2023

BAYESIAN REGRESSION
&
META-ANALYSIS

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The ‘data synthesis challenge’

As more and more data become available, how to conduct rigorous and comprehensive assessments on climate change?

Formal methods are needed to help researchers to conduct rigorous and comprehensive literature synthesis

Meta-analysis: a statistical approach for quantitative synthesis

« The analysis of analyses »

« The statistical analysis of a large collection of results from individual studies for the purpose of integrating the findings »

« Systematic review + statistical analysis »

Dictionary of epidemiology, 2001; Chalmers et al., 2002; Glass, 1976; Koricheva et al., 2013

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments

Individual experiment

Control	Treatment
X_C	X_T

Effect size = X_T / X_C

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments

Individual experiment

Absence of treatment	Tested treatment
Mortality rate X_C	Mortality rate X_T

Effect size = X_T / X_C

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments

Individual experiment

Reference feeding strategy	New feeding strategy
Animal weight X_C	Animal weight X_T

Effect size = X_T - X_C

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments

Individual experiment

Conventional cropping system	Organic cropping system
Crop yield X_C	Crop yield X_T

Effect size = X_T / X_C

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments

Individual experiment

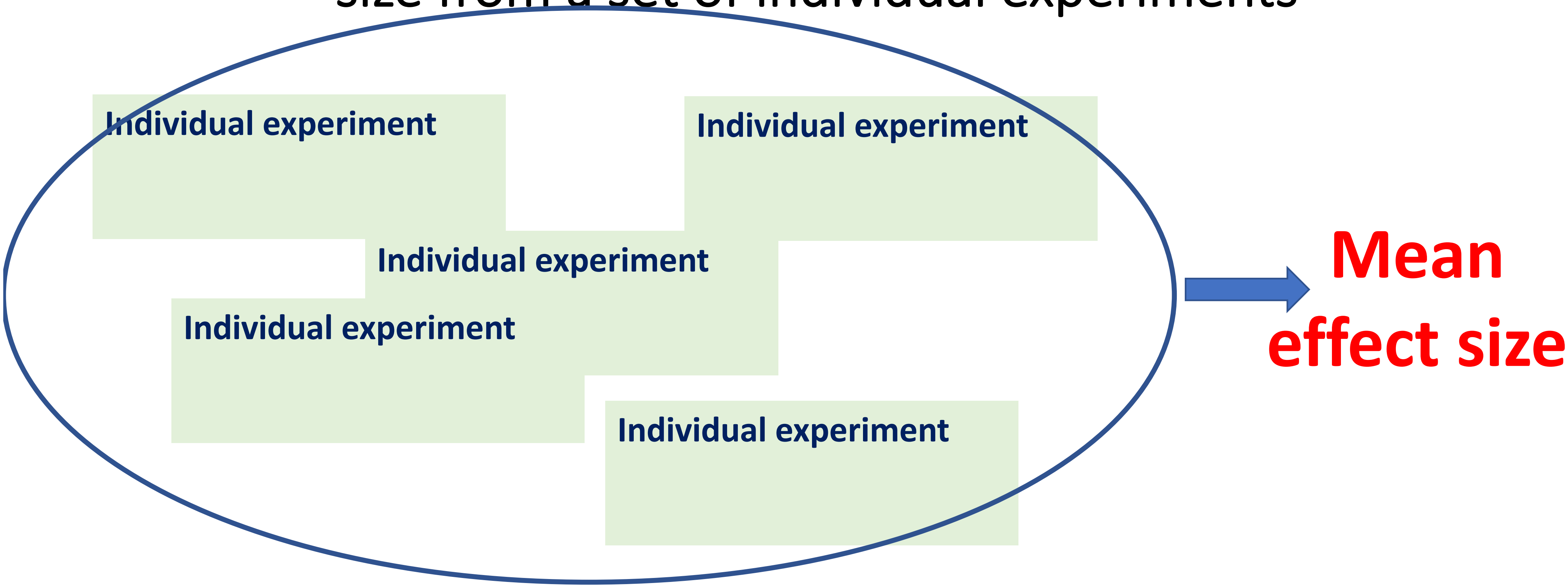
Individual experiment

Individual experiment

Individual experiment

Individual experiment

Meta-analysis main objective: estimating a mean effect size from a set of individual experiments



Main steps of a meta-analysis

- Scoping
- Literature search
- Paper selection
- Data extraction
- Statistical analysis
- Bias and uncertainty

Main steps of a meta-analysis

Step	Time
Scoping	A few hours
Literature search	One day
Paper selection	One day, up to a few days
Data extraction	Two papers per day
Statistical analysis	One to two days
Bias and uncertainty	One to a couple of hours

Main steps of a meta-analysis

Because of the steps « Paper selection » and « Data extraction », a meta-analysis often require several months of work.

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Main steps of a meta-analysis

Systematic review

Scoping

Literature search

Paper selection

Data extraction

Statistical analysis

Bias and uncertainty

Main steps of a meta-analysis

Systematic review

- Scoping
- Literature search
- Paper selection

Quantitative estimation

- Data extraction
- Statistical analysis
- Bias and uncertainty

Main steps of a meta-analysis

Meta-analysis
=
Systematic review +
Quantitative estimation
=
Quantitative systematic review

Scoping

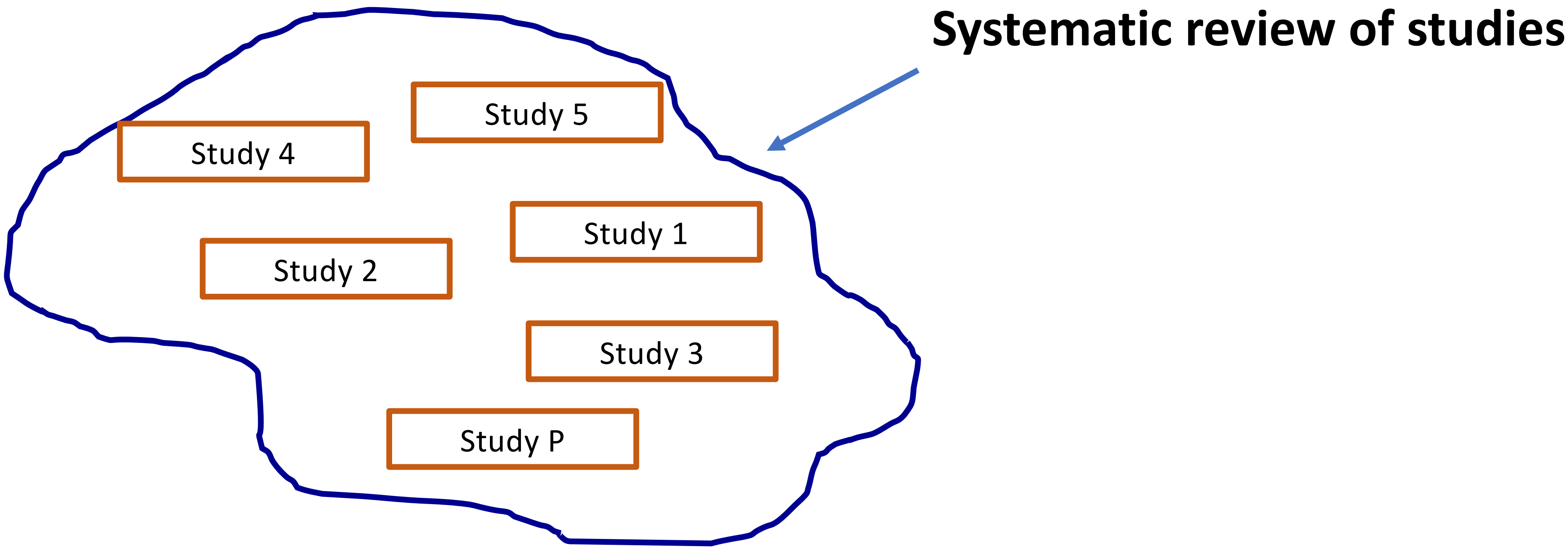
Literature search

Paper selection

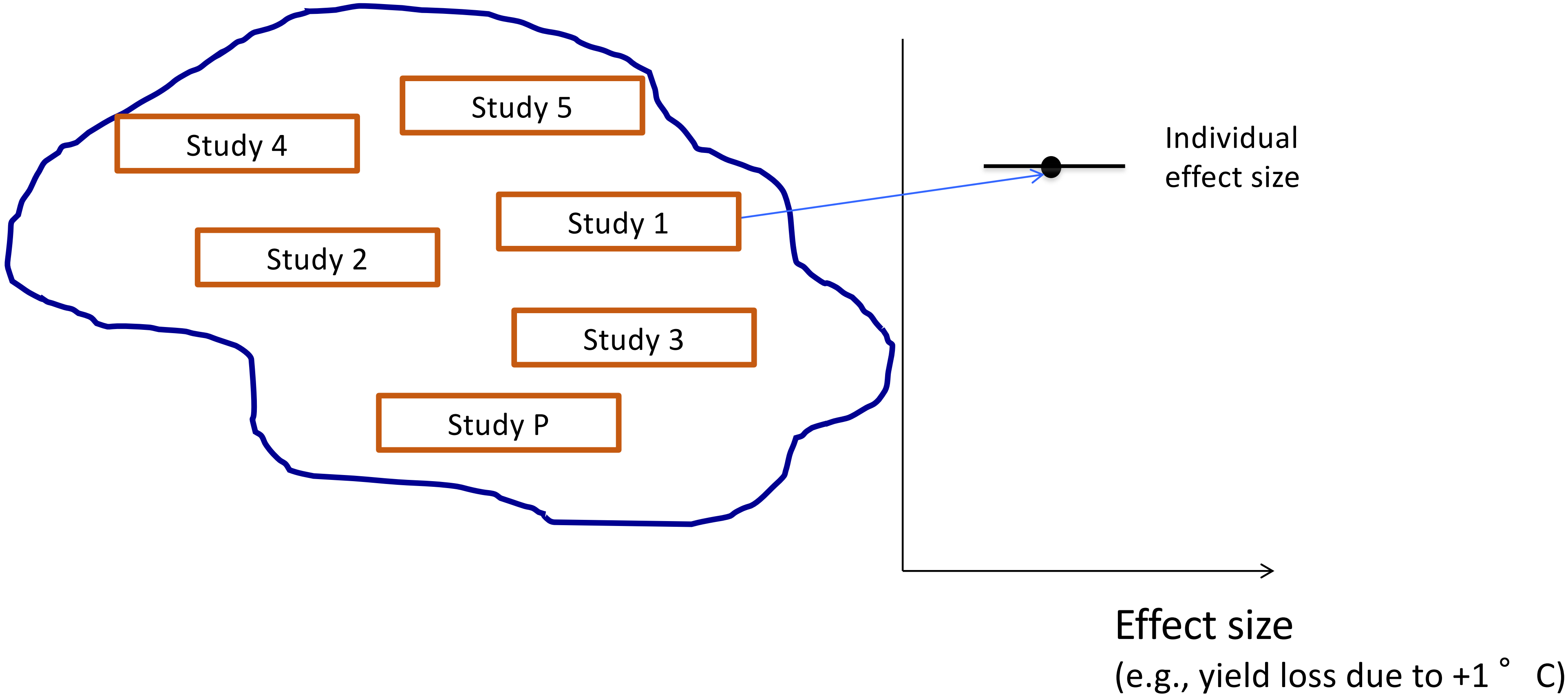
Data extraction

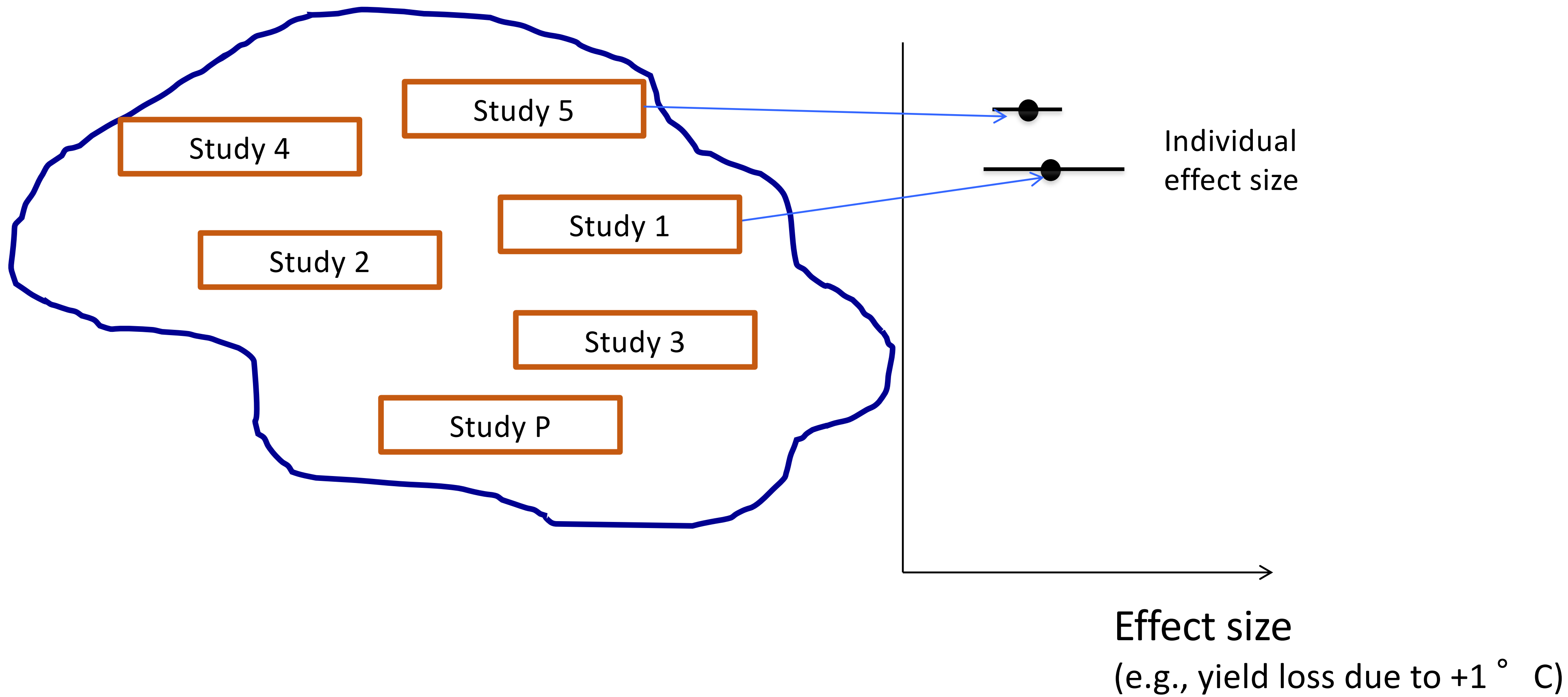
Statistical analysis

Bias and uncertainty



Set of studies dealing with a specific topic (e.g., %yield loss due to +1°C in China)


Study 5
Study 4
Study 1
Study 2
Study 3
Study P
Individual
effect size
Effect size
(e.g., yield loss due to +1 ° C)



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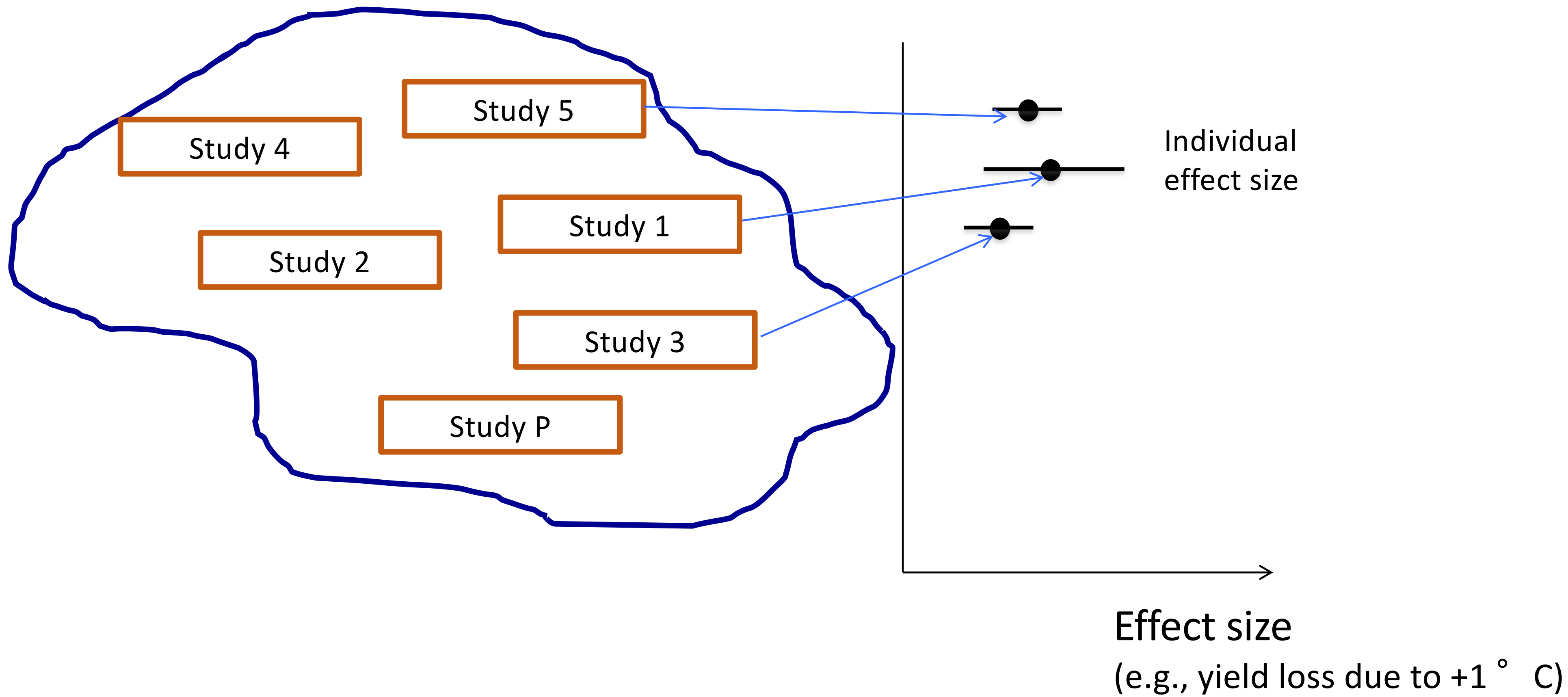
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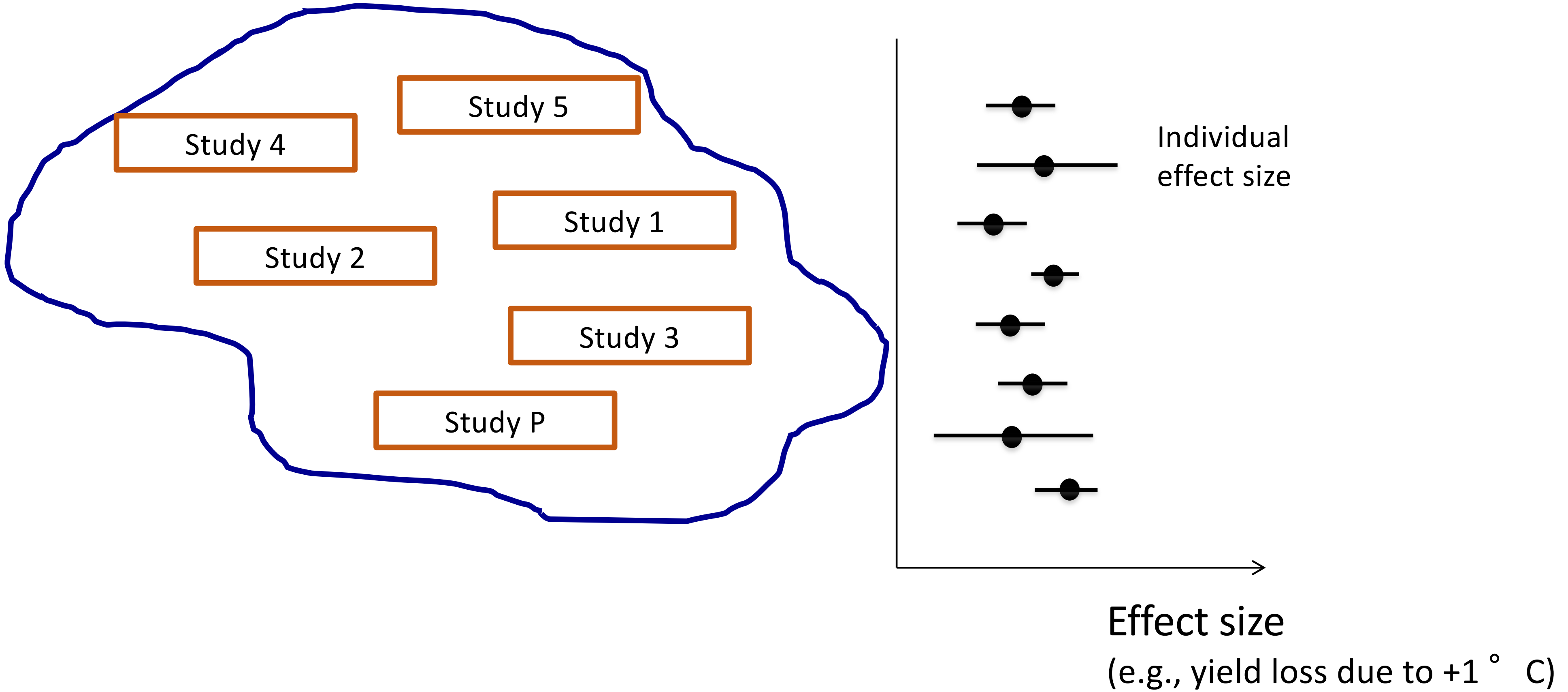
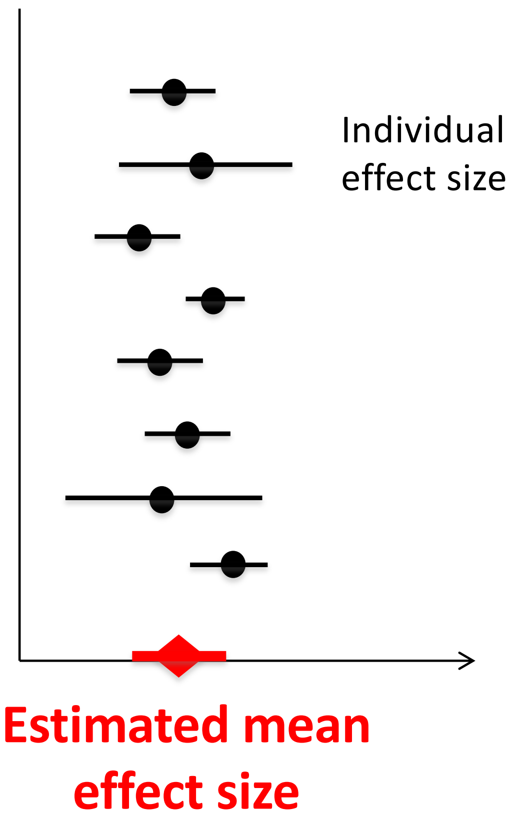
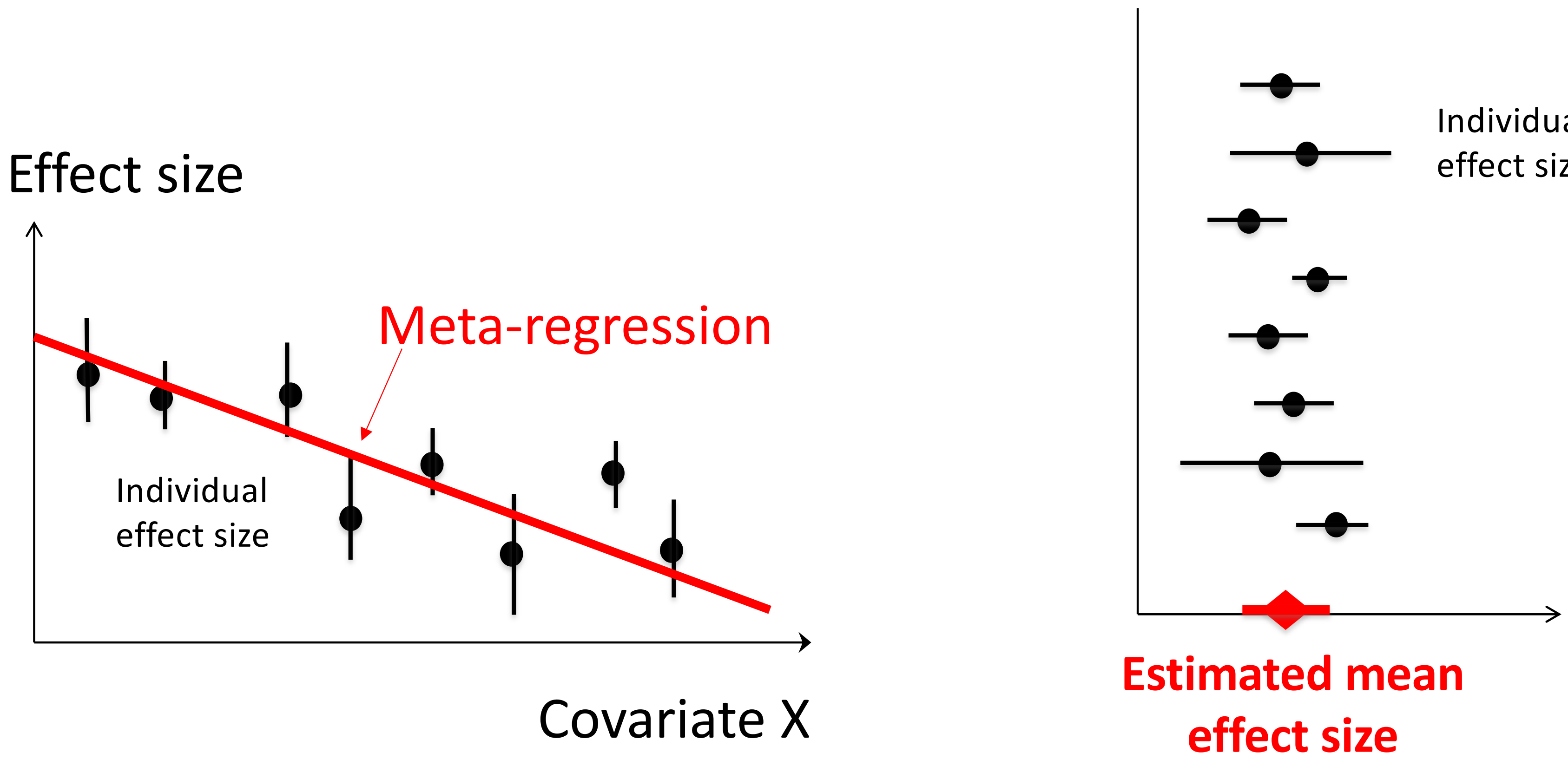

Study 5
Study 4
Study 1
Study 2
Study 3
Study P
Individual
effect size
Effect size
(e.g., yield loss due to +1 ° C)

Use of simple hierarchical models to estimate the mean effect size from the individual effect sizes




Effect size
Meta-regression
Individual
effect size
Covariate X
Individual
effect size
Estimated mean
effect size

Meta-analysis often used to deal with controversial topics in human health…

Review

Exposure to glyphosate-based herbicides and risk for non-Hodgkin lymphoma: A meta-analysis and supporting evidence

Luoping Zhang [a], Iemaan Rana [a], Rachel M. Shaffer [b], Emanuela Taioli [c], Lianne Sheppard [b, d]

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Systematic review

Effect of hydroxychloroquine with or without azithromycin on the mortality of coronavirus disease 2019 (COVID-19) patients: a systematic review and meta-analysis

Thibault Fiolet[1,2,*], Anthony Guihur[3], Mathieu Edouard Rebeaud[3], Matthieu Mulot[4], Nathan Peiffer-Smadja[5,6,7], Yahya Mahamat-Saleh[1,2]

… and in agriculture as well

A Meta-Analysis of the Impacts of Genetically Modified Crops

Wilhelm Klümper, Matin Qaim*

… and in agriculture as well

A Meta-Analysis of the Impacts of Genetically Modified Crops

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A meta-analysis of experiments testing the effects of a neonicotinoid insecticide (imidacloprid) on honey bees

James E. Cresswell

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A meta-analysis of experiments testing the effects of a neonicotinoid insecticide (imidacloprid) on honey bees

James E. Cresswell

A meta-analysis of crop yield under climate change and adaptation

A. J. Challinor, J. Watson, D. B. Lobell, S. M. Howden, D. R. Smith & N. Chhetri

Example: Assessment of the impact of temperature increase on crop yield

Two sources of information:

- Experiments
- Crop model simulations

Example: Assessment of the impact of temperature increase on crop yield

Two sources of information:

- **Experiments**
- Crop model simulations

Field warming experiment

Individual experiment

Field with infrared heater	Field with 'dummy' heater
Crop yield X_C	Crop yield X_T

Effect size = Yield sensitivity

from Chi et al. 2013

doi.org/10.1371/journal.pone.0056482

$$\Delta Y = (Yield_{warm} \text{-} Yield_{control})/Yield_{control}$$

$$Sensitivity = Yield\ \%\ change\ per\ ^{\circ}\mathrm{C} = 100\,\frac{\Delta Y}{\Delta T}$$

nature COMMUNICATIONS

Article | OPEN | Published: 17 November 2016

Field warming experiments shed light on the wheat yield response to temperature in China

Chuang Zhao, Shilong Piao, Yao Huang, Xuhui Wang, Philippe Ciais, Mengtian Huang, Zhenzhong Zeng & Shushi Peng

Compilation of 46 results of field warming experiments located in 11 sites in China

Field experiment
Ambient CO_2

Objectives

- Estimate the mean yield sensitivity to +1°C
- Estimate the effect of the mean local temperature on the sensitivities
- Analyse uncertainties

Site_name	Lat	Long	Design	Temp_increase	Irrigtion_mm	Nitrogen_kgha	TGS	PGS	DGS	Sensitivity	Ref
Nanjing	32.03	118.86	Infrared heaters	1.5	0	225	10.3	393	8	11.1	1
Nanjing	32.03	118.86	Infrared heaters	1.5	0	225	10.6	476	7.6	10.5	1
Nanjing	32.03	118.86	Infrared heaters	1.5	0	225	11.6	341	8.3	14.5	1
Nanjing	32.03	118.86	Infrared heaters	1.5	0	225	10.0	387	7.9	6.7	1
Nanjing	32.03	118.86	Infrared heaters	1.5	0	225	10.5	325	8.1	11.8	1
Tongwei County	35.22	105.23	Heating cable	1.4	0	NA	5.7	288	6.6	2.21	2
Tongwei County	35.22	105.23	Heating cable	2.2	0	NA	5.7	288	6.6	1.18	2
Lulu Mountain	35.29	105.26	Heating cable	0.6	0	NA	5.7	288	6.6	0.83	2
Lulu Mountain	35.29	105.26	Heating cable	1.4	0	NA	5.7	288	6.6	2.57	2
Lulu Mountain	35.29	105.26	Heating cable	2.2	0	NA	5.7	288	6.6	2.73	2
Guyuan	36.03	106.46	Greenhouse	0.5	0	NA	12.4	97	11.2	-9.33	3
Guyuan	36.03	106.46	Greenhouse	1.2	0	NA	12.4	97	11.2	-11.03	3
Guyuan	36.03	106.46	Greenhouse	2.0	0	NA	12.4	97	11.2	-7.4	3
Guyuan	36.03	106.46	Greenhouse	0.5	60	NA	12.4	97	11.2	-11.84	3
Guyuan	36.03	106.46	Greenhouse	1.2	60	NA	12.4	97	11.2	-10.65	3
Guyuan	36.03	106.46	Greenhouse	2.0	60	NA	12.4	97	11.2	-6.78	3
Dingxing	39.13	115.66	Infrared heaters	2.9	500	210	8.9	77	9.3	-3.1	4
Dingxing	39.13	115.66	Infrared heaters	2.9	500	210	6.4	113	8.2	8.25	4
Xidatan	38.80	106.30	Infrared heaters	0.5	NA	136	14.3	23	12.2	-1.0	5
Xidatan	38.80	106.30	Infrared heaters	1.0	NA	136	14.3	23	12.2	-0.8	5
Xidatan	38.80	106.30	Infrared heaters	1.5	NA	136	14.3	23	12.2	-3.33	5
Xidatan	38.80	106.30	Infrared heaters	2.0	NA	136	14.3	23	12.2	-8.25	5
Xidatan	38.80	106.30	Infrared heaters	2.5	NA	136	14.3	23	12.2	-7.4	5
Shanghai	31.21	121.13	Infrared heaters	1.5	NA	NA	11.0	433	6.1	3.46	6
Shanghai	31.21	121.13	Infrared heaters	1.5	NA	NA	11.4	343	6.6	3.33	6
Shanghai	31.21	121.13	Infrared heaters	1.5	NA	NA	10.2	476	5.7	3.45	6
Yucheng	36.83	116.57	Infrared heaters	1.3	150	285	8.6	118	10	1.23	7
Yucheng	36.83	116.57	Infrared heaters	1.3	150	285	8.6	118	10	-2.54	7
Yucheng	36.83	116.57	Infrared heaters	1.3	150	285	8.8	160	8.8	-1.15	7
Yucheng	36.83	116.57	Infrared heaters	1.3	150	285	8.8	160	8.8	-4.69	7
Lianyungang	34.55	119.39	Infrared heaters	2.2	0	0	7.9	372	7.3	12.92	8
Lianyungang	34.55	119.39	Infrared heaters	2.2	0	150	7.9	372	7.3	6.31	8
Lianyungang	34.55	119.39	Infrared heaters	2.2	0	225	7.9	372	7.3	2.95	8
Lianyungang	34.55	119.39	Infrared heaters	2.2	0	300	7.9	372	7.3	5.82	8
Luancheng	37.88	114.68	Infrared heaters	2.0	160	240	9.6	131	9.5	-13.0	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	240	8.9	65	9.8	-13.5	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	240	8.8	153	8.6	-6.0	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	0	9.6	131	9.5	-12.5	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	0	8.9	65	9.8	-3.5	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	0	8.8	153	8.6	-4.5	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	240	7.3	167	8.5	15.5	9
Luancheng	37.88	114.68	Infrared heaters	2.0	160	0	7.3	167	8.5	0.5	9
Dingxi	35.58	104.62	Infrared heaters	1.0	0	NA	13.6	171	8.6	-8.37	10
Dingxi	35.58	104.62	Infrared heaters	2.0	0	NA	13.6	171	8.6	-7.55	10
Dingxi	35.58	104.62	Infrared heaters	3.0	0	NA	13.6	171	8.6	-7.27	10

Steps

- Explore the dataset
- Define and fit Bayesian models
- Conclude

Steps

- Explore the dataset
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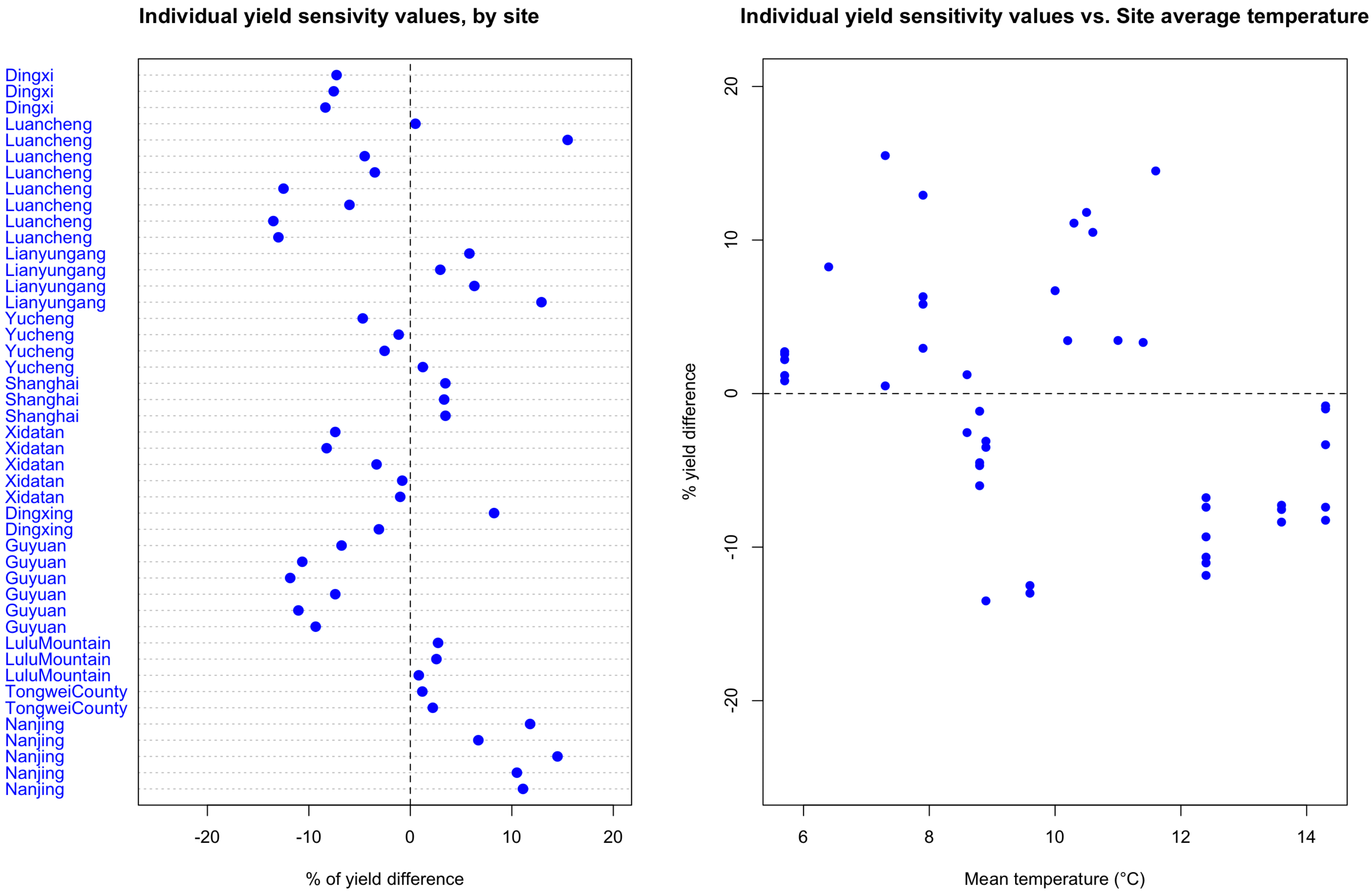
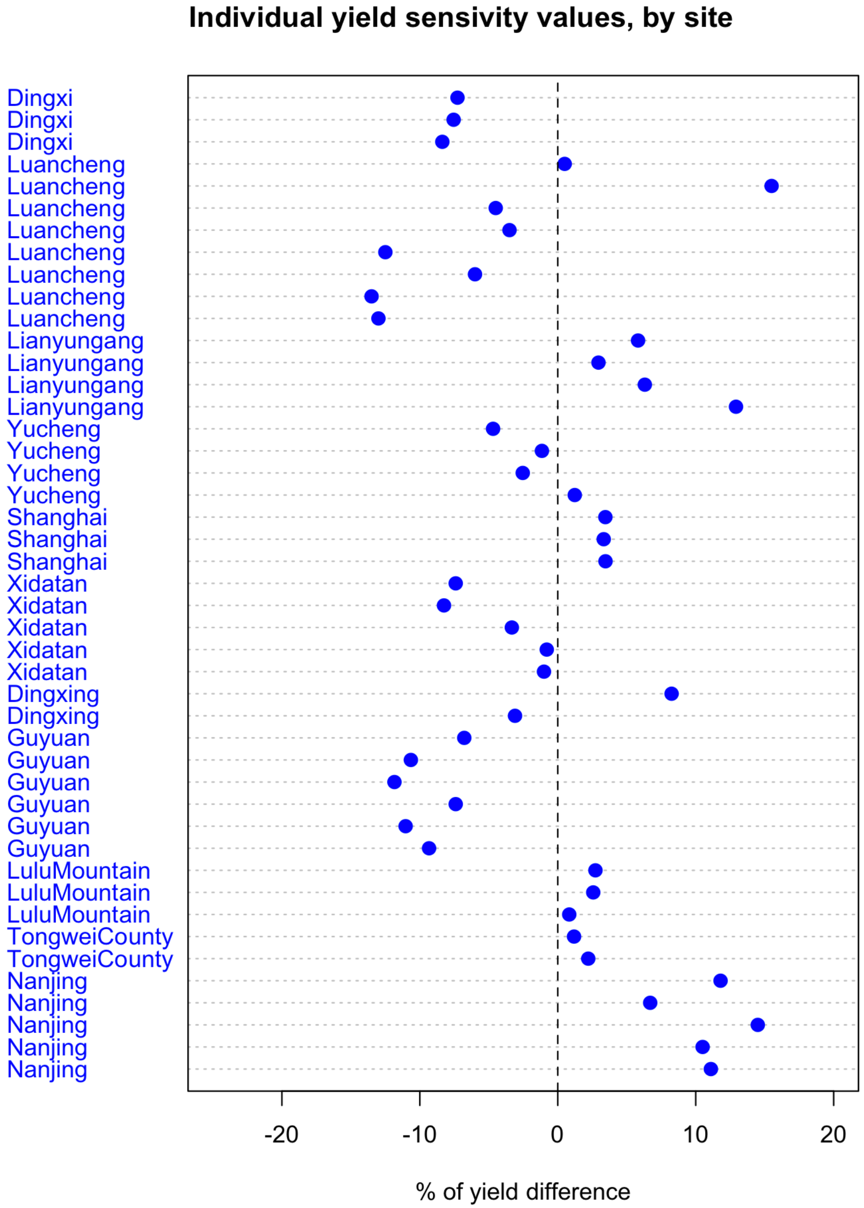
- Conclude

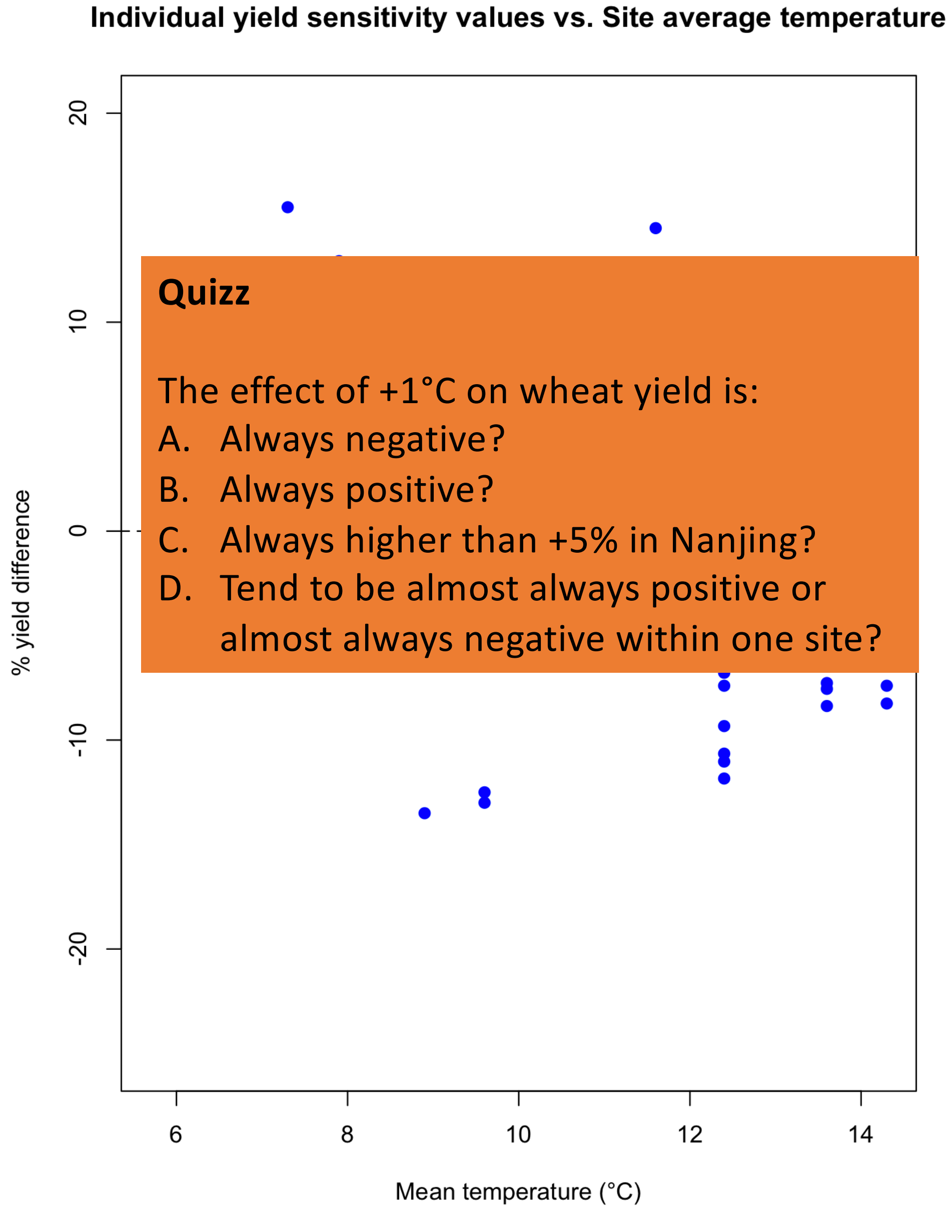
Open and run **BayesianRegression_1.R**

```
      Wheat_type         Site_name        Lat             Long                          Design     Temp_increase  Irrigtion_mm    Nitrogen_kgha        TGS              PGS
 Springwheat:14   Luancheng  : 8   Min.   :31.21   Min.   :104.6   Greenhouse      : 6   Min.   :0.5   Min.   :  0.0   Min.   :  0.0   Min.   : 5.70   Min.   : 23
 Winterwheat:31   Guyuan     : 6   1st Qu.:34.55   1st Qu.:106.3   Heatingcable    : 5   1st Qu.:1.3   1st Qu.:  0.0   1st Qu.:136.0   1st Qu.: 7.90   1st Qu.: 97
                  Nanjing    : 5   Median :36.03   Median :114.7   Infraredheaters:34   Median :1.5   Median :  0.0   Median :225.0   Median : 9.60   Median :160
                  Xidatan    : 5   Mean   :35.86   Mean   :112.5                        Mean   :1.7   Mean   : 82.7   Mean   :178.6   Mean   :10.04   Mean   :202
                  Lianyungang: 4   3rd Qu.:37.88   3rd Qu.:118.9                        3rd Qu.:2.0   3rd Qu.:160.0   3rd Qu.:240.0   3rd Qu.:12.40   3rd Qu.:325
                  Yucheng    : 4   Max.   :39.13   Max.   :121.1                        Max.   :3.0   Max.   :500.0   Max.   :300.0   Max.   :14.30   Max.   :476
                  (Other)    :13                                                                       NA's   :8       NA's   :17
      DGS          Sensitivity           Ref
 Min.   : 5.700   Min.   :-13.5000   Min.   : 1.000
 1st Qu.: 7.300   1st Qu.: -7.4000   1st Qu.: 3.000
 Median : 8.600   Median : -1.0000   Median : 5.000
 Mean   : 8.942   Mean   : -0.8364   Mean   : 5.467
 3rd Qu.:10.000   3rd Qu.:  3.4500   3rd Qu.: 8.000
 Max.   :12.200   Max.   : 15.5000   Max.   :10.000
```



Individual yield sensivity values, by site
Dingxi
Dingxi
Dingxi
Luancheng
Luancheng
Luancheng
Luancheng
Luancheng
Luancheng
Luancheng
Luancheng
Lianyungang
Lianyungang
Lianyungang
Lianyungang
Yucheng
Yucheng
Yucheng
Yucheng
Shanghai
Shanghai
Shanghai
Xidatan
Xidatan
Xidatan
Xidatan
Xidatan
Dingxing
Dingxing
Guyuan
Guyuan
Guyuan
Guyuan
Guyuan
Guyuan
LuluMountain
LuluMountain
LuluMountain
TongweiCounty
TongweiCounty
Nanjing
Nanjing
Nanjing
Nanjing
Nanjing
-20
-10
0
10
20
% of yield difference
Individual yield sensitivity values vs. Site average temperature
20
10
0
-10
-20
% yield difference
6
8
10
12
14
Mean temperature (°C)



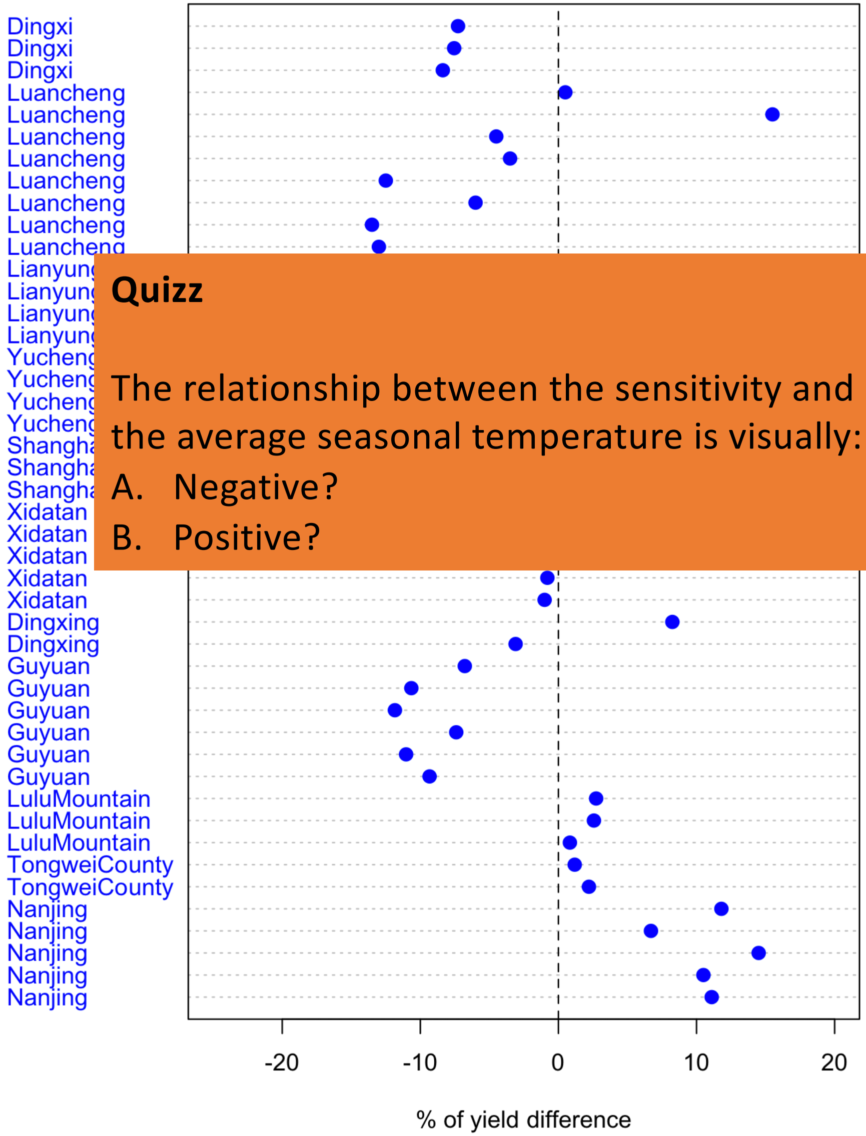


Quizz

The effect of +1°C on wheat yield is:

A. Always negative?
B. Always positive?
C. Always higher than +5% in Nanjing?
D. Tend to be almost always positive or almost always negative within one site?

Individual yield sensivity values, by site

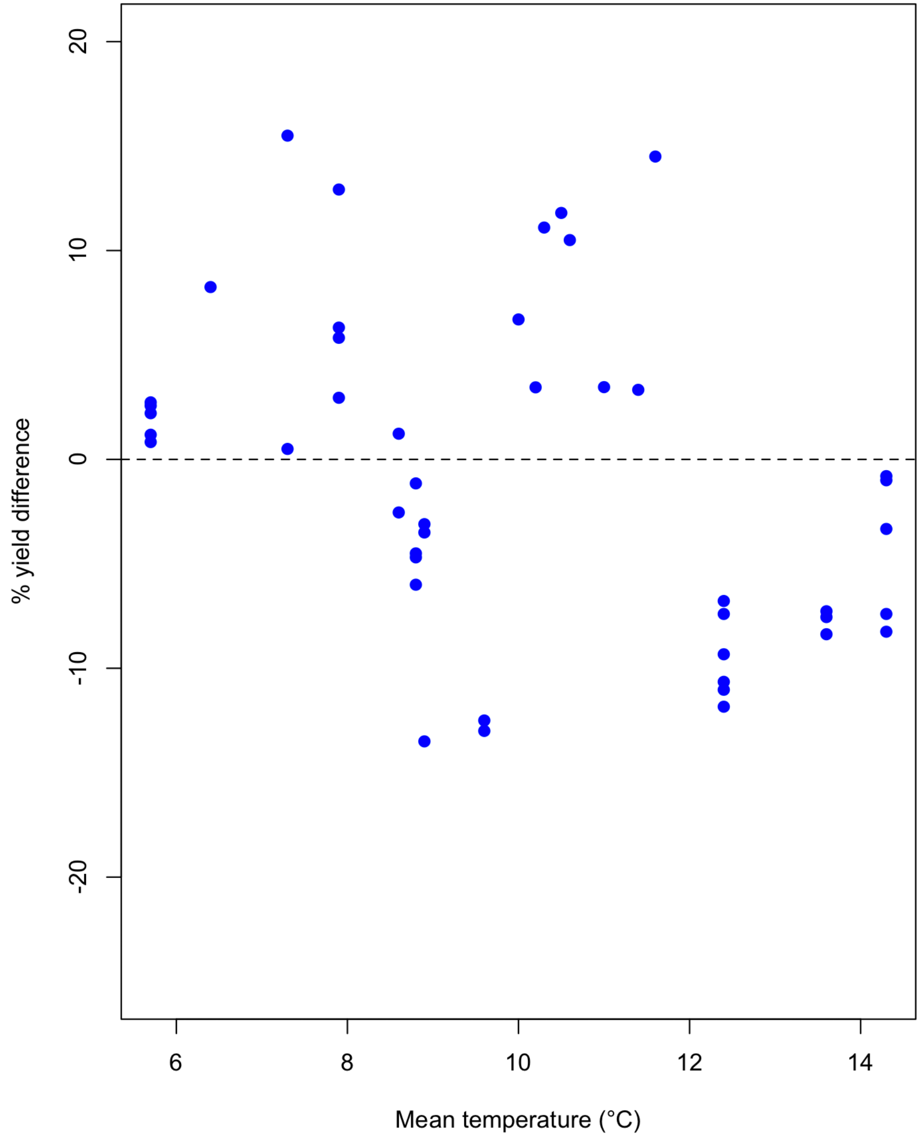


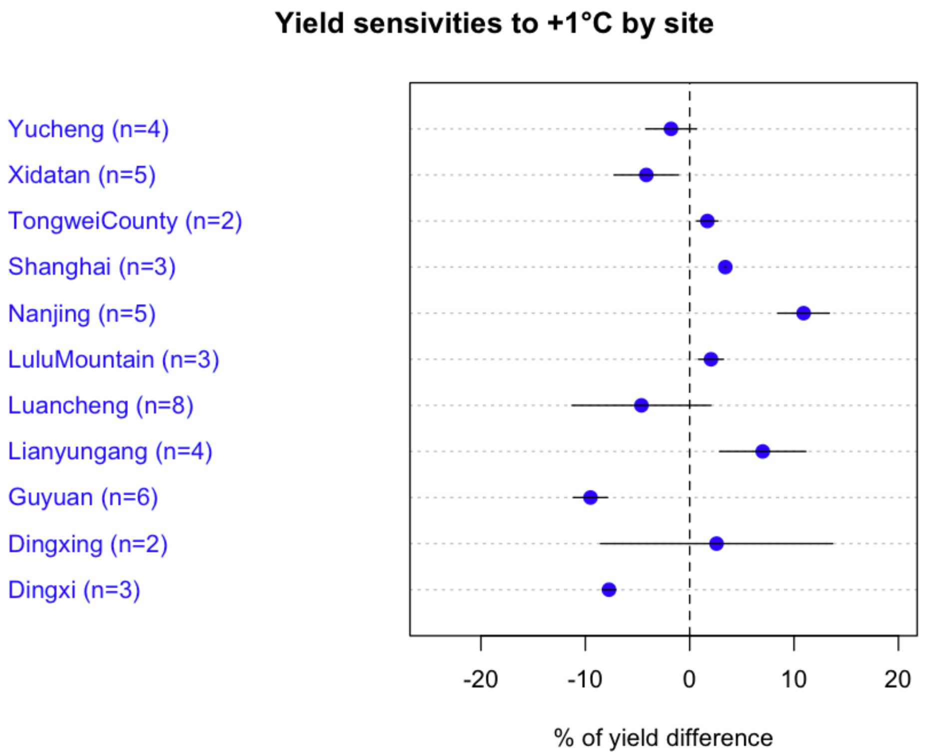
Quizz

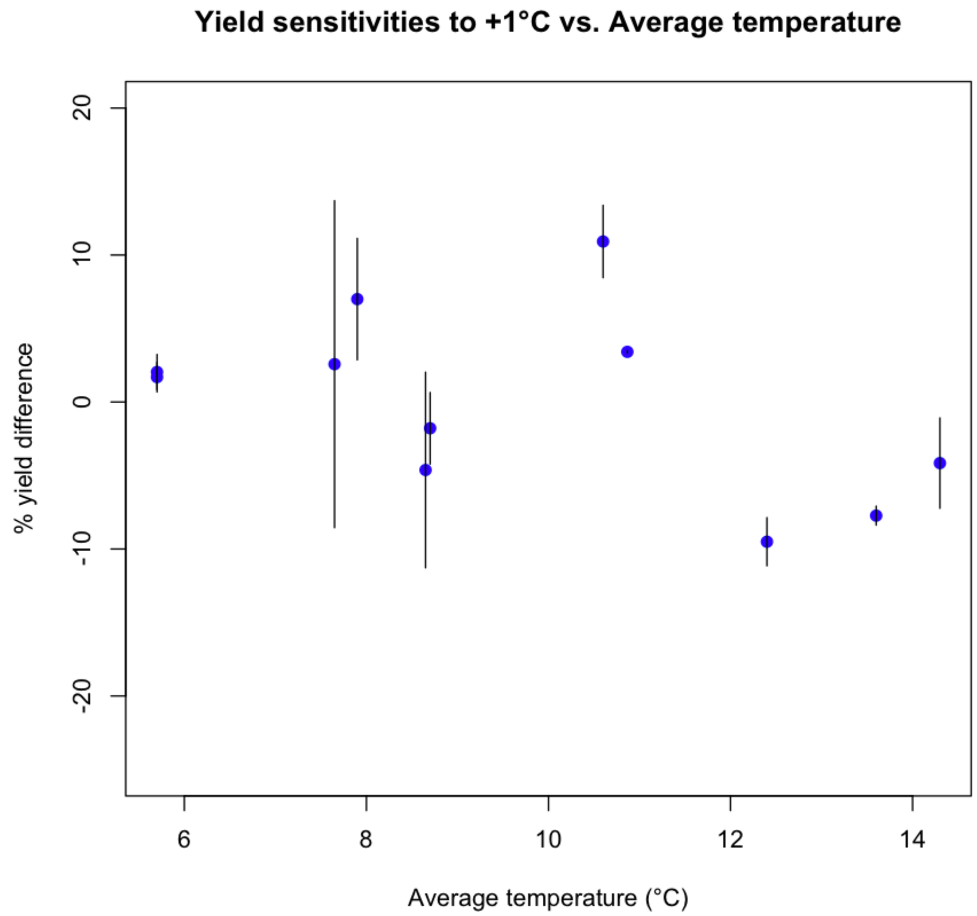
The relationship between the sensitivity and the average seasonal temperature is visually:

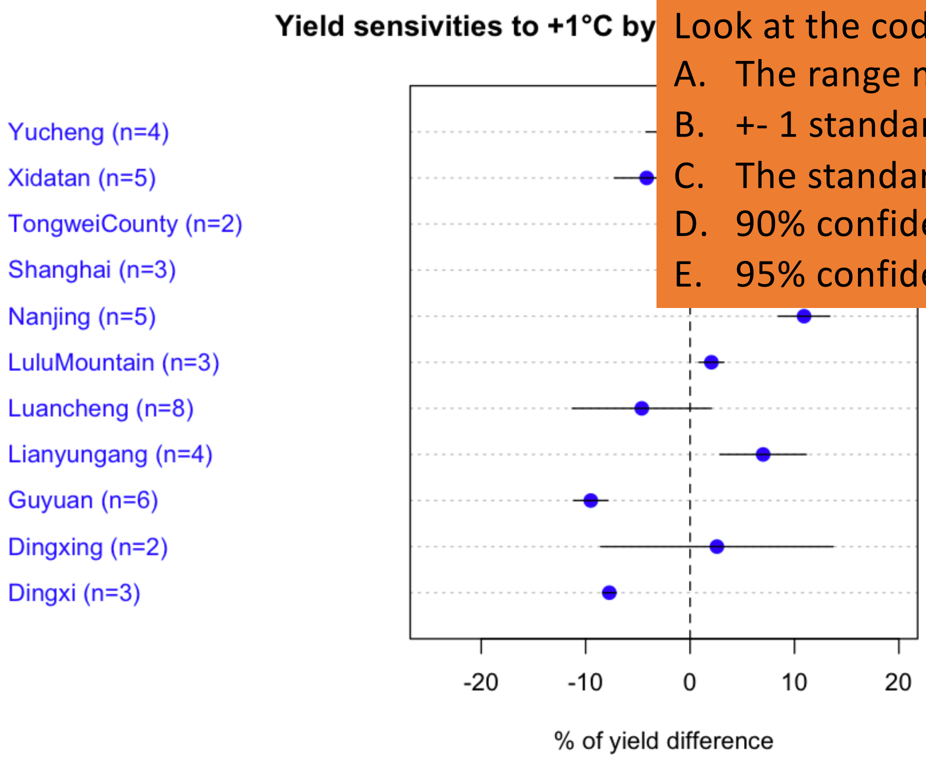
A. Negative?
B. Positive?

Individual yield sensitivity values vs. Site average temperature




Yield sensivities to +1°C by site
Yucheng (n=4)
Xidatan (n=5)
TongweiCounty (n=2)
Shanghai (n=3)
Nanjing (n=5)
LuluMountain (n=3)
Luancheng (n=8)
Lianyungang (n=4)
Guyuan (n=6)
Dingxing (n=2)
Dingxi (n=3)
-20
-10
0
10
20
% of yield difference

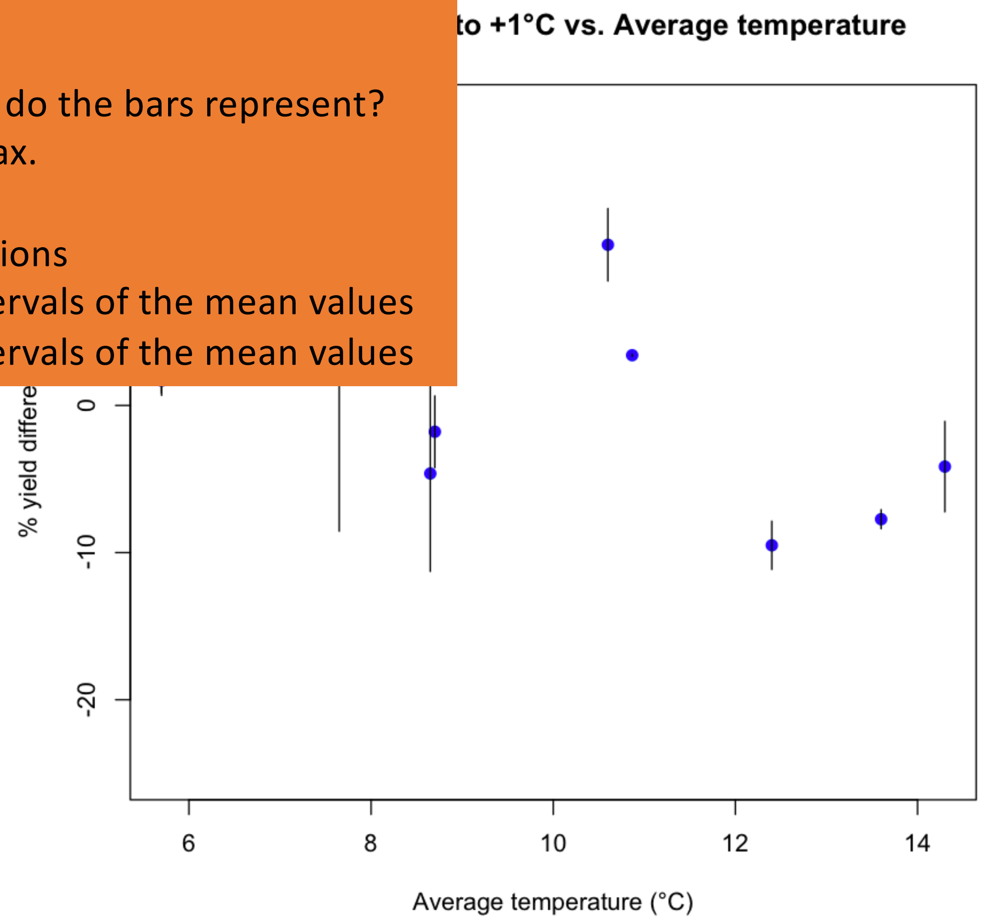

Yield sensitivities to +1°C vs. Average temperature
% yield difference
20
10
0
-10
-20
6
8
10
12
14
Average temperature (°C)

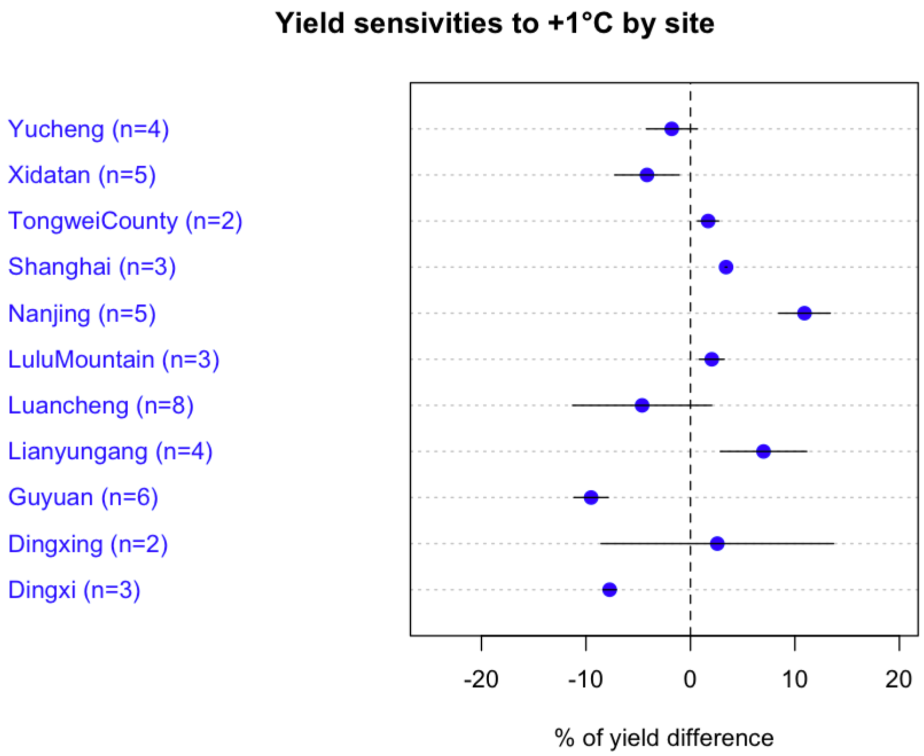


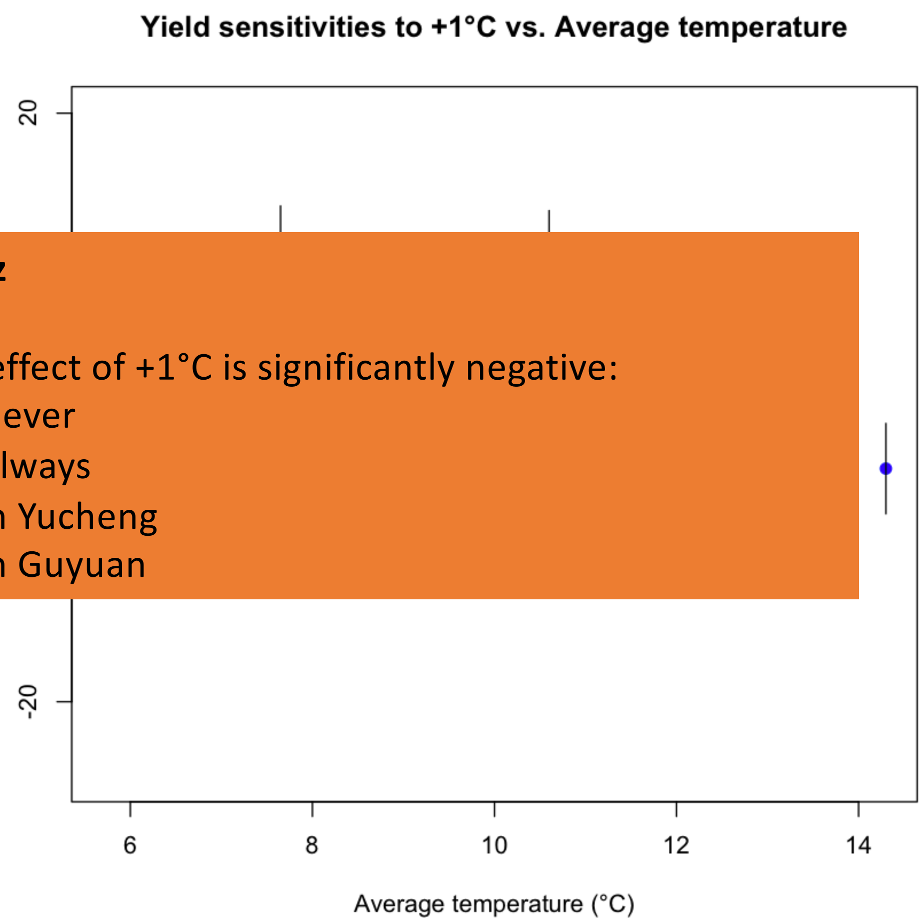
Quizz

Look at the code. What do the bars represent?

A. The range min. – max.
B. +- 1 standard errors
C. The standard deviations
D. 90% confidence intervals of the mean values
E. 95% confidence intervals of the mean values



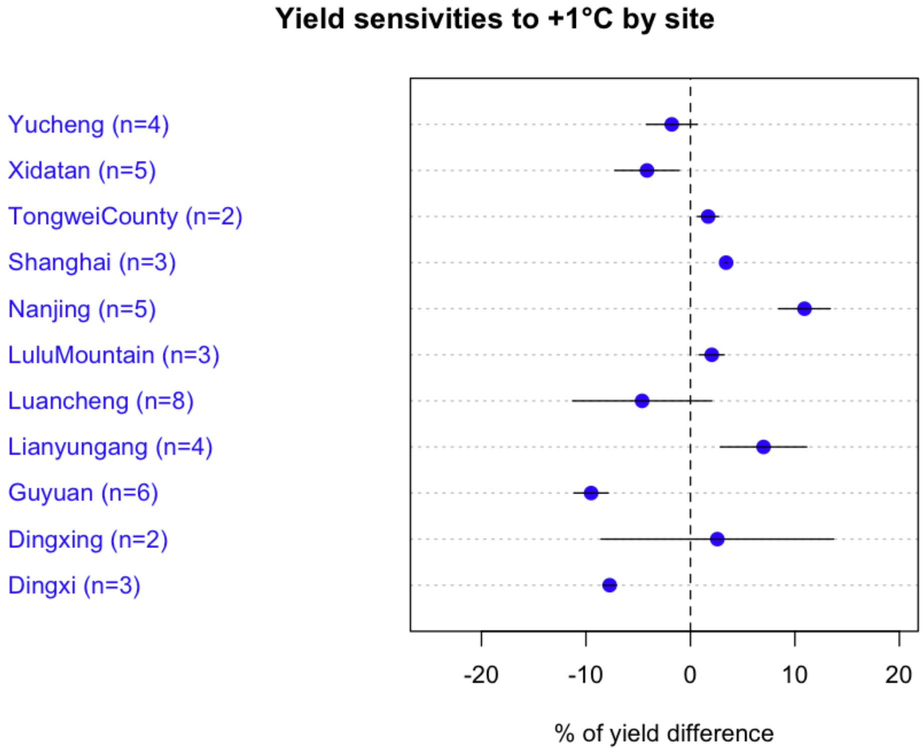


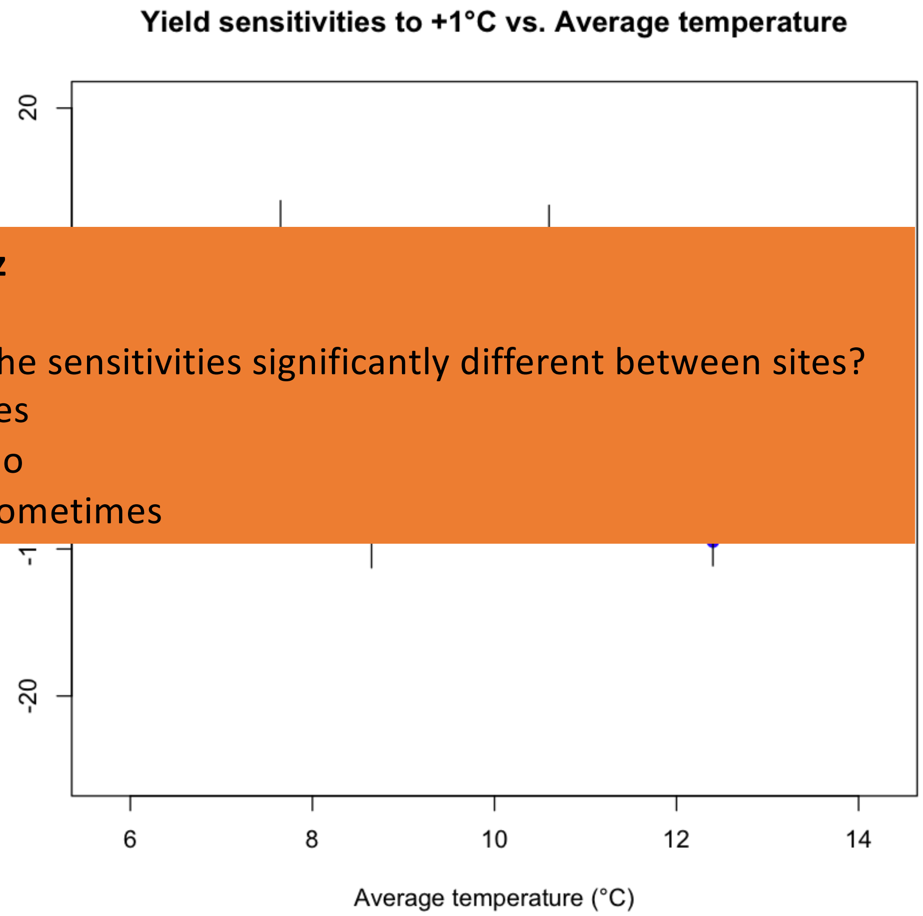


Quizz

The effect of +1°C is significantly negative:

A. Never
B. Always
C. In Yucheng
D. In Guyuan

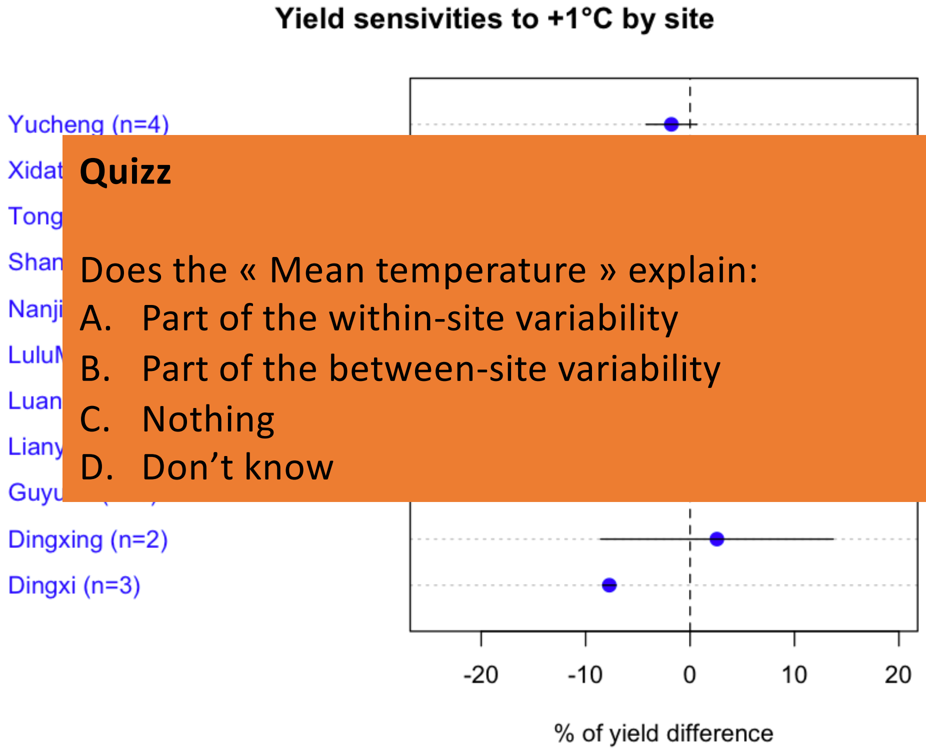




Quizz

Are the sensitivities significantly different between sites?

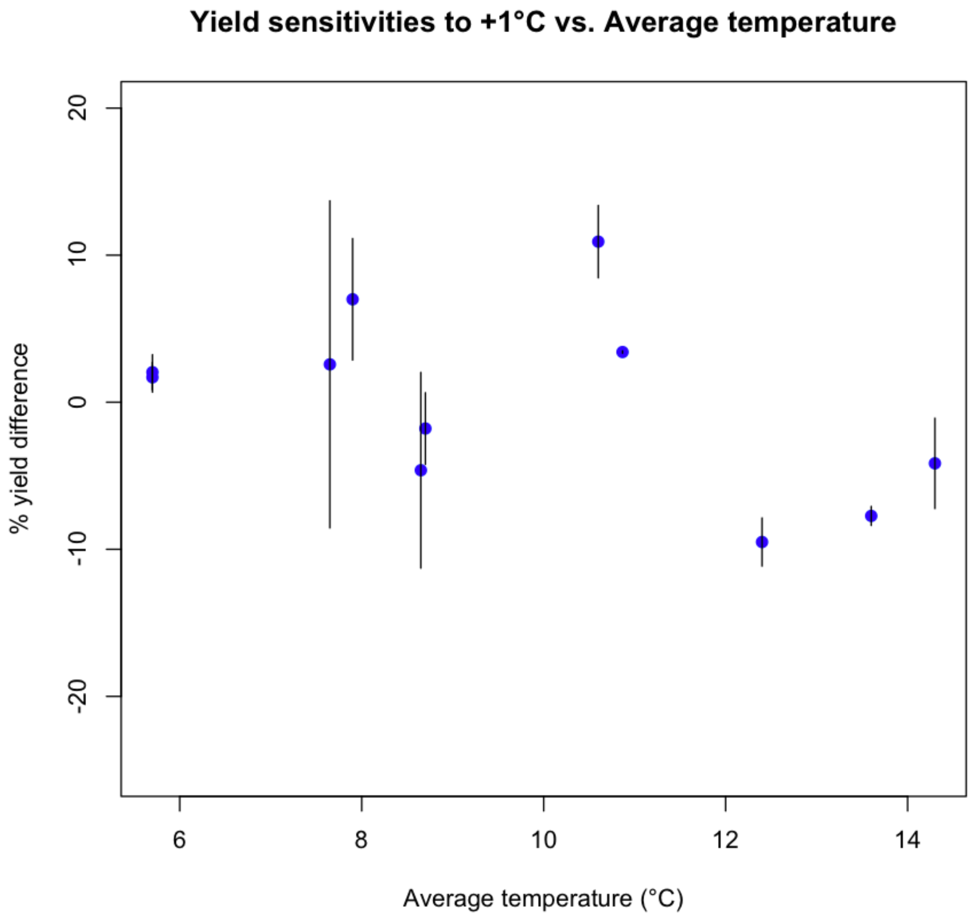
A. Yes
B. No
C. Sometimes



Quizz

Does the « Mean temperature » explain:

A. Part of the within-site variability
B. Part of the between-site variability
C. Nothing
D. Don’t know



Steps

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- Define and fit Bayesian models
- Conclude

Hierarchical statistical model
« Random-effect model »

Overall mean

Effect of study i

Within-study level: $$S_{ij} = \mu + b_i + \varepsilon_{ij} \qquad \varepsilon_{ij} \sim N\left(0, \sigma^2_{\varepsilon i}\right)$$

Yield sensitivity in study i, replicate j

Residual for jth data in study i

Hierarchical statistical model
« Random-effect model »

Overall mean Effect of study i

Within-study level: $S_{ij} = \mu + b_i + \varepsilon_{ij}$ $\qquad \varepsilon_{ij} \sim N\left(0, \sigma^2_{\varepsilon i}\right)$

Yield sensitivity in study i, replicate j

Residual for jth data in study i

Between-study level: $b_i \sim N\left(0, \sigma^2_b\right)$

Prior: Gaussian and Uniform

Hierarchical statistical model
« Random-effect model »

Overall mean

Effect of study i

Within-study level: $$S_{ij} = \mu + b_i + \varepsilon_{ij} \qquad \varepsilon_{ij} \sim N\left(0, \sigma_{\varepsilon i}^2\right)$$

Yield sensitivity in study i, replicate j

Residual for jth data in study i

Between-study level: $$b_i \sim N\left(0, \sigma_b^2\right)$$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Hierarchical statistical model (with covariate)
« Random-effect model »

Covariate value for study i replicate j

Within-study level: $S_{ij} = \mu + \alpha X_{ij} + b_i + \varepsilon_{ij}$ $\qquad \varepsilon_{ij} \sim N\left(0, \sigma_{\varepsilon i}^2\right)$

Yield sensitivity in study i, replicate j

Between-study level: $b_i \sim N\left(0, \sigma_b^2\right)$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Open **BayesianRegression_2.R**

This file includes four Bayesian models coded with **rjags**.
How to install Jags: https://wiki.student.info.ucl.ac.be/Logiciels/JAGS

- How many models are coded?
- What are the differences between these models?
- Which one is the simplest?
- Which one is the most complex?

Model 1

Within-study level: $S_{ij} = \mu + b_i + \varepsilon_{ij}$ $\varepsilon_{ij} \sim N\left(0, \sigma_{\varepsilon i}^2\right)$

Between-study level: $b_i \sim N\left(0, \sigma_b^2\right)$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Model 2

Within-study level: $S_{ij} = \mu + \alpha X_{ij} + b_i + \varepsilon_{ij}$ $\varepsilon_{ij} \sim N\left(0, \sigma_{\varepsilon i}^2\right)$

Between-study level: $b_i \sim N\left(0, \sigma_b^2\right)$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Model 1bis

Within-study level: $S_{ij} = \mu + b_i + \varepsilon_{ij}$ $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$

Between-study level: $b_i \sim N\left(0, \sigma_b^2\right)$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Model 2bis

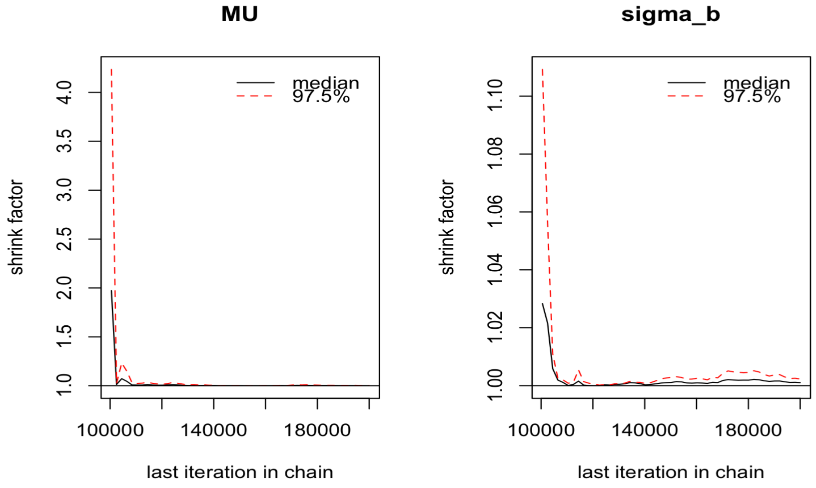
Within-study level: $S_{ij} = \mu + \alpha X_{ij} + b_i + \varepsilon_{ij}$ $\quad \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$

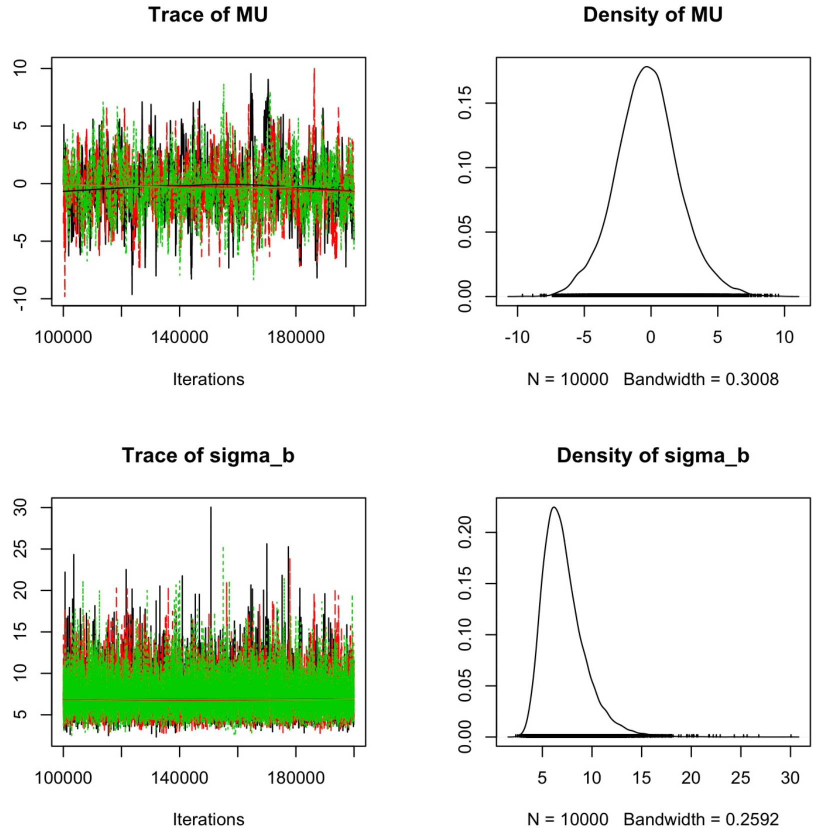
Between-study level: $b_i \sim N\left(0, \sigma_b^2\right)$

Prior: Gaussian and Uniform

Fitting algorithm: MCMC

Run model 1

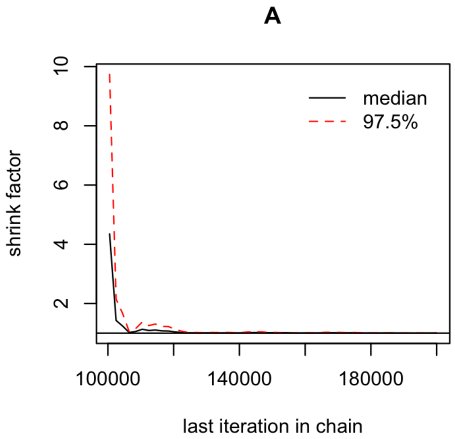


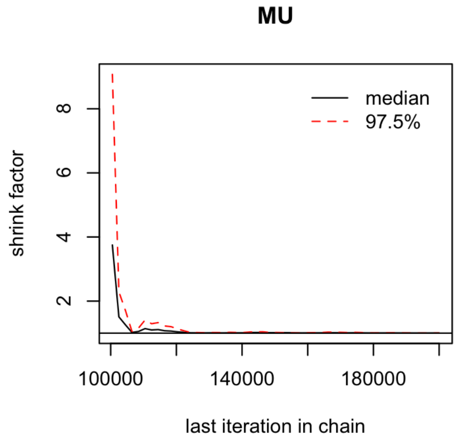


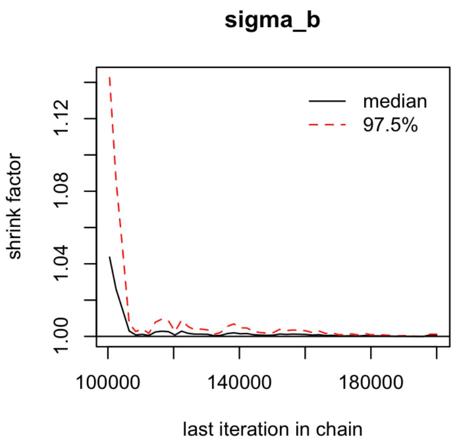

```
Iterations = 100010:2e+05
Thinning interval = 10
Number of chains = 3
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

           Mean    SD Naive SE Time-series SE
MU      -0.2577 2.362  0.01364        0.09475
sigma_b  7.1907 2.205  0.01273        0.02351

2. Quantiles for each variable:

          2.5%    25%    50%   75%  97.5%
MU      -4.985 -1.784 -0.286 1.205  4.668
sigma_b  4.086  5.677  6.798 8.252 12.614

> dic.samples(model, 10000)
  |**************************************************| 100%
Mean deviance:  236.2
penalty 437.3
Penalized deviance: 673.5
```

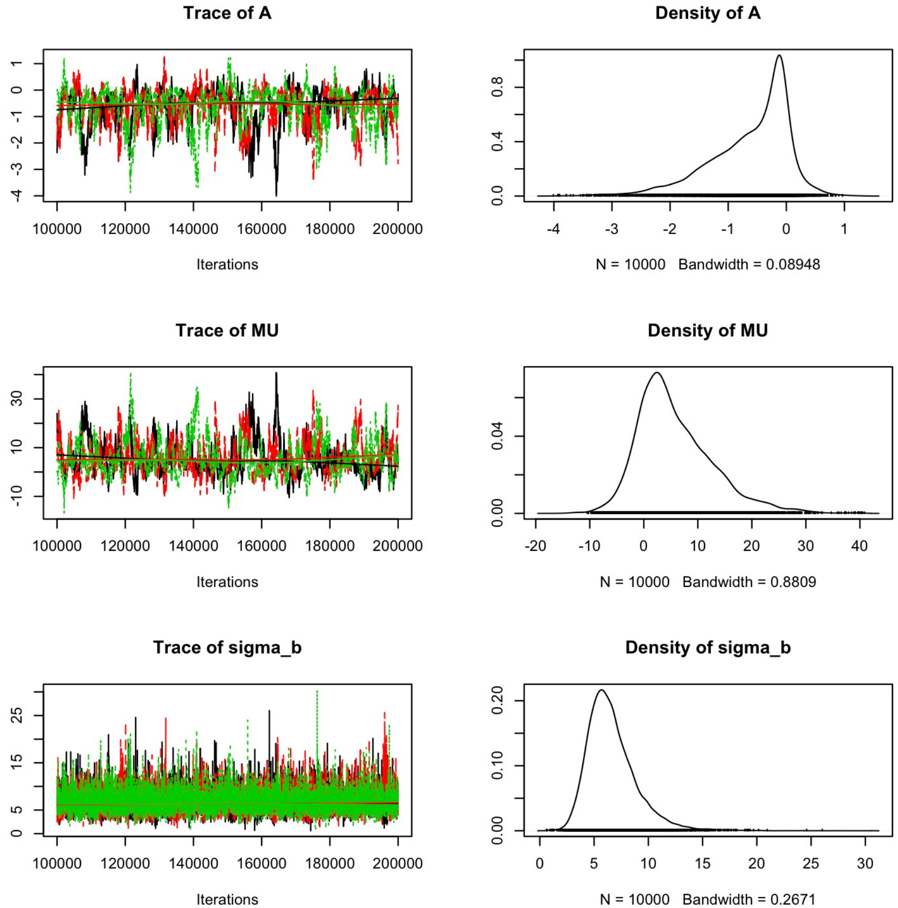
- What is the posterior mean of the yield sensitivity?
- What is the 95% credibility interval?
- Practical conclusion?
- Modify the code to look at the within-study variances

Run model 2





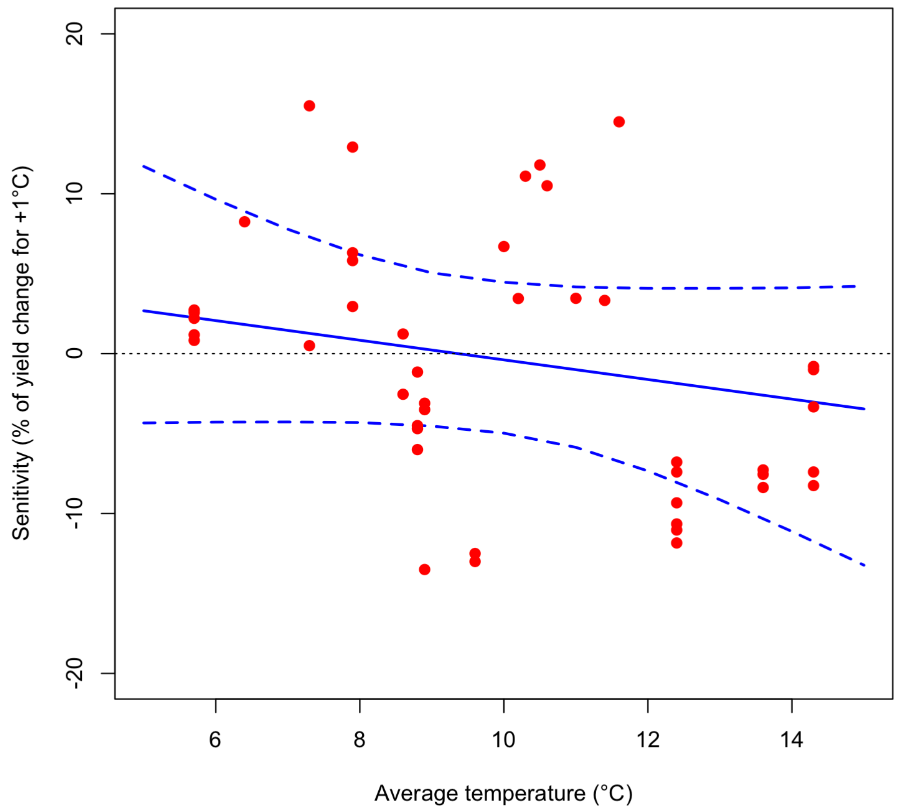




- What are the meanings of A, MU, sigma_b?
- Does the model includes other parameters?
- Modify the code to look at the within-study variances

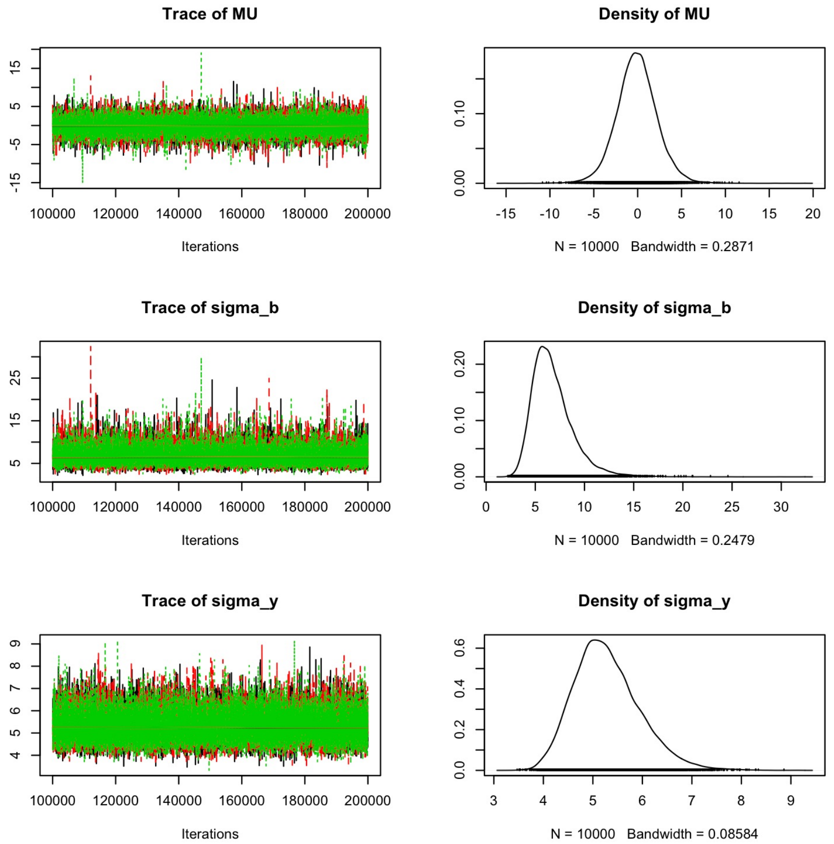
Run model 2

```
> summary(samples)

Iterations = 100010:2e+05
Thinning interval = 10
Number of chains = 3
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

           Mean    SD Naive SE Time-series SE
A       -0.6139 0.689 0.003978        0.04979
MU       5.7506 7.015 0.040500        0.49460
sigma_b  6.6276 2.280 0.013165        0.04103

2. Quantiles for each variable:

          2.5%     25%     50%     75%   97.5%
A       -2.293 -1.0024 -0.4249 -0.1132  0.3409
MU      -4.960  0.8774  4.3804  9.6299 22.8198
sigma_b  3.276  5.0932  6.2541  7.7468 12.1035

> dic.samples(model, 10000)
  |**************************************************| 100%
Mean deviance:  243.4
penalty 1146
Penalized deviance: 1389
```



- What is the effect of the average temperature?
- Does this variable explain a big part of the between-site variability?

Run model 1bis



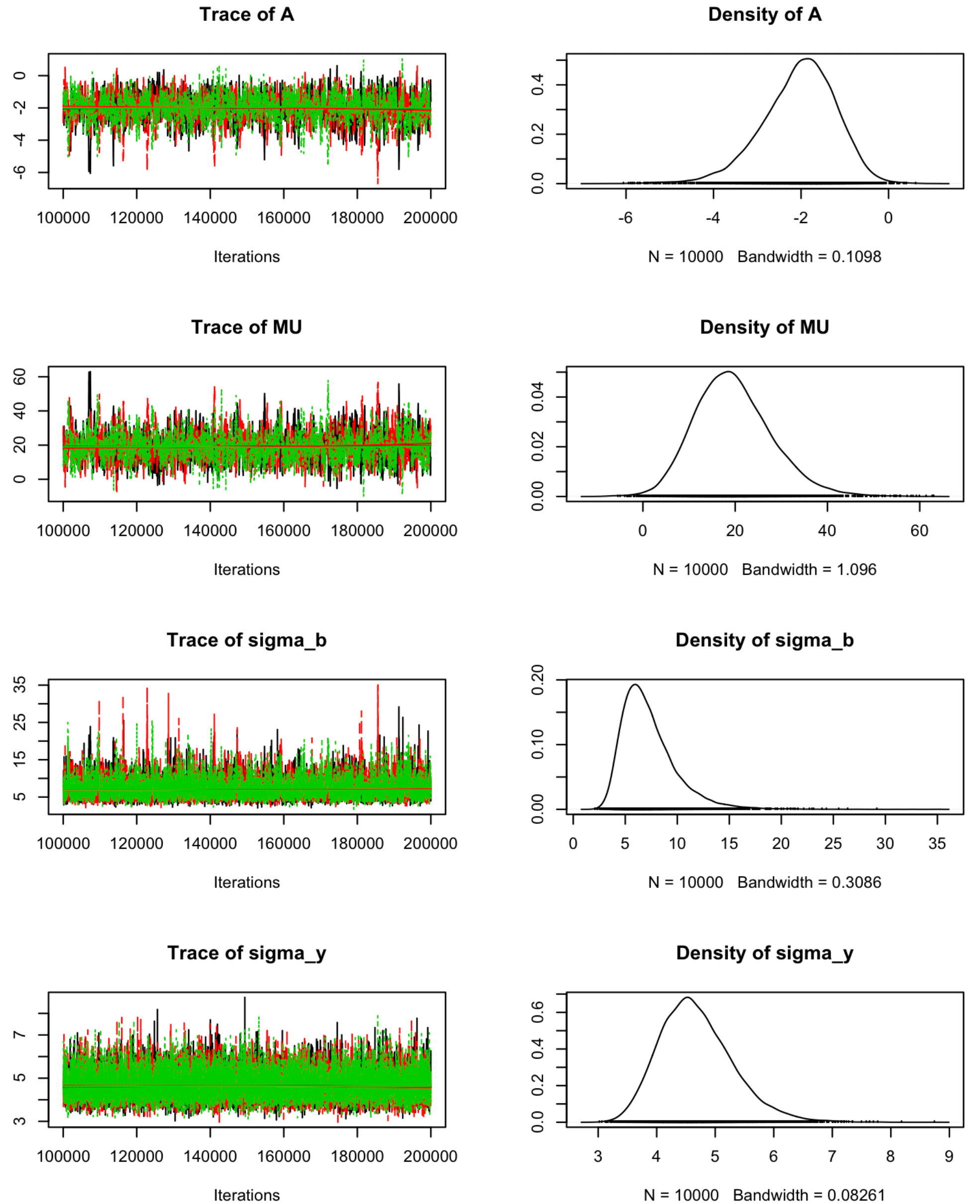

```
> summary(samples)

Iterations = 100010:2e+05
Thinning interval = 10
Number of chains = 3
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

            Mean     SD Naive SE Time-series SE
MU      -0.09876 2.2985 0.013270        0.01889
sigma_b  6.76543 2.1056 0.012157        0.01323
sigma_y  5.28503 0.6643 0.003836        0.00380

2. Quantiles for each variable:

          2.5%    25%     50%   75%  97.5%
MU      -4.593 -1.536 -0.1248 1.317  4.528
sigma_b  3.751  5.320  6.4026 7.783 11.902
sigma_y  4.173  4.823  5.2160 5.676  6.772

> dic.samples(model, 10000)
  |**************************************************| 100%
Mean deviance:  275.9
penalty 11.3
Penalized deviance: 287.2
```

- How does this model compare with model 1?
- Is it relevant to consider different within-site variances?

Run model 2bis

```
> summary(samples)

Iterations = 100010:2e+05
Thinning interval = 10
Number of chains = 3
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

          Mean     SD Naive SE Time-series SE
A       -2.045 0.8503 0.004909       0.027451
MU      19.581 8.3981 0.048487       0.266484
sigma_b  7.273 2.6770 0.015456       0.054430
sigma_y  4.679 0.6271 0.003621       0.006523

2. Quantiles for each variable:

          2.5%    25%    50%    75%   97.5%
A       -3.917 -2.552 -1.974 -1.461 -0.5895
MU       5.001 13.773 18.979 24.668 37.8528
sigma_b  3.719  5.435  6.748  8.501 13.8902
sigma_y  3.632  4.234  4.614  5.055  6.0909
```

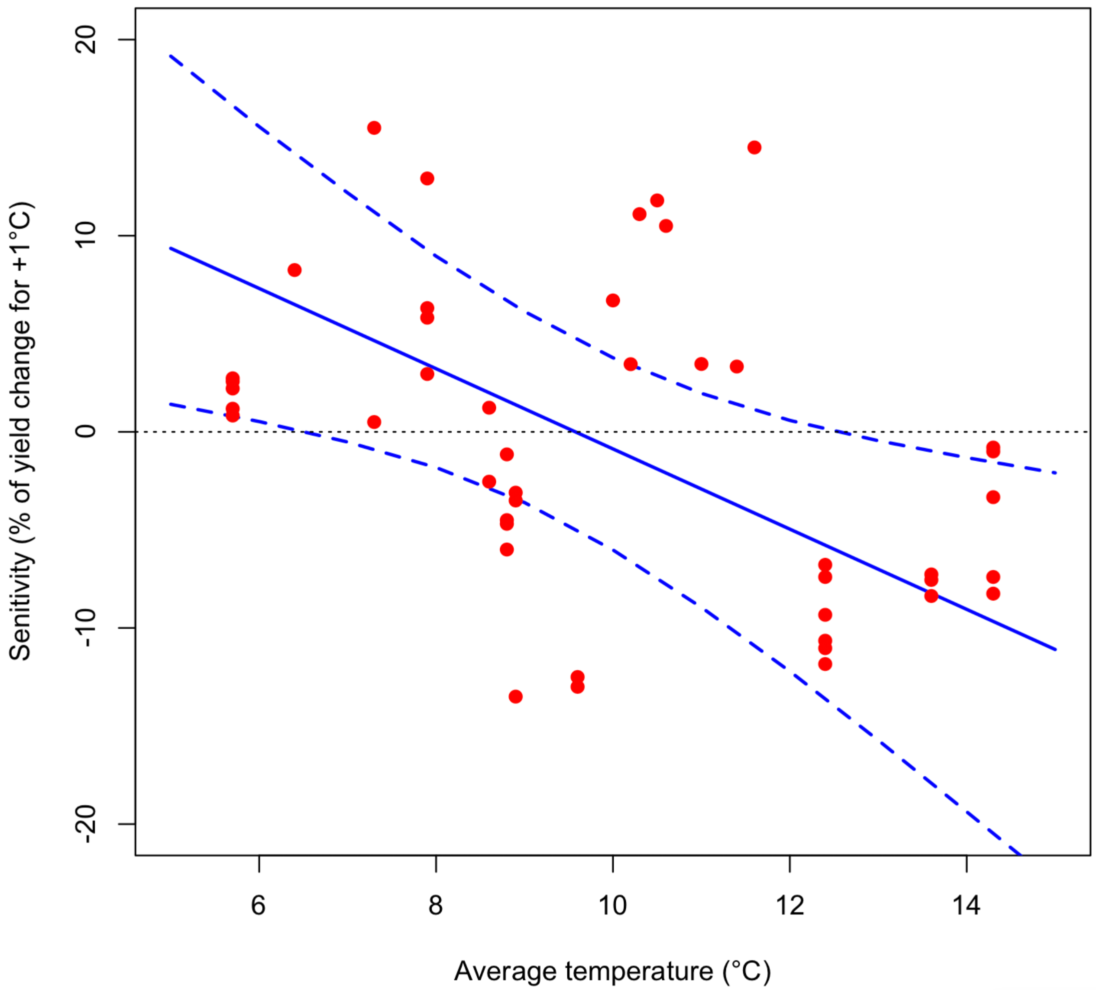
```
> dic.samples(model, 10000)
  |**************************************************| 100%
Mean deviance:  264.7
penalty 12.43
Penalized deviance: 277.2
```



- How does this model compare with the other models?
- What is the effect of the average temperature?

Run model 2bis

- What is the yield sensitivity when the average temperature is of 6°C or of 14°C?
- Can you anticipate a shift of the wheat growing area due to climate change based on this model?



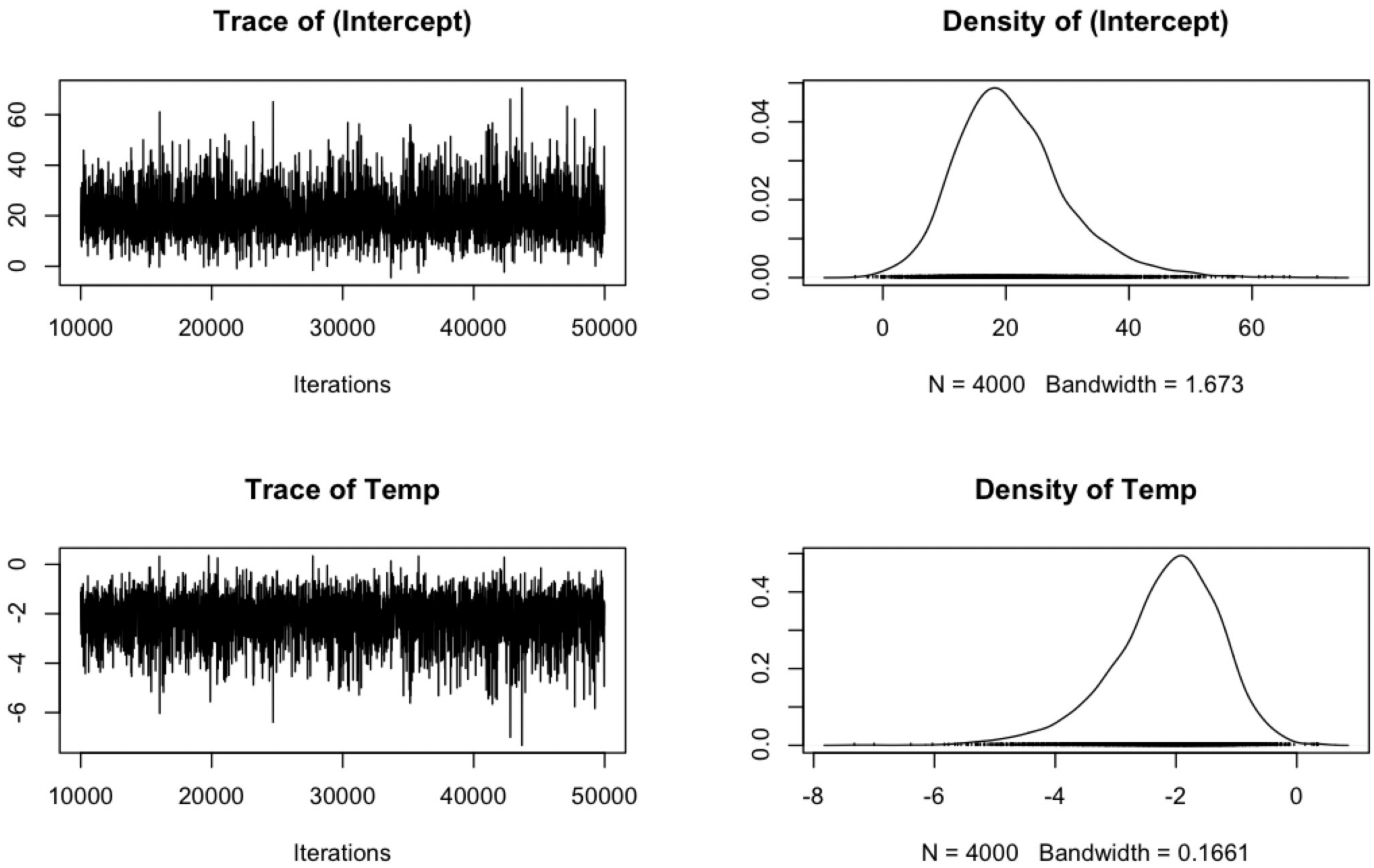
If you had 2 days available, how will you improve this model?

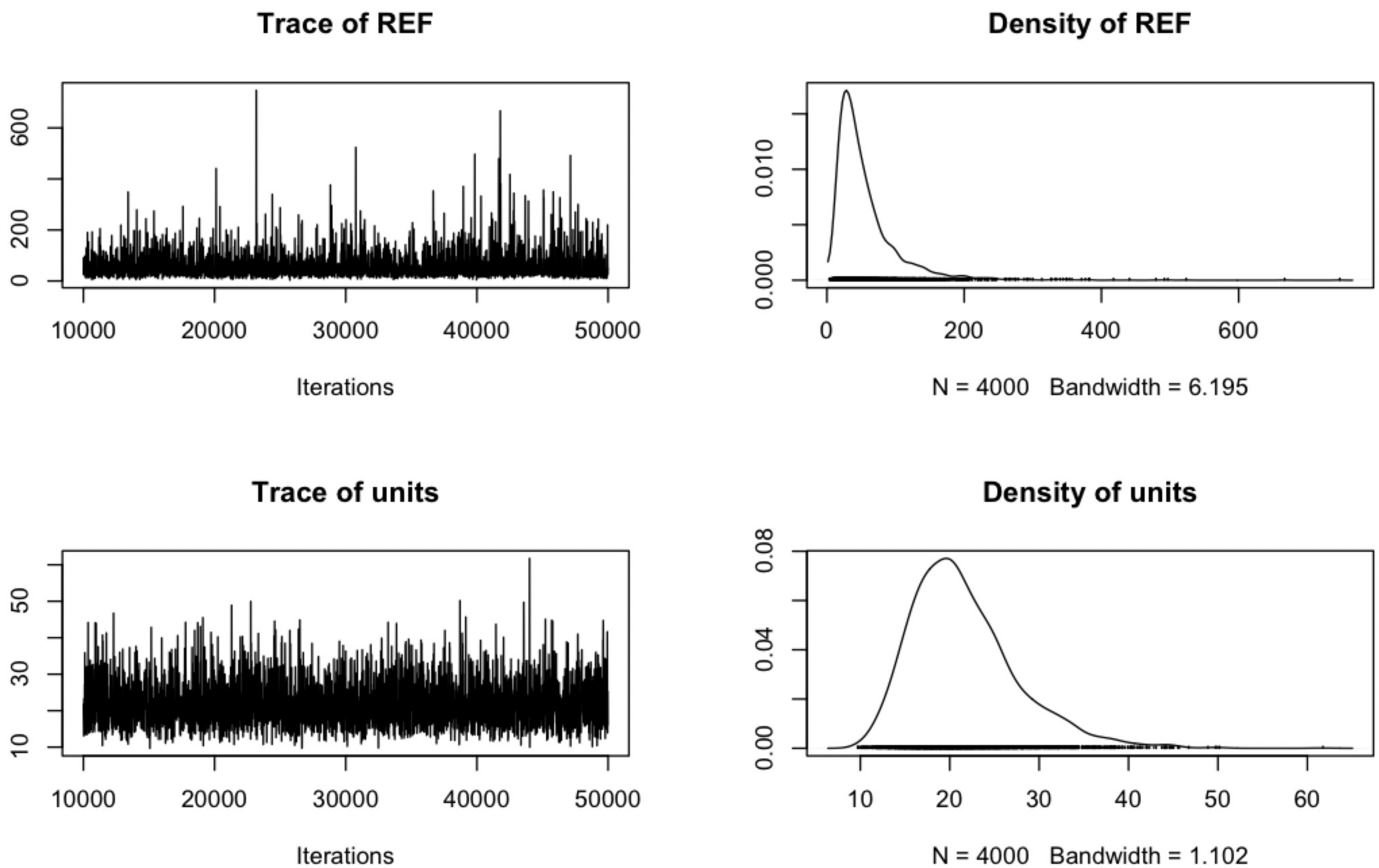
Another package with a different phylosophy: MCMCglmm

```r
#Data and their summaries
TAB<-read.table("Data_Zhao_Wheat.txt", sep="\t", header=T)
summary(TAB)

###Data###

Y=TAB$Sensitivity
Temp=TAB$TGS
REF=as.numeric(as.factor(as.character(TAB$Site_name)))

DATA=data.frame(REF, Y, Temp)
```

```r
library(MCMCglmm)

Mod_mcmc<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000, thin=10, burnin=10000, pr=TRUE)

#Visualisation du résumé de l'ajustement
summary(Mod_mcmc)

#Graphique de la chaine de valeur pour le paramètre mu
plot(Mod_mcmc)
```

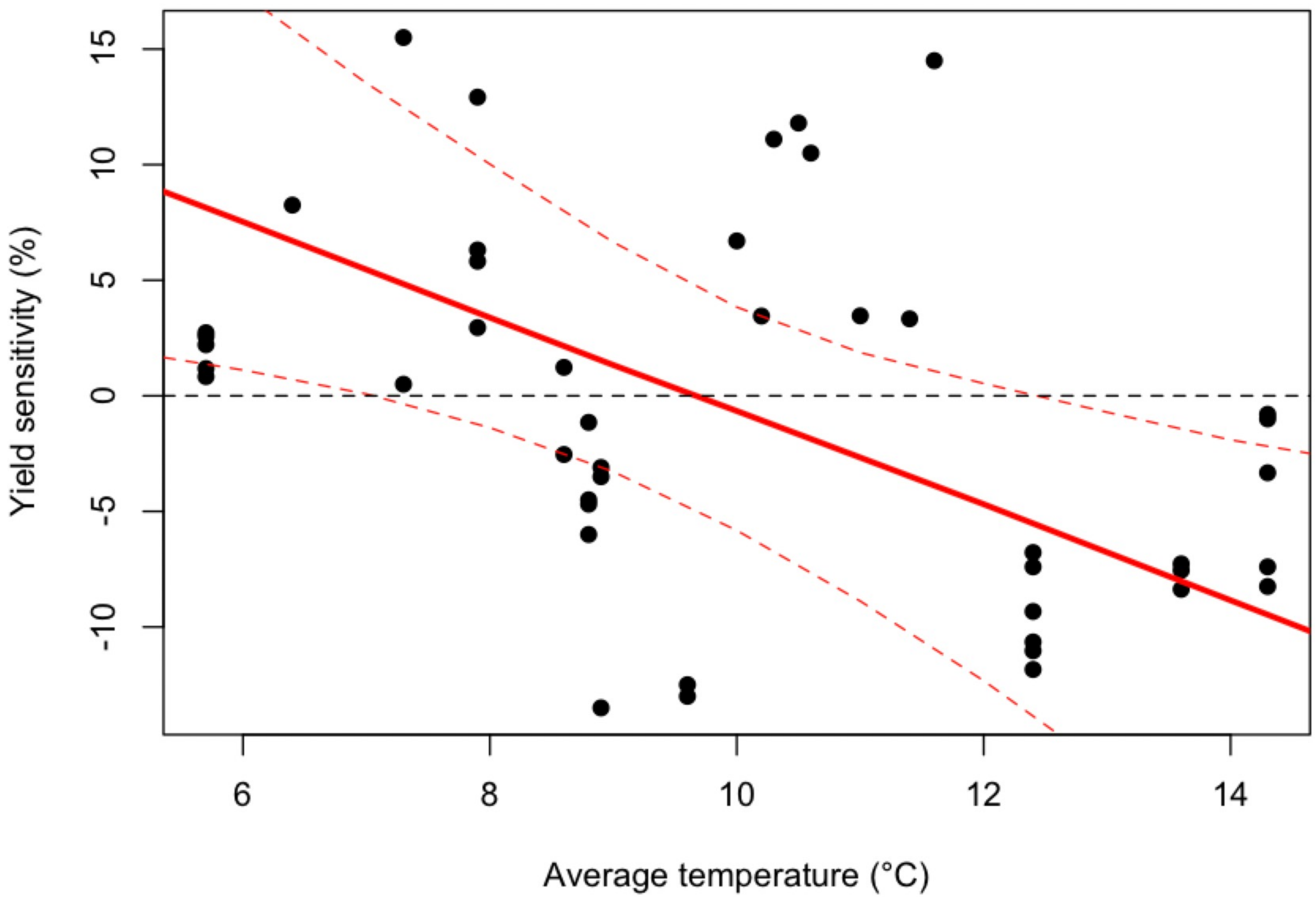
```
> summary(Mod_mcmc)

 Iterations = 10001:49991
 Thinning interval  = 10
 Sample size  = 4000

 DIC: 274.9552

 G-structure:  ~REF

    post.mean l-95% CI u-95% CI eff.samp
REF      56.5    7.024    145.6     3454

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units      21.6    11.74    33.75     3512

 Location effects: Y ~ 1 + Temp

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)   20.8669   4.6540  40.0615     4000 0.0050 **
Temp          -2.1598  -4.0221  -0.5079     4000 0.0035 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Trace of (Intercept)
60
40
20
0
10000
20000
30000
40000
50000
Iterations
Density of (Intercept)
0.04
0.02
0.00
0
20
40
60
N = 4000   Bandwidth = 1.673
Trace of Temp
0
-2
-4
-6
10000
20000
30000
40000
50000
Iterations
Density of Temp
0.4
0.2
0.0
-8
-6
-4
-2
0
N = 4000   Bandwidth = 0.1661



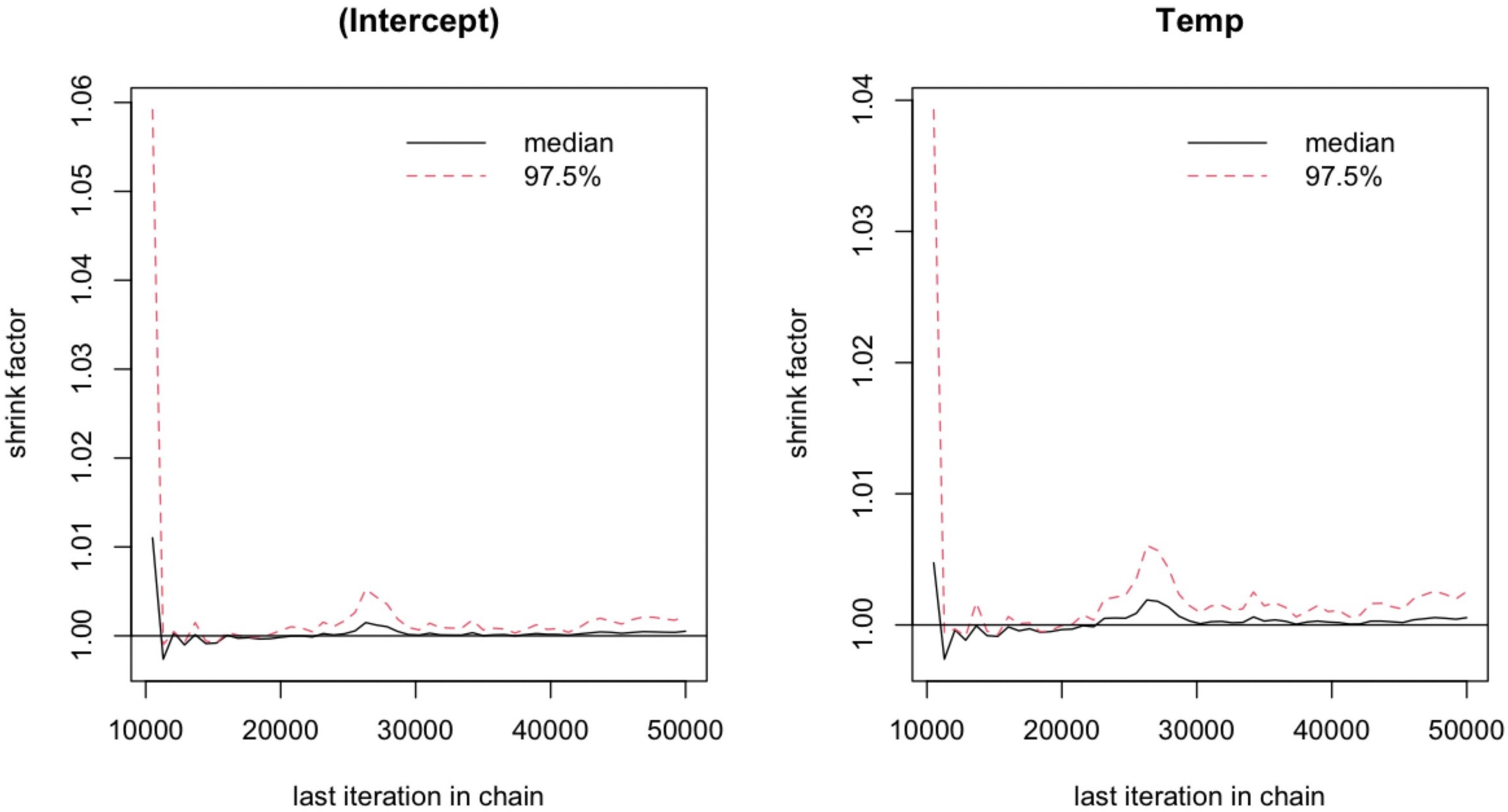
Trace of REF
Iterations
Density of REF
N = 4000 Bandwidth = 6.195
Trace of units
Iterations
Density of units
N = 4000 Bandwidth = 1.102

```
plot(Temp,Y, xlab="Average temperature (°C)", ylab="Yield sensitivity (%)", pch=19)

Temp_vec=5:15
Response=Mod_mcmc$Sol[,1]+as.matrix(Mod_mcmc$Sol[,2])%*%t(Temp_vec)

med_rep=apply(Response, 2, median)
q2.5_rep=apply(Response, 2, quantile, 0.025)
q97.5_rep=apply(Response, 2, quantile, 0.975)

lines(Temp_vec, med_rep, col="red", lwd=3)
lines(Temp_vec, q2.5_rep, col="red", lty=2)
lines(Temp_vec, q97.5_rep, col="red", lty=2)
abline(h=0, lty=2)
```



15
10
5
0
-5
-10
Yield sensitivity (%)
6
8
10
12
14
Average temperature (°C)

```r
Mod_mcmc_1<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000, thin=10, burnin=10000)
Mod_mcmc_2<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000, thin=10, burnin=10000)
Mod_mcmc_3<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000, thin=10, burnin=10000)

ChainList<-mcmc.list(Mod_mcmc_1$Sol,Mod_mcmc_2$Sol,Mod_mcmc_3$Sol)
gelman.plot(ChainList)
```


(Intercept)
Temp
shrink factor
last iteration in chain
median
97.5%
1.00
1.01
1.02
1.03
1.04
1.05
1.06
10000
20000
30000
40000
50000

Prior

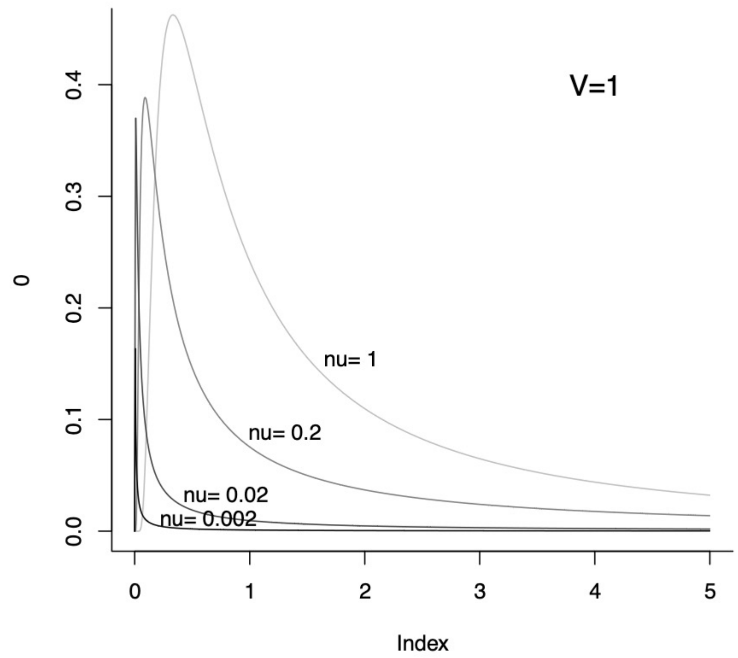


Figure 1.5: Probability density function for a univariate inverse Wishart with the variance at the limit set to 1 (`V=1`) and varying degree of belief parameter (`nu`). With `V=1` these distributions are equivalent to inverse gamma distributions with shape and scale parameters set to `nu`/2.

```
prior1<-list(B=list(mu=c(0,0),V=diag(c(10^8,10^8))), R=list(V=1,nu=1),G=list(G1=list(V=1,nu=1)))

Mod_mcmc<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000,
                   thin=10, burnin=10000, pr=TRUE, prior=prior1)
```

```
 Iterations = 10001:49991
 Thinning interval  = 10
 Sample size  = 4000

 DIC: 275.2081

 G-structure:  ~REF

    post.mean l-95% CI u-95% CI eff.samp
REF     43.94    5.529      106     3705

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units     21.23    11.38     32.2     4000

 Location effects: Y ~ 1 + Temp

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)   19.4293   4.6279  36.3057     3489 0.0040 **
Temp          -2.0153  -3.6662  -0.5011     3075 0.0025 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

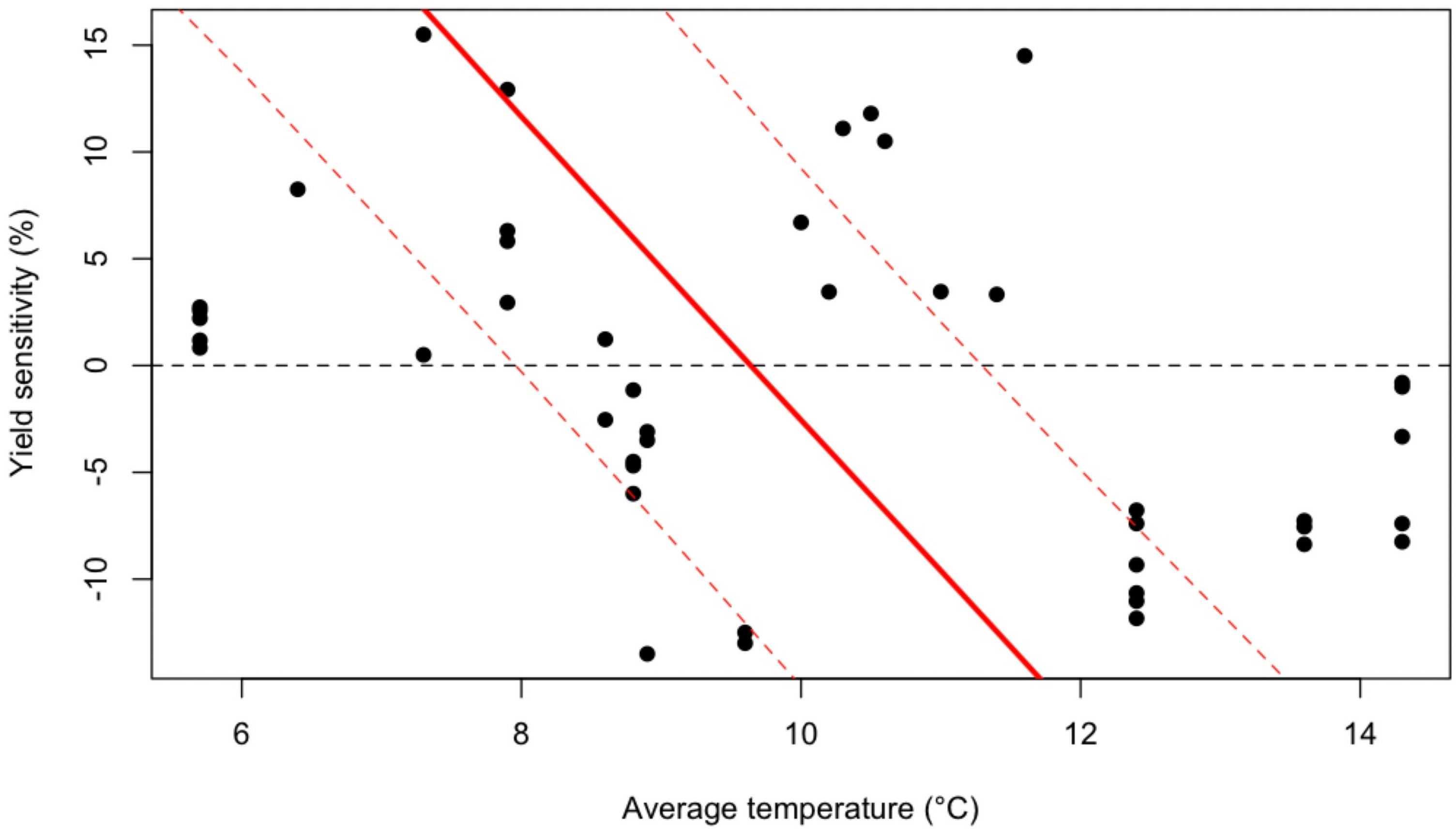
```
prior1<-list(B=list(mu=c(0,0),V=diag(c(10^8,10^8))), R=list(V=1,nu=1),G=list(G1=list(V=1,nu=1)))

Mod_mcmc<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000,
                   thin=10, burnin=10000, pr=TRUE, prior=prior1)

 Iterations = 10001:49991
 Thinning interval  = 10
 Sample size  = 4000

 DIC: 275.2081

 G-structure:  ~REF

    post.mean l-95% CI u-95% CI eff.samp
REF     43.94    5.529      106     3705

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units     21.23    11.38     32.2     4000

 Location effects: Y ~ 1 + Temp

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)   19.4293   4.6279  36.3057     3489 0.0040 **
Temp          -2.0153  -3.6662  -0.5011     3075 0.0025 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```r
prior1<-list(B=list(mu=c(0,-10),V=diag(c(10^8,1))), R=list(V=1,nu=1),G=list(G1=list(V=1,nu=1)))

Mod_mcmc<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000,
                   thin=10, burnin=10000, pr=TRUE, prior=prior1)

summary(Mod_mcmc)
```

```
 Iterations = 10001:49991
 Thinning interval  = 10
 Sample size  = 4000

 DIC: 269.5588

 G-structure:  ~REF

    post.mean l-95% CI u-95% CI eff.samp
REF     409.2    79.22    861.1     4000

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units     18.04    10.04    27.53     3780

 Location effects: Y ~ 1 + Temp

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)    69.792   50.498   90.452     4000 <3e-04 ***
Temp           -7.238   -8.905   -5.572     4000 <3e-04 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

```r
prior1<-list(B=list(mu=c(0,-10),V=diag(c(10^8,1))), R=list(V=1,nu=1),G=list(G1=list(V=1,nu=1)))

Mod_mcmc<-MCMCglmm(Y~1+Temp,random=~REF, data=DATA,verbose=F, nitt=50000,
                   thin=10, burnin=10000, pr=TRUE, prior=prior1)

summary(Mod_mcmc)
```

```
 Iterations = 10001:49991
 Thinning interval  = 10
 Sample size  = 4000

 DIC: 269.5588

 G-structure:  ~REF

    post.mean l-95% CI u-95% CI eff.samp
REF     409.2    79.22    861.1     4000

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units     18.04    10.04    27.53     3780

 Location effects: Y ~ 1 + Temp

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)    69.792   50.498   90.452     4000 <3e-04 ***
Temp           -7.238   -8.905   -5.572     4000 <3e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Yield sensitivity (%)
15
10
5
0
-5
-10
6
8
10
12
14
Average temperature (°C)