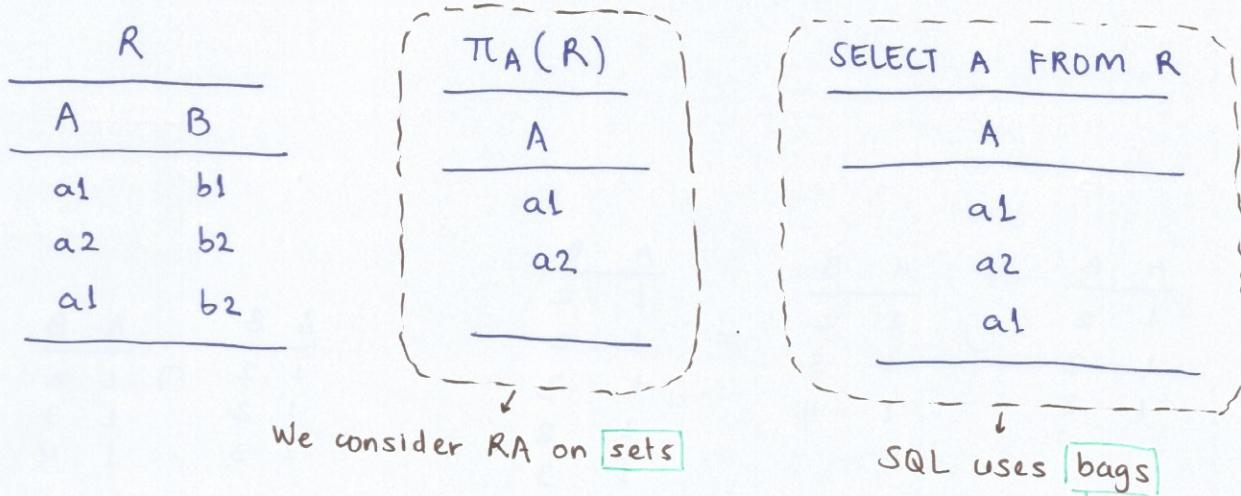


MULTISETS & AGGREGATION

MULTISETS (a.k.a. BAGS)

- are sets where the same element can occur multiple times



- The no. of occurrences of an element is called its **multiplicity**

NOTATION	
$a \in_k B$	a occurs k times in bag B
$a \in B$	a occurs in B w/ multiplicity ≥ 1
$a \notin B$	a does not occur in $B \rightarrow a \in_0 B$

RELATIONAL ALGEBRA ON BAGS

- We now consider relations as bags of tuples

PROJECTION

- Keeps duplicates
i.e.

$$\pi_A \left(\begin{array}{c|c} A & B \\ \hline 2 & 3 \\ 1 & 1 \\ 2 & 2 \end{array} \right) = \begin{array}{c} A \\ \hline 2 \\ 1 \\ 2 \end{array}$$

CARTESIAN PRODUCT

- Concatenates tuples as many times as they occur

i.e.

$\begin{array}{c c} A & B \\ \hline 1 & 1 \end{array}$	\times	$\begin{array}{c c} C \\ \hline 2 \\ 2 \end{array}$
--	----------	---

↓

$\begin{array}{c c c} A & B & C \\ \hline 1 & 1 & 2 \\ 1 & 1 & 2 \end{array}$

SELECTION

→ Takes all occurrences of tuples satisfying the condition

i.e.

$$\sigma_{A>1} \left(\begin{array}{cc} A & B \\ 2 & 3 \\ 1 & 2 \\ 2 & 3 \end{array} \right) = \begin{array}{cc} A & B \\ 2 & 3 \\ 2 & 3 \end{array}$$

DUPLICATE ELIMINATION

→ removes duplicates

If $\bar{a} \in R$, then $\bar{a} \in \epsilon(R)$

i.e.

$$\epsilon \left(\begin{array}{cc} A & B \\ 2 & 3 \\ 2 & 3 \\ 1 & 1 \end{array} \right) = \begin{array}{cc} A & B \\ 2 & 3 \\ 1 & 1 \end{array}$$

UNION

→ Adds multiplicities

If $\bar{a} \in_k R$ and $\bar{a} \in_n S$, then $\bar{a} \in_{k+n} R \cup S$

i.e.

$$\begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \end{array} \cup \begin{array}{cc} A & B \\ 1 & 2 \\ 2 & 3 \\ 1 & 4 \end{array} = \begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{array}$$

INTERSECTION

→ Takes the minimum multiplicity

If $\bar{a} \in_k R$ and $\bar{a} \in_n S$,

then $\bar{a} \in_{\min\{k,n\}} R \cap S$

i.e.

$$\begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \end{array} \cap \begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{array} = \begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{array}$$

DIFFERENCE

→ Subtracts multiplicities up to zero — If $\bar{a} \in_k R$ and $\bar{a} \in_n S$,

i.e.

$$\begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \end{array} - \begin{array}{cc} A & B \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \end{array} = \begin{array}{cc} A & B \\ 1 & 2 \end{array}$$

then $\begin{cases} \bar{a} \in_{k-n} R-S & \text{if } k > n \\ \bar{a} \notin R-S & \text{otherwise} \end{cases}$

RA ON SETS vs. RA ON BAGS

→ Equivalences of RA on sets do NOT necessarily hold on bags

→ i.e. On bags, $\sigma_{\theta_1 \vee \theta_2}(R) \not\equiv \sigma_{\theta_1}(R) \cup \sigma_{\theta_2}(R)$

R	A
2	

$\sigma_{(A>1) \vee (A<3)}(R)$	A
	2

$\sigma_{(A>1)}(R) \cup \sigma_{(A<3)}(R)$	A
	2

→ Notice that $\epsilon(\sigma_{\theta_1 \vee \theta_2}(R)) \equiv \epsilon(\sigma_{\theta_1}(R) \cup \sigma_{\theta_2}(R))$ holds

SQL and RA ON BAGS

$Q := \underbrace{\text{SELECT [DISTINCT] } \alpha \text{ FROM } \tau \text{ WHERE } \theta}_{\text{keeps duplicates and need to explicitly say to remove them.}}$

| $Q_1 \text{ UNION [ALL] } Q_2$
| $Q_1 \text{ INTERSECT [ALL] } Q_2$
| $Q_1 \text{ EXCEPT [ALL] } Q_2$

} when you do these operations, the default is that duplicates are removed and you need to explicitly say when you want to keep them

SQL	RA on bags
SELECT $\alpha \dots$	$\pi_\alpha(\cdot)$
SELECT DISTINCT $\alpha \dots$	$\epsilon(\pi_\alpha(\cdot))$
$Q_1 \text{ UNION ALL } Q_2$	$Q_1 \cup Q_2$
$Q_1 \text{ INTERSECT ALL } Q_2$	$Q_1 \cap Q_2$
$Q_1 \text{ EXCEPT ALL } Q_2$	$Q_1 - Q_2$
$Q_1 \text{ UNION } Q_2$	$\epsilon(Q_1 \cup Q_2)$
$Q_1 \text{ INTERSECT } Q_2$	$\epsilon(Q_1 \cap Q_2)$
$Q_1 \text{ EXCEPT } Q_2$	$\epsilon(Q_1) - Q_2 \neq \epsilon(Q_1 - Q_2)$ \equiv $\epsilon(Q_1) - \epsilon(Q_2)$

AGGREGATE FN_S IN SQL

- COUNT no. of elems. in a column
 - AVG avg. value of elems in a column
 - SUM adds up all elems. in a column
 - MIN min. value of elems. in a column
 - MAX max. value of elems. in a column
- } Using DISTINCT here makes no difference

→ COUNT (*) ✓ counts all rows in a table

COUNT (DISTINCT *) X

↳ To count all distinct rows of a table T use:

{ SELECT COUNT (DISTINCT T.*)
FROM T }

✓

EMPTY TABLES

Suppose table T has a column of numbers called A

```
SELECT MIN(A), MAX(A), AVG(A), SUM(A), COUNT(A), COUNT(*)  
FROM T  
WHERE 1 = 2;
```

min	max	sum	avg	count	count
NULL	NULL	NULL	NULL	0	0

There is one row and the value is NULL