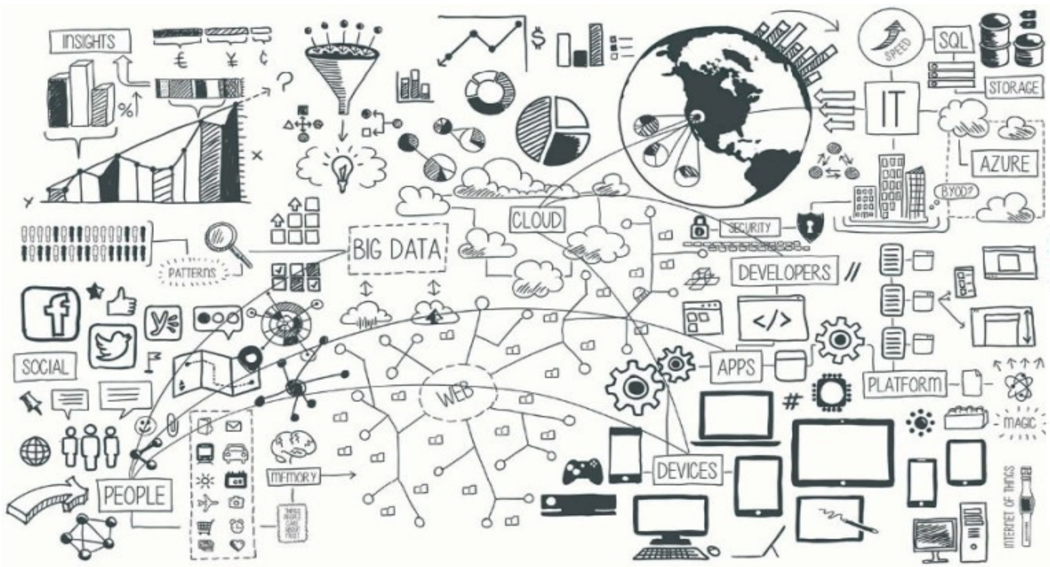
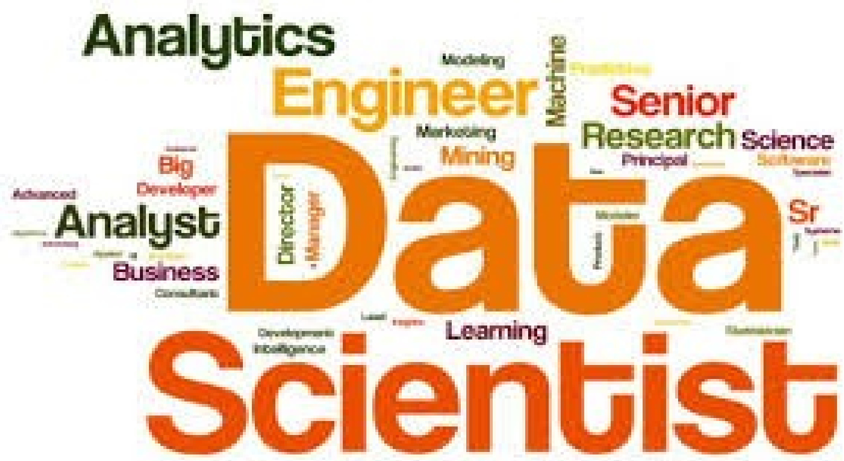
M1970 – Machine Learning II
Redes Probabilísticas Discretas





Sixto Herrera (sixto.herrera@unican.es)
Mikel Legasa

Grupo de Meteorología

Univ. de Cantabria – CSIC
MACC / IFCA

M1970 – Machine Learning (L 16:00-18:00; X 16:00-18:00)			
Mar	2	L	Introducción - Redes Probabilísticas Discretas (2h-T)
	4	X	Redes Bayesianas: Creación e Inferencia (2h-L)
	9	L	Clasificacidores Bayesianos. Naive Bayes (2h-L)
	11	X	Redes Bayesianas: Aprendizaje Estructural (2h-T)
	16	L	Redes Bayesianas: Aprendizaje Paramétrico – R. Gaussianas/Mixtas (2h-TL)
	18	X	Redes Bayesianas: Aprendizaje (2h-L)
	23	L	Evaluación (2h)

NOTA: Las líneas de código de R en esta presentación se muestran sobre un fondo gris.

Con carácter obligatorio todas las tareas se realizarán o entregarán usando la plataforma virtual de la asignatura. Por tanto es responsabilidad del alumno, asegurarse de que puede acceder a la plataforma virtual de la asignatura, antes del comienzo de las sesiones en las que se realicen las pruebas.

Todas las entregas deberán incluir el **nombre y apellidos** del alumno que realiza la entrega.

La plataforma usada es Moodle y podéis acceder a ella usando el Aula Virtual de la Universidad de Cantabria. Para ello es imprescindible vuestro usuario y contraseña.

Si fuese necesario comunicarse mediante correo electrónico con el profesorado, es obligatorio usar el correo con el cual se ha inscrito en Moodle.

Para cualquier problema con vuestro correo, poneros en contacto con el Servicio de Informática de la Universidad de Cantabria.

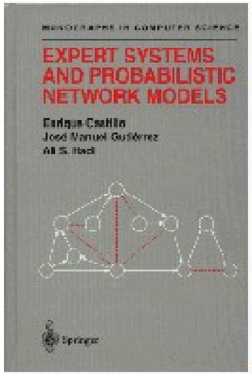
Cualquier duda, anuncio o comentario sobre el desarrollo y actividades de la asignatura se hará mediante el Foro disponible en el Moodle.

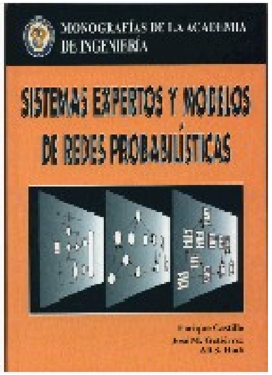
Muchas veces las dudas también las puede tener otro compañero y de este modo toda la información relevante sobre el desarrollo de la asignatura estará a disposición de vuestros compañeros.

El Foro no se usa para evaluaros. Es una herramienta que os permite compartir información con vuestros compañeros.

El Foro no está moderado, pero sí supervisado por el profesorado, de tal forma que se resuelvan o aclaren dudas si ningún compañero vuestro lo hace.

Sólo se permite el uso de mensajes personales o correo electrónico para situaciones personales muy excepcionales.

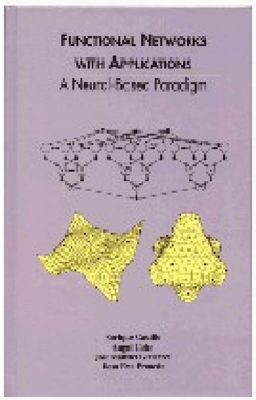


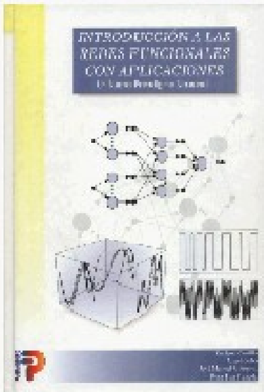


Expert Systems and Probabilistic Network Models.
E. Castillo, J.M. Gutiérrez, y A.S. Hadi
Springer-Verlag, New York.

Monografías de la Academia Española de Ingeniería

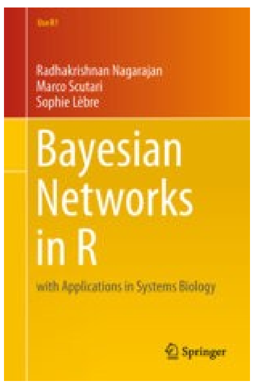
Spanish version free at: http://personales.unican.es/gutierjm

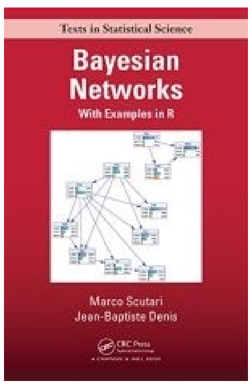




An Introduction to Functional Networks
E. Castillo, A. Cobo, J.M. Gutiérrez and E. Pruneda
Kluwer Academic Publishers (1999).

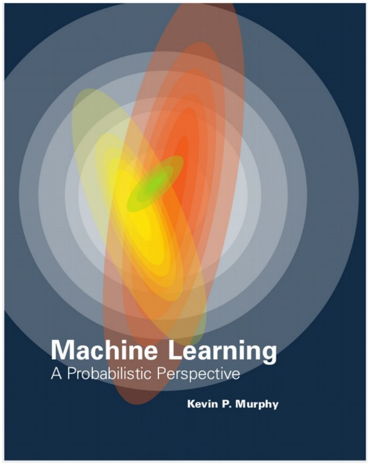
Paraninfo/International Thomson Publishing





Marco Scutari: *Bayesian networks in R & Bayesian networks with examples in R*

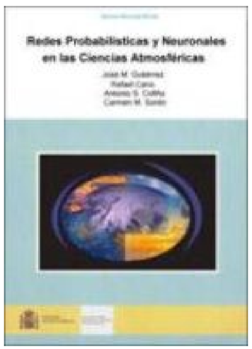
http://www.bnlearn.com/



Machine Learning: A Probabilistic Perspective

Kevin P. Murphy, The MIT Press, Cambridge (2013)

Disponible on-line en:
http://liuchengxu.org/books/src/Theory/Machine-Learning-A-Probabilistic-Perspective.pdf



Una aplicación en Ciencias Atmosféricas

LIBRO

J.M. Gutiérrez, R. Cano, A.S. Cofiño, and C. Sordo
Redes Probabilísticas y Neuronales en las Ciencias Atmosféricas
Ministerio de Medio Ambiente (*Monografías del Instituto Nacional de Meteorología*), Madrid. 350 páginas, 2004

7. MÉTODOS DE LA EVALUACIÓN				
Descripción	Tipología	Eval. Final	Recuper.	%
Valoración de informes y trabajos escritos	Actividad de evaluación con soporte virtual	Sí	Sí	60,00

Calif. mínima	3,00
Duración	
Fecha realización	Durante el periodo de impartición de la asignatura.
Condiciones recuperación	
Observaciones	Evaluación de los trabajos de grupo e individuales entregados por el alumno.

Descripción	Tipología	Eval. Final	Recuper.	%
Examen (escrito, oral y/o práctico en el aula de computación)	Actividad de evaluación con soporte virtual	Sí	Sí	40,00

Calif. mínima	0,00
Duración	Un máximo de dos horas
Fecha realización	Durante el periodo de impartición de la asignatura.
Condiciones recuperación	
Observaciones	

TOTAL	100,00

Observaciones

Si la nota final del alumno fuese menor que 5 sobre 10, entonces la recuperación consistirá en la realización de cada una de las tareas en las que hubiera obtenido una calificación menor que 5 sobre 10. El procedimiento de evaluación de una actividad recuperable será equivalente al de la actividad original.

Observaciones para alumnos a tiempo parcial

M1970 – Machine Learning (L 16:00-18:00; X 16:00-18:00)

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Observaciones	Evaluación de los trabajos de grupo e individuales entregados por el alumno.

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TOTAL	100,00
Observaciones	

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Observaciones para alumnos a tiempo parcial

Examen tipo test a desarrollar en el aula a través de la plataforma Moodle
El examen incluirá cuestiones de ambas partes de la asignatura.

7. MÉTODOS DE LA EVALUACIÓN				
Descripción	Tipología	Eval. Final	Recuper.	%
Valoración de informes y trabajos escritos	Actividad de evaluación con soporte virtual	Sí	Sí	60,00

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Duración	
Fecha realización	Durante el periodo de impartición de la asignatura.
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T01 – Redes Bayesianas

actividad recuperable será equivalente al de la actividad original.

Observaciones para alumnos a tiempo parcial

A nivel global, el valor de esta tarea se corresponde con el 30% de la nota final

1.1 Dataset de ejemplo: 'survey'

CATEGÓRICO

A partir de los datos de campo recogidos por la encuesta, se investigará la selección de medios de transporte por distintos perfiles de usuarios, y particularmente a la preferencia de tren o coche. Este tipo de análisis se utilizan con frecuencia en la planificación de infraestructuras. Para cada individuo encuestado, se han recopilado datos referenctes a 6 variables discretas. Las abreviaturas de dichas variables se muestran entre paréntesis, y se utilizarán a lo largo de la práctica para referirse a los nodos de la red creada. Tanto las abreviaturas como los nombres de las variables preservan la nomenclatura original del dataset en inglés.

- Edad (`A`): Edad del encuestado, agrupado en los siguientes estados: joven (`young` , < 30 años), adulto (`adult` , 30 < edad <= 60) y anciano (`old` , edad > 60).
- Sexo (`S`): Sexo del encuestado, con sus dos posibles estado: masculino (`M`) y femenino (`F`).
- Educación (`E`): Nivel más alto de educación alcanzado. Hasta educación secundaria (`high`) o título universitario (`uni`).
- Ocupación (`O`): Considera dos estados: trabajador por cuenta ajena (`emp`) o autónomo (`self`).
- Residencia (`R`): El tamaño de la población de residencia del individuo. Estados posibles: `big` y `small` .
- Transporte (`T`): El medio de transporte más utilizado por el encuestado para acudir al trabajo, diferenciando 3 posibles estados: `car` , `train` y `other` .

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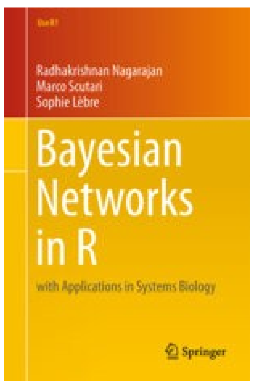
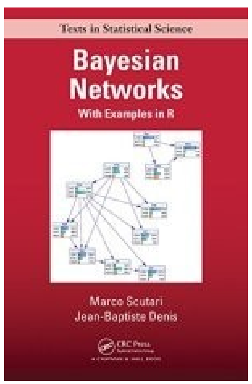
MIXTO

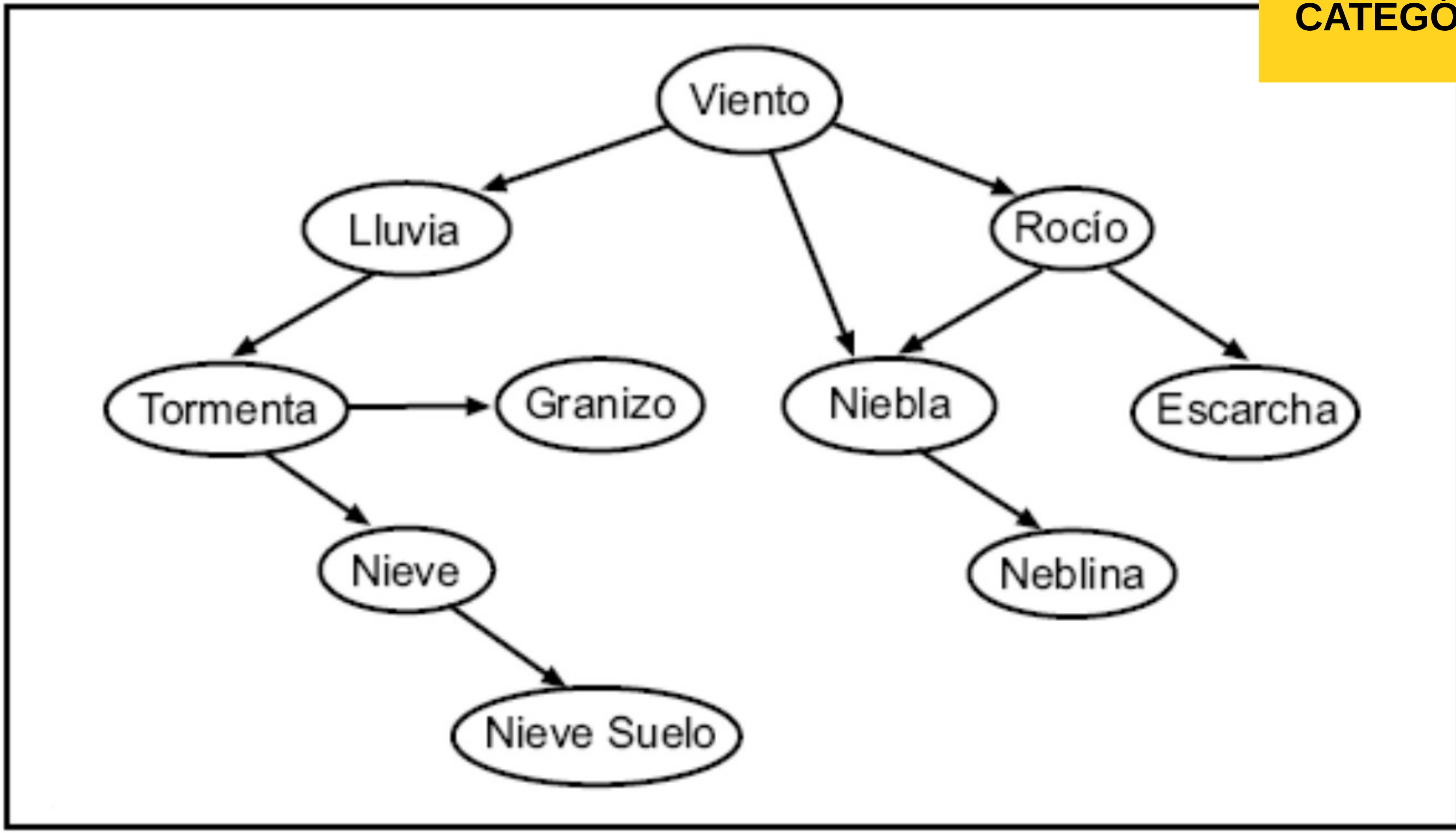




Marco Scutari: *Bayesian networks in R & Bayesian networks with examples in R*

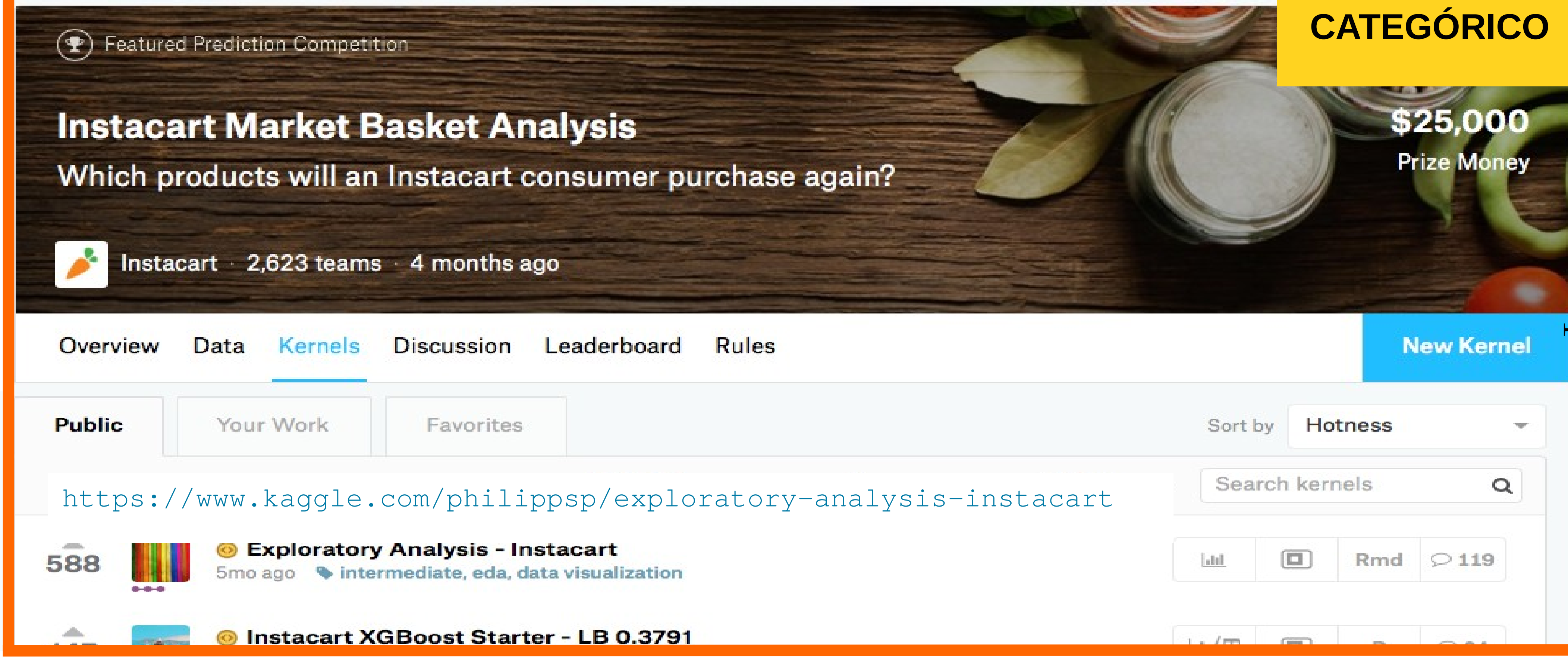
http://www.bnlearn.com/

CATEGÓRICO




```
Lluvia nieve granizo tormenta niebla rocio escarcha nieveSuelo neblina viento
s     n     n     n     n     n     n     n     n     s
s     n     n     n     n     n     n     n     n     s
s     n     n     s     n     n     n     n     n     s
s     n     n     n     n     n     n     n     n     s
```

CATEGÓRICO



En el curso utilizaremos un dataset más pequeño, “Groceries”, disponible en el paquete de R **arulesViz**.

Attribute characteristics	Categorical
Number of instances	9835
Number of attributes	169

```
install.packages("arulesViz")
data(“Groceries”)
```

M1970 – Machine Learning (L 16:00-18:00; X 16:00-18:00)

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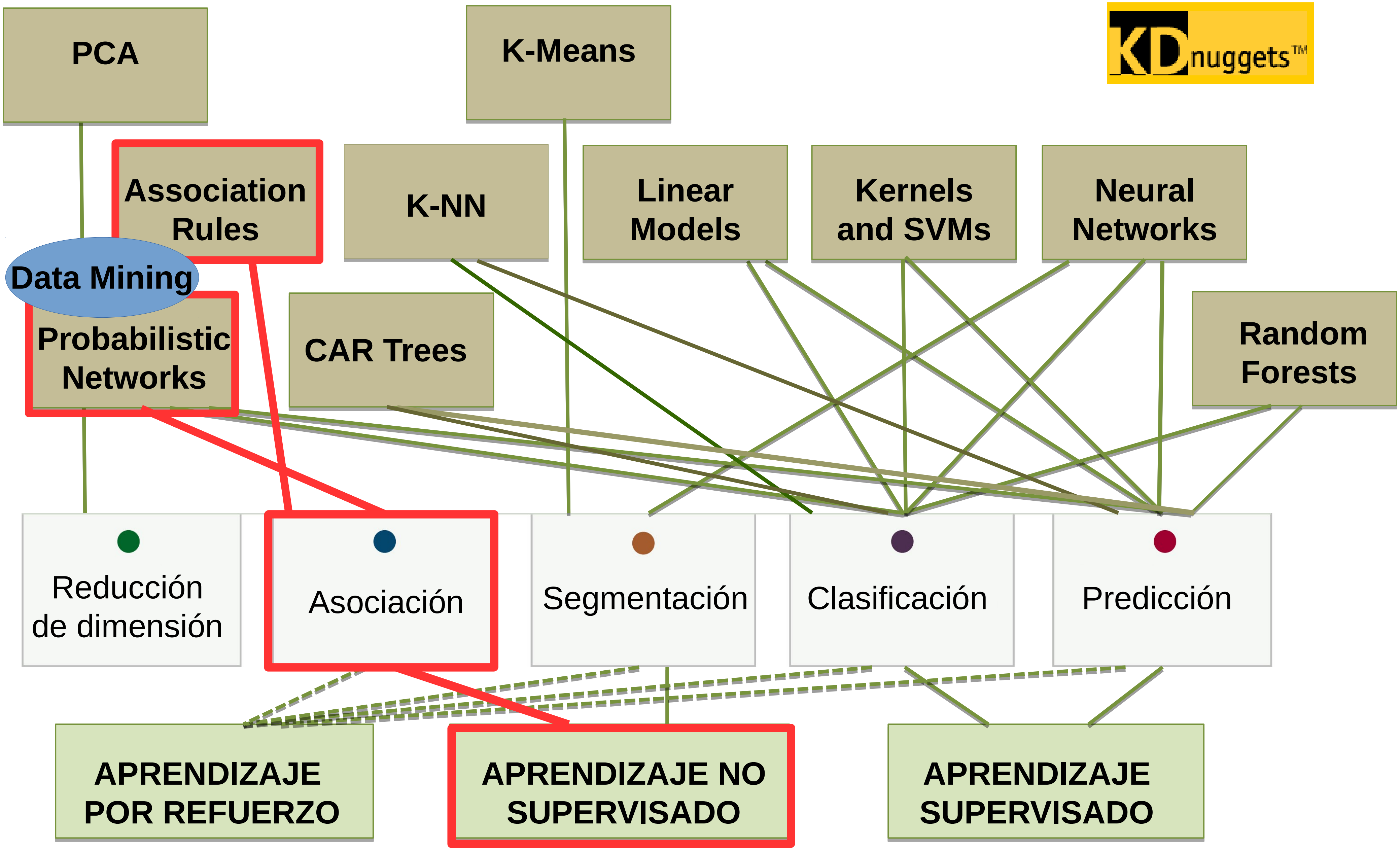
	23	**L**	**Evaluación (2h)**

NOTA: Las líneas de código de R en esta presentación se muestran sobre un fondo gris.

https://www.kdnuggets.com/2017/10/top-10-machine-learning-algorithms-beginners.html



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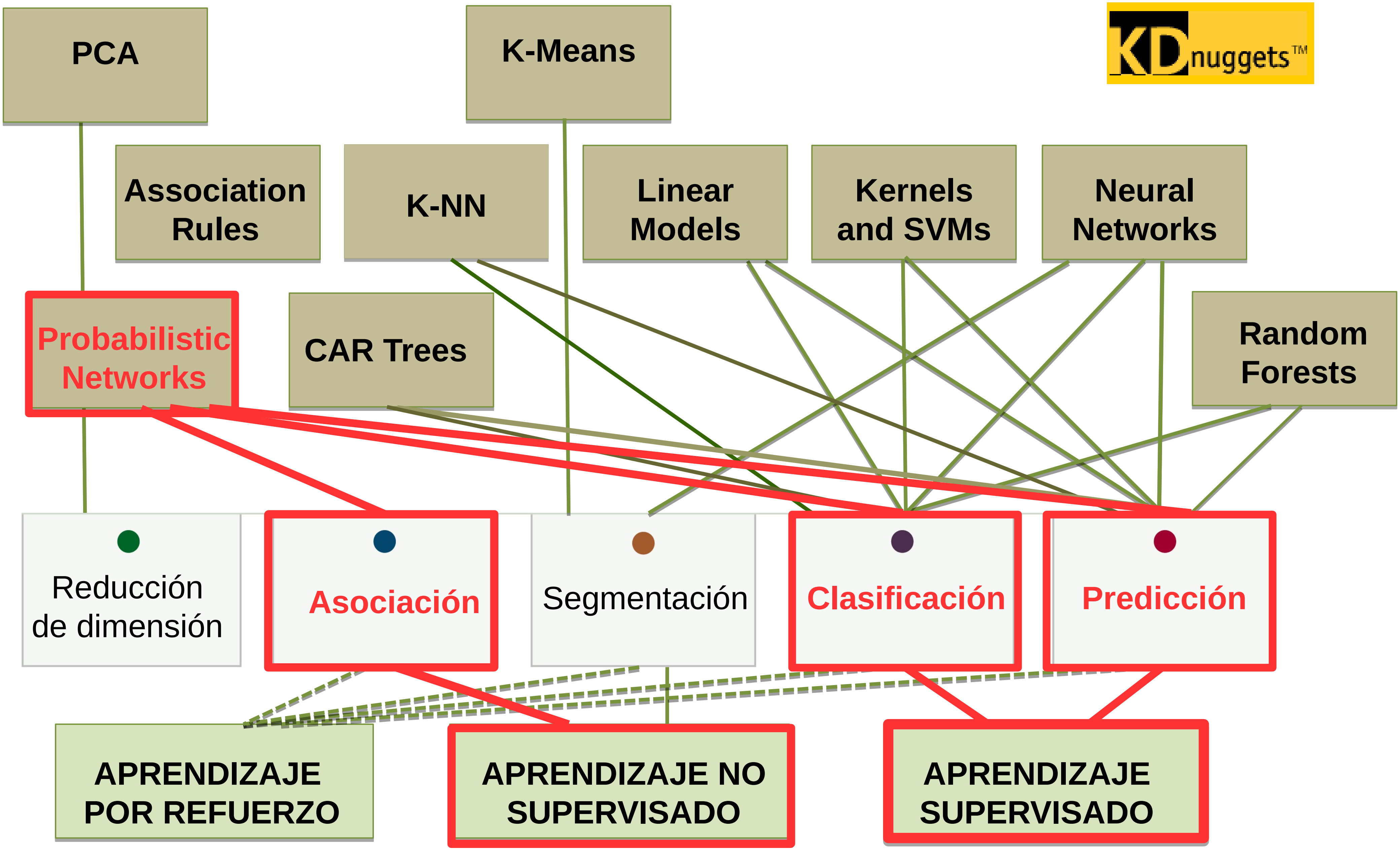
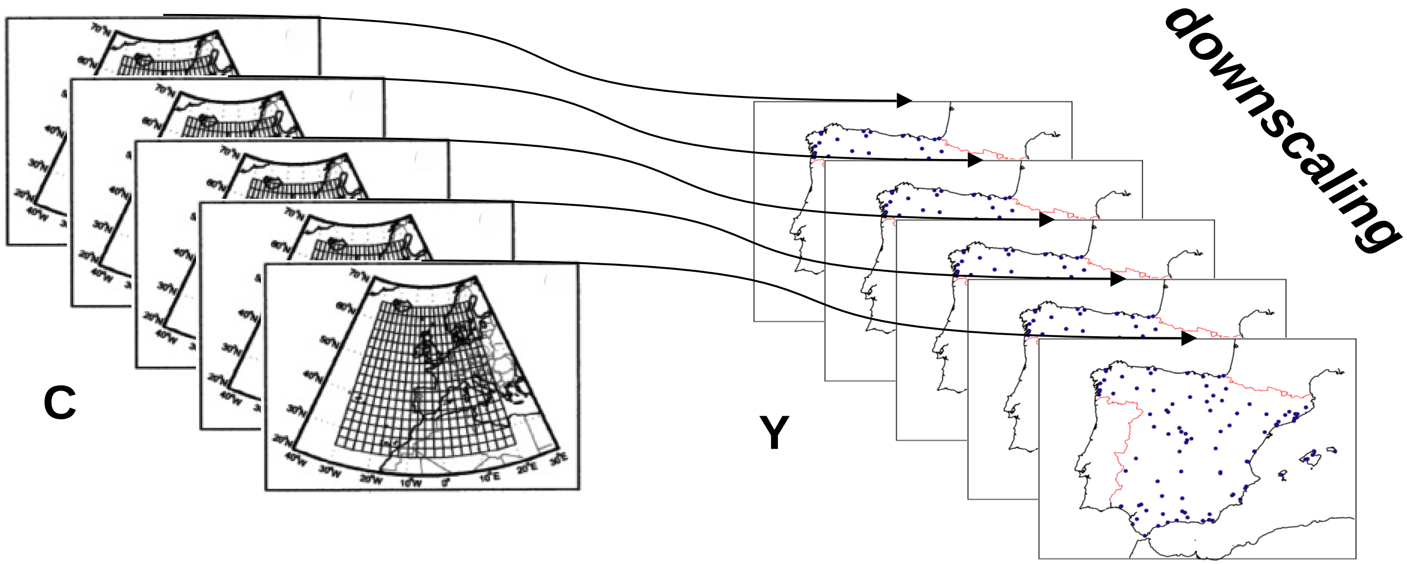


Clasificación
Predicción



$$P(\mathbf{y}|\mathbf{c}) = P(y_1, \ldots, y_n \mid c_1, \ldots, c_m)$$

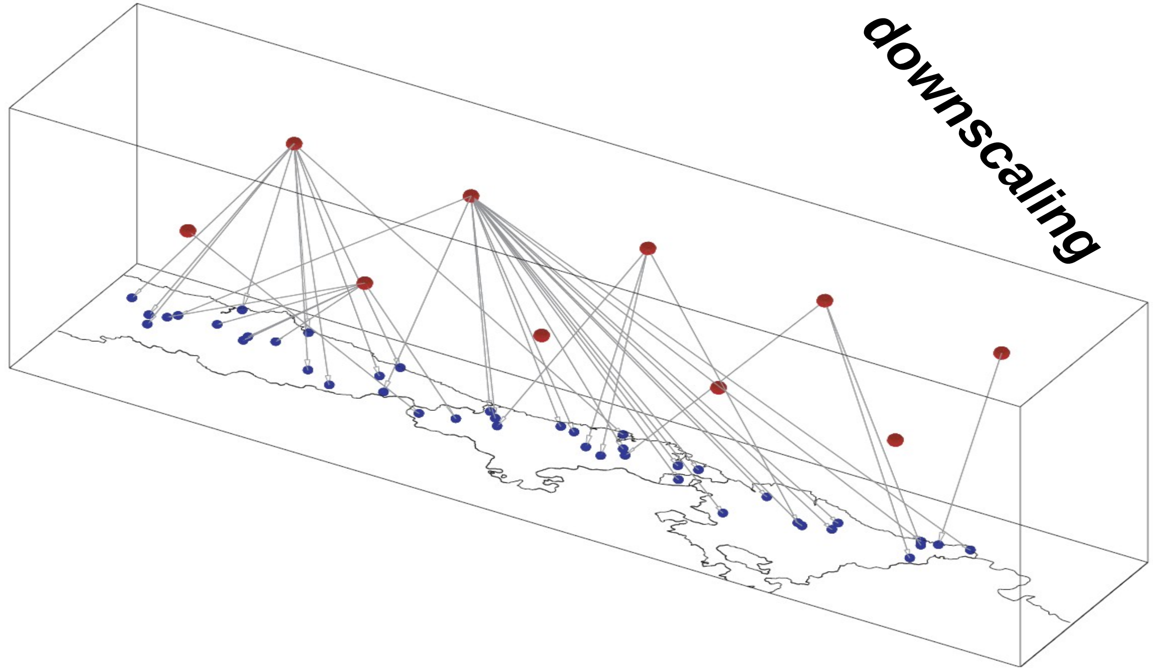
Probabilistic Networks

- Descripción y visualización
- **Asociación**
- Segmentación
- **Clasificación**
- **Predicción**

APRENDIZAJE POR REFUERZO

APRENDIZAJE NO SUPERVISADO

APRENDIZAJE SUPERVISADO



downscaling

Clasificación
Predicción
Asociación

Probabilistic Networks

Descripción y visualización

Asociación

Segmentación

Clasificación

Predicción

APRENDIZAJE POR REFUERZO

APRENDIZAJE NO SUPERVISADO

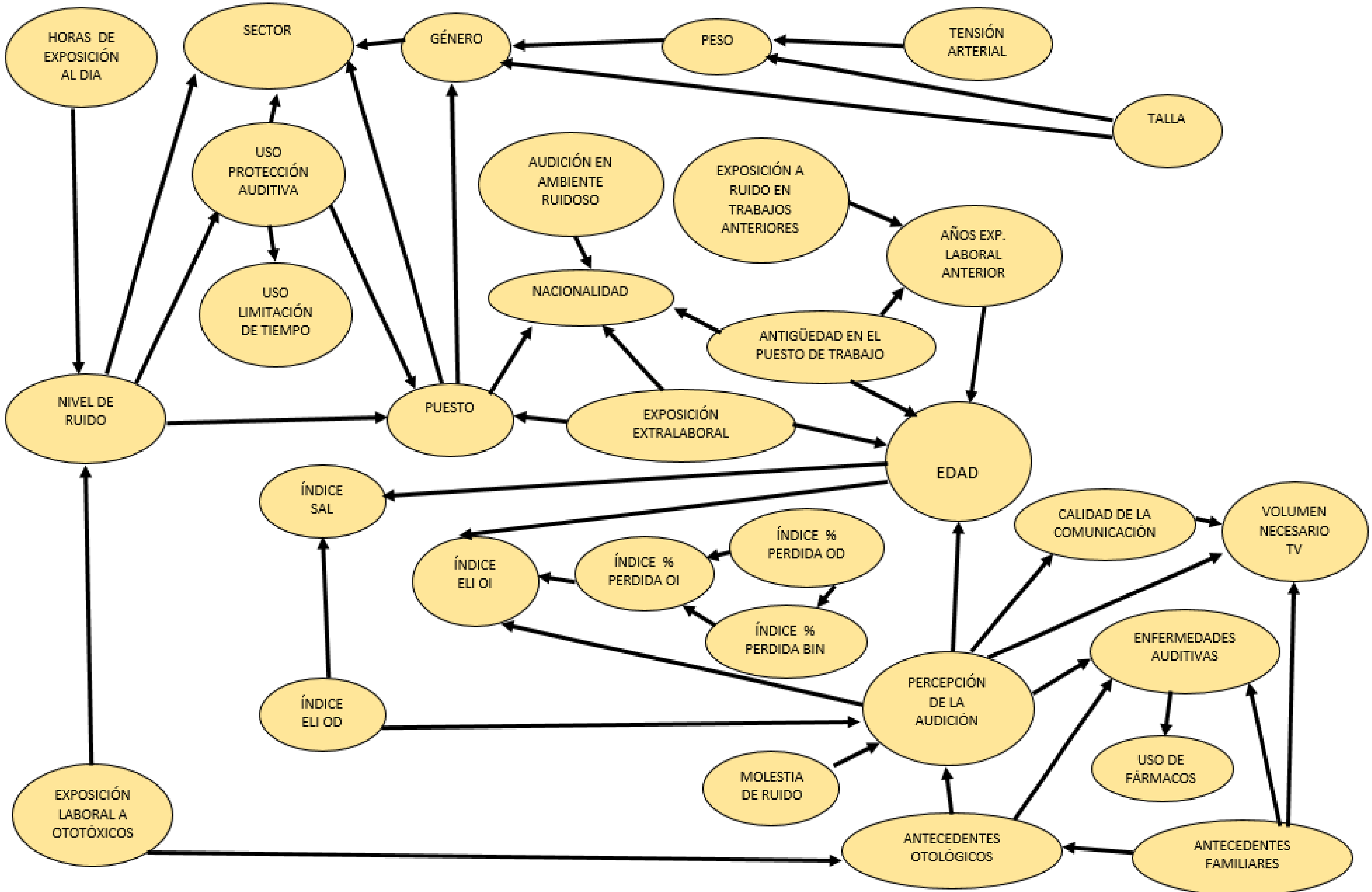
APRENDIZAJE SUPERVISADO

Work conditions & Health

Asociación

Probabilistic Networks

Descripción y visualización



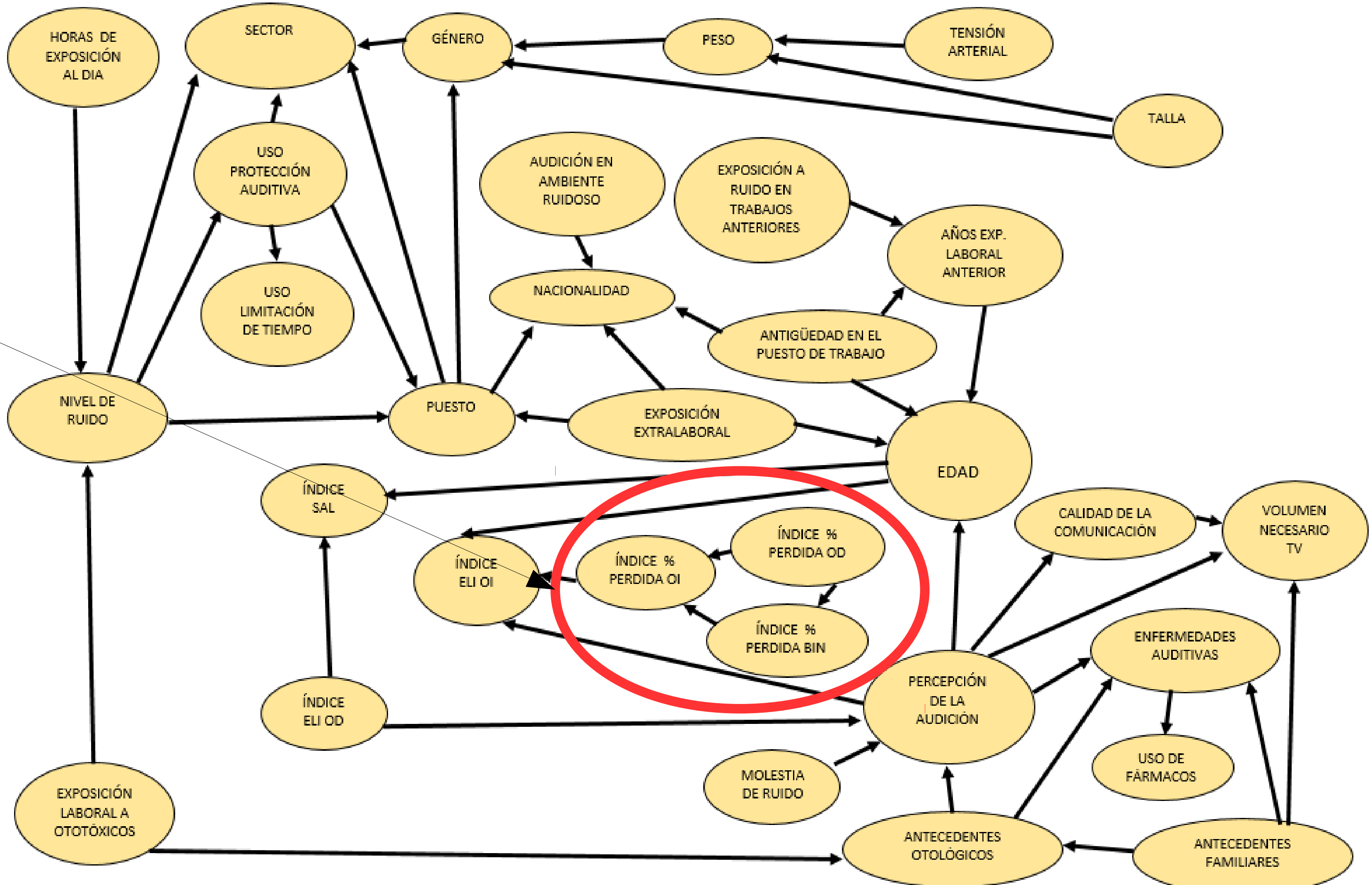
APRENDIZAJE POR REFUERZO

APRENDIZAJE NO SUPERVISADO

APRENDIZAJE SUPERVISADO

Work conditions & Health

Asociación
Clasificación



Probabilistic Networks

Descripción y visualización

APRENDIZAJE POR REFUERZO

APRENDIZAJE NO SUPERVISADO

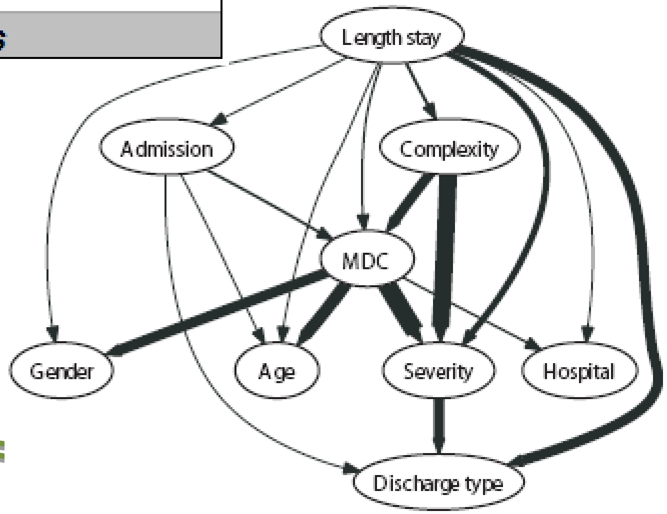
APRENDIZAJE SUPERVISADO

Healthcare management (ICMBD)

1. Estancia Media
2. Tasa de Mortalidad
3. Tasa de Reingresos (a 30 días)
4. Tasa de Infección Nosocomial
5. Estancia Media Preoperatoria
6. Tasa de Cesáreas
Conjunto de indicadores

RELACIONADAS CON LA ENFERMEDAD
1. Complejidad (medido a través del peso español de GRD-AP v 18)
2. Severidad (medido a través de GRD refinados)
3. Categoría Diagnostica Mayor de GRD-AP v 18
4. Tipo de GRD: medico, quirúrgico, indeterminado (Solo aplicado al indicador de infección nosocomial)
RELACIONADAS CON EL PACIENTE, O CON EL FUNCIONAMIENTO HOSPITALARIO
1. Edad
2. Sexo
3. Tipo de ingreso
4. Tipo de alta
5. Tipo de hospital
Variables de influencia propuestas para el análisis

Probabilistic Networks



Descripción y visualización

Asociación

Segmentación

Clasificación

Predicción

APRENDIZAJE POR REFUERZO

APRENDIZAJE NO SUPERVISADO

APRENDIZAJE SUPERVISADO

$$P: \mathbf{M} \longrightarrow [0,1]$$
$$A \longmapsto a$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
SE	64	57	24	18	6	4	1	9	33	26
SW	225	661	98	223	18	119	15	71	94	248
NW	288	825	49	150	95	277	108	251	36	147
Total	1591	2059	361	490	406	566	484	493	340	510

$$P(X)\in[0,1],\ X\subseteq M$$
$$P(\varnothing)=0 \wedge P(M)=1$$
$$P(X_1\cup X_2)=P(X_1)+P(X_2)-P(X_1\cap X_2)$$
$$P(X_1\cap X_2)=P(X_1)*P(X_2)\Rightarrow X_1\wedge X_2\, independent$$

```
## States of the variables:
estados.Wind < - c("NE","SE","SW","NW")
estados.Season < - c("Anual","Invierno","Primavera","Verano","Otono")
estados.Precip < - c("Seco","Lluvioso")

## Table of Absolute frequencies:
table.freq < - array(c(1014, 64, 225, 288, 190, 24, 98, 49, 287, 6, 18, 95, 360, 1, 15, 108,
                       177, 33, 94, 36, 516, 57, 661, 825, 99, 18, 223, 150,
                        166, 4, 119, 277, 162, 9, 71, 251, 89, 26, 248, 147), dim = c(4,5,2),
                    dimnames = list(W=estados.Wind, S=estados.Season, P = estados.Precip))
```

$$P: \mathbf{M} \longrightarrow [0,1]$$
$$A \longmapsto a$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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Total	1591	2059	361	490	406	566	484	493	340	510

$$P(X) \in [0,1], X \subseteq M$$
$$P(\emptyset) = 0 \wedge P(M) = 1$$
$$P(X_1 \cup X_2) = P(X_1) + P(X_2) - P(X_1 \cap X_2)$$
$$P(X_1 \cap X_2) = P(X_1) * P(X_2) \Rightarrow X_1 \wedge X_2 \, independent$$

$$X = \{Inv, \mathrm{Ll}, NW\} \Rightarrow P(X) = \frac{freq(Inv, \mathrm{Ll}, NW)}{N} = \frac{150}{3650} = 0.041$$

```
## States of the variables:
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estados.Precip < - c("Seco","Lluvioso")

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table.freq < - array(c(1014, 64, 225, 288, 190, 24, 98, 49, 287, 6, 18, 95, 360, 1, 15, 108,
                    177, 33, 94, 36, 516, 57, 661, 825, 99, 18, 223, 150,
                     166, 4, 119, 277, 162, 9, 71, 251, 89, 26, 248, 147), dim = c(4,5,2),
                dimnames = list(W=estados.Wind, S=estados.Season, P = estados.Precip))
## Obtain the probability:
table.freq["NW","Invierno","Lluvioso"]/sum(table.freq[,"Anual",])
```

P: **M** $\longrightarrow$ [0,1]

A $\longrightarrow$ a

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
SE	64	57	24	18	6	4	1	9	33	26
SW	225	661	98	223	18	119	15	71	94	248
NW	288	825	49	150	95	277	108	251	36	147
Total	1591	2059	361	490	406	566	484	493	340	510

$$P(X)\in[0,1], X\subseteq M$$

$$P(\emptyset)=0 \wedge P(M)=1$$

$$P(X_1\cup X_2)=P(X_1)+P(X_2)-P(X_1\cap X_2)$$

$$P(X_1\cap X_2)=P(X_1)*P(X_2)\Rightarrow X_1\wedge X_2\,independent$$

$$X=\{Inv,\mathrm{Ll},NW\}\Rightarrow P(X)=\frac{freq(Inv,\mathrm{Ll},NW)}{N}=\frac{150}{3650}=0.041$$

$$\boxed{X=\{Inv\}}\Rightarrow P(X)=\frac{freq(Inv)}{N}=\frac{\sum_{p\in Pr}\sum_{v\in Vi} freq(Inv,p,v)}{N}=0.233$$

```
## Obtain the probability:
sum(table.freq[,"Invierno",])/sum(table.freq[,"Anual",])
```

$$P: \mathbf{M} \longrightarrow [0,1]$$
$$A \longrightarrow a$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
SE	64	57	24	18	6	4	1	9	33	26
SW	225	661	98	223	18	119	15	71	94	248
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$$\boxed{X=\{NW\} \Rightarrow P(Y|X)=\frac{P(Y,X)}{P(X)}}$$

$$Y=\{Inv\} \Rightarrow P(Y|X)=\frac{freq(Y,X)}{freq(X)}=\frac{199}{1113}=0.179$$

P: **M** $\longrightarrow$ [0,1]

A $\longrightarrow$ a

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
SE	64	57	24	18	6	4	1	9	33	26
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$$\boxed{X=\{NW\}\Rightarrow P(Y|X)=\frac{P(Y,X)}{P(X)}} \Rightarrow Y=\{Inv\}\Rightarrow P(Y|X)=\frac{freq(Y,X)}{freq(X)}=\frac{199}{1113}=0.179$$

```
## New probability-space:
cond.table.freq <- table.freq["NW",,]
print(cond.table.freq)
```

P: **M** $\longrightarrow$ [0,1]

A $\longrightarrow$ a

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
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```
## Obtain the probability:
sum(cond.table.freq["Invierno",])/sum(cond.table.freq["Anual",])
```

$$P:\ \mathbf{M} \longrightarrow [0,1]$$
$$A \longmapsto a$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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Bayes' Theorem (Predictands vs. Predictors)

$$\{A \wedge B \subseteq M\} \Rightarrow P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$X = \{NW\} \Rightarrow P(Y|X) = \frac{P(Y,X)}{P(X)}$$

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$$P: \mathbf{M} \longrightarrow [0,1]$$
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	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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Bayes' Theorem (Predictands vs. Predictors)

Probability "*a priori*"

$$\{A \wedge B \subseteq M\} \Rightarrow \boxed{P(A|B)} = \frac{P(B|A)\boxed{P(A)}}{P(B)}$$

Probability "*a posteriori*"

$$X = \{NW\} \Rightarrow P(Y|X) = \frac{P(Y,X)}{P(X)}$$

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$$P: \mathbf{M} \longrightarrow [0,1]$$
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	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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Bayes' Theorem (Predictands vs. Predictors) **Verosimilitud** **Probability "*a priori*"**

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Probability "*a posteriori*"

$$X=\{NW\}\Rightarrow P(Y|X)=\frac{P(Y,X)}{P(X)} \quad \Rightarrow \quad Y=\{Inv\}\Rightarrow P(Y|X)=\frac{freq(Y,X)}{freq(X)}=\frac{199}{1113}=0.179$$

$$P: \mathbf{M} \longrightarrow [0,1]$$
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	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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$$\{X_1, \ldots, X_n : X_1 \cup \ldots \cup X_n = M \wedge X_i \cap X_j = \emptyset \; \forall i \neq j\} \Rightarrow P(X_i|B)=\frac{P(B|X_i)P(X_i)}{P(B)}$$

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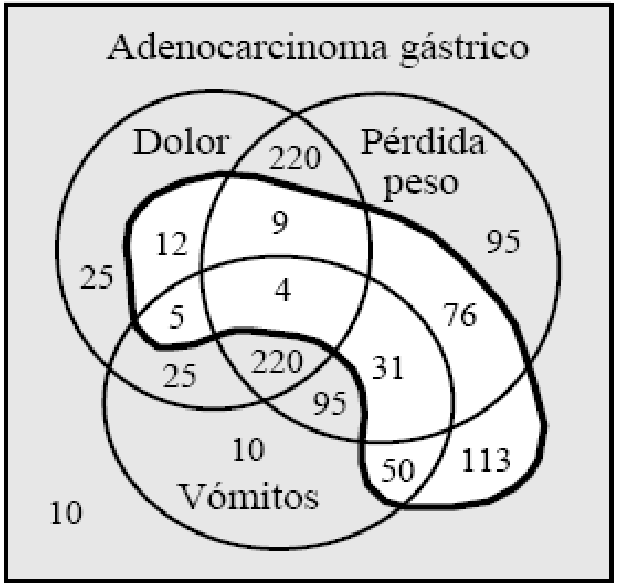


Initial Probabilities:

Gray → Adenocarcinoma

White → Not Adenocarcinoma

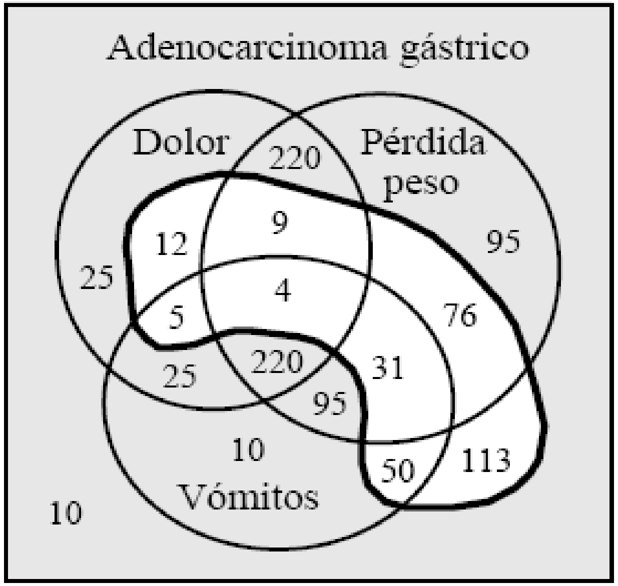
$$P(g) = \frac{700}{700+300} = \frac{700}{1000} = 0.7$$

$$P(\neg g) = 1 - P(g) = 1 - 0.7 = 0.3$$

Could we predict the probability of a disease based on the symptoms?

Bayes' Theorem (Predictands vs. Predictors), Factorization, etc.

$$\{X_1, \ldots, X_n : X_1 \cup \ldots \cup X_n = M \wedge X_i \cap X_j = \emptyset \; \forall i \neq j\} \Rightarrow P(X_i | B) = \frac{P(B|X_i)P(X_i)}{\sum_{j=1}^{n} P(B|X_j)P(X_j)}$$



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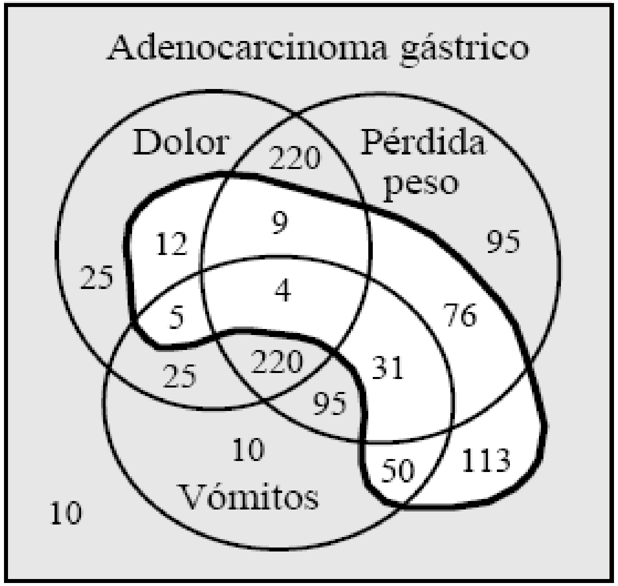
Patient has suffered threw up:

$$\{V=v\}\Rightarrow P(g|v)=\frac{P(g)P(v|g)}{P(g)P(v|g)+P(\neg g)P(v|\neg g)}=\frac{0.7*0.5}{0.7*0.5+0.3*0.3}=0.795$$



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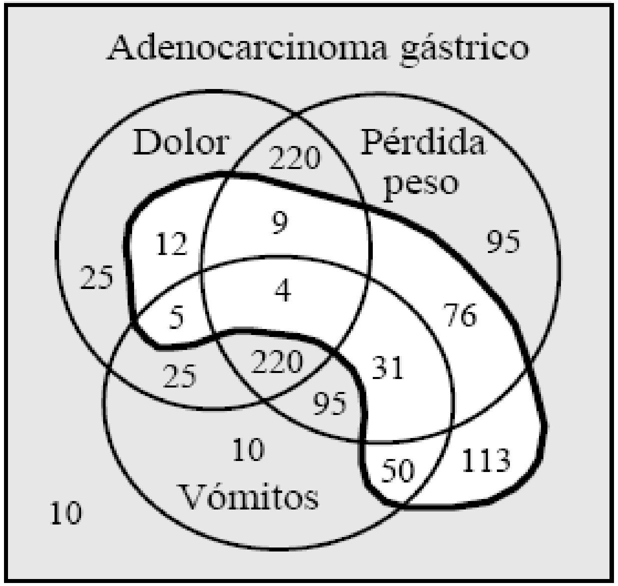
Patient has suffered of weight loss and threw up:

$$\{P=p\wedge V=v\}\Rightarrow P(g|v,p)=\frac{P(g)P(v,p|g)}{P(g)P(v,p|g)+P(\neg g)P(v,p|\neg g)}=\frac{0.7*0.45}{0.7*0.45+0.3*0.12}=0.9$$



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Significant changes in the probabilities reflect the dependence between predictand and predictors.

→ **Predictability**

Could we predict the probability of a disease based on the symptoms?

Patient has suffered threw up:

$$\{V=v\}\Rightarrow P(g|v)=\frac{P(g)P(v|g)}{P(g)P(v|g)+P(\neg g)P(v|\neg g)}=\frac{0.7*0.5}{0.7*0.5+0.3*0.3}=\boxed{0.795}$$

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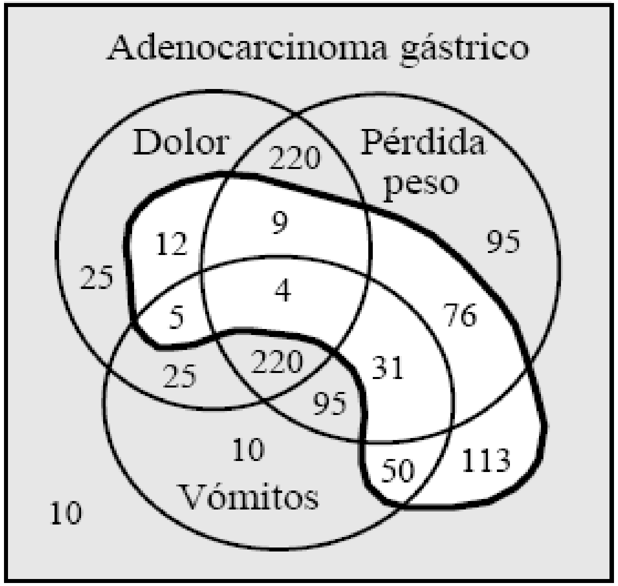

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Significant changes in the probabilities reflect the dependence between predictand and predictors.

↓

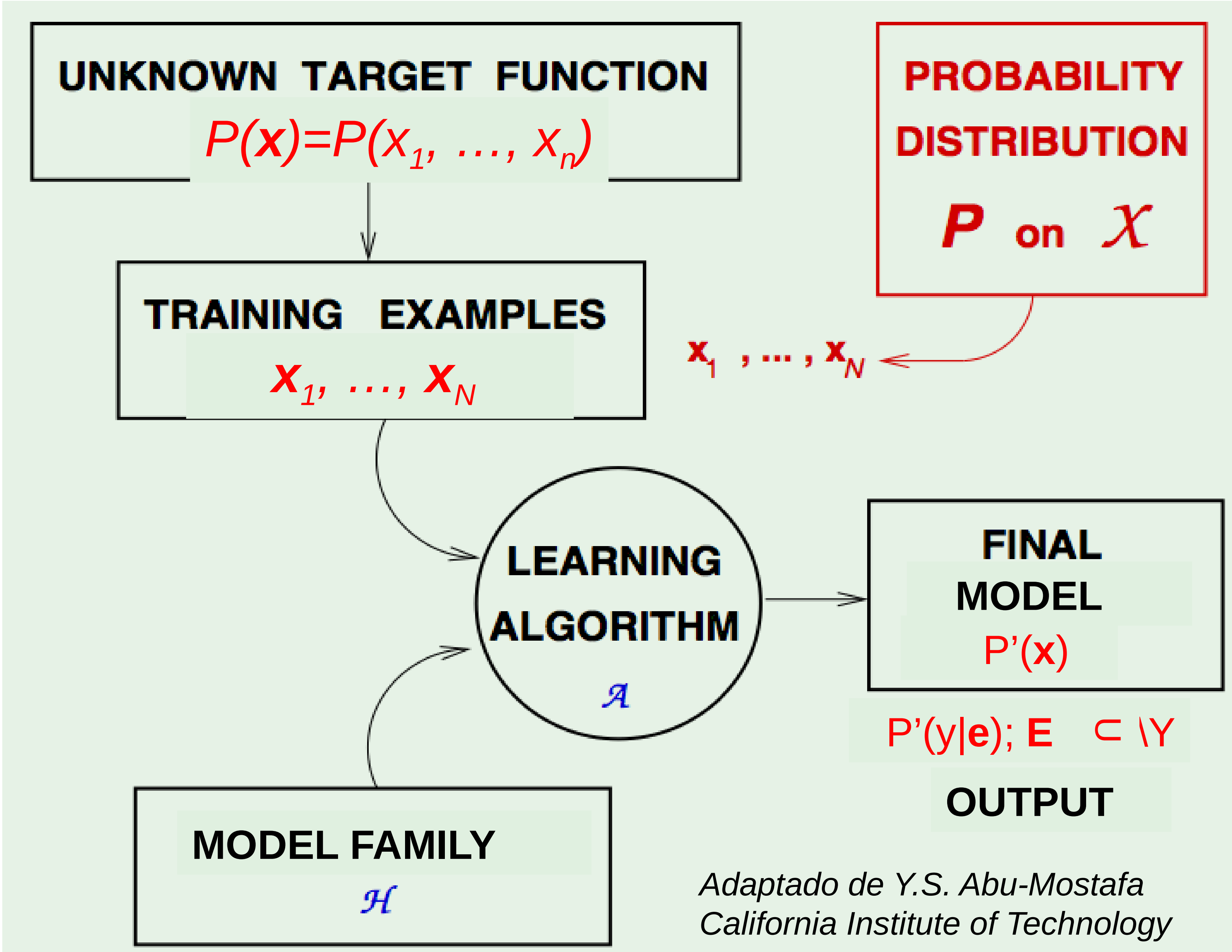
Predictability

Hypothesis Testing to Compare Two Population Proportions

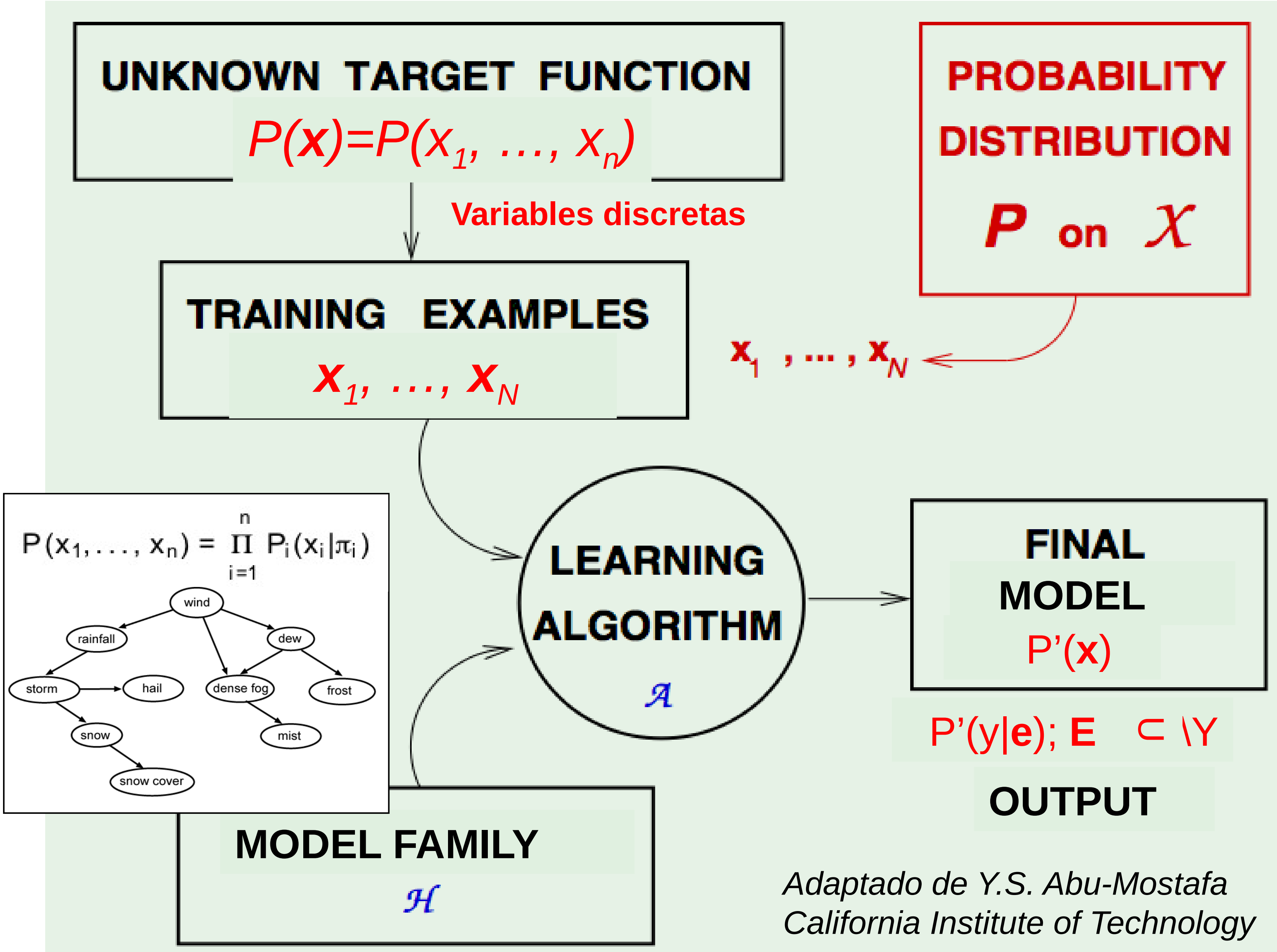
$$Z=\frac{p_1-p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$p=\frac{x_1+x_2}{n_1+n_2}$$

⇨ $If\ Z>N^{-1}_{(0,1)}(\alpha)\Rightarrow p_1\neq p_2$


UNKNOWN TARGET FUNCTION
$P(\mathbf{x})=P(x_1, \ldots, x_n)$
PROBABILITY DISTRIBUTION
$\mathbf{P}$ on $\mathcal{X}$
TRAINING EXAMPLES
$\mathbf{x}_1, \ldots, \mathbf{x}_N$
$\mathbf{x}_1, \ldots, \mathbf{x}_N$
LEARNING ALGORITHM
$\mathcal{A}$
FINAL MODEL
P'(**x**)
P'(y|**e**); **E** ⊂ \Y
OUTPUT
MODEL FAMILY
$\mathcal{H}$
Adaptado de Y.S. Abu-Mostafa
California Institute of Technology

Graphical Probabilistic Models



x	y	z	$p(x,y,z)$
0	0	0	0.12
0	0	1	0.18
0	1	0	0.04
0	1	1	0.16
1	0	0	0.09
1	0	1	0.21
1	1	0	0.02
1	1	1	0.18

To explicitly define the joint probability function is not possible in most of the real cases due to the large amount of parameters (e.g. 10^{25} parameters for 100 variables).

x	y	z	$p(x,y,z)$
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Bayes's Theorem → Factorization

$$P(X_i|B)=\frac{P(B|X_i)\,P(X_i)}{\sum_{j=1}^{n} P(B|X_j)\,P(X_j)}$$

$$\underbrace{B \wedge X_i\ independent \Rightarrow P(X_i|B)=P(X_i) \wedge P(B|X_i)=P(B)}$$

.

Reduction of parameters

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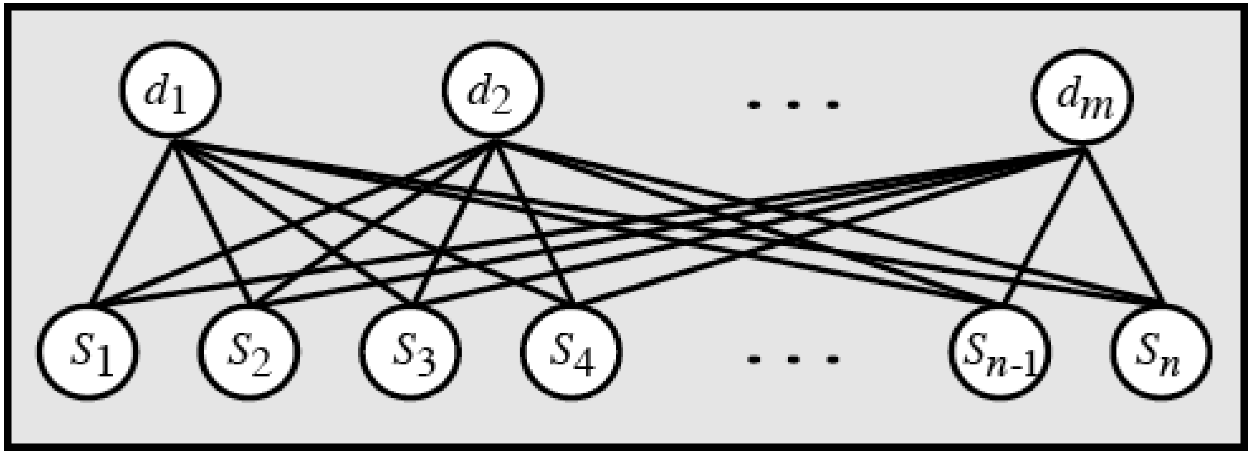
.

Reduction of parameters

Can we build a model including some pre-defined independences?

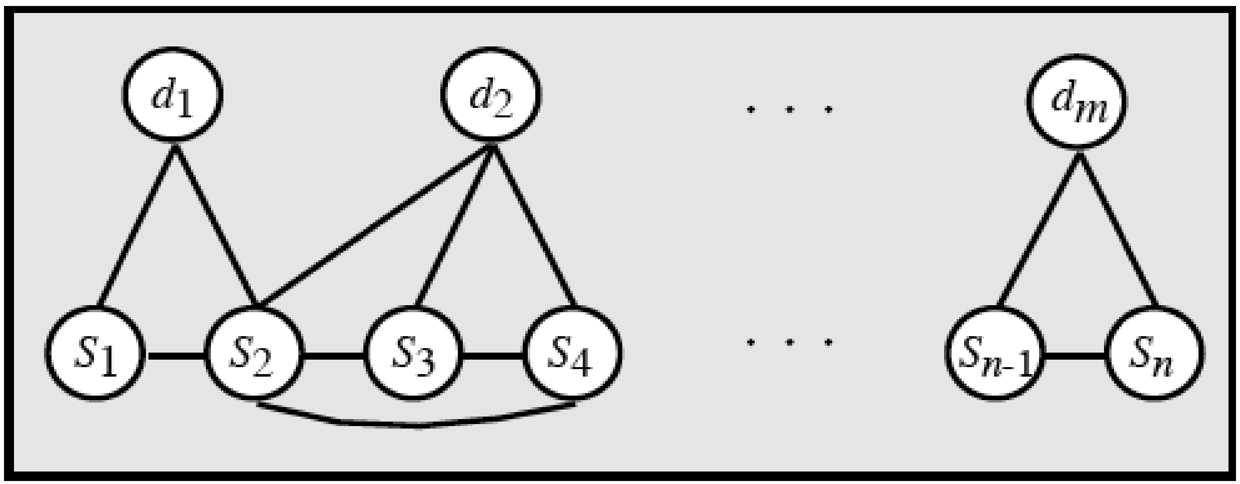
Firstly, unrealistic models “*ad-hoc*” were proposed.

$$P(s_1, \ldots, s_n, d_1, \ldots, d_m) = P(s_1, \ldots, s_n | d_1, \ldots, d_m)\, P(d_1, \ldots, d_m)$$



Independent symptoms model → Independent symptoms given a disease

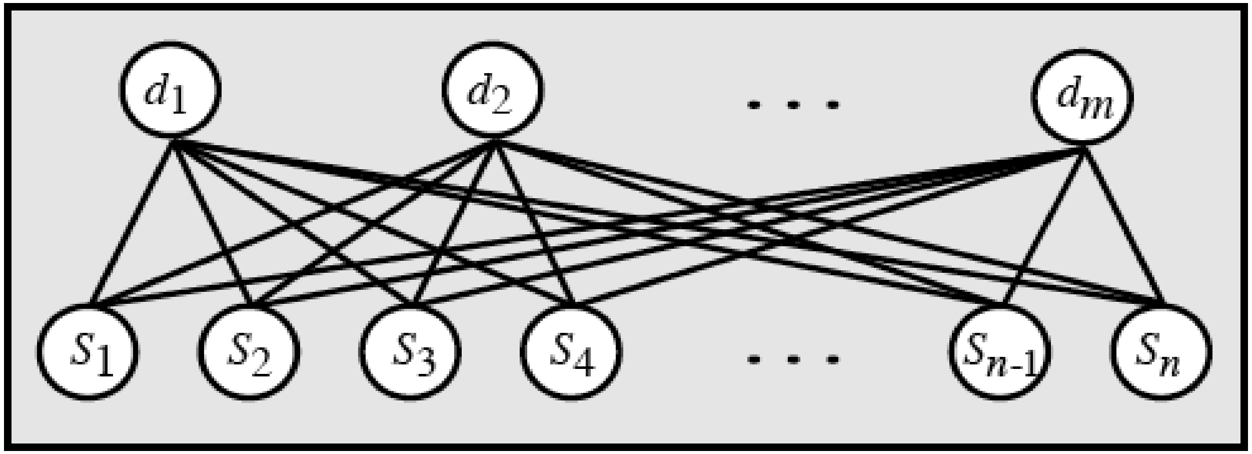
$$p(s_1, \ldots, s_n | d_i) = \prod_{j=1}^{n} p(s_j | d_i).$$



Syndrome model → for each disease there are a relevant subset of dependent symptoms.

Firstly, unrealistic models “*ad-hoc*” were proposed.

$$P(s_1, \ldots, s_n, d_1, \ldots, d_m) = P(s_1, \ldots, s_n | d_1, \ldots, d_m)\, P(d_1, \ldots, d_m)$$



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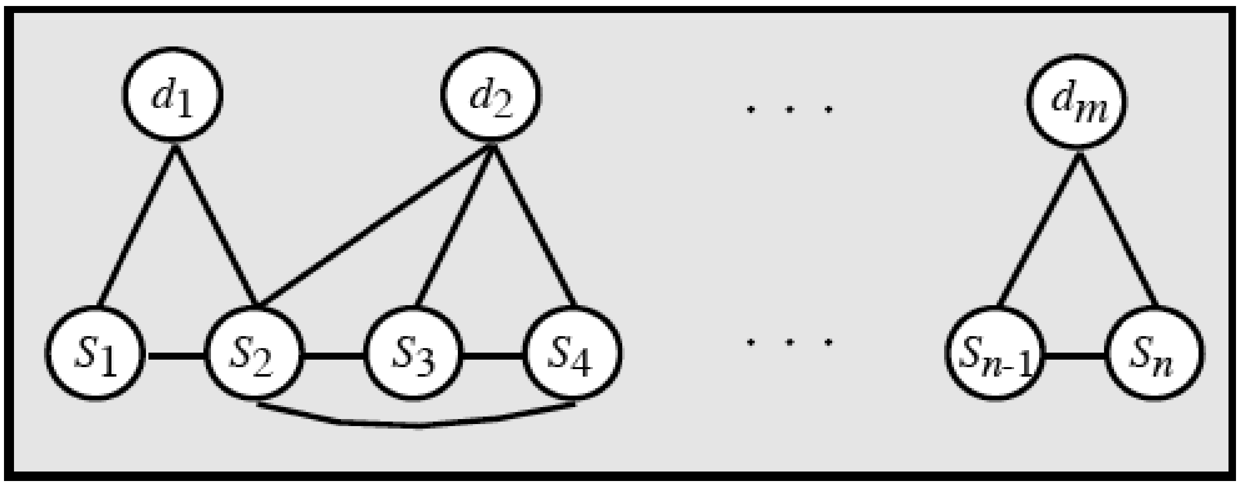


Is there any method to objectively define dependences between the variables and reduce the number of parameters?

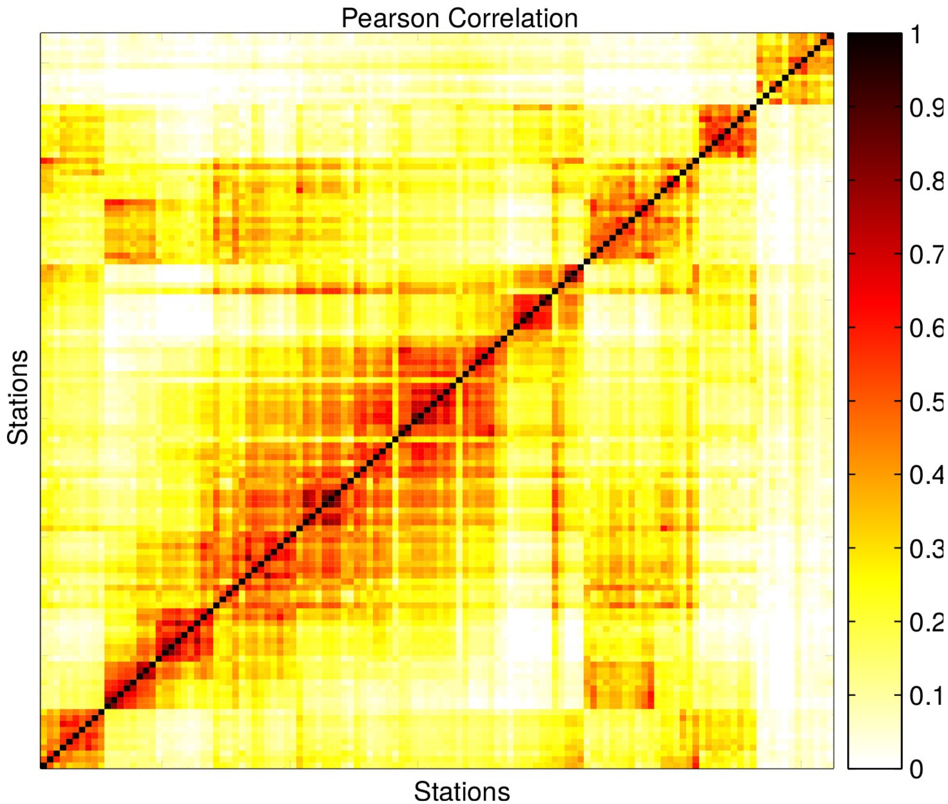
Subjective/ad-hoc approach
Large amount of parameters

Modelo	Número de parámetros	
	Fórmula	Valor
DSM	$m2^n - 1$	$> 10^{62}$
ISM	$m(n+1) - 1$	20,099
IRSM	$m(r+1) + n - 1$	1,299
DRSM	$m2^r + n - 1$	102,599

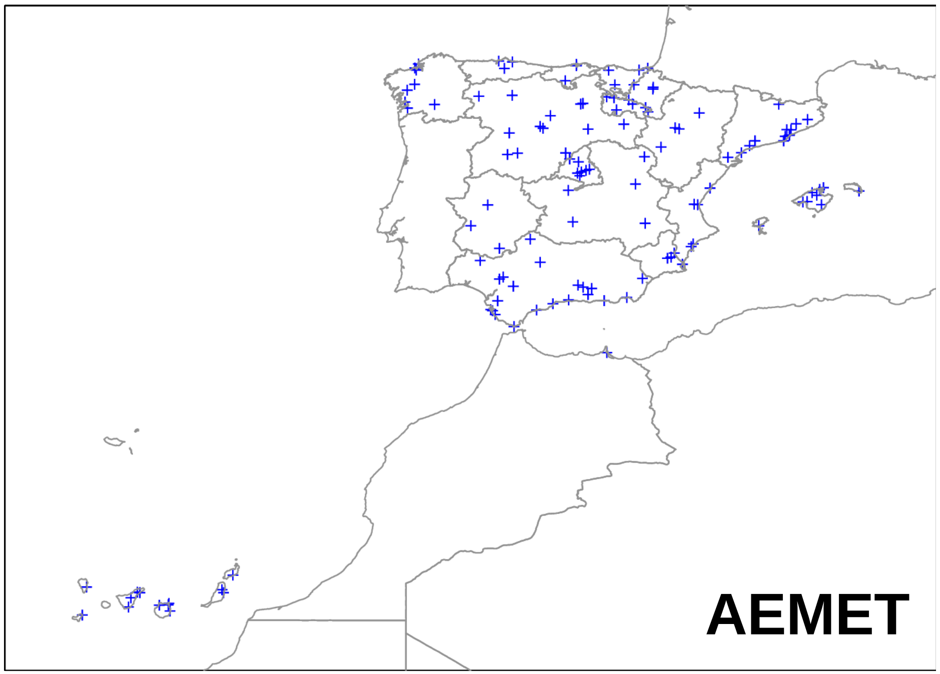
Information theory has defined several measures to evaluate de dependence between two variables:

Pearson Correlation

$$r(X,Y)=\frac{\sigma_{xy}}{\sigma_x \sigma_y}$$



Precip. Occurrence



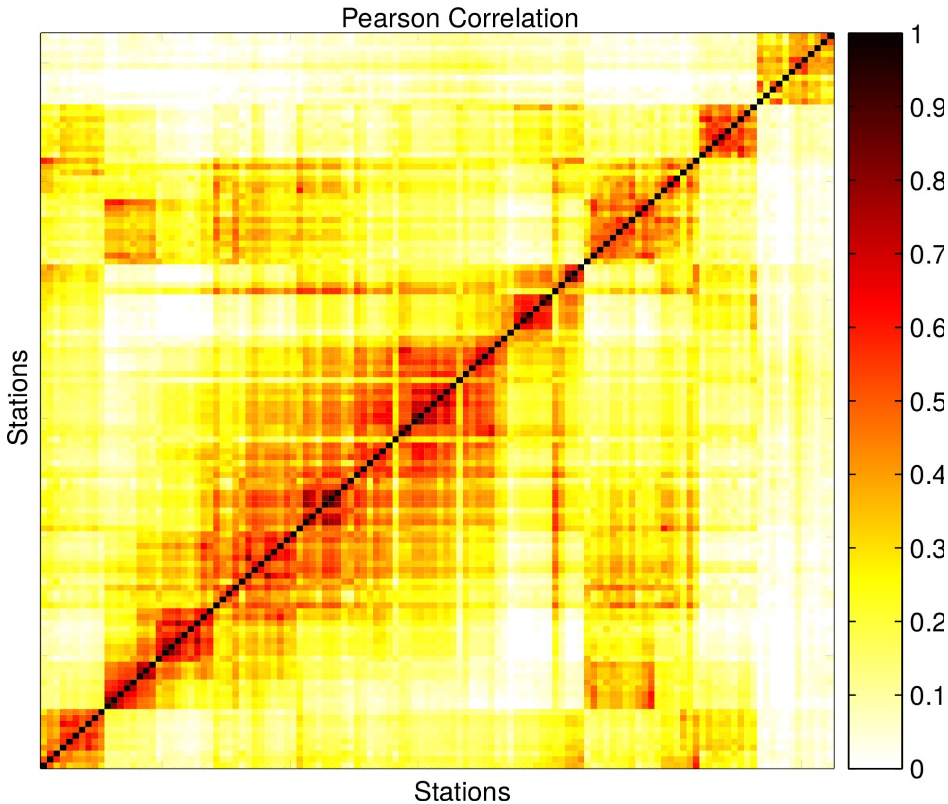
Information theory has defined several measures to evaluate de dependence between two variables:

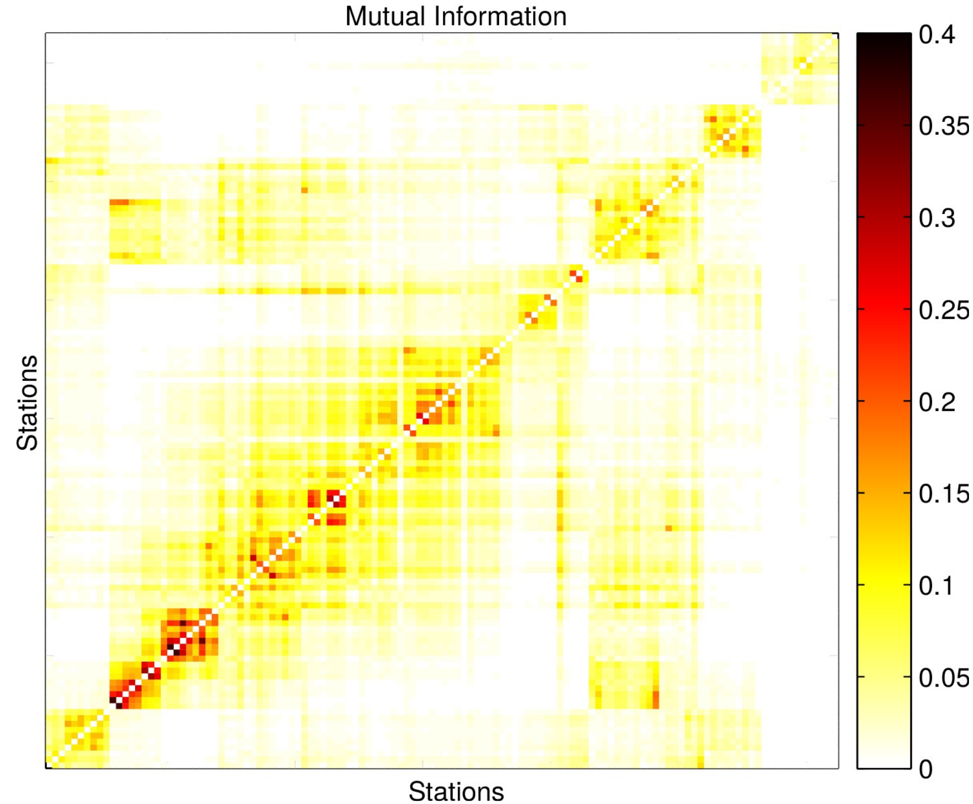
Pearson Correlation

$$r(X,Y)=\frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Mutual Information

$$I(X,Y)=\sum_{x\in X}\sum_{y\in Y}P(x,y)\log_b\left(\frac{P(x,y)}{(P(x)P(y))}\right)$$





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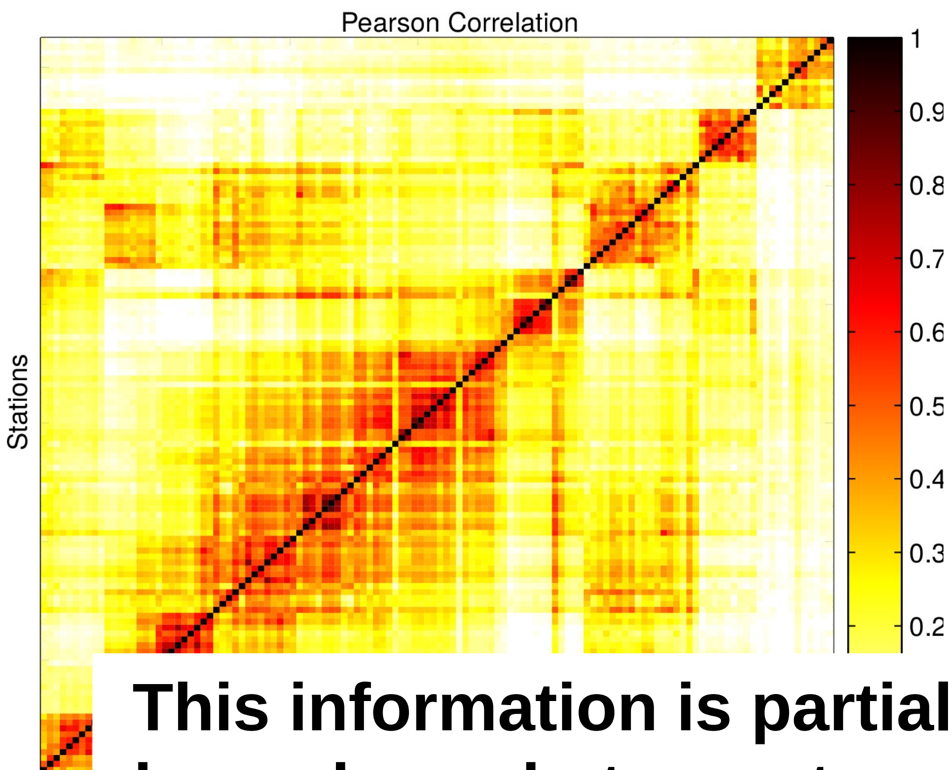
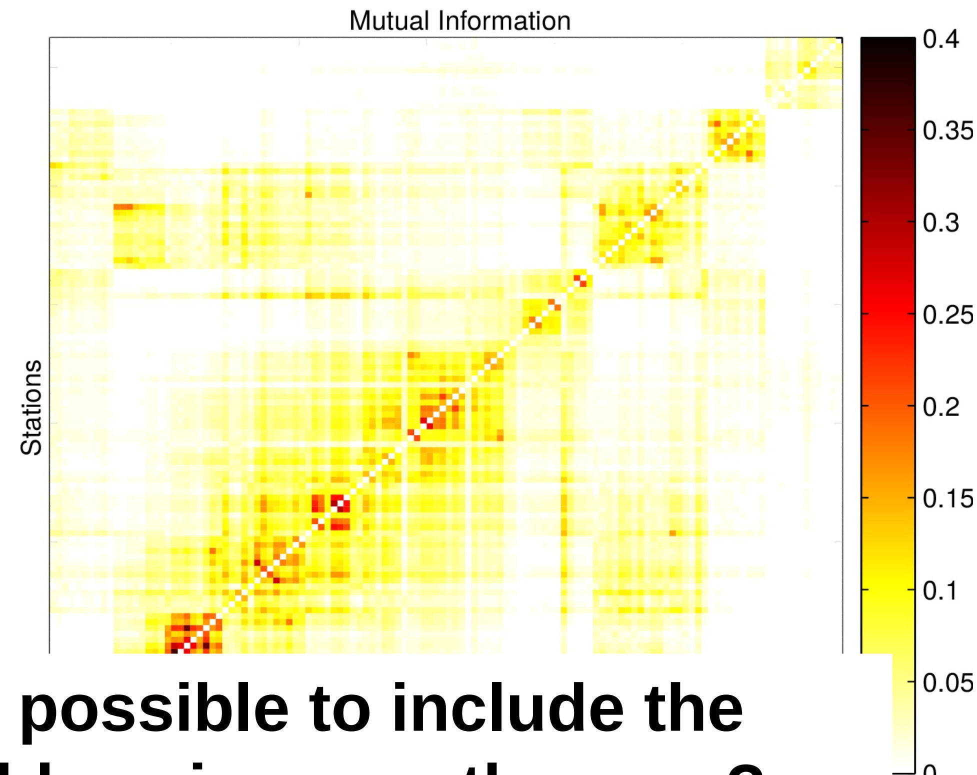




This information is partial, is it possible to include the dependence between two variables given another one?

Information theory has defined several measures to evaluate de dependence between two variables:

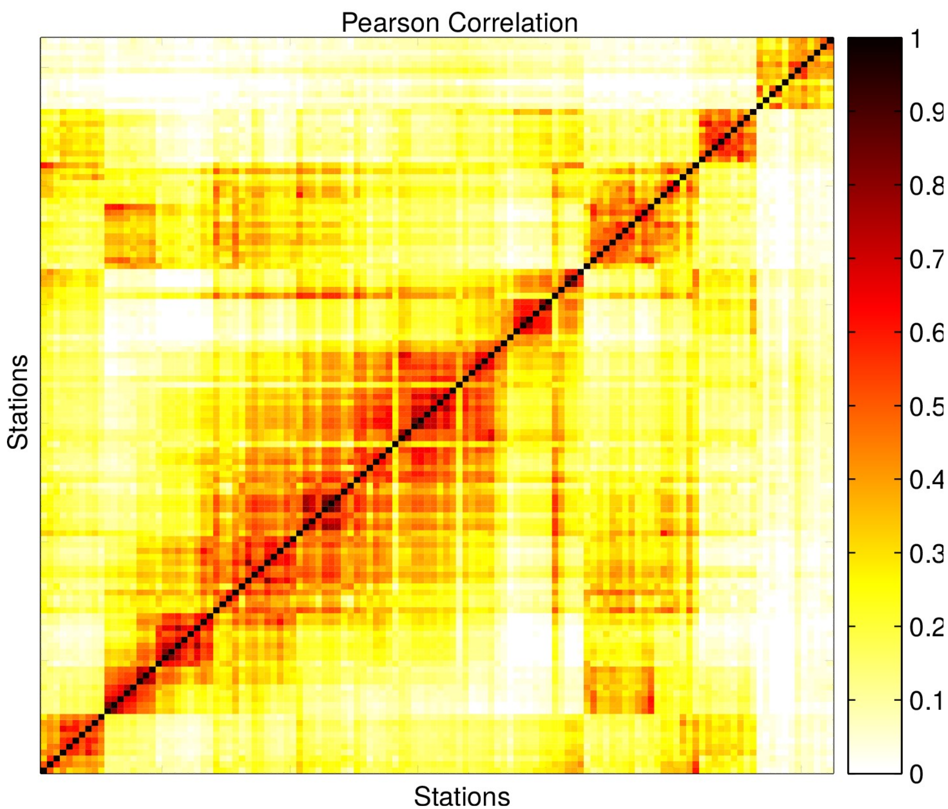
Pearson Correlation

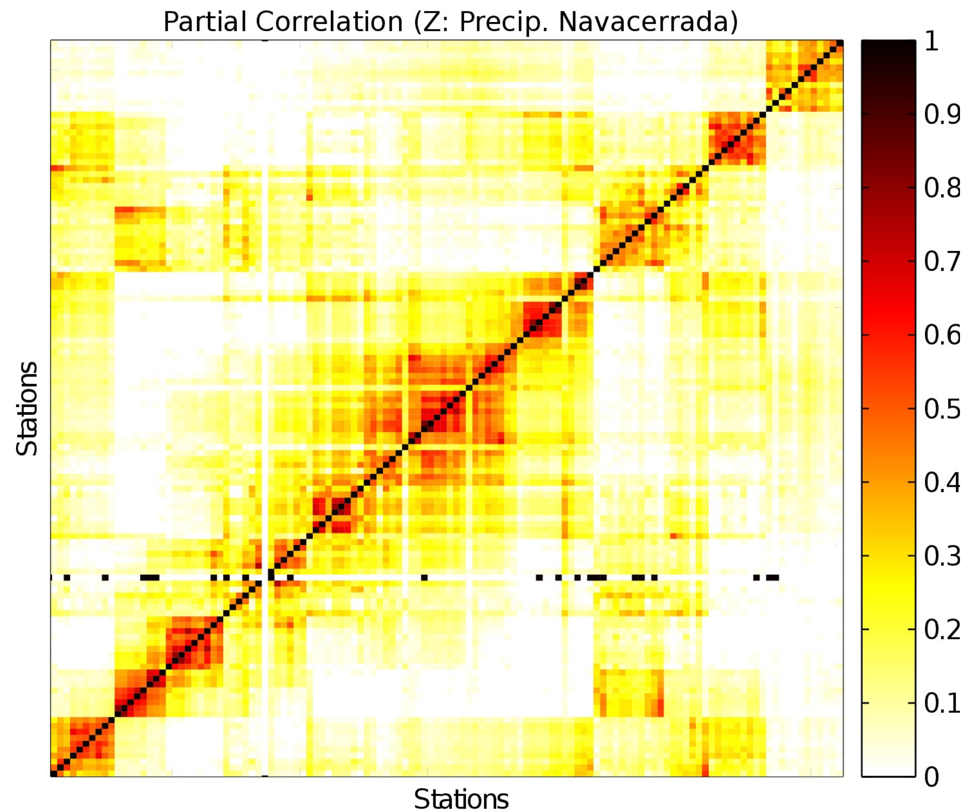
$$r(X,Y)=\frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Partial Correlation

$$r(X,Y|Z)=\frac{r(X,Y)-r(X,Z)*r(X,Z)}{\sqrt{1-r(X,Z)^2}*\sqrt{1-r(Y,Z)^2}}$$

Recursively applied to Z involving more than one variable





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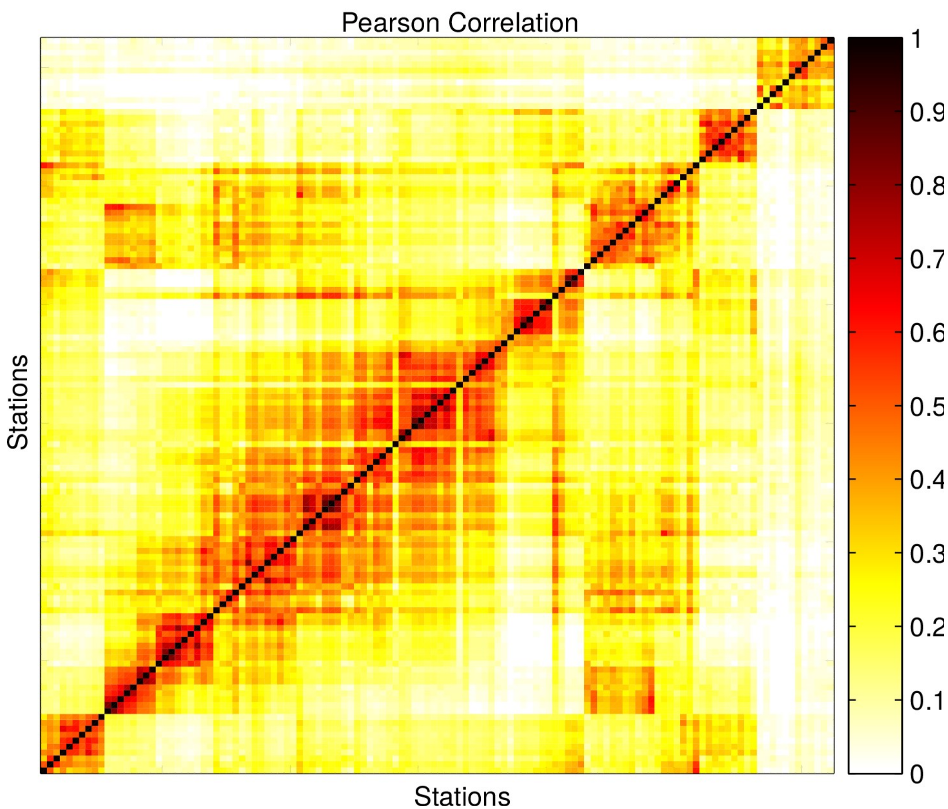
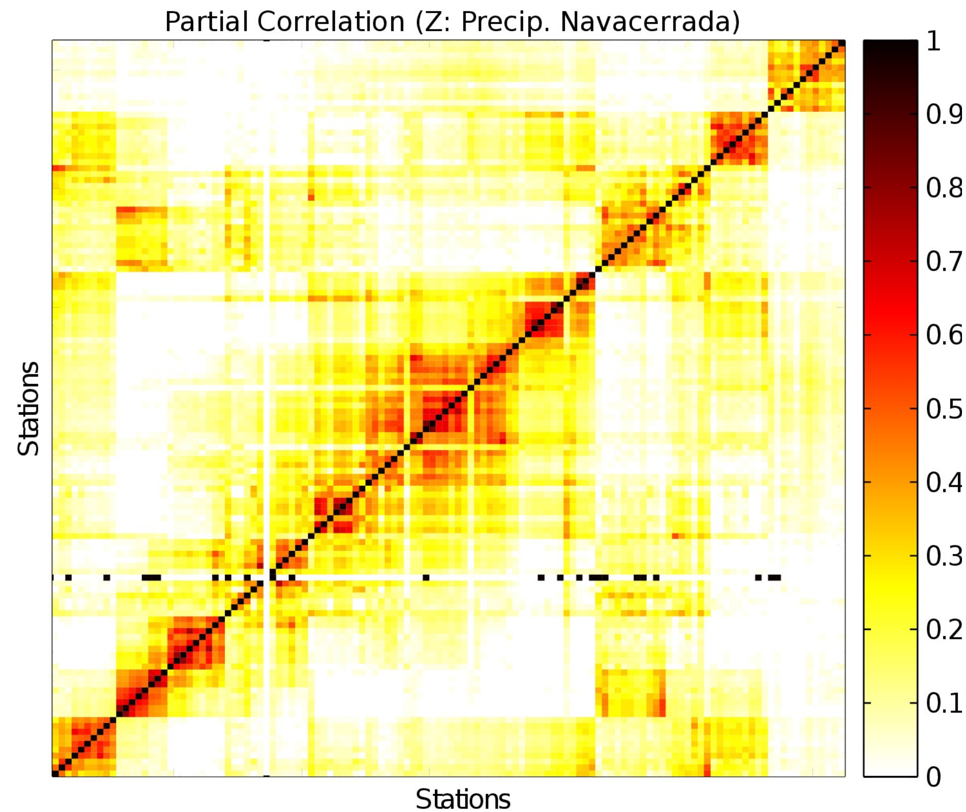
How to condition to a particular event (e.g. pr > 1mm in Navacerrada)?





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Filtering by the occurrence of the target event

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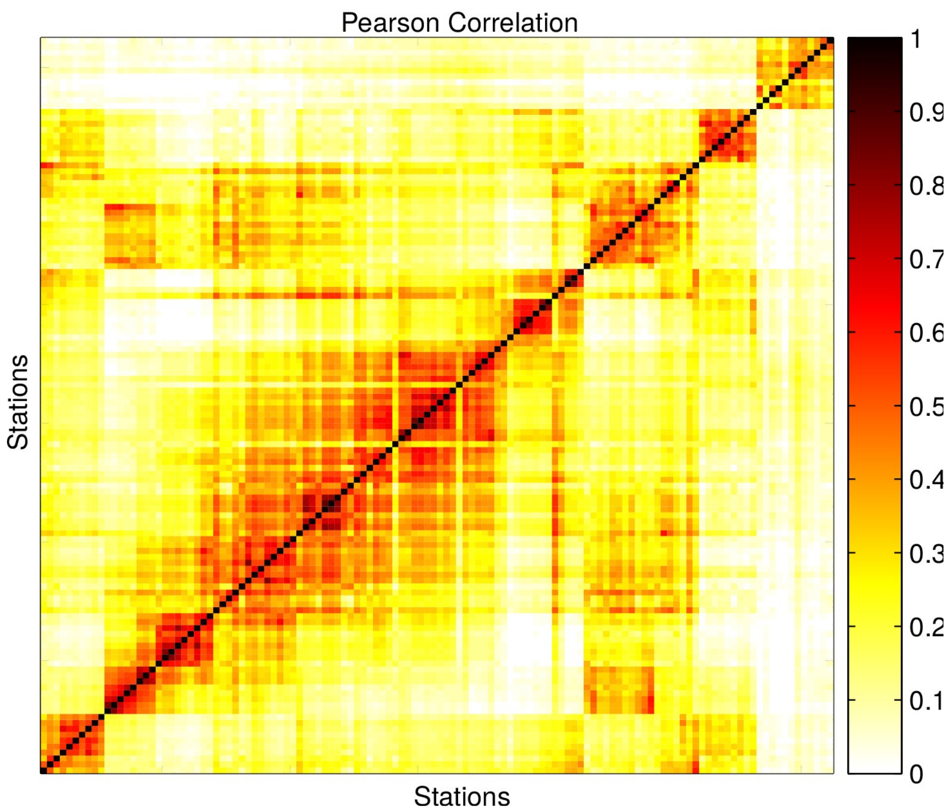

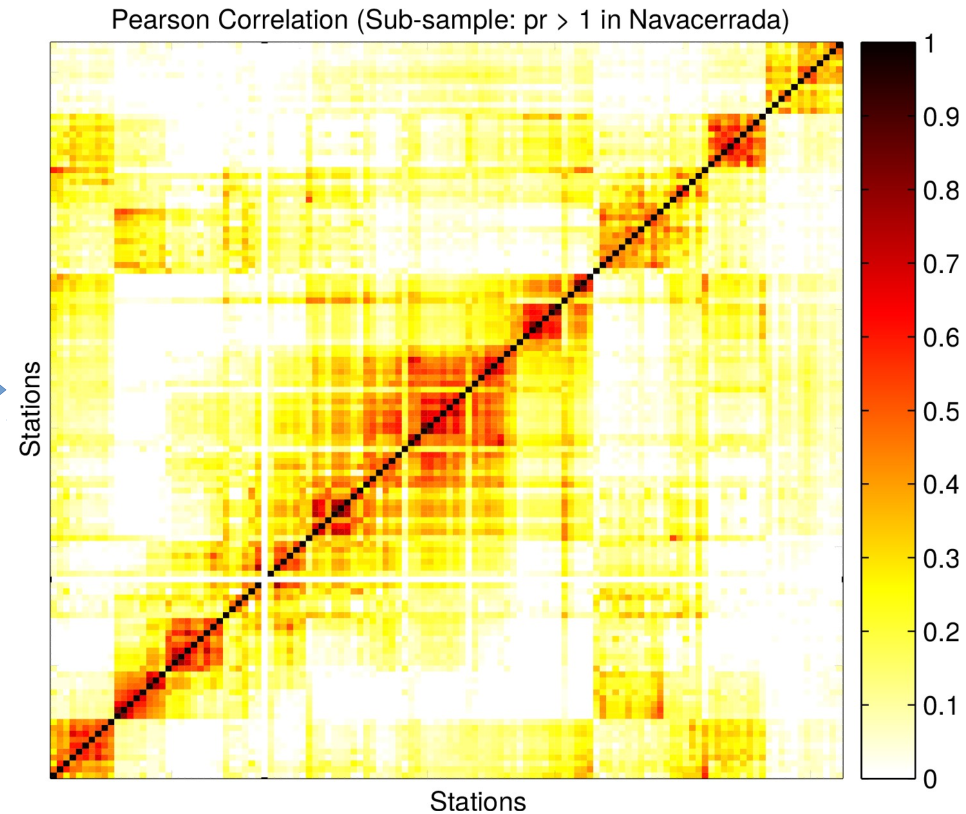

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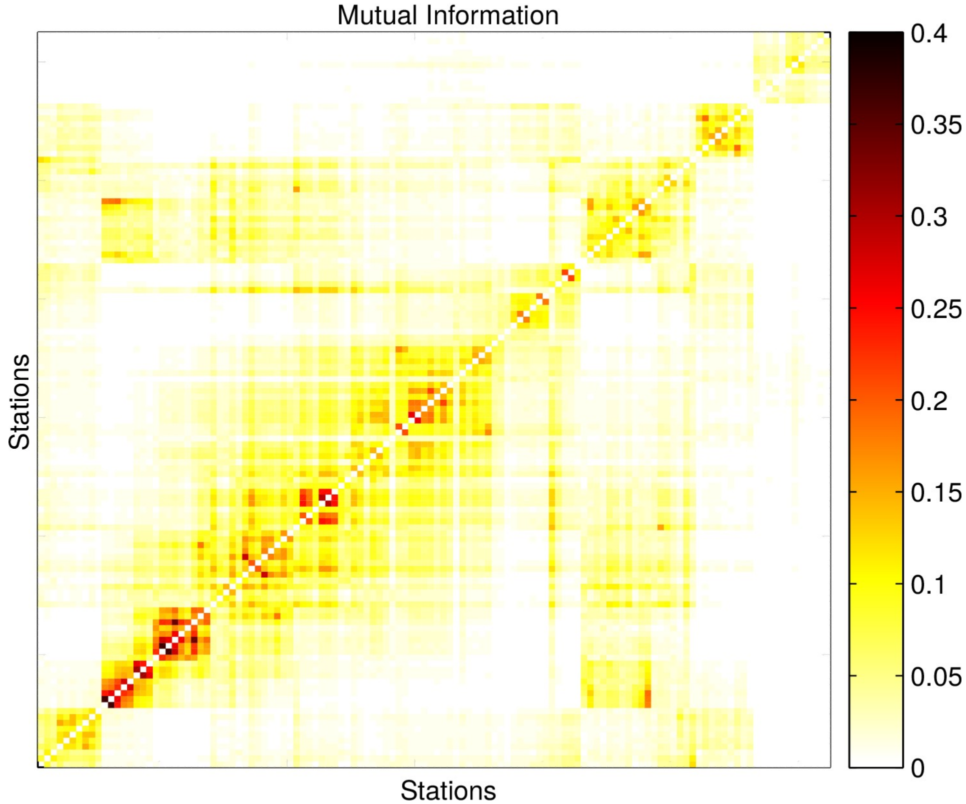

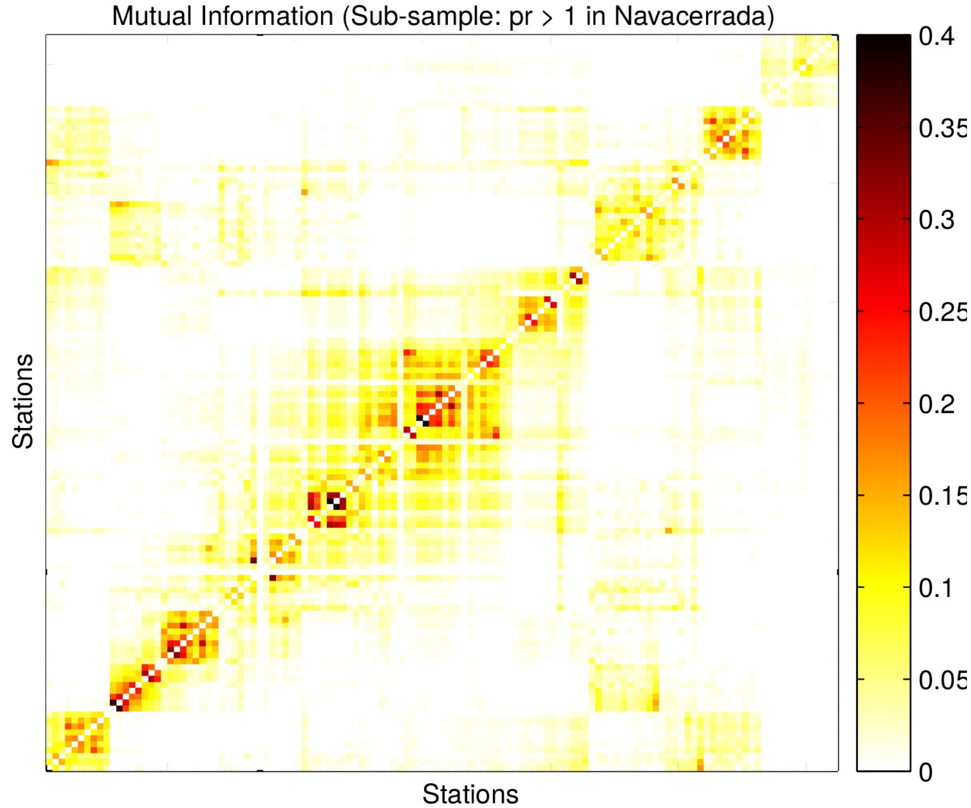

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Filtering by the occurrence of the target event

Remember the definition of condioned probability

$$X=\{NW\}\Rightarrow P(Y|X)=\frac{P(Y,X)}{P(X)}$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
SE	64	57	24	18	6	4	1	9	33	26
SW	225	661	98	223	18	119	15	71	94	248
NW	288	825	49	150	95	277	108	251	36	147
Total	1591	2059	361	490	406	566	484	493	340	510

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This information is partial and does not include the dependence between two variables given another one!!!!

A definition of conditional (in)dependence surges naturally from the conditional probability definition

$$P_Z(Y|X)=P(Y|X,Z)=P(Y|Z)=P_Z(Y)\Rightarrow I(X,Y|Z)$$

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	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
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Total	1591	2059	361	490	406	566	484	493	340	510

P (Ll / Primavera) = ?
P (Ll / Invierno) = ?

P (Ll) = ?

```
## States of the variables:
estados.Moon <- c("Anual","Llena","Menguante","Creciente","Nueva")

## Table of Absolute frequencies:
table2.freq <- array(c(1014, 64, 225, 288, 255, 12, 59, 51, 208, 16, 65, 77,
                    297, 22, 58, 82, 254, 14, 43, 78, 516, 57, 661, 825, 137, 12, 165, 192,
                    106, 16, 166, 231, 132, 12, 175, 225, 141, 17, 155, 177), dim = c(4,5,2),
          dimnames = list(W=estados.Wind, S=estados.Moon, P = estados.Precip))

## Obtain the probabilities:
```

	Anual		Llena		C. Menguante		C. Creciente		Nueva	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	255	137	208	106	297	132	254	141
SE	64	57	12	12	16	16	22	12	14	17
SW	225	661	59	165	65	166	58	175	43	155
NW	288	825	51	192	77	231	82	225	78	177
Total	1591	2059	377	506	366	519	459	544	389	490

P (Ll) = ?
P (Ll / Cc) = ?
P (Ll / Ln) = ?
P (Ll / Cm) = ?
P (Ll / LL) = ?

$$P_Z(Y|X)=P(Y|X,Z)=P(Y|Z)=P_Z(Y)\Rightarrow I(X,Y|Z)$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	190	99	287	166	360	162	177	89
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Total	1591	2059	361	490	406	566	484	493	340	510

P (Ll / Primavera) = 0.576
P (Ll / Invierno) = 0.582

Direct independence variables → Involve only two variables

P (Ll) = 0.564

Ll

Luna

Est.

	Anual		Llena		C. Menguante		C. Creciente		Nueva	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
NE	1014	516	255	137	208	106	297	132	254	141
SE	64	57	12	12	16	16	22	12	14	17
SW	225	661	59	165	65	166	58	175	43	155
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Total	1591	2059	377	506	366	519	459	544	389	490

P (Ll) = 0.564
P (Ll / Cc) = 0.557
P (Ll / Ln) = 0.542
P (Ll / Cm) = 0.586
P (Ll / LL) = 0.573

$$P_Z(Y|X)=P(Y|X,Z)=P(Y|Z)=P_Z(Y)\Rightarrow I(X,Y|Z)$$

	Anual		Invierno		Primavera		Verano		Otoño	
	S	Ll	S	Ll	S	Ll	S	Ll	S	Ll
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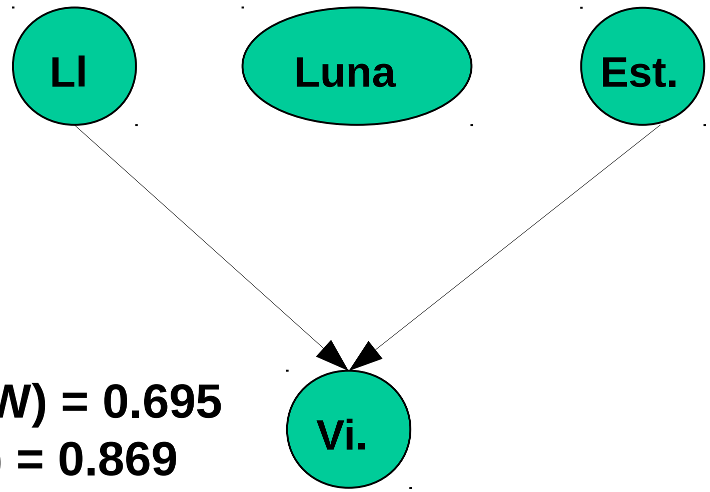

P (Ll / Primavera, SW) = 0.695
P (Ll / Invierno, SW) = 0.869

Conditional dependence between rainfall and season, given the wind

P (Ll) = 0.564
P (Ll / Cc) = 0.557
P (Ll / Ln) = 0.542
P (Ll / Cm) = 0.586
P (Ll / LL) = 0.573

x	y	z	$p(x,y,z)$
0	0	0	0.12
0	0	1	0.18
0	1	0	0.04
0	1	1	0.16
1	0	0	0.09
1	0	1	0.21
1	1	0	0.02
1	1	1	0.18

To explicitly define the joint probability function is not possible in most of the real cases due to the large amount of parameters (e.g. 10^{25} parameters for 100 variables).

Bayes's Theorem → Factorization

$$P(X_i|B)=\frac{P(B|X_i)\,P(X_i)}{\sum_{j=1}^{n} P(B|X_j)\,P(X_j)}$$

$$\underbrace{B \wedge X_i \; independent \Rightarrow P(X_i|B)=P(X_i) \wedge P(B|X_i)=P(B)}$$

Can we build a model including some pre-defined independences? **YES**

$$\boxed{I(X_3, X_1|X_2) \text{ and } I(X_4, \{X_1, X_3\}|X_2).}$$

$$\begin{cases} p(x_3|x_1,x_2) = p(x_3|x_2), \\ p(x_4|x_1,x_2,x_3) = p(x_4|x_2). \end{cases}$$

4 tables instead 16 → $\boxed{p(x_1,\ldots,x_4) = p(x_1)p(x_2|x_1)p(x_3|x_2)p(x_4|x_2).}$

x	y	z	$p(x, y, z)$
0	0	0	0.12
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Can we build a model including some pre-defined independences? YES

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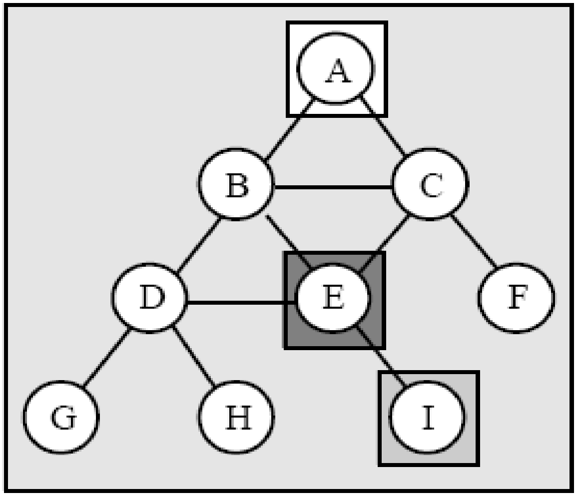
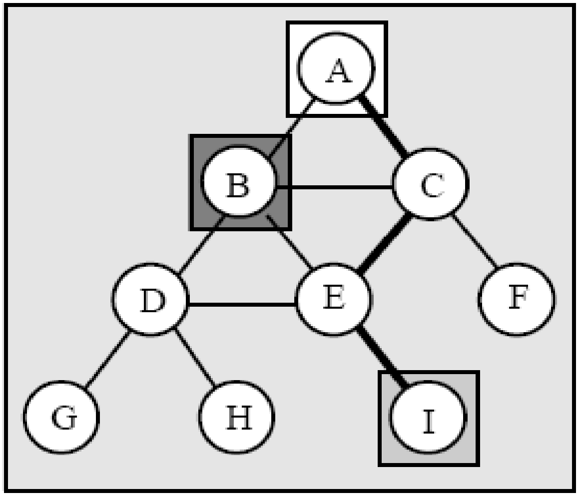
$$\boxed{p(x_1, \ldots, x_4) = p(x_1)p(x_2|x_1)p(x_3|x_2)p(x_4|x_2).}$$

Can we obtain efficiently these (in)dependences? → Graphs



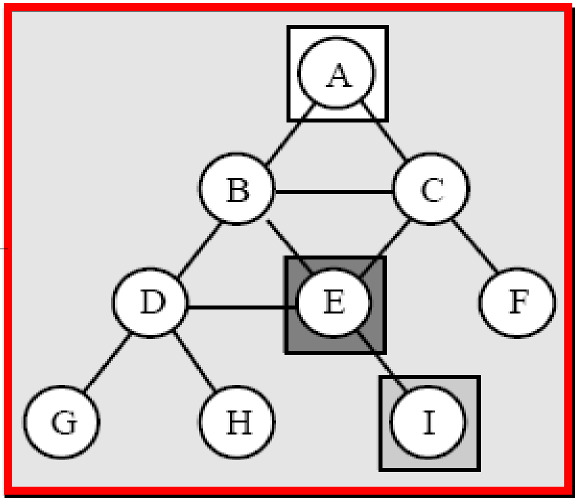
(a) $I(A, I \mid E)$



(b) $D(A, I \mid B)$

Links of the graph reflect **dependences** between the linked variables.

Non directed graphs define the conditional dependence through the **d-separation** concept.



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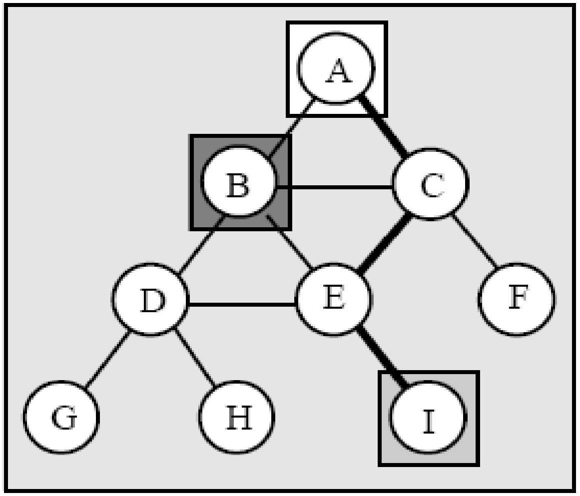
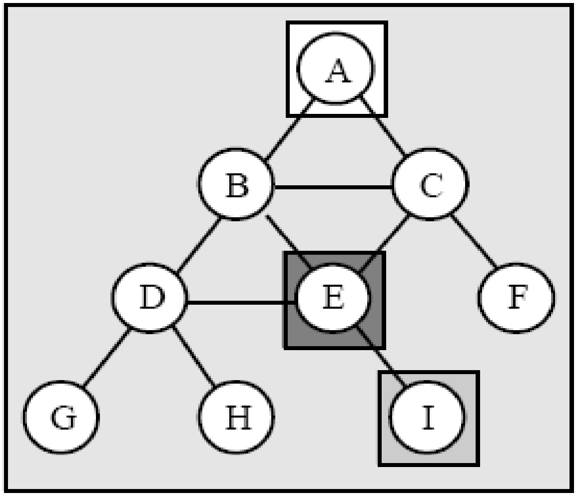


(b) $D(A, I \mid B)$

Links of the graph reflect **dependences** between the linked variables.

Non directed graphs define the conditional dependence through the **d-separation** concept.

There is not a path linking A and I not passing for E.
Thus A and I are dependent but conditional independent given E.



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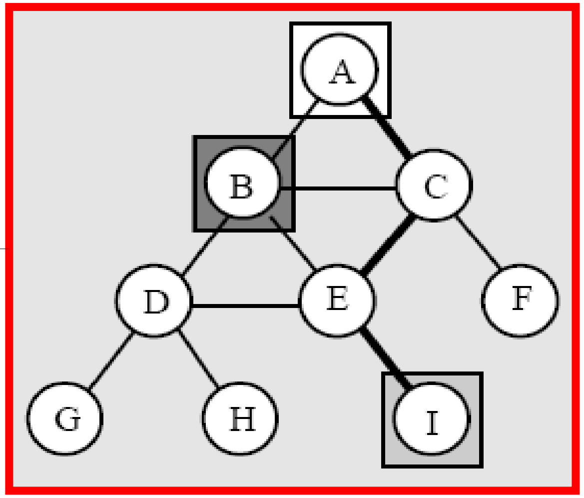
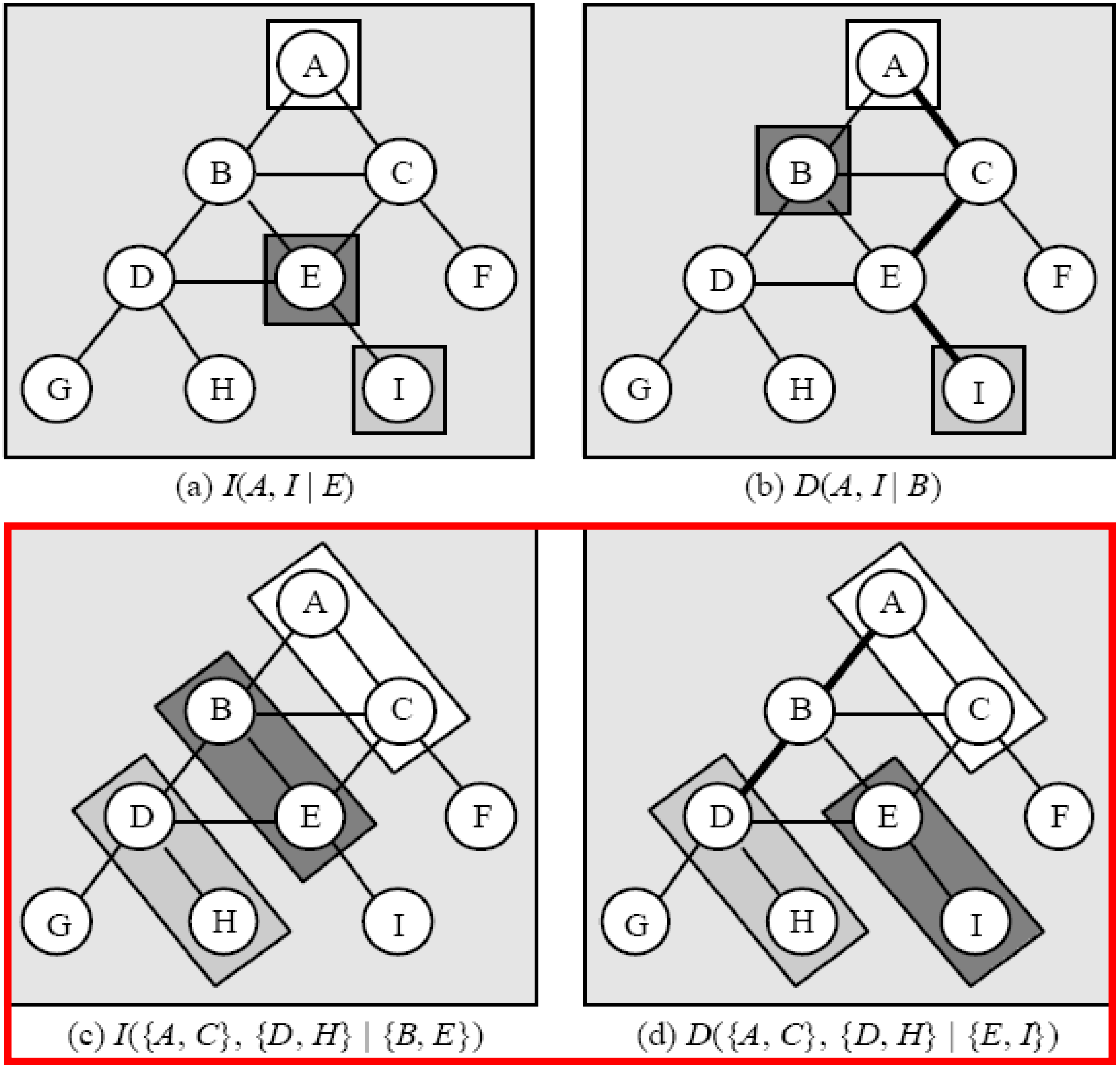


(b) $D(A, I \mid B)$

Links of the graph reflect **dependences** between the linked variables.

Non directed graphs define the conditional dependence through the **d-separation** concept.

There is a path linking A and I not passing for B (A->C->E->I).
Thus A and I are dependent given B and B doesn't d-separate A and I.



(a) $I(A, I \mid E)$ (b) $D(A, I \mid B)$

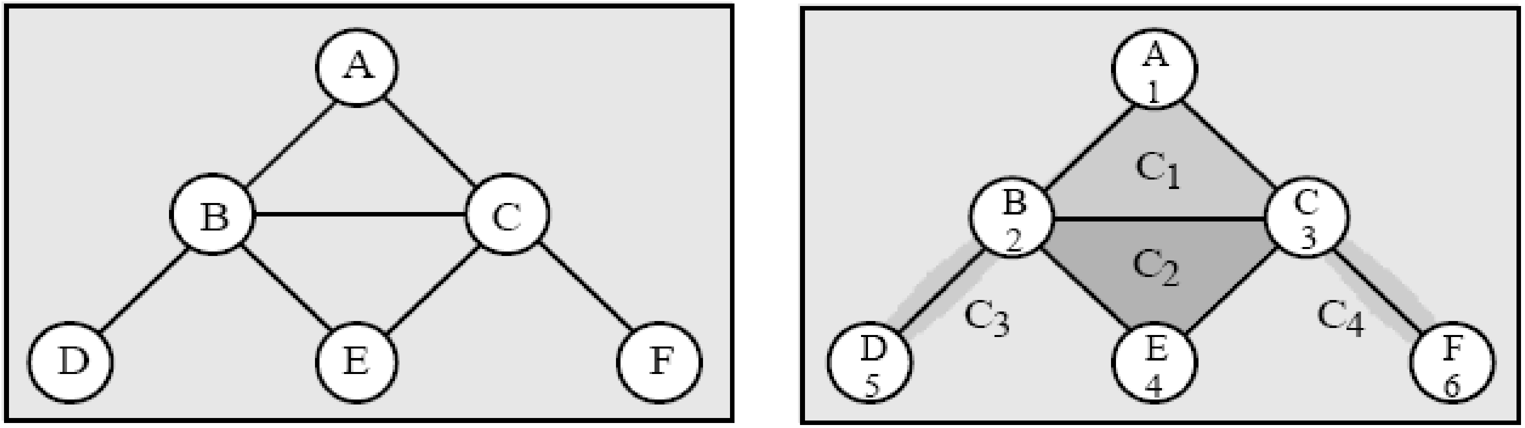
(c) $I(\{A, C\}, \{D, H\} \mid \{B, E\})$ (d) $D(\{A, C\}, \{D, H\} \mid \{E, I\})$

Links of the graph reflect **dependences** between the linked variables.

Non directed graphs define the conditional dependence through the **d-separation** concept.

D-separation is extended to set of variables.

Non-directed graphs define a graphical probabilistic model family based on the **cliques** of the graph and the factorization of the joint probability function given by them.



(a) (b)

$$C_1 = \{A, B, C\}, \quad C_2 = \{B, C, E\},$$
$$C_3 = \{B, D\}, \qquad C_4 = \{C, F\}.$$

i	Clique C_i	Separator S_i	Residual R_i
1	A, B, C	ϕ	A, B, C
2	B, C, E	B, C	E
3	B, D	B	D
4	C, F	C	F

$$\begin{aligned} p(a,b,c,d,e,f) &= \psi_1(c_1)\psi_2(c_2)\psi_3(c_3)\psi_4(c_4) \\ &= \psi_1(a,b,c)\psi_2(b,c,e)\psi_3(b,d)\psi_4(c,f). \end{aligned}$$

$$p(a,b,c,d,e,f) = \prod_{i=1}^{4} p(r_i|s_i) = p(a,b,c)p(e|b,c)p(d|b)p(f|c).$$

$$P_Z(Y|X)=P(Y|X,Z)=P(Y|Z)=P_Z(Y)\Rightarrow I(X,Y|Z)$$

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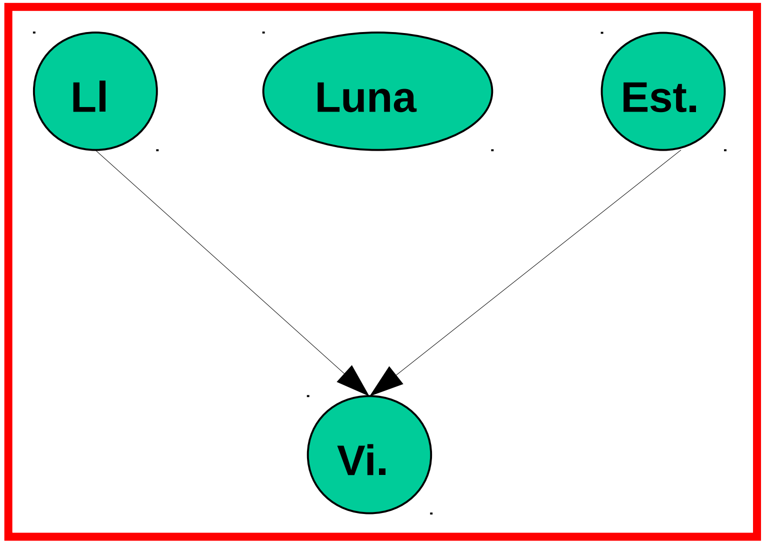
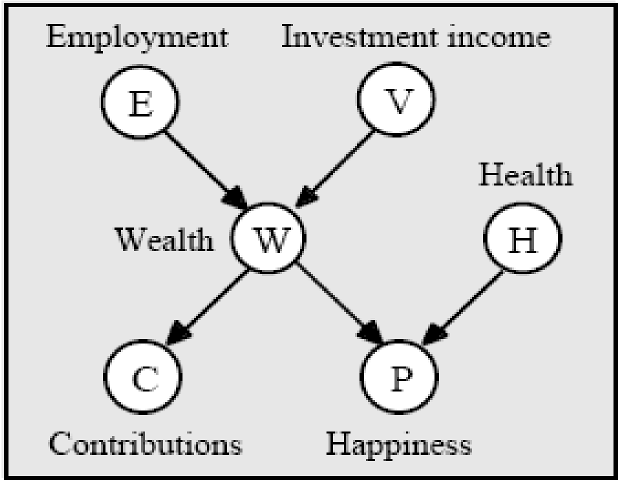
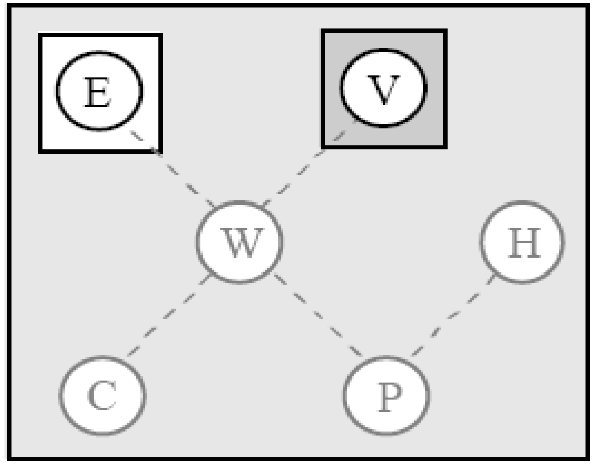


Non-directed graphs are not able to represent this kind of dependence!!!

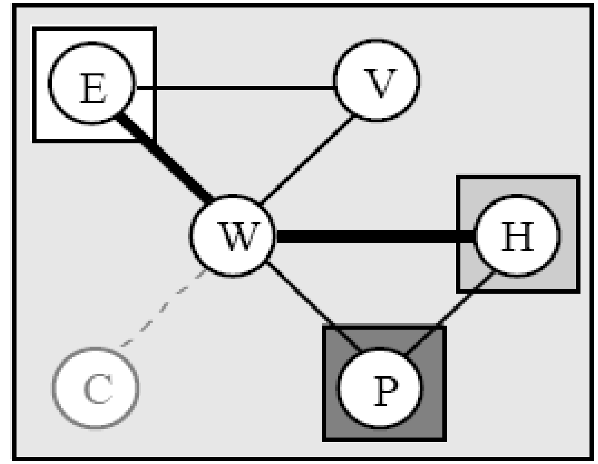
Conditional dependence between rainfall and season, given the wind

D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**





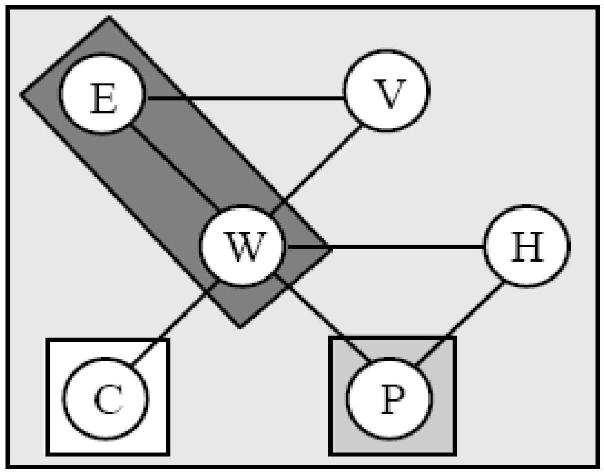
(a) $I(E, V \mid \emptyset)$



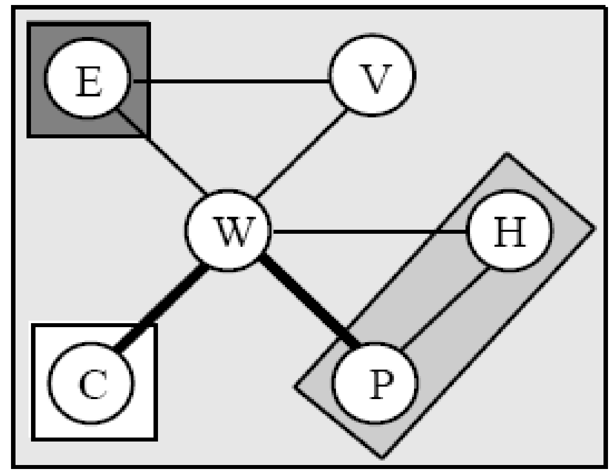
(b) $D(E, H \mid P)$

Links between variables imply probabilistic dependence **NOT CAUSALITY !!!!!!**

Causal Networks (not seen)

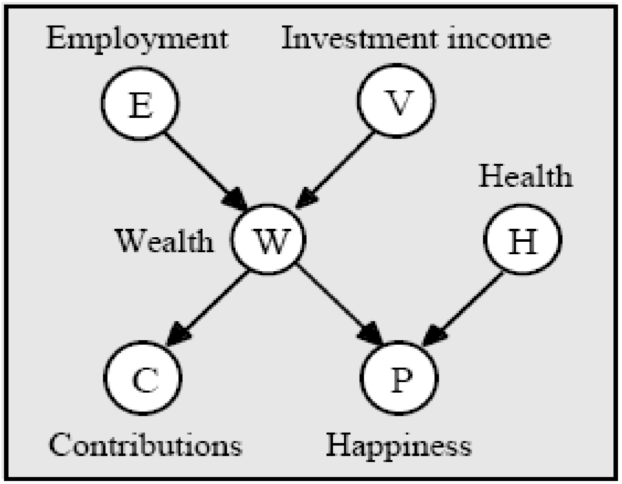


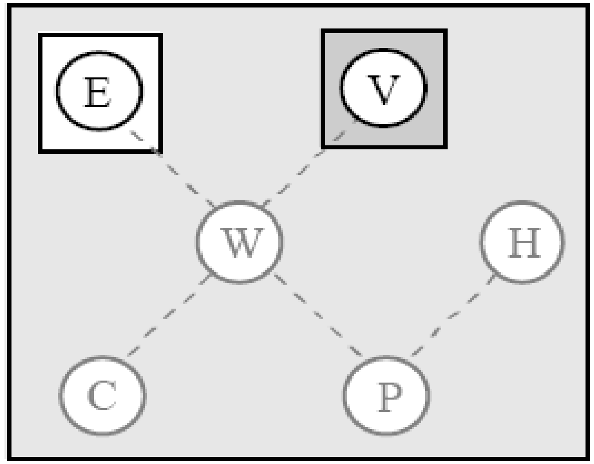
(c) $I(C, P \mid \{E, W\})$



(d) $D(C, \{H, P\} \mid E)$

D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**





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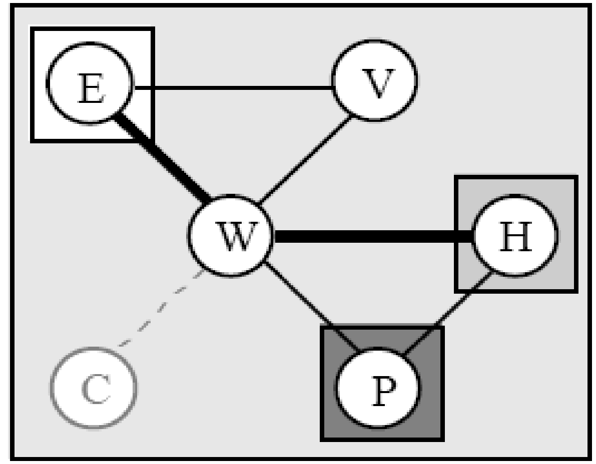


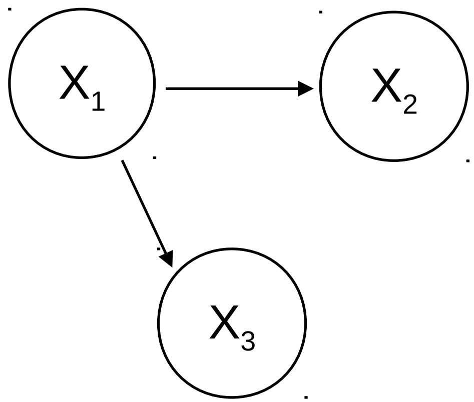
(b) $D(E, H \mid P)$

```
## Load bnlearn:
library(bnlearn)
## Defining an empty graph:
dag<-empty.graph(nodes=c("E","V","W","H","C","P"))
class(dag)
print(dag)
plot(dag)
## Adding link between nodes:
dag<-set.arc(dag,from="E",to="W")
dag<-set.arc(dag,from="V",to="W")
## Complete and plot the graph:
## Evaluate the separation included in the previous slide (See ? dsep and ?path):
```

D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**

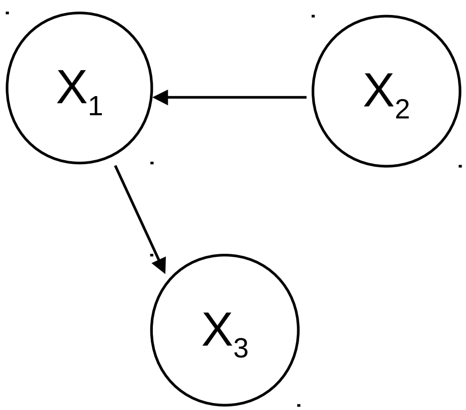
Two directed graph are **equivalents** when they lead to the same probabilistic model:

$$P(X_1, X_2, X_3) = P(X_1) P(X_2|X_1) P(X_3|X_1) = P(X_1, X_2) P(X_3|X_1)$$



Equivalents

$$P(X_1, X_2, X_3) = P(X_2) P(X_1|X_2) P(X_3|X_1) = P(X_1, X_2) P(X_3|X_1)$$

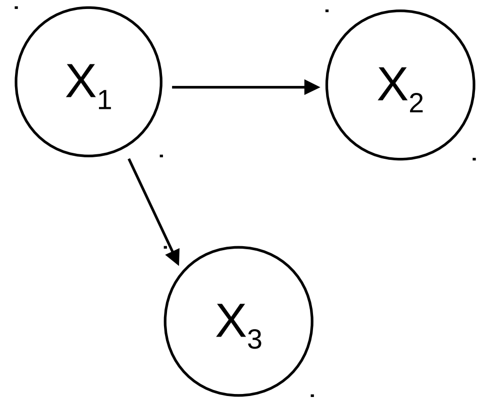


D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**

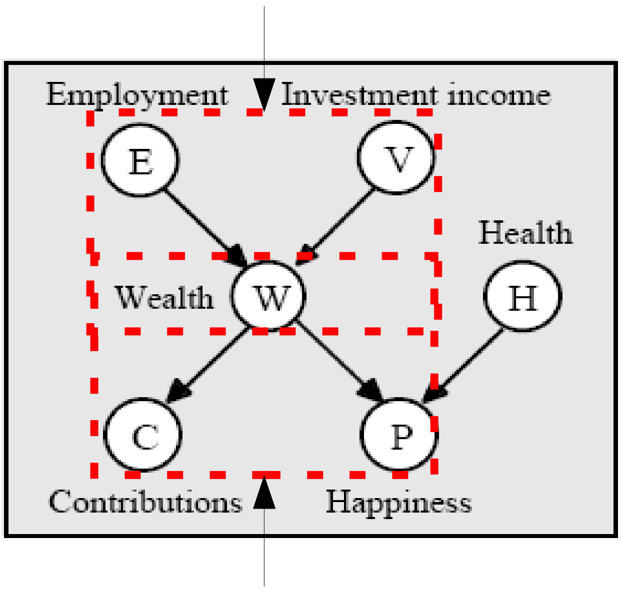
Two directed graph are **equivalents** when they lead to the same probabilistic model.

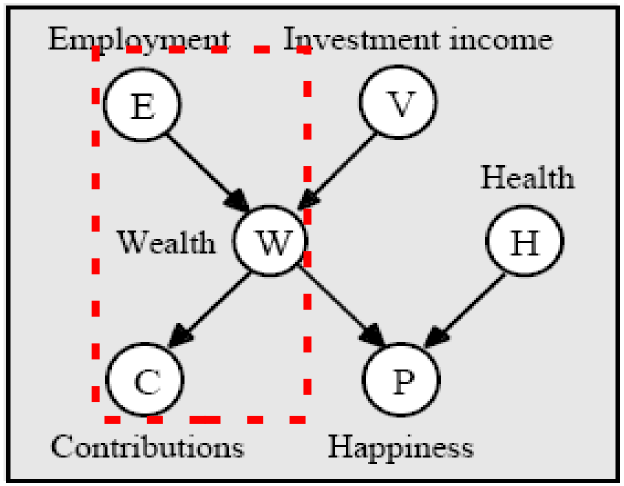
This occurs when the **subyacent non-directed graph** is the same and include the same **V-structures**.

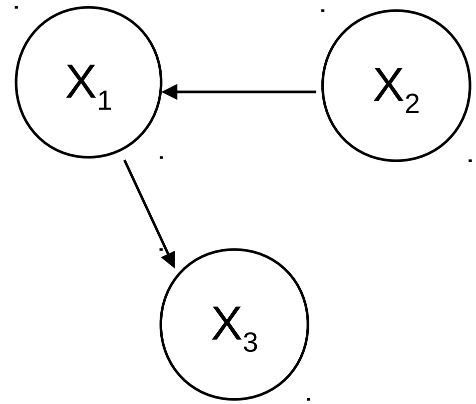
$$P(X_1, X_2, X_3) = P(X_1, X_2) P(X_3 | X_1)$$



Common effect







Common cause **Indirect evidential/causal effect**

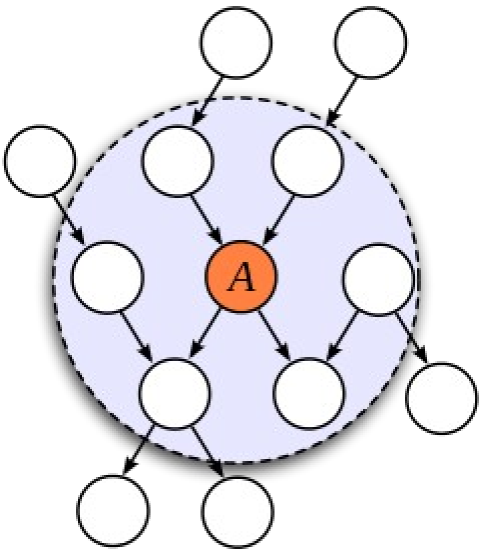
D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**

Two directed graph are **equivalents** when they lead to the same probabilistic model.

This occurs when the **subyacent non-directed graph** is the same and include the same **V-structures**.

The **Skeleton** of the graph is the undirected graph underlying.

The **Markov Blanket** of a node ***A*** is the set of nodes that completely separates ***A*** from the rest of the graph. In particular, it includes the parents and childrens of the node ***A***, and those children's other parents.



Source: Image from https://en.wikipedia.org/wiki/Markov_blanket

D-separation concept for directed graphs enrich the representativity of the model → **Moral graph.**

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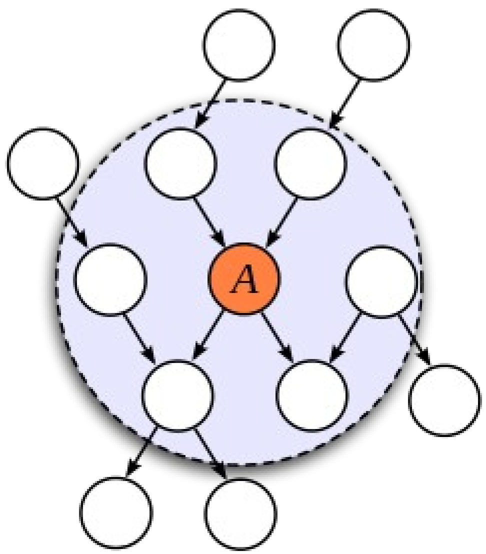
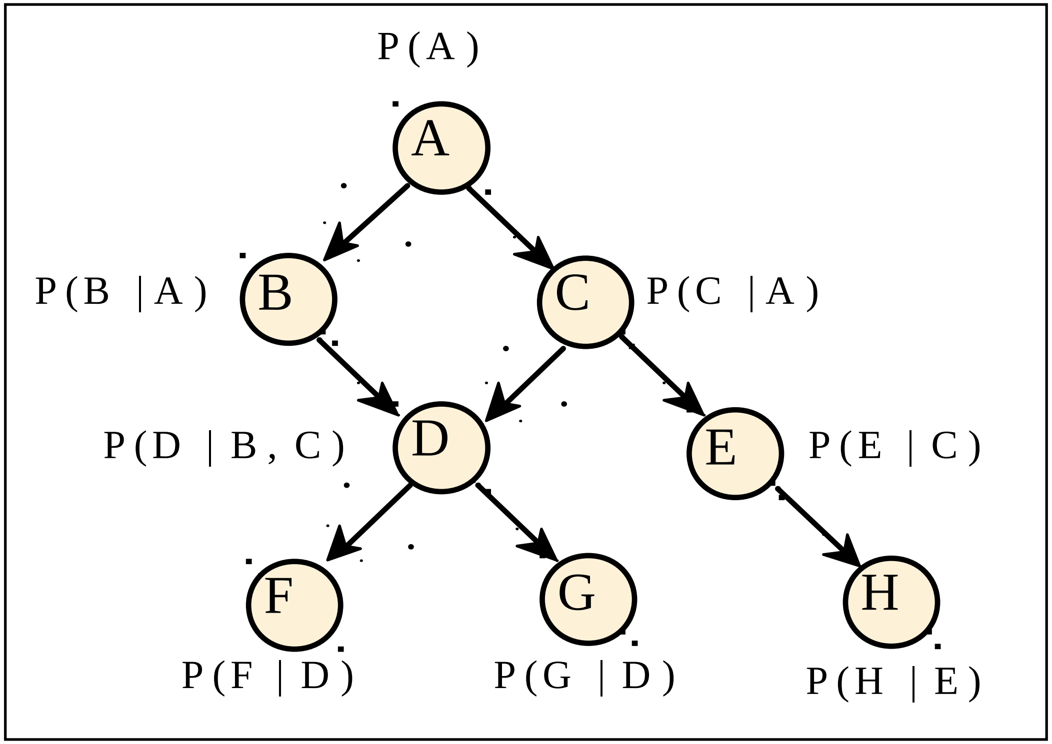


The **Markov Blanket** of is the set of nodes that includes all the knowledge needed to do inference on the node ***A***, from estimation to hypothesis testing to prediction.

Source: Image from https://en.wikipedia.org/wiki/Markov_blanket

Directed graphs lead to a probabilistic model directly obtained from the graph, defining the factorization of the joint probability function as product of conditional probabilities of each node x_i given his parents π_i.

$$P(X)=\prod_{i=1}^{n} P(X_i|\pi_i)$$

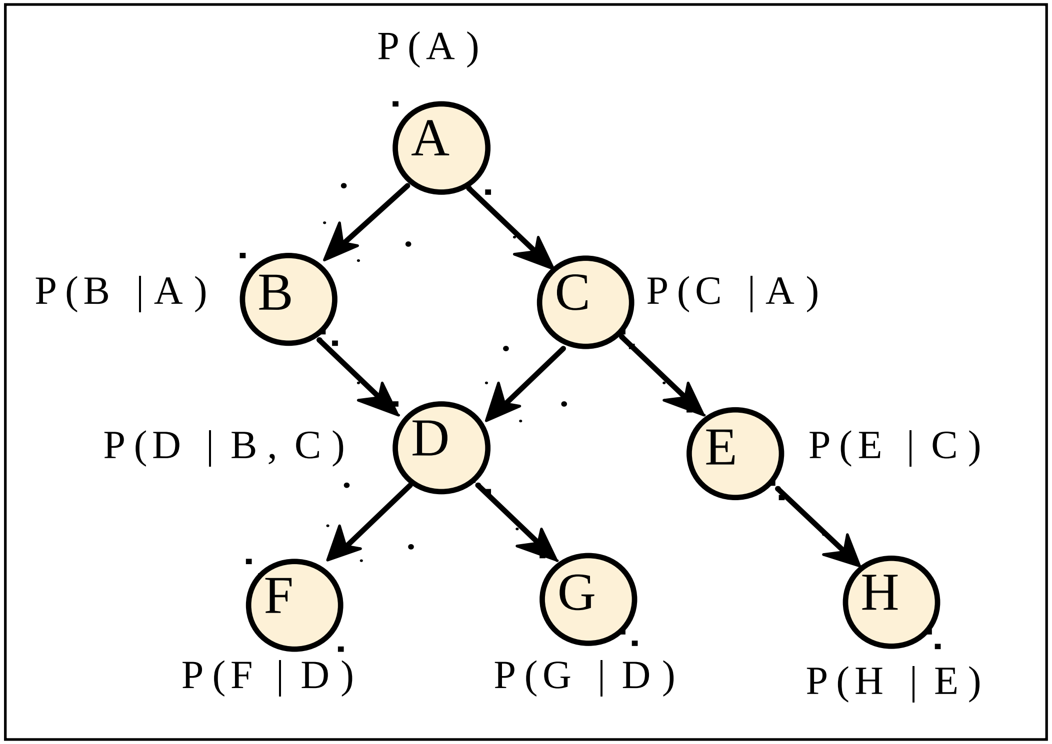


$$P(A,B,C,D,E,F,G,H)=P(A)P(B|A)P(C|A)P(D|B,C)x\ldots$$
$$xP(E|C)P(F|D)P(G|D)P(H|E)$$

```
## Define the graph using both the graph and the factorization expression (See ?modelstring):

## Plot both graphs, is there any difference between them?
```

$$P(X) = \prod_{i=1}^{n} P(X_i | \pi_i)$$



$$P(A,B,C,D,E,F,G,H) = P(A)P(B|A)P(C|A)P(D|B,C)\, x \ldots$$

$$xP(E|C)P(F|D)P(G|D)P(H|E)$$

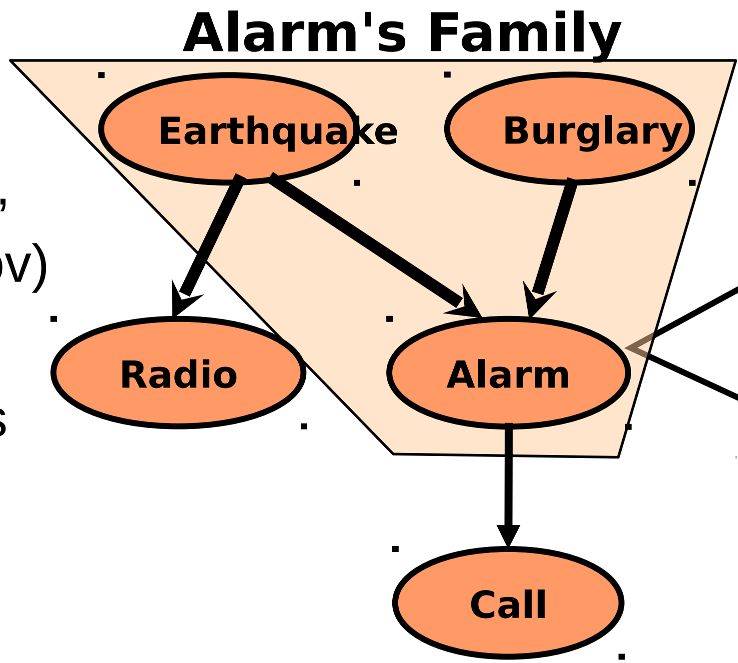
Bayesian Networks obtain a compact representation of the joint probability function through the conditional independences.

Structure:

Acyclic Directed Graph (DAG),

or non-directed graphs (Markov)

- Nodes – variables
- Links – direct dependences



E	B	P(A | E,B)	
e	b	**0.9**	**0.1**
e	$\bar{b}$	**0.2**	**0.8**
$\bar{e}$	b	**0.9**	**0.1**
$\bar{e}$	$\bar{b}$	**0.01**	**0.99**

Factorization of the joint probability function.

$$P(B,E,A,R,C)=P(E)P(B)P(R|E)P(A|E,B)P(C|A)$$

Parameters: Probabilities and tables.