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A historical review of evolutionary learning methods for Mamdani-type fuzzy rule-based systems: Designing interpretable genetic fuzzy systems

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ABSTRACT

The need for trading off interpretability and accuracy is intrinsic to the use of fuzzy systems. The obtaining of accurate but also human-comprehensible fuzzy systems played a key role in Zadeh and Mamdani's seminal ideas and system identification methodologies. Nevertheless, before the advent of soft computing, accuracy progressively became the main concern of fuzzy model builders, making the resulting fuzzy systems get closer to black-box models such as neural networks. Fortunately, the fuzzy modeling scientific community has come back to its origins by considering design techniques dealing with the interpretability-accuracy tradeoff. In particular, the use of genetic fuzzy systems has been widely extended thanks to their inherent flexibility and their capability to jointly consider different optimization criteria. The current contribution constitutes a review on the most representative genetic fuzzy systems relying on Mamdani-type fuzzy rule-based systems to obtain interpretable linguistic fuzzy models with a good accuracy.

1. Introduction

System identification involves the use of mathematical tools and algorithms to build dynamical models describing the behavior of real-world systems from measured data [150]. There are always two conflicting requirements in the modeling process: the model capability to faithfully represent the real system (accuracy) and its ability to express the behavior of the real system in an understandable way (interpretability). Obtaining high degrees of accuracy and interpretability is a contradictory aim and, in practice, one of the two properties prevails over the other. Before the advent of soft computing, and in particular of fuzzy logic, accuracy was the main concern of model builders, since interpretability was practically a lost cause [26]. Notice that, in traditional control theory approaches, the models' interpretability is very limited, given the rigidity of the underlying representation language.

Fuzzy systems have demonstrated their superb ability as system identification tools [22,61,107]. The use of fuzzy rule-based systems (FRBSs) for system identification can be considered as an approach used to model a system making use of a descriptive language based on fuzzy logic with fuzzy predicates [152]. This paradigm has proven its ability to automatically generate different kinds of fuzzy models from data, permitting the incorporation of human expert knowledge, and integrating numerical and symbolic processing into a common scheme [132].

When a Mamdani-type FRBS [112,113] is considered to compose the model structure, the linguistic fuzzy model so obtained consists of a set of linguistic descriptions regarding the behavior of the system being modeled. It thus becomes a highly interpretable grey-box model [60]. However, the relatively easy design of FRBSs, their attractive advantages, and their emergent proliferation have made fuzzy modeling suffer a deviation from the seminal purpose directed towards exploiting the descriptive power of the linguistic variable concept [167]. Instead, during the 90s, much of the research developed in

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fuzzy modeling focused on increasing the accuracy as much as possible paying little attention to the interpretability of the final model. The Takagi-Sugeno-Kang (TSK) FRBS structure [151,153] played a pivotal role in the latter research.

Nevertheless, as stated by Bonissone in [26], soft computing provides the (fuzzy) model designer with a much richer repertoire to represent the structure, to tune the parameters, and to iterate the process within the equation "*model = structure + parameters*', classically followed by the traditional system identification approaches. A new tendency in the fuzzy modeling scientific community that looks for a good balance between interpretability and accuracy has thus increased in importance in the last few years [1,34,37,81,118,148,155]. The term *fuzzy modeling interpretability-accuracy tradeoff* [34,37] has been coined to define this discipline, collecting two different perspectives: the interpretability improvement of accurate (usually TSK) fuzzy models, or the accuracy improvement of linguistic fuzzy (Mamdani-type) models with a good interpretability.

One of the most successful fuzzy system identification methodologies within the realm of soft computing are genetic fuzzy systems (GFSs) [53,54,85] where genetic (and, in general, evolutionary) algorithms [63] are considered to learn the components of a FRBS. A GFS is basically a fuzzy system augmented by a learning process based on a genetic or an evolutionary algorithm (GA/EA). A large amount of research has been developed in the design of Mamdani-type GFSs to deal with the interpretability-accuracy tradeoff. The aim of the current contribution is to develop a historical review on the most representative proposals of this kind.

To do so, this contribution is structured as follows: The next section introduces some preliminaries including the Mamdani-type FRBS structure, some basic aspects on the interpretability-accuracy tradeoff, and a brief overview of GFSs. Then, Section 3 constitutes the core of the contribution by reviewing most of the Mamdani-type GFSs existing in the literature by especially focusing on the way they deal with the interpretability-accuracy tradeoff. Finally, Section 4 collects some concluding remarks and describes some current research trends and open issues in the area.

2. Preliminaries

2.1. Mamdani-type fuzzy rule-based systems for control, modeling, and classification. Pros and cons

As any FRBS, Mamdani-type FRBSs [112,113] present two main components: (1) the fuzzy inference system,[1] which implements the fuzzy reasoning process to be applied on the system input to get the system output and (2) the fuzzy knowledge base (KB), which represents the knowledge about the problem being solved. Fig. 1 graphically represents this framework.

The KB contains fuzzy *IF − THEN* rules composed of linguistic variables [166] which take values in a term set with a real-world meaning. The fuzzy sets defining the semantics of the linguistic labels are uniformly defined for all the rules included in the KB, thus easing the readability of the system for human beings. As said, this collection of fuzzy linguistic rules constitute a *descriptive* approach since the KB becomes a qualitative expression of the system. Besides, this division between the fuzzy rule structures and their meaning allows us to distinguish two different components, the fuzzy rule base (RB), containing the collection of fuzzy rules, and the data base (DB), containing the membership functions of the fuzzy partitions associated to the linguistic variables. This specifies a clear distinction between the fuzzy model structure and parameters as defined in classical system identification.

There are different kinds of linguistic fuzzy rules proposed in the specialized literature mainly depending on the rule consequent structure directly affected by the system output nature. The most usual rule structure is that of linguistic fuzzy models/controllers which considers a linguistic variable in the consequent (to finally provide a real-valued output) as follows:

$$\textit{If } X_1 \textit{ is } A_1 \textit{ and } \ldots \textit{ and } X_n \textit{ is } A_n \textit{ then } Y \textit{ is } B,$$

with X_i and Y being the system linguistic input and output variables, respectively, and with A_i and B being the linguistic labels associated with fuzzy sets specifying their meaning. These fuzzy sets are defined in their respective universes of discourse $U_1, \ldots, U_n, V$, and are characterized by their membership functions:

$$\mu_{A_i(B)} : \mu_{U_i(V)} \rightarrow [0, 1], \quad i = 1, \ldots, n.$$

Different fuzzy membership function shapes can be considered. Fig. 2 shows an example of a strong fuzzy partition (SFP) [139] with triangular-shaped membership functions.

In addition, fuzzy rule-based classification systems (FRBCSs) [94,107] consider a linguistic fuzzy rule structure where the output involves a discrete value, the class associated to the patterns matching the rule antecedent. Three different fuzzy classification rule structures can be distinguished depending on the use of a certainty factor associated to the class in the consequent [44]: (i) class label only, (ii) class label and certainty degree, and (iii) certainty degree for every class. The most extended is the second one which shows the following structure:

$$\textit{If } X_1 \textit{ is } A_1 \textit{ and } \ldots \textit{ and } X_n \textit{ is } A_n \textit{ then } Y \textit{ is } C \textit{ with } r,$$

with $C \in \{C_1, \ldots, C_M\}$ being the rule class and $r \in [0, 1]$ being the rule certainty degree.

[1] The analysis of the fuzzy inference system of linguistic fuzzy models/controllers/classifiers is out of the scope of this contribution. The interested reader is referred to [51,61,44,94], respectively, for a deep introduction.

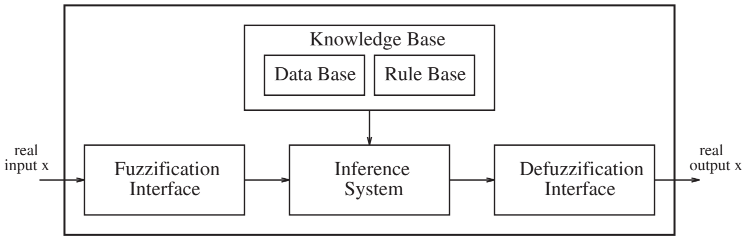

Fig. 1. General structure of a Mamdani-type FRBS.

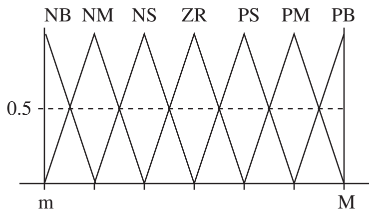

Fig. 2. Example of a strong fuzzy partition composed of seven linguistic terms with triangular membership functions associated.

In view of the latter, it can be clearly recognized that the Mamdani-type FRBS structure demonstrates several interesting features. On the one hand, it provides a natural framework to include expert knowledge in the form of linguistic fuzzy rules. This knowledge can be easily combined with rules which are automatically generated from data sets that describe the relation between system input and output [17]. On the other hand, there are many different design issues for the fuzzy inference mechanism, making a full use of the power of fuzzy logic-based reasoning, opposite to TSK FRBSs which apply a single and simplified kind of fuzzy inference. Moreover, Mamdani-type FRBSs provide a highly flexible means to formulate knowledge, while at the same they remain interpretable, as long as a proper design is developed (see Section 2.2).

However, although Mamdani FRBSs possess several advantages, they also come with some drawbacks. One of their main pitfalls is the lack of accuracy when modeling some complex, high-dimensional systems. This is due to the inflexibility of the linguistic variable concept, which imposes hard restrictions to the fuzzy rule structure [23]. The descriptive power is obtained at the cost of an exponentially increasing model complexity. This means that many rules may be needed to approximate a system to a given degree of accuracy (especially with many input variables) as a consequence of the rigid partitioning of the input and output spaces.

Due to the latter reasons, some extensions have been considered on the classical linguistic fuzzy rule structure to relax it in an attempt to increase the accuracy of Mamdani-type FRBSs. The most extreme extension involves the use of scatter fuzzy partitions instead of the classical grid-based ones, in such a way that every single rule has its own meaning (its own fuzzy sets associated) [8,22]. Scatter fuzzy partitions are of course more suitable to generate accurate fuzzy models since they are not subject to the rigid input space partitioning of grid-based ones. Hence, the number of fuzzy rules required to approximate a real system to the desired accuracy degree could be smaller. However, they carry a strong interpretability reduction as a different linguistic term has to be assigned to each fuzzy set in each rule, thus losing the global semantic of the classical Mamdani FRBS. Hence, readable and distinguishable rules can only be obtained when compact RBs are considered and when there is not a large number of similar fuzzy sets composing them.

Some other extensions of the Mamdani-type fuzzy rule structure have been proposed keeping its global semantics and thus being generally more interpretable. They include double-consequent rules [66,126], weighted rules [40,127], and rules with linguistic hedges [79,165]. In all the cases, the linguistic variable restrictions are relaxed obtaining higher degrees of freedom to increase the accuracy of the obtained linguistic fuzzy model/classifier/controller.

Another quite extended variant is that of the DNF (disjunctive normal form) linguistic fuzzy rule [78,111]. Regardless the composition of the consequent, the antecedent is extended by allowing each input variable X_i to take a disjunction of linguistic terms as a value. The complete syntax for the rule antecedent is as follows:

$$IF\ X_1\ is\ \widetilde{A_1}\ and\ \ldots\ and\ X_n\ is\ \widetilde{A_n},$$

where

$$\widetilde{A_1} = \{A_{11}\ or\ \ldots\ or\ A_{1l_1}\},\ \ldots,\ \widetilde{A_n} = \{A_{n1}\ or\ \ldots\ or\ A_{nl_n}\}.$$

The DNF rule structure shows several advantages. First, it relaxes the grid-based partitioning constraints. Besides, it permits value grouping (e.g. "smaller than Positive_Big"), thus making the rules more interpretable. Finally, it allows the design methods to perform feature selection at rule level: if a variable takes all the possible values from its domain, it is considered to be irrelevant as rule premise. Due to the latter reasons, they are usually considered in classification problems.

In addition to the use of extended Mamdani-type fuzzy rule structures, advanced design methods keeping the basic rule structure or considering any of the latter extensions have been proposed. Section 3 in this contribution reviews most of the existing proposals based on the use of GFSs which aim to improve the accuracy of the fuzzy linguistic model without significantly losing its interpretability.

2.2. The interpretability-accuracy tradeoff

Reviewing the historical development of fuzzy modeling, it can be easily recognized that the original aim of using fuzzy techniques for system modeling was the obtaining of human interpretable models [166]. The classical Mamdani-type linguistic fuzzy rule structure [112,113] described in the previous subsection was considered for that aim. This was the reason why less accurate but more interpretable (grey-box) fuzzy rule-based models were sometimes preferred to other more accurate (black-box) models (such as neural network-based ones) for some specific problems.

Then, during the 80s, the TSK fuzzy rule structure was proposed in several works by Takagi, Sugeno, and Kang [151,153]. The new fuzzy model structure showed some interesting characteristics, namely its higher system approximation ability due to the presence of a larger number of freedom degrees in the rule consequent, and the chance to directly derive it from examples by means of numerical approximation techniques. This fact caused the research in the fuzzy modeling community to shift to the design of highly accurate models using TSK FRBSs.

Nevertheless, this accuracy increase is actually obtained at the expenses of some interpretability loss: by definition, the TSK fuzzy rule structure, having a polynomial function in the consequent, is less interpretable for the user than a Mamdani-type fuzzy linguistic rule.[2] This made fuzzy modeling suffer a deviation from its seminal purpose directed towards exploiting the descriptive power of the linguistic variable concept.

In the last few years, it has been shown an increasing interest on considering fuzzy techniques to design both accurate and interpretable fuzzy models [1,34,37,81,118,148,155]. As these two requirements are usually contradictory for any kind of system identification methodology, this framework resulted in the so called *interpretability-accuracy tradeoff* which must be considered when tackling the design of a fuzzy model for a specific application [35,36]. This tradeoff can be managed in two different ways:

(1) Flexibilizing the most interpretable fuzzy model structures (as the Mamdani-type one) to make them as accurate as possible without losing their interpretability to a high degree [34,35].
(2) Imposing restrictions to the most accurate fuzzy model structures to make them as interpretable as possible [36,37].

Of course, the two said approaches have their pros and cons. At first sight, it can be recognized that applying the latter will usually lead to the obtaining of more accurate but less interpretable models and *vice versa*. This contribution is focused on the former alternative, considering a GFS as the fuzzy system identification methodology.

To properly understand the interpretability-accuracy framework, it is important to keep in mind that a fuzzy model is not interpretable *per se*. Instead, there are many different issues which must be taken into account in order to obtain a human interpretable structure (such as, for example, the RB compactness or the semantic comprehensibility of the fuzzy partitions, as we will see as follows) [81,101,102,118,148]. Hence, a fuzzy system identification process aiming to properly deal with the interpretability-accuracy tradeoff must impose a set of constraints in order to guarantee the interpretability of the finally derived fuzzy system. With this aim, some classical proposals such as [155] introduced several useful interpretability constraints for fuzzy membership function optimization such as natural zero positioning, limited overlap between neighboring fuzzy sets (distinguishability), coverage of the universe of discourse, and unimodality of fuzzy sets. In [117], Mencar and Fanelli presented a review of the different fuzzy system interpretability constraints which has been proposed in the specialized literature and classify them through a taxonomy considering six different families, namely constraints for: (i) fuzzy sets, (ii) universes of discourse, (iii) fuzzy information granules, (iv) fuzzy rules, (v) fuzzy systems, and (vi) learning methods.

We must actually notice that the measurement of the interpretability degree of a fuzzy system is an unsolved problem currently, as it is strongly affected by subjectivity. Opposite to accuracy evaluation, where everybody commonly accepts the use of any error measure such as the mean square error, no general way to measure interpretability is available. In fact, even the terminology in the area is sometimes confusing and terms as interpretability, comprehensibility, readability, and transparency are used as synonyms when they refer to different concepts. In order to fix the terminology considered in this contribution, we will consider *fuzzy system interpretability* involves two different issues:

(1) The *readability of the KB*, which is mainly related to the complexity of this fuzzy system structure. It includes criteria such as the compactness of the RB (low number of rules and premises) and the DB (low number of linguistic labels).
(2) The *comprehensibility of the fuzzy system*, which concerns the semantic interpretability of the fuzzy system structure and reasoning method for the human user. It considers criteria such as the fuzzy rules consistency or the fuzzy partitions integrity.

[2] Of course, the interpretability of a FRBS does not only depend on the kind of fuzzy rule used, but also in other aspects such as the complexity of the RB or the comprehensibility of the DB, as we will recall in the current section.

Classically, interpretability indexes have only focused on the former issue, KB readability, to evaluate the overall fuzzy system interpretability. Some complexity measures have been considered such as the number of rules in the RB (rule compactness) [90] or the total rule length (number of antecedent conditions involved in each rule, i.e. rule simplicity) [39,93]. However, these measures are too simple as they only focus on the RB complexity and ignore both the readability of the remaining components and the FRBS comprehensibility.

Hence, the definition of more complex and plausible interpretability indexes has become a hot topic in the fuzzy modeling community in the last few years and some proposals have been made [16,17,74,121,168] with the aim of solving the latter problem. In [168], Zhou and Gan introduced a global framework distinguishing two different levels for fuzzy system interpretability, *high-level* and *low-level interpretability*. While the former accounts for the interpretability of the fuzzy rules in the RB, considering criteria as the already mentioned ones (complexity-based interpretability) as well as others dealing with the coverage, completeness, and consistency of the fuzzy rules, the latter is related to the other Mamdani-type FRBS component, the DB, and measures the interpretability of the fuzzy sets in the fuzzy partitions (semantics-based interpretability). Up to our knowledge, the first interpretability index combining measures from both high and low interpretability levels was that proposed by Naück in [121] which integrates a membership function coverage measure and two complexity measures, the ratio between the number of classes and the total number of premises, and the average-normalized fuzzy partition granularity.

In [16], Alonso et al. defined a general framework for characterizing FRBS interpretability. The authors start from the classifications proposed in previous works [117,168] and perform an experimental analysis (in the form of a web poll with real users) in order to evaluate the most used indexes and characterize their actual capability for interpretability assessment. Results extracted from the poll show the inherent subjectivity of the measure. The main conclusion obtained is that defining a numerical index is not enough to get a widely accepted index but there is a need to define a fuzzy index easily customizable to the context of each problem as well as to the user's quality criteria. With that aim, the same authors designed a FRBS for measuring the interpretability degree of a Mamdani-type KB in [17]. Six main input variables, total number of rules, total number of premises, number of rules using one, two or three or input variables, and total number of labels per input, are considered. Notice that, all the interpretability criteria considered are complexity-based as the overall index assumes the use of SFPs. The single output is the interpretability degree of the evaluated KB, computed as the result of a fuzzy reasoning process. The proposed FRBS shows a hierarchical structure composed of four different modules which group the latter six criteria into four different families according to the information they convey, namely, RB dimension, RB complexity, RB interpretability (which combines the outputs of the latter two), and joint DB–RB interpretability (which combines the output of the former and the total number of labels per input criterion). Notice that, each of the latter interpretability evaluation subsystems is guided by an expert-defined KB which thus allows to directly express the user preferences in the interpretability evaluation. Nevertheless, the latter FRBS for interpretability evaluation shows the problem of its difficulty to be adapted to different problems and different user's preferences as the whole fuzzy index must be defined from scratch. As an alternative, Alonso and Magdalena introduced another framework in [14] allowing us to define a fuzzy system quality index (including both accuracy and interpretability). The new index is easily customizable to the context of each system identification problem by incorporating the user's preferences and different kinds of quality criteria. To do so, all the desired criteria (chosen from the different families of interpretability criteria, considering both readability and comprehensibility-based) are combined into a decision hierarchy framework. The process of assessing interpretability to a set of KBs is seen as a multi-criteria decision making problem with the final goal of setting a ranking of KBs according to their interpretability degree. The top of the hierarchy represents the quality index while the bottom includes all the fuzzy systems to be evaluated to select the most appropriate one for the problem solving requirements. The hierarchy consists of k decision levels structured as suggested by the classical analytic hierarchy process defined by Saaty [140]. The aggregation process is made using Yager's ordered weighted averaging (OWA) operators [164].

Although the definition of semantic interpretability indexes has been less extended in the area, it has been recently tackled in works such as [28,68,73,115,116]. A very novel approach also considering human intervention is that presented in [116], where the authors define a strategy for assessing comprehensibility of FRBCSs based on the so called "cointension degree" between the explicit semantics, defined by the formal parameter settings of the model, and the implicit semantics conveyed to the reader by the linguistic representation of knowledge. The strategy is evaluated on a set of pre-existent FRBCSs concluding that the linguistic representation of some of them is not appropriate as they are not cointensive with user's knowledge, even though they can be tagged as readable from a complexity viewpoint.

Finally, Gacto et al. presents a further taxonomy in [74] where the existing interpretability measures are classified based on two different criteria, kind of interpretability index (complexity vs. semantic) and FRBS component where it is applied (RB vs. DB). This leads to the creation of four different groups combining the latter two criteria: (i) complexity at RB level, (ii) complexity at DB level, (iii) semantic interpretability at RB level, and (iv) semantic interpretability at DB level. The aim of the authors is to provide the user with a more complete interpretability assessment framework which can be used to guide the definition of multicriteria functions for Mamdani-type FRBS design with a good interpretability-accuracy tradeoff in the short future.

2.3. Genetic fuzzy systems

Despite the previous successful history of FRBS design, the lack of learning capabilities characterizing most of the works in the field generated a certain interest for the study of FRBSs with added learning capabilities during the early 90s. That was also one of the reasons for the huge development of the TSK fuzzy model structure, which incorporated that characteristic since

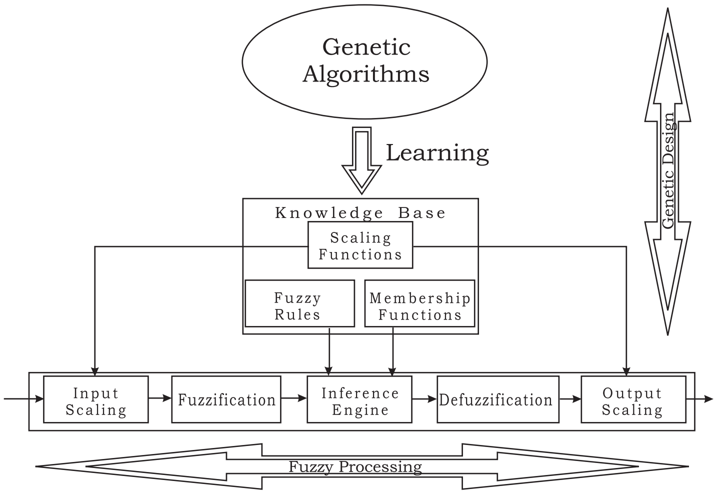

Fig. 3. General structure of a GFS.

its proposal. During that decade, two very successful approaches arose in the framework of soft computing by integrating the learning capabilities of neural networks, on the one hand, and GAs/EAs, on the other hand, into the approximate reasoning method of FRBSs. While the former hybridization lead to the field of neuro-fuzzy systems [120], the latter resulted in the creation of GFSs [54].

Genetic learning processes cover different levels of complexity according to the structural changes produced by the algorithm [59], from the simplest case of parameter optimization to the highest level of complexity of learning the rule set of a rule-based system. The KB is usually the object of study in the GFS framework (see Fig. 3). When considering a GA/EA to design a FRBS, the latter two tasks respectively stand for parameter estimation (DB) and structure identification (RB or DB+RB), following the classical system identification terminology. From the optimization viewpoint, the task of finding an appropriate KB for a particular problem is equivalent to parameterize the considered KB components and to find those parameter values that are optimal with respect to one or several optimization criteria. The KB parameters constitute the search space, which is transformed into a suitable genetic representation on which the search process operates [54]. This provides the GA/EA with enough flexibility to tackle the interpretability-accuracy tradeoff by considering optimization criteria of different nature.

Developing a deep revision of the GFS area is out of the scope of the current contribution. The interested reader is referred to [53,54,85].

3. Historical review of evolutionary learning methods for Mamdani-type fuzzy rule-based systems

This section is devoted to review most of the existing GA/EA-based approaches to design Mamdani-type FRBSs (Mamdani-type GFSs) dealing with the interpretability-accuracy tradeoff. Different related proposals will be grouped in subsections in order to reach a coherent taxonomy. In each subsection, classical approaches will be presented first to later describe more advanced and recent proposals.

3.1. Genetic tuning

A genetic tuning process assumes a previous definition of the structure of the FRBS and then adapts some of its parameters, such as the scaling functions, the universes of discourse, or the membership function definitions, with the latter being one of the most common choice. The role of genetic tuning can thus be recognized as one of the main *parameter estimation* approaches in fuzzy system identification using GFSs.

Genetic tuning methods are inherent to GFSs since their creation. During the first decade of GFS development, several methods of this kind were proposed for Mamdani-type FRBSs considering different membership function shapes and coding schemes. One of the first pioneering GFS proposals by Karr in 1991 was a GA to adapt the membership function shapes for a previously defined Mamdani-type FRBS [103]. It was based on a binary-coded GA which encoded candidate definitions for SFPs of triangular-shaped fuzzy sets. Only the crossing points between successive fuzzy sets were encoded, thus composing a compact representation. In this way, the SFP nature of the adapted membership functions was directly ensured, thus keeping the linguistic fuzzy model interpretability level. A similar representation but based on an integer coding was proposed in [77], while [27,104] considered the use of trapezoidal-shaped membership functions. Besides, Gaussian functions not associated to a SFP were adapted in [82] by means of a binary coding.

A more natural real-coded representation for triangular or trapezoidal-shaped and Gaussian membership function parameters was considered in [48,67,84], respectively. As many later proposals, these approaches were not based on the use of a SFP but on directly encoding the two, three, or four (depending on their Gaussian, triangular, or trapezoidal-shaped nature) real-valued definition parameters for each fuzzy set in each fuzzy partition. This coding scheme presents both pros and cons. On the one hand, the genetic tuning has a higher number of freedom degrees in comparison with those based on SFPs. In this way, more accurate linguistic fuzzy models can be obtained. On the other hand, it develops more significant modifications, thus reducing the interpretability of the resulting Mamdani-type FRBS.

In order to ensure keeping an appropriate interpretability after the application of a genetic tuning process, the use of semantic interpretability constraints was extended in the area (see Section 2.2). Valente de Oliveira presented a study of semantically driven conditions to constrain the optimization process in such a way that the resulting membership functions can still represent human readable linguistic terms [155]. Cordón et al. already considered some of these properties in their real-coded genetic tuning process for Mamdani-type FRBSs [48] and FRBCSs [43] within the MOGUL GFS framework [45].

An alternative approach in the Mamdani-type FRBS genetic tuning literature has been that considering the adaptation of the *linguistic variables context*. This notion comes from the observation that, in real life, the same basic concept can be perceived differently in different situations. Instead of individually adapting the membership functions shapes, the fuzzy partitions are globally adapted by scaling the fuzzy sets from one universe of discourse to another by means of linear or non-linear scaling functions whose parameters are identified from data. In many cases, this global adjustment constitutes a better approach to deal with the interpretability-accuracy tradeoff than an isolated membership function tuning as the obtained fuzzy partitions are more interpretable.

Genetic tuning of linear scaling functions was proposed in the early times of GFSs [111,122] for Mamdani-type fuzzy controllers. Later, more advanced proposals for non-linear context adaptation were introduced [80,110]. The usual approach in these kinds of processes is the adaptation of one to four parameters (defining the scaling function) per variable: one when using a scaling factor, two for linear scaling, and three or four in non-linear scaling. Most of the cited works consider a real coding scheme but the oldest method [122], where a three bits binary representation of each scaling factor is used.

More sophisticated genetic tuning processes have been developed in the last few years. On the one hand, we find those proposals considering linguistic fuzzy rule extensions and/or combining the tuning method with a rule selection (see Sections 3.4 and 3.5). On the other hand, new coding schemes have been proposed such as that in [4] based on the use of the linguistic 2-tuples representation model [86] to perform what the authors called *lateral tuning*. Instead of making use of the classical three parameter representation to encode the triangular-shaped membership function definition points, they introduced a novel single point coding scheme only allowing the lateral displacement of the fuzzy sets, i.e., slight displacements of the original fuzzy sets to the left or to the right. This resulted in a reduction of the search space tackled by the genetic tuning method thus easing the derivation of linguistic fuzzy models, especially in complex or high-dimensional problems. The same authors later extended this coding scheme in [2] by adding one more parameter per membership function taking the linguistic 3-tuples approach as a base. In this way, they can perform both *lateral and amplitude tuning* by adjusting the lateral displacement and the amplitude variation of the support of this fuzzy set. Tuning of both parameters also involves a reduction of the search space that eases the derivation of optimal models with respect to the classical methods.

In addition, a GA to optimize the linguistic terms of a FRBCSs was introduced in [157] for a real-world ecological problem. It considers the use of semantic interpretability constraints in the two different variants based on the use of two different coding schemes, binary and real-coded. The technique's differential characteristic is that it considers two novel fuzzy accuracy criteria for fuzzy ordered classifiers. Another advanced genetic tuning process considering interpretability issues was introduced in [28]. In this case, the method focuses on context adaptation in linguistic fuzzy models. The use of real coding, parametric orthogonal fuzzy modifiers, a flexible non linear scaling function, and specifically designed genetic operators is considered for the context tuning. Besides, a novel proposal for an specific index to measure the semantic interpretability of a fuzzy partition in this framework based on fuzzy ordering relations is introduced within the GFS.

3.2. Genetic rule selection

As seen in Section 2.1, when tackling a high-dimensional problem with a Mamdani-type FRBS, the number of rules in the RB grows exponentially as more inputs are added. Hence, a fuzzy rule generation method is likely to derive fuzzy rule sets including undesired rules degrading both the accuracy and the interpretability of the fuzzy linguistic models. Among those rules, we can find redundant rules, whose actions are covered by other rules in the RB; wrong rules, badly defined and perturbing the system performance; and conflicting rules, which worsen the system performance when co-existing with other rules in the RB.

Rule reduction methods are used as postprocessing techniques to solve the latter problems, both in Mamdani-type and in other FRBS structures [37]. Rule selection is the most extended rule reduction method for linguistic fuzzy models and EAs are the most usual optimization procedure to put it into effect. Hence, genetic rule selection is among the oldest and more extended GFS proposals. All those approaches share a fixed-length, binary coding where the chromosomes consider one bit for each rule in the initial RB. Only those linguistic fuzzy rules whose associated allele takes value 1 are considered to belong to the final RB. This approach constitutes a good way to deal with the interpretability-accuracy tradeoff as fuzzy rule subsets can be derived with both a better accuracy (thanks to their good cooperation level) and a better readability (due to the reduction in the RB complexity) than the original RB.

The first genetic rule selection method was introduced by Ishibuchi et al. in [97] in a fuzzy classification framework. Cordón et al. introduced a genetic multi-selection process within the MOGUL GFS design framework [45]. The basic genetic selection procedure for fuzzy linguistic models proposed in [48] is wrapped by a niching procedure [63] with the aim of not only obtaining a single best fuzzy rule subset but rather a variety of potential solutions of comparable performance. In [43], the latter method is applied to design FRBCSs by both removing unnecessary rules from the initial RB and refining them by means of a linguistic hedge learning process, considering one of the extensions described in Section 3.4. In [161], Wang et al. proposed a genetic integration process of multiple knowledge bases that can also be considered as a particular case of genetic rule selection.

All the latter methods were initially only focused on accuracy as the fitness function was only composed of criteria of that kind (usually, a single error criterion). Later, the incorporation of basic complexity criteria (such as the minimization of the number of rules, total rule length, etc.) arose, thus dealing with a multicriteria optimization problem within the interpretability-accuracy tradeoff framework [90]. Up to our knowledge, that was the first application of evolutionary multiobjective optimization [42] to fuzzy linguistic modeling (in this case, as a multiobjective genetic rule selection process for linguistic FRBCSs). More recent approaches will be reviewed in Secion 3.5.

In addition, genetic rule selection methods have commonly formed part of more sophisticated GFSs for Mamdani-type FRBSs, either in multi-stage structures or in a joint evolutionary learning processes. Some of these approaches will be reviewed in the remainder of this contribution.

3.3. Evolutionary learning methods for Mamdani-type knowledge bases and rule bases

Some of the first Mamdani-type GFSs aimed to learn both KB components, DB and RB, in order to deal with the strong synergy existing between them. That was done mainly by following two of the classical GFS learning approaches, Pittsburgh (where the whole KB definition is encoded in each chromosome) and iterative rule learning (IRL) (where a chromosome encodes a single rule, the GA/EA is sequentially applied to obtain the whole RB, and the learning process considers independent stages to learn each KB component) [54].[3] Of course, the computational cost of the genetic search grows with the increasing complexity of the solution space required to deal with the whole KB derivation. Park et al. [131], Homaifar et al. [87], and Magdalena et al. [111] constitute three classical examples of Pittsburgh-based GFSs for designing Mamdani-type fuzzy logic controllers by jointly learning membership function shapes and linguistic fuzzy rules, in the former two cases, and contexts and linguistic DNF fuzzy rules, in the latter. Besides, SLAVE [78] is a typical example of an IRL-based Mamdani-type GFS for classification problems while MOGUL [45] is another one for both modeling/control [48] and classification [43] problems.

Later, more advanced GFSs were proposed to design more accurate Mamdani-type FRBSs with a high degree of interpretability. On the one hand, a novel approach to properly deal with the joint learning of DB and RB is that called *embedded KB learning*. It is based on an evolutionary DB learning process which wraps a basic RB generation method. The GA/EA function is to derive the DB definition by learning components such as scaling functions/contexts, membership functions, and/or granularity parameters. A subsequent linguistic fuzzy rule generation method, which must be simple and efficient, derives the RB for the DB definition encoded in each chromosome. The chromosome evaluation thus measures the performance of the whole KB so obtained and it is usually based on a weighted sum of accuracy and interpretability criteria (such as the minimization of the number of rules in the RB). Notice that, this operation mode constitutes and efficient and effective way to tackle the interpretability-accuracy tradeoff as it involves a partitioning of the KB learning problem. The synergy between both KB components is properly accounted, while reducing the huge search space size tackled in Mamdani-type GFSs based on the Pittsburgh approach.

Three different GFSs of this family to learn fuzzy linguistic models (the former two) and classifiers (the latter) are respectively proposed in [50,52,89]. The method in [52] encodes the fuzzy partitions' granularity and the definition parameters for each triangular-shaped membership function in each fuzzy partition while that in [50] learns the variables' domain, the fuzzy partitions' granularity, and the non-linear scaling functions to define their contexts. In both cases, a hybrid representation scheme considering integer and real coding is used. Alternatively, the proposal in [89] also jointly derives the granularity and the triangular-shaped membership functions' definition parameters but taking a different coding scheme based on a single information level. Binary chromosomes of fixed length including a segment per variable are employed. Each segment has a predefined length that determines the maximum granularity allowed. In the chromosome, a one indicates the peak value of a triangular membership function and both extremes of the neighbor membership functions to define a SFP.

Besides, another recent linguistic fuzzy modeling proposal is also to be found in [5] presenting some differential characteristics. First, the DB encoding is based on the linguistic 2-tuples representation model (see Section 3.1). Both the fuzzy partition granularity and the definitions of the membership functions are encoded. The latter ones are represented by a single parameter per fuzzy set defining its lateral deviation with respect to its original support in an initial uniform fuzzy partition. Second, the authors test the performance of three different ad-hoc data-driven RB generation methods in the embedded process, the classical and very extended Wang and Mendel's algorithm [163] (which is the rule learning method also considered in [50,52]) and other two methods originally proposed in this paper. Finally, the considered EA is the good performing real-coded CHC [65]. The Mamdani-type FRBSs derived by means of this GFS show a very high interpretability as they are

[3] The use of the third classical GFS learning approach, the Michigan approach (also referred to as fuzzy classifier systems), is less extended in the area and mainly focus on Mamdani-type FRBSs with scatter partitions (see Section 2.1) [76,156].

composed of compact RBs whose semantic is defined by fuzzy partitions with isosceles triangular-shaped membership functions.

On the other hand, the cooperative coevolutionary paradigm [134] has constituted the base of other kinds of Mamdani-type GFSs learning the whole KB definition. Coevolutionary algorithms are advanced evolutionary techniques proposed to solve decomposable complex problems. They involve several species (populations) that permanently interact among them by a coupled fitness cooperating to build the problem solution. Hence, they are able to properly deal with huge search spaces thanks to the problem decomposition. This decomposition is quite natural in Mamdani-type KB learning as each of its two components, DB and RB, can be easily assigned to a different species in an efficient and effective search process. Two examples of this group are *Fuzzy CoCo* [133] for fuzzy classification and the proposal in [32] for fuzzy modeling.

In addition, there is a third very representative family of GFSs for learning Mamdani-type KBs composed of multiobjective evolutionary learning processes. It will be reviewed in Section 3.5.

Alternatively, some other Mamdani-type GFSs have been proposed which exclusively focus on the RB design and keeps the DB invariable, thus ensuring a good interpretability level. These family of methods consider sophisticated RB learning approaches. That is the case of the COR (cooperative rules) methodology [33] which follows the primary goal of inducing a better cooperation among the linguistic fuzzy rules in the derived RB. To do so, the genetic learning process is guided by global criteria that jointly consider the action of the different rules. The main advantages of the COR methodology are its capability to include heuristic information to guide the search, its flexibility to be used not only with GAs/EAs but also with other kinds of metaheuristics, and its easy integration with other FRBS design processes (see Section 3.4).

In [100], it was proposed what, up to our knowledge, constitutes the only GFS considering the hybridization of the two classical learning approaches. A Pittsburgh-Michigan hybrid genetic learning algorithm was designed to learn linguistic fuzzy classification rules for high-dimensional problems in an efficient and effective way. The method incorporates “don't care” conditions into its rule coding scheme in order to remove not necessary rule premises.

Another family of advanced GFSs for learning Mamdani-type RBs is that following the novel *genetic cooperative-competitive learning* (GCCL) approach. It is based on a coding scheme where each chromosome encodes a single rule (as in the classical Michigan or IRL approaches [54]) but either the complete population (as in Michigan) or only a subset of it (new capability) encodes the final RB. Hence, in this learning model the chromosomes compete and cooperate simultaneously in order to reach a Mamdani-type FRBS with a good interpretability-accuracy tradeoff. These kinds of GFSs have been mainly designed for classification problems. In [92], a proposal is made dealing with classical fuzzy classification rules while that in [25] considers the use of the DNF fuzzy rule structure.

3.4. Extensions of the classical fuzzy linguistic rule structure and hybrid learning methods

Different Mamdani-type GFSs have been proposed based on the three linguistic fuzzy rule structure extensions described in Section 2.1: double-consequent rules [49], weighted rules [10], and linguistic hedges [109]. As a consequence of the higher complexity level introduced in the linguistic fuzzy model identification when considering these kinds of rule structures, the associated learning methods are usually complemented by a rule selection mechanism. This is done in order to both increase the cooperation level of the resulting RB and keep the derived FRBS interpretability as high as possible (it is of course reduced due to the use of a extended rule structure [1,121]).

In this way, these kinds of GFSs commonly involve a genetic rule selection method and a process to estimate the numerical parameters of the rule, either in two independent stages or in a single one. The former is the case of the GFS proposed in [49], where an initial set of candidate double-consequent fuzzy rules for modeling problems are generated by an ad-hoc data-driven method and the most cooperative subset is finally obtained by genetic selection. The selection method properly tackles the interpretability-accuracy tradeoff as for each specific fuzzy input subspace it is able to either: (i) remove the existing rules, (ii) keep a single-consequent rule (the first or the second in importance), or (iii) keep a double-consequent rule. That decision is taken according to the complexity of the modeling task in each local subspace.

One particularly interesting Mamdani-type FRBS extension requiring the associated GFSs to have an independent genetic rule selection stage is that of hierarchical KBs. The hierarchical Mamdani-type KB is composed of a set of layers where each layer in a deeper level in the hierarchy contains linguistic partitions with an increasing granularity (a layer of the hierarchical DB) and fuzzy rules whose linguistic variables take values in the latter partitions (a layer of the hierarchical RB) [55].

At least two GFSs to derive hierarchical KBs have been proposed in the specialized literature. The method introduced by Ishibuchi et al. in [97] operates by creating several hierarchical linguistic partitions with different granularity levels (e.g., from 2 two 15 labels, including the “don't care" condition to allow it to perform feature selection at rule level), generating the complete set of linguistic fuzzy classification rules in each of these partitions, taking the union of all of these sets, and performing a genetic rule selection process on the whole candidate rule set to obtain the final hierarchical RB structure.

Alternatively, the GFS proposed by Cordón et al. in [55] is designed as a strategy to improve simple linguistic FRBSs, preserving their structure and descriptive power. It is based on only reinforcing the modeling/classification of those problem subspaces with more difficulties by a hierarchical treatment of the rules generated in these regions. It uses a linguistic fuzzy rule generation method to progressively refine the controversial regions (those covered by rules with a bad performance in the original FRBS) by defining new rules in a deeper layer. The obtained hierarchical RB is compacted by a subsequent genetic selection process which plays a key role as it obtains the most cooperative rule subset composed of rules of different granularity levels. The proposal was originally devoted to modeling problems but it has been recently applied to deal with a hard classification problem variant, imbalanced classification [69].

Therefore, the latter method follows a descending approach refining the required regions by only increasing the granularity in them. In this way, the combinatorial explosion in the number of linguistic fuzzy rules derived from the fuzzy partitions of the largest granularity is avoided as those rules are only considered in those fuzzy input subspaces where it is actually needed. In a nutshell, while Ishibuchi et al.'s method apply a global (top-down) approach for the hierarchical KB generation, Cordón et al.'s one follows a local (bottom-up) approach.

Nevertheless, the most common situation is that where the rule selection and the estimation of the numerical parameters associated to the new rule structures (e.g., the rule weights) are jointly developed by a hybrid learning method. This allows us to make the best possible use of the synergy existing among structure and parameters in order to derive both accurate and interpretable linguistic fuzzy models. EAs are a very powerful tool to put this learning task into effect, thanks to their large flexibility to encode chromosomes composed of different information levels.

An example of these kinds of GFSs is the genetic multiselection process to design FRBCSs proposed in [43] (see Section 3.2) which has the capability of refining an initial RB of classical linguistic fuzzy classification rules by both removing unnecessary rules and including linguistic hedges in the rule antecedents. This novel method considers a double coding scheme, with the first chromosome part being associated to rule reduction and the second one to linguistic hedge learning. Another example is the genetic rule weighting and selection process introduced in [7] to refine a human expert-derived Mamdani-type fuzzy logic controller for heating, ventilating, and air conditioning (HVAC) systems in large buildings. It is based on another two-level coding scheme where the selected rules are encoded in a first binary-coded chromosome part and the weight vector is encoded in a second real-coded part. Both parts have a fixed-length, corresponding to the number of rules in the original RB. The GFS is guided by a multicriteria fitness function including five different performance criteria (related to thermal comfort, indoor air quality, energy consumption, and system stability) combined by means of a weighted sum.

In addition, these flexible coding schemes have also been considered by Mamdani-type GFSs to jointly develop other kinds of learning tasks such as:

(1) *Considering advanced genetic RB learning methods to derive extended rule structures*, as the GFS to extract weighted fuzzy linguistic rules following the COR methodology (see Section 3.3) introduced in [9].
(2) *Performing DB tuning while deriving RBs composed of extended rules*, as the advanced genetic tuning approach presented in [31], which jointly considers linear and/or non-linear adjustments of the membership functions and slight refinements of the fuzzy rule structures by having the chance to include the linguistic hedges "very" and "more-or-less".
(3) *Deriving fuzzy linguistic models whose rule structure considers two different extensions*, as the GFS to learn weighted double-consequent fuzzy linguistic rules by means of coevolutionary algorithms proposed in [10], or that generating weighted hierarchical linguistic fuzzy rules introduced in [6].

Finally, another very extended hybrid learning model involves the refinement of a previous definition of a whole Mamdani-type KB by means of a joint selection and tuning process. These two learning tasks have demonstrated a strong synergy, thus being a proper way to perform both fuzzy linguistic model structure identification and parameter estimation leading to a good interpretability-accuracy tradeoff. In all the cases analyzed, the linguistic fuzzy models obtained by means of the joint selection-tuning process outperformed those derived from a sequential combination of both methods.

Up to our knowledge, the first hybrid Mamdani GFS applying this approach was the tuning method introduced in [31] which was also combined with rule selection in a complex coding scheme containing four information levels: linear and non-linear membership function adjustments (DB level), and linguistic hedge addition and rule selection (RB level). Recently, another two variants were developed combining the advanced tuning methods described in Section 3.1, lateral and lateral-and-amplitude tuning, with a rule selection process in [4,2], respectively. These two GFSs were applied to the refinement of the fuzzy controller for the HVAC system in [3]. Finally, the same authors have proposed another some other genetic learning methods combining Mamdani-type fuzzy rule selection and tuning in a multiobjective fashion which will be described in the next section.

A detailed experimental summary of some of the Mamdani-type GFSs described in this section is to be found in [1].

3.5. Multiobjective genetic fuzzy systems

Evolutionary multiobjective optimization (EMO) [42] is an important research area in evolutionary computation to deal with optimization problems involving the satisfaction of several conflicting criteria. EAs are outstanding tools to solve multiobjective optimization problems since their population-based nature allows them to provide a set of nondominated solutions (Pareto set) in a single run with a different tradeoff in the satisfaction of the tackled objectives. The use of EMO algorithms to design FRBSs has been largely extended in the last few years and this is currently a hot topic in the GFS area [88].[4] Multiobjective GFSs are the most natural approach to face the interpretability-accuracy tradeoff as both requirements are clearly in conflict. The multiobjective genetic learning process allows us to jointly consider the optimization of different accuracy and interpretability measures (see Section 2.2). In addition, they show another very interesting feature: the learning mechanism

[4] A website on the topic is maintained by Marco Cococcioni at http://www.iet.unipi.it/m.cococcioni/emofrbss.html.

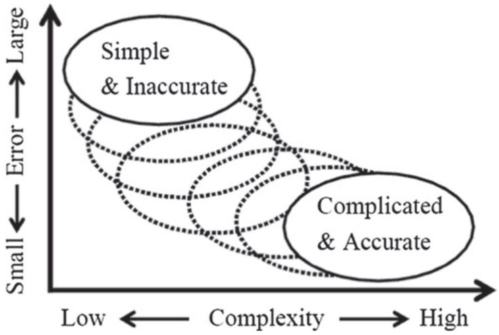

Fig. 4. Nondominated FRBSs along the accuracy-complexity tradeoff curve (reprint from [88]).

is able to find an appropriate balance between interpretability and accuracy by its own definition. This is a consequence of the output of the multiobjective GFS which directly provides a number of FRBSs with a different interpretability-accuracy tradeoff and not only one as in single-objective GFSs. As seen in Fig. 4, simple and inaccurate FRBSs are located in the top left part of the space while complicated (and thus less interpretable) and accurate ones are in the bottom right part. In this way, the model designer can then choose the most appropriate FRBS structure among those nondominated ones in the obtained accuracy-complexity (or accuracy-interpretability, if other kinds of interpretability indexes are considered) tradeoff curve according to her/his current modeling requirements.

We will briefly review a wide range of multiobjective Mamdani-type GFSs in various research areas in the following subsections.

3.5.1. Multiobjective genetic tuning

Some proposals adapting different KB components can be found within this category. In [119], the authors introduce a multiobjective genetic tuning process for the fuzzy membership functions of a fuzzy visual system for autonomous robots. This fuzzy visual system is based on a hierarchical structure comprised by three different linguistic fuzzy classifiers, whose combined action allows the robot to detect the presence of doors in the images captured by its camera. The whole fuzzy visual system DB is represented using a single chromosome encoding the four parameters defining each trapezoidal-shaped membership function in the three FRBCSs. In order to ensure the obtaining of interpretable SFPs, each linguistic fuzzy partition is encoded based on the crossing points of its membership functions and the separation between them using a real coding scheme. BLX-α crossover and random mutation are considered as genetic operators while the true positive and false positive detection rates are directly taken as the two conflicting objectives to be optimized. Three different multiobjective GAs are used (SPEA [170], SPEA2 [169], and NSGA-II [58]) and benchmarked against two single-objective EAs (a generational GA and CHC [64]), with NSGA-II reporting the best performance.

The method proposed in [136] aims to tune both the rule antecedent and the membership functions of a preliminary FRBCS structure by means of NSGA-II. The initial KB is obtained from the transformation of a C4.5 decision tree. The tuned definitions are represented by means of a double real-coding scheme including a chromosome part for the rule antecedents and another part for the fuzzy sets. Gaussian membership function shapes specified by three parameters are considered. Polynomial mutation and simulated binary crossover are taken as genetic operators, while the three objectives to be minimized are the number of misclassified patterns (accuracy), and the number of rules and the total rule length (complexity). In [135] this multiobjective genetic tuning process is applied to a real-world bioaerosol detector problem by customizing the three objectives. In this case, the true positive and false positive detection rates are considered as accuracy criteria while a membership function similarity metric composes the semantic interpretability measure.

Finally, in [29] the authors introduce a multiobjective version of their genetic context adaptation method described in Section 3.1. The new multiobjective genetic tuning process is also based on NSGA-II and uses two objectives, their previous proposal of a semantic interpretability index based on fuzzy ordering relations [28] and the mean square error, aimed at generating a set of Pareto-optimal context-adapted Mamdani-type FRBSs with different trade-offs between accuracy and interpretability. The context adaptation is obtained through the same procedures considered in the single-objective version, i.e., specifically designed operators that adjust the universe of the input and output variables, and modify the core, the support and the shape of the fuzzy sets in the fuzzy partitions. The EMO algorithm showed a very good performance in four different modeling problems.

3.5.2. Multiobjective genetic rule selection

The two-objective genetic rule selection process for the design of linguistic FRBCSs introduced in [90] is one of the earliest studies on multiobjective GFSs. It is a direct variant of the single-objective genetic selection process in [97] based on a weighted sum with fixed weights (see Section 3.2) by considering the joint optimization of an error criterion and a RB complexity measure (number of rules). This two-objective formulation was later extended to a three-objective one in [93] by introducing the total number of antecedent conditions (i.e., the total rule length) as an additional complexity index. Both

GFSs are implemented by means of a multiobjective GA based a scalar fitness function (weighted sum) with random weight values although any other, more advanced EMO algorithm can be used (e.g., a Pareto-based algorithm such as NSGA-II or SPEA).[5] A multiobjective memetic algorithm (i.e., a hybrid algorithm of EMO and local search) was considered with the latter three-objective genetic rule selection in [99]. The three-objective genetic selection process was also used in [83] to build linguistic fuzzy classifiers for a real-world application, the test of the capability of inter-vehicle communication to avoid traffic congestion. The incorporation of user preferences into the multiobjective genetic rule selection process was tackled in [124]. Finally, in [95] the same authors also considered its use to design linguistic fuzzy classifier ensembles by combining the obtained nondominated FRBCSs.

3.5.3. Multiobjective joint genetic selection and tuning

Several multiobjective Mamdani-type GFSs have been introduced to jointly perform rule selection and tuning to make use of the positive synergy between both post-processing approaches. The proposal in [13] is devoted to the design of fuzzy linguistic models where the fuzzy partitions are composed of triangular-shaped membership functions. It considers the classical coding scheme with two information levels, a binary string for the rule selection and a real-coded array for the three definition parameters of each fuzzy set. Two objectives are to be minimized, the mean square error (accuracy) and the number of rules (complexity). The most distinguishing characteristic of this multiobjective GFS is its accuracy-oriented nature. As it is considered that, between the two modeling goals, accuracy and interpretability, the former could be more important for the model designer, the multiobjective process focuses the search on the most accurate part of the accuracy-complexity tradeoff curve. Hence, it obtains a Pareto set approximation composed of a small number of very fine solutions in terms of accuracy which still present the lowest possible number of rules. To do so, the authors apply two different modifications on the classical SPEA2 multiobjective EA (they check the fact that SPEA2 is better adapted for this learning task than NSGA-II) by: (i) restarting the population at the middle of the run time, keeping the individual with the highest accuracy as the only one in the external population and generating all the new individuals with the same number of rules it has and (ii) decreasing the number of chromosomes in the external population considered for the binary tournament in each iteration, focusing the selection on the most accurate ones. Hence, the designed multiobjective EA (called Accuracy-Oriented SPEA2, $SPEA2_{Acc}$) progressively concentrates the search in the most promising solutions, allowing exploration at the first stages of the search and favoring the exploitation of the most accurate solutions at the later stages. Later, in [72], the authors analyze the performance of six different multiobjective EAs in the joint selection and tuning of linguistic fuzzy models using the same coding scheme and objective functions. Among them, they test a new version of $SPEA2_{Acc}$ based on the use of a specifically designed crossover operator, which obtains the best performance in the experiments developed.

In [73] the authors extend the latter proposal by adding a third objective, a novel interpretability index measuring the semantic integrity of the FRBS fuzzy partitions. It is based on computing the "amount of adjustment" developed on a fuzzy partition by comparing the tuned membership functions with those in a SFP, which is considered as the highest interpretability definition. The index is computed as the geometric mean of three similarity metrics accounting for the displacement of the modal points of the fuzzy sets, and the variation in their lateral amplitude rates and areas. In this way, the proposed multiobjective EA jointly optimizes one accuracy (mean square error) and two interpretability (the classical RB complexity and the new the new semantic interpretability) criteria, with the latter two belonging to a different kind, readability and comprehensibility (see Section 2.2). In addition, the authors introduced another multiobjective Mamdani-type GFS for joint rule selection and tuning in [75]. In this case, the fuzzy membership function definitions are encoded using the 2-tuples representation and only adjusted via lateral tuning (see Section 3.1). The method is again based on SPEA2 and incorporates specific mechanisms to maintain the population diversity and to expend few evaluations in the optimization process. This requirement is a consequence of being applied to refine the fuzzy controller for the time-consuming HVAC systems application. The maximization of the fuzzy controller performance, based on the weighted sum of the five performance criteria (see Section 3.4), and the minimization of the number of rules are considered to guide the multiobjective search. A large experimental setup including all the previous GFSs for the problem as well as all the different variants for multiobjective joint rule selection and tuning reviewed in this subsection showed how the new proposal provided the state-of-the-art performance.

3.5.4. Multiobjective genetic rule base and knowledge base learning

This is another very prolific topic in the multiobjective Mamdani-type GFS research area. The techniques described in the previous subsections are very useful to improve the accuracy (and, in many cases, also the interpretability) of a previously designed Mamdani-type KB but they cannot generate it from scratch as those analyzed in the current one. First, a three-objective fuzzy classification rule learning algorithm was compared with its rule selection version (see Section 3.5.2) in [93] with the aim to build comprehensible FRBCSs for high dimensional problems using a small number of short linguistic fuzzy classification rules with clear linguistic interpretations. The multiobjective genetic learning process was based on the Pittsburgh-Michigan hybrid approach later published in [100] (see Section 3.3) and considered the use of a scalar fitness function with random weights. In [96], the latter algorithm was generalized as a Pareto-based multiobjective method for interpretability-accuracy tradeoff analysis using NSGA-II. In every case, each RB definition is represented as a concatenated integer string of variable length (considering "don't care" conditions) which only encodes the rule antecedents (the class

[5] Notice that, studies on multiobjective GFSs started in the mid 90s, when EMO algorithms were still at a preliminary stage of development.

and the certainty factor in the rule consequent are computed from a heuristic procedure). The accuracy and complexity of the resulting FRBCSs are jointly optimized by measuring the number of correctly classified training patterns, and the total number of fuzzy rules and premises.

One of the first multiobjective Mamdani-type GFS to learn a whole KB definition was introduced in [46] based on the embedded learning approach (see Section 3.3). The classical, first generation Fonseca and Flemming's multiobjective GA (MOGA) [71] is used for jointly performing feature selection and fuzzy partition granularity learning in order to obtain FRBCSs with a good tradeoff between classification ability and RB complexity. Two objectives are jointly minimized, the classification error and a product of the total number of selected features and the average granularity of their fuzzy partitions. The method is later extended in [47] by also incorporating the learning of a non-linear scaling function. Besides, Alonso et al. has also proposed some multiobjective Mamdani-type GFSs following the same learning approach. In [15], they present a NSGA-II EMO algorithm to learn the optimal granularity for the SFPs in linguistic fuzzy classifiers derived by the HILK heuristic method [17]. This GFS is strongly concerned on the design of interpretable FRBCSs as that is already the aim of the HILK methodology. The embedded learning technique is guided by a three-objective fitness function composed of one accuracy, the maximization of the right classification rate, and two interpretability criteria. The two interpretability criteria belong to the two existing families: (i) readability, with the minimization of the total rule length and (ii) comprehensibility, with the minimization of the average number of rules fired at the same time. Up to our knowledge, this is the first time that a comprehensibility criteria of the latter kind, i.e., related to the FRBS reasoning mechanism, is considered in the specialized literature. Recently, the latter proposal has been extended in [30] by considering a novel comprehensibility index called logical view index, which is based on a semantic cointension approach (see Section 2.2). In this new multiobjective GFS variant, the average number of rules fired at the same time is substituted by the logical view index as a better FRBCS comprehensibility measure.

A very elaborated technique following a different approach is presented in [162]. The learning process takes the Pittsburgh approach as a base and uses NSGA-II to derive different definitions of the whole KB including both the linguistic fuzzy rule structures in the RB and the granularity and Gaussian-shaped membership function shapes in the DB. "Don't care" conditions and different semantic interpretability indexes are considered to increase the interpretability of the obtained FRBSs. The multiobjective GFS can derive both linguistic fuzzy controllers and classifiers. Another multiobjective approach to learn FRBCS KBs based on SPEA2 is to be found in [149]. It also presents some distinguishing characteristics such as the use of a tailor-made two-information level representation scheme, which helps to maintain the interpretability and allows the application of problem-specific variation operators; the use of different membership function shapes; the consideration of the area under the receiver operating characteristic curve (AUC) as accuracy criterion (the two complexity criteria are the usual total number of rules and premises); and the inclusion of a self-adaptation parameter mechanism in the multiobjective EA.

Some other proposals for multiobjective GFSs to learn linguistic fuzzy model KBs are those developed by Ducange et al. In [41], they adopt a variant of the $(2+2)$-Pareto Archived Evolutionary Strategy ($(2+2)$-PAES) [105] to only learn the RB structure. The method considers the integer coding proposed in [93] for both the rule antecedent and consequent, one-point crossover, and two appropriately defined mutation operators. The tackled optimization criteria are the root mean square error of the model (accuracy) and the total number of premises in the RB (complexity). This method is extended in [11] to allow it to learn a whole KB structure by encoding the fuzzy partition triangular-shaped membership functions using the linguistic 2-tuples representation model (see Section 3.1). Both the $(2+2)$-PAES and the NSGA-II EMO algorithms are considered to put the learning task into effect by optimizing the same two criteria. Later, in [18], the authors present a more sophisticated technique to concurrently learn the RB structure and the granularity of the uniform partitions in the DB. To this aim, the concepts of virtual and concrete RBs are introduced in order to tackle a reduced search space exploiting a two information level-chromosome encoding both the variables' partition granularities and the virtual RB. The RB is thus defined on fictitious partitions with a maximum fixed granularity and only when accuracy and complexity (measured through the same criteria considered in the previous contributions) have to be evaluated this sort of virtual RB is mapped to a concrete RB by using the number of fuzzy sets determined by the first chromosome part. The considered genetic operators manage virtual RBs.

The latter Mamdani GFS is extended in [19] by considering a similar concept but applied to fuzzy partitions. Virtual and concrete partitions are considered by an $(2+2)$-PAES EMO algorithm to derive different KB definitions by concurrently learning the RB, the granularity of the fuzzy partitions, and the membership function definitions. While virtual partitions are defined by uniformly partitioning each linguistic variable using a fixed maximum number of fuzzy sets, concrete partitions considers the specific granularity determined by the evolutionary process for each linguistic variable. To encode the possible KB definitions, a specific three information level-chromosome structure is proposed where virtual RBs and membership function parameters are defined on the virtual partitions and mapped to the concrete partitions for their bi-criteria evaluation. Further, the membership function definitions are learned through a piecewise linear scaling function (see Section 3.1), thus resulting in a compact representation. In this case, the *ad hoc* designed genetic operators handle both the virtual fuzzy partitions and the virtual RBs. Later, in [20], the latter method is extended into a three-objective framework concerning the joint optimization of accuracy, RB complexity, and fuzzy partition integrity. The RB complexity index accounts for the total rule length and the partition integrity is evaluated by means of a specifically designed index based on the piecewise linear transformation, which takes the semantic integrity index proposed in [29] as a base (see Section 3.5.1).

Finally, in [62], the first multiobjective Mamdani-type GFSs described, that presented in [41], is extended to a three-objective NSGA-II-based framework to learn the RB of FRBCSs applied to imbalanced classification problems. Two accuracy (sensitivity and specificity, i.e., true positive and false positive rates) and one complexity (total rule length) criteria are considered, and the ROC convex hull method is taken to select the potentially optimal FRBCS design from the projection of the Pareto front approximation onto the ROC plane. All the FRBCSs obtained in the ROC convex hull are characterized by the lowest classification errors and low values of complexity when benchmarked against other fuzzy and non-fuzzy classifiers in the experimentation developed.

3.6. Genetic fuzzy systems for low quality data

This subsection covers a recent but very promising topic within the GFSs area, that considering evolutionary learning processes to derive Mamdani-type FRBSs from low-quality data (i.e., data observed in an imprecise way, such as noisy or vague data). As advocated by Sánchez and Couso in their pioneering work [142], this is a system identification branch where fuzzy systems can actually make a difference with classical approaches. The reason is that this constitutes a specific domain of competence for the application of Zadeh and Mamdani's seminal ideas based on the use of fuzzy sets to represent knowledge and perform fuzzy reasoning.

It is well known that real-world problems are inherently affected by uncertainty, vagueness, and imprecision. System identification techniques consider data sets obtained from measurements taken on the attributes of a physical system to derive the related model. Of course, these measurements are not precise due to many different reasons such as the sensors' tolerance or the presence of noise in the environment. Sometimes, the difference between an attribute observation and its actual value can be ignored and the model quality can be quantified by the accuracy in the prediction of such observations. On the opposite, when the discrepancy between the measured and the actual values is significant, the latter approach is not appropriate to obtain models from the available uncertain data. In such situations, the classical approach is to define a probability distribution on the error values. This allows the designer to apply classical robust statistics techniques to derive models by optimizing their average quality on the distribution of the differences between the observations and the actual values [24,160].

Nevertheless, there are many real-world domains where the observation error does not follow a single probability distribution, thus making the average quality of the derived models to become undetermined (that is the case, for example, of global positioning systems (GPSs)). In those cases, a classical stochastic model of the error presents a limited validity. The identification technique can only rely on the optimization of the best or worst case average quality, which is not a very selective criterion, thus reducing the chance to obtain high quality, robust models.

Hence, the use of fuzzy logic-based models becomes a promising alternative to deal with these kinds of problems. In fact, fuzzy data is the main object of study in fuzzy statistics [56] and, according to fuzzy statistics viewpoint, the primary use of fuzzy sets in system identification problems is for the treatment of vague data. Using vague data to train and test fuzzy models and classifiers allows us to analyze the performance of these FRBSs on the type of problems for which they are expected to be superior. However, fuzzy logic techniques are not, generally speaking, compatible with those fuzzy statistical techniques used for modeling vague observations of variables. This is the reason why fuzzy models and classifiers have only been derived from crisp data up to now.

Some work has been developed in the area to bridge the latter gap. Different formalizations for the definition of fuzzy classifiers, based on the relationships between random sets and fuzzy sets were proposed in [141]. In that contribution, it was also shown that a GA can perform rule selection in a random sets rule-based system with the resulting classifiers being competitive with state-of-the-art Mamdani-type genetic fuzzy classifiers in both accuracy and interpretability. Later, Sánchez and Couso established the basis to derive linguistic fuzzy models and classifiers from low quality data using EAs (low quality data-based GFSs) [142]. First, they chose the possibilistic interpretation of the meaning of fuzzy membership among the different definitions existing in the specialized literature [114] as the most appropriate definition to relate fuzzy data and low quality data in the current scenario. This interpretation consists of understanding a fuzzy membership function as a nested family of sets, each one of them containing the true value of the variable with a probability greater than or equal to certain bound [56]. Hence, it matches a large amount of practical situations. For instance, it can be used to model data sets with missing values (one interval that spans the whole range of the variable), left and right censored data (the value is greater or lower than a cutoff value, or it is between a couple of bounds), compound data (each item comprises a disperse list of values), mixes of punctual and set-valued measurements (as produced by certain sensors, for instance GPS receivers), etc. All these cases share in common a certain degree of ignorance about the actual value of a variable and assume less prior knowledge than the standard model.

In addition, they focused on the required adaptations to allow an EA to derive a Mamdani-type FRBS from low quality data. The key question is the change in the objective function, which must handle an imprecise evaluation of the FRBS error. Thus, this function becomes interval-valued or fuzzy as the model/classifier error can be defined as an interval or a fuzzy set. As an example, for the case of a fuzzy classifier, the overall number of errors can be obtained by adding the individual errors with interval or fuzzy arithmetic operators. In this way, estimating a fuzzy model or classifier from data requires a numerical technique that finds the minimum of the fuzzy modeling or classification error with respect to the free parameters of the FRBS. EAs were selected for that task thanks to their inherent flexibility which allowed the definition of evolutionary mechanisms able to deal with these kinds of fuzzy fitness functions. These mechanisms are based on the use of either precedence operators between imprecise values [106,108,154] or EMO algorithms capable of optimizing a mix of crisp and fuzzy objectives [144].

Since the definition of that framework, Sánchez et al. have developed many different GFSs to derive linguistic fuzzy models and classifiers from low quality data. In fact, all the existing proposals in the area are based on the use of the Mamdani-type FRBS structure as the linguistic interpretability of the obtained model is a requirement for the real-world problems tackled. Regarding modeling problems, a first step was taken in [147] by demonstrating that the use of these kinds of GFSs on crisp data sets with a small amount of artificially added fuzziness resulted in the obtaining of more robust fuzzy models. The latter contribution was later extended in [146] to focus on data sets with medium to high discrepancies between the observed and the actual values of the variables, such as those containing missing values and coarsely discretized data. The GFS considered the incremental learning of Mamdani-type fuzzy models with backfitting algorithms. The negative effect the outliers can have in such learning approach was solved by fuzzifying by hand the crisp data set and optimizing the subsequent fuzzy-valued fitness function with a multicriteria simulated annealing algorithm [147] and a new multiobjective GCCL method (see Section 3.3) based on NSGA-II [146], respectively. This mechanism acted as a regularization process in the system identification as candidate fuzzy models with high slopes were penalized because a small deviation in the input will mean they have a higher upper bound in the proposed fuzzy error measure. This work thus showed collateral advantages of the proposed low quality data-based Mamdani GFS framework.

In [38] the methodology was applied to problems where each pattern comprises a set of values, such as multiple-query questionnaires. Instead of considering averaged answers, which discards potentially useful information and can lead to the arising of contradictions in the data, these lists of answers are represented by means of fuzzy sets following the possibilistic interpretation. A linguistic fuzzy model of preferences in a marketing problem is obtained with a Mamdani-type GFS. The fuzzy-valued fitness function was optimized by means of a GA that used a fuzzy ranking to select the best of any two fuzzy intervals. Later, in [145], the same problem was tackled with an EMO algorithm, derived from NSGA-II, that does not use the fuzzy ranking but a true dominance relation based on the fuzzy fitness. The validation error of the low quality data-based fuzzy model was found to be lower than that of the fuzzy model derived from crisp data (computed as the average of the value of the answers).[6]

Concerning the design of Mamdani-type FRBCSs from low quality data, the first example of a genetic fuzzy classifier of this kind was introduced in [129] based on the GCCL approach. A new extension principle-based fuzzy reasoning method that is compatible with the possibilistic view of the imprecision in the data is proposed in such a way that the derived linguistic fuzzy classifier is able to operate even when we cannot accurately observe all the properties of the tackled object. The proposed low quality data-based GFS is a generalization to vague data of the classical genetic fuzzy classifier proposed in [91], which was chosen because of its good tradeoff between simplicity and performance. The latter crisp data-based GFS is based on only encoding the rule antecedents and adjusting the class and the certainty degree in the consequent in each population by a reward and punishment scheme. The low quality data adaptation comprises: (i) the definition of new procedures to assign the consequents, (ii) the computation of fuzzy fitness functions, and (iii) the genetic selection and replacement of the worst individuals under the newly defined fuzzy fitness functions.

The new method was tested in four different families of problems, synthetic data sets, realistic problems, real-world problems, and classical UCI crisp data sets, obtaining outstanding results in comparison with the crisp data-based GFS. The method was applied to tackle the modeling of the future performance of athletes to assist coaches in the configuration of an athletics team in [130]. The resulting Mamdani-type fuzzy KBs combine the experience of the trainer and his personal knowledge of the athletes with genetically mined information from past training sessions and competitions, thus highlighting the importance of the linguistic interpretability. The considered low quality data sets integrate subjective perceptions of athletes' mistakes, different kinds of measurements taken by different observers, and interval-valued attributes. The performance of the fuzzy classifiers finally obtained was compared with that of several classical methods derived from the crisp data sets resulting from taking the average for each imprecise attribute value, clearly showing the performance advantage of the former ones.

Finally, in [128], the proposal was extended to deal with other real-world problem, the design of an intelligent system for the diagnosis of dyslexia in early childhood to assist psychologists. The diagnosis is based on non-writing-based graphical tests solved by the children which are later scored by the human experts. Thus, the automation of this task is hampered by multiple sources of uncertainty. The extension involved the use of fuzzy classification rules with a class and a certainty factor in the consequent (see Section 2.1), which was not supported by the previous version. The new design for the low quality data-based genetic fuzzy classifier clearly outperformed both the crisp GFS and the previous version in the experimental study developed with five different real-world data sets showing different characteristics (the first three of them contain vague data in both the input and the output variables while the last two consider precise inputs and vague outputs). The algorithm was integrated in a web-based, automated prescreening software tool to be used by the parents for detecting those symptoms that advise taking the children to a psychologist for an individual examination.

[6] In addition, in [158,159] the authors dealt with a modeling problem with inherently fuzzy data, the calibration of taximeters with a GPS. The output of a standard GPS receiver comprises a set of confidence intervals for the expected position of the vehicle, obtained at different significance levels. Thus, it directly matches the same semantic interpretation of a fuzzy set. To succeed in the calibration task, there was a need to compute the lowest upper bound of all the trajectories compatible with a set of fuzzy coordinates. A modified NSGA-II EMO algorithm was used to search for a fuzzy model that minimized the fuzzy error between that set of vague coordinates taken from the GPS and the model-based trajectory of the taxi. However, in this case that fuzzy model was not based on a rule-based structure.

In addition, in [143] the learning of a Mamdani-type fuzzy classifier is formulated as a problem of statistical inference. To do so, fuzzy memberships are understood as coverage functions of random sets and the rules are learned by maximizing the likelihood of the classifier. Interval-censored data and upper and lower bounds of the likelihood are considered to evolve the RBs using a new low quality data-based GFS definition. The new method is an extension of the GA to generate random sets rule-based systems proposed in [141]. It is based on the combination of an advanced co-evolutionary multiobjective algorithm with a gradient descent method which is able to produce not only a nondominated linguistically understandable classifier, but also the list of the instances of the training set that contribute the most to the uncertainty about the fitness of the classifier. The new approach was able to obtain better performing FRBCSs than different existing statistical and fuzzy classifiers in synthetic low quality data problems and crisp problems with missing data.

As seen in this subsection, exploiting the information in low quality data sets has been recently acknowledged as a new challenge in GFSs. In our opinion, low quality data-based Mamdani-type GFSs are a very useful system identification tool which will become a hot topic in the area in the next few years.

4. Concluding remarks and future prospects

Linguistic FRBSs have demonstrated their outstanding capability for system identification along their almost forty years of development. They have been successfully applied to a large amount of real-world problems in many different modeling, classification, and control domains. Mamdani-type FRBSs, which put Zadeh's seminal ideas into effect, allows comprehensible grey-box models comprised by a set of linguistic descriptions regarding the system behavior to be obtained if a proper design is developed. Linguistic fuzzy models are thus very appropriate tools to face the two system identification requirements, accuracy and interpretability, permitting both an automatic derivation from data and the incorporation of human expert knowledge, and integrating numerical and symbolic processing into a common scheme. Besides, the goodness of the Mamdani-type fuzzy rule structure has opened the door for its use in other artificial intelligence fields such as data mining [70].

Nevertheless, Mamdani-type FRBSs present some pitfalls related to their lack of accuracy when modeling some complex, high-dimensional systems as a consequence of the hard restrictions imposed by the use of linguistic variables. The descriptive power is obtained at the cost of an exponentially increasing model complexity. To solve this problem, different extensions and advanced design methods have been proposed during the last two decades within the realm of the fuzzy modeling interpretability-accuracy tradeoff. GFSs have become one of the main tools to develop these advanced linguistic FRBS derivation approaches. This contribution has reviewed the different Mamdani-type GFSs proposed in the specialized literature with the aim of improving the accuracy of linguistic fuzzy models while preserving the interpretability unaltered or reducing it to the lower possible degree.

Although the field of Mamdani-type GFSs has reached its maturity after twenty years of development, there are still many current and future trends to be considered. Some of them have already been mentioned through this contribution and were also identified by Herrera in his survey contribution on GFSs [85]. Among them, we can highlight the proposal of new interpretability assessment indexes for Mamdani FRBSs (see Section 2.2) or the design of new Mamdani GFS frameworks to cope with the additional learning complexity when dealing with large and/or high-dimensional datasets. As discussed in Section 2.1, the latter problem has a strong influence on the design of Mamdani-type FRBS due to the rigid partitioning of the input and output spaces made by linguistic variables. Some recent approaches to the problem are to be found in [12,21,57,123,125,138]. In particular, multiobjective Mamdani GFSs show a large number of hot lines. On the one hand, there is a strong interest on analyzing new ways to incorporate user preferences to the learning process in such a way that the EMO algorithm constrains the search on a specific region of the Pareto front [90]. That is especially the case when dealing with a multiobjective interpretability-accuracy space (see for example the proposal in [13], reviewed in Section 3.5.3). On the other hand, there is another research trend, shared with the EMO community, on the consideration of more than just two/three objectives (the current multiobjective GFS standard) in the learning process, which would require the definition of more sophisticated Pareto-based EMO algorithms than the current state of the art. The problem states on the fact that, for high dimensional objective vectors, the probability that a solution dominates another becomes very small and this may lead to a large number of Pareto-optimal solutions. Nevertheless, recent proposals in the EMO field have managed to deal with a significantly large number of objectives in what is called evolutionary many-objective optimization [98,137] and their adaptation to multiobjective GFS has become feasible.

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