Practical Questions

CSCA48 – Week 8

a) Implement a Heap class, with all the methods described in class. We know you have the code already, but make sure you can do it on your own without looking at Nick's code. Try to do it both with and without recursion.

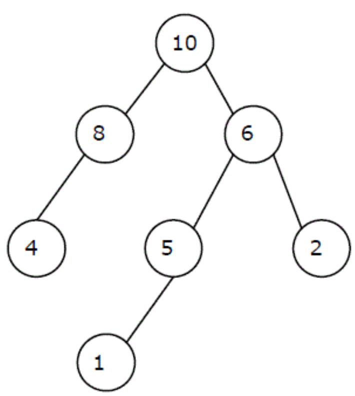
b) Implement a Priority Queue class, using the Heap class from part a, with all the methods describe in class. We know you have the code already, but make sure you can do it on your own without looking at Nick's code.

c) Let's practice more BANANA game we played in class. List the moves of following using priority queue, if it is impossible then just write impossible. (A < B < N)

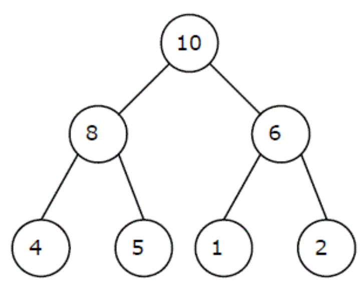
AAABNN   NNBAAA   ANANAB   ANNABA   NAANAB   BNAANA

d) A max-heap is a heap where the value of each parent is greater than or equal to the values of its children. Which of the following is a max-heap?

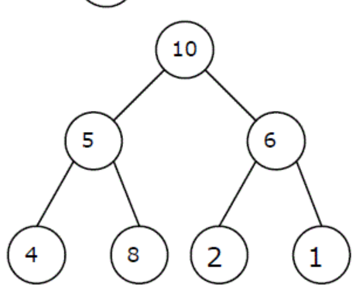
(A)



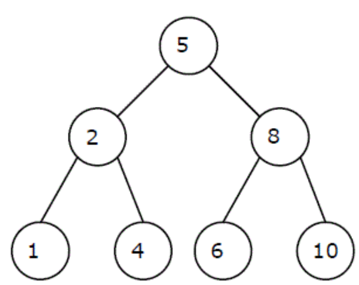
(B)



(C)



(D)



e)

i. Draw the binary min heap (smallest value at the root) that results from inserting: 77, 22, 9, 68, 16, 34, 13, 8 in that order into an initially empty binary heap. You do not need to show the list representation of the heap. You are only required draw the final heap.

ii. Draw the binary heap that results from doing 2 delete mins on the heap you created in part i. You are only required draw the final heap.

iii. What is the null path length of the root node in the last heap you drew in part ii above?

f)

i. Draw the binary min heap (smallest value at the root) that results from inserting: 4, 9, 3, 7, 2, 5, 8, 6 in that order into an initially empty binary heap. You do not need to show the list representation of the heap. You are only required draw the final heap.

ii. Draw the binary heap that results from doing 2 delete mins on the heap you created in part i. You are only required draw the final heap.

iii. What is the null path length of the root node in the last heap you drew in part ii above?

g)

i. Arrange the following values so that they fit into a min heap:
3, 37, 28, 71, 10, 69, 73

ii. Show the result of after inserting 1 into the min heap of part a

iii. Show the result of delete_ min on the min-heap of part a

h) Consider a binary max-heap implemented using an array. Which one of the following array represents a binary max-heap?

a. 25, 12, 16, 13, 10, 8, 14
b. 25, 12, 16, 13, 10, 8, 14
c. 25, 14, 16, 13, 10, 8, 12
d. 25, 14, 12, 13, 10, 8, 16

i) What is the content of the array after two delete operations on the correct answer to the previous question?
   a. 14, 13, 12, 10, 8
   b. 14, 12, 13, 8, 10
   c. 14, 13, 8, 12, 10
   d. 14, 13, 12, 8, 10

j) Suppose a priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:
   a. 10, 8, 7, 3, 2, 1, 5
   b. 10, 8, 7, 2, 3, 1, 5
   c. 10, 8, 7, 1, 2, 3, 5
   d. 10, 8, 7, 5, 3, 2, 1

k) Consider any array representation of an n element binary heap where the elements are stored from index 1 to index n of the array. For the element stored at index i of the array (i <= n), the index of the parent is
   a. i – 1
   b. floor(i/2)
   c. ceiling(i/2)
   d. (i+1)/2

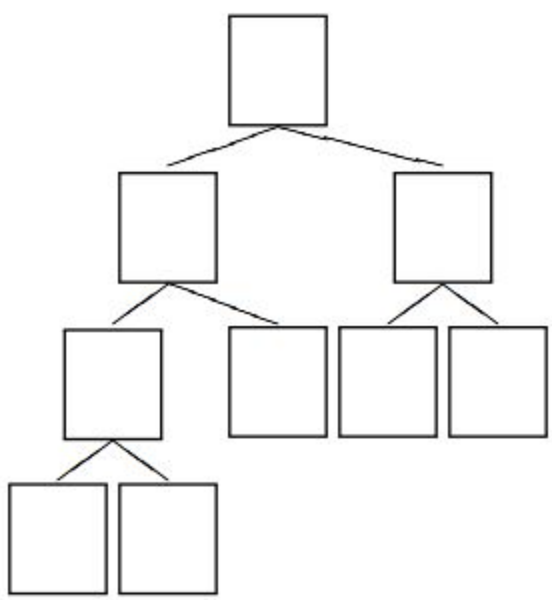
l) A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, a[0], nodes in the next level, from left to right, is stored from a[1] to a[3]. The nodes from the second level of the tree from left to right are stored from a[4] location onward. An item x can be inserted into a 3-ary heap containing n items by placing x in the location a[n] and pushing it up the tree to satisfy the heap property. Which one of the following is a valid sequence of elements in an array representing 3-ary max heap?
   a. 1, 3, 5, 6, 8, 9
   b. 9, 6, 3, 1, 8, 5
   c. 9, 3, 6, 8, 5, 1
   d. 9, 5, 6, 8, 3, 1

m) Suppose the elements 7, 2, 10 and 4 are inserted, in that order, into the valid 3-ary max heap found in the above question. Which one of the following is the sequence of items in the array representing the resultant heap?
   a. 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
   b. 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
   c. 10, 9, 4, 5 ,7 ,6, 8, 2, 1, 3
   d. 10, 8, 6, 9, 7, 2, 3, 4, 1, 5

n) A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is ____________ .

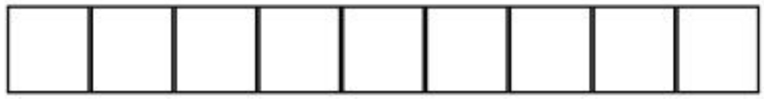
o) A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 36 can appear is ____________ .

p)
   i. Suppose the numbers 1, 2, 3, 4, 5, 6 ,7, 8, 9 were found in a min heap which is shaped like the following tree. The numbers could have been inserted in any order.



Mark with an X all the nodes where it is possible to find the number 4.

ii. Instead of using a tree, we can hold the same heap in the list below.



Mark with an X all the places in the list where it is possible to find the number 6

q) <u>(Challenge)</u>
Suppose we have a list of nonempty lists, each of which is sorted from smallest to largest. Give an algorithm that uses min-heap that merges all the lists into one big sorted list.

r) <u>(Challenge)</u>
Suppose we are implementing a binary heap, based on binary trees (not lists). We want to implement a delete operation which, given a reference to a node in the tree, can delete that node from the heap while maintaining the heap-order property—even if the node isn't the root and it isn't the minimum. Write a method called delete().

<u>Logic Question (Challenge)</u>

You are the last person in line for boarding a plane that holds 100 people. There are 99 people in front of you in line, but you're not worried, because has an assigned seat. Unfortunately, the first person in line is a terrible human being, and just sits in a random seat. Since everyone getting on the flight is Canadian, they're all too polite to ask anyone to move. So if they get on the plane, and someone else is sitting in their seat, they will pick another seat at random. If their seat is empty, they will sit in it.

What are the odds that when it's your turn to get on the plane, your seat will be empty?