Automated Reasoning: Solutions for Tutorial 2

Exercise 1

An interpretation with a domain containing a single member (i.e. a singleton set) will satisfy the formula. For example, if the domain is $\{a\}$ then either $p(a)$ is true or $p(a)$ is false. In either case, the statement is true.

Note: There are many other possible solutions.

Exercise 2

See the Isabelle theory file.

Note: The statements from this exercise can be proved in several ways. Moreover, Isabelle has pre-proved lemmas that can make some of these proofs fairly trivial. You should attempt to prove the statements using only the basic Natural Deduction rules for classical FOL.

Exercise 3

1. $(\forall x.P\,x \to Q) \to (\exists x.P\,x \to Q)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P\,a \to Q \vdash P\,a \to Q}\,\mathit{assumption}}{\forall x.P\,x \to Q \vdash P\,a \to Q}\,\mathit{allE}}{\forall x.P\,x \to Q \vdash \exists x.P\,x \to Q}\,\mathit{exI}}{\vdash (\forall x.P\,x \to Q) \to (\exists x.P\,x \to Q)}\,\mathit{impI}$$

2. $\forall x.\neg P\,x$, assuming that $\neg\exists x.P\,x$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P\,x_0 \vdash P\,x_0}\,\mathit{assumption}}{P\,x_0 \vdash \exists x.P\,x}\,\mathit{exI}}{\neg\exists x.P\,x, P\,x_0 \vdash \bot}\,\mathit{notE}}{\neg\exists x.P\,x \vdash \neg P\,x_0}\,\mathit{notI}}{\neg\exists x.P\,x \vdash \forall x.\neg P\,x}\,\mathit{allI}$$

3. $\exists x. \neg P\, x$, assuming that $\neg \forall x. P\, x$ is true

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P\, x_0 \vdash \neg P\, x_0}\ \mathit{assumption}}{\neg P\, x_0 \vdash \exists x. \neg P\, x}\ \mathit{exI}}{\neg \exists x. \neg P\, x, \neg P\, x_0 \vdash \bot}\ \mathit{notE}}{\neg \exists x. \neg P\, x \vdash P\, x_0}\ \mathit{ccontr}}{\neg \exists x. \neg P\, x \vdash \forall x. P\, x}\ \mathit{allI}}{\neg \forall x. P\, x, \neg \exists x. \neg P\, x \vdash \bot}\ \mathit{notE}}{\neg \forall x. P\, x \vdash \exists x. \neg P\, x}\ \mathit{ccontr}
$$