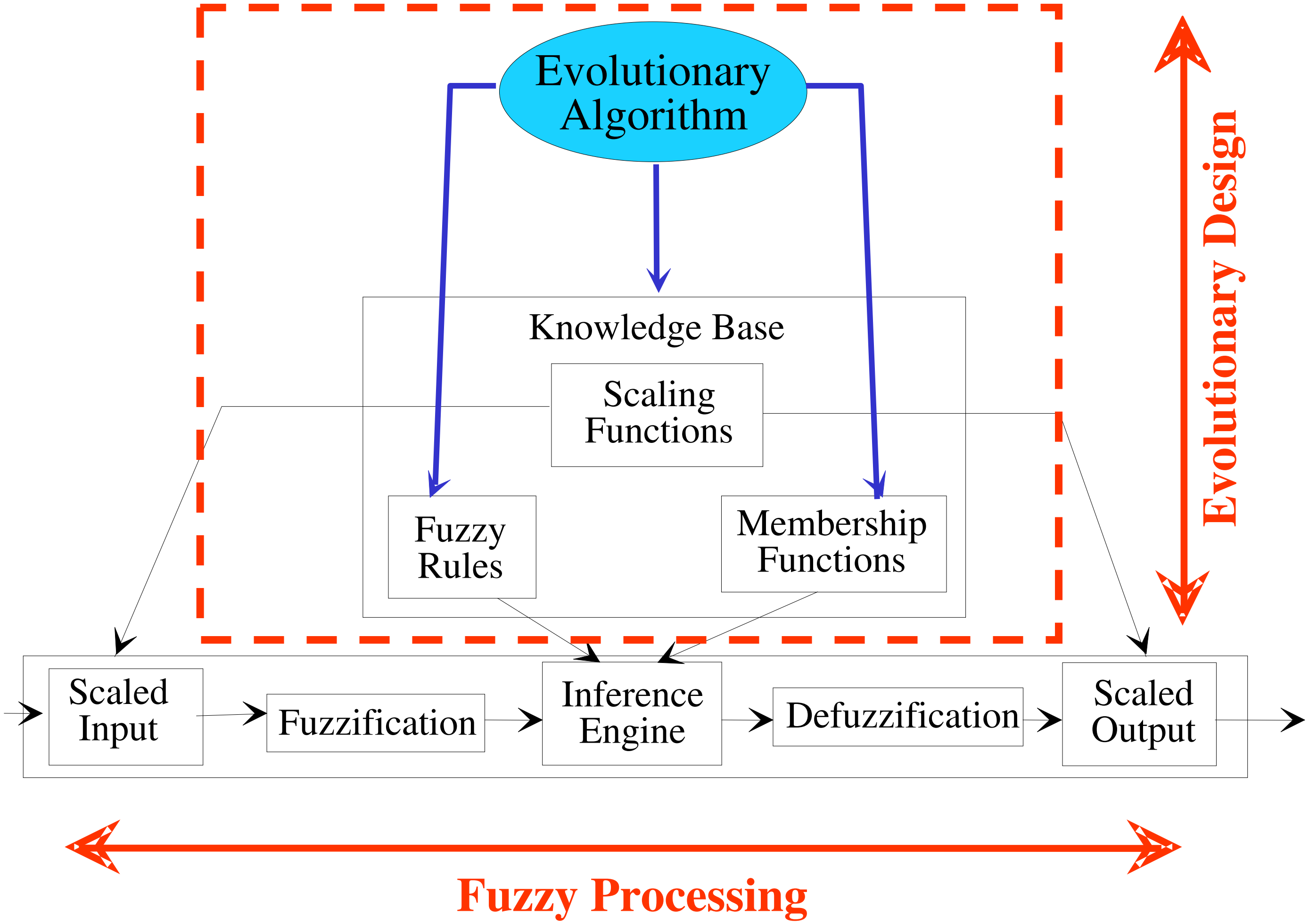
7. Conclusions. What's next?



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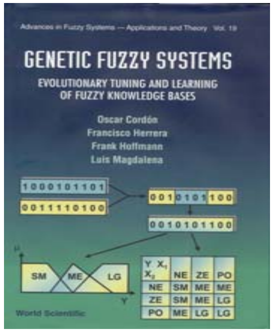

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- O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena, Ten Years of Genetic Fuzzy Systems: Current Framework and New Trends, FSS 141 (1) (2004) 5-31
- F. Hoffmann, Evolutionary Algorithms for Fuzzy Control System Design, Proceedings of the IEEE 89 (9) (2001) 1318-1333

2. The birth of GFSs: 1991

OUTLINE

1. Introduction to GFSs
2. **GFSs roadmap and milestones**
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6. Advanced GFS approaches
7. Conclusions. What's next?

- Thrift's ICGA91 paper (Mamdani-type Rule Base Learning. Pittsburgh approach)
- Valenzuela-Rendón's PPSN-I paper (Scatter Mamdani-type KB Learning. Michigan approach)
- Pham and Karaboga's Journal of Systems Engineering paper (Relational matrix-based FRBS learning. Pittsburgh approach)
- Karr's AI Expert paper (Mamdani-type Data Base Tuning)

Almost the whole basis of the area were established in the first year!

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1991-1996/7: INITIAL GFS SETTING: KB LEARNING:

- Establishment of the three classical learning approaches in the GFS field: Michigan, Pittsburgh, and IRL
- Different FRBS types: Mamdani, Mamdani DNF, Scatter Mamdani, TSK
- Generic applications: Classification, Modeling, and Control

1995-...: FUZZY SYSTEM TUNING:

- First: Membership function parameter tuning
- Later: other DB components adaptation: scaling factors, context adaptation (scaling functions), linguistic hedges, ...
- Recently: interpretability consideration

OUTLINE

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1998-...: APPROACHING TO MATURITY? NEW GFS LEARNING APPROACHES:

- New EAs: Bacterial genetics, DNA coding, Virus-EA, genetic local search (memetic algorithms), ...
- Hybrid learning approaches: a priori DB learning, GFNNs, Michigan-Pitt hybrids, ...
- Multiobjective evolutionary algorithms
- Interpretability-accuracy trade-off consideration
- Course of dimensionality (handling large data sets and complex problems):
  - Rule selection (1995-...)
  - Feature selection at global level and fuzzy rule level
  - Hierarchical fuzzy modeling
- "Incremental" learning

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GLOBAL GFS EVOLUTION SNAPSHOT:

From:	To:
Binary coding	Real coding
Simple/basic EAs	Sophisticated EAs
Accuracy-driven GFSs	Accuracy-interpretability trade-off in GFSs
Single-objective optimization	Multi-objective optimization
Strict GFRBS structures	Relaxed GFS structures: • Fuzzy logic for knowledge representation and reasoning • EAs for learning and tuning fuzzy models
Small data sets – simple problems	Large data sets (DM applications) and complex problems ??????

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- 1991: four pioneering papers
- 1995: Geyer-Schulz's book: "Fuzzy Rule-Based Expert Systems and Genetic Machine Learning". Physica-Verlag
  - First GFSs book. Very specific: fuzzy classifier systems (Michigan approach) and RB learning with genetic programming
- 1996: Herrera and Verdegay's edited book "Genetic Algorithms and Soft Computing". Physica-Verlag
- 1997:
  - Sanchez, Shibata and Zadeh's edited book "Genetic Algorithms and Fuzzy Logic Systems. Soft Computing Perspectives". World Scientific
  - Pedrycz's edited book "Fuzzy Evolutionary Computation". Kluwer
  - Herrera's special issue on "GFSs for Control and Robotics", IJAR 17:4
  - Herrera and Magdalena's two special sessions on "GFSs" at IFSA'97
- 1998: Herrera and Magdalena's special issue on "GFSs", IJIS 13:10-11

2. GFSs milestones

OUTLINE

1. Introduction to GFSs
2. **GFSs roadmap and milestones**
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- 2000: Cordón and Herrera's two special sessions on "GFSs: Issues and Applications" at IPMU'2000

- 2001:
  - Cordón-Herrera-Hoffmann-Magdalena's book on "GFSs. Evolutionary Tuning and Learning of Fuzzy Knowledge Bases", World Scientific

    First general GFSs book, covering the overall state of the art of GFSs by that time
  - 2001: Cordón-Herrera-Hoffmann-Magdalena's special issue on "Recent Advances in GFSs", Information Science 136:1-4
  - 2001: Cordón-Gomide-Herrera-Hoffmann-Magdalena's minitrack on "GFSs: New Developments" at Joint IFSA-NAFIPS

- 2002: Angelov's book "Evolving Rule-Based Models. A Tool for Design of Flexible Adaptive Systems". Physica-Verlag

- 2003: Carse-Pipe's two special sessions on "Evolutionary Fuzzy Systems" at EUSFLAT'2003

OUTLINE

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7. Conclusions. What's next?

- 2004: Cordón-Gomide-Herrera-Hoffmann-Magdalena's special issue on "GFSs: New Developments", FSS 141:1
  - Position paper from the editors: "Ten years of GFSs: current framework and new trends". 35 citations (June 2007)
- 2005:
  - Carse-Casillas-Pipe's three special sessions on "Evolutionary Fuzzy Systems: Models and Applications" at EUSFLAT'2005
  - Ishibuchi-Nakashima-Nii's book on "Classification And Modeling With Linguistic Information Granules: Advanced Approaches To Linguistic Data Mining", Springer
  - **First International Workshop on GFSs. Granada (Spain)**
- 2006-...:
  - 2 workshops (EFS'06, GEFS'08)
  - 4 special issues (IJAR, IJIS, IEEE TFS, Soft Computing)
  - 6 special sessions (WCCI'06, FuzzIEEE2007), ...

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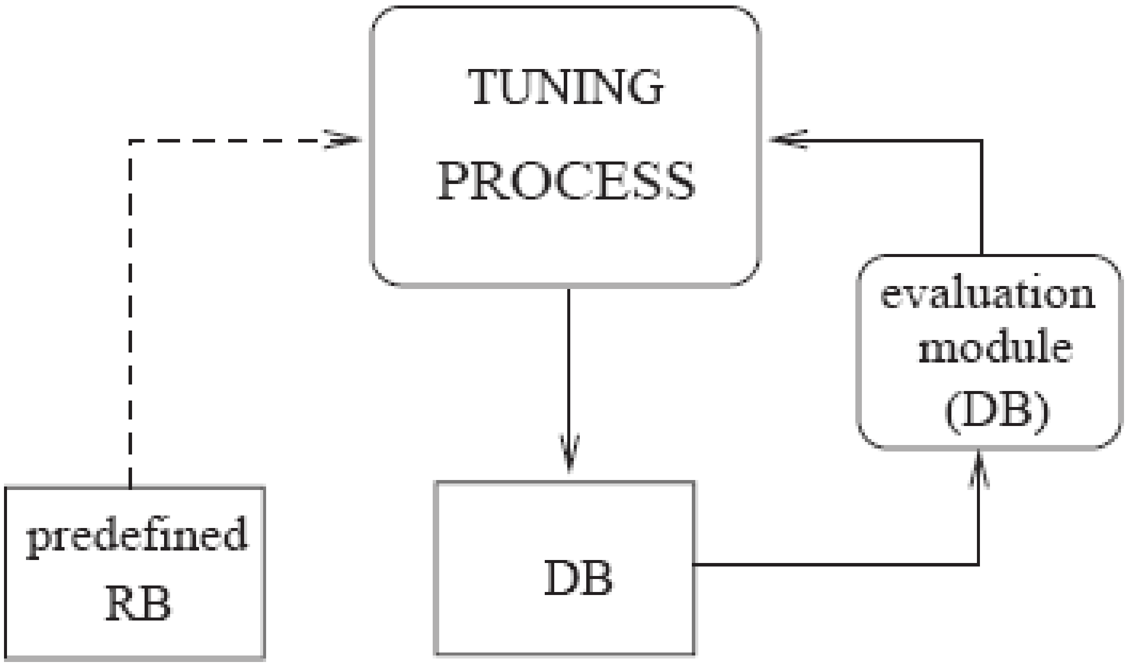
6. Advanced GFS approaches
7. Conclusions. What's next?



Evolutionary Data Base Tuning

1. Tuning of scaling functions
2. Tuning of membership functions

OUTLINE

1. Introduction to GFSs
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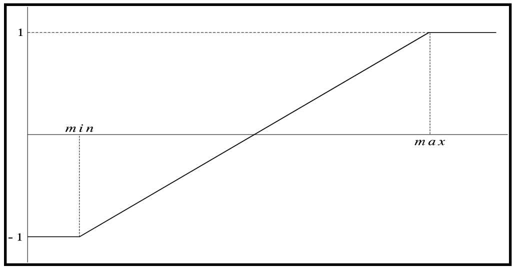
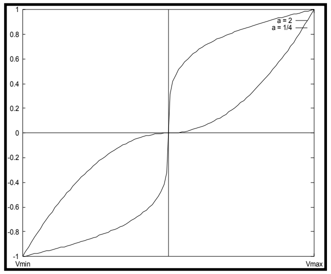
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

1. Tuning of scaling functions

- They apply the universes of discourse of the input and output variables to the domain where the fuzzy sets are defined
- Their adaptation allows the scaled universe of discourse to match the variable range in a better way



- Definition parameters:
  - Scaling factor
  - Upper and lower bounds (linear scaling function)
  - Contraction/dilation parameters (non linear scaling function)



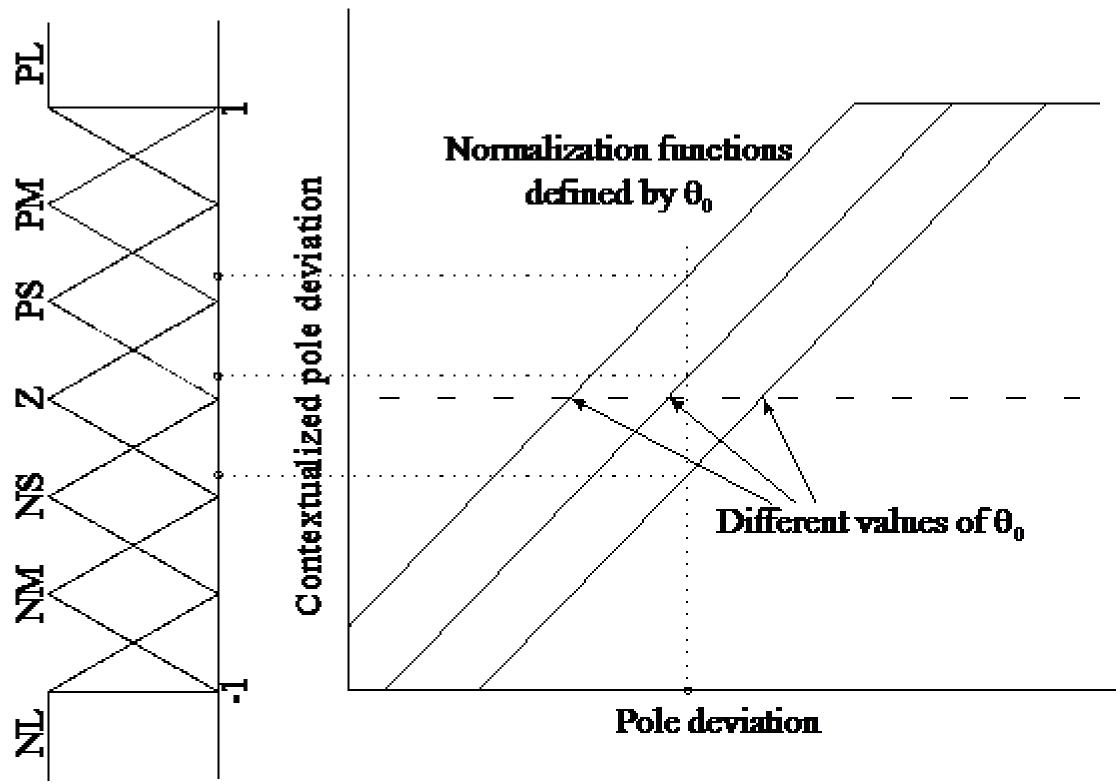
- Coding scheme: fixed length real-coded chromosome

3. Evolutionary Tuning of FRBSs

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. **Evolutionary tuning of FRBSs**
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Especially useful for fuzzy control applications, where the scaling function represents the gain from a control viewpoint



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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. **Evolutionary tuning of FRBSs**
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- L. Magdalena, F. Monasterio, A fuzzy logic controller with learning through the evolution of its knowledge base, Intl. J. Approx. Reasoning 16 (3–4) (1997) 335–358
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- L. Magdalena, Adapting the gain of an FLC with genetic algorithms, Intl. J. Approximate Reasoning 17 (4) (1997) 327–349

OUTLINE

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2. GFSs roadmap and milestones
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5. Some real-world applications
6. Advanced GFS approaches
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2. Tuning of membership functions

- A genetic tuning process that slightly adjusts the shapes of the membership functions of a preliminary DB definition
- Each chromosome encodes a whole DB definition by joining the partial coding of the different membership functions involved
- The coding scheme depends on:
  - The kind of membership function considered (triangular, trapezoidal, bell-shaped, ...) $\rightarrow$ different real-coded definition parameters
  - The kind of FRBS:
    - Grid-based: Each linguistic term in the fuzzy partition has a single fuzzy set definition associated
    - Non grid-based (free semantics, scatter partitions, fuzzy graphs): each variable in each rule has a different membership function definition

OUTLINE

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6. Advanced GFS approaches
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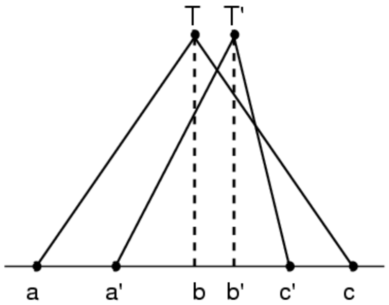
- Example: Tuning of the triangular membership functions of a grid-based SISO Mamdani-type FRBS, with three linguistic terms for each variable fuzzy partition

- Each chrosome encodes a different DB definition:
  - 2 (variables) · 3 (linguistic labels) = 6 membership functions
  - Each triangular membership function is encoded by 3 real values (the three definition points):
  - So, the chromosome length is 6 · 3 = 18 real-coded genes (binary coding can be used but but is not desirable)



- Either definition intervals have to be defined for each gene and/or appropriate genetic operators in order to obtain meaningful membership functions

3. Evolutionary Tuning of FRBSs

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
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5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

-0.5	0	0.5	**0**	**0.5**	**1**	0.5	1	1.5	-0.5	0	0.5	0	0.5	1	0.5	1	1.5

-0.5	0	0.5	0.25	0.5	0.75	0.5	1	1.5	-0.35	0	0.35	**0.3**	**0.5**	**0.7**	0.5	1	1.5

The RB remains unchanged!

R1: IF X1 is Small THEN Y is Large
R2: IF X1 is Medium THEN Y is Medium
. . .

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
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4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- P.P. Bonissone, P.S. Khedkar, Y. Chen, Genetic algorithms for automated tuning of fuzzy controllers: a transportation application, in Proc. Fifth IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE'96), New Orleans, USA, 1996, pp. 674–680
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- H.B. Gurocak, A genetic-algorithm-based method for tuning fuzzy logic controllers, FSS 108 (1) (1999) 39–47
- O. Cordón, F. Herrera, A two-stage evolutionary process for designing TSK fuzzy rule-based systems, IEEE TSMC 29 (6) (1999) 703–715

4. Classical GFS learning approaches

OUTLINE

1. Introduction to GFSs
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5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Genetic derivation of the FRBS Rule Base
  - Michigan learning approach
  - Pittsburgh learning approach
  - Iterative Rule learning approach
  - Fuzzy rule coding
  - Examples
- Genetic selection of fuzzy rule sets
- Genetic derivation of the FRBS Knowledge Base
  - Single-stage GFSs
  - Multi-stage GFSs

OUTLINE

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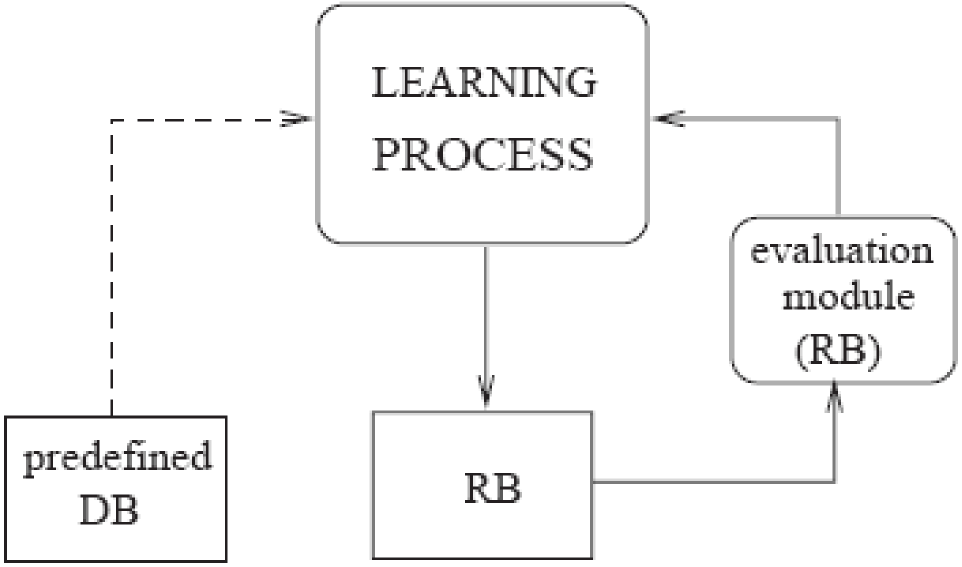
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

1. Genetic derivation of the FRBS Rule Base



- The genetic learning of the RB assumes the existence of a predefined DB definition and looks for an optimal fuzzy rule set
- It only deals with grid-based Mamdani-type FRBSs

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
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5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Michigan Learning Approach:

- Each chromosome encodes a single fuzzy rule and the derived RB is composed of the whole population
- Reinforcement mechanisms (reward (credit apportion) and weight penalization) are considered to adapt the rules through a GA
- Low weight (bad performing) rules are substituted by new rules generated by the GA
- The key question is to induce collaboration in the derived RB as the evaluation procedure is at single rule level (cooperation vs. competition problem (CCP))
- Mainly used in on-line learning (fuzzy control applications)

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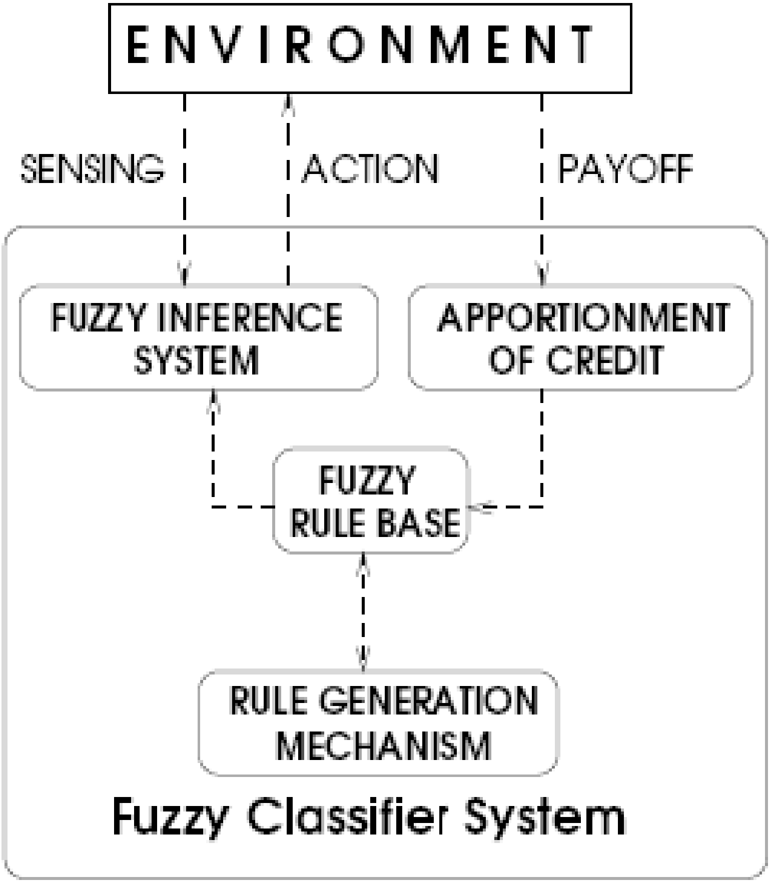
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Michigan Learning Approach:



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Pittsburgh Learning Approach:

- Each chromosome encodes a whole fuzzy rule set and the derived RB is the best individual of the last population
- The fitness function evaluates the performance at the complete RB level, so the CCP is easy to solve
- However, the search space is huge, thus making difficult the problem solving and requiring sophisticated GFS designs
- Mainly used in off-line learning (fuzzy modeling and classification applications)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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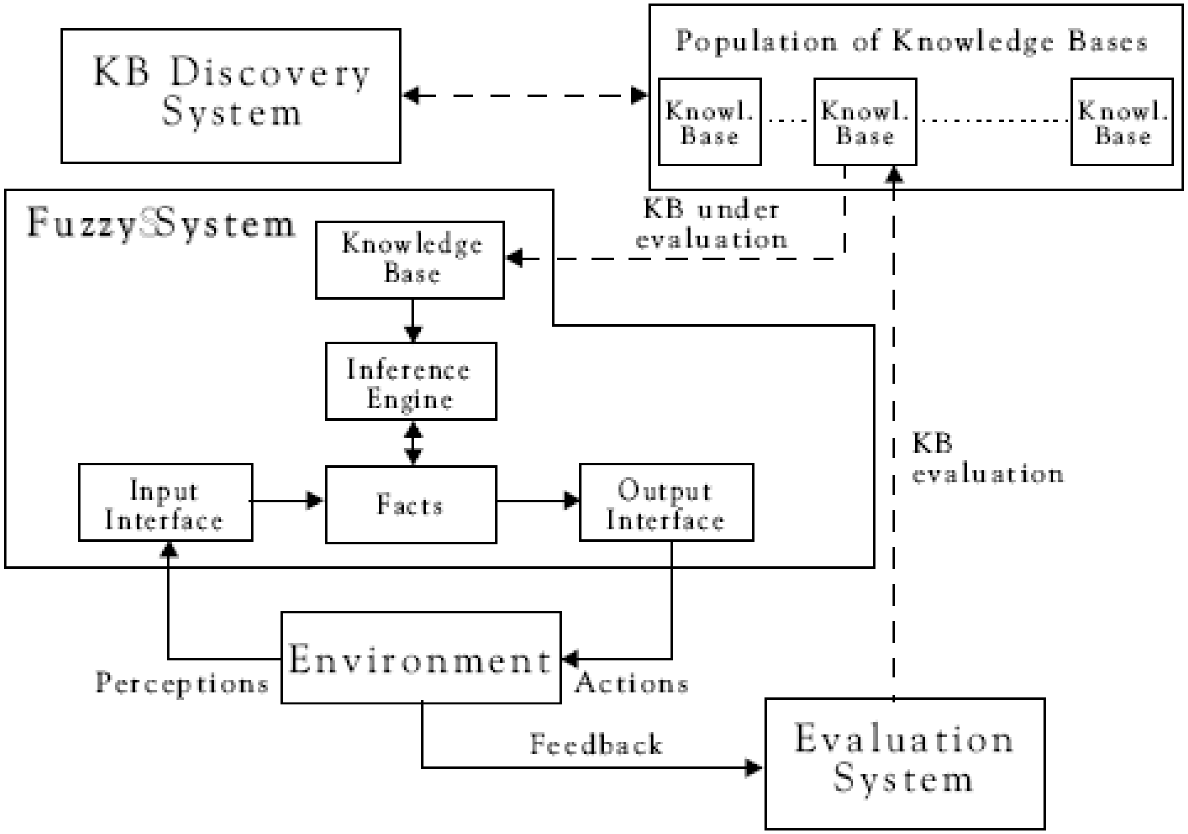


OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Iterative Rule Learning Approach:

- Intermediate approach between the Michigan and Pittsburgh ones, based on partitioning the learning problem into several stages and leading to the design of multi-stage GFSs

- As in the Michigan approach, each chromosome encodes a single rule, but a new rule is learnt by an iterative fuzzy rule generation stage and added to the derived RB, in an iterative fashion, in independent and successive runs of the GA

- The evolution is guided by data covering criteria (rule competition). Some of them are considered to penalize the generation of rules covering examples already covered by the previously generated fuzzy rules (soft cooperation)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Iterative Rule Learning Approach:

- A second post-processing stage is considered to refine the derived RB by selecting the most cooperative rule set and/or tuning the membership functions (cooperation induction)

- Hence, the CCP is solved taking the advantages of both the Michigan and Pittsburgh approaches (small search space and good chances to induce cooperation)

- Mainly used in off-line learning (fuzzy modeling and classification applications). Not applicable for fuzzy control

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Fuzzy rule coding:

- The RB can be represented as a relational matrix, a decision table, or a list of rules
- The two former ones are only useful when the FRBS has a reduced number of variables (huge chromosomes with more than two or three input variables)
- The list of rules is the most used representation and can be adapted to the three classical genetic learning approaches: Michigan, Pittsburgh and IRL

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Thrift's GFS:

P. Thrift, Fuzzy logic synthesis with genetic algorithms, Proc. Fourth Intl. Conf. on Genetic Algorithms (ICGA'91), San Diego, USA, 1991, pp. 509–513

- Classical approach: Pittsburgh – the decision table is encoded in a rule consequent array
- The output variable linguistic terms are numbered from 1 to n and comprise the array values. The value 0 represents the rule absence, thus making the GA able to learn the optimal number of rules
- The ordered structure allows the GA to use simple genetic operators

X_2 \ X_1	S	M	L
S	R_1 B	R_2 —	R_3 M
M	R_4 —	R_5 M	R_6 —
L	R_7 M	R_8 —	R_9 A

1 2 3

Y → {B, M, A}

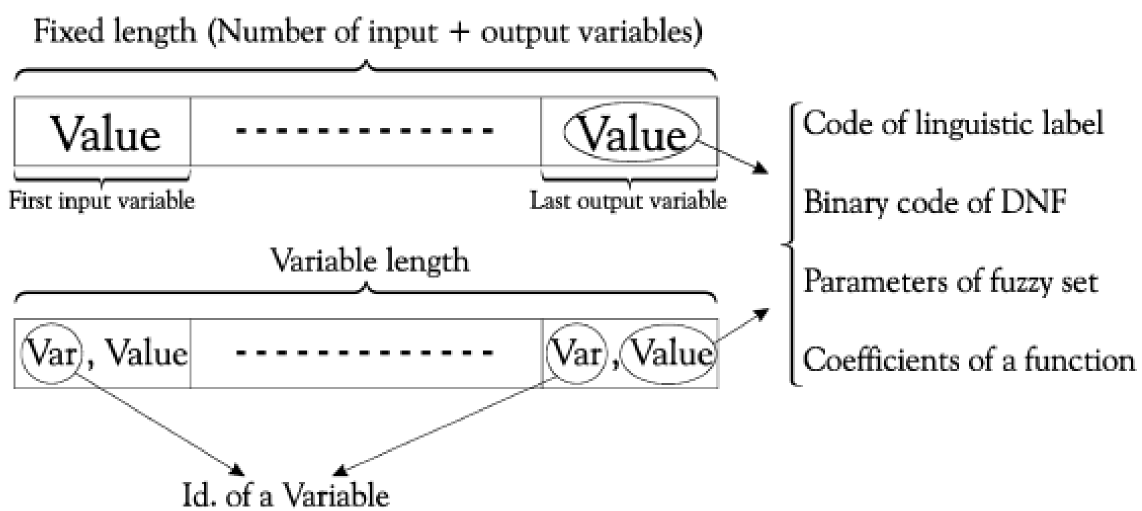
1	0	2	0	2	0	2	0	3

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Coding by a list of fuzzy rules:

- The problem of Thrift's decision table coding scheme is that it is difficult to reduce the RB size by only using the null value
- A good solution is to consider the list of rules representation, where each rule is individually coded and then the partial encodings are concatenated (Pittsburgh approach)



4. Classical GFS learning approaches

OUTLINE

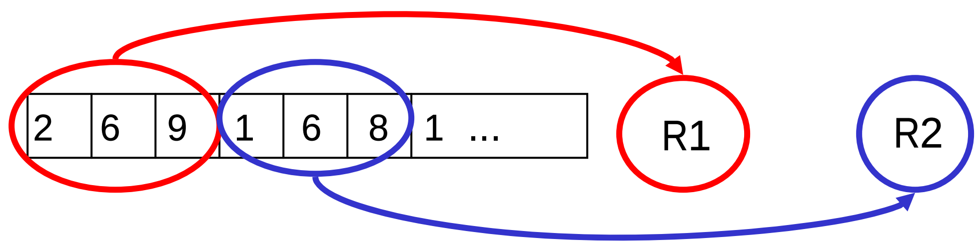
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Example: Two inputs-one output fuzzy control problem with an existing DB definition:

Error → {N, Z, P} (1 2 3)	ΔError → {N, Z, P} (4 5 6)	Power → {S, M, L} (7 8 9)

2	6	9

R1: IF **Error** is *Zero* (2) and **ΔError** is *Positive* (6) THEN **Power** is *Large* (9)



Permutation of clauses results in the same rule!

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Often the number of rules in the list is variable (having in some cases an upper limit)
- Other chance is to use variable-length chromosomes: the population encode RBs with different number of rules
- The problem anyway is that the genetic operators are more complicated since no rule ordering happens in the coding
- Other chance is that the individual contains the code of a single rule (Michigan and IRL approaches)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- A common approach to code individual rules is the use of the disjunctive normal form (DNF) represented in the form of a fixed length binary string

- A DNF fuzzy rule allows an antecedent variable to take a disjunction of linguistic terms from its domain as a value:

IF Femur_length is (medium or big-medium or big) and Head_diameter is (medium) and Foetus_sex is (male or female or unknown) THEN Foetus_weight is normal

0	1	1	1	0	1	0	1	1	1	0	1	1

Femur_length ={small,small-medium, medium, big-medium,big}

Head_diameter ={small,medium,big}

Foetus_sex ={male,female,unknown}

Foetus_weight = {low, normal, high}

4. Classical GFS learning approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- They carry some advantages such as the variable selection at rule level:

0	1	1	1	0	1	0	~~1~~	~~1~~	~~1~~	0	1	1

IF Femur_length is (medium or big-medium or big) and Head_diameter is (medium) and ~~Foetus_sex is (male or female or unknown)~~ THEN Foetus_weight is normal

or the label groupings making the rules more interpretable:

IF Femur_length is (**not small**) and Head_diameter is (medium) THEN Foetus_weight is normal

They are thus usually considered for classification problems

- DNF rules have also been derived when working with variable length codes based on messy GAs

OUTLINE

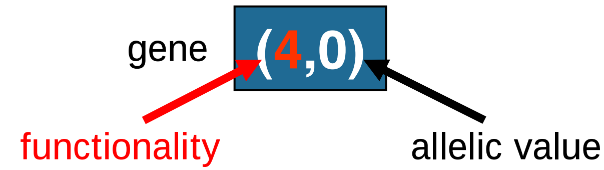
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Hoffmann-Pfister's GFS:

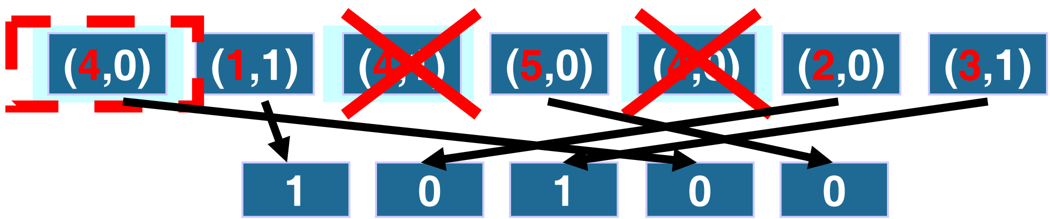
F. Hoffmann, G. Pfister, Evolutionary design of a fuzzy knowledge base for a mobile robot, IJAR 17 (4) (1997) 447–469

- Variable-length Pittsburgh GA to learn DNF fuzzy rules with the list of rules representation

- Messy GAs:



  - position independent encoding
  - gene functionality defined by additional enumeration
  - variable length chromosome
  - Genetic crossover → cut and splice
  - Over- and under-specification



4. Classical GFS learning approaches

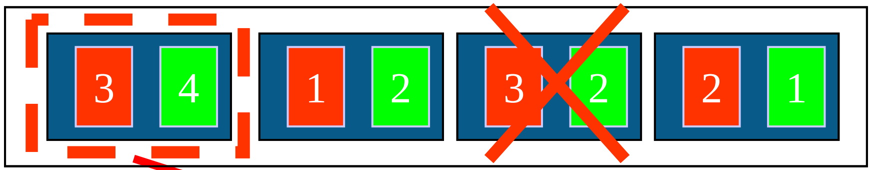
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Fuzzy rule over-specification:

- Multiple output terms:

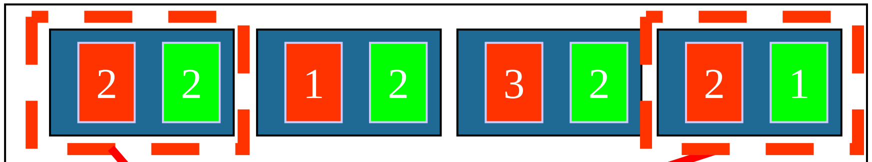
  Positional dominance



IF X1 is medium and X2 is small THEN Y is large

- Multiple input terms:

  Or-combination of terms for the same variable



IF X1 is medium and X2 is (small or medium) THEN Y is large

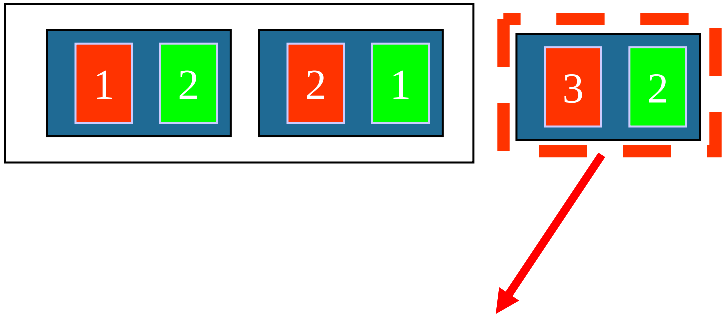
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Fuzzy rule under-specification:

- Missing output term:

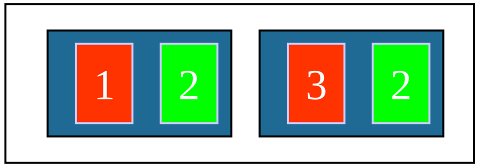
  Randomly generate output clause



IF X1 is medium and X2 is small THEN Y is small

- Missing input variable:

  DNF rule variable selection



IF X1 is medium THEN Y is small

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- A. Bonarini, Evolutionary learning of fuzzy rules: competition and cooperation, In: W. Pedrycz (Ed.): Fuzzy Modelling: Paradigms and Practice, Kluwer, 2006, 265-284 (Mamdani fuzzy rules for mobile robotics).
- M. Valenzuela-Rendón, Reinforcement learning in the fuzzy classifier system, Expert Systems with Applications 14 (1998) 237-247 (scatter Mamdani fuzzy rules for control/modeling problems)
- J.R. Velasco, Genetic-based on-line learning for fuzzy process control, IJIS 13 (10–11) (1998) 891–903 (scatter Mamdani fuzzy rules for control problems)
- H. Ishibuchi, T. Nakashima, T. Murata, Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems, IEEE TSMC 29 (1999) 601–618 (Mamdani fuzzy rules for classification problems)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- D.T. Pham, D. Karaboga, Optimum design of fuzzy logic controllers using genetic algorithms, J. System Eng. 1 (1991) 114–118 (relational matrix)
- F. Hoffmann, G. Pfister, Evolutionary design of a fuzzy knowledge base for a mobile robot, IJAR 17 (4) (1997) 447–469 (list of DNF fuzzy rules)
- L. Magdalena, Crossing unordered sets of rules in evolutionary fuzzy controllers, IJIS 13 (10-11) (1998) 993-1010 (list of Mamdani fuzzy rules)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- A. González, R. Pérez, A fuzzy theory refinement algorithm, IJAR 19 (1998) 193-200 (DNF fuzzy rules for classification and control problems)
- A. González, R. Pérez, SLAVE: a genetic learning system based on an iterative approach, IEEE TFS 7 (2) (1999) 176–191 (DNF fuzzy rules for classification problems)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

2. Genetic selection of fuzzy rule sets

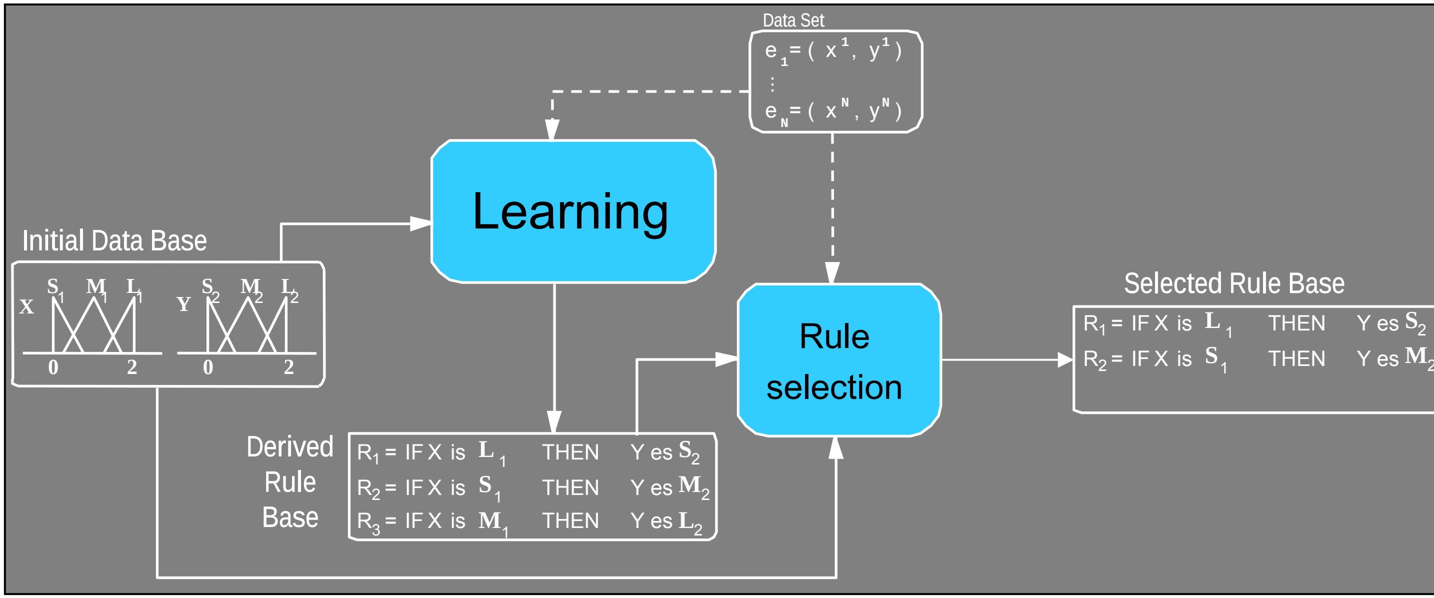
MOTIVATION:

- In high-dimensional problems, the number of rules in the RB grows exponentially as more inputs are added
- Hence, a fuzzy rule generation method is likely to derive fuzzy rule sets including:
  - redundant rules: whose actions are covered by other rules,
  - wrong rules: badly defined and perturbing the system performance, and
  - conflicting rules: that worsen the system performance when co-existing with other rules in the RB
- Rule reduction methods are used as postprocessing techniques to solve the latter problems

4. Classical GFS learning approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?



There are two different rule reduction approaches:

- Combination of the membership functions of two or more rules, reducing them to a single ones (scatter partition FRBSs)
- Selection of fuzzy rules, getting rule subsets with a good cooperation from the initial RB (descriptive and scatter FRBSs)

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4. Classical GFS learning approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Example: Binary GA for rule selection

- The coding scheme considers binary strings of fixed length m (number of rules of the initial RB):

  Allele '0' $\Rightarrow$ the corresponding rule IS NOT selected

  Allele '1' $\Rightarrow$ the corresponding rule IS NOT selected

- Initial population generation:

$$C_1^1[k] = 1, \forall k \{1, \ldots, m\}$$
$$C_1^p[k] = 0, \forall k \{1, \ldots, m\}, \; p \neq 1$$

- Genetic operators:
  - Two-point crossover
  - Bit flipping mutation

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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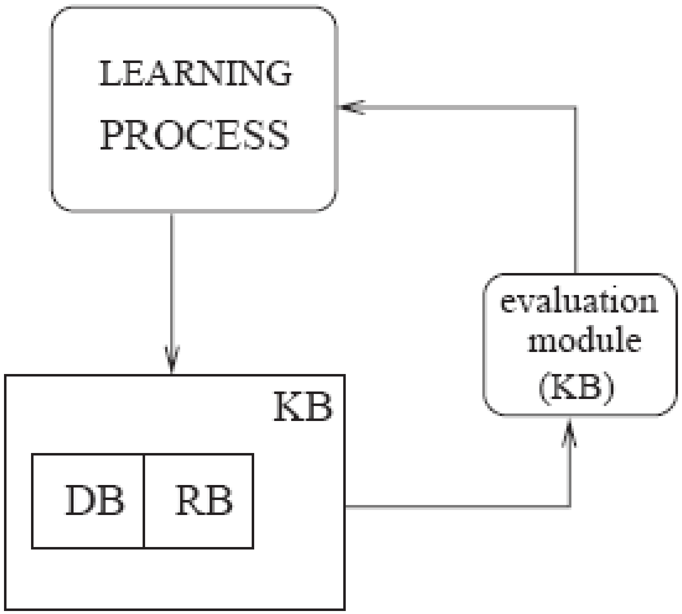
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

3. Genetic derivation of the FRBS Knowledge Base



- The genetic learning process of the KB must jointly determine:
  - Membership function definitions
  - Fuzzy rules

  and sometimes also:
  - Scaling factors/functions
  - Linguistic terms (fuzzy partition granularity)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Information items to be encoded into a chromosome:
  - Scaling factors
  - Membership functions
  - Fuzzy rules

  → Fixed or variable-length coding scheme

- Each information level is an independent chromosome part (multi-chromosomes)

- Different ways to adapt this two-level structure (DB and RB information) through crossover:
  - As a single one, by merging the substructures
  - As two unrelated substructures, applying a parallel process
  - As two related substructures, applying a sequential process where the result of crossing over one of them affects the crossover of the other

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

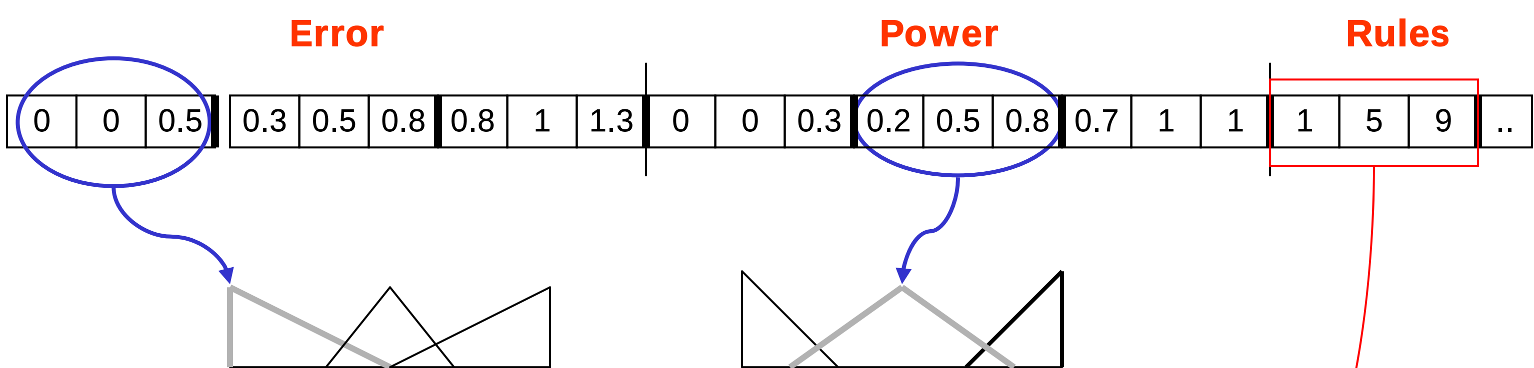
5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

- Example: SISO fuzzy control problem with 3 labels per variable:

Error → {N, Z, P}   Power → {S, M, L}



R1: IF Error is Negative THEN Power is Large

R2: ...

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- The search space is thus very large and complex, causing problems to the Pittsburgh approach:
  - Variable-length chrosomes, or
  - one rule per chromosome (Michigan or IRL) with scatter partitions, or
  - multi-stage GFSs

- The problem is simpler for the case of scatter partition Mamdani-type FRBSs, since each rule has its own semantics and so the chromosome has a single information level (list of rules representation)

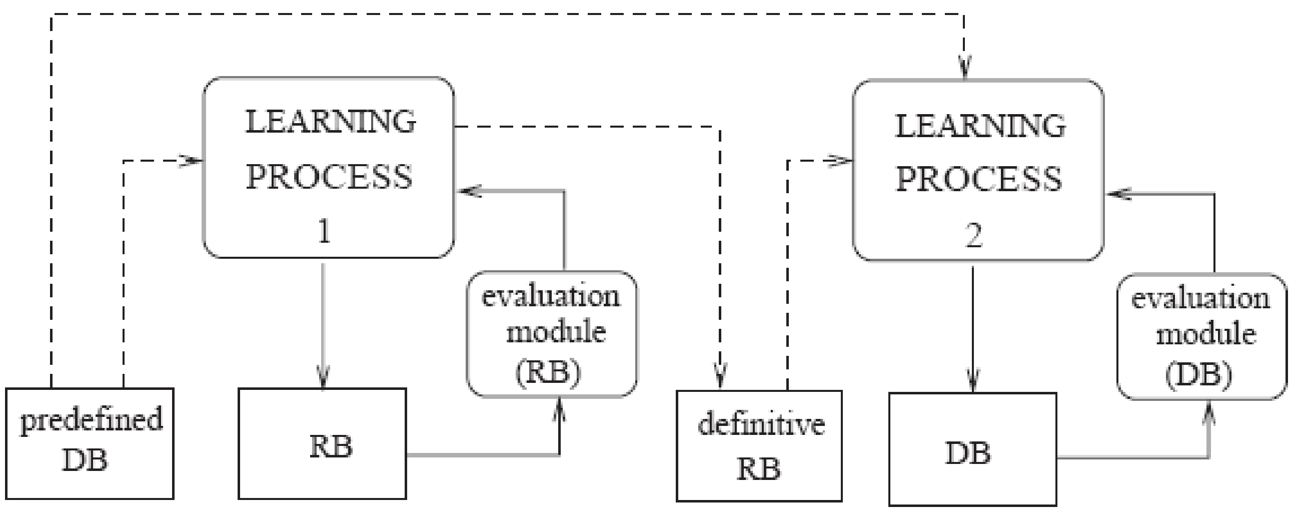
4. Classical GFS learning approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

- Some approaches partition the learning problem and try to improve the DB definition, once the RB has been derived (multi-stage GFSs):

1. Initial genetic RB learning (*predefined DB*)
2. Genetic DB learning (tuning) (*derived RB from the previous step*)



- This is the usual case for GFSs based on the IRL approach

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- L. Magdalena, F. Monasterio, A fuzzy logic controller with learning through the evolution of its knowledge base, IJAR 16 (3–4) (1997) 335–358 (DNF Mamdani fuzzy rules for control problems)
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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. **Classical GFS learning approaches**
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

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- O. Cordón, F. Herrera, A two-stage evolutionary process for designing TSK fuzzy rule-based systems, IEEE TSMC 29 (6) (1999) 703–715 (TSK fuzzy rules for classification problems)
- O. Cordón, M.J. del Jesus, F. Herrera, M. Lozano, MOGUL: a methodology to obtain genetic fuzzy rule-based systems under the iterative rule learning approach, IJIS 14 (11) (1999) 1123–1153 (generic methodology for different kinds of fuzzy rules and problems)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Control:

- Inverted pendulum, Cart-pole
- Biped robot walking (Magdalena, 1994)
- Fossil power plant operation supervision (Magdalena-Velasco, 1995)
- Control strategies for trains (Bonissone, 1996; Hwang, 1998)
- Industrial processes (Huang, 1998)
- Mobile Robotics: basic behaviors (obstacle avoidance, wall following, ...); behavior coordination, visual systems (Bonarini, 1996,1997; Hoffmann, 1996; Muñoz-Salinas, 2006; ...)
- Helicopter control (Hoffmann, 2001)
- Photovoltaic Systems (Magdalena, 2001)
- HVAC systems (Alcalá, 2003, 2005)
- Hybrid resonant-driven linear piezoelectric ceramic motor (Wai, 2007)
- F16 aircraft flight controller (Stewart, 2007)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Modeling/Forecasting:

- Food quality evaluation by sensorial tests (Ishibuchi, 1994; Guilleaume, 2002)
- Dental development age prediction (Lee, 1996)
- Electrical distribution problems (Sanchez, 1997; Cordón, 1999)
- Intelligent consumer products (dish washer, microwave oven, ...) (Shim, 1999)
- Color prediction for paint production (Mizutani, 2000)
- Wind forecasting for power generation in wind farms (Damousis, 2001)
- Decision systems for insurance risk assessment (Bonissone, 2002)
- Ecological problems (Van Broekhoven, 2007)
- Environmental modeling (Nebot, 2007)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Classification/Diagnosis:

- Myocardial infarction diagnosis (González, 1995)
- Classification of defects in sheets of glass (Sánchez, 1998)
- Breast cancer diagnosis (Peña-Reyes, 1999)
- Cardio-vascular diseases risk prediction (Cordón, 2002)
- Classification of amino acid sequences (Bandyopadhyay, 2005)
- Matrix crack detection in thin-wafled composite beam (Pawar, 2005)
- Intrusion detection (Abadeh, 2007)
- Microcalcification classification in digital mammograms (Jiang, 2007)
- Structural health monitoring of helicopter rotor blades (Pawar, 2007)

Optimization:

- Railway networks timetable (Voget, 1998)
- Supply strategies for the electrical market (Sánchez, 2003)
- Scheduling (Gomide, 2000; Franke, 2007)

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

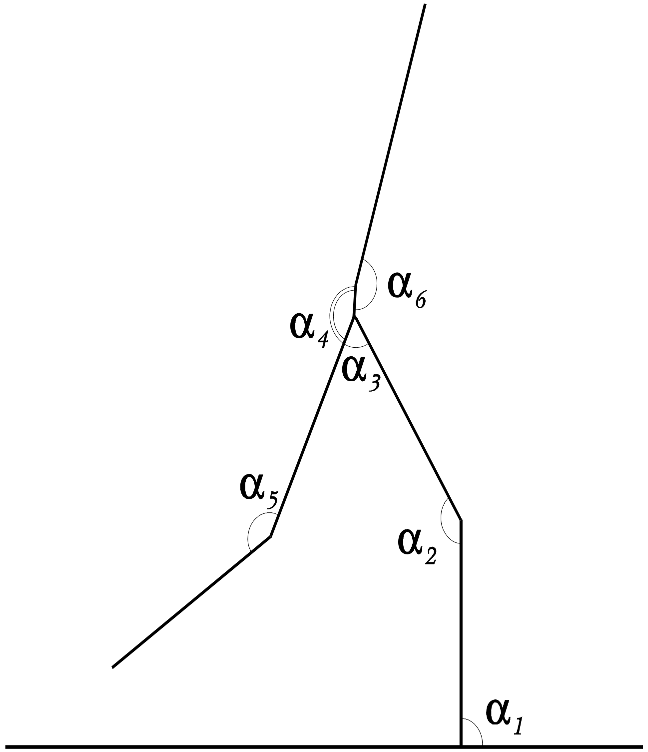
6. Advanced GFS approaches

7. Conclusions. What's next?

Biped robot walking control

L. Magdalena, F. Monasterio, A fuzzy logic controller with learning through the evolution of its knowledge base, IJAR 16 (3–4) (1997) 335–358

- Anthropomorphic structure
- Searching for the sequence of movements allowing continuous and regular walking
- Magdalena's Pittsburgh GFS to learn different gait controls

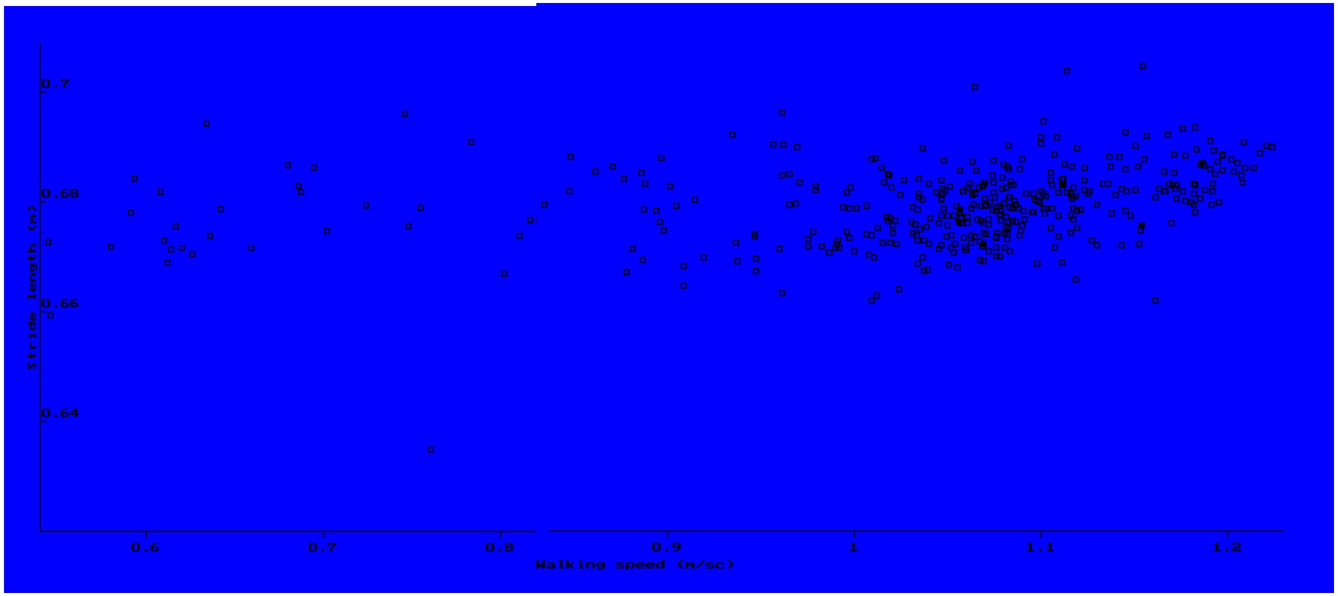


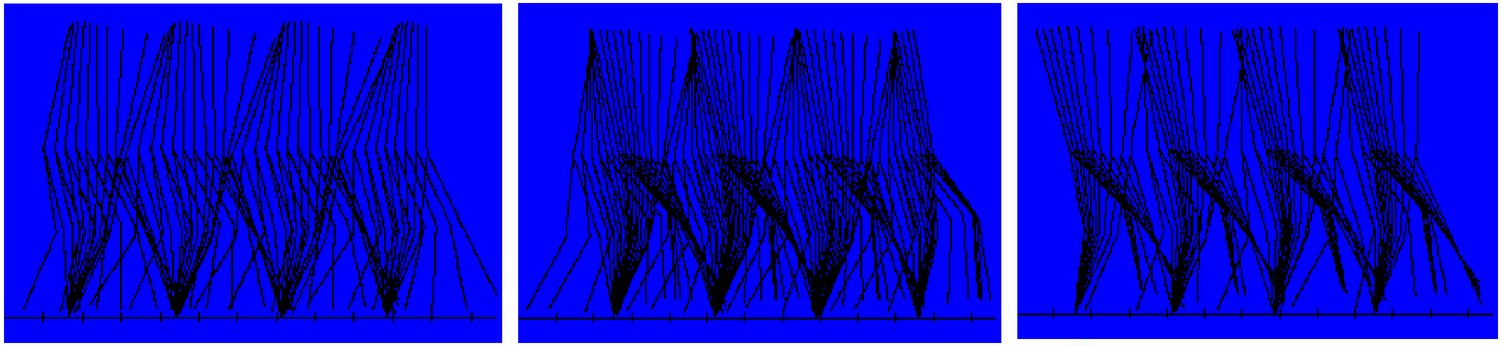
5. Some real-world applications

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Biped robot walking control

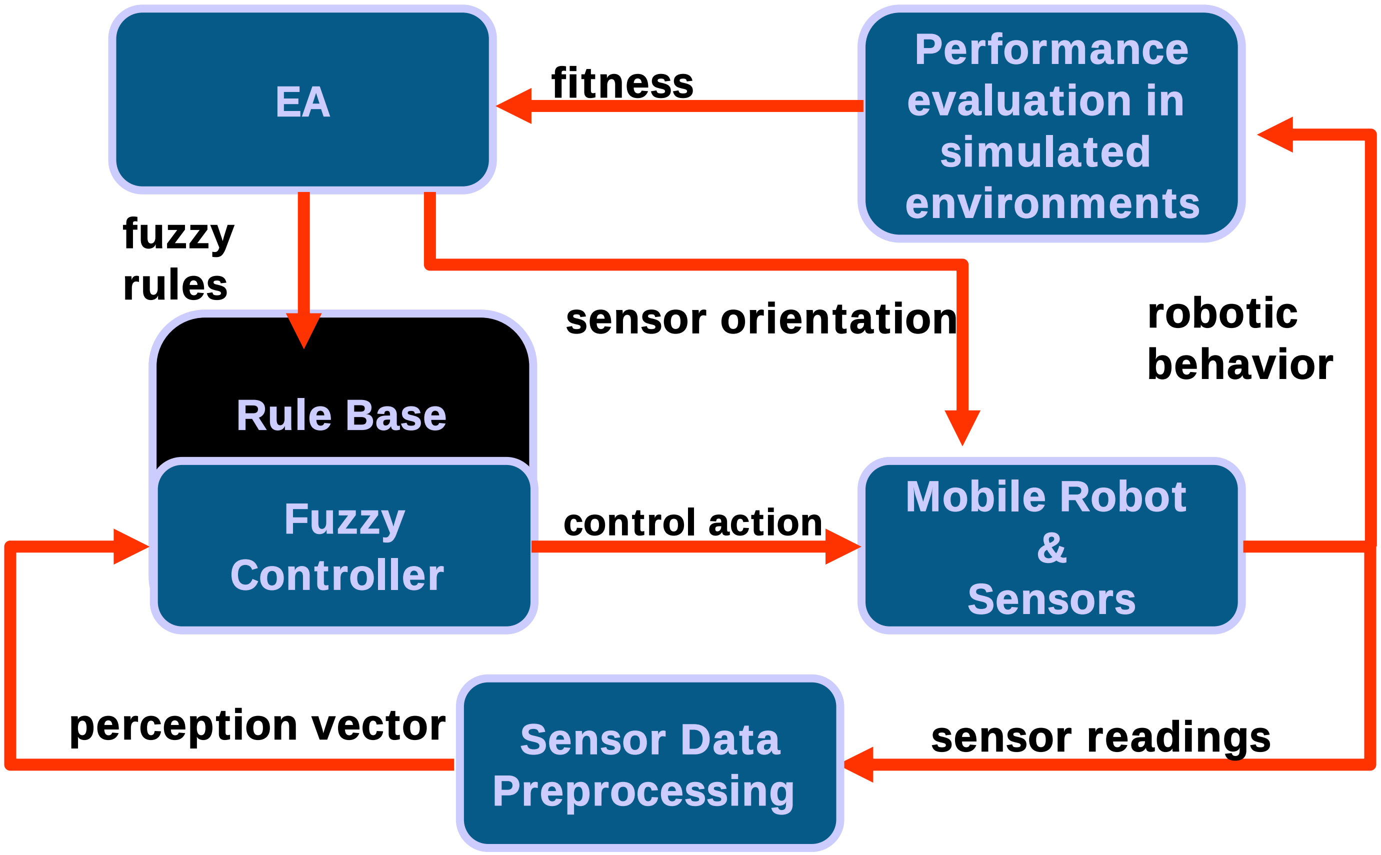




OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Mobile robotics: obstacle avoidance

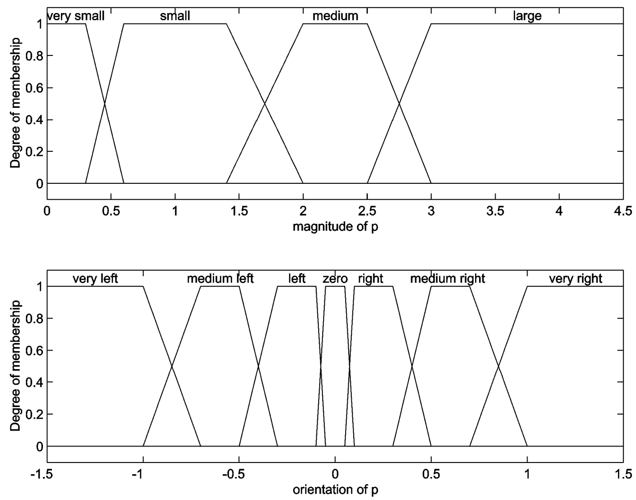


OUTLINE

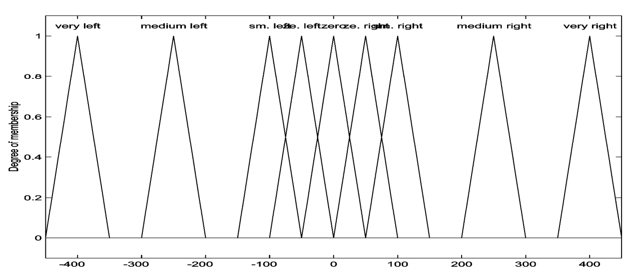
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Mobile robotics: obstacle avoidance

Perception



Action



Initial RB

ω		$\angle\vec{P}$						
		VL	ML	SL	ZE	SR	MR	VR
$\lvert\vec{P}\rvert$	VS	ZE	ZE	ZE	NZ	ZE	ZE	ZE
	SM	ZE	NZ	NZ	NZ	PZ	PZ	ZE
	ME	ZE	NZ	NS	NM	PS	PZ	ZE
	LG	NZ	NS	NM	NB	PM	PS	PZ

Evolved RB

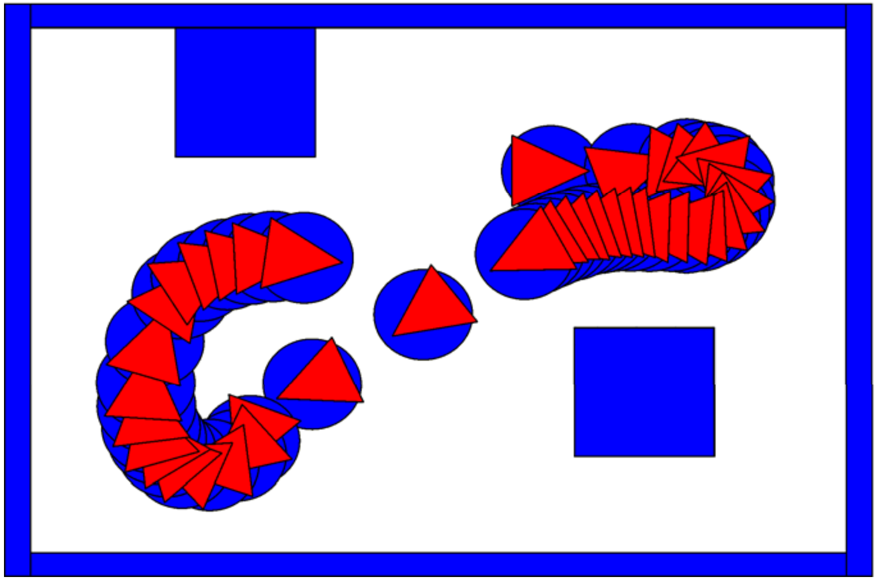
ω		$\angle\vec{P}$						
		VL	ML	SL	ZE	SR	MR	VR
$\lvert\vec{P}\rvert$	VS	ZE	PS	NM	NB	NS	NB	PM
	SM	NZ	NZ	NB	PZ	NB	NM	NZ
	ME	NZ	PS	NS	PZ	NM	ZE	NB
	LG	PS	PZ	NZ	NZ	ZE	NB	PZ

Thrift's GFS

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Obtained results in the real environment



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Maintenance cost estimation for low and medium voltage lines in Spain:

O. Cordón, F. Herrera, L. Sánchez, Solving electrical distribution problems using hybrid evolutionary data analysis techniques, Appl. Intell. 10 (1999) 5-24

- Spain's electrical market (before 1998): Electrical companies shared a business, Red Eléctrica Española, receiving all the client fees and distributing them among the partners
- The payment distribution was done according to some complex criteria that the government decided to change
- One of them was related to the maintenance costs of the power line belonging to each company
- The different producers were in trouble to compute them since:
  - As low voltage lines are installed in small villages, there were no actual measurement of their length
  - The goverment wanted the maintenance costs of the optimal medium voltage lines installation and not of the real one, built incrementally

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Low voltage line maintenance cost estimation:

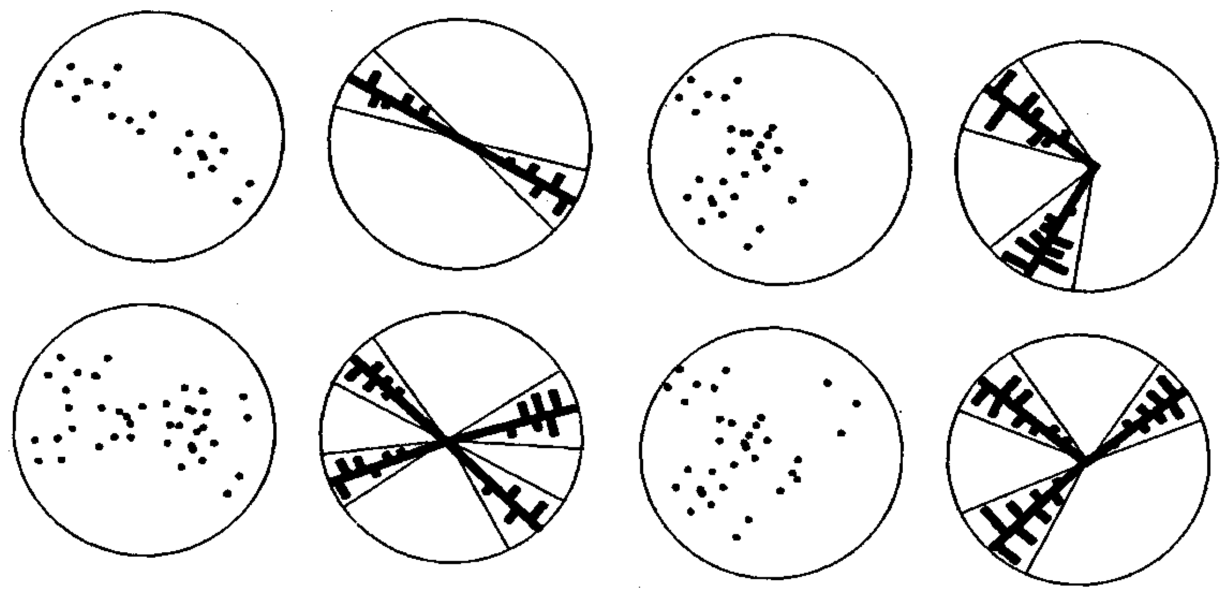
- Goal: estimation of the low voltage electrical line length installed in 1000 rural towns in Asturias
- Two input variables: number of inhabitants and radius of village
- Output variable: length of low voltage line
- Data set composed of 495 rural nuclei, manually measured and affected by noise
- 396 (80%) examples for training and 99 (20%) examples for test randomly selected
- Seven linguistic terms for each linguistic variable

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Low voltage line maintenance cost estimation:

- Classical solution: numerical regression on different models of the line installation in the villages



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Performance comparison of different fuzzy modeling methods

Method	#R	MSE_{tra}	MSE_{test}
Wang-Mendel	24	222,623	240,566
Cordón-Herrera	32	267,923	249,523
Ishibuchi (simp. TSK)	32	173,230	190,808
Thrift	47	185,204	168,060
Shan-Fu	45	1,281,547	1,067,993
ANFIS	49	256,605	268,451
FCM	49	163,615	198,617
Chiu+FCM	37	200,999	222,362
3rd order polynomial regression	49 nodes, 2 pars.	235,934	202,991
NN 2-25-1	102 par.	169,399	167,092

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Medium voltage line maintenance cost estimation:

- Goal: estimation of the maintenance cost of the optimal medium voltage electrical line installed in the Asturias' towns
- Four input variables: street length, total area, total area occupied by buildings, and supplied energy
- Output variable: medium voltage line maintenance costs
- Data set composed of 1059 simulated cities
- 847 (80%) examples for training and 212 (20%) examples for test randomly selected
- Five linguistic terms for each linguistic variable

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Performance comparison of different fuzzy modeling methods

Method	#R	MSE_{tra}	MSE_{test}
Wang-Mendel (3 labels)	28	197,313	174,400
Wang-Mendel	66	71,294	80,934
Cordón-Herrera (TSK) multi-stage GFS	268	11,073	11,836
Thrift	534	34,063	42,116
2nd order polynomial regression	77 nodes, 15 par.	103,032	45,332
NN 4-5-1	35 par.	86,469	33,105

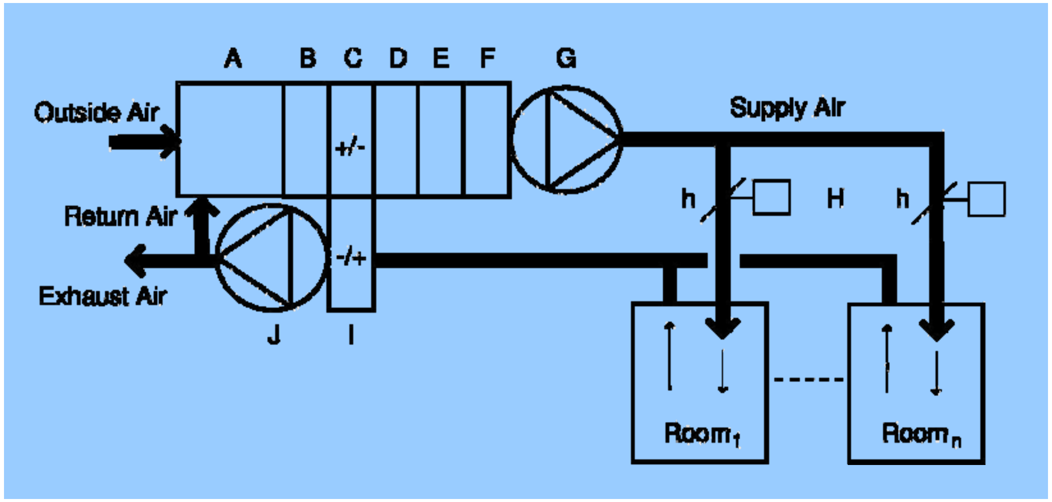
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

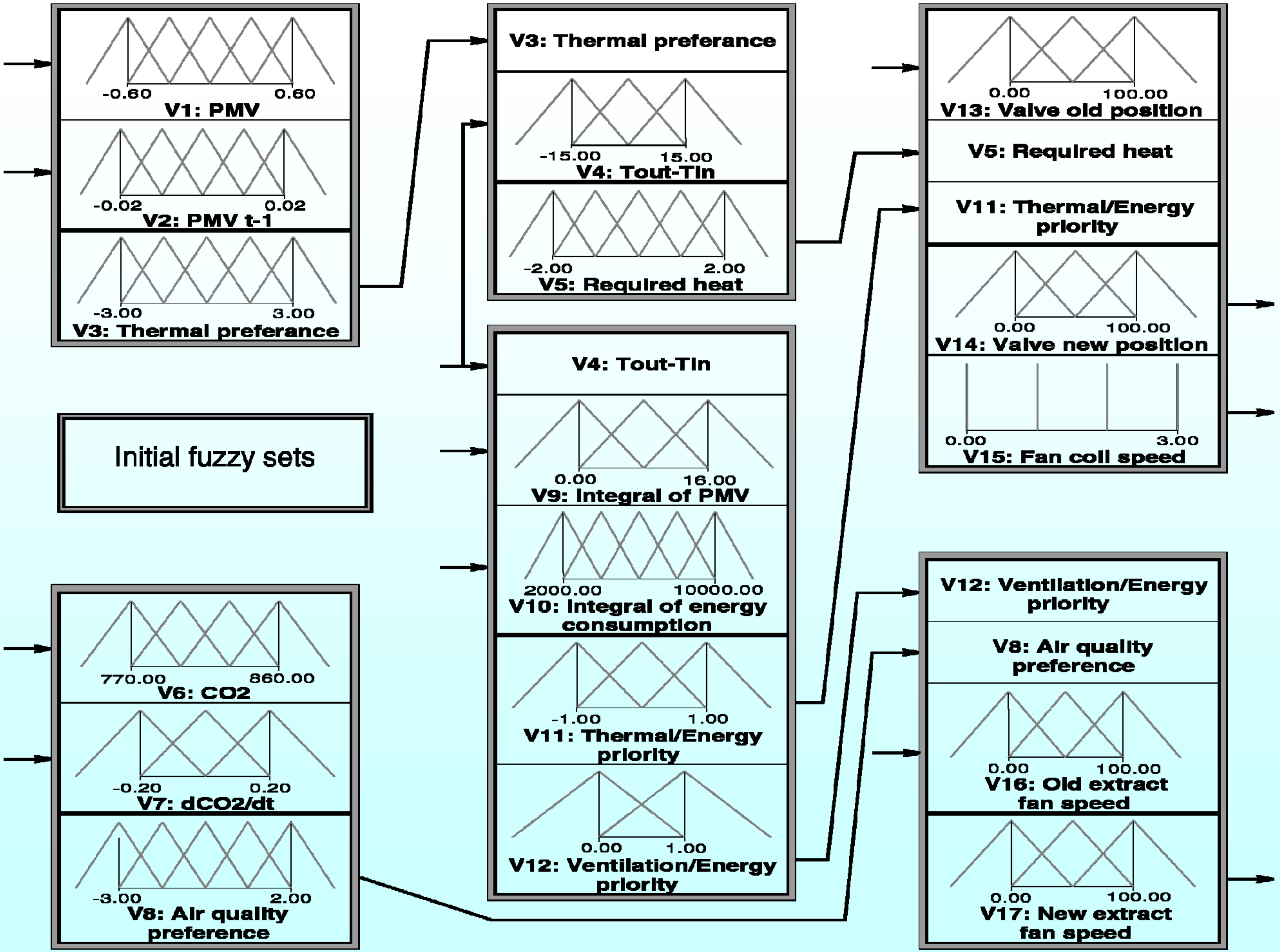
Fuzzy control of Heating Ventilating and Air Conditioning (HVAC) systems:

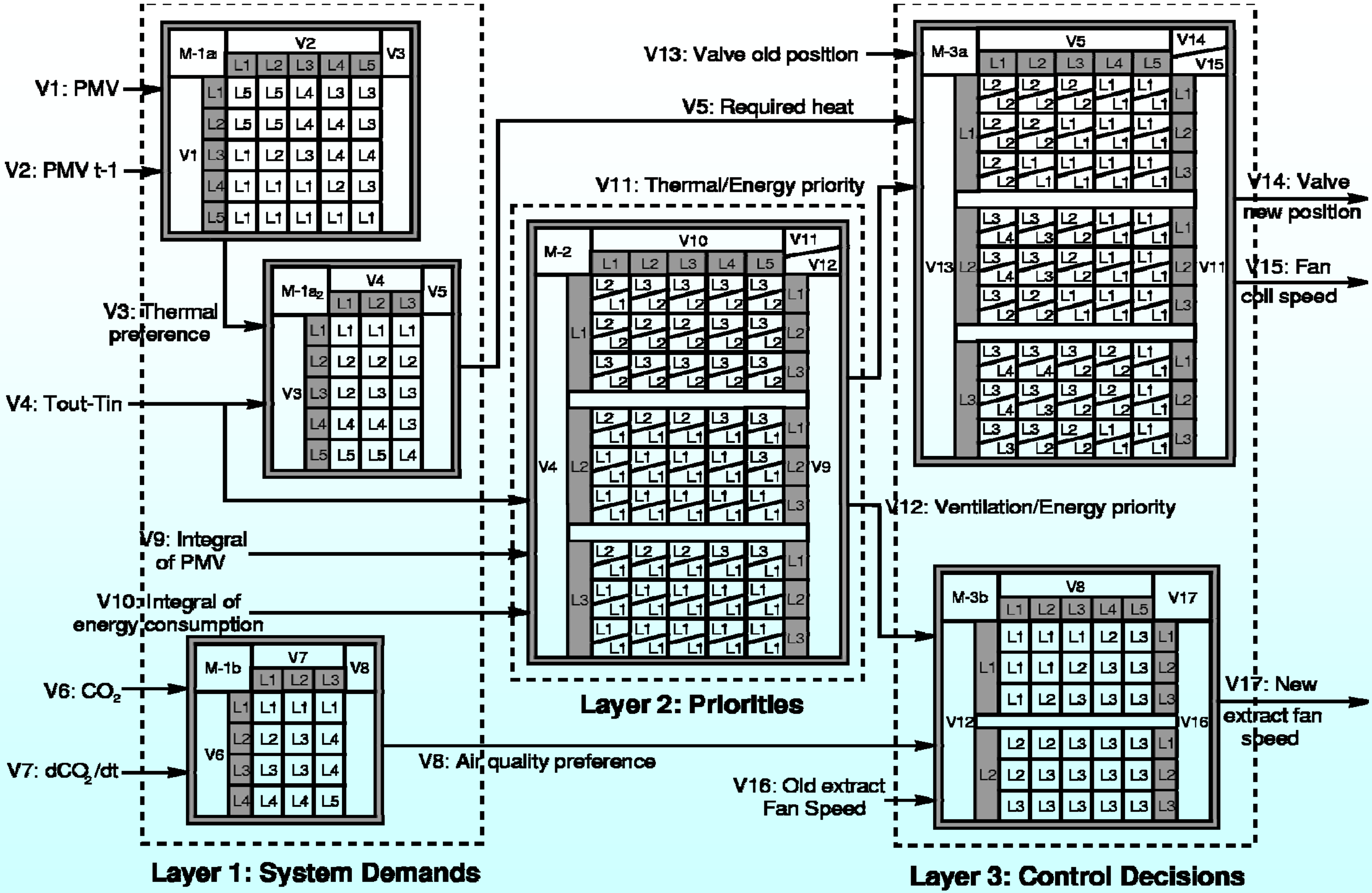
R. Alcalá, J.M. Benítez, J. Casillas, O. Cordón, R. P&erez, Fuzzy control of HVAC systems optimised by genetic algorithms, Appl. Intell. 18 (2003) 155–177

- Goal: multi-criteria optimization of an expert FLC for an HVAC system: reduction of the energy consumption but maintaining the required indoor comfort levels



HVAC system structure


-0.60
0.60
V1: PMV
-0.02
0.02
V2: PMV t-1
-3.00
3.00
V3: Thermal preferance
Initial fuzzy sets
770.00
860.00
V6: CO2
-0.20
0.20
V7: dCO2/dt
-3.00
2.00
V8: Air quality preference
V3: Thermal preferance
-15.00
15.00
V4: Tout-Tin
-2.00
2.00
V5: Required heat
V4: Tout-Tin
0.00
16.00
V9: Integral of PMV
2000.00
10000.00
V10: Integral of energy consumption
-1.00
1.00
V11: Thermal/Energy priority
0.00
1.00
V12: Ventilation/Energy priority
0.00
100.00
V13: Valve old position
V5: Required heat
V11: Thermal/Energy priority
0.00
100.00
V14: Valve new position
0.00
3.00
V15: Fan coil speed
V12: Ventilation/Energy priority
V8: Air quality preference
0.00
100.00
V16: Old extract fan speed
0.00
100.00
V17: New extract fan speed



Module $1a_1$: Thermal Demands
Module $1a_2$: Thermal Preference
Module 1b: Air Quality Demands
Module 2: Energy Priorities
Module 3a: Required HVAC System Status
Module 3b: Required Ventilation System Status

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Genetic tuning of the HVAC FLC:

- Goals to optimize:

  O_1 Upper thermal comfort limit [3]: $if\ PMV > 0.5, O_1 = O_1 + (PMV - 0.5)$.

  O_2 Lower thermal comfort limit: $if\ PMV < -0.5, O_2 = O_2 + (-PMV - 0.5)$.

  O_3 IAQ requirement: $if\ CO_2\ conc. > 800ppm, O_3 = O_3 + (CO_2 - 800)$.

  O_4 Energy consumption: $O_4 = O_4 +$ Power at time t.

  O_5 System stability: $C_5 = C_5 +$ System change from time t to $(t-1)$.

- Hence, the fitness function is multi-criteria. In this case, an aggregation approach is preferred to a Pareto-based one since:
  - Weight values are provided by the human experts defining the importance of each objective
  - The search space is smaller
  - Quicker GAs can be designed

$$\boxed{F(x) = w_1 \cdot f_1(x) + \ldots + w_n \cdot f_n(x)}$$

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Genetic tuning of the HVAC FLC (2):

- Problem restriction: the simulation model used to evalute the performance of a DB definition takes 3-4 minutes
- An efficient genetic tuning methodology is mandatory:
  - Local adjustment of each membership function definition parameter
  - GA with quick convergence: steady-state (just 2000 evaluations will take around 4 days)
  - Small population size (31 individuals)
- Real-coded steady-state GA:
  - Two parents are selected and crossed over (max-min-arithmetical) and mutated (Michalewicz), obtaining four offspring
  - The two best of them compete with the two worst individuals in the population to enter into it
  - A restart is applied if the GA has stagnated

OUTLINE

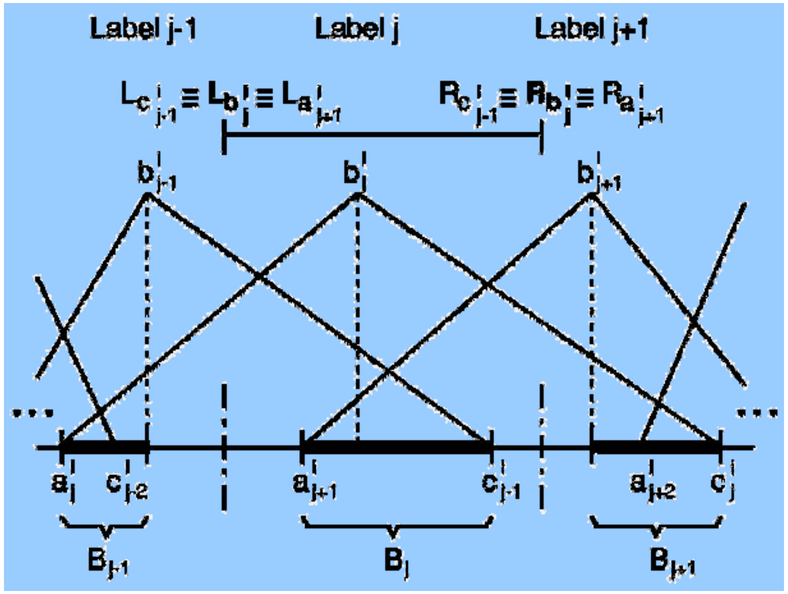
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Genetic tuning of the HVAC FLC (3):

- Coding scheme: n variables and L_i linguistic terms

$$C_i = \left(a_1^i, b_1^i, c_1^i, \ldots, a_{L_i}^i, b_{L_i}^i, c_{L_i}^i\right),\ i = 1, \ldots, n$$

$$C = C_1 C_2 \ldots C_n$$



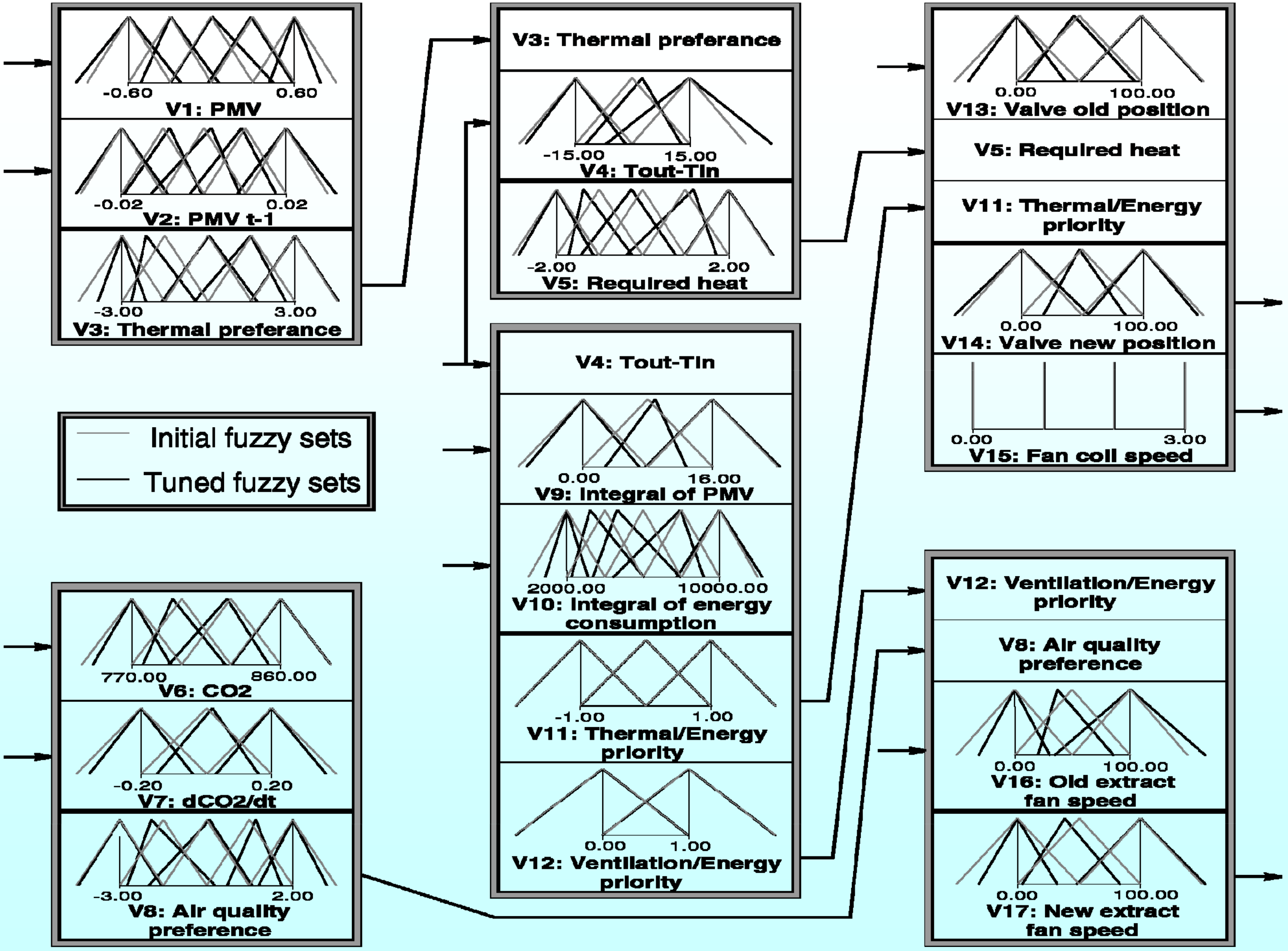
5. Some real-world applications

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. **Some real-world applications**
6. Advanced GFS approaches
7. Conclusions. What's next?

Obtained results:

	#R	PMV>0.5		PMV<-0.5		CO_2		Energy		Stability	
MODEL	–	O_1	%	O_2	%	O_3	%	O_4	%	O_5	%
Goals (g_i)	–	1.0	–	1	–	7	–	2000000	–	1000	–
ON-OFF	–	0.0	–	0	–	0	–	3206400	–	1136	–
FLC	172	0.0	–	0	–	0	–	2901686	9.50	1505	-32.48
Considering Data Base tuning											
DB 1	172	0.0	–	0	–	0	–	2575949	19.66	1115	1.85
DB 2	172	0.0	–	0	–	0	–	2587326	19.31	1077	5.19
DB 3	172	0.0	–	0	–	0	–	2596875	19.01	1051	7.48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2.00
V5: Required heat
V4: Tout-Tin
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16.00
V9: Integral of PMV
2000.00
10000.00
V10: Integral of energy consumption
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1.00
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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

New learning schemes

- KB derivation through *a priori* genetic DB learning
- Coevolutionary GFSs
- Incremental Learning

Interpretability-Accuracy trade-off

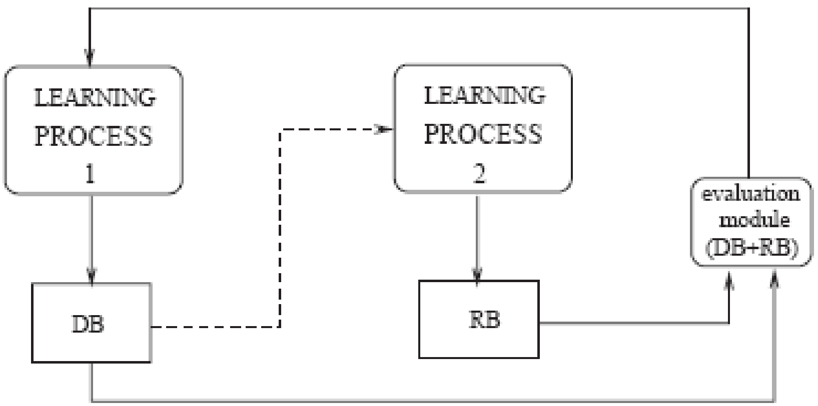
- Multi-objective genetic learning and selection of fuzzy rules
- New fuzzy model structures. Combined parameter learning and rule selection
- Advanced tuning approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

KB derivation through *a priori* genetic DB learning:

- GFS based on the decomposition of the learning problem in two intertwined stages:
  - Learning of the DB
  - Derivation of the RB



- The DB learning algorithm wraps the RB derivation method. The quality of each candidate DB is given by the performance of the whole KB

- Advantages (with respect to the joint DB+RB generation):
  - Reduction of the search space
  - More chances to find optimal solutions

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

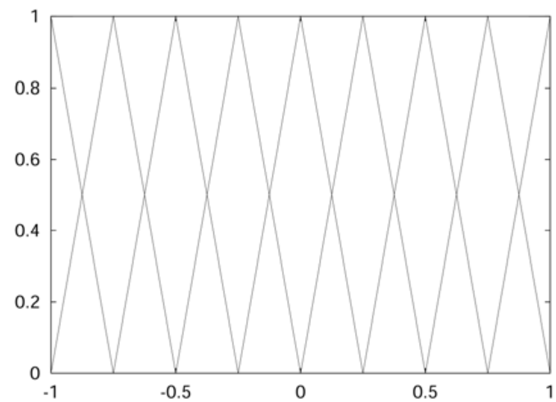
- The GA used to learn the DB can consider any of the following components:
  - The variable domain (scaling factor allowing a brief enlargement)
  - The non-linear scaling function for each fuzzy partition including areas with different "sensibility" in the variable domain
  - The number of labels per variable (granularity)
  - The membership function shapes
- The rule generation method must be quick, since the evaluation of each DB definition requires its run
- Due to this, ad-hoc data-driven method are usually considered, such as Wang y Mendel's

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Non-linear scaling function for context definition:

f: [-1,1] → [-1,1] $f(x) = \mathrm{sign}(x) \cdot |x|^a$ with $a > 0$



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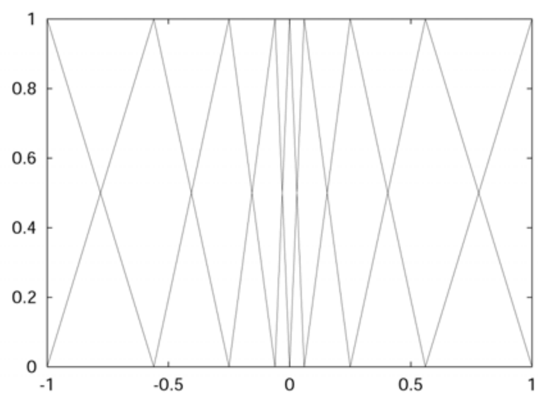

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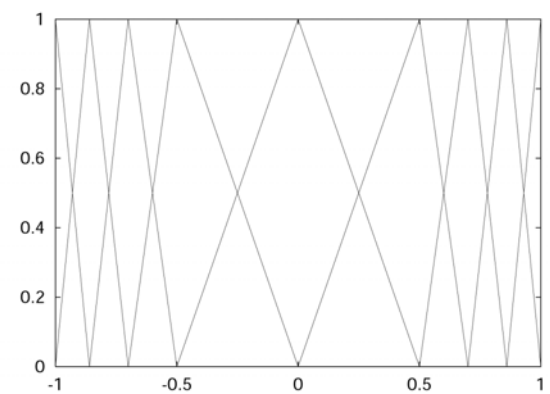

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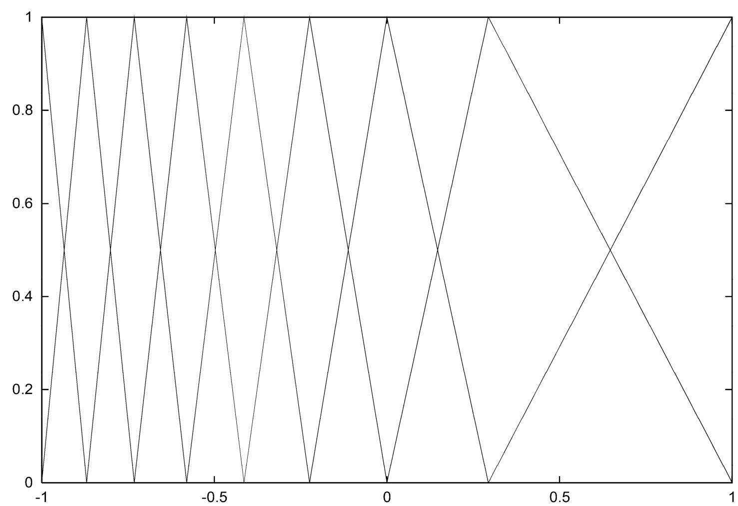
OUTLINE

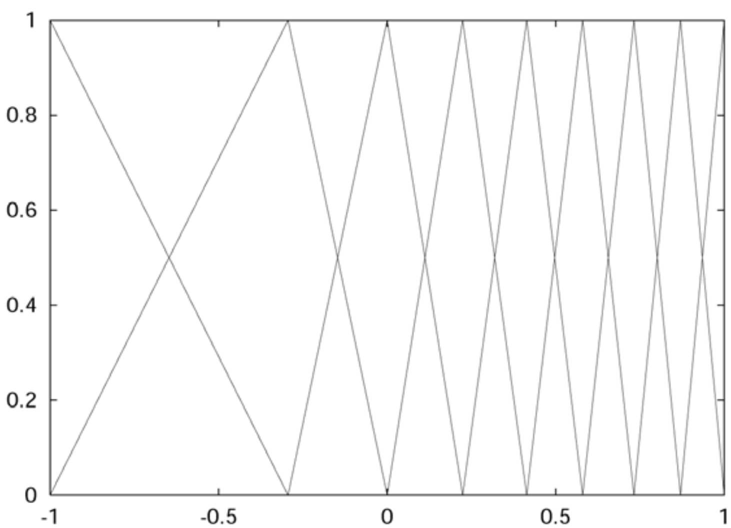
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Non-linear scaling function for context definition:

- That scaling function is good for symmetrical fuzzy partitions
- We add a new parameter to distinguish non-linearities with asymmetric shape ($S \in \{0,1\}$)



S=1 , a>1



S=1 , a<1

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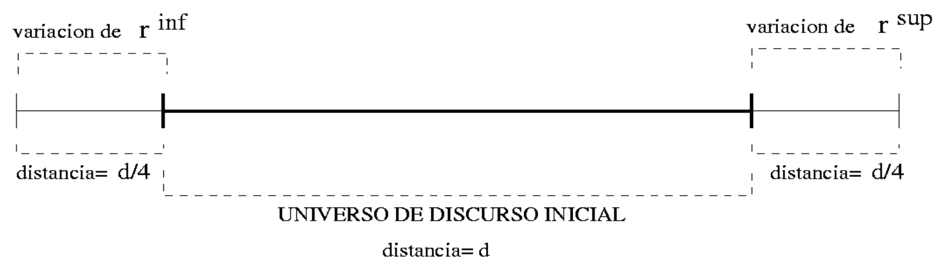
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Chromosome structure:

- Scaling factors (C_1): $C_1 = (R_1, R_2, \ldots, R_N)$; $R_i = (r_i^{inf}, r_i^{sup})$



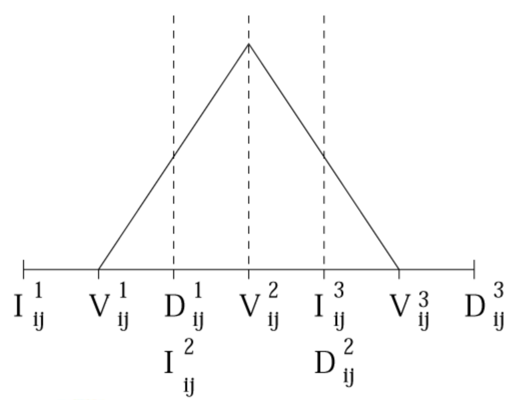
- Sensibility parameters (C_2): $C_2 = (a_1, a_2, \ldots, a_N, S_1, S_2, \ldots, S_N)$

- Granularity (C_3): $C_3 = (E_1, E_2, \ldots, E_N)$ (integer coding)

- Membership function shapes (C_4):

  $C_{4i} = (V1_{i1}, V2_{i1}, V3_{i1}, \ldots, V1_{iE}, V2_{iE}, V3_{iE})$

  $C_4 = (C_{41}, C_{42}, \ldots, C_{4N})$



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

DB learning options:

BASIC OPTIONS:

- Only linear scaling functions (variable domains): $C = C_1 = (R_1, R_2, \ldots, R_N)$
- Only sensibility parameters: $C = C_2 = (a_1, a_2, \ldots, a_N)$ or $C = C_2' = (a_1, a_2, \ldots, a_N, S_1, S_2, \ldots, S_N)$
- Only granularity: $C = C_3 = (E_1, E_2, \ldots, E_N)$
- Only membership function shapes: $C = C_4 = (C_{41}, C_{42}, \ldots, C_{4N})$

COMBINATIONS:

- Scaling factors + Granularity: $C = (C_1, C_3)$
- Non-linear scaling functions + membership functions: $C = (C_2, C_4)$
- …

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Chromosome evaluation:

- Build the DB from the parameters encoded in the chromosome
- Run the RB generation on that DB definition
- Compute the performance measure (MSE_{tra}, classification error or control error) of the obtained KB (DB+RB)
- To improve the generalization capability in modeling/ classification, KBs with a large number of rules (NR) can be slightly penalized:

$$F(C) = \omega_1 \cdot MSE_{tra} + \omega_2 \cdot NR$$

with $\omega_1 = 1$ and ω_2 computed from the results of the FRBS with the maximum granularity

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results in the medium voltage line problem:

Method	Granul.	NR	MSE_{tra}	MSE_{test}
WM	9 9 9 9 9	130	32.337,4	33.504,9
WM + Tun	9 9 9 9 9	130	13.442,5	17.585,7
FJ	9 9 9 9 9	133	17.441,1	21.184,6
	9 9 9 9 9	139	18.654,5	19.112,8
Gr.+m.f. (C_3+C_4)	4 3 9 9 9	96	**9.163,5**	11.121,3
	3 3 9 7 9	68	9.987,7	10.414,1
Scaling factor + Gr + Scal. function 1 (C_1+C_2+C_3)	5 4 9 9 9	**65**	9.799,3	9.966,9
Scaling factor + Gr + Scal. function 2 (C_1+C_2+C_3)	4 5 9 9 9	82	9.424,2	**9.312,9**

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

References:

- O. Cordón, F. Herrera, L. Magdalena, P. Villar, A genetic learning process for the scaling factors, granularity and contexts of the fuzzy rule-based system data base, Inf. Sci. 136 (1-4) (2001) 85-107
- O. Cordón, F. Herrera, P. Villar, Generating the knowledge base of a fuzzy Rule-based system by the genetic learning of data base. IEEE TFS 9 (4) (2001) 667-674
- O. Cordón, F. Herrera, J. De la Montaña, A.M. Sánchez, P. Villar, A prediction system of cardiovascularity diseases using genetic fuzzy rule-based systems. In: F.J. Garijo et al. (Eds.), Advances in Artificial Intelligence IBERAMIA 2002, LNCS 2527, Springer, 2002, pp. 381-391

OUTLINE

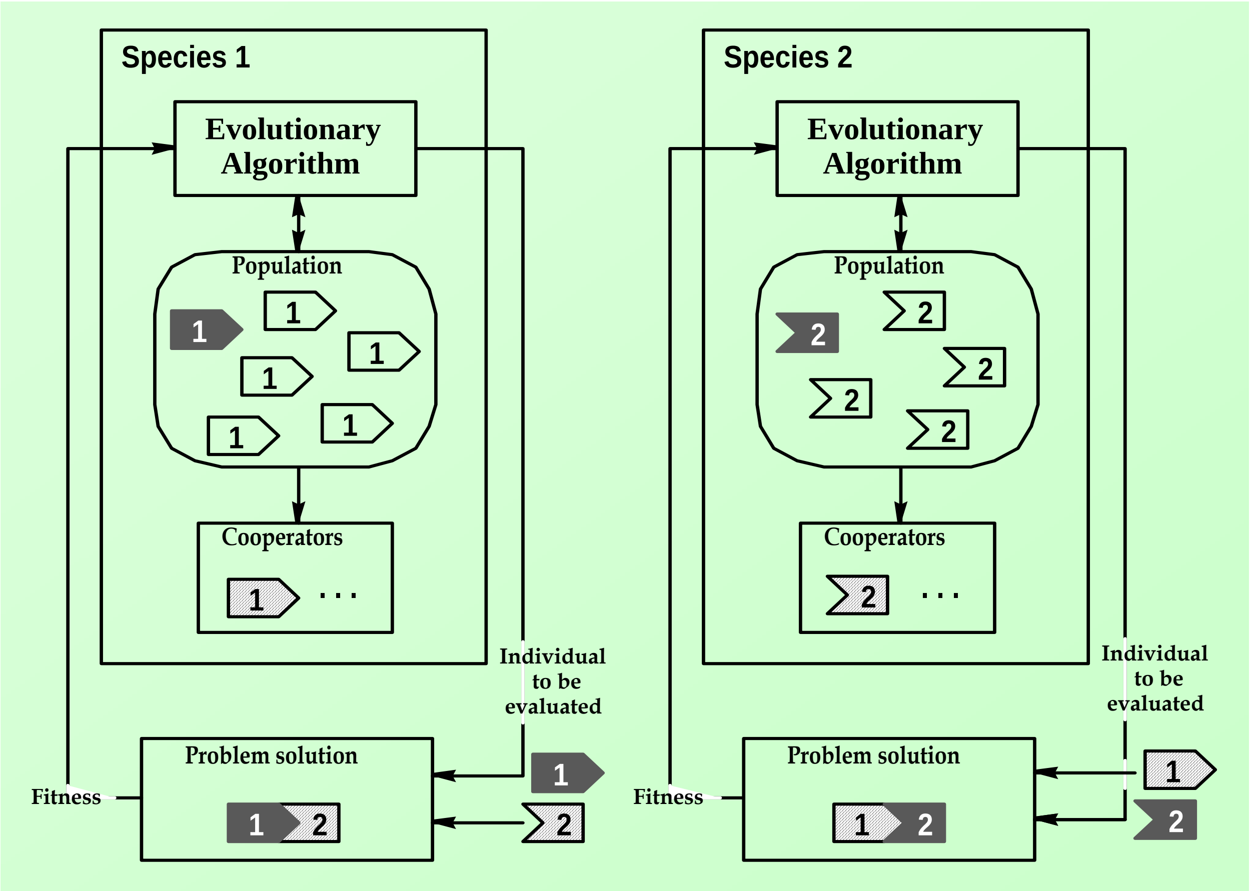
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Coevolutionary genetic fuzzy systems:

- Coevolutionary algorithms are advanced evolutionary techniques proposed to solve decomposable complex problems
- They involve several species (populations) that permanently interact among them by a coupled fitness
- In the cooperative approach all the species cooperate to build the problem solution
- They are recommendable when:
  - The search space is huge
  - The problem may be decomposable in subcomponents
  - Different coding schemes are used
  - The subcomponents present strong interdependencies

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?



Cooperative coevolutionary algorithm

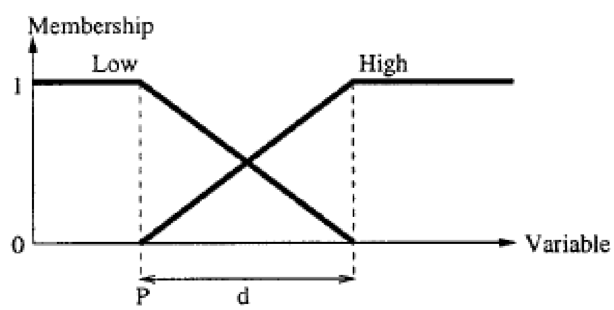
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Peña-Reyes' Fuzzy CoCo GFS:

Peña-Reyes, C.A., Sipper, M., Fuzzy CoCo: a cooperative-coevolutionary approach to fuzzy modeling, IEEE TFS 9 (5) (2001) 727-737

- Coevolutionary GFS with two binary-coded species:
  - Data Base: definition of all the membership functions
  - Rule Base: fuzzy rules



Species 1: Membership functions

Parameter	Values	Bits	Qty	Total bits
P	$\{1,2,\ldots,8\}$	3	9	27
d	$\{1,2,\ldots,8\}$	3	9	27
	Total			54

Species 2: Rules

Parameter	Values	Bits	Qty	Total bits
A	$\{0,1,2,3\}$	2	$9 \times N_r$	$18 \times N_r$
C	$\{1,2\}$	1	$N_r + 1$	$N_r + 1$
	Total			$19 \times N_r + 1$

- Designed for the Breast cancer classification problem: 9 inputs
- Two linguistic labels per variable (genes 1 and 2). Genes 0 and 3 are used for feature selection at rule level

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results:

Rules per system	Neuro-Rule [41]	Single population GA [30]	Fuzzy CoCo	
	best	best	average	best
1	97.36% (4)	97.07% (4)	97.36% (4)	97.36% (4)
2	–	97.36% (4)	97.73% (3.9)	98.54% (5)
3	98.10% (4)	97.80% (6)	97.91% (4.4)	98.54% (4)
4	–	98.24% (7)	98.12% (4.2)	98.68% (5)
5	98.24% (5)	97.95% (7)	98.18% (4.6)	98.83% (5)
6	–	98.10% (9)	98.18% (4.3)	98.83% (5)
7	–	97.95% (8)	98.25% (4.7)	$\boxed{98.98\%}$ (5)

- Results from 495 runs
- The number between parenthesis is the number of variables of the most complex rule in the RB

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results:

Best evolved KB with 2 rules:

Database

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
P	3		1	3	4	5		7	2
d	8		3	1	2	2		4	1

Rule base

Rule 1	**if** (v_1 **is** *Low*) **and** (v_3 **is** *Low*) **and** (v_5 **is** *Low*) **then** (*output* **is** *benign*)
Rule 2	**if** (v_1 **is** *Low*) **and** (v_4 **is** *Low*) **and** (v_6 **is** *Low*) **and** (v_8 **is** *Low*) **and** (v_9 **is** *Low*) **then** (*output* **is** *benign*)
Default	**else** (*output* **is** *malignant*)

Classification rate: 98.54%

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results:

Best evolved KB with 7 rules:

Database

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
P	2	1	1	1	6	1	3	5	2
d	7	8	4	8	1	4	8	4	1

Rule base

Rule 1	if (v_1 is *Low*) and (v_3 is *Low*) then (*output* is *benign*)
Rule 2	if (v_4 is *Low*) and (v_6 is *Low*) and (v_8 is *Low*) and (v_9 is *Low*) then (*output* is *benign*)
Rule 3	if (v_1 is *Low*) and (v_3 is *High*) and (v_5 is *High*) and (v_8 is *Low*) and (v_9 is *Low*) then (*output* is *benign*)
Rule 4	if (v_1 is *Low*) and (v_2 is *High*) and (v_4 is *Low*) and (v_5 is *Low*) and (v_8 is *High*) then (*output* is *benign*)
Rule 5	if (v_2 is *High*) and (v_4 is *High*) then (*output* is *malignant*)
Rule 6	if (v_1 is *High*) and (v_3 is *High*) and (v_6 is *High*) and (v_7 is *High*) then (*output* is *malignant*)
Rule 7	if (v_2 is *High*) and (v_3 is *High*) and (v_4 is *Low*) and (v_5 is *Low*) and (v_7 is *High*) then (*output* is *malignant*)
Default	else (*output* is *malignant*)

Classification rate: 98.98%

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Incremental learning:

Hoffmann, F., Koo, T.-J., Shakernia, O., Evolutionary design of a helicopter autopilot, In: Advances in Soft Computing - Engineering Design and Manufacturing, Part 3: Intelligent Control, Springer-Verlag, 1999, pp. 201-214

- GFS that learns TSK fuzzy rules incrementally:

  IF X_1 IS A_1 AND X_2 IS A_2 ... THEN $y=c_0 + c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_n \cdot x_n$

- The system starts from a single, very simple rule, covering the whole input space and with a linear output

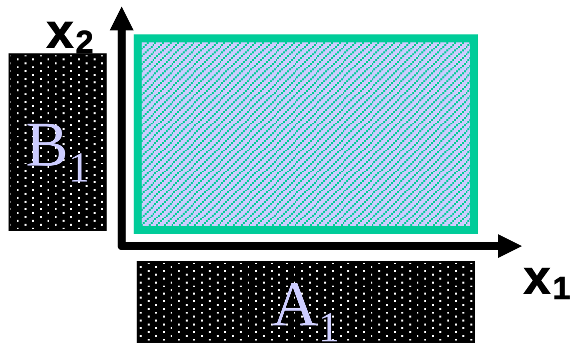
- An evolution strategy is considered to iteratively partition the fuzzy input subspaces, keeping the linear outputs

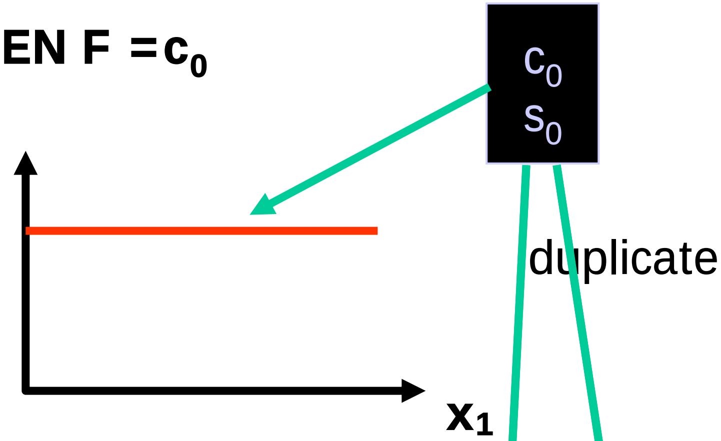
- Alternatively, new terms are added to the consequent weighted combination to get a non linear mapping in the output

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

IF x_1 IS A_1 AND x_2 IS B_1 … THEN F $= c_0$

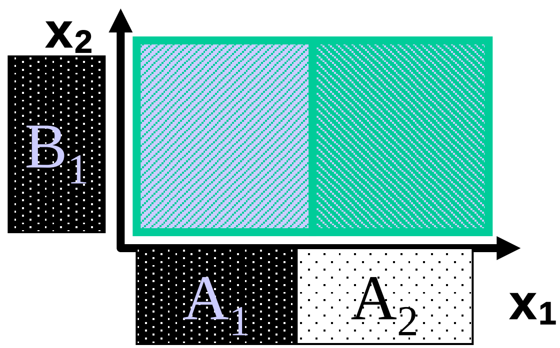


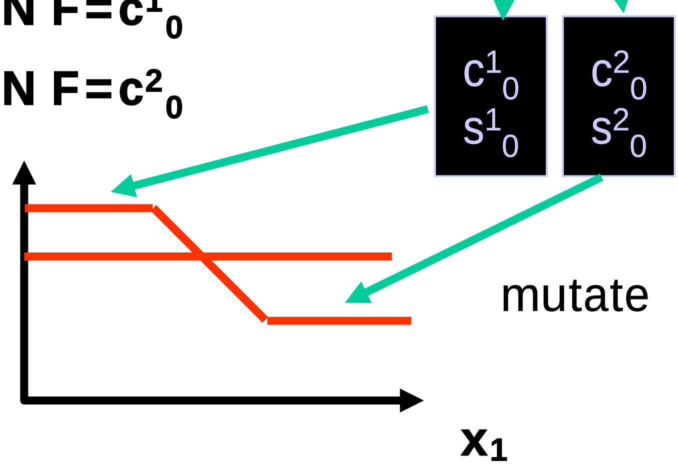


partition input space along one variable

IF x_1 IS A_1 AND x_2 IS B_1 … THEN F$=c^1_0$

IF x_1 IS A_2 AND x_2 IS B_1 … THEN F$=c^2_0$

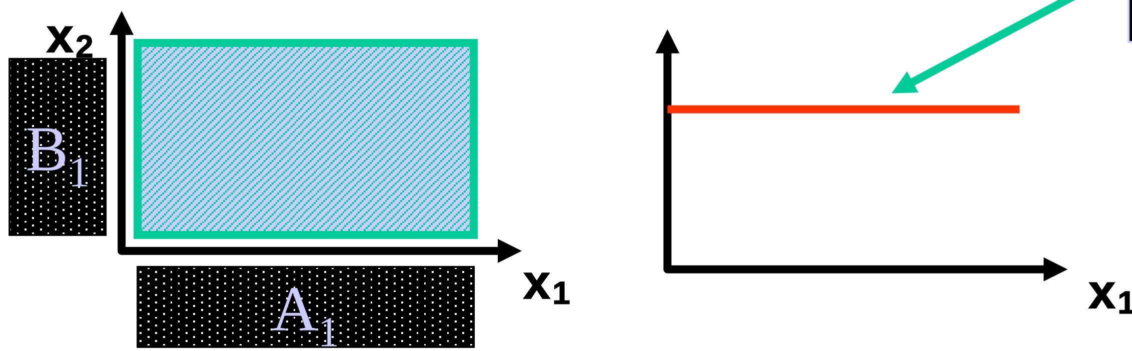


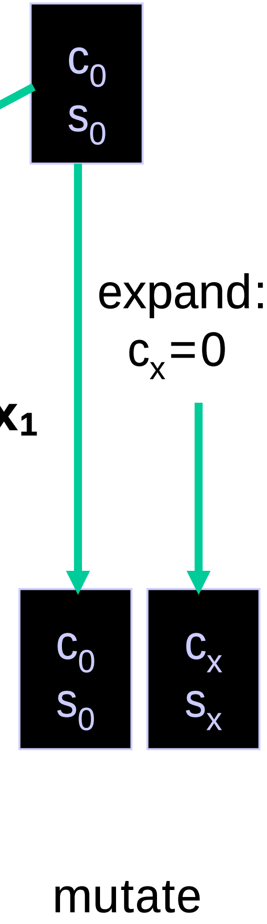


OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

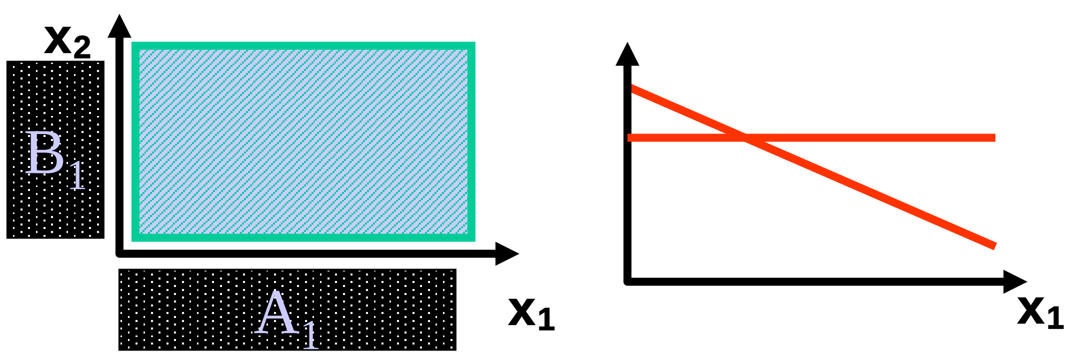
IF x_1 IS A_1 AND x_2 IS B_1 ... THEN F $= c_0$





add a term to the linear output

IF x_1 IS A_1 AND x_2 IS B_1 ... THEN F$=c_0$ $\boxed{+c_x}$

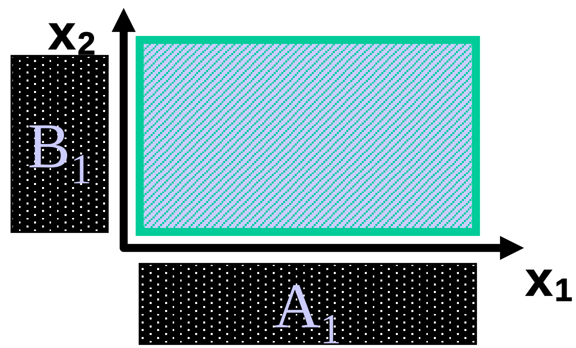


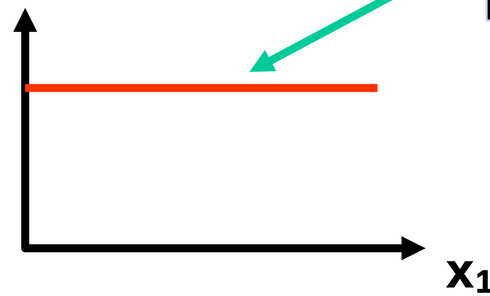
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

IF x_1 IS A_1 AND x_2 IS B_1 … THEN F = c_0

c_0
s_0

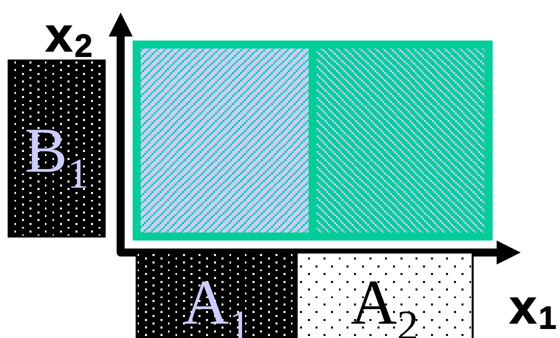


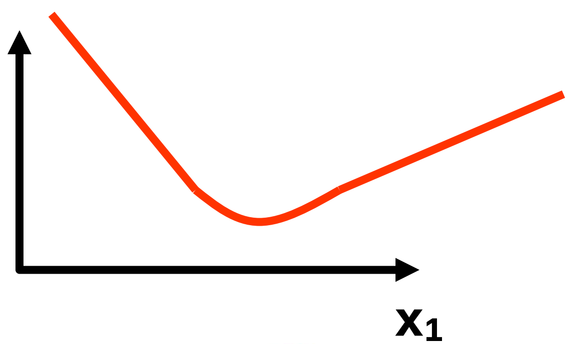


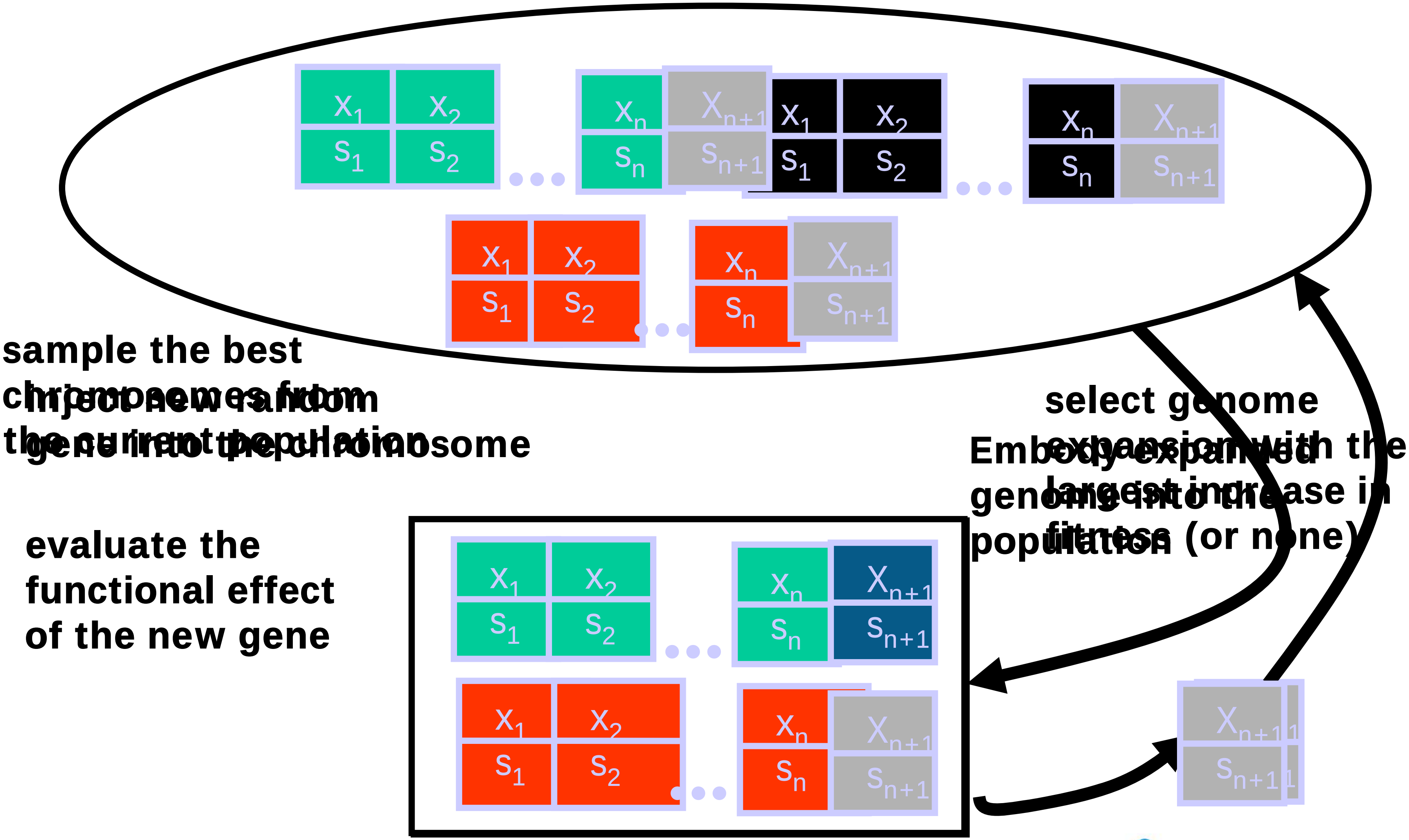
IF x_1 IS A_1 AND x_2 IS B_1 … THEN F=c^1_0+c^1_x

IF x_1 IS A_2 AND x_2 IS B_1 … THEN F=c^2_0+c^2_x

c^1_0 s^1_0	c^2_0 s^2_0	c^1_x s^1_x	c^2_x s^2_x





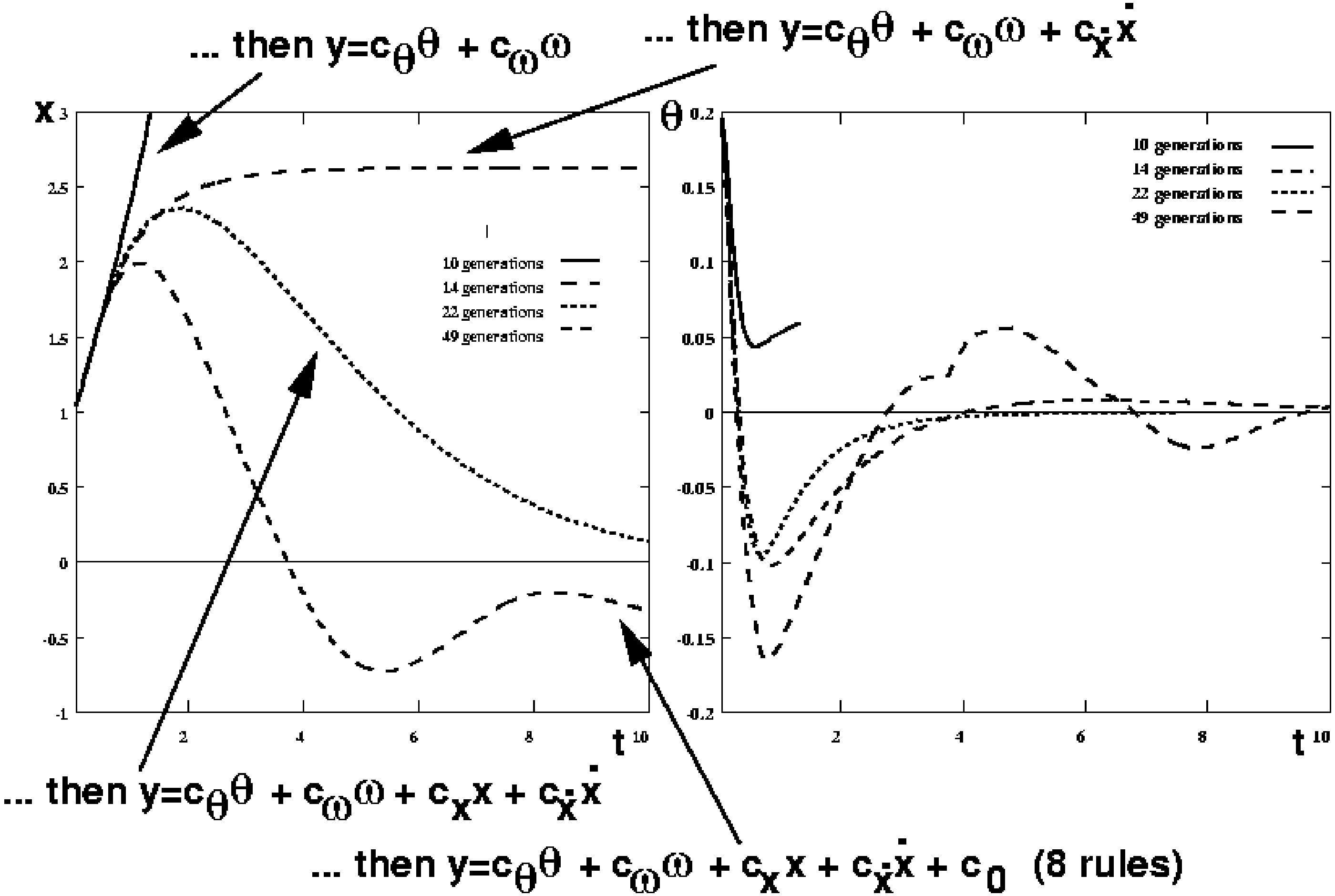


sample the best chromosomes from the current population
Inject new random gene into the chromosome
evaluate the functional effect of the new gene
select genome expansion with the largest increase in fitness (or none)
Embody expanded genome into the population
X_1 X_2 S_1 S_2
X_n X_{n+1} S_n S_{n+1}

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results in the cart pole problem:

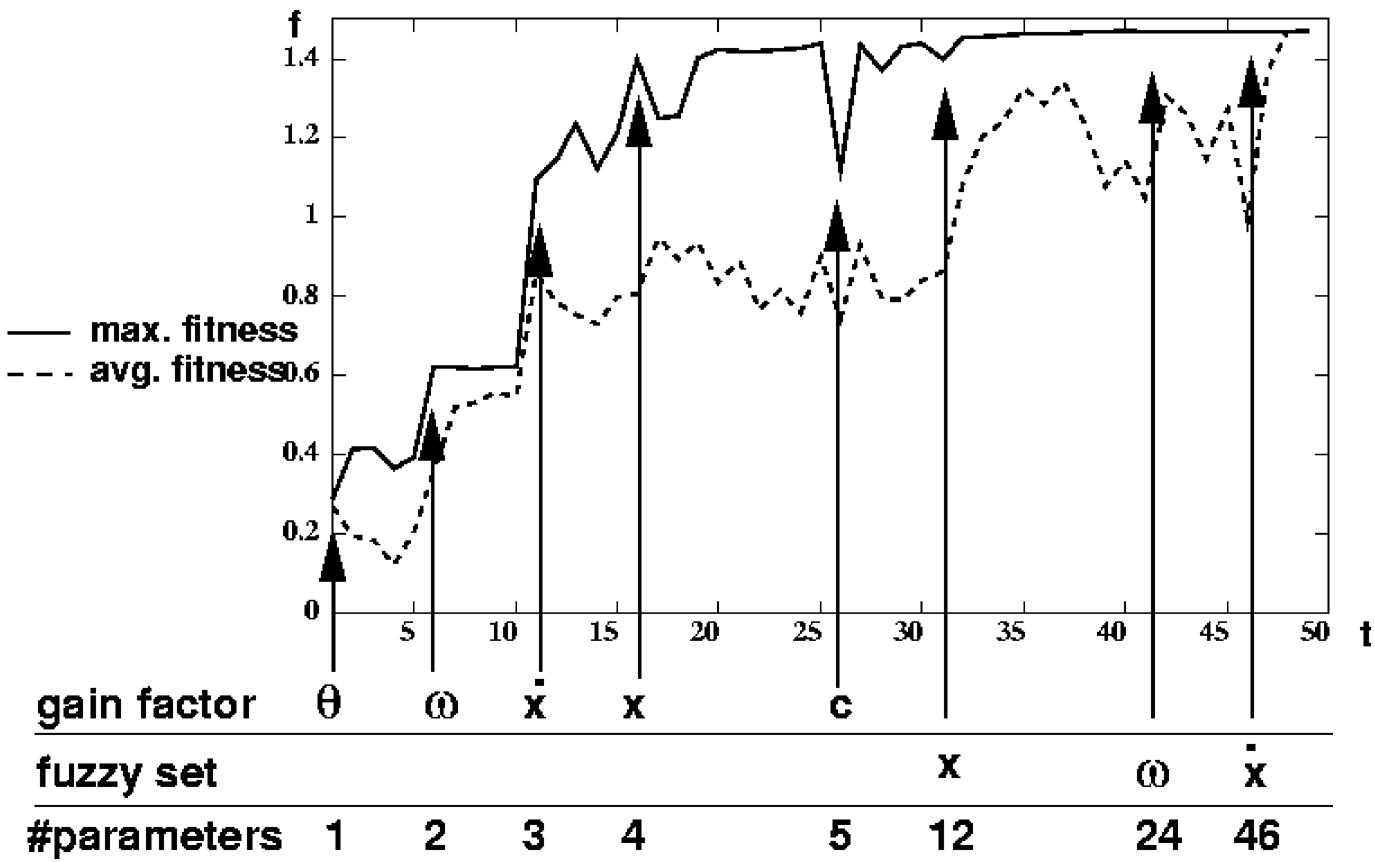


6. Advanced GFS approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

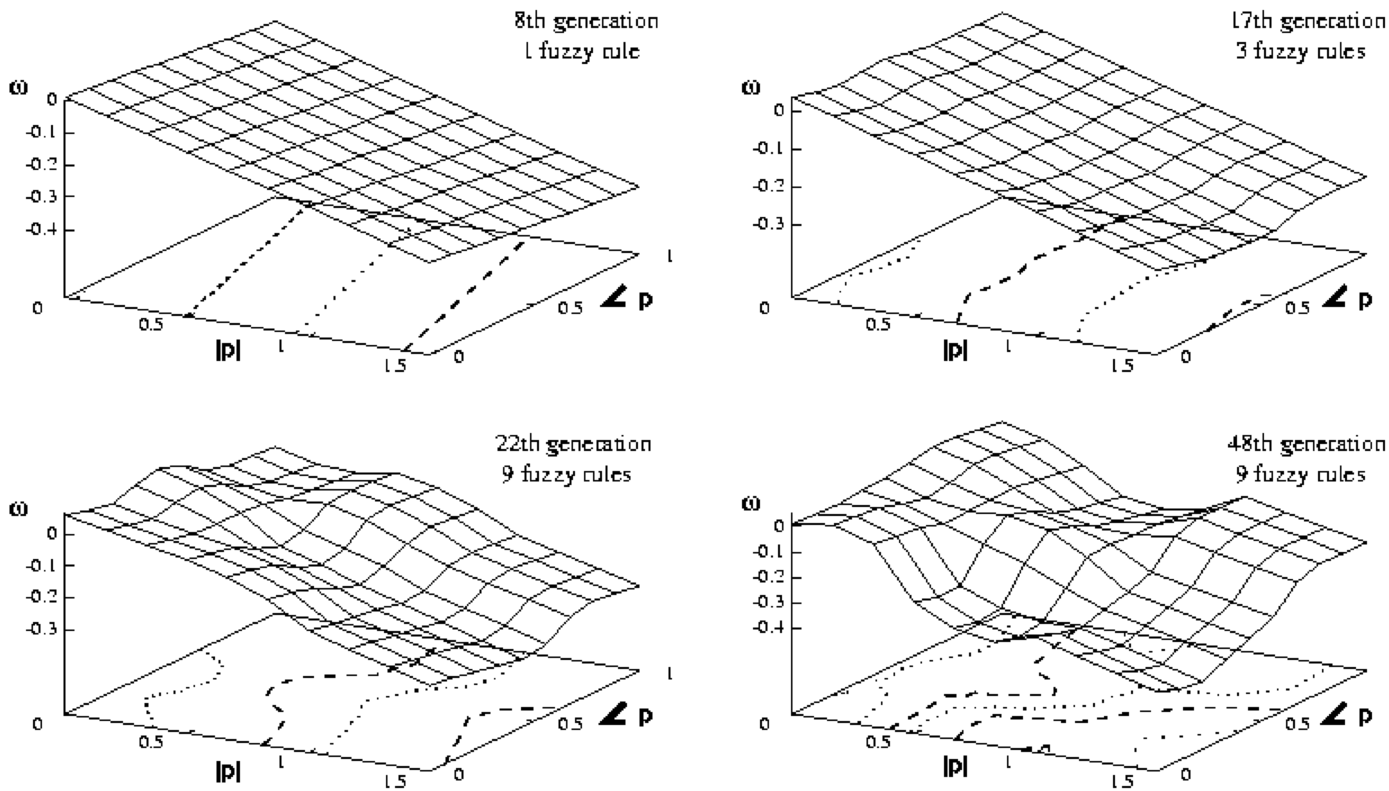
Order of genome expression in the cart pole problem:



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results in a mobile robot problem:



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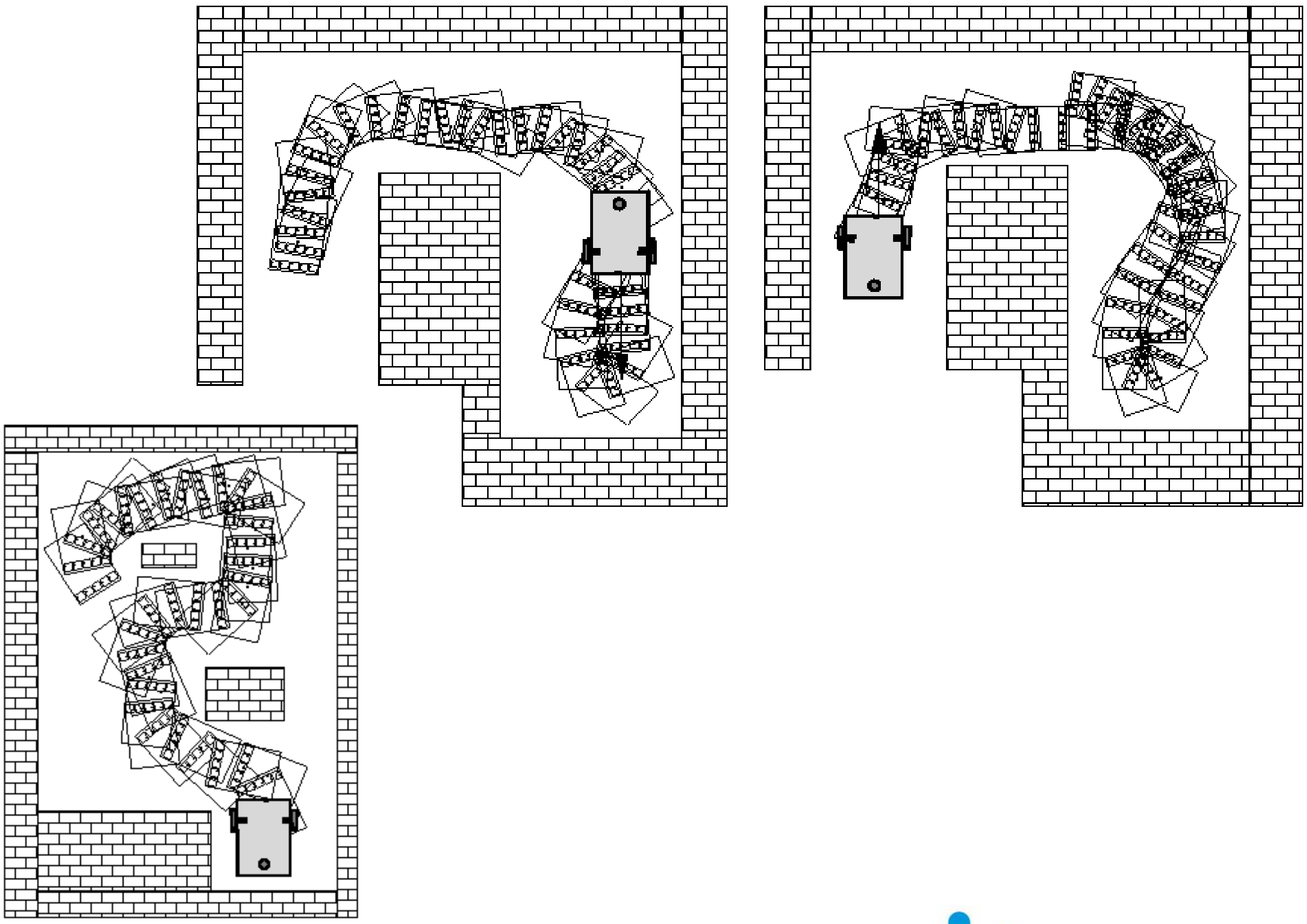
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Experiments on the real robot:



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

New learning schemes

- Learning KBs through *a priori* genetic DB learning
- Coevolutionary GFSs
- Incremental Learning

Interpretability-Accuracy trade-off

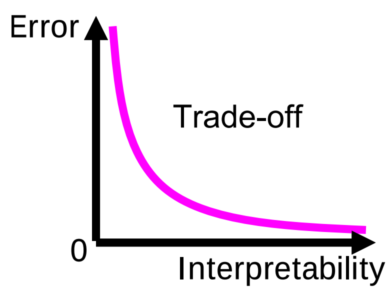
- Multi-objective genetic learning and selection of fuzzy rules
- New fuzzy model structures. Combined parameter learning and rule selection
- Advanced tuning approaches

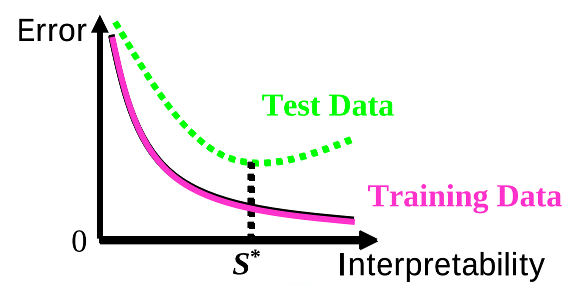
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Interpretability-accuracy trade-off in fuzzy system design

- Every model must satisfy two basic requirements:
  - Accuracy: Actually represent the modeled reality
  - Interpretability: Describe the system in a readable way
- To obtain high degrees for both is a contradictory purpose and, in practice, one of the two properties prevails over the other
- A very simple model does not properly represent the system and a complex model is difficult to understand and generalizes badly
- Obtaining accurate and comprehensible fuzzy models/classifiers/controllers is known as the interpretability-accuracy trade-off



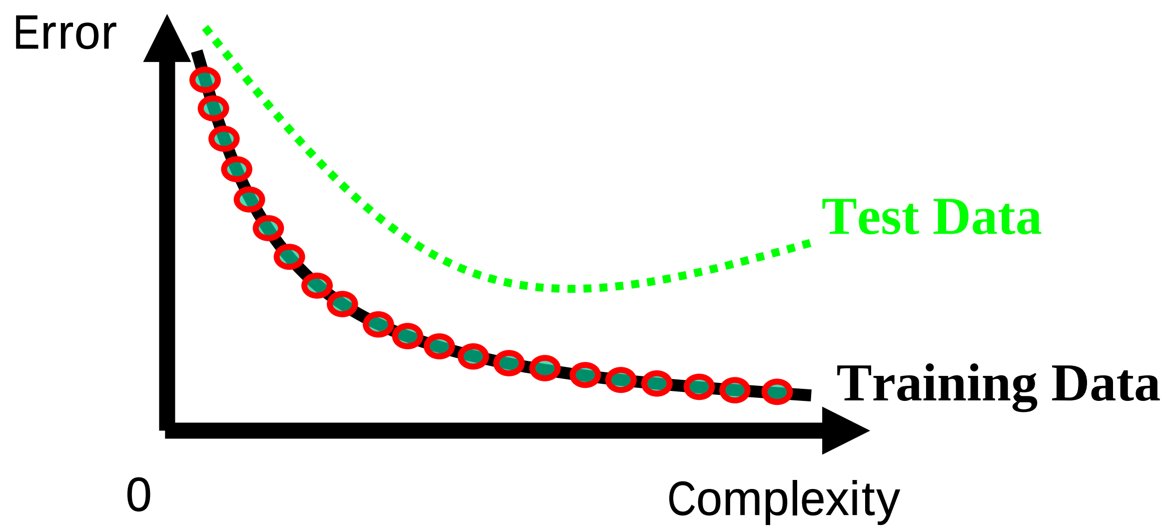


OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Multi-objective genetic learning and selection of fuzzy rules:

- Goal: To find a large number of fuzzy rule sets with different interpretability-accuracy trade-offs



- Problem: Classification problems present a large number of input variables → many rule antecedents and huge number of possible Mamdani fuzzy rules

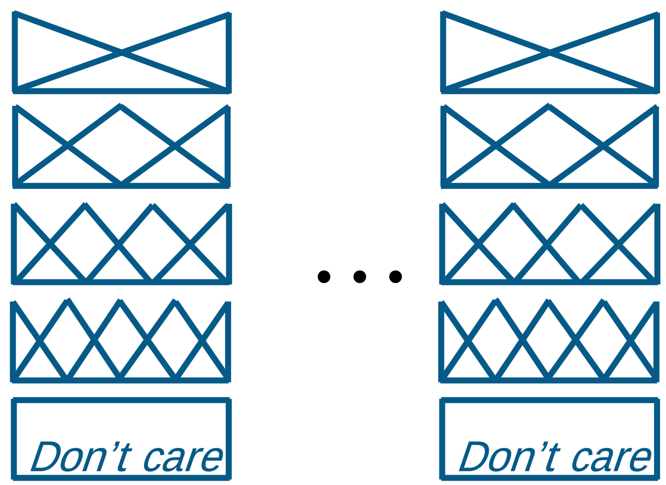
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Two-stage genetic fuzzy system:

H. Ishibuchi, T. Yamamoto, Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining, FSS 141 (2004) 59-88

1. Heuristic Rule Extraction: A pre-specified number of candidate fuzzy rules of different granularity are extracted from numerical data using a heuristic rule evaluation criterion



# of possible rules:

x_1 ... x_n

$(14+1) \times \ldots \times (14+1) = 15^n$

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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

2. Genetic Rule Selection: A small fuzzy rule set is selected from the extracted candidate rules using a multi-objective GA

- Binary coding scheme

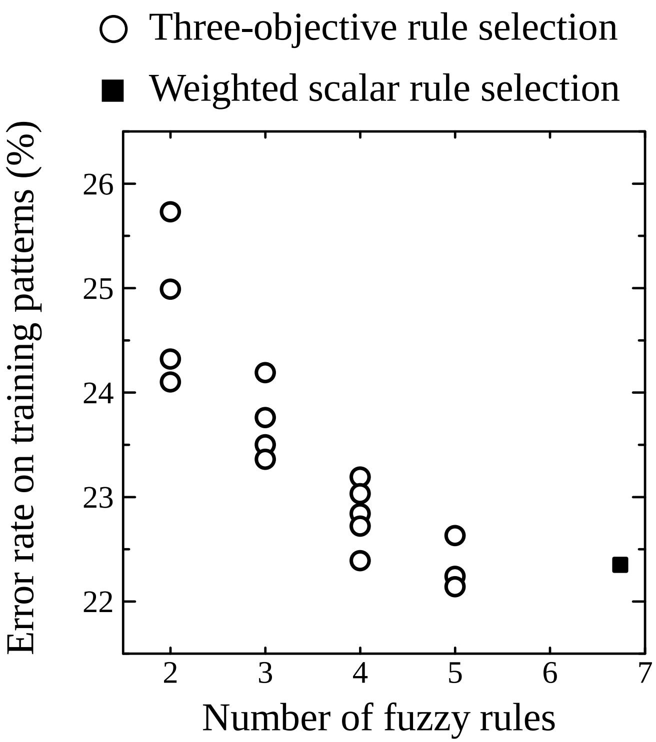
- Objectives:
  - $f_1(S)$: Number of correctly classified patterns by S
  - $f_2(S)$: Number of selected rules in S
  - $f_3(S)$: Total number of antecedent conditions in S

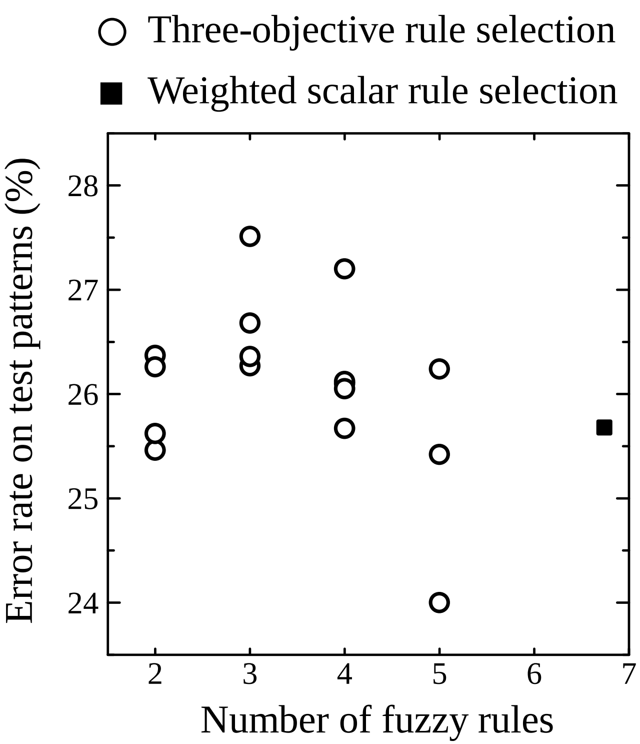
- Multicriteria approaches:
  1. Two-objective approach: Maximize $f_1(S)$ and minimize $f_2(S)$
  2. Weighted sum of the two objectives: Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S)$
  3. Three-objective approach: Maximize $f_1(S)$ and minimize $f_2(S)$, $f_3(S)$
  4. Weighted sum of the three objectives: Max $w_1 \cdot f_1(S) - w_2 \cdot f_2(S) - w_3 \cdot f_3(S)$

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Example of the obtained results (Diabetes):





A single rule set is obtained by the weighted sum approach

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

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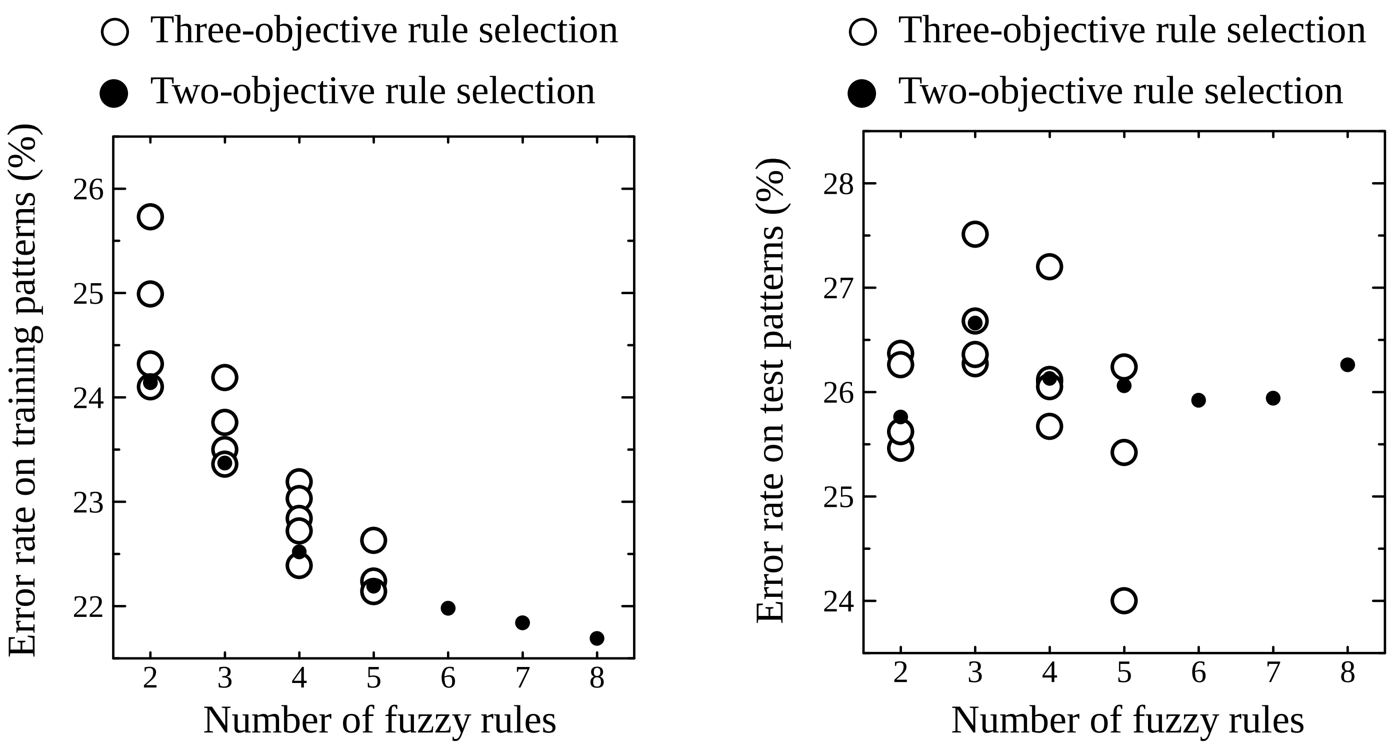

The effect in the rule increase is not clear

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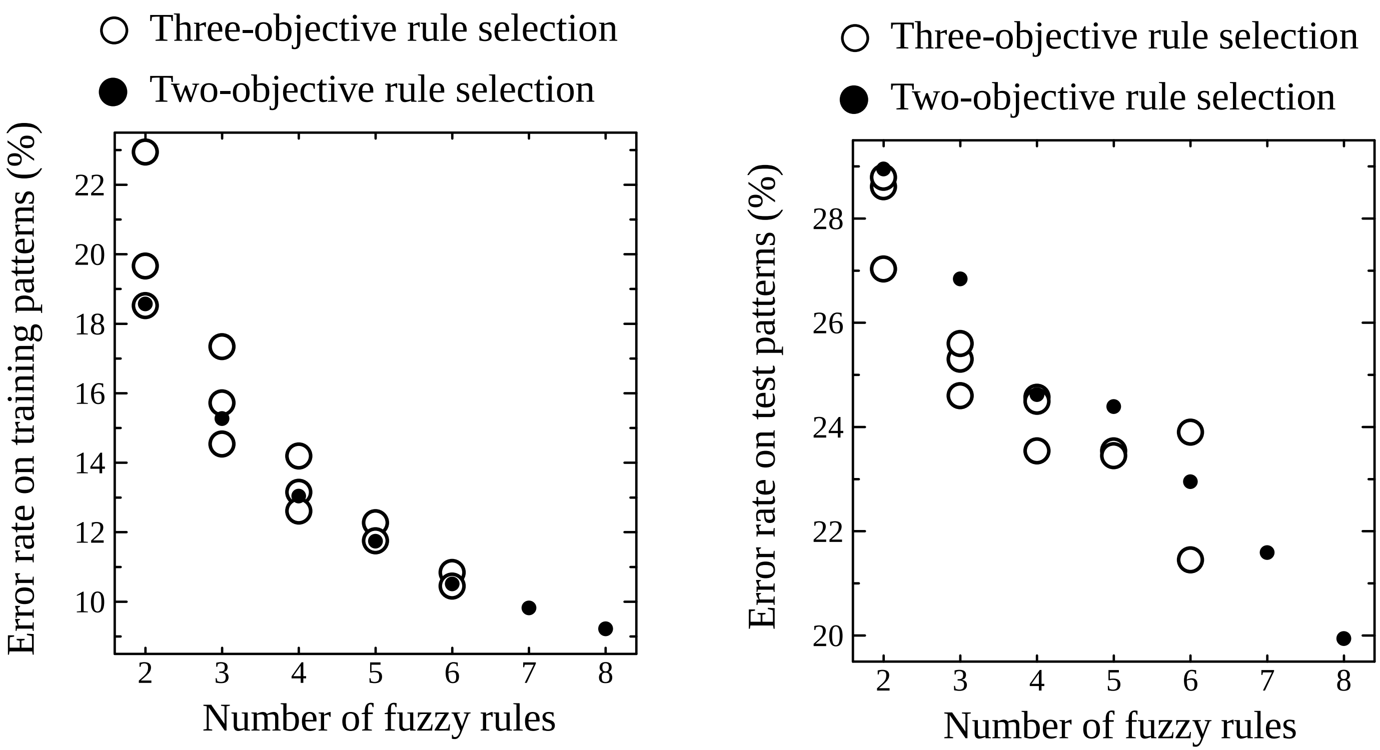
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Example of the obtained results (Sonar):



The generalization ability is increased by increasing the number of fuzzy rules (i.e., the overfitting is not observed)

6. Advanced GFS approaches

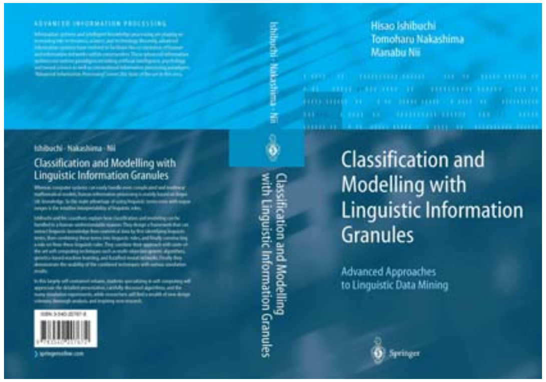
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

References:

- H. Ishibuchi, T. Murata, I.B. Türksen, Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems, FSS 89 (1997) 135–150
- H. Ishibuchi, T. Nakashima, T. Murata, Three-objective genetics-based machine learning for linguistic rule extraction, Inform. Sci. 136 (1–4) (2001) 109–133
- H. Ishibuchi, T. Yamamoto, Rule weight specification in fuzzy rule-based classification systems, IEEE TFS 13 (4) (2005) 428-435

H. Ishibuchi, T. Nakashima, M. Nii, Classification and Modeling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer (2005)



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

New fuzzy model structures:

- Use of linguistic hedges:

IF X_1 is lh_{X1} A_1 AND ... AND X_n IS lh_{Xn} A_n THEN Y IS lh_Y B

- Use of more than one consequent for each rule:

IF X_1 is A_1 AND ... AND X_n IS A_n THEN Y IS {B_1, ..., B_c}

- Use of weighted rules:

IF X_1 is A_1 AND ... AND X_n IS A_n THEN Y IS B with [*w*]

The creation of this new fuzzy rule models require sophisticated (genetic) learning approaches and selection methods to promote rule cooperation

R. Alcalá, J. Alcalá-Fdez, J. Casillas, O. Cordón, F. Herrera, Hybrid learning models to get the interpretability-accuracy trade-off in fuzzy modelling, Soft Computing 10 (9 (2006) 717-734

OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

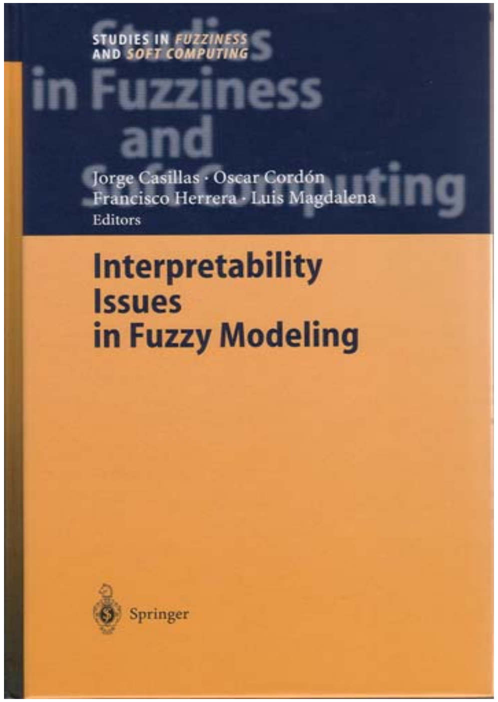
5. Some real-world applications

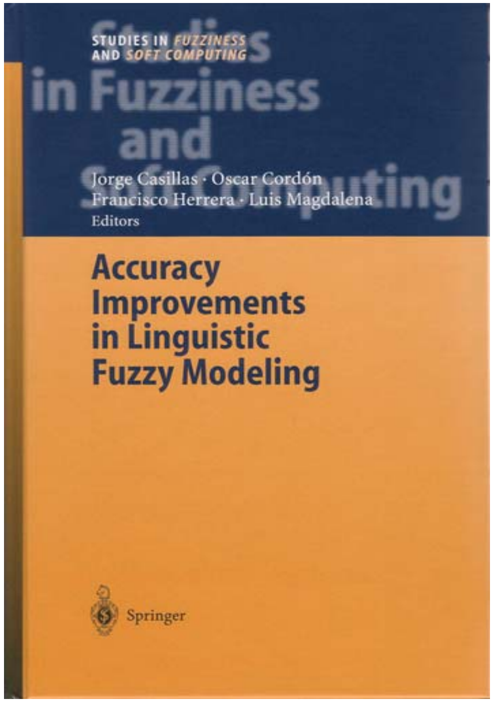
6. Advanced GFS approaches

7. Conclusions. What's next?

References:

- J. Casillas, O. Cordón, F. Herrera, L. Magdalena (Eds.). Springer-Verlag, 2003





OUTLINE

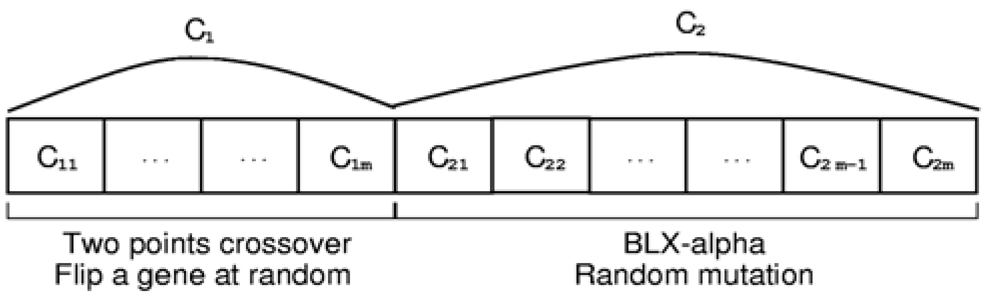
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Joint weight derivation-rule selection process:

R. Alcalá, J. Casillas, O. Cordón, A. González, F. Herrera, A genetic rule weighting and selection process for fuzzy control of Heating, Ventilating and Air Conditioning Systems, Engineering Applications of Artificial Intelligence 18 (3) (2005) 279-296

GA with a two-level coding scheme: $C = (C_1,C_2)$

- C_1 (selection): binary chromosome of length *m* (# of simple Mamdani-type rules derived in a first learning stage)
- C_2 (weights): real-coded chromosome of length *m*. Each gene encodes the weight ([0,1]) for the corresponding rule
- Genetic operators: cooperatively working in the two-level structure:

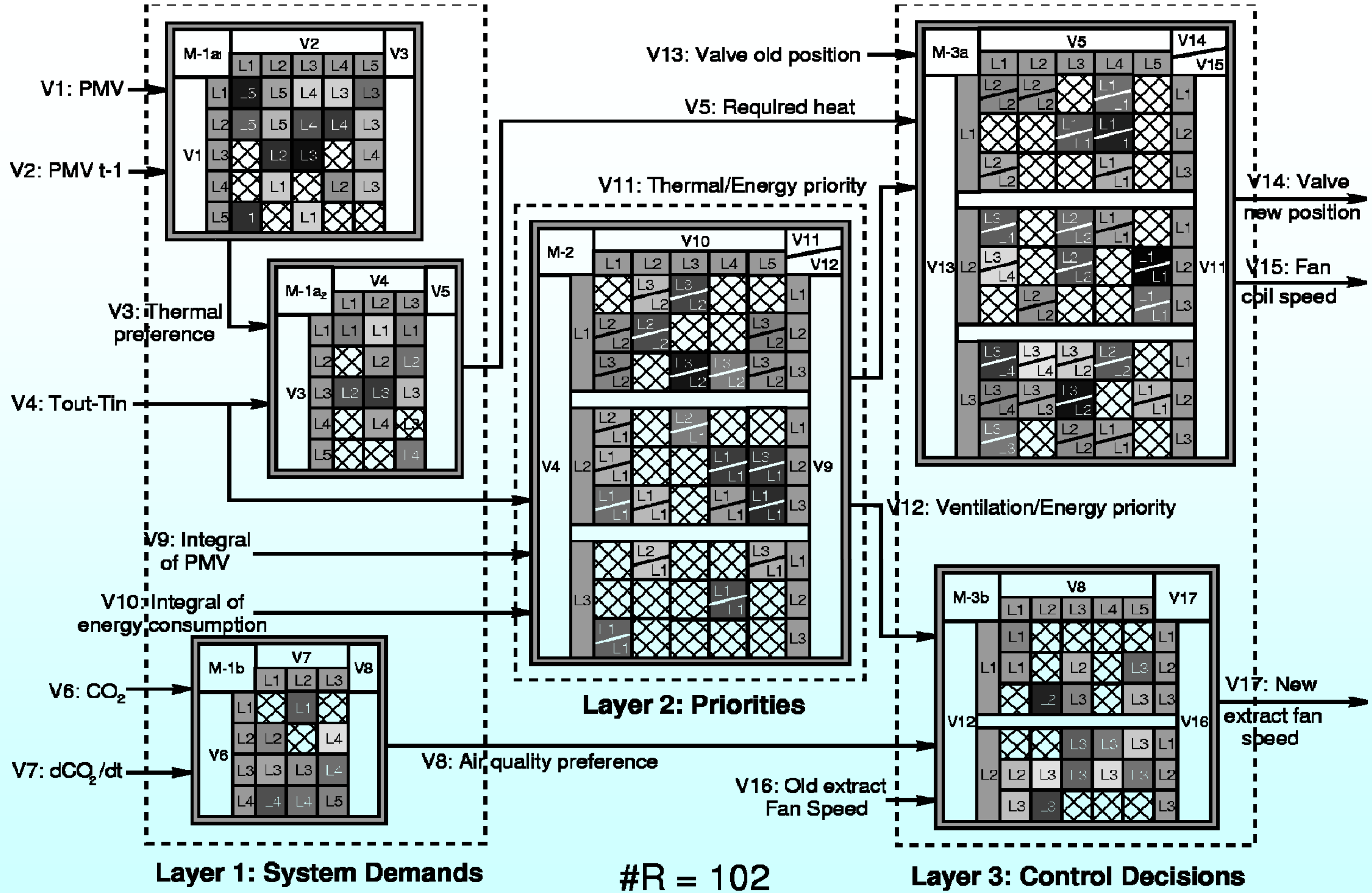


OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results for the HVAC FLC tuning problem:

	#R	PMV>0.5		PMV<-0.5		CO_2		Energy		Stability	
MODEL	–	O_1	%	O_2	%	O_3	%	O_4	%	O_5	%
Goals (g_i)	–	1.0	–	1	–	7	–	2000000	–	1000	–
ON-OFF	–	0.0	–	0	–	0	–	3206400	–	1136	–
FLC	172	0.0	–	0	–	0	–	2901686	9.50	1505	-32.48
Considering Data Base tuning											
DB 3	172	0.0	–	0	–	0	–	2596875	19.01	1051	7.48
Considering rule weights and rule selection											
Seed 1	123	0.9	–	0	–	0	–	2769621	13.62	970	14.61
Seed 2	102	0.7	–	0	–	0	–	2731798	14.80	942	17.08
Seed 3	103	0.2	–	0	–	0	–	2766135	13.73	936	17.61



Module $1a_1$: Thermal Demands
Module $1a_2$: Thermal Preference
Module 1b: Air Quality Demands

Module 2: Energy Priorities
Module 3a: Required HVAC System Status
Module 3b: Required Ventilation System Status

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Joint coevolutionary double consequent fuzzy rule weight derivation- selection process:

R. Alcalá, J. Casillas, O. Cordón, F. Herrera, Linguistic modeling with weighted double-consequent fuzzy rules based on cooperative coevolutionary learning. Integrated Computer Aided Engineering 10 (4) (2003) 343-355

Cooperative coevolutionary GA with two species:

- S_1 (rule selection): binary chromosome of length m (# of rules derived in a first learning stage). Double-consequent rules are reduced to simple rules

  Two-point crossover and flip mutation

- S_2 (weight derivation): real-coded chromosome of length m. Each gene encodes the weight ([0,1]) for the corresponding rule

  Max-min-arithmetical crossover and random mutation

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Experimental study for the low voltage power line problem:

Method	Description
WM	Ad hoc data-driven method
ALM	The ALM double-consequent fuzzy rule method
WRL	The WRL weighted fuzzy rule method
WALM	A simple GA that learns weighted double-consequent fuzzy rules as a first approximation to the problem
WALM-CC	The proposed cooperative coevolutionary GA to learn weighted double-consequent fuzzy rules

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results for the low voltage power line problem:

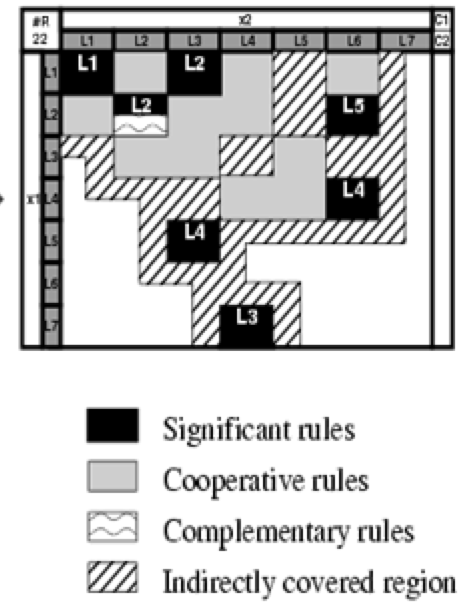
Method	#R	MSE_{tra}	MSE_{tst}
WM	24	222,654	239,962
ALM	20	155,866	178,601
WRL	24	149,303	182,249
WALM	26	151,359	182,997
WALM-CC	22	144,290	176,057
NN 2-25-1	102 par.	169,399	167,092

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained fuzzy model:

#R 22		x2 L1	x2 L2	x2 L3	x2 L4	x2 L5	x2 L6	x2 L7	C1 C2
x1	L1	L1 - 0.8	L2 - 0.4 L1 - 0.5	L2 - 0.7	L2 - 0.4		L2 - 0.1		
x1	L2	L1 - 0.6	L2 - 0.9 L3 - 0.1	L2 - 0.5 L4 - 0.5	L5 - 0.3 L3 - 0.2		L5 - 0.2		
x1	L3		L2 - 0.3	L4 - 0.2		L3 - 0.1			
x1	L4				L6 - 0.2	L7 - 0.2	L4 - 1.0		
x1	L5			L4 - 0.1					
x1	L6								
x1	L7				L3 - 0.1				



OUTLINE

1. Introduction to GFSs

2. GFSs roadmap and milestones

3. Evolutionary tuning of FRBSs

4. Classical GFS learning approaches

5. Some real-world applications

6. Advanced GFS approaches

7. Conclusions. What's next?

References:

- O. Cordón, F. Herrera, A proposal for improving the accuracy of linguistic modeling, IEEE TFS 8 (3) (2000) 335-344
- J. Casillas, O. Cordón, F. Herrera, COR: A methodology to improve ad hoc data-driven linguistic rule learning methods by inducing cooperation among rules. IEEE TSMC. Part B: Cybernetics 32 (4) (2002) 526-537
- O. Cordón, F. Herrera, I. Zwir, Linguistic modeling by hierarchical systems of linguistic rules, IEEE TFS 10 (1) (2002) 2-20
- R. Alcalá, J.R. Cano, O. Cordón, F. Herrera, P. Villar, I. Zwir, Linguistic Modeling with Hierarchical Systems of Weighted Linguistic Rules, IJAR 32 (2-3) (2003) 187-215

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Genetic tuning of DB and RB using linguistic hedges:

J. Casillas, O. Cordón, M.J. del Jesus, F. Herrera, Genetic tuning of fuzzy rule deep structures preserving interpretability and its interaction with fuzzy rule set reduction, IEEE TFS 13 (1) (2005) 13-29

Genetic tuning process that refines a preliminary KB working at two different levels:

- DB level: Linearly or non-linearly adjusting the membership function shapes
- RB level: Extending the fuzzy rule structure using automatically learnt linguistic hedges

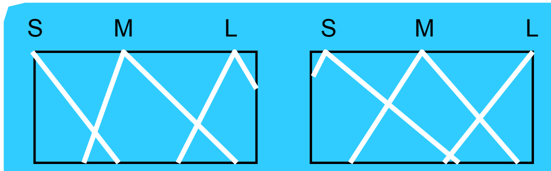
6. Advanced GFS approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What’s next?

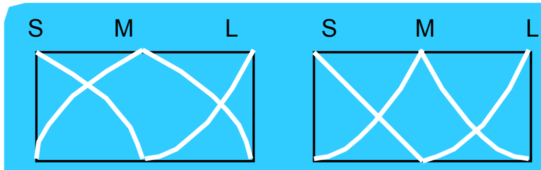
- Tuning of the DB:

Linear tuning



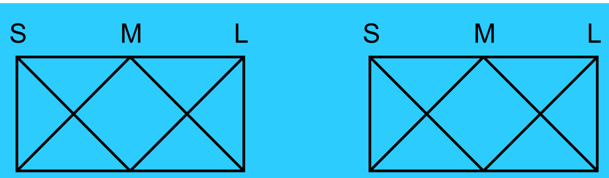
R_1= IF X is S THEN Y es M
R_2= IF X Is M THEN Y es L
R_3= IF X Is L THEN Y es S

Non-linear tuning



R_1= IF X is S THEN Y es M
R_2= IF X Is M THEN Y es L
R_3= IF X Is L THEN Y es S

- Tuning of the RB: linguistic hedges ‘very’ and ‘more-*or-less*’



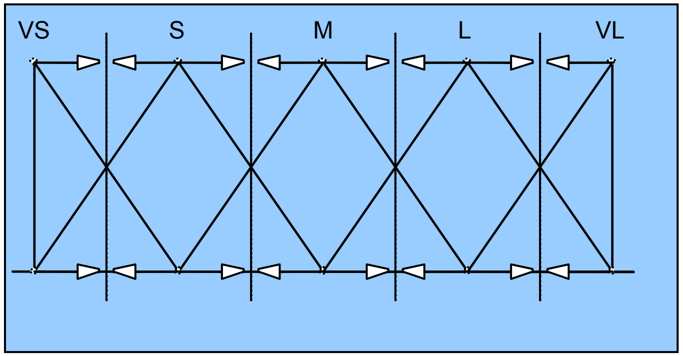
R_1= IF X is **more-or-less** S THEN Y is M
R_2= IF X is **very** M THEN Y is **more-or-less** L
R_3= IF X is **very** L THEN Y is **very** S

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Triple coding scheme:

- Membership function parameters (P) (DB linear tuning): real coding



- Alpha values (A) (DB non linear tuning): real coding

$$\alpha = \begin{cases} 1 + c_{ij}^{A}, & \text{si } c_{ij}^{A} \in [-1,0] \\ 1 + 4 \cdot c_{ij}^{A}, & \text{si } c_{ij}^{A} \in]0,1] \end{cases}$$

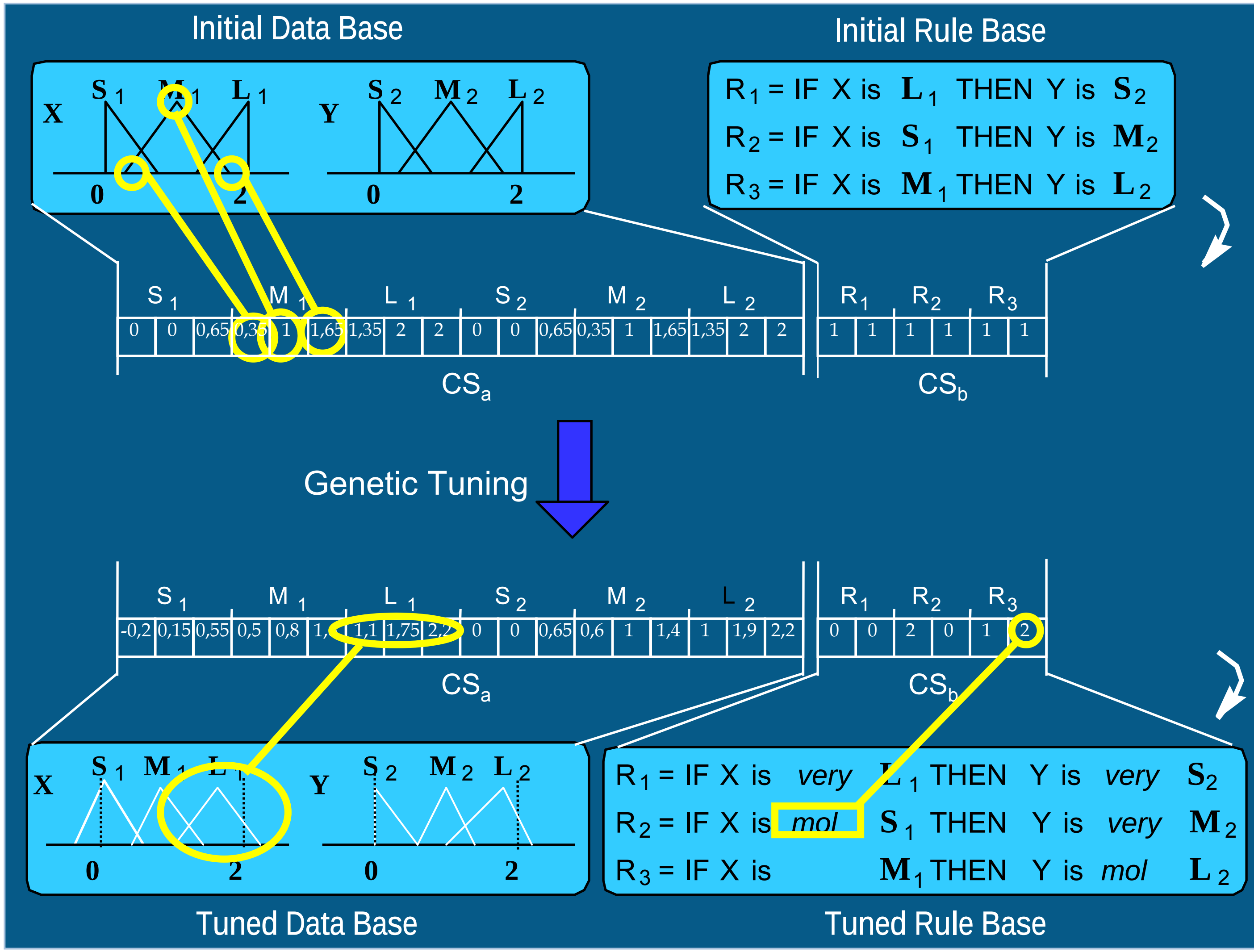
- Linguistic hedges (L) (RB tuning): integer coding

$$\begin{array}{lll} c_{ij} = 0 & \leftrightarrow & \text{'very'} \\ c_{ij} = 1 & \leftrightarrow & \text{no hedge} \\ c_{ij} = 2 & \leftrightarrow & \text{'more-or-less'} \end{array}$$

6. Advanced GFS approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?



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OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Experimental study for the medium voltage line problem:

- Learning method considered: Wang-Mendel
- Tuning method variants:

TUNING PROCESSES CONSIDERED IN THIS EXPERIMENTAL STUDY

Method	Basic m.f. parameters	α m.f. parameter	Surface structure with linguistic hedges
P-tun	✓		
A-tun		✓	
L-tun			✓
PA-tun	✓	✓	
PL-tun	✓		✓
AL-tun		✓	✓
PAL-tun	✓	✓	✓

- Evaluation methodology: 5 random training-test partitions 80-20% (5-fold cross validation) $\times$ 6 runs = 30 runs per algorithm

6. Advanced GFS approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results for the medium voltage line problem:

Tuning methods:

	Electrical Problem									
	$\bar{x}$				σ_{x_i}			$\sigma_{\bar{x}_i}$		
Method	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**h:m:s**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**
WM	**65**	56,135	56,359	0:00:00	**0.0**	1,498	4,685	—	—	—
WM+P-tun	65	18,395	22,136	**0:22:41**	0.0	778	3,200	—	1,110	1,988
WM+A-tun	65	37,243	38,837	0:33:58	0.0	455	1,816	—	**125**	**572**
WM+L-tun	65	20,967	23,420	0:25:16	0.0	632	3,207	—	336	1,439
WM+PA-tun	65	17,967	21,377	0:38:02	0.0	1,078	1,625	—	2,133	2,628
WM+PL-tun	65	**9,617**	**13,519**	0:25:33	0.0	**263**	3,153	—	694	1,509
WM+AL-tun	65	20,544	23,207	0:34:55	0.0	834	2,701	—	797	1,430
WM+PAL-tun	65	11,222	14,741	0:38:12	0.0	380	**1,315**	—	801	2,136

Other fuzzy modeling techniques and GFS:

	Electrical Problem									
	$\bar{x}$				σ_{x_i}			$\sigma_{\bar{x}_i}$		
Method	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**h:m:s**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**
Nozaki [5]	**532**	**26,705**	**27,710**	**0:00:00**	**0.0**	**764**	**2,906**	—	—	—
Thrift [38]	565.3	31,228	37,579	3:13:25	2.6	1,018	7,279	6.1	**2,110**	**3,609**
Liska [45]	624.9	49,263	56,089	7:13:34	0.1	2,356	4,628	**0.1**	7,522	11,191

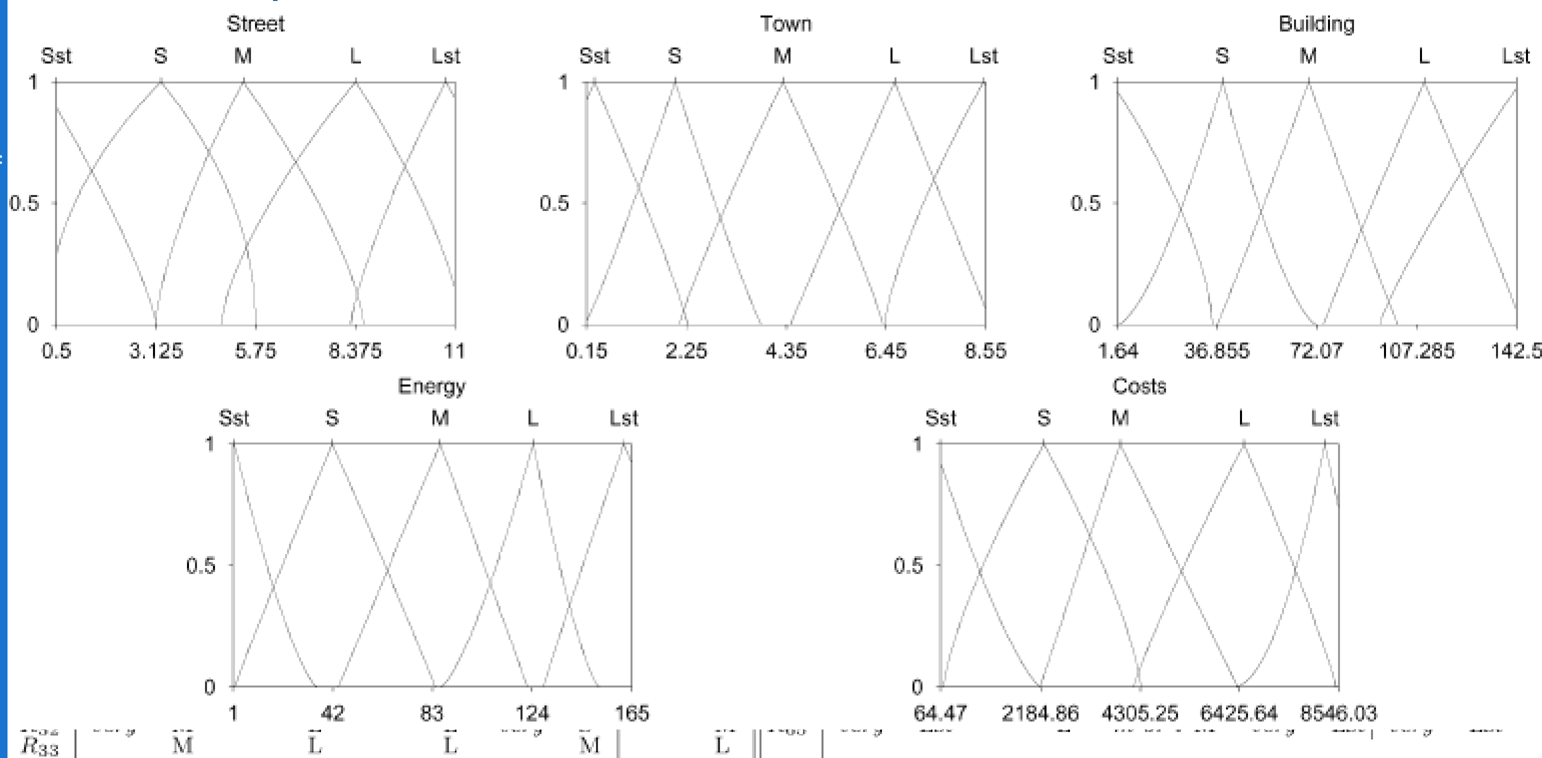
6. Advanced GFS approaches

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. **Advanced GFS approaches**
7. Conclusions. What's next?

Obtained results for the medium voltage line problem:

Example of one KB derived from the WM+PAL-tun method:



Before tuning: $MSE_{tra/test}$ = 58032 / 55150

After tuning: $MSE_{tra/test}$ = 11395 / 14465

OUTLINE

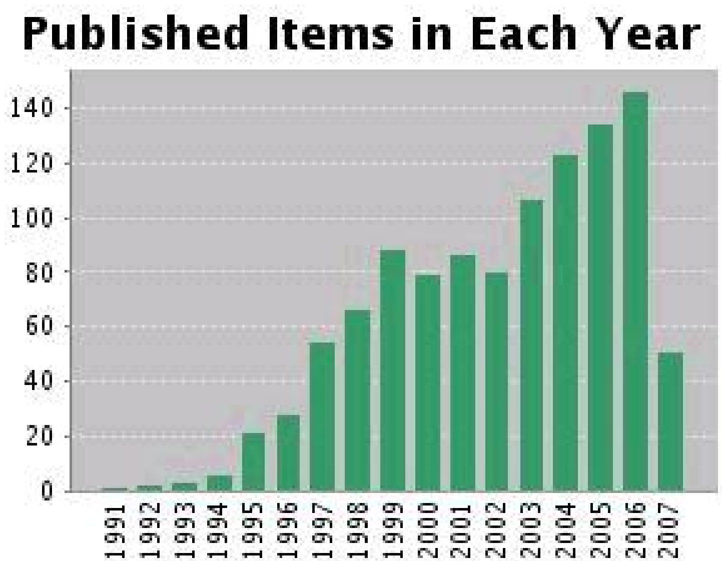
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

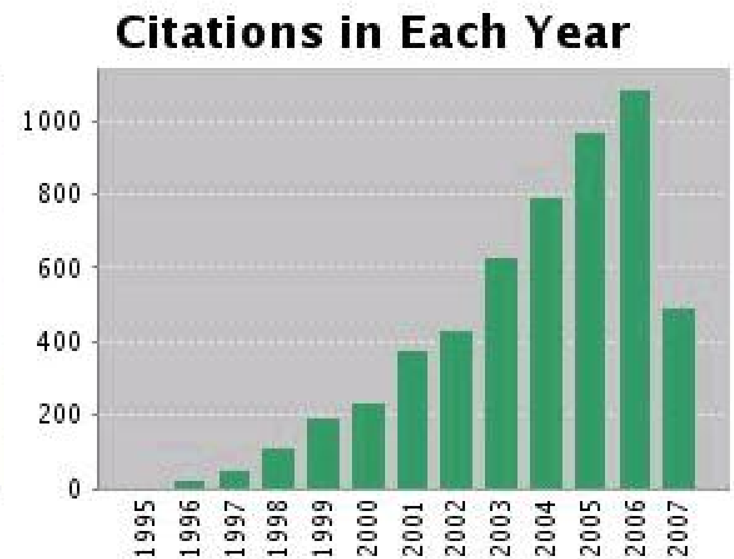
- **GFSs statistics**
- **Critical view of GFSs**
- **What's next?**

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Number of papers on GFSs published in JCR journals:





Source: The Thomson Corporation ISI Web of Knowledge

Query: (evolutionary OR "genetic algorithm*" OR "genetic programming" OR "evolution strate*") AND ("fuzzy rule*" OR "fuzzy system*" OR "fuzzy neural" OR "neuro-fuzzy" OR "fuzzy control*" OR "fuzzy logic control*" OR "fuzzy classif*")

Date: July, 5, 2007

Number of papers: 1080

Number of citations: 5403

Average citations per paper: 5.0

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Most cited papers on GFSs:

1. Homaifar, A., McCormick, E., Simultaneous Design of Membership Functions and rule sets for fuzzy controllers using genetic algorithms, IEEE TFS 3 (2) (1995) 129-139. **Citations: 161**
2. Ishibuchi, H., Nozaki, K., Yamamoto, N., Tanaka, H., Selecting fuzzy if-then rules for classification problems using genetic algorithms, IEEE TFS 3 (3) (1995) 260-270. **Citations: 151**
3. Setnes, M., Roubos, H., GA-fuzzy modeling and classification: complexity and performance, IEEE TFS 8 (5) (2000) 509-522 . **Citations: 86**
4. Park, D., Kandel, A., Langholz, G., Genetic-based new fuzzy reasoning models with application to fuzzy control, IEEE TSMC B 24 (1) (1994) 39-47. **Citations: 82**
5. Ishibuchi, H., Nakashima, T., Murata, T., Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems, IEEE TSMC B 29 (5) (1999) 601-618. **Citations: 79**
6. Herrera, F., Lozano, M., Verdegay, J.L., Tuning fuzzy-logic controllers by genetic algorithms, IJAR 12 (3-4) (1995) 299-315. **Citations: 64**
7. Shi, Y.H., Eberhart, R., Chen, Y.B., Implementation of evolutionary fuzzy systems, IEEE TFS 7 (2) (1999) 109-119. **Citations: 57**
8. Carse B., Fogarty, TC., Munro, A., Evolving fuzzy rule based controllers using genetic algorithms, FSS 80 (3) (1996) 273-293. **Citations: 54**
9. Linkens, D.A., Nyongesa, H.O., Genetic algorithms for fuzzy control. 1. Offline system-development and application. IEE PROCEEDINGS-CONTROL THEORY AND APPLICATIONS 142 (3) (1995) 161-176. **Citations: 51**
10. Cordon, O., Herrera, F., A three-stage evolutionary process for learning descriptive and approximate fuzzy-logic-controller knowledge bases from examples, IJAR 17 (4) (1997) 369-407. **Citations: 50**

Date: July, 5, 2007

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Authors with the largest publication record on GFSs:

Order	Author	Record count	% of 1080	GFS h index
1	Oh, S.K.	36	3.3333%	6
2	Herrera, F.	33	3.0556%	11
3	Pedrycz, W.	30	2.7778%	6
4	Cordón, O.	25	2.3148%	11
5	Ishibuchi, H.	18	1.6667%	8
6	Wong, C.C.	13	1.2037%	5
7	Rojas, I.	12	1.1111%	4
8	Ahn, T.C.	11	1.0185%	1
8	Hoffmann, F.	11	1.0185%	6
8	Linkens, D.A.	11	1.0185%	5
8	Pratihar, D.K.	11	1.0185%	3

Date: July, 5, 2007

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Critical view of GFSs:

What is the actual GFS competence?

- Advantages and drawbacks with respect to other Computational Intelligence techniques
- Capability to solve real-world problems
- Visibility of GFSs outside the fuzzy community
- Impact of GFSs in a broader research community

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

What are GFS researchers doing badly?

Experimental setup:

- Extended use of toy problems in journal papers
- Just one (or at most a few) algorithm run. No statistical test use for the performance checking
- "Soft comparison" against other classical and Computational Intelligence tools for the problem tackled
- Need of benchmark problem databases (only existing for classification applications (UCI))

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

What's next in GFSs:

Forecasting:

- New learning approaches and coding schemes
- New application areas: Internet, Bioinformatics, ...
- More multi-objective approaches
- Increasing interest on the interpretability-accuracy trade-off
- More real-world applications
- Scaling up to high-dimensional problems

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

- **New coding schemes**
  - GFSs based on 2 and 3-tuple fuzzy rule representation
- **New learning schemes**
  - New Michigan GFS
  - New multi-objective GFS for the interpretability-accuracy trade-off
- **New kinds of problems**
  - GFSs for handling inherently fuzzy data

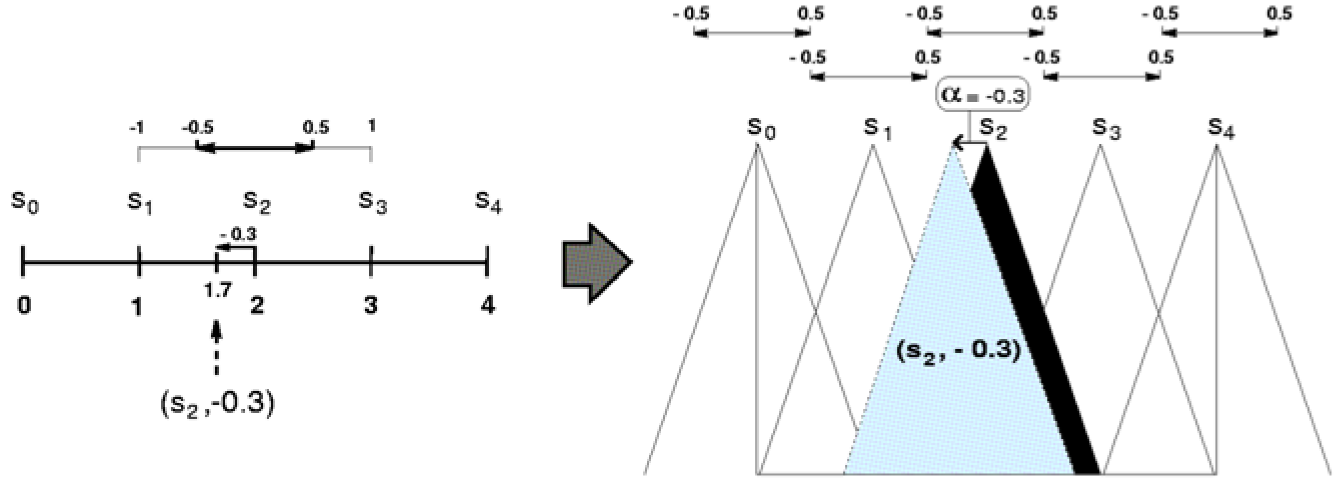
OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

New coding schemes: 2- and 3-tuples:

IDEA: New fuzzy rule representation model permitting a more flexible definition of the fuzzy sets of the linguistic labels

- **2-tuples**: label id. **i** and a displacement parameter $\alpha_i \in [-0.5,0.5]$



a) Simbolic Translation of a label

b) Lateral Displacement of a Membership function

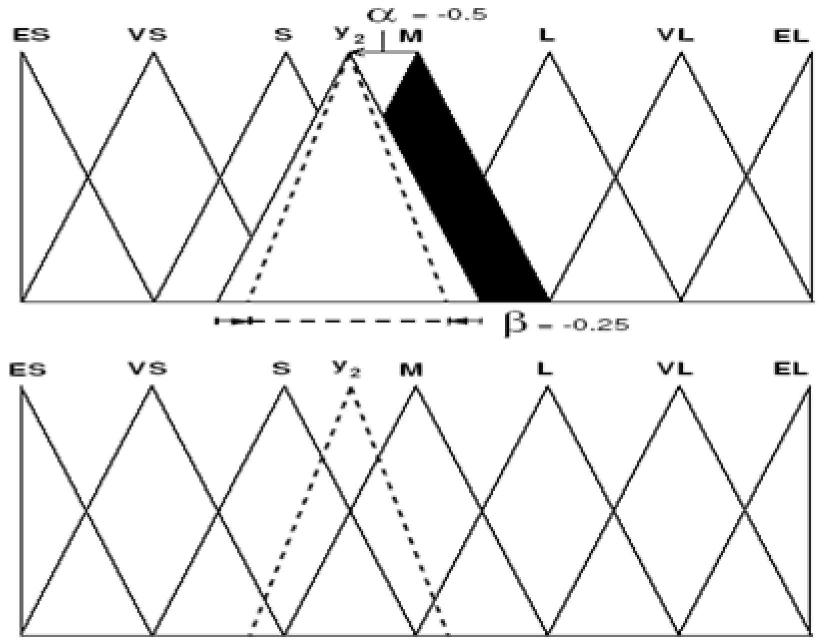
- New rule structure:

$$\text{IF } X_1 \text{ IS } (S^1_i, \alpha_1) \text{ AND ... AND } X_n \text{ IS } (S^n_i, \alpha_n) \text{ THEN } Y \text{ IS } (S^y_i, \alpha_y)$$

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

- **3-tuples**: label id. **i**, a displacement parameter $\alpha_i \in [-0.5,0.5]$, and a width parameter $\beta_i \in [-0.5,0.5]$



- New rule structure:

IF X_1 IS (S^1_i,α_1,β_1) AND ... AND X_n IS (S^n_i,α_n,β_n) THEN Y IS (S^y_i,α_y,β_y)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

New coding schemes: 2- and 3-tuples:

COLLATERAL PRO: Both structures decreases the KB learning/tuning complexity, since the fuzzy sets are encoded using a lower number of parameters

Existing proposals:

- Genetic 2-tuple/3-tuple DB global tuning: adjustment of the global fuzzy sets → full interpretability (usual fuzzy partitions)
- Genetic 2-tuple/3-tuple DB tuning at rule level→ lower interpretability, higher flexibility (like scatter Mamdani FRBSs)
- Genetic 2-tuple/3-tuple DB tuning + rule selection
- KB derivation through a priori genetic 2-tuple/3-tuple DB learning: granularity and 2-tuple/3-tuple parameter learning → full interpretability (usual fuzzy partitions)

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Obtained results for the medium voltage line problem:

Genetic 2-tuple tuning + rule selection method:

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
Approaches without tuning							
WM	12.4	234712	32073	+	242147	24473	+
S	10.0	226135	19875	+	241883	19410	+
Approaches with global semantics							
T	12.4	158662	6495	+	221613	29986	+
T+S	8.9	156313	2967	+	193477	49912	=
GL_{dd}	12.4	166674	11480	+	189216	14743	=
GL_{dd}+S	9.0	160081	7316	+	189844	22448	=
Approaches with local semantics							
PAL	12.4	141638	4340	+	189279	19523	=
PAL+S	10.6	145712	5444	+	191922	16987	=
LL_{dd}	12.4	**139189**	3155	⋆	191604	18243	=
LL_{dd}+S	10.5	141446	3444	=	**186746**	15762	⋆

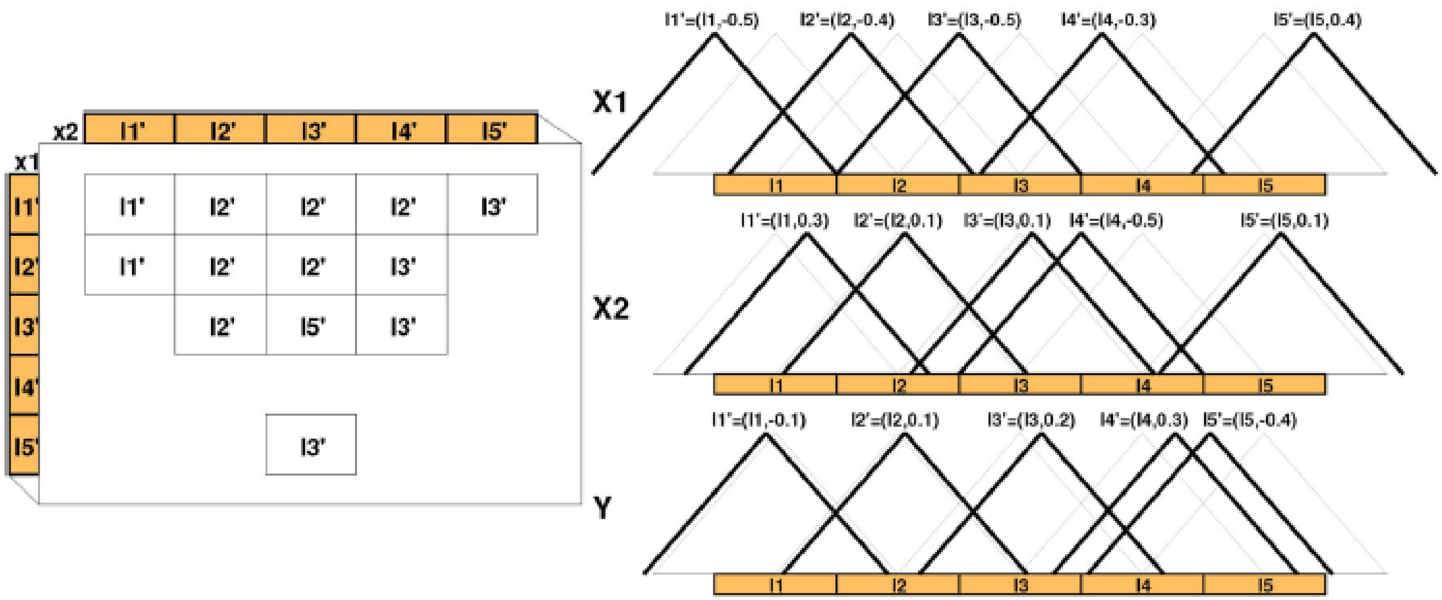
- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Obtained results for the medium voltage line problem:

Example of one KB derived from the global tuning method:



After tuning+rule selection: #R=13; $MSE_{tra/test}$ = 187494 / 176581

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

MODEL	#R	PMV O_1	O_2	CO_2 O_3	Energy O_4	%	Stability O_5	%
Rule Selection								
S1	147	0.2	0	0	2867692	10.56	991	12.76
S2	162	0.0	0	0	2889889	9.87	1441	-26.85
S3	172	0.0	0	0	2901686	9.50	1505	-32.48
Classic Tuning								
C1	172	0.0	0	0	2575949	19.66	1115	1.85
C2	172	0.0	0	0	2587326	19.31	1077	5.19
C3	172	0.0	0	0	2596875	19.01	1051	7.48
Selection with Classic Tuning								
C+S1	94	0.0	0	0	2540065	20.78	1294	-13.91
C+S2	109	0.1	0	0	2492462	22.27	989	12.94
C+S3	100	0.1	0	0	2578019	19.60	887	21.92
Global Lateral Tuning								
$GL_{ss}1$	172	0.7	0	0	2378784	25.81	1069	5.90
$GL_{ss}2$	172	1.0	0	0	2327806	27.40	1066	6.16
$GL_{ss}3$	172	0.9	0	0	2268689	29.25	1080	4.93
Local Lateral Tuning								
$LL_{ss}1$	172	0.9	0	0	2386033	25.59	896	21.13
$LL_{ss}2$	172	0.8	0	0	2343409	26.92	943	16.99
$LL_{ss}3$	172	0.3	0	0	2377596	25.85	938	17.43
Selection with Global Lateral Tuning								
GL_{ss}+S1	105	1.0	0	0	2218598	$\boxed{30.81}$	710	37.50
GL_{ss}+S2	115	0.4	0	0	2358405	26.45	818	27.99
GL_{ss}+S3	118	0.8	0	0	2286976	28.68	872	23.24
Selection with Local Lateral Tuning								
LL_{ss}+S1	133	0.5	0	0	2311986	27.90	788	30.63
LL_{ss}+S2	104	0.6	0	0	2388470	25.51	595	$\boxed{47.62}$
LL_{ss}+S3	$\boxed{93}$	0.5	0	0	2277807	28.96	1028	9.51

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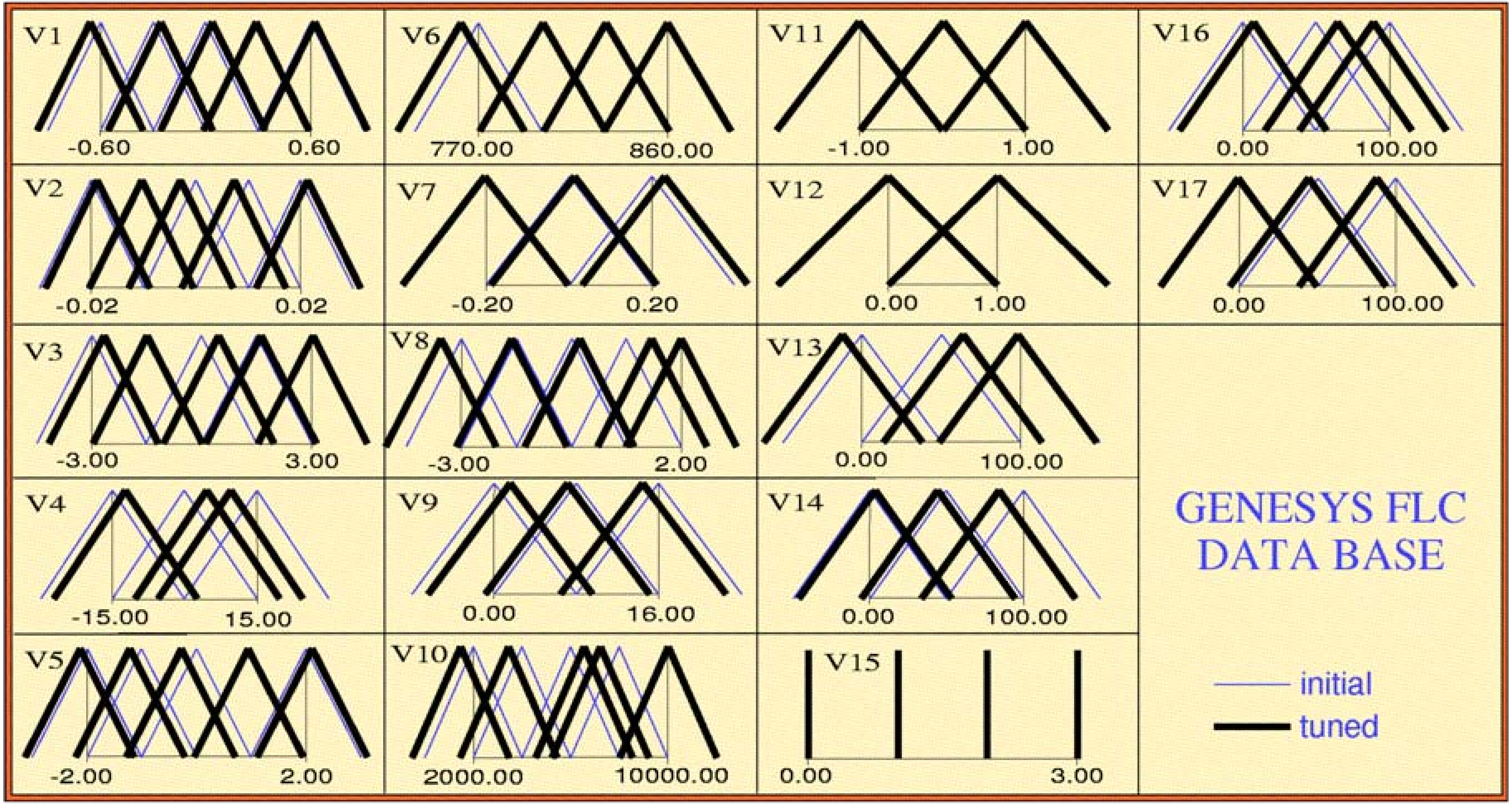
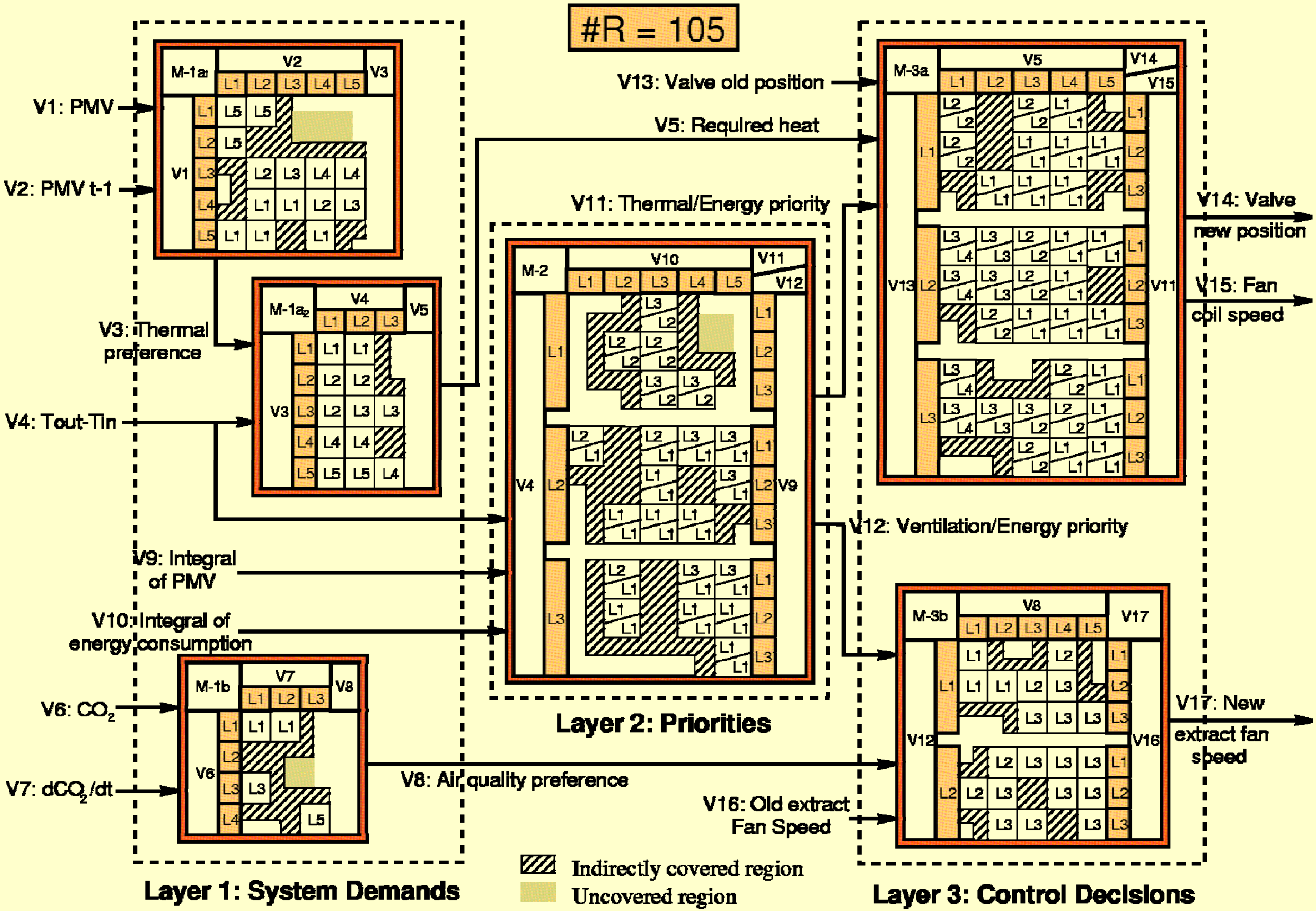
7. Conclusions. What's next?

Tuned Data Base (GL-SS_1):





Module 1a$_1$: Thermal Demands
Module 1a$_2$: Thermal Preference
Module 1b: Air Quality Demands

Module 2: Energy Priorities
Module 3a: Required HVAC System Status
Module 3b: Required Ventilation System Status

7. Conclusions. What's next?

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

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- R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving fuzzy logic controllers obtained by experts: a case study in HVAC systems, Appl. Intel, in press

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

New learning schemes: A new Michigan GFS:

J. Casillas, B. Carse, L. Bull, Fuzzy-XCS: a michigan genetic fuzzy system. IEEE TFS, in press

- Rule generalization (compact rule-based descriptions of state-action relationships) and the interplay between general and specific rules in the same evolving population have received a great attention in non-fuzzy classifier systems (e.g., XCS research)
- but not in Michigan-style fuzzy rule systems due to the difficulty in extending the discrete-valued system operation to the continuous case
- Generalized rules allow more compact rule bases, scalability to higher dimensional spaces, faster inference, and better linguistic interpretability
- It would be a nice solution to the GFS interpretability-accuracy trade-off

PROPOSAL: fuzzy XCS system for single-step reinforcement problems

7. Conclusions. What's next?

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

- Traditional evolutionary reinforcement learning algorithms are "strength-based": a rule accrues strength during interaction with the environment (through rewards and/or penalties)
- A different approach is that were a rule's fitness is based on its "accuracy", i.e. how well a rule predicts payoff whenever it fires
- This accuracy concept is different from the fuzzy modeling one
- Broadly speaking, the strength is the mean of the obtained payoffs and the accuracy is the corresponding standard deviation
- Pros of the accuracy-based approach: avoiding overgeneral rules, obtaining optimally general rules, and learning of a complete "covering map"

7. Conclusions. What's next?

OUTLINE

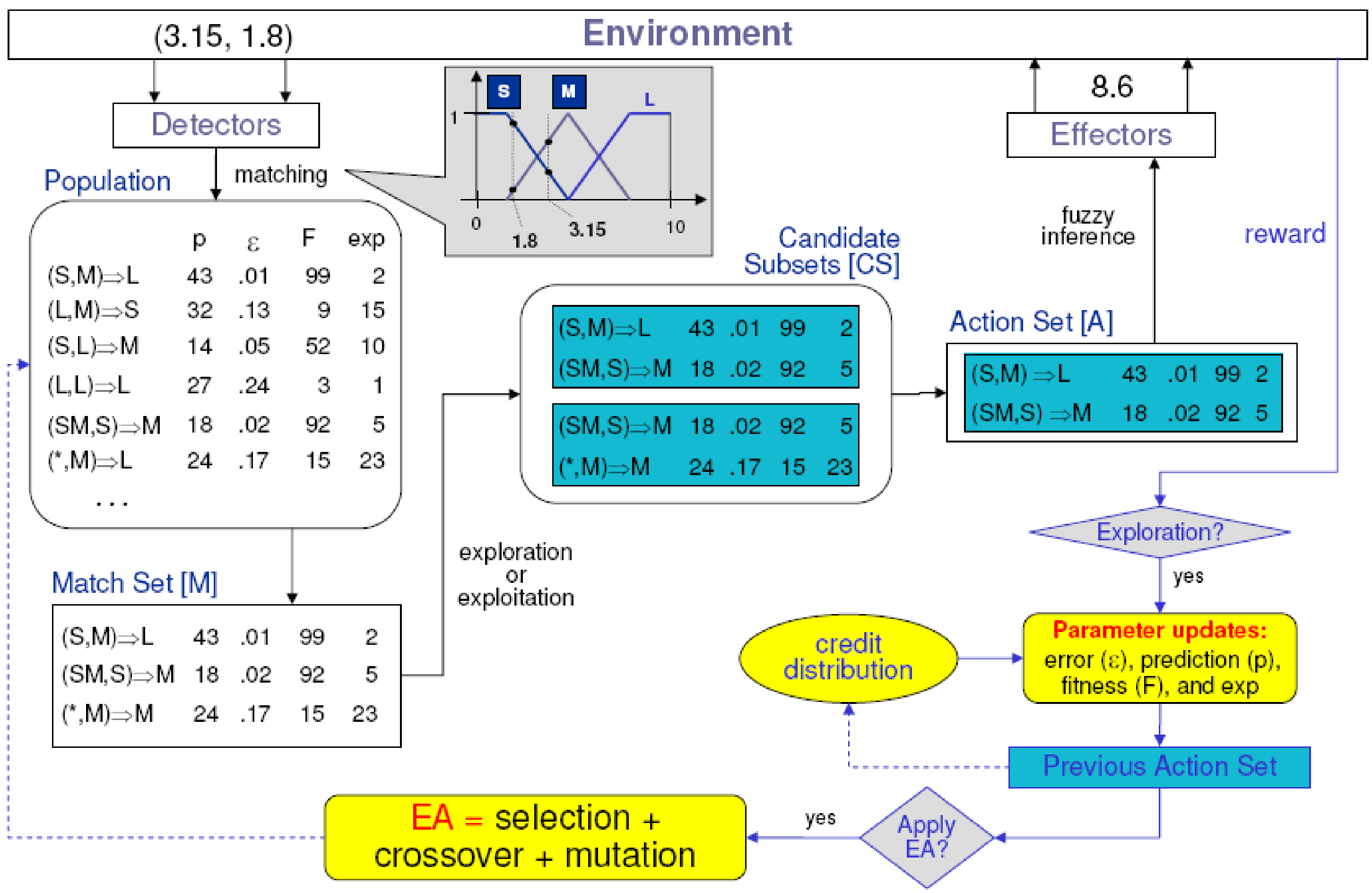
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

- XCS was the first accuracy-based EA and it is currently of major interest to the research community in this field. However, all the proposals of Michigan-style GFSs are strength-based
- Casillas et al. propose an accuracy-based Michigan-style GFS, Fuzzy-XCS, based on XCS but properly adapted to fuzzy systems
- The proposed system interacts with the environment by means of continuous actions
- The on-line behavior involves two cycles: action and learning
- A DNF rule representation is considered to maximize the payoff

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. Conclusions. What's next?

Accuracy-based Fuzzy XCS structure:



OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

New learning schemes: New multi-objective GFS for the interpretability-accuracy trade-off:

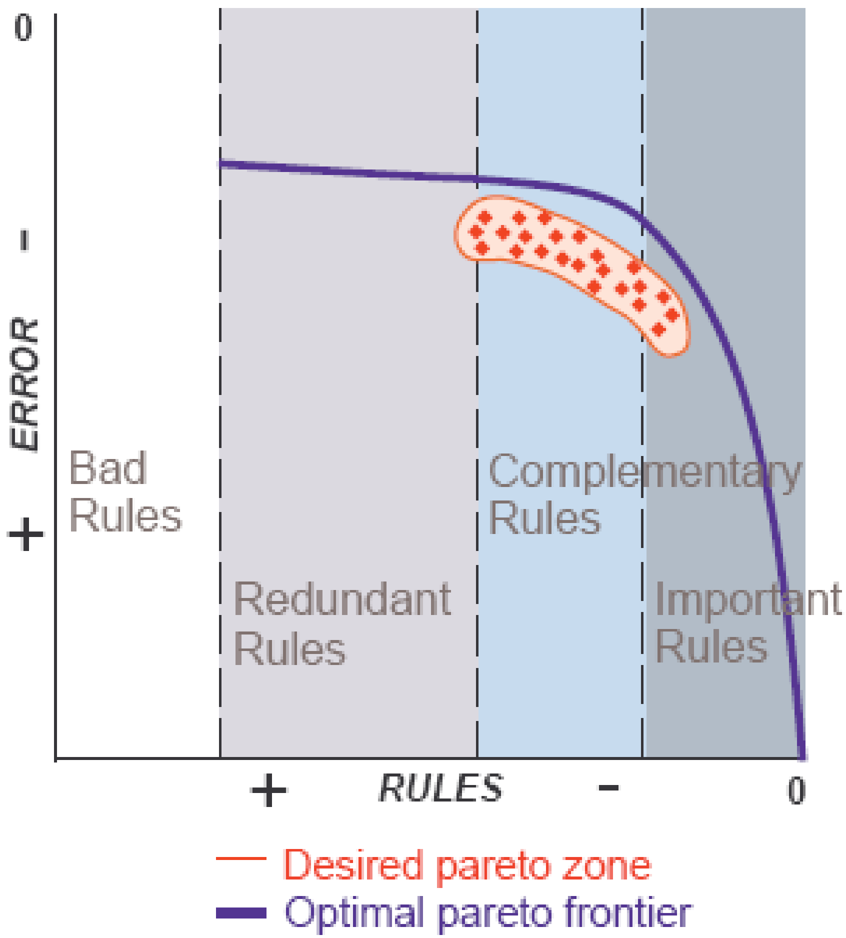
R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, IJUFKBS, 15(5) (2007) 539-557

- Multi-objective EAs are powerful tools to generate GFSs but they are based on getting a large, well distributed and spread off, Pareto set of solutions

- The two criteria to optimize in GFSs are accuracy and interpretability. The former is more important than the latter, so many solutions in the Pareto set are not useful

- Solution: Inject knowledge through the MOEA run to bias the algorithm to generate the desired Pareto front part

Pareto front classification in an interpretability-accuracy GFSs:



- Bad rules zone: solutions with bad performance rules. Removing them improves the accuracy, so no Pareto solutions are located here
- Redundant rules zone: solutions with irrelevant rules. Removing them does not affect the accuracy and improves the interpretability
- Complementary rules zone: solutions with neither bad nor irrelevant rules. Removing them slightly decreases the accuracy
- Important rules zone: solutions with essential rules. Removing them significantly decreases the accuracy

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Accuracy-oriented modifications performed:

- Restart the genetic population at the middle of the run time, keeping the individual with the highest accuracy as the only one in the external population and generating all the new individuals with the same number of rules it has
- In each MOGA step, the number of chromosomes in the external population considered for the binary tournament is decreased, focusing the selection on the higher accuracy individuals

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

Obtained results for the medium voltage line problem:

Multi-objective genetic tuning + rule selection method:

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
WM	65	57605	2841	+	57934	4733	+
WM+T	65	18602	1211	+	22666	3386	+
WM+S	40.8	41086	1322	+	59942	4931	+
WM+TS	41.9	14987	391	+	18973	3772	+
NSGAII	41.0	14488	965	+	18419	3054	+
$NSGAII_{ACC}$	48.1	16321	1636	+	20423	3138	+
SPEA2	$\boxed{33}$	13272	1265	+	17533	3226	+
$SPEA2_{ACC}$	34.5	$\boxed{11081}$	1186	*	$\boxed{14161}$	2191	*

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

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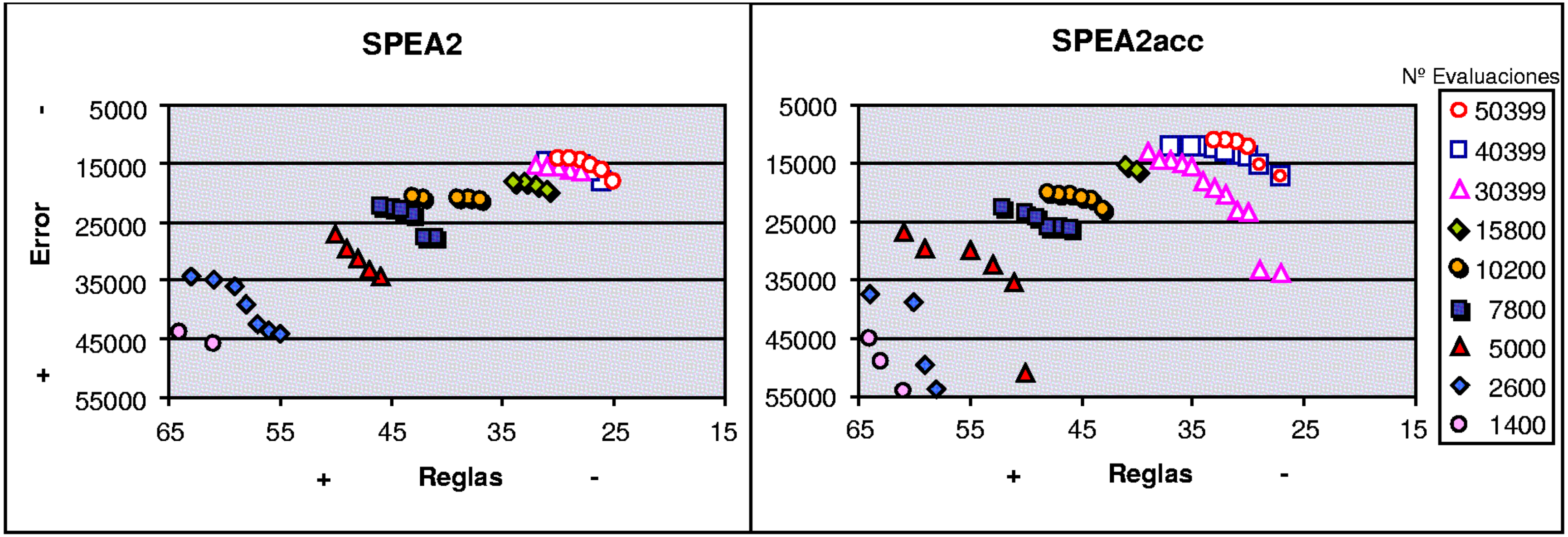
7. Conclusions. What's next?

Comparison of the SPEA2 – SPEA2acc convergence:



7. Conclusions. What's next?

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

GFSs for handling inherently fuzzy data:

- There are many practical problems requiring learning models from uncertain data:
  - Those with coarse-grained digital data, as obtained when weighing small objects in a low resolution scale, or
  - with values comprising both a numerical measure and one or more confidence intervals defining its imprecision (e.g., the position given by a GPS sensor)
- In either case, there is an unknown difference between the true measure and the observed one
- Assuming it to be stochastic noise is an oversimplification. Intervals or fuzzy sets are best suited to represent the uncertainty in the observation

7. Conclusions. What's next?

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

- Fuzzy systems have been extensively applied to learning problems dealing with crisp data, that can be also solved by many other classical (statistical) and computational intelligence techniques
- However, their intrinsic characteristics make them one of the few and most adapted tools to deal with the latter problems!
- Moreover, an interval or fuzzy-based representation can also be used to:
  - reconcile different measurements of a feature in a given object, and
  - to describe incomplete knowledge about a value (for example, a missing input value can be codified by an interval spanning the whole range of the variable)

IDEA:

- **advocate the use of fuzzy data to learn and evaluate GFSs, and**
- **raise the use of fuzzy-valued fitness functions to formulate that kind of problems**

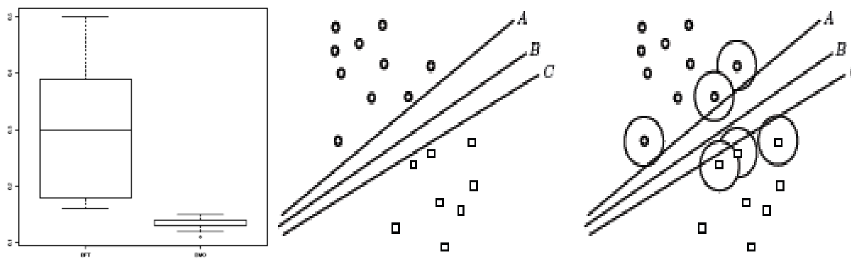
7. Conclusions. What's next?

OUTLINE

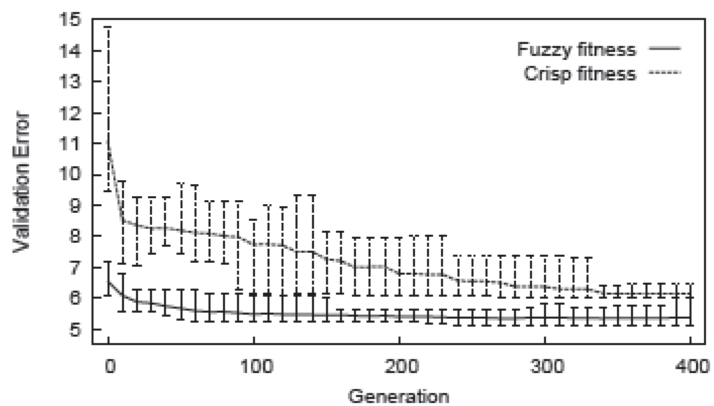
1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

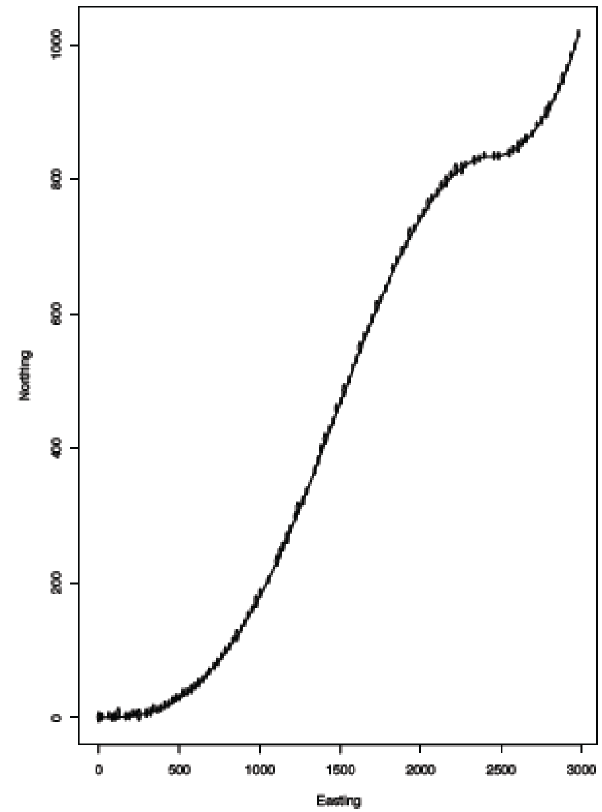
Some examples of practical applications:

- Crisp data with hand-added fuzziness: increase of fuzzy models/classifiers robustness:



- Transformations of data based on semantic interpretations of fuzzy sets: factor evaluation of questionnaires in marketing





- Inherently fuzzy data: taximeter calibration with a GPS

OUTLINE

1. Introduction to GFSs
2. GFSs roadmap and milestones
3. Evolutionary tuning of FRBSs
4. Classical GFS learning approaches
5. Some real-world applications
6. Advanced GFS approaches
7. **Conclusions. What's next?**

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