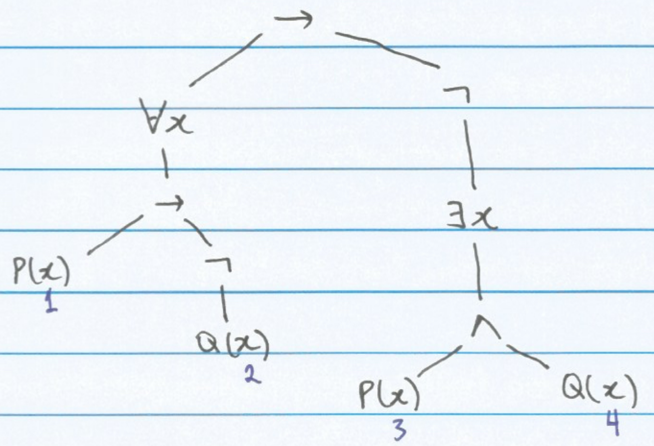
Q3

3(a).



Free variables are variables that are not in the scope of any quantifier

→ The variable x labeled 1 and 2 are <u>bound</u> by the universal quantification

→ The variable x labeled 3 and 4 are <u>bound</u> by the existential quantification

We know this as they are below $\forall x$ and $\exists x$ respectively in the parse tree.

(b). For formula (1), the result is:

$$(\exists z. P(z,x) \rightarrow \forall x. Q(x,z))$$

This is because $[x/y]$ means to replace all occurrences of y with x in the given pattern or formula

For formula (2), the result is:

$$(\forall x. (\exists y. (x = y + g(y))))$$

This is because the substitution $[g(y)/z]$ means to replace all occurences of z with $g(y)$ in the given pattern or formula.

3(c). → Matching is where given pattern and target, we find a substitution such that pattern[substitution] ≡ target, where ≡ means that terms are identical,

→ whereas unification is a two-way matching where there is no distinction between pattern and target. That is,

$term_1 [substitution] \equiv term_2 [substitution]$

unification succeeds to make $(s(X)+s(0))$ and $(s(0)+Y)$ identical but matching fails. This is because with unification, we have the switch rule which would make $s(0) \equiv Y$ into $Y \equiv s(0)$, whereas matching will fail with a conflict as we cannot match s with Y ($s \not\equiv Y$)

(d). $f(X, g(a,Y)) \equiv f(g(Z,b), X)$

$X \equiv g(Z,b) \wedge g(a,Y) \equiv X$ by Decompose

$X \equiv g(Z,b) \wedge g(a,Y) \equiv g(Z,b)$ by Eliminate

$X \equiv g(Z,b) \wedge a \equiv Z \wedge Y \equiv b$ by Decompose

$X \equiv g(Z,b) \wedge Z \equiv a \wedge Y \equiv b$ by Switch

$X \equiv g(a,b) \wedge Z \equiv a \wedge Y \equiv b$ by Eliminate

The unification process succeeds with the most general unifier

$\theta = \{ g(a,b)/X,\ b/Y,\ a/Z \}$

3(e).

$$\underline{s(s(0))\cdot s(0)}$$

$\to \underline{s(s(0))\cdot 0} + s(s(0))$ by (4)

$\to \underline{0 + s(s(0))}$ by (3)

$\to s(0 + \underline{s(0)})$ by (2)

$\to s(\underline{0+0})$ by (2)

$\to s(0)$ by (1)

The normal form is $s(0)$.

(f). → For these rules, define the measure of an expression as the number of functions, i.e., f and g

→ The first rule removes the use of function g

→ The second rule removes the use of function f

→ Since we have defined a natural number measure, and such that each rewrite rule decreases the measure, this set of rewrite rules is terminating.