Cosmological parameters estimation

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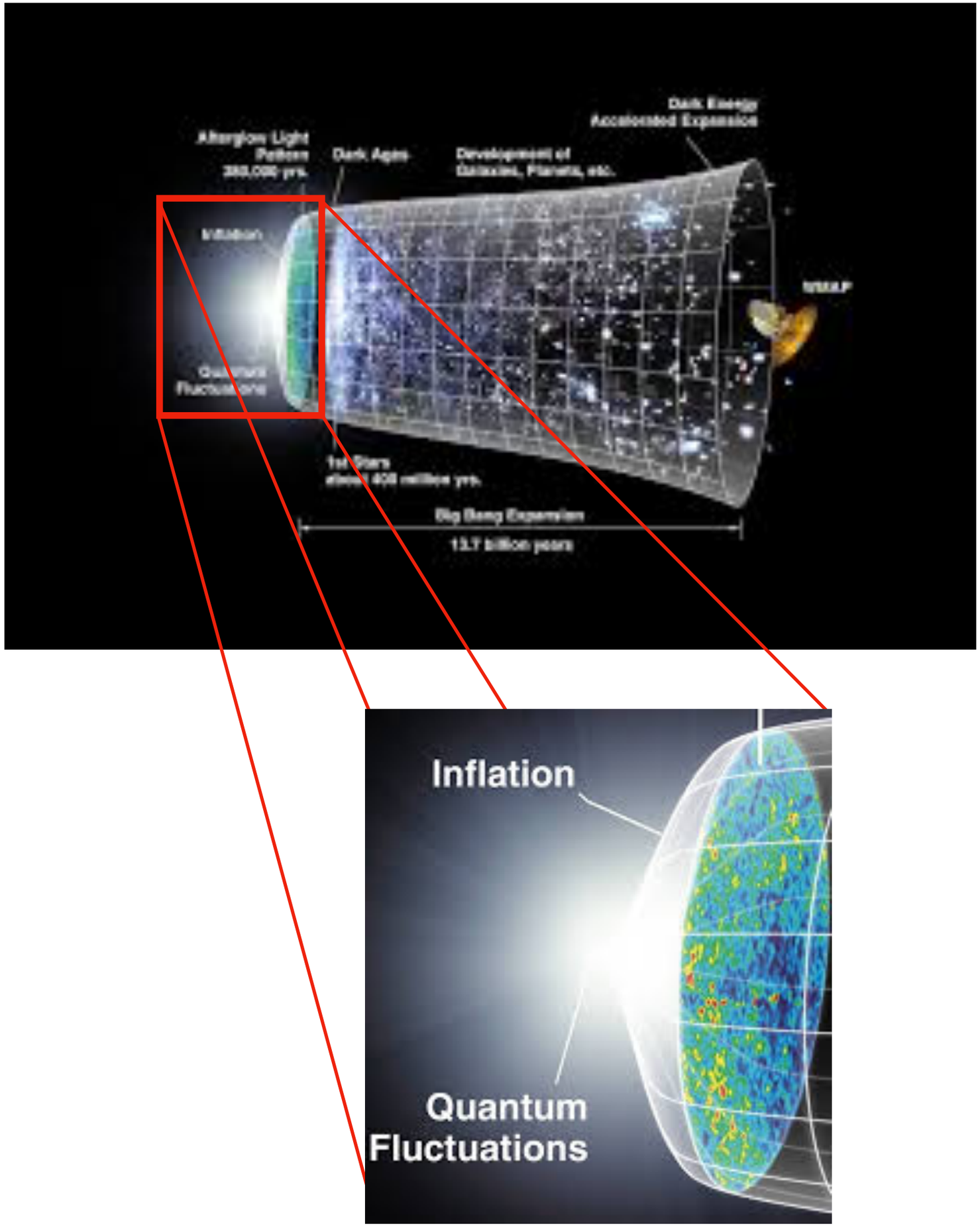

Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Dark Energy
Accelerated Expansion
WMAP
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
Inflation
Quantum
Fluctuations

Inflation: the theoretical model

Inflation provides a mechanism to generate primordial fluctuations

Scalar perturbations (curvature)

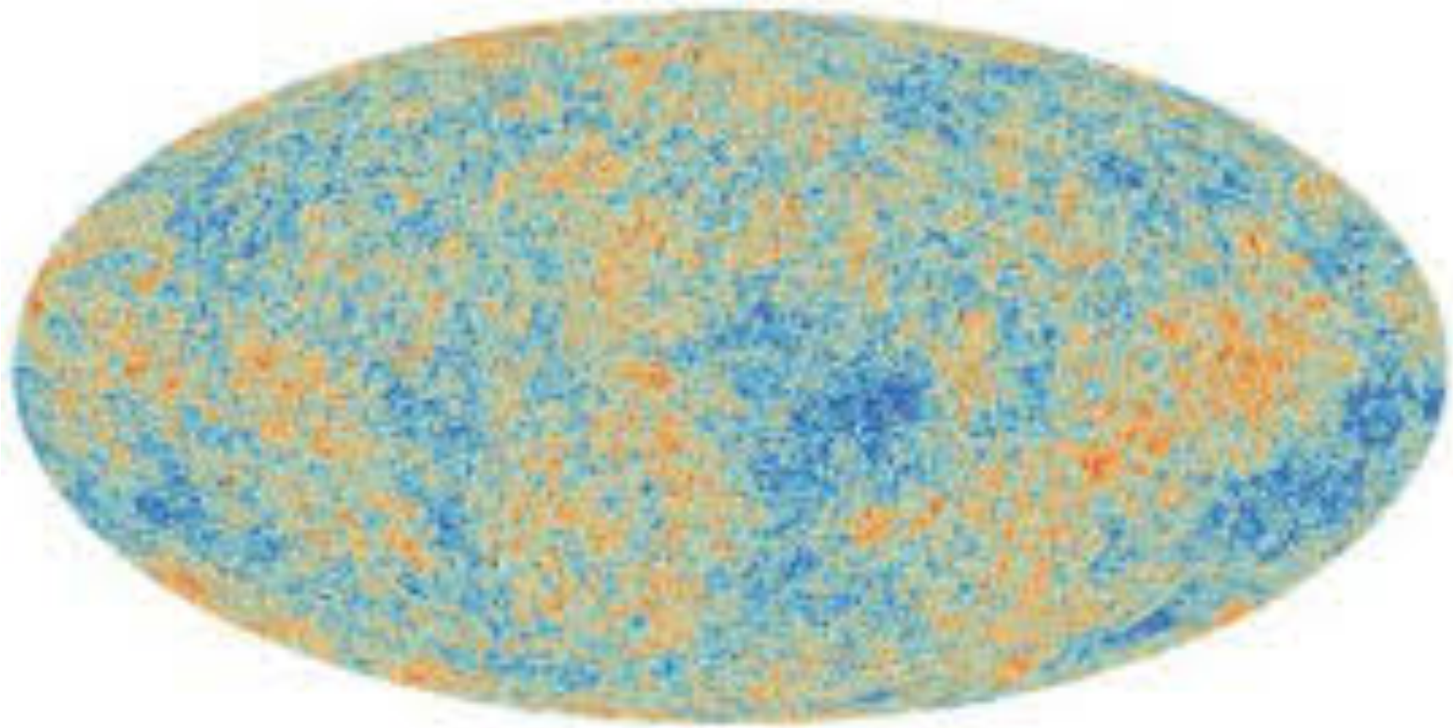
$$P_s(k) = A_s \left(\frac{k}{k_0} \right)^{n_s - 1}$$

Tensor perturbations (gravitational waves)

$$P_t(k) = A_t = r\, A_s$$

$$r = \frac{A_s}{A_t}$$

Cosmic Microwave Background (CMB)

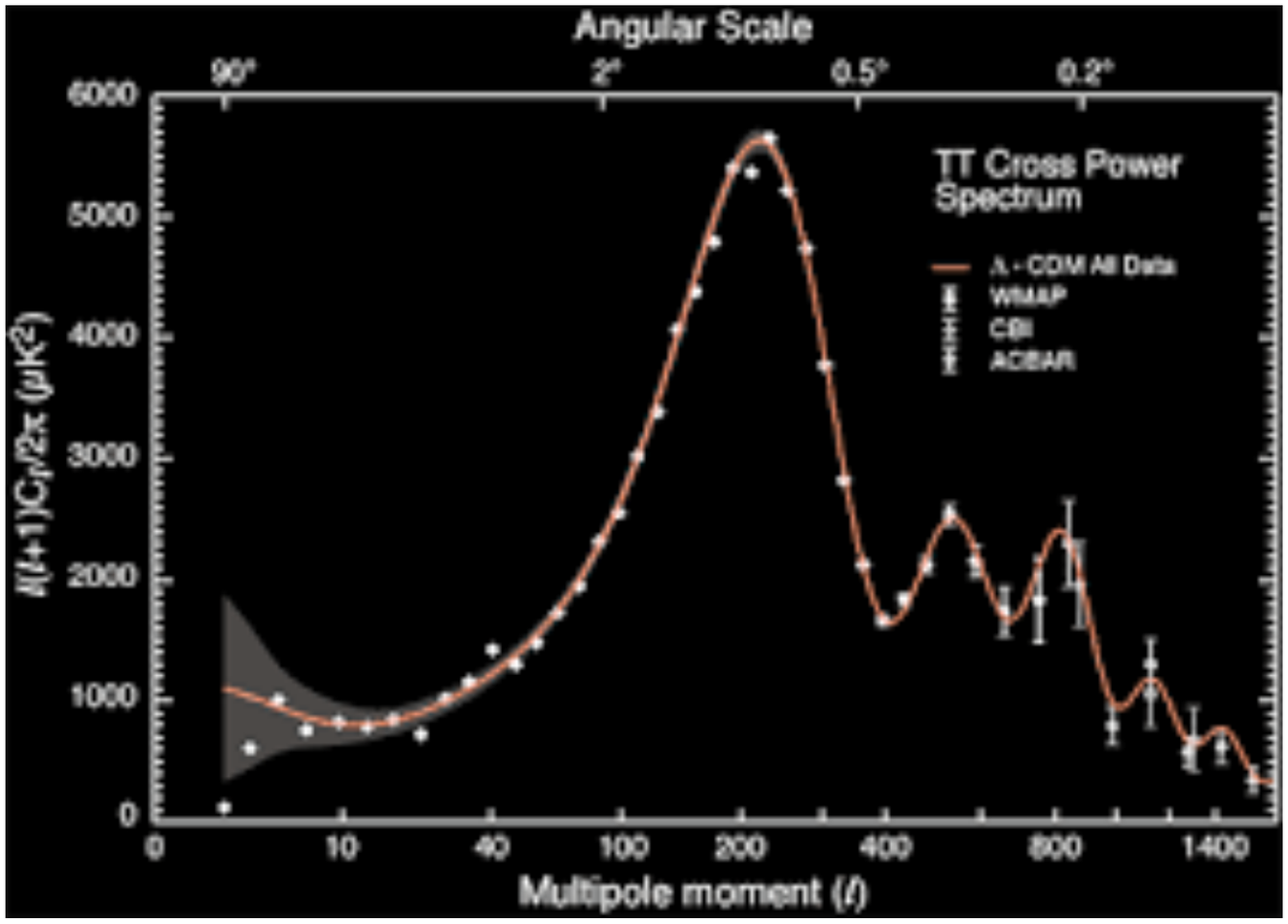


Spherical harmonics transform

$$T(\mathbf{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}\, Y_{\ell m}(\mathbf{n})$$

$$a_{\ell m} = \int T(\mathbf{n})\; Y^{*}_{\ell m}(\mathbf{n})\; \mathrm{d}^2 n$$

CMB angular power spectrum

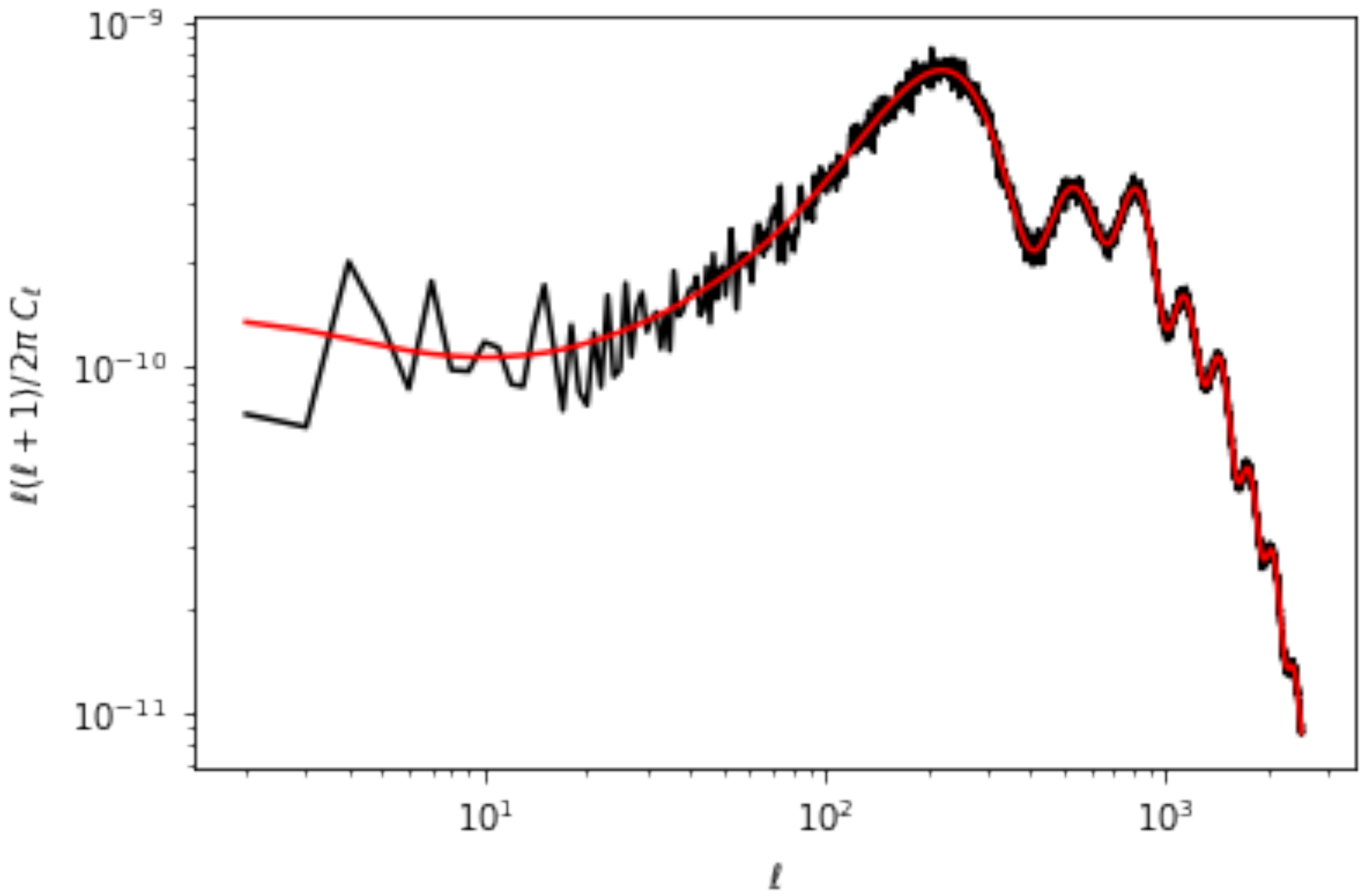


The CMB is an isotropic Gaussian random field

$$\langle a_{\ell m}\, a^*_{\ell' m'} \rangle = \delta_{\ell \ell'}\, \delta_{m m'}\, C_\ell$$

Angular power spectrum estimator

$$\hat{C}_\ell = \frac{1}{2\ell+1} \sum_{\ell=-m}^{m} |a_{\ell m}|^2$$



The theoretical model

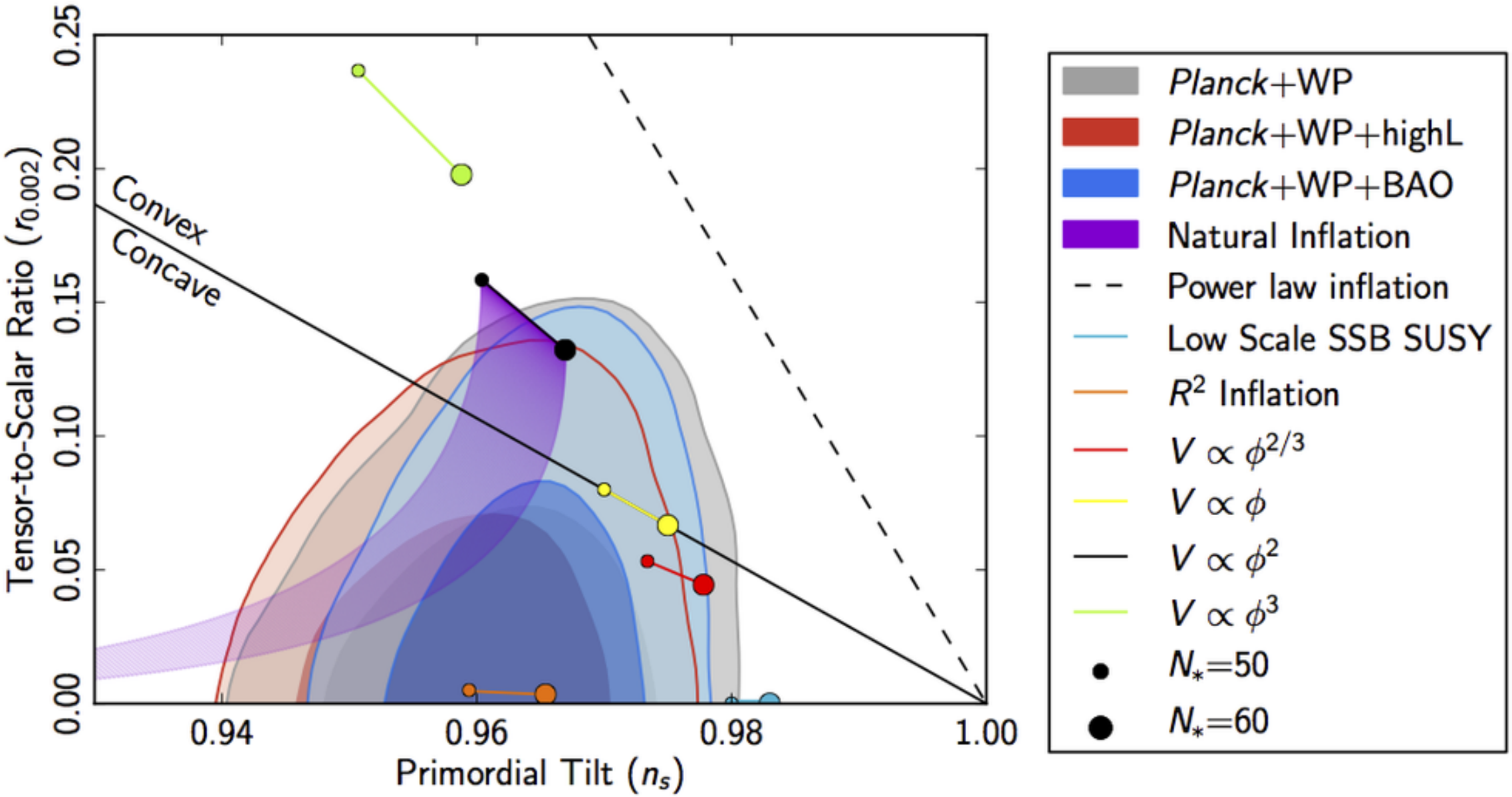
$$C_\ell(A_s, n_s, r) = A_s \left[C_\ell^s(n_s) + r\ C_\ell^t \right]$$

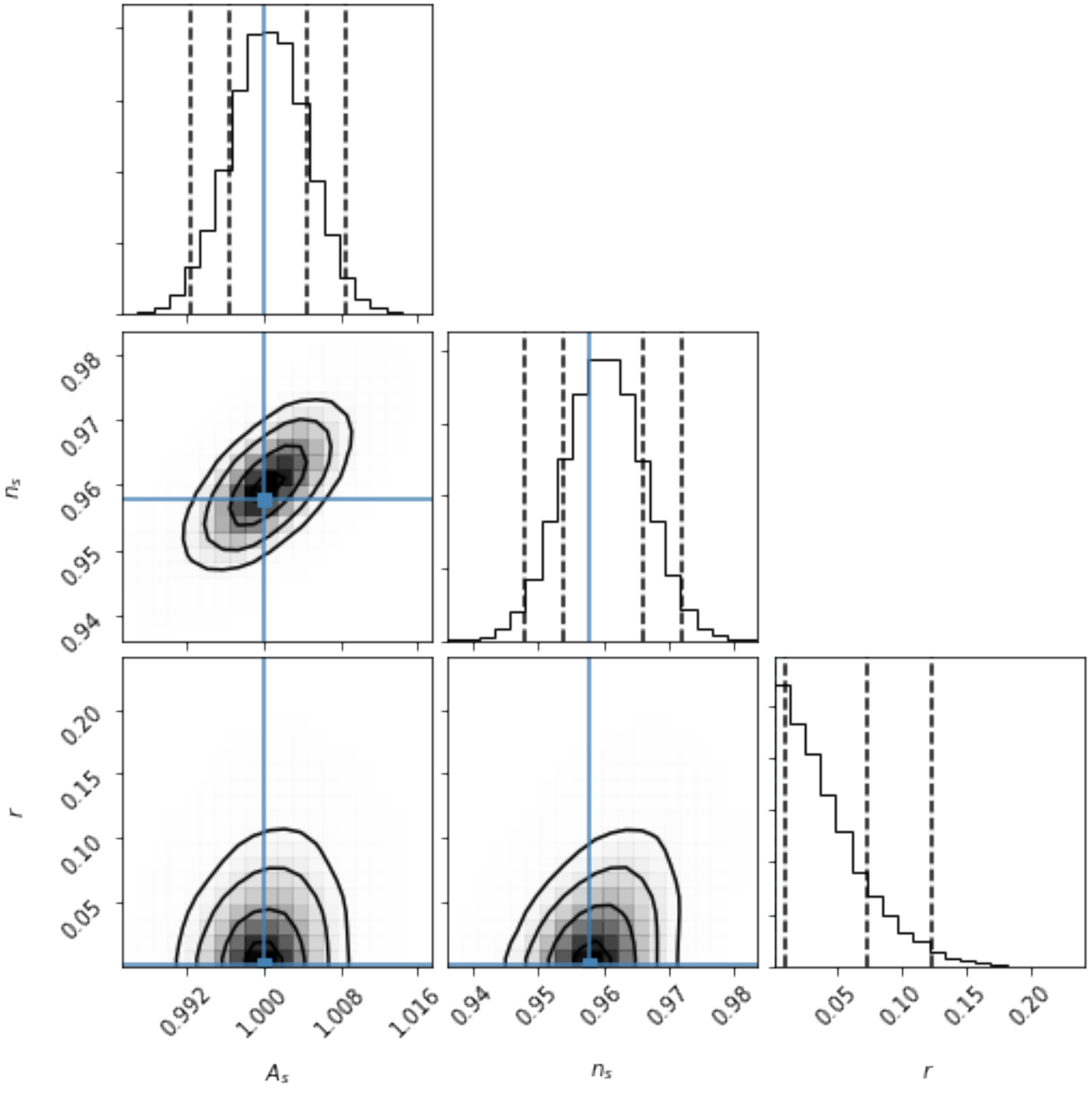
Gaussian likelihood

$$\ln L \sim -\frac{1}{2} \sum_{\ell=2}^{\ell_{\max}} \frac{\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2}{C_\ell}$$

$$\ln L(\hat{C}_\ell \,|\, \theta) = -\sum_{\ell=2}^{\ell_{\max}} \frac{2\ell+1}{2} \left(\frac{\hat{C}_\ell}{C_\ell(\theta)} + \ln C_\ell(\theta) \right)$$

Constraints on inflation




0.98
0.97
0.96
0.95
0.94
n_s
0.20
0.15
0.10
0.05
r
0.992
1.000
1.008
1.016
A_s
0.94
0.95
0.96
0.97
0.98
n_s
0.05
0.10
0.15
0.20
r

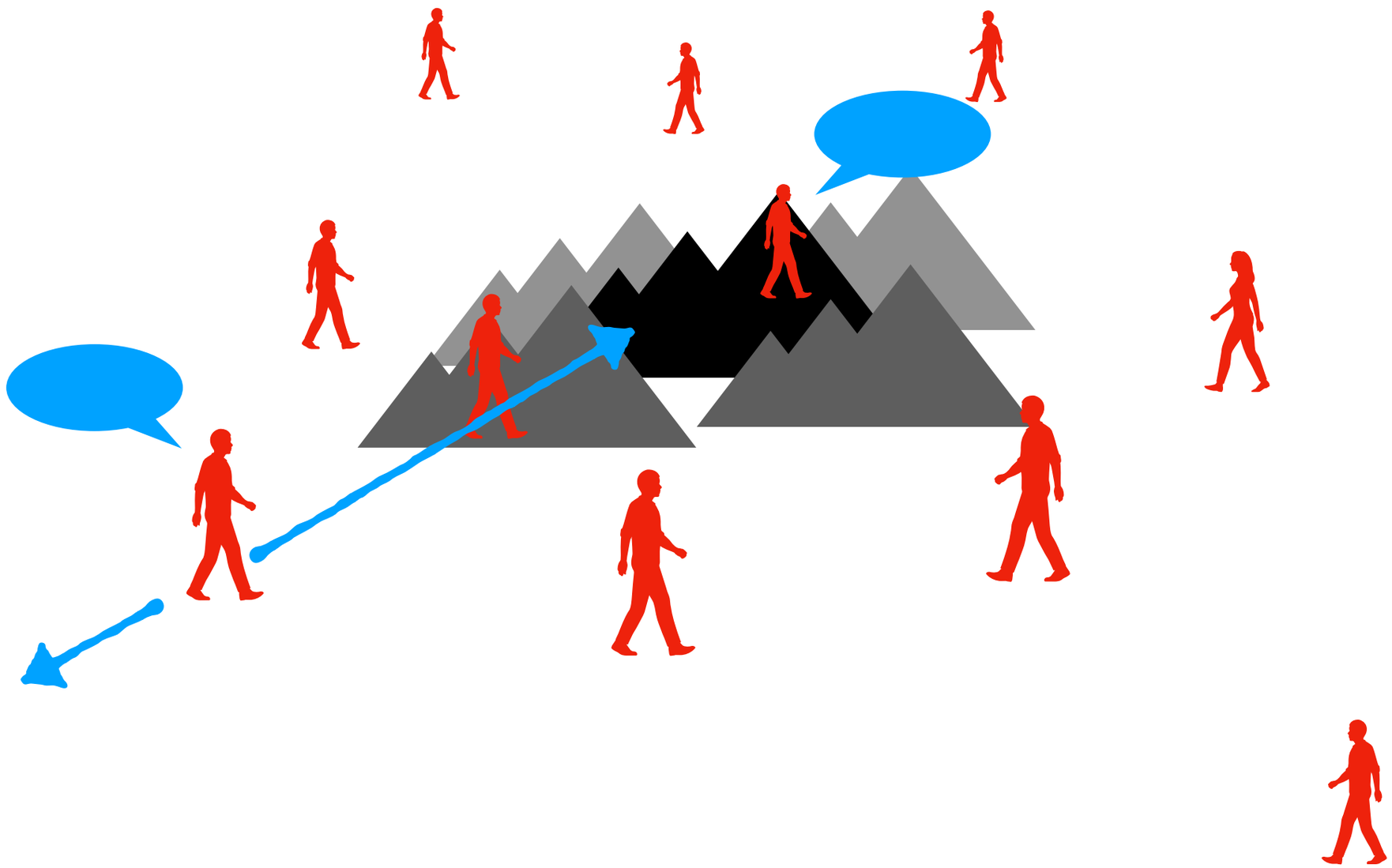
Cosmological parameters estimation

1. Programming the posterior function (likelihood + prior) given the theoretical model and the data
2. Running the MCMC algorithm for the posterior
3. Checking the convergence of the chains
4. Calculation of the different statistics of the parameters
5. Computation of the evidence of the model

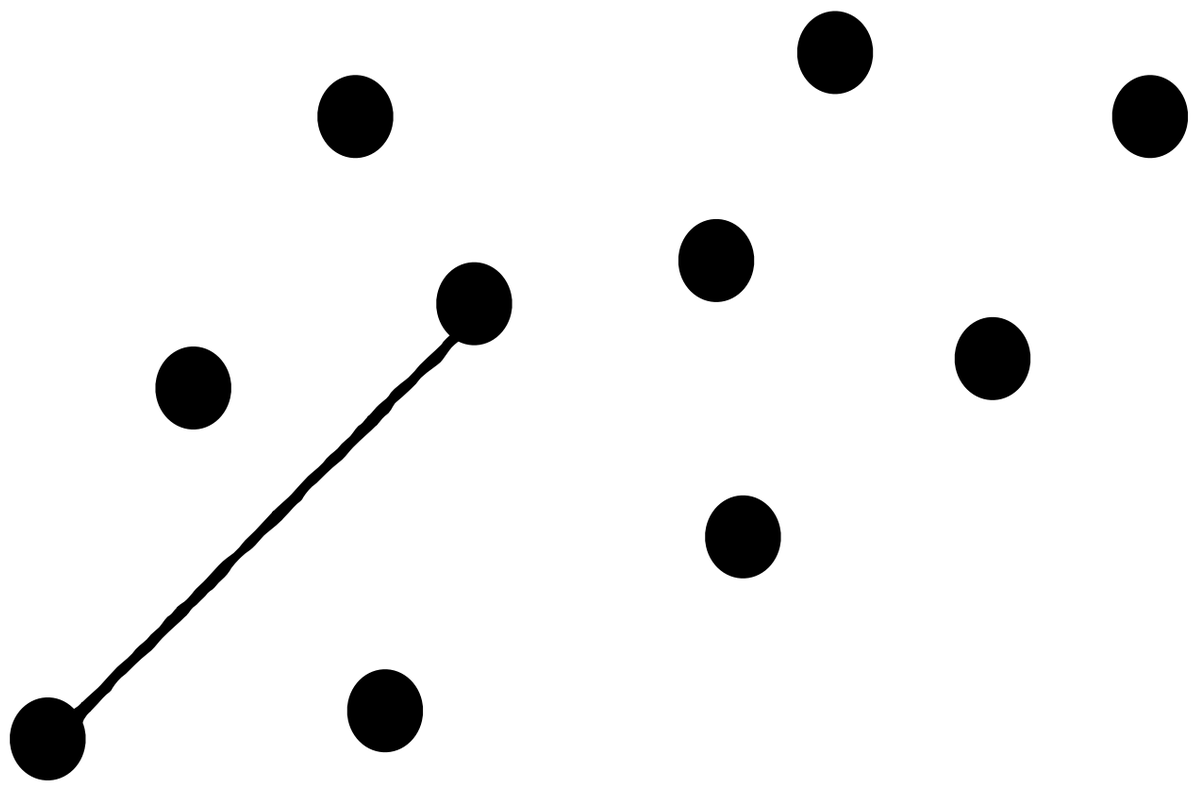
Python modules

- **emcee: MCMC sampler implementing an affine invariant algorithm (Goodman & Weare, 2010)**
- **corner: representing parameter space**
- **tqdm: toolkit for including progress bar**

Ensemble of “walkers”



Affine invariant MCMC ensemble sampler



$$X_{t+1}^k = X^j + Z\left(X_t^k - X^j\right)$$