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Nonlinear models
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Fit and compare models for optimizing N fertilizer dose

The objective is to estimate the parameters of three models simulating the response of wheat yield at the nitrogen fertilizer rate (d). The equations of the models are defined below.

Model 1: Quadratic model
$$f(d) = \theta_1 + \theta_2 d + \theta_3 d^2$$

Model 2: Linear-plus-plateau model
$$f(d) = \theta_1 \text{ if } d > \theta_2$$
$$f(d) = \theta_1 + \theta_3(d - \theta_2) \text{ othewise}$$

Model 3: Quadratic-plus-plateau model
$$f(d) = \theta_1 \text{ if } d > \theta_2$$
$$f(d) = \theta_1 + \theta_3(d - \theta_2)^2 \text{ othewise}$$

Yield measurements were carried out on one plot of wheat for six doses of fertilizer:
yield = 5.1 t/ha for d = 0
yield = 7.5 t/ha for d = 50 kg/ha
yield = 9.2 t/ha for d = 100 kg/ha
yield = 9.8 t/ha for d = 200 kg/ha
yield = 9.6 t/ha for d = 250 kg/ha
yield = 9.7 t/ha for d = 300 kg/ha

- Estimate the parameters of the quadratic model with the data using the R `lm` function. Display the results of the estimation.
- Make a graph presenting the data and the yield response curve calculated by the quadratic model.
- Define two new R functions for calculating a yield as a function of a fertilizer rate using the linear-plus-plateau and quadratic-plus-plateau models. The parameters of the models must appear as arguments of the functions.
- Estimate the parameters of the linear-plus-plateau and quadratic-plus-plateau models with the data using the `nls` R function. Analyze the estimated parameter values. Are they precisely estimated ?
- Graphically compare the calculated yield response curves using all three models.
- Compare the quality of fit of the models using several criteria.
- Determine the values of the smallest doses maximizing yield (optimal dose) using each model in turn.
- Discuss the consequences of using each model for N fertilizer management.