Q1.

AUTOMATED REASONING
B142849

1. (a). (i). $Learns(Alice, Programming) \rightarrow (Likes(Alice, Programming) \vee Takes(Alice, Programming)) \wedge \neg(Likes(Alice, Programming) \wedge Takes(Alice, Programming))$

(ii). $\exists x.\ Best\text{-}Score(x, Programming) \wedge x = Alice \wedge \forall y.\ y \neq x \rightarrow \neg Best\text{-}Score(y, Programming)$

(b).

$$\dfrac{}{P(x_0) \vdash P(x_0)}\ (\text{assumption}) \qquad \dfrac{\dfrac{}{[[P(x_0);\ Q(x_0)]] \vdash Q(x_0)}\ (\text{assumption})}{[[P(x_0);\ Q(x_0)]] \vdash \exists x.\ Q(x)}\ (\text{r exI } x_0)$$

$$[[P(x_0) \rightarrow Q(x_0);\ P(x_0)]] \vdash \exists x.\ Q(x) \quad (\text{e impE})$$

$$[[\forall x.\ P(x) \rightarrow Q(x);\ P(x_0)]] \vdash \exists x.\ Q(x) \quad (\text{e allE } x_0)$$

$$[[\forall x.\ P(x) \rightarrow Q(x);\ \exists x.\ P(x)]] \vdash \exists x.\ Q(x) \quad (\text{e exE})$$

$$[[\forall x.\ P(x) \rightarrow Q(x);\ \neg\exists x.\ Q(x);\ \exists x.\ P(x)]] \vdash \bot \quad (\text{e notE})$$

$$[[\forall x.\ P(x) \rightarrow Q(x);\ \neg\exists x.\ Q(x)]] \vdash \neg\exists x.\ P(x) \quad (\text{notI})$$

$$(\forall x.\ P(x) \rightarrow Q(x)) \vdash (\neg\exists x.\ Q(x)) \rightarrow (\neg\exists x.\ P(x)) \quad (\text{impI})$$

$$\vdash (\forall x.\ P(x) \rightarrow Q(x)) \rightarrow (\neg\exists x.\ Q(x)) \rightarrow (\neg\exists x.\ P(x)) \quad (\text{impI})$$

Q1

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(c).(i). The first critical pair (CP) arises from rules 1 and 2:

$$\overbrace{(x\cdot y)\cdot z}^{L_2} \Rightarrow \overbrace{x\cdot(y\cdot z)}^{R_2}$$

$$\underbrace{i(\underbrace{x'\cdot y'}_{s})}_{L_1} \Rightarrow \underbrace{i(y')\cdot i(x')}_{R_1}$$ [after renaming]

most general unifier

The mgu θ, given our choice of non-variable subterm s of L_1 is:

$\{(x\cdot y)/x',\ z/y'\}$ → With this, $s[\theta] = L_2[\theta]$

The CP is $\langle R_1[\theta],\ L_1[\theta]/\{R_2[\theta]/s[\theta]\}\rangle$

$= \underline{\langle i(z)\cdot i(x\cdot y),\ i(x\cdot(y\cdot z))\rangle}$

The second CP arises from rule 1 with itself:

$$\overbrace{(x\cdot y)\cdot z}^{L_2} \Rightarrow \overbrace{x\cdot(y\cdot z)}^{R_2}$$

$$\underbrace{(\underbrace{x'\cdot y'}_{s})\cdot z'}_{L_1} \Rightarrow \underbrace{x'\cdot(y'\cdot z')}_{R_1}$$ [after renaming]

The mgu θ, given our choice of non-variable subterm s of L_1 is:

$\{(x\cdot y)|x',\ z/y'\}$ → With this, $s[\theta] = L_2[\theta]$

The CP is $\langle R_1[\theta],\ L_1[\theta]/\{R_2[\theta]/s[\theta]\}\rangle$

$= \underline{\langle (x\cdot y)\cdot(z\cdot z'),\ (x\cdot(y\cdot z))\cdot z'\rangle}$

There are no other CPs that can arise.

(c).(ii). For the CP $\langle s_1, s_2\rangle = \langle i(z)\cdot i(x\cdot y),\ i(x\cdot(y\cdot z))\rangle$

→ Normal form of s_1 is $i(z)\cdot i(y)\cdot i(x)$ by rule 2

→ Normal form of s_2 is $i(z)\cdot i(y)\cdot i(x)$ by applying rule 2 twice

Since $s_1 \equiv s_2$, we can conflate this CP.

The intermediate step is $i(y\cdot z)\cdot i(x)$

continuation of (c).(ii).

(c)(ii). cont. For the second CP $\langle s_1, s_2 \rangle = \langle (x \cdot y) \cdot (z \cdot z'), (x \cdot (y \cdot z)) \cdot z' \rangle$

→ Normal form of s_1 is $x \cdot (y \cdot (z \cdot z'))$ by rule 1

→ Normal form of s_2 is $x \cdot ((y \cdot z) \cdot z') \rightarrow x \cdot (y \cdot (z \cdot z'))$ by applying rule 1 twice

Since $s_1 \equiv s_2$, we can conflate this CP.

Since S is terminating and all the critical pairs are conflatable, S is locally confluent.

(d). $\lambda a\, b.\ b(\lambda x.\ x\ a)$ is equivalent to $b(x(a))$
where x is a function that takes in type 'a and returns type 'x
and b is a function that takes in type 'x and returns type 'b
Therefore, the type is $('a \Rightarrow 'x) \Rightarrow 'b$

(e). datatype 'a listx = Nil | Cons "'a" "'a listx" "'a"